Taylor dispersion in premixed combustion: Questions from turbulent combustion answered for laminar flames

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We present a study of Taylor dispersion in premixed combustion and use it to clarify fundamental issues related to flame propagation in a flow field. In particular, simple analytical formulas are derived for variable density laminar flames with arbitrary Lewis number Le providing clear answers to important questions arising in turbulent combustion, when these questions are posed for the case of one-scale laminar parallel flows. Exploiting, in the context of a laminar Poiseuille flow model, a thick flame distinguished asymptotic limit for which the flow amplitude is large with the Reynolds number Re fixed, three main contributions are made. First, a link is established between Taylor dispersion [G. Taylor, Proc. R. Soc. London Ser. A **219**, 186 (1953)] and Damköhler's second hypothesis [G. Damköhler, Ber. Bunsen. Phys. Chem. **46**, 601 (1940)] by describing analytically the enhancement of the effective propagation speed U_T due to small flow scales. More precisely, it is shown that Damköhler's hypothesis is only partially correct for one-scale parallel laminar flows. Specifically, while the increase in U_T due to the flow is shown to be directly associated with the increase in the effective diffusivity as suggested by Damköhler, our results imply that $U_T \sim \mathrm{Re}$ (for $\mathrm{Re} \gg 1$) rather than $U_T \sim \sqrt{\mathrm{Re}}$, as implied by Damköhler's hypothesis. Second, it is demonstrated analytically and confirmed numerically that, when U_T is plotted versus the flow amplitude for fixed values of Re, the curve levels off to a constant value depending on Re. We may refer to this effect as the laminar bending effect as it mimics a similar bending effect known in turbulent combustion. Third, somewhat surprising implications associated with the dependence of U_T and of the effective Lewis number $\mathrm{Le}_{\mathrm{eff}}$ on the flow are reported. For example, $\mathrm{Le}_{\mathrm{eff}}$ is found to vary from Le to Le^{-1} as Re varies from small to large values. Also, U_T is found to be a monotonically increasing function of Re if $\mathrm{Le} < \sqrt{2}$ and a nonmonotonic function if $\mathrm{Le} > \sqrt{2}$.

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I. INTRODUCTION

In 1953, the British physicist G.I. Taylor published an influential paper describing the enhancement of diffusion processes by a shear flow [1], a phenomenon later termed Taylor dispersion. This

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has generated to date thousands of publications in various areas involving transport phenomena. Surprisingly, until recently [2], none of these publications appeared in the field of combustion.

In 1940, the German chemist G. Damköhler postulated two hypotheses which have largely shaped current views on the propagation of premixed flames in turbulent flow fields [3]. According to Damköhler's first hypothesis, the large scales in the flow merely increase the flame surface area by wrinkling it, without affecting its local normal propagation speed. According to Damköhler's second hypothesis, the small scales in the flow do not cause any significant flame wrinkling but do increase the normal propagation speed (and flame thickness). However, unlike the first hypothesis, the second one is more questionable as argued by Williams [4], most notably as far as theoretical or analytical work is concerned, in what he calls the "high-intensity, small-scale regime." Notwithstanding these limitations, a few studies can be found in the literature, which lend partial support to this hypothesis, e.g., [5–10].

In this paper, we will establish a link between Taylor dispersion and Damköhler's second hypothesis in the context of laminar parallel flows. In this simple context, we will assess the validity of Damköhler's hypothesis and clarify related issues, inspired by practically important fundamental questions which arise in turbulent combustion. Clear analytical answers to these questions, even when posed for the simplest laminar flows, are desirable but largely unavailable in the literature. In this paper, we will therefore attempt to answer such questions based on the derivation of analytical formulas for flame propagation described by a meaningful laminar flow model accounting for variable density and preferential diffusion effects. The derivation will be facilitated by the adoption of a distinguished asymptotic limit which may be viewed as a specific case of what we termed more broadly the thick flame asymptotic limit in previous work. The latter was first introduced theoretically by Daou *et al.* [7] and is relevant for situations where the flame can be considered thick compared to the typical scale of the system, such as for flames propagating in narrow channels [11]. Since its introduction, the thick flame asymptotic limit has been adopted in studies by various workers and has gained some popularity partially due to an emerging interest in micropower generation using combustion [12]. Several aspects of thick flames have thus been investigated to date including the effect of heat loss and preferential diffusion [7,11,13], the effect of gas expansion [2,14,15], and flame stability [16,17]. The reader is referred to these publications and references therein for a proper account regarding these aspects. The focus of this paper is different however; in particular, the effect of heat loss, which is particularly relevant for thick flames and which we explored in previous publications [7,11,18], will not be accounted for. The focus is mainly on three important questions, particularly relevant in turbulent combustion, which are posed and answered herein for flames in a laminar parallel one-scale flow.

The first question is related to the examination of Damköhler's second hypothesis and its link to Taylor dispersion, as discussed above.

The second question is related to the so-called bending effect of the turbulent burning velocity U_T, the effective burning velocity of a premixed flame in a turbulent flow, when plotted versus the flow turbulence intensity u'. This effect, shown in Fig. 1, has received considerable historical attention [19] and continues to be a topic of interest [26]. Although there seems to be no agreed upon definition of the bending effect, this generally refers to the fact, observed experimentally, that the turbulent burning velocity increases slower than linearly for high turbulence intensity [19,27]. In this paper, we will take it to mean the leveling off or the presence of a horizontal asymptote in the curve obtained by plotting $U_T \equiv S_T/S_L$ versus u'/S_L for a fixed valued of the Reynolds number as suggested by the (smoothed) experimental data in Fig. 1; here S_L refers to the laminar burning velocity (the dimensional speed of a planar flame whose thickness will be denoted by δ_L) and S_T the dimensional turbulent burning velocity. We will examine the validity of the bending effect, using mostly an analytical approach within an adiabatic laminar flow model in which the scaled amplitude

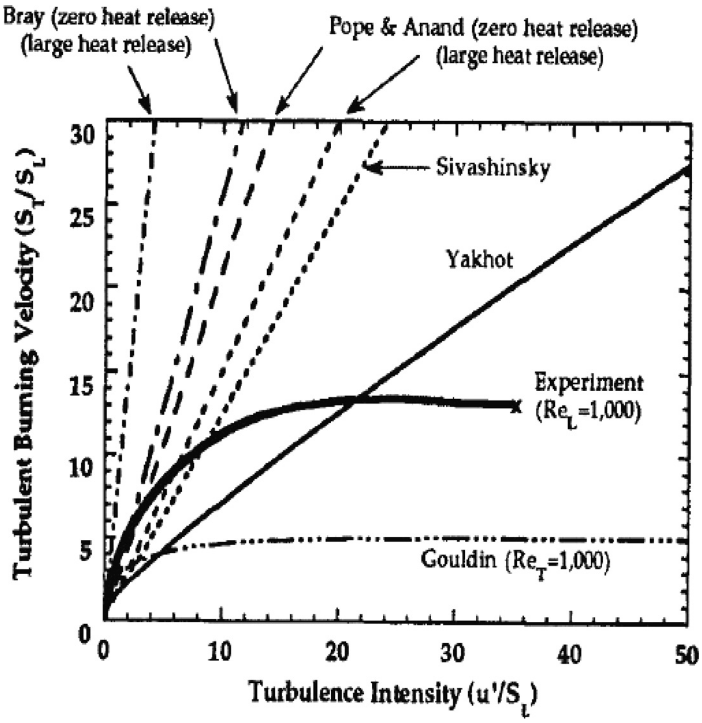

FIG. 1. Comparison between theoretical and experimental predictions of the (scaled) turbulent burning velocity S_T/S_L (U_T in our notation) as a function of the (scaled) turbulent intensity u'/S_L (A in our notation) [19]. Theoretical studies (carried out in the thin flame limit) include those of Bray [20] with zero heat release and large (density ratio = 7) heat release, Anand and Pope [21] with zero and infinite heat release, Yakhot [22], Sivashinsky [23], and Gouldin [24] with $\mathrm{Re}_L = 1{,}000$; experimental values are from Bradley [25] for $\mathrm{Re}_L = 1000$. Where Re_L is not specified, predictions are independent of Re_L. Here S_T is the turbulent flame speed, S_L the laminar flame speed, u' the turbulent intensity, and Re_L the turbulent Reynolds number (Re in our notation). (This figure is reproduced from [19] with permission from the publisher.)

$A \equiv \hat{A}/S_L$ will be identified with u'/S_L; here $\hat{A}$ refers to the dimensional[1] flow amplitude. We will demonstrate that our minimalist model is able to reproduce the bending effect as defined above without the need to include additional effects such as heat losses. On the other hand, the model is not expected to reproduce the practically important phenomenon of total flame extinction which may occur for very large values of A (or u'), as this is known indeed to require the inclusion of heat losses [19–29].

The third question is related to the characterization of the effective Lewis number in turbulent combustion. This controversial question, discussed in [30], concerns whether the effective Lewis number tends to one for large values of u', as argued in [6], or further deviates from one, as argued in [31]. We will provide a clear and somewhat surprising answer to this question posed within our laminar flow model based on a simple analytical formula to be derived.

[1]Throughout the text, when the same letter is used to describe a quantity in its dimensional and nondimensional forms, a circumflex is used to indicate the dimensional form.

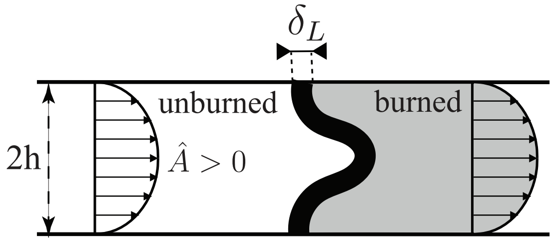

FIG. 2. Premixed flame (with thickness δ_L) propagating in a channel of height $2h$ against a flow of amplitude $\hat{A}$. The main parameter is $\epsilon = h/\delta_L$.

The results of the paper generate precise conclusions, extending those given in the abstract, which are based on transparent analytical formulas and which are also supported by an extensive set of numerical calculations. These conclusions, pertaining to our unidirectional one-scale flow model, provide definite insight into the fundamental question of flame-flow interaction for simple laminar flows. It is hoped, however, that they may also provide, in particular if incorporated in semianalytical cascade-renormalization theories such as in [23,32], some understanding of premixed flame propagation in more complex (turbulent) flows. No claim is made, however, that our results may be applied, without further investigations, outside the precise laminar-flow context in which they are derived and in particular to turbulent combustion. For a discussion of the issues and the progress in the vast field of turbulent combustion, the reader is referred to monographs such as [33] or specialized reviews such as [34].

The presentation is structured as follows. We begin in Sec. II by formulating a nondimensional model based on a scaling similar to that used in lubrication theory in fluid mechanics [35]. The scaling used (as well as some of the notation) has significant differences from our previous publications [2,7,11]. This results in nondimensional equations which are more transparent for the asymptotic treatment used in the present paper. In Sec. III, an asymptotic model is derived that is valid for arbitrary Lewis number. Analytical solutions and useful explicit formulas are obtained in Sec. IV. A discussion of the implication of the findings is given in Sec. V.

II. FORMULATION

We consider a flame propagating in a channel of half-width h against a Poiseuille flow of amplitude $\hat{A}$ as represented in Fig. 2. We will use a Cartesian coordinate system with x as a longitudinal coordinate and y as a transverse coordinate. The velocity field will be denoted by (u,v), the hydrodynamic pressure by p, and the density by ρ. We will scale x by δ_L and y by h, where δ_L represents the thickness of a planar flame in the reactive mixture (whose laminar burning speed will be denoted by S_L). Also, we will scale u by $\hat{A}$, v by $\epsilon\hat{A}$, ρ by $\hat{\rho}_u$, and p by $\mu\hat{A}\delta_L/h^2$; here $\epsilon = h/\delta_L$ is the inverse of the nondimensional flame thickness, $\hat{\rho}_u$ is the (dimensional) density in the unburned mixture, and μ is the dynamic viscosity which is assumed constant. We note that the scales for the longitudinal and transverse velocity components are chosen so as to ensure a balance of terms in the continuity equation, while the scale for pressure is chosen so as to ensure a balance between transverse viscous diffusion and the longitudinal pressure gradient.

In a frame of reference attached to the flame front, the nondimensional governing equations are then given by

$$\frac{\partial}{\partial x}[\rho(u+U)] + \frac{\partial}{\partial y}[\rho v] = 0, \tag{1}$$

$$\epsilon\,\mathrm{Re}\left\{\frac{\partial}{\partial x}[\rho(u+U)u] + \frac{\partial}{\partial y}[\rho v u]\right\} = -\frac{\partial p}{\partial x} + \frac{\partial^2 u}{\partial y^2} + \epsilon^2\frac{\partial^2 u}{\partial x^2}, \tag{2}$$

$$\epsilon^3\mathrm{Re}\left\{\frac{\partial}{\partial x}[\rho(u+U)v] + \frac{\partial}{\partial y}[\rho v v]\right\} = -\frac{\partial p}{\partial y} + \epsilon^2\frac{\partial^2 v}{\partial y^2} + \epsilon^4\frac{\partial^2 v}{\partial x^2}, \tag{3}$$

$$\epsilon\,\mathrm{Pe}\left\{\frac{\partial}{\partial x}[\rho(u+U)\theta] + \frac{\partial}{\partial y}[\rho v\theta]\right\} = \frac{\partial^2 \theta}{\partial y^2} + \epsilon^2\frac{\partial^2 \theta}{\partial x^2} + \epsilon^2\omega, \tag{4}$$

$$\epsilon\,\mathrm{Pe}\left\{\frac{\partial}{\partial x}[\rho(u+U)y_F] + \frac{\partial}{\partial y}[\rho v y_F]\right\} = \frac{1}{\mathrm{Le}}\left(\frac{\partial^2 y_F}{\partial y^2} + \epsilon^2\frac{\partial^2 y_F}{\partial x^2}\right) - \epsilon^2\omega, \tag{5}$$

$$\rho = \left(1 + \frac{\alpha}{1-\alpha}\theta\right)^{-1}, \tag{6}$$

$$\omega = \frac{\beta^2}{2\,\mathrm{Le}(1-\alpha)}\rho y_F \exp\left(\frac{\beta(\theta-1)}{1+\alpha(\theta-1)}\right) \tag{7}$$

and are subject to the boundary conditions

$$\frac{\partial\theta}{\partial y} = \frac{\partial y_F}{\partial y} = \frac{\partial u}{\partial y} = v = \frac{\partial p}{\partial y} = 0 \quad \text{at } y = 0, \tag{8}$$

$$\frac{\partial\theta}{\partial y} = \frac{\partial y_F}{\partial y} = u = v = 0 \quad \text{at } y = 1, \tag{9}$$

$$\theta = 0, \quad y_F = 1, \quad u = 1 - y^2, \quad v = 0 \quad \text{as } x \to -\infty, \tag{10}$$

$$\theta = 1, \quad y_F = 0, \quad u_x = v_x = p = 0 \quad \text{as } x \to \infty. \tag{11}$$

We have assumed that (traveling-wave) solutions to the problem are sought in the upper half domain $0 < y < 1$, that symmetry conditions apply at $y = 0$, and that the walls, located at $y = -1$ and $y = 1$, are rigid and adiabatic. In the equations above, U is an eigenvalue representing the flame propagation speed in the negative x direction with respect to the channel's walls, with $U > 0$ indicating propagation to the left. Also, we note that the pressure p appearing in (2) and (3) is in fact a modified nondimensional pressure p_m in which an unimportant viscous term has been absorbed.[2]

Furthermore, the flame is modeled by a single chemical reaction whose rate $\hat{\omega}$ follows an Arrhenius law with preexponential factor B and activation temperature E/R such that

$$\text{fuel} \Rightarrow \text{product} + q, \quad \hat{\omega} = \hat{\rho} B Y_F \exp\left(-\frac{E}{RT}\right).$$

Here q is the heat release per unit mass of the fuel (assumed to be deficient), Y_F the fuel mass fraction, and T temperature. The latter are used to define the scaled mass fraction y_F, the scaled temperature

[2]Specifically, p_m is obtained by nondimensionalization of the modified pressure $\hat{p}_m \equiv \hat{p} - \frac{\mu}{3}\nabla \cdot \mathbf{v}$; in this expression all terms are dimensional, with $\hat{p}$ being the hydrodynamic pressure and $\mathbf{v}$ the velocity field.

θ, and the Zeldovich number β, given by

$$y_F = \frac{Y_F}{Y_{Fu}}, \quad \theta = \frac{T - T_u}{T_{ad} - T_u}, \quad \beta = \frac{E(T_{ad} - T_u)}{RT_{ad}^2},$$

where $T_{ad} = T_u + qY_{Fu}/c_p$ is the adiabatic flame temperature (c_p being the mixture's heat capacity, assumed constant). We note that the subscript u is used (throughout) to indicate values in the unburned mixture (as $x \to -\infty$); similarly, when used, the subscript b indicates values in the burned mixture (as $x \to \infty$).

Furthermore, the main nondimensional parameters appearing in the equations are defined by

$$\epsilon = \frac{h}{\delta_L}, \quad \text{Pe} = \frac{h\hat{A}}{D_{T,u}} = \epsilon A, \quad \text{Re} = \frac{\text{Pe}}{\text{Pr}}, \quad \text{Le} = \frac{D_T}{D_F},$$

respectively, the flow scale, the Péclet number, the Reynolds number, and the Lewis number. For convenience, the Prandtl number $\text{Pr} = \nu/D_T$ (with ν being the kinematic viscosity) will be taken equal to one, so that Pe and Re may be used interchangeably. In the expressions above we have used the fact that $\delta_L \equiv D_{Tu}/S_L$. Given that the density is variable, with dimensional values equal to $\hat{\rho}_u$ in the unburned gas (where $\theta = 0$) and $\hat{\rho}_b$ in the burned gas (where $\theta = 1$), it is important to note the adoptions of the following definitions and assumptions:

$$\rho = \frac{\hat{\rho}}{\hat{\rho}_u}, \quad \alpha \equiv \frac{\hat{\rho}_u - \hat{\rho}_b}{\hat{\rho}_u}, \quad \hat{\rho} D_T = \text{const}, \quad \hat{\rho} D_F = \text{const}. \tag{12}$$

Therefore, the heat and mass diffusion coefficients D_T and D_F depend on density (and therefore on temperature) but their ratio, the Lewis number Le, does not. Of course, we have also adopted the perfect gas equation of state $\hat{\rho} T = \text{const} = \hat{\rho}_u T_u$, which is Eq. (6), when written in nondimensional form. Finally, we record for later reference the expression (for $\beta \gg 1$) of the laminar flame speed

$$S_L = \sqrt{\frac{2(1-\alpha)}{\beta^2} \text{Le} D_{T,u} B e^{-E/RT_{ad}}}. \tag{13}$$

We are now ready to tackle the problem. Our main aim is to determine the effective propagation speed U_T, defined as the flux of fuel per unit cross section normalized by $\hat{\rho}_u S_L$. On averaging[3] the continuity equation (1) (i.e., on integrating it with respect to y from 0 to 1) and due to our choice of scales, we have

$$\overline{\rho(u + U)} \equiv \frac{\epsilon}{\text{Pe}} U_T = U + \frac{2}{3}. \tag{14}$$

Before proceeding to the asymptotic analysis which is the main focus of the paper, a short comment on the numerical results which will also be included is in order. We note that, in general, the problem given by Eqs. (1)–(11) must be solved numerically. In this paper, such numerical solutions are obtained using the method described in [2,36,37], which we have tested extensively in several combustion applications. Briefly, the set of equations is solved using the finite-element package COMSOL MULTIPHYSICS on a nonuniform grid of triangular elements, with particular refinement around the reaction zone. The results are tested to ensure that they are not dependent on the mesh. All calculations are performed for $\beta = 10$, $\text{Pr} = 1$, $\text{Le} = 1$, and $\alpha = 0.85$, unless otherwise stated. Constant density results refer to the results of calculations where the density and flow velocities are fixed to be $\rho = 1$, $u = 1 - y^2$, and $v = 0$. In this case, only Eqs. (4) and (5) require numerical solution. When computed numerically, the effective propagation speed U_T is normalized by the propagation speed of the planar premixed flame, which is also computed numerically. The effective Lewis number Le_{eff} is calculated from numerical results as the average ratio between the flame

[3]We use common notation such that $\overline{\theta} \equiv \int_0^1 \theta \, dy$ and $\theta' \equiv \theta - \overline{\theta}$.

thickness and the mass fraction thickness and is normalized to be equal to the Lewis number at Pe = 0. Using the method of [30], for each y the flame thickness is defined as the distance between the point where $\theta = 0.9$ and the point where $\theta = 0.331$; the mass fraction thickness is defined as the distance between the point where $y_F = 0.669$ and the point where $y_F = 0.1$.

III. ASYMPTOTIC ANALYSIS

In this section, our aim is to reduce the problem to a one-dimensional one. As a preliminary step, we record that the average temperature $\bar{\theta}(x)$ and the average mass fraction $\overline{y}_F(x)$ are governed by

$$U_T\bar{\theta}_x + \frac{\mathrm{Pe}}{\epsilon}\frac{\partial}{\partial x}[\overline{\rho(u+U)\theta'}] = \bar{\theta}_{xx} + \omega(\bar{\theta},\overline{y}_F) + \mathcal{T}_{\mathrm{sm}}, \tag{15}$$

$$U_T\overline{y}_{Fx} + \frac{\mathrm{Pe}}{\epsilon}\frac{\partial}{\partial x}[\overline{\rho(u+U)y_F'}] = \frac{\overline{y}_{Fxx}}{\mathrm{Le}} - \omega(\bar{\theta},\overline{y}_F) + \mathcal{T}_{\mathrm{sm}}. \tag{16}$$

In obtaining these equations, we have anticipated, as we will confirm and explain below, that the fluctuation θ' and y_F' are small compared to $\bar{\theta}$ and $\overline{y}_F$, respectively, and we have denoted by $\mathcal{T}_{\mathrm{sm}}$ small terms, of quadratic order in θ' and y_F'. More precisely, the equations are obtained by averaging Eqs. (4) and (5), which yields

$$\epsilon\,\mathrm{Pe}\frac{\partial}{\partial x}[\overline{\rho(u+U)\theta}] = \epsilon^2\frac{d^2\bar{\theta}}{dx^2} + \epsilon^2\overline{\omega(\theta,y_F)},$$

$$\epsilon\,\mathrm{Pe}\frac{\partial}{\partial x}[\overline{\rho(u+U)y_F}] = \frac{\epsilon^2}{\mathrm{Le}}\frac{d^2\overline{y}_F}{dx^2} - \epsilon^2\overline{\omega(\theta,y_F)}$$

on using the boundary conditions (8) and (9) at $y = 0$ and $y = 1$. Equations (15) and (16) then follow on substituting $\theta = \bar{\theta} + \theta'$ and $y_F = \overline{y}_F + y_F'$ into the preceding expressions and using (14) along with

$$\overline{\omega(\theta,y_F)} = \omega(\bar{\theta},\overline{y}_F) + O(\theta'^2) + O({y_F'}^2) + O(\theta' y_F'),$$

which is obtained from a Taylor expansion of $\omega(\theta,y_F) = \omega(\bar{\theta}+\theta',\overline{y}_F+y_F')$ for small values of θ' and y_F'.

In Eqs. (15) and (16), we need an approximation for $\overline{\rho(u+U)\theta'}$ and $\overline{\rho(u+U)y_F'}$ where $U = -\frac{2}{3} + \frac{\epsilon}{\mathrm{Pe}}U_T$ on account of (14). To this end, we need to consider some asymptotic limits involving the parameters ϵ and ϵPe under which the problem is tractable. This typically requires, as known in lubrication theory, that both parameters be small [35]. Motivated by our intention to explain the bending effect in Fig. 1 (which is obtained for fixed values of Re), we will consider the distinguished limit $\epsilon \to 0$ with $\mathrm{Pe} = O(1)$, equivalent to $A \to \infty$ with $\mathrm{Re} = O(1)$, although other distinguished limits are possible. In this limit, we introduce expansions of the form

$$u = u_0 + \epsilon u_1 + \cdots, \quad v = v_0 + \epsilon v_1 + \cdots.$$

To leading order, we then find

$$\frac{\partial}{\partial x}\left[\rho_0\left(u_0 - \frac{2}{3}\right)\right] + \frac{\partial}{\partial y}[\rho_0 v_0] = 0, \tag{17}$$

$$\frac{\partial p_0}{\partial x} = \frac{\partial^2 u_0}{\partial y^2}, \tag{18}$$

$$\frac{\partial p_0}{\partial y} = 0, \quad \frac{\partial^2\theta_0}{\partial y^2} = 0, \quad \frac{\partial^2 y_{F0}}{\partial y^2} = 0. \tag{19}$$

Equations (19) imply, when used with the boundary condition (8) and Eq. (6), that

$$p_0 = p_0(x), \quad \theta_0 = \theta_0(x), \quad y_{F0} = y_{F0}(x), \quad \rho_0 = \rho_0(x).$$

Equation (18) may then be integrated with respect to y, which yields, on applying the boundary conditions (8) and (9) at $y = 0$ and $y = 1$,

$$u_0 = -\frac{1}{2}\frac{dp_0}{dx}(1 - y^2).$$

In this expression, which implies that $\overline{u_0} = -\frac{1}{3}dp_0/dx$, the function dp_0/dx is as yet unknown but can be determined by using (17) after averaging it, as is classically done in lubrication theory; hence

$$\frac{\partial}{\partial x}\left[\overline{\rho_0\left(u_0 - \frac{2}{3}\right)}\right] = 0 \Rightarrow \rho_0\left(\overline{u_0} - \frac{2}{3}\right) = \text{const} = 0,$$

where an integration constant has been set to zero by using the boundary condition (10) as $x \to -\infty$. Therefore, $\overline{u_0} = 2/3$ and $dp_0/dx = -2$, which fully specifies u_0. Equation (17) can now be reused, along with the boundary condition (8) or (9), to determine v_0. We thus find

$$u_0 = 1 - y^2, \quad v_0 = -\frac{1}{\rho_0}\frac{d\rho_0}{dx}\left(\frac{y}{3} - \frac{y^3}{3}\right).$$

To next order $O(\epsilon)$ we obtain

$$\text{Pe}\rho_0\left(u_0 - \frac{2}{3}\right)\frac{d\theta_0}{dx} = \frac{\partial^2\theta_1}{\partial y^2},$$
$$\text{Pe}\,\text{Le}\rho_0\left(u_0 - \frac{2}{3}\right)\frac{dy_{F0}}{dx} = \frac{\partial^2 y_{F1}}{\partial y^2},$$

which follows from Eqs. (4) and (5) and the fact that $U \sim -2/3$ implied by (14). Integrating with respect to y twice then yields

$$\theta_1 = \text{Pe}\rho_0\frac{d\theta_0}{dx}\left(\frac{y^2}{6} - \frac{y^4}{12}\right) + \check{\theta}_1(x),$$
$$y_{F1} = \text{Pe}\,\text{Le}\rho_0\frac{dy_{F0}}{dx}\left(\frac{y^2}{6} - \frac{y^4}{12}\right) + \check{y}_{F1}(x),$$

where $\check{\theta}_1(x)$ and $\check{y}_{F1}(x)$ are integration functions. We now compute $\theta' \equiv \theta - \bar{\theta}$ and $y'_F \equiv y_F - \overline{y}_F$,

$$\theta' = [\theta_0(x) + \epsilon\theta_1 + \cdots] - [\theta_0(x) + \epsilon\bar{\theta}_1 + \cdots] = \epsilon(\theta_1 - \bar{\theta}_1) + \cdots,$$

hence

$$\theta' = \epsilon\,\text{Pe}\rho_0\frac{d\theta_0}{dx}\left(\frac{y^2}{6} - \frac{y^4}{12} - \frac{7}{180}\right) + \cdots.$$

Similarly,

$$y'_F = \epsilon\,\text{Pe}\,\text{Le}\rho_0\frac{dy_{F0}}{dx}\left(\frac{y^2}{6} - \frac{y^4}{12} - \frac{7}{180}\right) + \cdots.$$

We note that these expressions confirm that the fluctuation θ' and y'_F, which are $O(\epsilon\text{Pe})$, are indeed small in the limit considered, as anticipated above. Physically, this means that the curved flame of Fig. 2 is in fact weakly curved (or almost planar) in the thick flame asymptotic limit considered.

We can now evaluate the second terms in Eqs. (15) and (16):

$$\overline{\rho(u+U)\theta'} \sim \epsilon\,\mathrm{Pe}\rho_0^2\frac{d\theta_0}{dx}\overline{\left(u_0-\frac{2}{3}\right)\left(\frac{y^2}{6}-\frac{y^4}{12}-\frac{7}{180}\right)}$$
$$= -\gamma_*\epsilon\,\mathrm{Pe}\rho_0^2\frac{d\theta_0}{dx} \quad \text{where } \gamma_* = \frac{8}{945}.$$

Similarly,

$$\overline{\rho(u+U)y_F'} \sim -\gamma_*\epsilon\,\mathrm{Pe}\,\mathrm{Le}\rho_0^2\frac{dy_{F0}}{dx}.$$

With these terms evaluated, the one-dimensional equations (15) and (16) are now fully specified to leading order.

To summarize, in the limit $\epsilon \to 0$ with $\mathrm{Pe} = O(1)$, the problem can be reduced to a one-dimensional problem for the leading-order temperature and mass fractions $\theta \sim \theta_0(x) \sim \overline{\theta}(x)$ and $y_F \sim y_{F0}(x) \sim \overline{y}_F(x)$. Dropping the subscript 0 and the overbar, we arrive at a one-dimensional eigenboundary value problem

$$U_T\frac{d\theta}{dx} = \frac{d}{dx}\left[(1+\gamma_*\mathrm{Pe}^2\rho^2)\frac{d\theta}{dx}\right] + \omega, \tag{20a}$$

$$U_T\frac{dy_F}{dx} = \frac{1}{\mathrm{Le}}\frac{d}{dx}\left[(1+\gamma_*\mathrm{Pe}^2\mathrm{Le}^2\rho^2)\frac{dy_F}{dx}\right] - \omega, \tag{20b}$$

$$\theta = 0, \quad y_F = 1 \quad \text{as } x \to -\infty, \tag{20c}$$

$$\theta = 1, \quad y_F = 0 \quad \text{as } x \to +\infty, \tag{20d}$$

where the functions $\rho = \rho(\theta)$ and $\omega = \omega(\theta, y_F)$ are given by (6) and (7).

The problem corresponds to a planar premixed flame with effective thermal diffusion coefficient $D_{T,\,\mathrm{eff}}$ and mass diffusion coefficients $D_{F,\,\mathrm{eff}}$ given by

$$D_{T,\,\mathrm{eff}} = D_T\left(1+\gamma_*\mathrm{Pe}^2\frac{\hat{\rho}^2}{\hat{\rho}_u^2}\right), \tag{21}$$

$$D_{F,\,\mathrm{eff}} = D_F\left(1+\gamma_*\mathrm{Pe}^2\mathrm{Le}^2\frac{\hat{\rho}^2}{\hat{\rho}_u^2}\right). \tag{22}$$

This is an important result because, in addition to providing a rational reduction of the two-dimensional problem to a one-dimensional one, it corresponds to a generalized form (accounting for variable density and Lewis number effects) of the effective diffusion coefficients found in the nonreactive Taylor dispersion problem.

Before turning to the solution of the problem and a discussion of the findings, we close this section by a few useful preliminary remarks. Specifically, we note that the ratios $D_{T,\,\mathrm{eff}}/D_T$, $D_{F,\,\mathrm{eff}}/D_F$, and $D_{T,\,\mathrm{eff}}/D_{F,\,\mathrm{eff}}$ depend on the local temperature θ, since $\rho \equiv \hat{\rho}/\hat{\rho}_u$ is a function of θ given by (12). The latter ratio defines a θ-dependent effective local Lewis number, say, $\mathrm{Le}' = D_{T,\,\mathrm{eff}}/D_{F,\,\mathrm{eff}}$. The value of Le' at $\theta = 1$ (where $\rho = 1-\alpha$), which we denote by $\mathrm{Le}_{\mathrm{eff}}$, is crucial for the determination of the propagation speed as we will demonstrate. It is given by

$$\frac{\mathrm{Le}_{\mathrm{eff}}}{\mathrm{Le}} = \frac{1+\gamma\,\mathrm{Pe}^2}{1+\gamma\,\mathrm{Pe}^2\mathrm{Le}^2}, \tag{23}$$

where

$$\gamma = \gamma_*(1-\alpha)^2 = \tfrac{8}{945}(1-\alpha)^2. \tag{24}$$

Similarly, we record for later reference the values of the effective mass and heat diffusivity corresponding to $\theta = 1$, which according to (21) and (22) are given by

$$D_{T,\,\mathrm{eff}} = D_T(1 + \gamma\,\mathrm{Pe}^2) \quad (\theta = 1), \tag{25}$$

$$D_{F,\,\mathrm{eff}} = D_F(1 + \gamma\,\mathrm{Pe}^2\mathrm{Le}^2) \quad (\theta = 1). \tag{26}$$

IV. SOLUTION FOR LARGE ACTIVATION ENERGY $\beta \to \infty$

In the asymptotic limit $\beta \to \infty$, the problem (20) may be solved analytically following a familiar methodology. In this limit, a thin reaction zone of thickness $O(\beta^{-1})$ is indeed expected, located at $x = 0$, say, separating two outer zones known as the preheat zone ($x < 0$) and the burned gas zone ($x > 0$). In the outer zones the reaction rate can be set to zero. This leads to a simple outer solution in the burned gas, namely,

$$\theta^{\mathrm{outer}} = 1, \quad y_F^{\mathrm{outer}} = 0 \quad (x > 0). \tag{27}$$

In the unburned gas, a fully explicit outer solution is more difficult to obtain, especially for y_F. However, for the determination of U_T, the explicit solutions are not needed and first integrals are sufficient as they provide matching conditions for the inner problem which allows determination of U_T. The first integrals in question, which are easily obtained by integrating (20a) and (20b) in the unburned gas ($x < 0$) after setting $\omega = 0$ and taking into account boundary conditions (20c), are

$$U_T\theta^{\mathrm{outer}} - (1 + \gamma_*\mathrm{Pe}^2\rho^2)\frac{d\theta^{\mathrm{outer}}}{dx} = 0, \tag{28}$$

$$U_T y_F^{\mathrm{outer}} - \frac{1}{\mathrm{Le}}(1 + \gamma_*\mathrm{Pe}^2\mathrm{Le}^2\rho^2)\frac{dy_F^{\mathrm{outer}}}{dx} = U_T. \tag{29}$$

We turn now to the inner solution by introducing an inner variable X and inner expansions by

$$X = \frac{x}{\beta^{-1}}, \quad \theta^{\mathrm{inner}} \sim 1 + \frac{\Theta^1(X)}{\beta}, \quad y_F^{\mathrm{inner}} \sim \frac{F^1(X)}{\beta}.$$

The inner problem to leading order is then given by

$$\Theta^1_{XX} + \Lambda F^1 \exp(\Theta^1) = 0, \tag{30}$$

$$\frac{1}{\mathrm{Le_{eff}}}F^1_{XX} - \Lambda F^1 \exp(\Theta^1) = 0, \tag{31}$$

$$\Theta^1 = 0, \quad F^1 = 0 \quad \text{as } X \to +\infty, \tag{32}$$

$$\Theta^1 = \frac{U_T X}{1 + \gamma\,\mathrm{Pe}^2}, \quad F^1 = \frac{-\mathrm{Le}U_T X}{1 + \gamma\,\mathrm{Pe}^2\mathrm{Le}^2} \quad \text{as } X \to -\infty, \tag{33}$$

where $\mathrm{Le_{eff}}$ is given by (23) and

$$\Lambda \equiv \frac{1}{2\,\mathrm{Le}(1 + \gamma\,\mathrm{Pe}^2)}. \tag{34}$$

We note that conditions (32) and (33) follow from the matching requirement

$$\left(\theta^{\mathrm{inner}}, y_F^{\mathrm{inner}}\right)(X \to \pm\infty) = (\theta^{\mathrm{outer}}, y_F^{\mathrm{outer}})(x \to 0^{\pm}).$$

Indeed, the zeroth-order matching is ensured by requiring $\theta^{\mathrm{outer}}(0\pm) = 1$ and $y_F^{\mathrm{outer}}(0\pm) = 0$. The first-order matching for θ is ensured by requiring $\Theta^1(X) = \theta_x^{\mathrm{outer}}(0\pm)X$ as $X \to \pm\infty$ and using (27) and (28) to evaluate $\theta_x^{\mathrm{outer}}(0\pm)$. The first-order matching for y_F is obtained similarly.

The inner problem can now be simplified further, by noting that $F^1 = -\mathrm{Le}_{\mathrm{eff}}\Theta^1$ identically; this is seen by adding (30) to (31) and using (32). Eliminating F^1, the inner problem becomes

$$\Theta^1_{XX} = \Lambda\,\mathrm{Le}_{\mathrm{eff}}\Theta^1\exp(\Theta^1), \tag{35}$$

$$\Theta^1(X \to -\infty) = \frac{U_T X}{1+\gamma\mathrm{Pe}^2}, \quad \Theta^1(X \to \infty) = 0. \tag{36}$$

Now U_T can be found by integrating Eq. (35) subject to boundary conditions (36). Multiplying (30) by Θ^1_X and integrating with respect to X from $X = -\infty$ to $X = +\infty$ yields

$$\left[\frac{(\Theta^1_X)^2}{2}\right]_{X=-\infty}^{X=+\infty} = \Lambda\,\mathrm{Le}_{\mathrm{eff}}\int_{\Theta^1(-\infty)}^{\Theta^1(+\infty)}\Theta^1\exp(\Theta^1)d\Theta^1.$$

Thus, using (36) and evaluating the integral, we obtain

$$\frac{U_T^2}{(1+\gamma\mathrm{Pe}^2)^2} = \Lambda\,\mathrm{Le}_{\mathrm{eff}}. \tag{37}$$

Hence, referring to (23) and (34), we have

$$U_T = \frac{1+\gamma\,\mathrm{Pe}^2}{(1+\gamma\,\mathrm{Pe}^2\mathrm{Le}^2)^{1/2}}, \tag{38}$$

where γ is a number given by (24). Equation (38) gives the effective flame speed U_T, to leading order, in terms of Le and Pe in the limits $\epsilon \to 0$ and $\beta \to \infty$, with $\mathrm{Pe} = O(1)$. For a unit Lewis number, (38) implies that

$$U_T = \sqrt{1+\gamma\mathrm{Pe}^2} \quad (\mathrm{Le} = 1), \tag{39}$$

in agreement with the result of [2].

V. IMPLICATIONS OF THE FINDINGS AND CONCLUSIONS

We now discuss the key implications of the results, with the main focus being on answering the questions raised in Sec. I. We emphasize, once again, that these questions, which are inspired from turbulent combustion, are posed and answered for the specific context of premixed flames in a laminar one-scale parallel flow. Any extrapolation to more complex flow situations is beyond the scope of this investigation.

A. Bending effect

Referring to the notation of Sec. II, our limit $\epsilon \to 0$ with $\mathrm{Pe} = O(1)$ is seen to be equivalent to $A \to \infty$ with Re fixed. Thus our results show that the bending effect, exhibited in Fig. 1 when U_T (the turbulent burning velocity S_T scaled by S_L) is plotted versus u'/S_L for fixed Re, is also exhibited by laminar premixed flames. We may therefore refer to this effect as the laminar bending effect. Indeed, the bending effect is associated with the apparent existence of a horizontal asymptote in the experimental curve in Fig. 1 and the laminar bending effect is associated with the existence of a finite value of U_T, given by (38), in our asymptotic limit $A \to \infty$ with Pe fixed. Therefore, the laminar bending effect observed in our analytical study shows parallels with the bending effect observed for turbulent premixed flames. Our analytical observation of the laminar bending effect has also been consolidated by extensive numerical calculations, in both the constant density case and the variable density case. For illustration, Fig. 3, where U_T is plotted against A, exhibits the bending effect for $A \to \infty$ and Re fixed (solid curves). Parenthetically, we make the following three observations pertaining to these fixed-Re solid curves. First, we note the interesting nonmonotonic

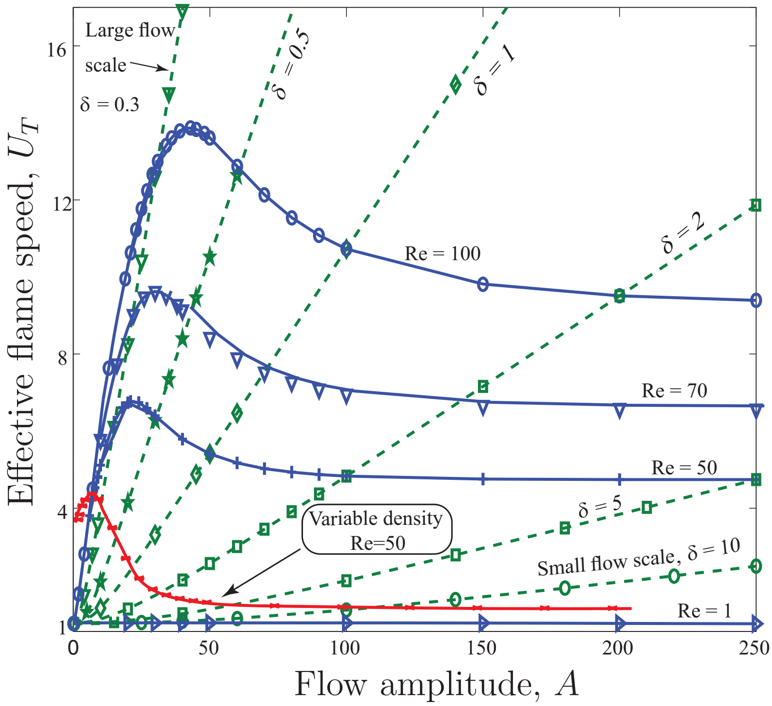

FIG. 3. Propagation speed U_T versus the flow amplitude A of a Poiseuille flow for fixed values of Re (solid curves) or fixed values of $\delta \equiv \epsilon^{-1}$ (dashed curves). Results of constant density numerical calculations are shown, except for the solid red curve labeled "Variable density," which shows variable density calculations for fixed Re = 50.

behavior of U_T versus A, with U_T attaining a maximum for a finite value of A. Second, we note that the asymptotic value of U_T as $A \to \infty$ in the variable density case is less than the corresponding value in the constant density case; this is consistent with formulas (24) and (39). Third, we note that in the variable density case U_T does not tend to unity (the normalized planar flame speed) as $A \to 0$, but rather to a higher value; this is because a planar solution does not exist in this case and the flame is necessarily curved unless the no-slip boundary condition at the wall is replaced by the less physically realistic slip boundary condition [37].

Returning to the bending effect, it is important to point out that this effect is not found to depend qualitatively on the form of the one-scale parallel flow and has also been observed for other flows [36]. Note, however, that the distinguished limit $A \to \infty$ with Re fixed is essential to this conclusion and that other limits may lead to a different behavior; for example, the limit $A \to \infty$ with ϵ fixed exhibits an apparently linear behavior (dashed curves in Fig. 3). Comparison between Figs. 1 and 3 shows striking similarities and suggests perhaps that a possible reason for the failing of previous theories to explain the experimentally observed bending effect may well be their lack of adoption of the proper distinguished limit. Most previous theories have indeed adopted the thin flame or large flow scale assumption, $\epsilon \gg 1$, while our distinguished limit requires the thick flame or small flow scale assumption, $\epsilon \ll 1$. We may therefore conclude that, at least for parallel flows, the bending effect cannot be explained without allowing small scales.

B. Damköhler's second hypothesis

This brings us to the next important point, namely, that formulas (38) and (39) pertain to small flow scales and directly relate, as we will elaborate below, the increase in the propagation speed to the increase in the effective diffusivity. This increase is therefore in line with the ideas put forward by Damköhler [3] in what is commonly referred to as Damköhler's second hypothesis [27]. Our formulas establish, in the simple context of one-scale parallel flows, a direct link between Taylor dispersion and Damköhler's second hypothesis. Furthermore, the formulas may also serve in this context as a means to test the validity of this hypothesis. This latter point was touched upon in [7] and later in [2,18], but the explicit influence of the Lewis number exhibited by (39) seems to be a significant feature revealed by this study with implications discussed in the next two sections. In fact, in his original seminal contribution [3] Damköhler did not consider Lewis number effects on the basis that all diffusivities are practically equal in usual gas mixtures.

To make the discussion more concrete, it is worth noting that Damköhler's second hypothesis may be stated as

$$U_T \sim \sqrt{\mathrm{Re}}, \tag{40}$$

where $U_T \equiv S_T/S_L$ is the scaled turbulent flame speed and $\mathrm{Re} \equiv u'l/\nu$ the (turbulent) Reynolds number given in terms of the turbulence intensity u', the turbulence (integral) scale l, and the kinematic viscosity ν. Damköhler arrived at this simple formula by making essentially two arguments. His first argument is that the turbulent flame speed $S_T = \sqrt{D_{\mathrm{eff}}/\tau}$, where D_{eff} is an effective (turbulent) diffusivity and τ the same chemical time which enters the laminar flame speed formula $S_L = \sqrt{D_T/\tau}$ [see, e.g., (13)]. This first argument implies of course that

$$U_T \equiv \frac{S_T}{S_L} = \sqrt{\frac{D_{\mathrm{eff}}}{D_T}}. \tag{41}$$

His second argument is that in a gas all diffusivities are of the same order of magnitude $D_T \sim D_F \sim \nu$ and when the flow is turbulent any effective diffusivity may be estimated by $D_{\mathrm{eff}} \sim u'l$. This argument implies that

$$\frac{D_{\mathrm{eff}}}{D_T} = \mathrm{Re} \tag{42}$$

and immediately leads, when used together with (40), to (41).

We are now ready to assess the applicability of Damköhler's second hypothesis to parallel one-scale flows. Specifically, our findings demonstrate that the hypothesis is only partially correct, in that Damköhler's first argument leading to (41) is correct but his second argument leading to (42) is not. This is most clearly seen in the $\mathrm{Le} = 1$ case (the only case considered by Damköhler) for which the effective propagation speed U_T is given by formula (39). This formula clearly satisfies (41) provided D_{eff} is identified with $D_{T,\mathrm{eff}}$ given by (25). There is a clear disagreement however between our expression for the effective diffusivity (25) and Damköhler's expression (42), resulting in a significant disagreement between our expression for U_T, given by (39), and his, given by (40). For example, making no distinctions between Pe and Re and assuming $\mathrm{Re} \gg 1$, our formula implies that $U_T \sim \mathrm{Re}$, which is to be compared to Damköhler's formula $U_T \sim \sqrt{\mathrm{Re}}$.

With these precise conclusions, based on our analytical findings, we close our discussion on the applicability of Damköhler's second hypothesis in our well defined one-dimensional flow context. It is worth pointing out, however, that experimental evidence [19] and direct numerical simulations [9] seem to indicate that Damköhler's formula is qualitatively correct in turbulent flows; this suggest that flows more complex than the one used here are necessary for the formula to apply. We will not attempt herein to extrapolate the discussion of our conclusions to such more complex and realistic flows as this will involve some speculations which are beyond the scope of this study.

C. Dependence of the effective Lewis number on the flow

Another interesting aspect we address now is the dependence of the effective Lewis number on the flow, for a mixture with an arbitrary Lewis number. The reader is referred to [34] for a scholarly review

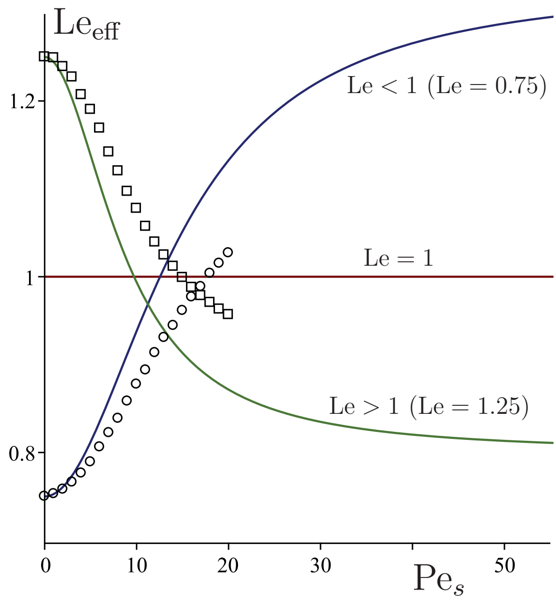

FIG. 4. Effective Lewis number $\mathrm{Le}_{\mathrm{eff}}$ versus the scaled Péclet number $\mathrm{Pe}_s \equiv \mathrm{Pe}(1-\alpha)$ for fixed values of Le. Solid lines denote analytical results from Eq. (23). Circles and squares denote numerical results for $\mathrm{Le} = 0.75$ and $\mathrm{Le} = 1.25$, respectively, computed in the constant density approximation (where $\mathrm{Pe}_s = \mathrm{Pe}$) and for fixed $\epsilon = 0.01$.

on Lewis number effects in turbulent combustion. Here our objective is more limited and addresses the question, discussed in [30], whether the effective Lewis number tends to one for large values of u', as argued in [6], or further deviates from one, as argued in [31]. This controversial question has a clear answer in our well defined framework. Specifically, the dependence of the effective Lewis number on the parallel flow is given by the simple formula (23). This formula leads to a striking and unexpected behavior, which is illustrated in Fig. 4, where $\mathrm{Le}_{\mathrm{eff}}$ is plotted versus the scaled Péclet number $\mathrm{Pe}_s \equiv \mathrm{Pe}(1-\alpha)$. Specifically, $\mathrm{Le}_{\mathrm{eff}}$ varies from Le to Le^{-1} as Pe varies from zero to large values. Thus a mixture whose Lewis number $\mathrm{Le} < 1$ sees its effective Lewis number increase with increasing Pe, so $\mathrm{Le}_{\mathrm{eff}} > 1$ for large Pe. Similarly, a mixture whose Lewis number $\mathrm{Le} > 1$ sees its effective Lewis number decrease, so $\mathrm{Le}_{\mathrm{eff}} < 1$ for large Pe. The dependence of the effective Lewis number on the flow is therefore a different and somewhat counterintuitive result. The result is readily explained as being a direct consequence of Taylor dispersion applied simultaneously to heat and mass diffusions in a nonunit Lewis number mixture; specifically, it follows from the definition of the effective Lewis number as the ratio $D_{T,\mathrm{eff}}/D_{F,\mathrm{eff}}$ and from Taylor dispersion formulas for $D_{T,\mathrm{eff}}$ and $D_{F,\mathrm{eff}}$ given by (21) and (22). The relationship between $\mathrm{Le}_{\mathrm{eff}}$ and Pe_s is confirmed, at least qualitatively, by a few numerical simulations which are also reported in Fig. 4. In these simulations, $\mathrm{Le}_{\mathrm{eff}}$ is computed as outlined at the end of Sec. II following the method of [30], where the difficulties in estimating $\mathrm{Le}_{\mathrm{eff}}$ numerically are discussed. In view of these difficulties, the quantitative agreement between the asymptotic and numerical results exhibited in Fig. 4 is reasonable.

It is worth noting that the dependence of $\mathrm{Le}_{\mathrm{eff}}$ on the flow has significant influence on U_T, as described in the following section. It is also anticipated to be an important factor to consider when studying the stability of a (thick) flame propagating against a flow between two parallel plates as in a Hele-Shaw cell [38–40]; the exploration of this specialized topic within a generalization of the current analytical approach is left for future investigation.

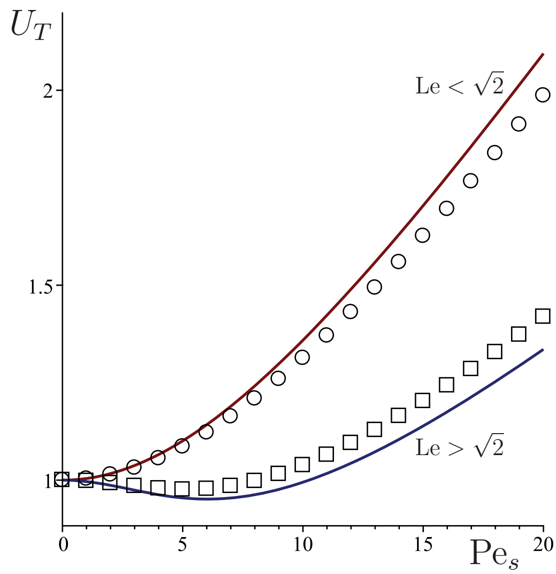

FIG. 5. Propagation speed U_T versus the scaled Péclet number $\mathrm{Pe}_s \equiv \mathrm{Pe}(1-\alpha)$ for fixed values of Le (Le $= 1$ and Le $= 1.7$). Solid lines denote analytical results from Eq. (38). Circles and squares denote numerical results for Le $= 1$ and Le $= 1.7$, respectively, computed in the constant density approximation (where $\mathrm{Pe}_s = \mathrm{Pe}$) and for fixed $\epsilon = 0.01$.

D. Dependence of U_T on the Péclet and Lewis numbers

Formula (38) for U_T leads to another surprising behavior shown in Fig. 5. Specifically, for a fixed value of Le, U_T is found to be a monotonically increasing function of Pe if Le $< \sqrt{2}$. On the other hand, if Le $> \sqrt{2}$, a nonmonotonic behavior is obtained as Pe is increased, corresponding to an initial decrease of U_T, which is followed by an increase. These dependences of U_T on Pe are confirmed by our numerical results (Fig. 5). At this point, one may wonder why U_T has the specific functional form given in (38), including the square root in the denominator, and why this form is a consequence of Taylor dispersion. The answer is quite simple, if one examines the formula for the planar flame speed (13) which, along with (12), implies that

$$S_L = \sqrt{\frac{D_{T,b}^2}{D_{F,b}}\tau^{-1}} \propto \frac{D_{T,b}}{D_{F,b}^{1/2}}. \tag{43}$$

Here τ is a characteristic chemical time given by

$$\tau = \frac{\hat{\rho}_u^2}{\hat{\rho}_b^2}\frac{\beta^2}{2B}e^{E/RT_{ad}}$$

and $D_{T,b}$ and $D_{F,b}$ refer to the values of the thermal and mass diffusivities in the burned gas (where $\theta = 1$ and hence $\rho = 1-\alpha$). Then replacing these diffusivities by their effective values given by Taylor dispersion formulas (25) and (26) applicable at $\theta = 1$ (where $D_T = D_{T,b}$ and $D_F = D_{F,b}$) provides the dimensional form of the effective propagation speed S_T. In other words, S_T can be obtained from S_L by replacing the diffusion coefficients (evaluated at the reaction zone or the burned gas) in (43) by their effective values given by (25) and (26). This results in an expression for $U_T \equiv S_T/S_L$ given by formula (38). This formula therefore quantifies the enhancement of the propagation speed due to small flow scales for mixtures with arbitrary Lewis numbers in our one-dimensional flow model. In such context, it provides an accurate description of such enhancement anticipated by heuristic arguments encapsulated by Damköhler's second hypothesis.

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