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A three-level consensus model for large-scale multi-attribute group decision analysis based on distributed preference relations under social network analysis

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ABSTRACT

Large-scale multi-attribute group decision analysis (LS-MAGDA) is common in practical problems. As a type of preference relation, distributed preference relation (DPR) can express the preferred, non-preferred, indifferent, and uncertain degrees of one alternative over another. In LS-MAGDA, conflict between assessment-based clustering analysis and consensus reaching process (CRP) may occur. Different levels of consensus measurement and feedback mechanism are not fully discussed in previous studies. To solve these problems, a trust-confidence analysis (TCA) framework, which takes into consideration both the trust relationship and self-confidence based on social network analysis (SNA), is proposed to let clustering analysis and CRP not influence with each other. Decision makers' social status and willingness to modify opinions can be reflected in TCA, which facilitates consensus adjustment and reaching process. A consensus measure framework at attribute, alternative and global levels is then proposed. Additionally, consensus feedback mechanism with different identification and direction rules from attribute level to global level is analyzed considering the consensus degree and importance of attributes. The identification rule becomes looser with the increasing of consensus status and decreasing of attribute weights. An illustrative example of product life cycle design is presented to demonstrate the validity and effectiveness of the proposed method in dealing with realistic problems.

1. Introduction

Large-scale multi-attribute group decision analysis (LS-MAGDA) refers to the situation in which a large number of decision makers (DMs) evaluate a set of alternatives on multiple attributes (Tang & Liao, 2021; Tang et al., 2020). LS-MAGDA is involved in every field of social and economic development (Li et al., 2021b; Rodríguez et al., 2021; Wang et al., 2022; Xu et al., 2018). Accordingly, LS-MAGDA has played a more and more important role in decision making processes and attracted enormous attentions.

Pairwise comparison based preference relations are common representation of assessment information in LS-MAGDA. Compared with providing direct assessments of alternatives on attributes such as belief distribution (BD) (Deng, 2020; Fu & Yang, 2011; Xiao, 2021a, 2021b; Yang & Xu, 2013; Zhou et al., 2020; Zhou et al., 2019), preference relations allow DMs to concentrate on two alternatives simultaneously, which needs less accurate assessment information. Considering complex decision-making circumstances, linguistic-based preference relation has attracted the attention of many scholars (Li et al., 2021a; Wu et al., 2015; Wu et al., 2021c; Zhang et al., 2014; Zhang et al., 2016; Zhang et al., 2020c). Distribution linguistic preference relation (DLPR) is firstly proposed by Zhang et al. (2014) to enable DMs to assign different probabilities on a set of linguistic terms. Probabilistic linguistic preference relation (PLPR) (Zhang et al., 2016) can also express DMs' preferences by using multiple linguistic terms with different probabilities.

Similar to DLPR or PLPR, distributed preference relation (DPR) (Fu et al., 2016; 2021) uses a set of linguistic evaluation grades to describe the preferred, non-preferred, indifferent, and uncertain degrees of one

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alternative over another. Although there are many similarities between DPR and DLPR or PLPR such as information representation, Xue et al. (2021) pointed out that the mechanisms that they handle uncertainty, consistency measure, information aggregation and solution generation process are fundamentally different. Under the theory of probability, DPR can model preference relation with incomplete information and generate consistent pairwise comparisons more flexibly, especially under the circumstance of LS-MAGDA (Zhou et al., 2022).

Social network analysis (SNA) discusses the relationship among various social actors and has become a key tool in LS-MAGDA. Trust relationship among DMs is commonly employed to address LS-MAGDA problems when conducting SNA (Du et al., 2020; Victor et al., 2011; Wang et al., 2017; Wu et al., 2021a). For example, Wu et al. (2021b) proposed a maximum self-esteem degree based feedback mechanism for consensus reaching with the distributed linguistic trust propagation. Tian et al (2019) proposed a social network analysis-based consensus-supporting framework with incomplete interval type-2 fuzzy information for LSGDM. SNA seems to be an indispensable analytical paradigm in order to meet the requirement of optimization and consensus reaching. Moreover, since individuals are a group of decision-making participants with social characteristics, it makes SNA an important component of assessing DMs' willingness to change opinions (Chu et al., 2020; Ding et al., 2019; Gai et al., 2020; Liu et al., 2019b; Lu et al., 2021). Although numerous studies have been proceeded with SNA, the joint influence of trust relationship and self-confidence to decision-making and consensus reaching process in LS-MAGDA should be further discussed.

Consensus reaching process (CRP) is one of the important issues in group decision-making. Considering a large number of DMs under the background of LS-MAGDA, dimension reduction of DMs is essential in order to reduce the complexity and cost in LS-MAGDA (Du et al., 2020). In general, clustering analysis is commonly used to reduce the dimension of DMs (Tang & Liao, 2021). The consistent attitude of each subgroup can be obtained by clustering rather than individuals' divergent opinions, which will facilitate the CRP. In addition, CRP is usually an iterative and dynamic process, which may require a major expenditure of time and cost in LS-MAGDA. A feedback mechanism which should calculate consensus degree and provide corresponding suggestions for adjustment can improve the efficiency of CRP (Wang et al., 2022; Wu et al., 2019). As for the mechanism to increase the consensus degree in each iteration, the key idea is to modify judgments and increase similarity among DMs' or subgroups' opinions. The general methods use group's collective assessment as benchmark and direction to modify DMs' opinions (Tang et al., 2019; Wu et al., 2018; Zhang et al., 2018). Moreover, some research focuses on minimum adjustment or cost feedback mechanism (Dong et al.,2010; Sun et al., 2021; Zhang et al., 2011; Zhang et al., 2019; Zhang et al., 2020a). However, the conflict that may be incurred by clustering analysis and CRP should be dealt with carefully.

Based on the above analysis, there are research gaps that limit the construction of consensus model for LS-MAGDA with DPRs under SNA as follows:

(1) Conflict between clustering analysis and CRP. The common method of clustering analysis is to use dissimilarity among DMs' assessments as clustering basis (Liu et al., 2014; Wang et al., 2018). This simple and efficient clustering method may conflict with subsequent CRP. The adjustment of assessments for the purpose of obtaining a higher consensus degree may destroy the pre-clustered subgroups based on assessments. Meanwhile, changes in clustering results may also influence the modification of assessments in CRP. Even if some research (Ding et al., 2019; Du et al., 2020) proposed clustering methods which embody both opinion similarity and social relations among DMs, this problem may not be completely solved. Therefore, repeated iterations between group clustering analysis and CRP may be unavoidable, which leads to a high instability of decision-making process.

(2) Different levels of consensus requirement. Up to now, lots of attention have been paid to CRP at alternative level (Cao et al., 2021; Rodríguez et al., 2018; Triantaphyllou et al., 2020). Nevertheless, multiple attributes are often included in decision problems, which means more precise consensus requirements at the attribute level need to be solved. Moreover, for important attributes, corresponding consensus requirements are supposed to be stricter. Therefore, consensus measurement and adjustment at different levels (Fu & Yang, 2011) are necessary. However, different levels of consensus requirement in pairwise comparison structure, including attribute, alternative and global levels are not fully discussed in existing literature.

(3) Rational feedback mechanism under SNA. Trust relationship plays an essential rule in the construction of SNA and subsequent CRP in existing literature. As a person's belief that a statement represents the best possible response, self-confidence is also influential to DM's willingness to change opinion (Liu et al., 2019a; Liu et al., 2019b; Zarnoth & Sniezek, 1997). Considering the social traits of DMs, suggestions generated by feedback mechanism should be rational and acceptable. However, the joint influence of trust relationship and self-confidence to the construction of feedback mechanism has not been fully discussed.

After taking the above-mentioned key issues into consideration, the main innovations and contributions of this paper can be summarized as follows:

(1) For solving the conflict between group clustering analysis and CRP, a trust-confidence analysis (TCA) which combines trust relationship and self-confidence of DMs under SNA is proposed. Together with the K-means algorithm, TCA is applied to divide DMs into different clusters according to their trust relationship and self-confidence status. The clustering analysis and consensus feedback mechanism can be separated to two non-interfering processes.

(2) Based on the TCA under SNA, the characteristics of DMs in different clusters representing diverse social properties are discussed. Clusters with different levels of trust relationship and self-confidence determine different prestige status and willingness to change opinions, which assures the acceptability and validity of feedback mechanism.

(3) Based on the proposed concept of collective DPR (CDPR), a three-level consensus model is developed to meet different consensus requirements in various LS-MAGDA scenarios. Accordingly, consensus feedback mechanisms corresponding to different levels of consensus requirement are also proposed by considering both consensus degree and attribute weights.

The reminder of the paper is organized below. Section 2 reviews some basic concepts used in this paper. In Section 3, the clustering analysis based on TCA is proposed. The features of different clusters are analyzed. Consensus measurement, identification and direction rules for LS-MAGDA based on CDPRs at the attribute, alternative and global levels are discussed in Section 4. Section 5 introduces an illustrative example to demonstrate the validity of the proposed method along with the comparative analysis with some state-of-the-art methods. This paper is concluded in Section 6.

2. Preliminaries

In this section, social network analysis (SNA) is introduced briefly. In addition, the definition of distributed preference relation (DPR) is reviewed.

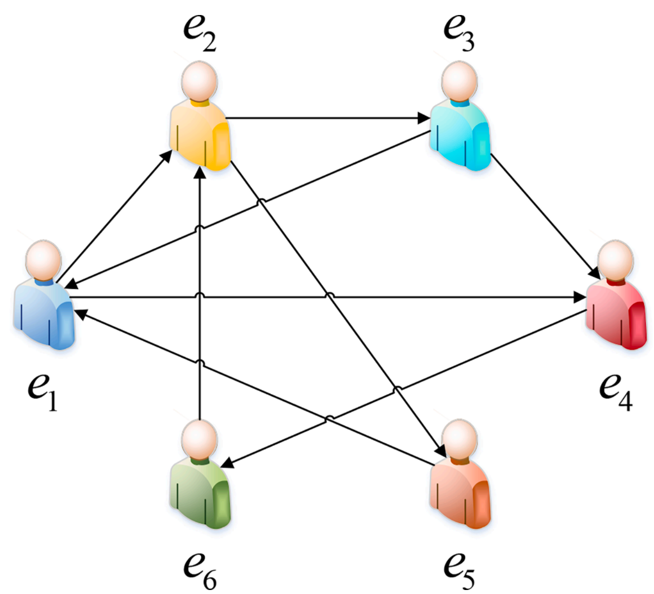


Fig. 1. Social network with direct/indirect relationships.

2.1. Social network analysis

Three main notational schemes are usually adopted to present the social actors and relationships in SNA as follows: (1) Graph: depicted as a graph which is made up of nodes linked by edges; (2) Algebraic: different relationships and a combination of relationships are distinguished via this presentation; (3) Sociometric: presented in the form of a matrix (Gai et al., 2020).

Definition 1. (*(2-tuple trust relationship)*) (Victor et al., 2009) *Suppose there are T experts represented as* $e = \{e_1, e_2, \cdots, e_T\}$. *The trust relationship from* e_r *to* e_s *is defined in the form of a 2-tuple style as* $\lambda_{rs} = (T_{rs}, D_{rs})$ $(r, s = 1, 2, \ldots, T)$, *where* T_{rs}, $D_{rs} \in [0, 1]$. T_{rs} *and* D_{rs} *represent the trust and distrust degree from* e_r *to* e_s *respectively. The set of this kind of trust functions (TFs) is denoted by* $\Lambda = \{\lambda_{rs} = (T_{rs}, D_{rs}) | T_{rs}, D_{rs} \in [0, 1]\} (r, s = 1, 2, \ldots, T; r \neq s)$.

The character of transitivity ensures the construction of trust relationships between experts who have indirect relationships (See Fig. 1). We can achieve this propagation process via an indirect chain of trusted third partners (TTPs) through the propagation operators.

Definition 2. (*(Dual trust propagation operator)*) (Wu et al., 2017) *The dual trust propagation operator is a mapping* $P_D : \Lambda \times \Lambda \to \Lambda$, *containing two TFs' information* $\lambda_{rs} = (T_{rs}, D_{rs})$, $\lambda_{st} = (T_{st}, D_{st})$, *which is defined as follows*:

$$P_D(\lambda_{rs}, \lambda_{st}) = \left(\frac{T_{rs} T_{st}}{1 + (1 - T_{rs})(1 - T_{st})}, \frac{D_{rs} + D_{st}}{1 + D_{rs} \times D_{st}} \right) \quad (1)$$

Fig. 2 (a) shows the relationship among three experts in which there is no direct trust function from e_1 to e_3, just like a one-way chain. We can use the dual trust propagation operator to complete the relationship (i.e. trust function) from e_1 to e_3 shown in Fig. 2 (b).

2.2. Distributed preference relation

Definition 3. (*(Distributed preference relation)*) (Fu et al., 2016) *Let* $A = \{A_1, A_2, \cdots, A_M\}$ *be a set of alternatives. Experts denoted as* $e = \{e_1, e_2, \cdots, e_T\}$ *give their comparisons between alternatives* A_i *and* A_j $(i, j = 1, 2, \cdots, M)$ *by a set of discrete linguistic grades* $\Omega = \{H_1, H_2, \cdots, H_N\}$, *which forms DPR matrix* $D^t = (d^t(A_{ij}))_{M \times M} \subset A \times A$ $(t = 1, 2, \ldots, T)$ *representing DPRs on A by* e_t. $d^t(A_{ij})$ *is presented below:*

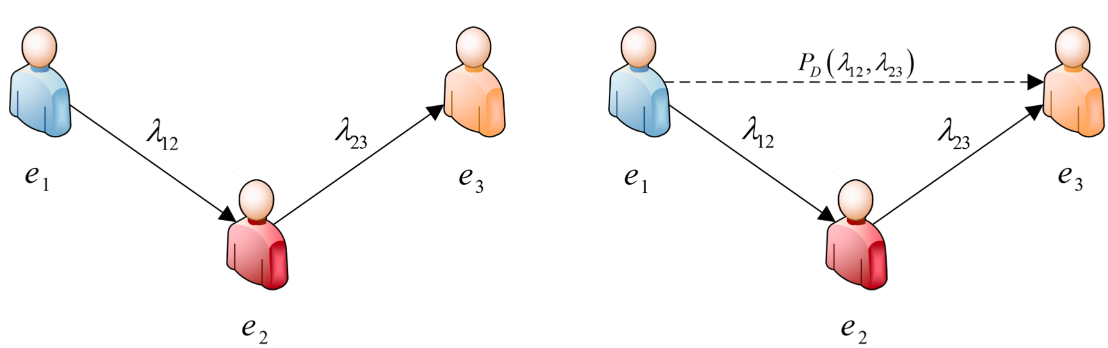
$$d^t(A_{ij}) = \{ (H_n, \beta_n^t(A_{ij})), n = 1, 2, \cdots, N; (\Omega, \beta_\Omega^t(A_{ij})) \} \quad (2)$$

DPR matrix D^t representing the comparisons between each pair of alternatives by expert e_t is shown in Table 1. Here, N is an odd number, the median grade $H_{(N+1)/2}$ represents the priority of indifference, $H_{(N+3)/2}, \cdots, H_N$ stand for the preferred grades with the intensity increasing when the subscripts growing, $H_1, \cdots, H_{(N-1)/2}$ signify the non-preferred grades with the intensity decreasing on account of the subscripts growing. $\beta_n^t(A_{ij})$ indicates the belief degree assigned to H_n when comparing A_i over A_j by expert e_t, and $\beta_\Omega^t(A_{ij})$ represents the global ignorance when comparing A_i over A_j. $\beta_\Omega^t(A_{ij})$ can be assigned to $\beta_X^t(A_{ij})$ where X denotes any subset of Ω. $0 \leqslant \beta_n^t(A_{ij}) \leqslant 1$, $0 \leqslant \beta_\Omega^t(A_{ij}) \leqslant 1$ and $\sum_{n=1}^N \beta_n^t(A_{ij}) + \beta_\Omega^t(A_{ij}) = 1$ are the basic conditions for DPR. Obviously, the following features are satisfied: $\beta_n^t(A_{ij}) = \beta_{N-n+1}^t(A_{ji})$ $(n = 1, 2, \ldots, N)$, $\beta_\Omega^t(A_{ij}) = \beta_\Omega^t(A_{ji})$ and $\beta_{(N+1)/2}^t(A_{ii}) = 1$.

For the convenience of comparing alternatives explicitly, a score value function of grades $H_n (n = 1, 2, \cdots, N)$ should be defined to transform DPR matrix into its corresponding score value matrix. Suppose the score value on grade $H_n (n = 1, 2, \cdots, N)$ is $S(H_n) (n = 1, 2, \cdots, N)$ which

Table 1
DPR matrix D^t given by expert e_t.

Alternative	A_1	A_2	…	A_M
A_1	$d^t(A_{11}) = \{(H_{(N+1)/2}, 1)\}$	$d^t(A_{12})$	…	$d^t(A_{1M})$
A_2	$d^t(A_{21})$	$d^t(A_{22}) = \{(H_{(N+1)/2}, 1)\}$	…	$d^t(A_{2M})$
…	…	…	…	…
A_M	$d^t(A_{M1})$	$d^t(A_{M2})$	…	$d^t(A_{MM}) = \{(H_{(N+1)/2}, 1)\}$



(a) No direct TF from e_1 to e_3 (b) Trust propagation from e_1 to e_3

Fig. 2. Dual trust propagation for indirect relationship.

satisfies the condition as follows:

$$-1\leqslant S(H_1)\leqslant\cdots\leqslant S(H_{(N-1)/2})\leqslant S(H_{(N+1)/2})\leqslant S(H_{(N+3)/2})\leqslant\cdots\leqslant S(H_N)\leqslant 1 \tag{3}$$

Naturally, the score of median grade is set to be zero such that $S(H_{(N+1)/2}) = 0$. And $S(H_n) = -S(H_{N-n+1})(n=1,2,\cdots,N)$. Then Eq.(2) can be transformed into an interval value $[S_l^t(A_{ij})^-, S_l^t(A_{ij})^+]$ as follows (Xue et al., 2021):

$$S^t(A_{ij})^- = \sum_{n=1}^{N}\beta_n^t(A_{ij})S(H_n)+\beta_\Omega^t(A_{ij})S(H_1) \tag{4}$$

$$S^t(A_{ij})^+ = \sum_{n=1}^{N}\beta_n^t(A_{ij})S(H_n)+\beta_\Omega^t(A_{ij})S(H_N) \tag{5}$$

where $S^t(A_{ij})^- + S^t(A_{ji})^+ = 0$ and $S^t(A_{ij})^+ + S^t(A_{ji})^- = 0$.

To relieve the burden of providing pairwise comparisons between every two alternatives, experts are advised to give judgments between adjacent alternatives such as $d^t(A_{i,i+1})(i=1,2,\cdots,M-1)$(Herrera-Viedma et al., 2004). It should be mentioned that the original concept of DPR is the distribution of judgments from all experts instead of given by a single expert. Hence, there is no CRP in the decision process. Furthermore, the comparison in Def.3 is only implemented at the alternative level, which is not suitable for decision making problems with multiple attributes.

As we all know, the consistency condition is a necessary indicator when using the method of pairwise comparisons on alternatives. Therefore, it is essential to maintain the consistency of DPR in order to ensure the correctness and reliability of the final results. The following function can be used to guarantee the requirement of consistency.

Definition 4. (*(Score function)*) (Fu et al., 2021) *Let $f:[-1,1]\times[-1,1]\to[-1,1]$ be a function to generate $S(A_{ki})$ from $S(A_{ij})$ and $S(A_{jk})$, then we have*:

$$f(S(A_{ij})^+, S(A_{jk})^+)+S(A_{ki})^+ = 0(i,j,k=1,2,\cdots,M) \tag{6}$$

$$f(S(A_{ij})^-, S(A_{jk})^-)+S(A_{ki})^- = 0(i,j,k=1,2,\cdots,M) \tag{7}$$

Any function satisfies Eqs.(6) and (7) can be applied to the construction of consistent score matrix, such as $g(y,z)=((y+z-(1+b)\cdot yz)/(1-b\cdot yz))$ with the parameter $b\in(-\infty,1)$. The specific process of confirmation on parameter b can be referred to Fu et al. (2016).

2.3. Research framework

The research framework of the paper is specified as follows:

2.3.1. Preparation

Determine the basic parameters of LS-MAGDA, including discrete linguistic evaluation grades $\Omega=\{H_1,H_2,\cdots,H_N\}$, a set of alternatives $A=\{A_1,A_2,\cdots,A_M\}$ and the associated attributes $a=\{a_1,a_2,\cdots,a_L\}$, invited experts $e=\{e_1,e_2,\cdots,e_T\}$.

2.3.2. Clustering procedure [Section 3]

1) Social network analysis (Section 3.1).

Obtain origin trust relationships among experts and self-confidence degree of each expert. Generate complete trust relationships among all experts via SNA. Calculate trust score and self-confidence score of each expert.

2) Trust-confidence analysis (Section 3.2).

Conduct trust-confidence analysis (TCA) from the dimensions of trust relationship and self-confidence of expert. Experts with different trust scores and self-confident levels are located in different regions in two-dimensional TCA coordinate system plot.

3) Group clustering based on K-means (Section 3.4).

Classify experts into different clusters by K-means algorithm and TCA structure.

2.3.3. Consensus reaching process for LS-MAGDA [Section 4]

1) Construct collective DPR (Section 4.1).

Each cluster of experts provides collective assessment at each attribute on every pair of adjacent alternatives in the form of DPR.

2) Calculate consensus at different levels (Section 4.2).

Calculate consensus at three levels, i.e. the attribute level, alternative level and global level.

3) Consensus feedback mechanism (Section 4.3).

Identification rules: Identify assessments of clusters that break consensus status by identification rules. Adjust assessments that has been identified by direction rules until consensus requirement has reached.

2.3.4. Final decision

1) Aggregation of assessments.

Aggregate the CDPRs on attribute by different clusters for each pair of adjacent alternatives by using ER algorithm.

2) Generate the result.

Generate a solution of LS-MAGDA problem by considering consistency and get the ranking order of alternatives.

3. The clustering analysis of DMs based on trust-confidence analysis (TCA)

Clustering analysis can not only reduce the complexity and cost of LS-MAGDA, but also find out representative opinion leaders (OLs), which is conducive to the smooth progress of subsequent CRP. In this section, a novel clustering analysis framework is given based on TCA.

3.1. Establishment of the trust relationship based on SNA

Trust relationship has become an increasingly essential information resource in LSGDA process under SNA. In order to rank the trust functions given in Def.1, a trust score (*TS*) function which maps *TF* to an interval value [0,1] is given as follows (Wu et al., 2017):

$$TS(\lambda_{rs}) = \frac{T_{rs}-D_{rs}+1}{2},(r,s=1,2,\cdots,m;r\neq s) \tag{8}$$

It is well known that trust information is transitive in social network. Besides the direct relationships between different pairs of experts, indirect relationships are also common in the context of social network, especially in the trend of increasing scale of experts during decision-making process. Therefore, to complete the construction of social connection between experts with indirect relationships, propagation operators are chosen to play the role of trust third partners (TTPs).

There have already been many propagation operators, such as the uninorm propagation operators with t - normand t - conorm (Victor

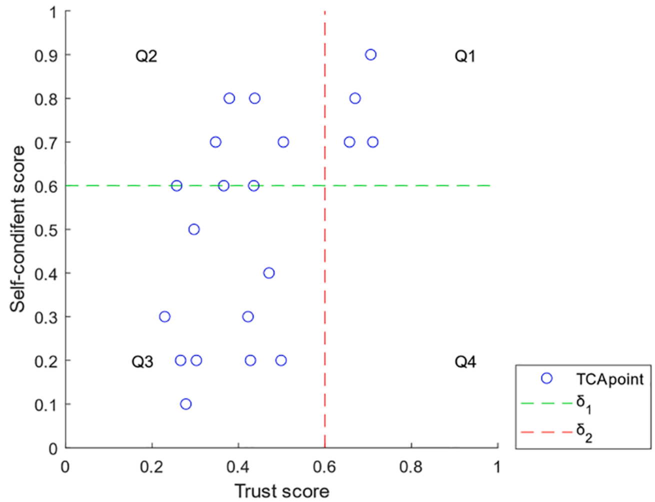


Fig. 3. The TCA plot with two dimensions.

et al., 2009), bilattice-based aggregation approaches (Victor et al., 2011), TS-UTOWA operator for multipath propagation (Wang et al., 2017), dual trust propagation operator based on the Einstein sum operator and the Einstein product operator (Wu et al., 2017). In this paper, the above-mentioned dual trust propagation operator is applied as shown in Def.2.

Obviously, the number of trust propagation chains from one expert to another may be more than one in real social network. Considering the attenuation in realistic trust propagation process, choosing the shortest path of propagation can yet be regarded as an effective and appropriate way (Ghavipour & Meybodi, 2018). In some special conditions, if there are several shortest paths of propagation from e_r to e_t, the average value of trust function is generated by dual trust propagation operator as the final trust function (Wu et al., 2017).

As such, we can obtain the complete trust relationship between each pair of experts under social network by using indirect chains of TTPs through the dual trust propagation operator. When the sociometric is constructed, a comprehensive trust level of each expert can be generated by aggregating the trust function from others, which is defined as follows:

Definition 5. (*(Aggregated trust function)*) (Wu et al., 2015) *Let* $e = \{e_1, e_2, \cdots, e_T\}$ *be the set of experts and* $\Lambda = \{\lambda_{ts} = (T_{ts}, D_{ts}) | T_{ts}, D_{ts} \in [0, 1]\}(t, s = 1, 2, \ldots, T)$ *be the set of trust functions from one expert to another. For* $t = 1, 2, \cdots T$, $\widetilde{\lambda}_t = \left(\widetilde{T}_t, \widetilde{D}_t\right)$ *is the aggregated trust function of* e_t *which is generated as follows*:

$$\widetilde{\lambda}_t = \left(\widetilde{T}_t, \widetilde{D}_t\right) = \frac{1}{T-1}\sum_{s=1, s\neq t}^{T} \lambda_{st} = \left(\frac{1}{T-1}\sum_{s=1, s\neq t}^{T} T_{st}, \frac{1}{T-1}\sum_{s=1, s\neq t}^{T} D_{st}\right) \tag{9}$$

where λ_{st} represents the trust function from expert e_s to e_t.

3.2. Construction of TCA framework

In addition to the trust relationship among experts, self-confidence is also a key element to reflect DM's individual trait in SNA. Self-confidence is defined as a person's belief that a statement represents the best possible response (Zarnoth & Sniezek, 1997). It plays an important role when experts are involved in the CRP of LS-MAGDA problem. Briefly, due to diverse professional background, risk attitude, knowledge level and working experience, experts with different self-confidence levels give different preference information. Moreover, self-confidence directly affects the willingness to change opinion and may influence final decision. Self-confidence can be defined by linguistic term as follows:

Definition 6. (*(Linguistic term set for self-confidence)*) *Suppose a linguistic term set is denoted by* $C = \{c_g | g = 1, 2, \cdots, G\}$, *in which* c_g *indicates crisp linguistic variable, representing a specific level of self-confident degree.*

For example, if $G = 9$, C can be defined as:

$$C = \{c_1 = ExtremelyLow, c_2 = VeryLow, c_3 = Low, c_4 = SlightlyLow, c_5 = Medium, c_6 = SlightlyHigh, c_7 = High, c_8 = VeryHigh, c_9 = ExtremelyHigh\}$$

Considering that each linguistic term represents a specific self-confidence level, a score function which can convert each linguistic variable into its corresponding score value is proposed.

Definition 7. (*(Score value of linguistic term set)*) *Let* $C = \{c_g | g = 1, 2, \cdots, G\}$ *be a set of linguistic terms, in which each linguistic variable represents a specific level of self-confidence. Suppose the score of linguistic variable* c_g *is denoted by* $S(c_g) \in [0, 1]$ *with.*

$$S(c_{g+1}) \rangle S(c_g) \tag{10}$$

where c_{g+1} represents the higher level of self-confidence than c_g.

For simplicity, the value of different linguistic variables in Def.7 are assumed to be equidistantly distributed in the range of [0,1]. Consequently, a framework of the TCA by combining the two dimensions of trust and self-confident scores can be constructed in a comprehensive way.

Definition 8. (*(Trust-confidence relationship model)*) *Let* TS_t *be the trust score of* $\widetilde{\lambda}_t = \left(\widetilde{T}_t, \widetilde{D}_t\right)$ *by aggregating trust functions from other* T*-1 experts to* e_t, *and* $S_t(c_g)$ *the self-confident score of expert* e_t *given in Def.7. Then, the trust-confidence relationship model is defined as*:

$$TC_t = (TS_t, S_t(c_g))(t = 1, 2, \cdots, T) \tag{11}$$

$$TS_t = \frac{\widetilde{T}_t - \widetilde{D}_t + 1}{2} \tag{12}$$

where TS_t represents the comprehensive trust score to expert e_t by other T-1 experts, and $S_t(c_g)$ signifies the self-confident level of e_t. It seems natural to map this trust-confidence relationship model into a two-dimensional coordinate system, i.e. TCA plot, for analysis, where the X and Y axes represent trust and self-confident score respectively (see Fig. 3). According to the different trust and self-confident scores of experts, we can set vertical and horizontal thresholds δ_1 and δ_2 to roughly evaluate the level of self-confidence and trust respectively. Experts with different trust and self-confident levels are shown in different positions in the figure, which sets the basis for the following GC analysis.

Based on TCA and the two-dimensional coordinate system plot, we can intuitively classify the mapping of the trust-confidence degree of each expert (i.e. the blue dot in the plot) in a specific social network into four quadrants via two thresholds. The details are specified as follows:

(1) The first quadrant (Q1) is described as the "authority region". The closer mapping of experts is to the upper right in TCA plot, the higher degree of self-confidence and trust within a social network. That is to say, these experts are highly authoritative and respected by others in practical decision-making scenario. They may have been engaged in the field related to the problem for years. As such, they have relatively professional knowledge system and experience with considerable achievements, which makes other experts to trust in them to a great extent. Therefore, they can make the most reliable judgment and evaluation for the problem, making them usually have high prestige and play the

role of OLs in decision-making process. In real world, they are generally senior specialist or scholars whose reputation is distinguished.

(2) The second quadrant (Q2) is defined as the “confident region”. Experts who are closer to the upper left in TCA plot are more characterized by their high degree of self-confidence compared with their relatively low degree of trust by other experts. Intuitively, these experts seem a little overconfident. They already have a certain amount of research and understanding in the relevant fields of the decision problem, so they are confident that they can make correct and effective assessments to solve the problem. Nevertheless, due to various reasons, such as inadequate work experience or less social activities, they do not have a desired level of trust and prestige from other experts. In real world, they are common experts who have accumulated some research experience, and have a tendency to move to Q1 in the near future.

(3) The third quadrant (Q3) is called the “ordinary region”. The closer mapping of experts is to the bottom left in TCA plot, the lower degree of self-confidence and trust. Compared with the experts lie in the “authority region” and “confident region”, experts located in the “ordinary region” neither have quite deep understanding nor high level of prestige in this field. Hence, their influences over others are relatively ordinary and they don’t obtain significant achievements, eventually manifesting as the ordinary judgment and evaluation for the decision problem. They are probably distinguished as young experts in real world.

(4) The fourth quadrant (Q4) is termed as the “abnormal region”. Experts who are closer to the bottom right in TCA plot are more characterized by their high degree of trust from other experts but low degree of self-confidence. This is a counter-intuitive situation because experts who represent authority and expertise in a particular field are generally trusted in a higher degree by others. The low degree of self-confidence of this kind of experts is illogical and not in line with reality.

3.3. Discussion about the K-means algorithm

Among so many clustering methods, such as fuzzy clustering method (Baraldi & Blonda, 1999), agglomerative hierarchical clustering algorithm (Wang et al., 2018) and fuzzy equivalence relation algorithm (Wu et al., 2018), K-means algorithm is chosen in this paper due to the following reasons. First of all, K-means algorithm is based on Euclidean distance. It is recognized as one of the simplest and most effective clustering algorithms. Secondly, it is an unsupervised algorithm that can iterate multiple times to optimize the unreasonable places in the initial sample classification. Thirdly, since the time complexity of the algorithm is $O(n)$, it is efficient to process large data sets. Nevertheless, K-means also has some disadvantages, such as the initial number of clustering K and the selection of initial centroid needs to be determined subjectively.

Here, two issues are to be solved: (1) how to confirm the number of clusters$K$; (2) the determination of initial cluster centroid$\mu_k^{(1)}$. It is too arbitrary to determine these parameters according to subjective judgment or past experience because they are not necessarily the true cluster number of the data we obtain. Consequently, it is natural to determine K and $\mu_k^{(1)}$ from data.

As for the selection of the number of centroids, elbow method, Silhouette Coefficient and Calinski-Harabasz criterion are generally used. Here, we use elbow method whose core index is sum of the squared errors (SSE) such that:

$$SSE = \sum_{k=1}^{K} \sum_{x_t \in cl_k} (x_t - \mu_k)^2 \tag{13}$$

where cl_k is the k - th cluster, xt denotes the sample point in cl_k, and μ_k signifies the center of mass of cl_k (i.e. the mean value of all samples in cl_k). SSE reflects the clustering error of all samples, representing the quality of clustering effect. The core of elbow method is that K-means algorithm takes minimizing the square error between the sample points and the centroid as objective function, and the sum of the square distance errors between the sample points in the cluster and the centroid of each cluster is called distortions. For a cluster, the lower its distortion degree is, the closer the members in the cluster are, and vice versa. The degree of distortion will decrease with the increase of categories (i.e. the number of clusters K). But for the data with a certain degree of differentiation, the degree of distortion will be greatly improved when K reaches a certain critical point, and then slowly decreases. This critical point can be considered as the point with good clustering performance.

There are also different ways for the determination of initial centroid. As we all know, the selection of initial centroids will not only affect the process of clustering iteration, but also influence the quality of clustering results. The core of the selection of initial centroids is to maximize the distance between them for the purpose to meet the requirements of strong intra-class homogeneity and strong inter-class heterogeneity of clustering results. Algorithm 1 shows a different rule to choose initial centroids.

Algorithm 1.. *The initial centroids selection process by K-means algorithm.*

Input: A set of sample points in TCA plot and the number of clusters$K$.

Output: Initial K centroids.

Step 1: For$t, s = 1, 2, \cdots, T$, calculate the average distance between each sample point$x_t = (TS_t, S_t(c_g))$ and the remaining sample points $x_s = (TS_s, S_s(c_g))(s \neq t)$ as follows:

$$\begin{aligned} dist_t^{(1)} &= \frac{1}{T-1} \sum_{s=1, s\neq t}^{T} dist(x_t, x_s) \\ &= \frac{1}{T-1} \sum_{s=1, s\neq t}^{T} \sqrt{(TS_t - TS_s)^2 + (S_t(c_g) - S_s(c_g))^2} \end{aligned} \tag{14}$$

Select the sample point corresponding to the maximum average distance as the first initial centroid$\mu_1^{(1)} = \left(TS_{(1)}^1, S_{(1)}^1(c_g)\right)$. $\left(TS_{(1)}^1, S_{(1)}^1(c_g)\right)$ represents the first centroid in first iteration.

Step 2: For$t = 1, 2, \cdots, T; x_t \neq \mu_1^{(1)}$, calculate the distance between the remaining sample points $x_t = (TS_t, S_t(c_g))$ and the first initial centroid $\mu_1^{(1)}$ as follows:

$$dist_t^{(2)} = \sqrt{\left(TS_t - TS_{(1)}^1\right)^2 + \left(S_t(c_g) - S_{(1)}^1(c_g)\right)^2} \tag{15}$$

Choose the sample point with maximum distance as the second initial centroid$\mu_2^{(1)} = \left(TS_{(1)}^2, S_{(1)}^2(c_g)\right)$.$\left(TS_{(1)}^2, S_{(1)}^2(c_g)\right)$ represents the second centroid in first iteration.

Step 3: Set number of iterations V, for$t = 1, 2, \cdots, T; x_t \neq \mu_v^{(1)} (v = 1, 2, \cdots, V)$, calculate the average distance between the remaining sample points $x_t = (TS_t, S_t(c_g))$ and the existed initial centroids $\mu_v^{(1)} (v = 1, 2, \cdots, V)$ as below:

$$\begin{aligned} dist_t^{(V)} &= \frac{1}{V} \sum_{v=1}^{V} dist\left(x_t, \mu_v^{(1)}\right) \\ &= \frac{1}{V} \left(\sum_{v=1}^{V} \sqrt{\left(TS_t - TS_{(1)}^v\right)^2 + \left(S_t(c_g) - S_{(1)}^v(c_g)\right)^2} \right) \end{aligned} \tag{16}$$

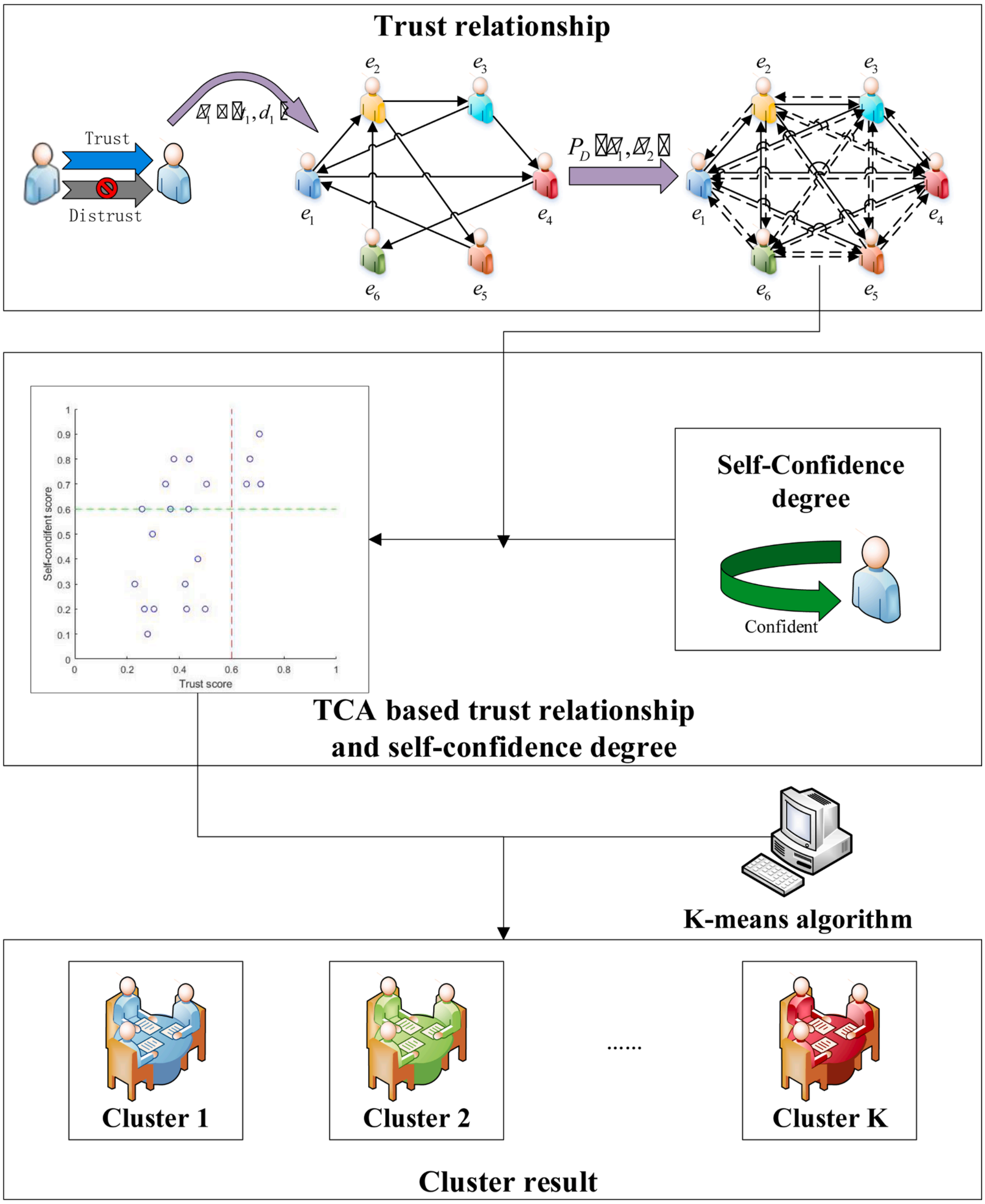


Fig. 4. TCA based clustering process.

Take the farthest sample point from existed initial centroids as the next initial centroid$\mu_V^{(1)} = \left(TS_{(1)}^V, S_{(1)}^V(C_g)\right)$. After that, $V = V + 1$.

Step 4: Repeat Step 3 until K initial centroids have been selected.

3.4. Experts clustering based on TCA by K-means algorithm

As aforementioned, clustering analysis can not only reduce the dimension and computational complexity of LS-MAGDA, but also improve the efficiency of assessment and reduce cost. Clustering experts with similar social trust-confidence relationship into a cluster seems a rational choice when considering the sociality of experts. Firstly, it can solve the conflict between group clustering analysis and CRP. If clustering is based on the assessments of experts, the subsequent consensus adjustment may probably alter the initial clusters. Meanwhile, changes of clusters may also influence the modification of assessments in CRP. This kind of clustering idea effectively avoids repeated iteration between the clustering process based on assessments and CRP. Moreover, experts with similar social characteristics and status in SNA may probably give compatible evaluation on the same decision making problem. Generally, trust relationship and self-confidence jointly reflect the authority of experts in a social network. In real world, experts with similar authority may have close understanding of a certain decision-making problem, which makes it possible for them to give high-consistent assessments. A relatively high consensus level may occur after clustering before CRP, which makes low cost for consensus adjustment process. Considering the advantages of K-means algorithm mentioned in Section 3.3, K-means algorithm based on TCA is suitable for group clustering analysis in this paper. The TCA based clustering process is depicted in Fig. 4.

Algorithm 2 gives a novel specific procedure for combining K-means algorithm with TCA.

Algorithm 2.. *The K-means clustering algorithm based on TCA for LS-MAGDA.*

Input: A set of trust functions between some pairs of experts in SNA,

Table 2
CDPRs between adjacent alternatives at the attribute level.

Attributes	$d(a_l(A_{12}))$	$d(a_l(A_{23}))$	$\cdots$	$d(a_l(A_{M-1,M}))$
a_1	$d^1(a_1(A_{12}))$	$d^1(a_1(A_{23}))$	$\cdots$	$d^1(a_1(A_{M-1,M}))$
	$d^2(a_1(A_{12}))$	$d^2(a_1(A_{23}))$		$d^2(a_1(A_{M-1,M}))$
	$\cdots$	$\cdots$		$\cdots$
	$d^K(a_1(A_{12}))$	$d^K(a_1(A_{23}))$		$d^K(a_1(A_{M-1,M}))$
a_2	$d^1(a_2(A_{12}))$	$d^1(a_2(A_{23}))$	$\cdots$	$d^1(a_2(A_{M-1,M}))$
	$d^2(a_2(A_{12}))$	$d^2(a_2(A_{23}))$		$d^2(a_2(A_{M-1,M}))$
	$\cdots$	$\cdots$		
	$d^K(a_2(A_{12}))$	$d^K(a_2(A_{23}))$		$d^K(a_2(A_{M-1,M}))$
$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$
a_L	$d^1(a_L(A_{12}))$	$d^1(a_L(A_{23}))$	$\cdots$	$d^1(a_L(A_{M-1,M}))$
	$d^2(a_L(A_{12}))$	$d^2(a_L(A_{23}))$		$d^2(a_L(A_{M-1,M}))$
	$\cdots$	$\cdots$		$\cdots$
	$d^K(a_L(A_{12}))$	$d^K(a_L(A_{23}))$		$d^K(a_L(A_{M-1,M}))$

self-confidence level derived from each expert.

Output: Different clusters of experts.

Step 1: For experts with direct connection, we construct the incomplete trust relationship matrix with initial trust function$\Lambda = \{\lambda_{rs} = (T_{rs}, D_{rs}) | T_{rs}, D_{rs} \in [0,1]\}$ $(r, s = 1, 2, \ldots, T; r \neq s)$.

Step 2: Use the dual trust propagation operator $P_D(\lambda_{rs}, \lambda_{st})$ to complete the trust relationship matrix as Eq.(1) shows.

Step 3: Generate the aggregated trust function $\widetilde{\lambda}_t = \left(\widetilde{t}_t, \widetilde{d}_t\right)$ by Eq.(9) to represent the general trust to expert e_t from other T-1 experts.

Step 4: Calculate the trust score TS_t of expert e_t by Eq.(12).

Step 5: Elicit the self-confidence degree c_g from each expert, and use score function $S(c_g)$ to convert linguistic variables to numerical scores.

Step 6: Construct the two-dimensional TCA plot with points$x_t = (TS_t, S_t(c_g))$.

Step 7: Set the number of clusters$K$, the set of initial clusters $CL = \{cl_1, cl_2, \cdots, cl_K\}$ and the initial K cluster centroids$\mu_1, \mu_2, \cdots, \mu_K$.

Step 8: Set the initial cluster $cl_k = \varnothing (k = 1, 2, \cdots, K)$ and initial centroids$\mu_k^{(1)} = \left(TS_{(1)}^k, S_{(1)}^k(c_g)\right)$, $\left(TS_{(1)}^k, S_{(1)}^k(c_g)\right)$ represent centroid of k-th cluster in first iteration.

Step 9: For$t = 1, 2, \cdots, T$, calculate the Euclidean distance for each sample point to each initial centroid as follows:

$$dist\left(x_t, \mu_k^{(1)}\right) = \sqrt{\left(TS_t - TS_{(1)}^k\right)^2 + \left(S_t(c_g) - S_{(1)}^k(c_g)\right)^2}\left(k = 1, 2, \cdots, K; x_t \neq \mu_k^{(1)}\right) \tag{17}$$

The sample points closest to a centroid $\mu_k^{(1)}$ are classified into corresponding cluster cl_k and update$cl_k = cl_k \cup \{x_t\}$.

Step 10: Recalculate the centroid $\mu_k^{(2)} = \left(TS_{(2)}^k, S_{(2)}^k(c_g)\right)$ for all the sample points in cl_k as follows:

$$\mu_k^{(2)} = \frac{1}{|cl_k|}\sum_{x_t \in cl_k} x_t = \left(\frac{1}{|cl_k|}\sum_{x_t \in cl_k} TS_t, \frac{1}{|cl_k|}\sum_{x_t \in cl_k} S_t(c_g)\right) \tag{18}$$

$|cl_k|$ represents number of experts in clustercl_k.

Step 11: For$t = 1, 2, \cdots, T$, recalculate the distance for sample points to each centroid in second iteration as follows:

$$dist\left(x_t, \mu_k^{(2)}\right) = \sqrt{\left(TS_t - TS_{(2)}^k\right)^2 + \left(S_t(c_g) - S_{(2)}^k(c_g)\right)^2}(k = 1, 2, \cdots, K) \tag{19}$$

The sample points closest to a centroid $\mu_k^{(2)}$ are classified into corresponding cluster cl_k and update$cl_k = cl_k \cup \{x_t\}$.

Step 12: Repeat operations in Steps 10 and 11 until all K clusters are unchanging or the number of iterations reaches a predetermined maximum.

4. CRP for LS-MAGDA based on CDPRs

Most of the research about CRP pay attention on GC at the alternative level (Tang et al., 2021; Tian et al., 2019; Triantaphyllou et al., 2020). However, in order to be applied to different LS-MAGDA scenarios and meet various requirements, the measure of GC at three levels based on the aforementioned clustering analysis is necessary. Here, the corresponding identification and direction rules at different levels are discussed.

4.1. The structure of LS-MAGDA based on collective DPR (CDPR)

Suppose there are M alternatives $A = \{A_1, A_2, \cdots, A_M\}$ with L attributes $a = \{a_1, a_2 \cdots, a_L\}$ which are evaluated by T experts $e = \{e_1, e_2, \cdots, e_T\}$ in a certain LS-MAGDA problem. The relative weights of L attributes are denoted by$\omega_a = \{\omega_{a_1}, \omega_{a_2}, \cdots, \omega_{a_L}\}$, which satisfies $\sum_{l=1}^{L}\omega_{a_l} = 1$ and$\omega_{a_l} \geqslant 0 (l = 1, 2, \cdots, L)$. The importance of T experts are signified as $\omega_e = \{\omega_{e_1}, \omega_{e_2}, \cdots, \omega_{e_T}\}$ where $0 \leqslant \omega_{e_t} \leqslant 1 (t = 1, 2, \cdots, T)$ and$\sum_{t=1}^{T}\omega_{e_t} = 1$. The T experts are classified into K clusters by the method discussed in Section 3 as$CL = \{cl_1, cl_2, \cdots, cl_K\}$. $|cl_k|$ $(k = 1, \ldots, K)$ denotes the cardinality of cl_k which means the number of experts included in the k-th cluster, and$\sum_{k=1}^{K}|cl_k| = T$.

In order to address the LS-MAGDA problem in complex circumstances more efficiently, collective DPR (CDPR) at the attribute level is proposed. It not only inherits the advantages of DPR, but also tackles with the decision-making problem in a more detailed manner.

Definition 9. ((*CDPRs at the attribute level*)) *Suppose T experts* $\{e_1, e_2, \cdots, e_T\}$ *are dealing with a common decision-making problem which involves M alternatives* $\{A_1, A_2, \cdots, A_M\}$ *on L attributes*$\{a_1, a_2, \cdots, a_L\}$. *They have been classified into K clusters by Algorithm 1 as*$CL = \{cl_1, cl_2, \cdots, cl_K\}$. *The pairwise comparisons between alternatives* A_i *and* $A_j (i, j = 1, 2, \cdots, M)$ *associated with attribute* $a_l (l = 1, 2, \cdots, L)$ *by cluster* $cl_k (k = 1, 2, \cdots, K)$ *is given as follows*:

$$d^k(a_l(A_{ij})) = \left\{\left(H_n, \beta_{n,l}^k(A_{ij})\right), n = 1, 2, \cdots, N; \left(\Omega, \beta_{\Omega,l}^k(A_{ij})\right)\right\} \quad (l = 1, 2, \cdots, L) \tag{20}$$

where $\beta_{n,l}^k(A_{ij})$ and $\beta_{\Omega,l}^k(A_{ij})$ represent the belief degree assigned to H_n and global ignorance in the comparison of A_i over A_j on attribute a_l given bycl_k.

In Def.9, $d^k(a_l(A_{ij}))$ is obtained by combining the assessments of $|cl_k|$ experts in clustercl_k. Specifically,$\beta_{n,l}^k(A_{ij}) = \frac{|H_{n,l}^k(A_{ij})|}{|cl_k|}$, $\beta_{\Omega,l}^k(A_{ij}) = \frac{|\Omega_l^k(A_{ij})|}{|cl_k|}$ in which $|H_{n,l}^k(A_{ij})|$ and $|\Omega_l^k(A_{ij})|$ denote the number of experts whose assessments in the comparison of A_i over A_j on a_l are H_n and global ignorance incl_k. Obviously,$\sum_{n=1}^{N}|H_{n,l}^k(A_{ij})| + |\Omega_l^k(A_{ij})| = |cl_k|$. Therefore, we can obtain CDPRs by comparing $M-1$ pairs of adjacent alternatives on L basic attributes by K clusters as shown in Table 2.

Definition 10. ((*The aggregated DPR*)) *Let* $d^k(a_l(A_{i,i+1}))$ *be the CDPR between adjacent alternatives* A_i *and* $A_{i+1} (i = 1, 2, \cdots, M-1)$ *on attribute* $a_l (l = 1, 2, \cdots, L)$ *by cluster* $cl_k (k = 1, 2, \cdots, K)$ *defined in Def.9. Then, the aggregated DPR on attribute* $a_l (l = 1, 2, \cdots, L)$ *and alternative pairs* A_i *and* A_{i+1} *is.*

Table 3
GC status at the alternative level.

Alternative	A_{12}	A_{23}	$\cdots$	$A_{M-1,M}$
$GC(A_{i,i+1})$	$GC(A_{12})$	$GC(A_{23})$	$\cdots$	$GC(A_{M-1,M})$

$d^c(a_l(A_{i,i+1})) = \{(H_n, \beta^c_{n,l}(A_{i,i+1})), n = 1,2,\cdots,N; (\Omega, \beta^c_{\Omega,l}(A_{i,i+1}))\}$ and $d^c(A_{i,i+1}) = \{(H_n, \beta^c_n(A_{i,i+1})), n = 1,2,\cdots,N; (\Omega, \beta^c_\Omega(A_{i,i+1}))\}$ respectively.

Here, the analytical ER algorithm (Wang et al., 2006) is applied as the aggregation function. It is applicable provided that each category of DMs makes judgment in a relatively independent manner. The aggregation function can be defined as:

$$d^c(a_l(A_{i,i+1})) = \gamma(d^1(a_l(A_{i,i+1})), d^2(a_l(A_{i,i+1})), \ldots, d^K(a_l(A_{i,i+1}))) \tag{21}$$

$$d^c(A_{i,i+1}) = \gamma(d^c(a_1(A_{i,i+1})), d^c(a_2(A_{i,i+1})), \ldots, d^c(a_L(A_{i,i+1}))) \tag{22}$$

where γ denotes the ER aggregation operator.

The score matrix $S = (S(A_{ij}))_{M\times M} = ([S(A_{ij})^-, S(A_{ij})^+])_{M\times M}$ of aggregated DPR at the alternative level can then be generated as$S(A_{i,i+1})^- = \sum_{n=1}^{N}\beta^c_n(A_{i,i+1})S(H_n) + \beta^c_\Omega(A_{i,i+1})S(H_1)$,$S(A_{i,i+1})^+ = \sum_{n=1}^{N}\beta^c_n(A_{i,i+1})S(H_n) + \beta^c_\Omega(A_{i,i+1})S(H_N)$. The values in the diagonal line of score matrix is$S(A_{ii})^- = S(A_{ii})^+ = 0$.

4.2. GC measure for CDPR at three levels

Due to the discrepancy of (Table 3) knowledge background, interest and work experience, there are often disagreements in the decision-making process among different categories of DMs, especially when the scale of group is large. Therefore, how to measure GC status has become a significant issue in LS-MAGDA. Generally, GC is measured either by using the direct judgments (Fu & Yang, 2011) or pairwise comparisons between alternatives (Wu & Xu, 2016, 2018; Xu et al., 2018) among DMs. This may bring about the problem that contradictory judgements exit on a minority of attributes although the general consensus has been reached on the global level. In order to deal with various requirements for different decision-making scenarios, such as manufacturing, logistics transportation and cost control (Chu et al., 2020; Lu et al., 2021), GC measure at different levels are particularly important, especially GC at the attribute level which has not been fully discussed in previous studies. Hence, we construct three levels of GC measure framework based on CDPR.

4.2.1. GC measure for CDPR at the attribute level

As mentioned above, GC at alternative or global level is not enough to deal with LS-MAGDA problem because multiple attributes also contain a great deal of information. Some important attributes not only have higher requirements for consensus degree, but also have a stronger impact on GC reaching process, thus affecting the whole decision making. Suppose an investment company wants to evaluate its future investment plan which includes five alternative manufacturers. The shareholders of investment company need to reach a high degree of consensus on the main business of these manufacturers. Accordingly, it may be unnecessary for them to reach an agreement on the non-dominant business.

Definition 11. (*(CC measure at the attribute level)*) *Let $d^k(a_l(A_{i,i+1}))$ be the CDPR between each pair of adjacent alternatives A_i and A_{i+1} $(i = 1,2,\cdots,M-1)$ on attribute $a_l (l = 1,2,\cdots,L)$ by cluster $cl_k (k = 1,2,\cdots,K)$ defined in Def.9. Then, the cluster consensus (CC) of cl_k on attribute a_l between the comparison of adjacent alternatives A_i and A_{i+1} is defined as the average similarity between $d^k(a_l(A_{i,i+1}))$ and CDPRs of other K-1 clusters $d^{k'}(a_l(A_{i,i+1}))(k' = 1,2,\cdots,K; k' \neq k)$ denoted by.*

$$CC^k(a_l(A_{i,i+1})) = \frac{1}{K-1}\sum_{k'=1,k'\neq k}^{K} SI^{k,k'}(a_l(A_{i,i+1})) \tag{23}$$

Here, $SI^{k,k'}(a_l(A_{i,i+1}))$ signifies the similarity measure between $d^k(a_l(A_{i,i+1}))$ and $d^{k'}(a_l(A_{i,i+1}))$ which is derived from$DI^{k,k'}(a_l(A_{i,i+1}))$. $DI^{k,k'}(a_l(A_{i,i+1}))$ denotes the numerical dissimilarity measure generated from$D^{k,k'}(a_l(A_{i,i+1}))$. And $D^{k,k'}(a_l(A_{i,i+1}))$ represents the distributed dissimilarity between $d^k(a_l(A_{i,i+1}))$ and $d^{k'}(a_l(A_{i,i+1}))$ (Fu et al., 2015).

$$D^{k,k'}(a_l(A_{i,i+1})) = \{(H_n, \beta^{k,k'}_{n,l}(A_{i,i+1})), n = 1,2,\cdots,N\} \tag{24}$$

$$\beta^{k,k'}_{n,l}(A_{i,i+1}) = \left|\beta^k_{n,l}(A_{i,i+1}) - \beta^{k'}_{n,l}(A_{i,i+1})\right|$$

$$DI^{k,k'}(a_l(A_{i,i+1})) = \frac{1}{2}\left(\sum_{n=1}^{N-1}\sum_{p=n+1}^{N}\beta^{k,k'}_{n,l}(A_{i,i+1})\cdot\beta^{k,k'}_{p,l}(A_{i,i+1})\cdot(s(H_p) - s(H_n))\right) \tag{25}$$

$$SI^{k,k'}(a_l(A_{i,i+1})) = 1 - DI^{k,k'}(a_l(A_{i,i+1})) \tag{26}$$

Definition 12. (*(GC measure at the attribute level)*) *Suppose the similarity measure between $d^k(a_l(A_{i,i+1}))$ and $d^{k'}(a_l(A_{i,i+1}))$ denoted as $SI^{k,k'}(a_l(A_{i,i+1}))$ is calculated by Eqs. (24)-(26), then the GC for CDPR at attribute a_l when comparing adjacent alternative A_i over A_{i+1} can be defined as follows*:

$$GC(a_l(A_{i,i+1})) = \sum_{k=1}^{K}\omega_{cl_k}\cdot CC^k(a_l(A_{i,i+1})) \tag{27}$$

where$\omega_{cl_k} = \sum_{t=1}^{|cl_k|}\omega_{e_{k_t}}$, and $\omega_{e_{k_t}}$ indicates the weight of the t-th $(t = 1,2,\ldots,|cl_k|)$ expert incl_k.$cl_k = \{e_{k_1}, e_{k_2}, \ldots, e_{k_{|cl_k|}}\}$ $(k = 1, 2, \ldots, K)$, and$\sum_{k=1}^{K}|cl_k| = T$. The cluster matrix of experts can then be represented below:

$$\begin{bmatrix} e_{1_1}\&e_{1_2}\&\ldots\&e_{1_{|cl_1|}} \\ e_{2_1}\&e_{2_2}\&\ldots\&e_{2_{|cl_2|}} \\ \ldots\&\ldots\&\ldots\&\ldots \\ e_{K_1}\&e_{K_2}\&\ldots\&e_{K_{|cl_K|}} \end{bmatrix} \begin{matrix} \omega_{cl_1} \\ \omega_{cl_2} \\ \ldots \\ \omega_{cl_K} \end{matrix}$$

As for the weights of clusters $\omega_{cl} = \{\omega_{cl_1}, \omega_{cl_2}, \cdots, \omega_{cl_K}\}$ in Eq.(27) and the aggregation function γ in Eq.(21), both trust scores and reliabilities of experts can be considered to generate the values. Trust score measures the degree of being trusted by others in social network, representing a kind of individual prestige in society. Experts are social beings, which is why their decision-making behaviors may be more or less influenced by others. Moreover, trust score of an expert is also the reflection of social status and authority. Hence, it is natural to use trust score as a component to generate expert's weight when solving LS-MAGDA problems. The other important factor to generate expert's weight is the reliability of expert, which reflects the ability to make correct judgment. It also characterizes the quality of information acquired by expert to some extent. If trust score is more of a subjective measure of how high others have faith in an expert, reliability may represent more of an objective factor, which can be measured by some objective existence or occurrence of affairs, such as the knowledge structure, age, position grade, research experience, etc. Therefore, reliability and trust score should be combined in a rational way for the computing of experts' weights.

Firstly, let $TS = \{TS_1, TS_2, \cdots, TS_T\}$ be a set of experts' trust scores generated by Eq.(12). The importance of trust score $\omega^t_{TS}(t = 1,2,\ldots,T)$ can be obtained by OWA based procedure with Basic Unit-interval

Monotone (BUM) membership function Q which can be defined as (Fu et al., 2016; Yager & Filev, 1999):

$$\omega_{TS}^{\sigma(t)} = Q\left(\frac{R(\sigma(t))}{R(\sigma(T))}\right) - Q\left(\frac{R(\sigma(t-1))}{R(\sigma(T))}\right) \quad (28)$$

Here, σ is permutation function, $R(\sigma(t))$ represents the sum of the top t trust scores $\sum_{h=1}^{t} TS_{\sigma(h)}$, and $TS_{\sigma(h)}$ signifies the h - th largest value in the set of experts' trust scores $\{TS_1, TS_2, \cdots, TS_T\}$. Additionally, $Q(U)$ can be set as $U^{2/3}$.

Example 1. *Five experts $e = \{e_1, e_2, e_3, e_4, e_5\}$ are involved in a social network. Their trust scores $TS = \{TS_1, TS_2, TS_3, TS_4, TS_5\}$ are shown as follows*:

$TS_1 = 0.51; TS_2 = 0.49; TS_3 = 0.58; TS_4 = 0.77;\ TS_5 = 0.39$

So we can obtain that:

$TS_{\sigma(1)} = 0.77; TS_{\sigma(2)} = 0.58; TS_{\sigma(3)} = 0.51; TS_{\sigma(4)} = 0.49;\ TS_{\sigma(5)} = 0.39$

$\omega_{TS}^{\sigma(1)} = \omega_{TS}^{4}; \omega_{TS}^{\sigma(2)} = \omega_{TS}^{3}; \omega_{TS}^{\sigma(3)} = \omega_{TS}^{1}; \omega_{TS}^{\sigma(4)} = \omega_{TS}^{2};\ \omega_{TS}^{\sigma(5)} = \omega_{TS}^{5}$

By using Eq.(28), the permuted importance weight can be determined as:

$\omega_{TS}^{\sigma(1)} = 0.429; \omega_{TS}^{\sigma(2)} = 0.195; \omega_{TS}^{\sigma(3)} = 0.149; \omega_{TS}^{\sigma(4)} = 0.130;\ \omega_{TS}^{\sigma(5)} = 0.097$

Then, the importance weight of each expert is:

$\omega_{TS}^{1} = 0.149; \omega_{TS}^{2} = 0.130; \omega_{TS}^{3} = 0.195; \omega_{TS}^{4} = 0.429;\ \omega_{TS}^{5} = 0.097$

Some studies have been done on the reliability of evidence. Smarandache et al. (2010) first distinguished the reliability and weight of evidence in the combination of different sources of evidence. Yang & Xu (2013) further provided a probabilistic reasoning approach to aggregate evidences with different reliabilities and weights. Liu et al. (2017) proposed a method to quantify experts' reliabilities under a certain decision-making scenario, i.e. project review, where the reliability of an expert can be generated by the true positive and true negative rate. To put it more broadly, Zhou et al. (2018) pointed out that the reliability of expert can be generated from historical or statistical data, along with the concept of reliability transfer be proposed. Considering the situation of data missing, the reliability of expert can also be determined by the familiarity to current problem compared with his/her past experience.

Finally, we can generate the comprehensive weight of each expert by combining the importance of trust score and reliability (Yang & Xu, 2013). For $t = 1, 2, \cdots, T$, let ω_{TS}^{t} and Re^{t} be the importance of trust score and reliability of expert e_t respectively. The comprehensive weight of e_t can be generated as follows:

$$\omega_{e_t} = \frac{\omega_{TS}^{t}}{1 + \omega_{TS}^{t} - Re^{t}} (t = 1, 2, \cdots, T) \quad (29)$$

Although we take reliability and importance weight based on trust scores as two components to measure the comprehensive weight of expert, it seems inappropriate to analyze them separately as independent concepts. Reliability has influence to trust relationship in a manner which affects final importance weight under the background of social network. Meanwhile, trust relationship also affects reliability in turn. They influence and complement with each other. Nevertheless, it may be more rational to distinguish them when tackling with realistic problems.

4.2.2. GC measure for CDPR at the alternative and global level

After constructing the framework of GC measure at the attribute level, GC measure at the alternative level should be implemented. GC measure at the attribute level can not only meet the requirement of consensus analysis at different levels under certain circumstances, but also lay the foundation for the construction of GC at the alternative level. It is natural to design the aggregation process of multiple attributes on alternatives given by different experts. Generally, GC measure at the alternative level can be carried out from two paths.

The first one is to aggregate experts' judgements on different attributes to generate the combined assessments of alternative by each expert through the ER rule. Then, GC at the alternative level can be measured by the method similar to Eqs.(23)-(27) where a_l is deleted. However, there are some problems with this process. In actual decision-making scenarios, GC identification should be oriented to attribute assessment by an expert or a cluster of experts on an alternative (i.e. $d^k(a_l(A_{i,i+1}))$), so as to facilitate the subsequent consensus adjustment process. Therefore, under the condition that different attributes are firstly aggregated to form a comprehensive assessment of alternatives by experts, if GC is not satisfied, only the comprehensive assessment that affects GC can be identified. Hence, assessments given by an expert or a cluster of experts are unable to be accurately identified on specific attribute.

The other approach is to directly aggregate consensus at attribute level to infer consensus at alternative level. Generally, suppose GC status for CDPR at different attributes are generated by Def.12, GC at alternative level can then be computed by using weighted average operator (WAO) to fuse GC status at attributes associated with the specific pair of alternatives (Fu & Yang, 2011). This is simpler and more efficient due to the following reason. Since GC at alternative level is computed based directly on GC at attribute level, when the GC at alternative level does not meet consensus requirement, we can quickly locate the attributes that affect the CRP. Consequently, it will be quick to identify expert assessment within the attribute that influences GC without redundant calculation process. Thus, this approach is chosen to measure GC status at alternative level.

Definition 13. (*(GC measure at the alternative level)*) *Let $GC(a_l(A_{i,i+1}))$ be GC for CDPR at attribute a_l when comparing adjacent alternative A_i and A_{i+1} given in Def.12. For $i = 1, 2, \cdots, M-1$, GC at alternative level is calculated as follows*:

$$GC(A_{i,i+1}) = \sum_{l=1}^{L} \omega_{a_l} GC(a_l(A_{i,i+1})) \quad (30)$$

where $GC(A_{i,i+1})$ denotes GC status in the comparison of A_i over A_{i+1}, $\omega_a = \{\omega_{a_1}, \omega_{a_2}, \cdots, \omega_{a_L}\}$ represents the vector of attribute weights, and $\sum_{l=1}^{L} \omega_{a_l} = 1$.

Definition 14. (*(GC measure at the global level)*) *Suppose GC level in the comparison of alternative A_i over A_{i+1} is $GC(A_{i,i+1})$, then GC measure at the global level is shown below*:

$$GC(gl) = \frac{1}{M-1} \sum_{i=1}^{M-1} GC(A_{i,i+1}) \quad (31)$$

4.3. The consensus feedback mechanism

In a LS-MAGDA problem, if the group fails to reach the consensus requirement at a particular level, a subsequent discussion and negotiation are required to obtain the opinion with a higher consensus degree. This process is generally carried out via a feedback mechanism. For the sake of rationality and efficiency of CRP, an adaptive feedback mechanism based on consensus measure is constructed, which includes two steps, identification and direction rules.

4.3.1. Identification rules

After measuring GC status at different levels, it is necessary to identify the assessments that break GC to the most extent. Corresponding to the three levels of GC measure, identification rules are also carried out at three levels. When constructing identification rules, the following aspects should be considered.

(1) Identification accuracy. In the GC measure at attribute level, it is reasonable to identify the cluster of experts who impedes CRP if requirement is not met. In practice, when consensus at alternative level does not satisfy the requirement, identification should be accurately located to assessments on certain attributes given by cluster. Hence, it is practical and feasible for a cluster of experts to modify the assessments of certain attributes, rather than to adjust judgments on alternative. As aforementioned, consensus at alternative level is directly aggregated by consensus at attribute level in order to provide convenience for identification rules. In this way, not only GC can be guaranteed, but also the original opinions of clusters can be preserved to the largest extent. Similarly, identification at global level ought to be accurately traced to the assessments of alternatives given by clusters on attributes. To sum up, the accuracy of identification rules is required to trace the non-consensus assessments of alternatives on attributes given by clusters, which has nothing to do with the level of consensus.

(2) Important attributes. In actual LS-MAGDA problems, different attributes play different roles. Important attributes which affect GC status at alternative and global levels to a great extent are the key objects that DMs should pay attention to. There are higher consensus requirements for important attributes, and the corresponding identification rules are supposed to be stricter than ordinary attributes. Therefore, attributes can be sorted according to their corresponding weights, and attributes that are ranked at the top of a specific proportion P_{imp} are regarded as important attributes. Considering different attributes in different decision scenarios, this proportion can vary depending on actual LS-MAGDA problems.

(3) Consensus threshold. The most straightforward way to determine whether consensus meets group requirement is to compare it with pre-set consensus threshold. Consensus degree directly affects the way and intensity of the subsequent identification and adjustment procedure. Therefore, setting multiple consensus thresholds and adopting different identification and direction rules in different threshold intervals can greatly improve efficiency, especially in the context of LS-MAGDA. There has been some research on the setting of consensus thresholds. Mata et al. (2009) divided consensus thresholds into four states: very low, low, medium and high enough. Rodríguez et al. (2018) constructed an adaptive consensus model which divided consensus degree into three levels through two consensus thresholds. Nevertheless, an excessively high level of consensus requirement, which is impractical when the scale of experts is large (Tang et al., 2019), may be a waste of time and can undermine the initial judgments of experts.

Here, two consensus thresholds ϑ_1 and ϑ_2 ($\vartheta_1 < \vartheta_2$) are chosen to divide consensus degree into three levels. Specifically, if $GC(a_l(A_{i,i+1}))\langle\vartheta_1$, it is said to be in a '*Low*' consensus degree at attribute level on a_l when comparing A_i and A_{i+1}; when $\vartheta_1 < GC(a_l(A_{i,i+1}))\langle\vartheta_2$, the consensus degree at attribute a_l is '*Slightly Low*'; $GC(a_l(A_{i,i+1}))\rangle\vartheta_2$ means the consensus degree is relatively '*High*' at a_l. Similarly, consensus degrees at alternative and global levels can be deduced by representing $GC(a_l(A_{i,i+1}))$ with $GC(A_{i,i+1})$ and $GC(gl)$.

(4) Opinion leader (OL). The concept of opinion leader is discussed in Section 3.2 that is an expert or a cluster of experts in the first quadrant of TCA analysis. Because of their knowledge, experience and social status, they usually have a comprehensive and deep understanding of problem, which can be regarded as the high reliability of assessment. It will provide other experts with benchmark and direction for adjustment. As such, it is reasonable to maintain the judgments of OLs as far as possible. In this paper, an expert or a cluster of experts who are located in the first quadrant and farthest from the origin are selected as OLs.

Corresponding to different levels of requirement, identification rules are built at three levels separately.

Definition 15. (*(Identification rules at the attribute level)*) *Let $GC(a_l(A_{i,i+1}))$ be consensus measure at attribute a_l when comparing adjacent alternatives A_i and A_{i+1}. The set of assessments to be adjusted at attribute level can be identified as:*

$$
\begin{aligned}
ATS_L = &\left\{d^k(a_l(A_{i,i+1}))(k=1,2,\cdots,K;k\neq k_{OL})\middle|GC(a_l(A_{i,i+1}))\leqslant\vartheta_1,\ \sigma_{\omega_a}(l)\leqslant P_{imp}\cdot L\right\}\\
&\cup\left\{d^k(a_l(A_{i,i+1}))(k\neq k_{OL})\middle|GC(a_l(A_{i,i+1}))\leqslant\vartheta_1,\ CC^k(a_l(A_{i,i+1}))\langle\right.\\
&\left.\frac{1}{K-1}\sum_{k=1,k\neq k_{OL}}^{K}CC^k(a_l(A_{i,i+1})),\sigma_{\omega_a}(l)\rangle P_{imp}\cdot L\right\}
\end{aligned}
\tag{32}
$$

$$
\begin{aligned}
ATS_{SL} = &\left\{d^k(a_l(A_{i,i+1}))(k\neq k_{OL})\middle|\vartheta_1 < GC(a_l(A_{i,i+1}))\leqslant\vartheta_2,\ CC^t(a_l(A_{i,i+1}))\right.\\
&\left.<\frac{1}{K-1}\sum_{k=1,k\neq k_{OL}}^{K}CC^k(a_l(A_{i,i+1})),\sigma_{\omega_a}(l)\leqslant P_{imp}\cdot L\right\}\cup\left\{d^k(a_l(A_{i,i+1}))\middle|\right.\\
&\vartheta_1 < GC(a_l(A_{i,i+1}))\leqslant\vartheta_2, CC^k(a_l(A_{i,i+1})) =\\
&\left.min\left\{IC^k(a_l(A_{i,i+1}))(k=1,2,\cdots,K;k\neq k_{OL})\right\},\sigma_{\omega_a}(l)\rangle P_{imp}\cdot L\right\}
\end{aligned}
\tag{33}
$$

where ATS_L and ATS_{SL} denote the set of assessments that are supposed to be modified at '*Low*' and '*Slightly Low*' degree of attribute consensus. $\sigma_{\omega_a}(l)$ represents a permutation function of attributes based on the weights of attributes. k_{OL} indicates the cluster of OLs in the group.

(1) When GC at a_l is '*Low*' in the comparison of A_i and A_{i+1} such that $GC(a_l(A_{i,i+1}))\leqslant\vartheta_1$, there are two situations to be considered. First, if an attribute is important such that $\sigma_{\omega_a}(l)\leqslant P_{imp}\cdot L$, the assessments on a_l for $A_{i,i+1}$ given by clusters except the cluster of OLs need to be updated. Second, if the attribute is ordinary such that $\sigma_{\omega_a}(l)\rangle P_{imp}\cdot L$, the mean value of the CC at a_l of $K-1$ clusters except the cluster of OLs denoted by $\frac{1}{K-1}\sum_{k=1,k\neq k_{OL}}^{K}CC^k(a_l(A_{i,i+1}))$ is calculated. Apart from the cluster of OLs, experts' assessments that are less than the above-mentioned mean value are recommended to be updated.

(2) When GC is '*Slightly Low*' but not high enough, a smaller range of assessments are supposed to be identified. First, for important attributes, the identification rule is equal to the above second rule. Second, for ordinary attributes, the assessment of the cluster whose CC at a_l for $A_{i,i+1}$ except the cluster of OLs is smallest needs to be modified. That is to say, the cluster other than cluster of OLs

Table 4
Identification rules for GC at the attribute level.

Consensus level	Importance of attribute	Identification rules
$GC(a_l(A_{i,i+1}))\langle\vartheta_1$	Important	Assessments given by all clusters except the cluster of OLs **(R 1)**
	Ordinary	Clusters whose CCs are less than the mean of the CCs of all clusters except the cluster of OLs **(R 2)**
$\vartheta_1 < GC(a_l(A_{i,i+1}))\langle\vartheta_2$	Important	**R 2**
	Ordinary	Cluster with minimum CC except the cluster of OLs **(R 3)**

Table 5
Identification rules for GC at the alternative level.

Consensus level	Importance of attribute	Identification rules
$GC(A_{i,i+1})\langle\vartheta_1$	Important	1. Identify the attributes whose GCs are less than the average of all attributes' GCs on $A_{i,i+1}$ **(Iden 1)** **R 1** on these identified attributes
	Ordinary	1. **Iden 1** **R 2** on these identified attributes
$\vartheta_1 < GC(A_{i,i+1})\langle\vartheta_2$	Important	1. **Iden 1** **R 2** on these identified attributes
	Ordinary	1. **Iden 1** **R 3** on these identified attributes

who gives the farthest opinion from other clusters needs to update the opinion.

Definition 16. (*(Identification rules at the alternative level)*) *Let* $GC(a_l(A_{i,i+1}))$ *and* $GC(A_{i,i+1})$ *be the GC measure at attribute and alternative level respectively. The set of assessments to be adjusted at alternative level can be identified as*:

$$ALS_L=\{d^k(a_l(A_{i,i+1}))(k=1,2,\cdots,K;k\neq k_{OL})|GC(A_{i,i+1})\leqslant\vartheta_1,GC(a_l(A_{i,i+1}))\leqslant \frac{1}{L}\sum_{l=1}^{L}GC(a_l(A_{i,i+1})),\sigma_{\omega_a}(l)\leqslant P_{imp}\cdot L\Bigg\}\cup\{d^k(a_l(A_{i,i+1}))(k\neq k_{OL})| GC(A_{i,i+1})\leqslant\vartheta_1,GC(a_l(A_{i,i+1}))\leqslant\frac{1}{L}\sum_{l=1}^{L}GC(a_l(A_{i,i+1})), CC^k(a_l(A_{i,i+1}))\langle\frac{1}{K-1}\sum_{k=1,k\neq k_{OL}}^{K}CC^k(a_l(A_{i,i+1})),\sigma_{\omega_a}(l)\rangle P_{imp}\cdot L\} \tag{34}$$

Table 6
Identification rules for GC at the global level.

Consensus level	Importance of attribute	Identification rules
$GC(gl)\langle\vartheta_1$	Important	1. Identify the alternatives and attributes whose GCs are less than the average of all alternatives and attributes' GCs **(Iden 2)** **R 1** on these identified alternatives and attributes
	Ordinary	1. **Iden 2** **R 2** on these identified alternatives and attributes
$\vartheta_1 < GC(gl)\langle\vartheta_2$	Important	1. **Iden 2** **R 2** on these identified alternatives and attributes
	Ordinary	1. **Iden 2** **R 3** on these identified alternatives and attributes

$$ALS_{SL}=\{d^k(a_l(A_{i,i+1}))(k\neq k_{OL})|\vartheta_1\leqslant GC(A_{i,i+1})\leqslant\vartheta_2,GC(a_l(A_{i,i+1}))\leqslant \frac{1}{L}\sum_{l=1}^{L}GC(a_l(A_{i,i+1})),CC^k(a_l(A_{i,i+1}))\langle \frac{1}{K-1}\sum_{k=1,k\neq k_{OL}}^{K}CC^k(a_l(A_{i,i+1})),\sigma_{\omega_a}(l)\leqslant P_{imp}\cdot L\Bigg\}\cup\{d^k(a_l(A_{i,i+1}))| \vartheta_1\leqslant GC(A_{i,i+1})\leqslant\vartheta_2,GC(a_l(A_{i,i+1}))\leqslant\frac{1}{L}\sum_{l=1}^{L}GC(a_l(A_{i,i+1})), CC^k(a_l(A_{i,i+1}))=min\{CC^k(a_l(A_{i,i+1}))(k=1,2,\cdots,K;k\neq k_{OL})\},\sigma_{\omega_a}(l)\rangle P_{imp}\cdot L\} \tag{35}$$

where ALS_L and ALS_{SL} denote the set of assessments that are supposed to be renewed at '*Low*' and '*Slightly low*' degree of alternative consensus.

When GC at alternative level (i.e. $A_{i,i+1}$) does not satisfy the requirement, it is necessary to identify which attributes break the GC. The attributes whose consensus is less than the average value of all attributes' GC associated with $A_{i,i+1}$ (i.e. $GC(a_l(A_{i,i+1}))\leqslant\frac{1}{L}\sum_{l=1}^{L}GC(a_l(A_{i,i+1}))$) are identified first. Then, we should determine which clusters should be notified to change their opinions except the cluster of OLs on the identified attributes. The similar process of different identification rules as Def. 15 can be applied according to the different levels of GC on alternatives.

Definition 17. (*(Identification rules at the global level)*) *Let*$GC(a_l(A_{i,i+1}))$, $GC(A_{i,i+1})$ *and* $GC(gl)$ *be the GC measure at attribute, alternative and global level respectively. The set of assessments to be adjusted at global level can be identified as:*

$$GLS_L=\{d^k(a_l(A_{i,i+1}))(k=1,2,\cdots,K;k\neq k_{OL})|GC(gl)\leqslant\vartheta_1,GC(A_{i,i+1})\leqslant \frac{1}{M-1}\sum_{i=1}^{M-1}GC(A_{i,i+1}),GC(a_l(A_{i,i+1}))\leqslant\frac{1}{L}\sum_{l=1}^{L}GC(a_l(A_{i,i+1})),\sigma_{\omega_a}(l)\leqslant P_{imp}\cdot L\Bigg\} \cup\{d^k(a_l(A_{i,i+1}))(k\neq k_{OL})|GC(gl)\leqslant\vartheta_1,GC(A_{i,i+1})\leqslant\frac{1}{M-1}\sum_{i=1}^{M-1}GC(A_{i,i+1}), GC(a_l(A_{i,i+1}))\leqslant\frac{1}{L}\sum_{l=1}^{L}GC(a_l(A_{i,i+1})),CC^k(a_l(A_{i,i+1}))\langle \frac{1}{K-1}\sum_{k=1,k\neq k_{OL}}^{K}CC^k(a_l(A_{i,i+1})),\sigma_{\omega_a}(l)\rangle P_{imp}\cdot L\} \tag{36}$$

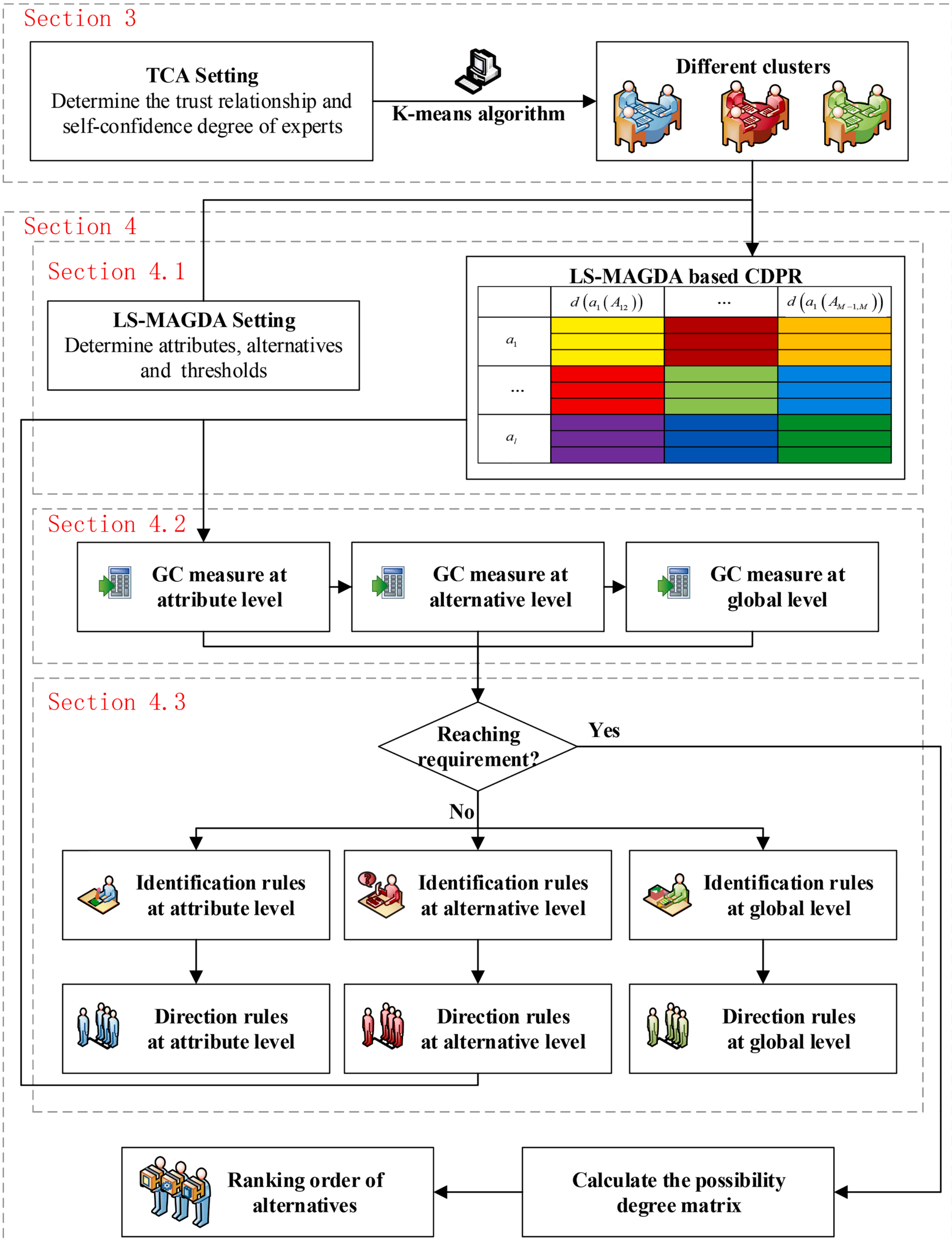


Fig. 5. The procedure for generating a solution in LS-MAGDA.

$$GLS_{SL} = \left\{ d^k\left(a_l\left(A_{i,i+1}\right)\right)(k \neq k_{OL}) \middle| \vartheta_1 \leqslant GC(gl) \leqslant \vartheta_2, GC\left(A_{i,i+1}\right) \leqslant \frac{1}{M-1}\sum_{i=1}^{M-1} GC\left(A_{i,i+1}\right), GC\left(a_l\left(A_{i,i+1}\right)\right) \leqslant \frac{1}{L}\sum_{l=1}^{L} GC\left(a_l\left(A_{i,i+1}\right)\right), CC^k\left(a_l\left(A_{i,i+1}\right)\right) \langle \frac{1}{K-1}\sum_{k=1,k\neq k_{OL}}^{K} CC^k\left(a_l\left(A_{i,i+1}\right)\right), \sigma_{\omega_a}(l) \leqslant P_{imp}\cdot L \right\} \cup \left\{ d^k\left(a_l\left(A_{i,i+1}\right)\right) \mid \vartheta_2 \leqslant GC(gl) \leqslant \vartheta_3, GC\left(A_{i,i+1}\right) \leqslant \frac{1}{M-1}\sum_{i=1}^{M-1} GC\left(A_{i,i+1}\right), GC\left(a_l\left(A_{i,i+1}\right)\right) \leqslant \frac{1}{L}\sum_{l=1}^{L} GC\left(a_l\left(A_{i,i+1}\right)\right), CC^k\left(a_l\left(A_{i,i+1}\right)\right) = \min\left\{CC^k\left(a_l\left(A_{i,i+1}\right)\right)(k = 1, 2, \cdots, K; k \neq k_{OL})\right\}, \sigma_{\omega_a}(l) \rangle P_{imp}\cdot L \right\} \tag{37}$$

where GLS_L and GLS_{SL} denote the set of assessments that are supposed to be renewed at '*Low*' and '*Slightly low*' degree of global consensus.

When the GC at global level does not satisfy the requirement, the primary task is to identify which alternatives break the consensus. The alternatives whose GCs are less than the average of all adjacent pairs of alternatives' GCs (i.e $GC(A_{i,i+1}) \leqslant \frac{1}{M-1}\sum_{i=1}^{M-1} GC(A_{i,i+1})$) are regarded as the cause to destroy CRP. After the identification of alternatives, the attributes that influence CRP can be identified as similar to Def. 16. Finally, the identification of attributes can be processed.

Remark 1. *From Tables 4 to 6, it is obvious that the identification rule becomes stronger when the consensus degree is lower and attribute is more important. In this case, the number of experts whose judgments need to be modified becomes larger.*

4.3.2. Direction rules

After the identification of assessments that decrease GC status, the subsequent procedure is to adjust the assessments for the purpose to narrow the contradictory opinions. Considering that consensus adjustments at the alternative and global levels are also oriented to the attribute level, only direction rules at the attribute level are discussed.

According to Defs. 11 and 12, the consensus degree at attribute level is measured by the weighted sum of CCs of all clusters. The cluster of OLs has larger weight in decision-making process, which means the assessment given by them has more influence on CRP. Naturally, to increase the degree of consensus at attribute level, the improvement of the similarity between the assessments given by OLs and other experts are supposed to be important. In conclusion, every assessment which is identified to be updated is recommended to adjust towards the assessments given by OLs.

Definition 18. (*(Direction rules)*) *Let $d^k(a_l(A_{ij}))$ be the assessment that are recommended to be updated. $d^{k_{OL}}(a_l(A_{ij}))$ is the assessment given by the cluster of OLs that is supposed to be used as a target for$d^k(a_l(A_{ij}))$. Then, cluster cl_k is supposed to increase or decrease the score values of their assessments as follows*:

$$\sum_{n=1}^{N}\beta_{n,l}^{k}(A_{ij})S(H_n) + \beta_{\Omega,l}^{k}(A_{ij})S\left(H_{(N+1)/2}\right) \langle \sum_{n=1}^{N}\beta_{n,l}^{k_{OL}}(A_{ij})S(H_n) + \beta_{\Omega,l}^{k_{OL}}(A_{ij})S\left(H_{(N+1)/2}\right) \tag{38}$$

$$\sum_{n=1}^{N}\beta_{n,l}^{k}(A_{ij})S(H_n) + \beta_{\Omega,l}^{k}(A_{ij})S\left(H_{(N+1)/2}\right) \rangle \sum_{n=1}^{N}\beta_{n,l}^{k_{OL}}(A_{ij})S(H_n) + \beta_{\Omega,l}^{k_{OL}}(A_{ij})S\left(H_{(N+1)/2}\right) \tag{39}$$

No matter what are the direction rules at attribute, alternative or global level, the assessments which are selected by identification rules that are recommended to be updated are all in the direction of the assessment given by OLs. It is obvious that the assessment with the largest dissimilarity must be adjusted towards the OLs' assessments at any level of consensus reaching. Considering the situation where more than one assessment needs to be adjusted, the adjustment process can be divided into multiple above-mentioned largest dissimilarity based assessment adjustment processes. It means the adjustment is a dynamic process that the assessment with the largest dissimilarity in the identified assessment set ought to be modified towards the cluster of OLs. Direction rules are guidelines that allow assessments to be modified to improve the level of consensus, which requires that there is a greater degree of consensus after every round of adjustment.

Theorem 1. *The modification of the assessment with the largest dissimilarity towards the cluster of OLs will enlarge GC status such that.*

$$GC^1\left(a_l\left(A_{i,i+1}\right)\right) \langle GC^2\left(a_l\left(A_{i,i+1}\right)\right) \tag{40}$$

where $GC^1(a_l(A_{i,i+1}))$ and $GC^2(a_l(A_{i,i+1}))$ represents the GC of initial assessments and one round adjusted assessments via direction rules respectively.

Proof of Theorem 1. See Supplementary Material.

4.4. The procedure for generating a solution to the LS-MAGDA based on experts' clustering and consensus feedback mechanism

In a LS-MAGDA problem, T experts are invited to evaluate a certain decision-making problem with M alternatives associated with L attributes. The whole procedure of clustering and GC reaching process is depicted in Fig. 5, which is illustrated in detail as follows:

Step 1: Prepare for the clustering of experts by K-means algorithm.

T experts are invited to evaluate a LS-MAGDA problem. The set of trust relationships Λ among different experts and self-confidence degree of each expert $S_t(c_g)$ $(t = 1, 2, ..., T)$ are provided.

Step 2: Propagate the trust relationships among experts.

Table 7
Trust score and self-confidence data of experts.

e	$\widetilde{\lambda}_t$	$TS\left(\widetilde{\lambda}_t\right)$	c_g	$S(c_g)$
1	(0.66,0.24)	0.71	*High*	0.7
2	(0.37,0.52)	0.42	*Low*	0.3
3	(0.59,0.28)	0.66	*High*	0.7
4	(0.22,0.68)	0.27	*Very Low*	0.2
5	(0.18,0.72)	0.23	*Low*	0.3
6	(0.39,0.52)	0.44	*Slightly High*	0.6
7	(0.37,0.49)	0.44	*Very High*	0.8
8	(0.20,0.65)	0.28	*None*	0.1
9	(0.64,0.23)	0.71	*Perfect*	0.9
10	(0.31,0.58)	0.37	*Slightly High*	0.6
11	(0.24,0.64)	0.30	*Medium*	0.5
12	(0.19,0.68)	0.26	*Slightly High*	0.6
13	(0.41,0.46)	0.47	*Slightly Low*	0.4
14	(0.61,0.27)	0.67	*Very High*	0.8
15	(0.32,0.56)	0.38	*Very High*	0.8
16	(0.39,0.53)	0.43	*Very Low*	0.2
17	(0.30,0.61)	0.35	*High*	0.7
18	(0.44,0.44)	0.50	*Very Low*	0.2

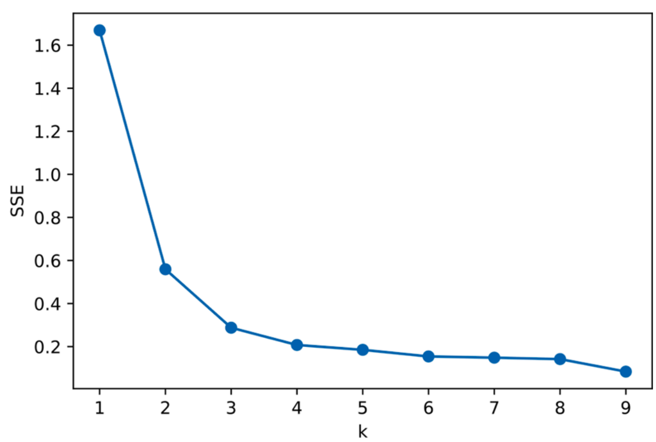


Fig. 6. The *SSE* of different number of clusters *K*.

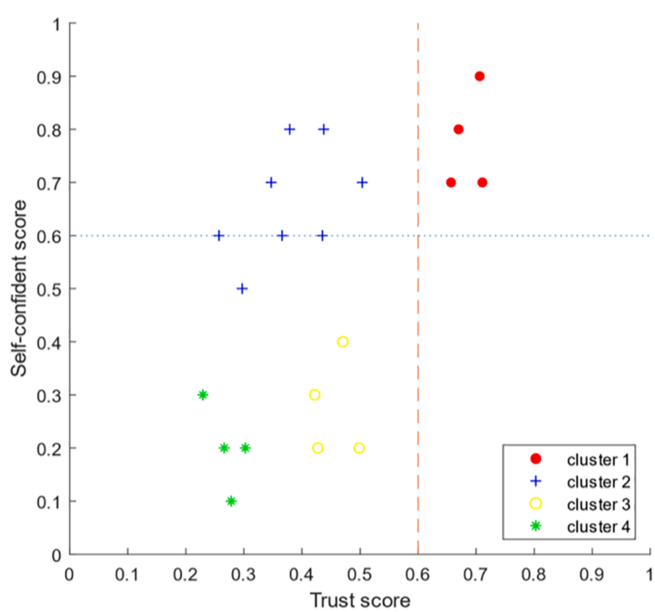


Fig. 7. Different clusters of experts by using *K*-means algorithm.

Table 8
The weight of each expert and cluster.

e	$TS\left(\widetilde{\lambda}_t\right)$	Reliability	Cluster	Weight ofexpert	Weight ofcluster
1	0.71	0.8	1	0.2009	0.56
3	0.66	0.8	1	0.1155	
9	0.71	0.7	1	0.147	
14	0.67	0.7	1	0.0939	
6	0.44	0.5	2	0.0311	0.22
7	0.44	0.6	2	0.0394	
10	0.37	0.5	2	0.0241	
15	0.38	0.5	2	0.0254	
17	0.35	0.5	2	0.0226	
20	0.50	0.6	2	0.0502	
11	0.30	0.4	2	0.0159	
12	0.26	0.4	2	0.0134	
2	0.42	0.6	3	0.0353	0.15
13	0.47	0.5	3	0.0355	
16	0.43	0.6	3	0.0365	
18	0.50	0.6	3	0.0475	
4	0.27	0.5	4	0.0167	0.07
5	0.23	0.4	4	0.0119	
8	0.28	0.5	4	0.0176	
19	0.30	0.5	4	0.0195	

Estimate the missing values in the trust relationships by dual trust propagation operator P_D through SNA.

Step 3: Classify experts by *K*-means algorithm and TCA structure. **(Proposed).**

Depict two-dimensional TCA plots reflecting the trust scores and self-confidence degrees of experts. These experts can be subsequently divided into different clusters by *K*-means algorithm. Meanwhile, the characteristics of different clusters such as OL can be identified by TCA.

Step 4: Generate the weights of experts and clusters.

The weights of different experts ω_{e_t} can be computed by the importance of trust relationship ω_{TS}^t and reliabilityRe^t. The weights of trust relationships are calculated by OWA based procedure with Basic Unit-interval Monotone (BUM) membership function Q by Eq.(28). The weights of different attributes ω_{a_l} are given and important attributes can also be identified.

Step 5: Construct CDPR between adjacent alternatives at the attribute level.

Extract the judgments of clusters to build CDPR denoted by $d^k(a_l(A_{ij}))\ \ (l=1,2,...,L;k=1,2,...,K;i,j=1,2,...,M)$ for $M-1$ pairs of alternatives associate with L different attributes and K clusters of experts. The group specifies $\Omega=\{H_1,H_2,\cdots,H_N\}$ and$s(H_n)(n=1,2,\cdots,N)$.

Step 6: Calculate a certain level of GC. **(Proposed).**

For attribute level, the average similarity between the assessments of cluster $c_k(k=1,2,...,K)$ and other K-1 clusters can be calculated as$\sum_{k=1,k\neq k'}^{K}\frac{1}{K-1}SI^{k,k'}(a_l(A_{i,i+1}))$. Then GC at attribute level $GC(a_l(A_{i,i+1}))$ can be defined as the weighted sum of the average similarity by Eq.(27). As for alternative level, $GC(A_{i,i+1})$ is computed as the weighted sum of the GC at each attribute by Eq.(30). With regard to global level, $GC(gl)$ can be calculated as the mean of the GC at each pair of adjacent alternatives by Eq.(31).

Step 7: Identification of cluster whose assessment breaks GC. **(Proposed).**

Table 9
GC at attribute and alternative level.

Attribute	$GC(a_l(A_{12}))$	$GC(a_l(A_{23}))$	$GC(a_l(A_{34}))$	$GC(a_l(A_{45}))$	$GC(a_l(A_{56}))$
a_1	0.755	0.69	0.704	0.613	0.697
a_2	0.621	0.681	0.74	0.783	0.668
a_3	0.763	0.724	0.689	0.742	0.771
a_4	0.615	0.744	0.699	0.83	0.642
a_5	0.713	0.832	0.741	0.866	0.616
a_6	0.707	0.808	0.737	0.730	0.597
a_7	0.556	0.717	0.614	0.879	0.605
a_8	0.631	0.815	0.61	0.777	0.746
a_9	0.568	0.695	0.763	0.826	0.791
a_{10}	0.618	0.764	0.711	0.953	0.617
$GC(A_{i,i+1})$	0.661	0.739	0.699	0.789	0.678

Table 10
GC at the alternative level in every iteration.

Iteration	$GC(a_l(A_{12}))$	$GC(a_l(A_{23}))$	$GC(a_l(A_{34}))$	$GC(a_l(A_{45}))$	$GC(a_l(A_{56}))$
1	0.661	0.739	0.699	0.789	0.678
2	0.740	0.786	0.746	0.825	0.762
3	0.772	0.850	0.812	0.825	0.813
4	0.814	0.850	0.812	0.825	0.813

Table 11
The aggregated interval score values on each attribute.

	$S(a_l(A_{12}))$	$S(a_l(A_{23}))$	$S(a_l(A_{34}))$	$S(a_l(A_{45}))$	$S(a_l(A_{56}))$
a_1	[−0.4323, −0.0649]	[−0.3205, −0.0225]	[0.3855, 0.5267]	[−0.2397, −0.1321]	[0.1878, 0.3998]
a_2	[−0.0680, 0.1182]	[0.2344, 0.4328]	[0.0665, 0.1755]	[0.3147, 0.4335]	[0.0953, 0.0953]
a_3	[−0.1402, 0.0936]	[−0.0998, 0.1962]	[0.2529, 0.5743]	[0.0480, 0.0592]	[−0.2197, 0.0668]
a_4	[−0.4977, −0.3557]	[−0.0552, 0.2444]	[−0.0624, 0.0460]	[−0.4794, −0.2054]	[0.0270, 0.0270]
a_5	[−0.2173, −0.1115]	[0.2878, 0.4730]	[−0.0206, 0.1974]	[−0.0710, 0.1310]	[0.1075, 0.5413]
a_6	[−0.4812, 0.0394]	[−0.0505, 0.2645]	[−0.0002, 0.3124]	[−0.1979, 0.0377]	[−0.4182, −0.167]
a_7	[0.4799, 0.4903]	[−0.3631, −0.1483]	[−0.0965, 0.0845]	[−0.3001, 0.2867]	[0.1202, 0.2454]
a_8	[−0.6063, −0.4181]	[−0.1134, 0.0232]	[−0.3547, −0.1581]	[0.0049, 0.1979]	[−0.1478, 0.0984]
a_9	[−0.3044, −0.2632]	[0.1940, 0.3898]	[−0.2254, −0.1864]	[−0.1980, 0.0820]	[0.0220, 0.1200]
a_{10}	[0.1757, 0.4785]	[−0.0305, 0.1713]	[0.0574, 0.2146]	v0.1514, 0.1454]	[−0.4604, −0.1394]

Table 12
The aggregated pairwise assessments at alternative level.

m	$d^c(A_{m,m+1})$ 1	2	3	4	5
	$\{(H_1,0.1231),$ $(H_2,0.1092),$ $(H_3,0.1838),$ $(H_4,0.2204),$ $(H_5,0.1421),$ $(H_6,0.0357),$ $(H_7,0.0901),$ $(\Omega,0.0956)\}$	$\{(H_1,0.0399),$ $(H_2,0.0986),$ $(H_3,0.1484),$ $(H_4,0.1985),$ $(H_5,0.2176),$ $(H_6,0.1250),$ $(H_7,0.0711),$ $(\Omega,0.1009)\}$	$\{(H_1,0.0303),$ $(H_2,0.0787),$ $(H_3,0.1487),$ $(H_4,0.1977),$ $(H_5,0.2246),$ $(H_6,0.1426),$ $(H_7,0.0988),$ $(\Omega,0.0786)\}$	$\{(H_1,0.1344),$ $(H_2,0.1153),$ $(H_3,0.1637),$ $(H_4,0.1350),$ $(H_5,0.1171),$ $(H_6,0.0933),$ $(H_7,0.1410),$ $(\Omega,0.1001)\}$	$\{(H_1,0.0703),$ $(H_2,0.0617),$ $(H_3,0.1290),$ $(H_4,0.2546),$ $(H_5,0.2302),$ $(H_6,0.1095),$ $(H_7,0.0595),$ $(\Omega,0.0853)\}$

DM decides whether the GC at certain level should be reached. If the requirement is satisfied, go to Step 8. Otherwise, the assessments which are recommended to be updated are identified via the identification rules by Defs. 15–17 and Tables 4–6.

Step 8: Adjust assessments via direction rules. **(Proposed)**.

After distinguishing the assessments which significantly decrease the consensus status, the clusters of experts are recommended to update their assessments through the direction rules by Def. 18 under different circumstances. Then, go to Step 9.

Step 9: Transform CDPR matrix into interval score value matrix and determine the parameter in transitive function.

The CDPR matrix can be transformed into interval score value matrix by using the value of grades $s(H_n)(n = 1, 2, \cdots, N)$ and Eqs.(4)-(5). An optimization model is constructed to determine the parameter b in function $g(y, z)$ which satisfies the transitivity of consistency.

Step 10: Aggregate the assessments of each attribute on different categories of experts for each pair of adjacent alternatives.

ER algorithm is applied to aggregate the K CDPRs by different clusters of experts on each attribute. Then, $L{\cdot}(M-1)$ aggregated CDPRs are further fused by the ER algorithm into $M-1$ comprehensive DPRs by Def. 10.

Step 11: Generate a solution to the LS-MAGDA.

The interval score values of $M-1$ comprehensive DPRs can be calculated by the comprehensive DPRs and$s(H_n)(n = 1, 2, \cdots, N)$. Then, the function which satisfies the transitivity of consistency with parameter b in Step 9 is used to complete the consistent score value matrix. Finally, the ranking order of alternatives can be generated by possibility degree (Li et al., 2018).

5. Illustrative example and comparative analysis

In this section, an example of vehicle model selection problem considering the product life cycle is investigated to discuss the LS-MAGDA based on DPR under social network analysis. Furthermore, comparative analysis is conducted to demonstrate the effectiveness and validity of the proposed method.

5.1. Description of the vehicle model selection problem

Life cycle of product is a comprehensive procedure of certain product or service, including the process of raw materials acquisition, product

Table 13
Complete interval score matrix of aggregated pairwise assessments at alternative level.

m	$S(A_{m,1})$	$S(A_{m,2})$	$S(A_{m,3})$	$S(A_{m,4})$	$S(A_{m,5})$	$S(A_{m,6})$
1	[0.0, 0.0]	[−0.1926, −0.0014]	[−0.0766, 0.1655]	[−0.1351, 0.6470]	[0.0899, 0.8524]	[−0.0102, 0.9669]
2	[0.0014, 0.1926]	[0.0, 0.0]	[−0.0305, 0.1713]	[−0.0141, 0.6551]	[−0.0931, 0.8568]	[−0.0521, 0.9680]
3	[−0.1655, 0.0766]	[−0.1713, 0.0305]	[0.0, 0.0]	[0.0574, 0.215]	[−0.2754, 0.4825]	[−0.0841, 0.8291]
4	[−0.6470, 0.1351]	[−0.65509, 0.0141]	[−0.2146, −0.0574]	[0.0, 0.0]	[−0.1229, 0.0773]	[−0.0579, 0.3680]
5	[−0.8524, −0.0899]	[−0.8568, 0.0931]	[−0.4825, 0.2754]	[−0.0773, 0.1229]	[0.0, 0.0]	[−0.0323, 0.1383]
6	[−0.9669, 0.0102]	[−0.9680, 0.0521]	[−0.8291, 0.0841]	[−0.3680, 0.0579]	[−0.1383, 0.0323]	[0.0, 0.0]

Table 14
Possibility degree matrix for the pairwise assessments of alternatives.

Vehicle Model	A_1	A_2	A_3	A_4	A_5	A_6
A_1	0.5	0.0035	0.7687	0.8956	0.9565	0.9947
A_2	0.9965	0.5	0.9111	0.9893	0.9457	0.9731
A_3	0.2313	0.0889	0.5	0.9129	0.7146	0.9493
A_4	0.1044	0.0107	0.0871	0.5	0.3146	0.9213
A_5	0.0435	0.0543	0.2854	0.6854	0.5	0.8833
A_6	0.0053	0.0269	0.0507	0.0787	0.1167	0.5

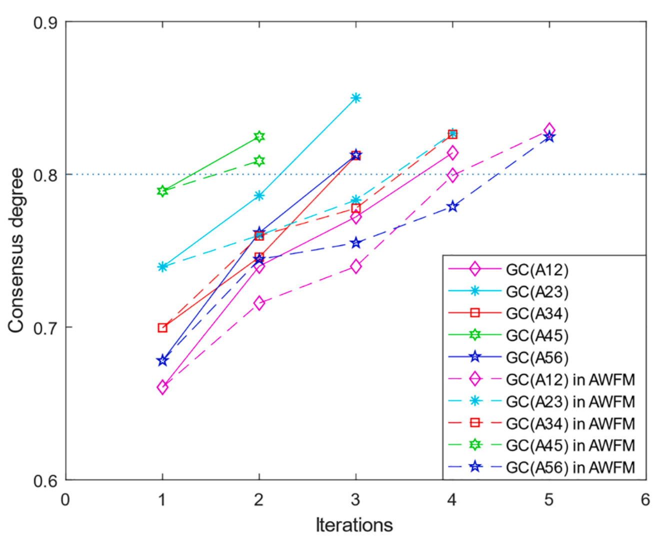


Fig. 8. Comparison of the consensus degree at alternative level in every iteration by the proposed method against AWFM.

manufacturing, distribution, usage, recycling and final disposal (Hellweg & Mila i Canals, 2014). Product life cycle cost (LCC) is an important component of product life cycle theory. In a narrow sense, LCC includes the costs created in the whole process from cradle to grave, i.e. market research and analysis, R&D, procurement, manufacturing and assembly, quality inspection, inventory, sales, transportation, operation and maintenance until the product is scrapped. Enterprises use product LCC theory to carry out cost management that can comprehensively analyze product cost structure and guide the implementation of cost control. It is just in line with the idea of strategic cost management and maintain long-term competitiveness. In this paper, we investigate the vehicle model selection problem which is based on LCC theory. Because this paper analyzes LCC management from the perspective of producer, costs in the scrapping phase and environmental costs at each stage are not taken into account.

Here, twenty employees from several departments of an automobile enterprise are invited to form a panel denoted by $e=\{e_1, e_2, \ldots, e_{20}\}$ to assess six vehicle models represented by $A=\{A_1, A_2, \ldots, A_6\}$. They are required to choose the best vehicle model or make a ranking order of these models. Ten attributes are extracted from LCC management phases denoted by $a=\{a_1, a_2, \ldots, a_{10}\}$, which signify product design, preliminary sample manufacturing, test and experiment, supplier selection, procurement strategy, production and assembly, marketing plan, promotion strategy, after-sales service, operation and maintenance, respectively. According to the different importance of each phase in LCC management, the weights of these attributes are determined by the panel of experts as w_a={0.15, 0.1, 0.12, 0.1, 0.08, 0.08, 0.11, 0.08, 0.09, 0.09}. Considering the characteristics of these attributes in this problem, the specific proportion P_{imp} is set as 0.3. A set of discrete evaluation grades denoted by $\Omega = \{H_1, H_2, \cdots, H_7\}$= {*Absolutely Worse*, *Worse*, *Slightly Worse*, *Indifferent*, *Slightly Better*, *Better*, *Absolutely Better*} are set to assess the alternative vehicles. Besides, a linguistic term set $C = \{c_1, c_2, \cdots, c_9\}$={*None*, *Very Low*, *Low*, *Slightly Low*, *Medium*, *Slightly High*, *High*, *Very High*, *Perfect*} is used as the level of self-confidence degree. In addition, the scores of evaluation grades and self-confidence linguistic terms are set as $s(H_n)(n = 1, 2, \cdots, 7)$ ={-1, −0.7, −0.3, 0, 0.3, 0.7, 1} and $S(c_g)$ ($g = 1, 2, \ldots, 9$)={0.1, 0.2, 0.3, 0.4, 0.5, 0.6, 0.7, 0.8, 0.9}. After discussion, the thresholds in TCA are set as $\delta_1 = \delta_2 = 0.6$.

Table 15
Comparison of the proposed method against several GDM methods.

	Process Zhang et al. 2016	Fu & Yang 2011	Method Zhang et al. 2018	Zhang et al. 2020b	Proposed
SNA	—	—	—	Trust-based social network	TCA based trust relationship and self-confidence
Expert clustering	—	—	Broad first search neighbors	Network partition algorithm	*K*-means algorithm based TCA
Preference representation	PLPR	BD	PO, UF, MPR, APR	IFPR	DPR
Consistency	√	√	—	—	√
Three levels of GC	—	√	—	—	√
Aggregation method	Linguistic terms subscript multiplying probabilities	ER algorithm	WAO	WAO	ER algorithm
Identification and direction rules	—	√	√	√	√

5.2. Adjustment of assessment based on feedback mechanism

5.2.1. Checking of GC at the alternative level

The corresponding initial incomplete trust relationships among experts $e=\{e_1, e_2, \ldots, e_{20}\}$ which are in the form of trust functions $\lambda_{rs} = (T_{rs}, D_{rs})$ ($r = s = 1,2,\ldots,20$; $r \neq s$) are shown in Table S.1 of Supplementary Material. The dual trust propagation operator in Def. 2 is applied to estimate the missing values in the initial trust relationship matrix as shown in Table S.2 of Supplementary Material. The trust score of each expert can be calculated by the aggregated trust function which is generated by Def. 5. The trust score and extracted self-confidence data of experts are shown in Table 7.

The 2-tuple plots $\left(TS\left(\widetilde{\lambda}_t\right), S(c_g)\right)$ are mapped into a two-dimensional coordinate system for analysis. After the selection process of centroid number and initial centroids as shown in Fig. 6, we determine the number of clusters as$K = 4$.

Then, K-means algorithm is used to classify these experts into four clusters as shown in Fig. 7. The result indicates that cluster 1 which contains four experts belongs to "authority region" because trust score and self-confidence degree of these four experts are larger than the preset thresholds $\delta_1 = \delta_2 = 0.6$. Therefore, we can regard cluster 1 as OLs in the decision-making problem. Steps 1 to 3 in Section 4.4 are completed (the step mentioned below refers to the step in Section 4.4).

From the above results, we can obtain the weights of different experts and clusters from trust scores and reliabilities by using Eqs.(28)-(29) which are shown in Table 8. Thus, Step 4 is completed.

Then, the initial pairwise comparisons from different clusters of experts on each attribute (i.e. CDPR) are collected and presented in Table S.3 of Supplementary Material. Step 5 is completed.

For the ranking of vehicle models, GC requirements are chosen at alternative level after considering the characteristics of this problem. GC at attribute and alternative levels can be calculated according to Eqs. (23)-(27), (30) which are shown in Table 9. Step 6 is completed. Here, two consensus thresholds $\theta_1 = 0.7$ and $\theta_2 = 0.8$ are set. As the results show, GC at the alternative level doesn't completely meet the consensus requirement. Next, feedback mechanism proposed in Section 4.3 is used to guide the adjustment of clusters' opinions.

5.2.2. Reaching of GC at the alternative level

When the feedback mechanism is started, manager recognizes the assessments which break the GC by the identification rules and recommends relevant experts in specific cluster to update their judgments through the direction rules. After these processes, GC at attribute and alternative levels are calculated with every iteration (the first iteration means initial condition) until the consensus requirement is satisfied. The assessments after every iteration are shown in Tables S.4-S.6 of Supplementary Material. Also, GC at the alternative level in every iteration is shown in Table 10. Consequently, GC at the alternative level has been reached. Steps 7 and 8 are completed.

5.3. Generation of solution to the vehicle model selection problem

After the GC adjustment process by using feedback mechanism, the assessments updating is finished. The assessments of four clusters on each attribute in Table S.6 of Supplementary Material are fused to generate the combined DPR on each attribute, which are then transformed into interval scores by using Eqs.(4)-(5). The results are shown in Table 11 where each line represents the interval scores of combined DPR on an attribute from the four clusters.

Then, another pairwise assessment needs to be given that can be determined by the result of Table 11. After $d(a_l(A_{35}))$ is provided in the last column of Table S.6, an optimization model is constructed to determine the parameter b in function $g(y,z)$ which satisfies the transitivity of consistency. The specific process can be referred to Fu et al. (2016). As such, we can obtain the parameter such that$b = -24.12$. Step 9 is completed.

The combined DPR on each attribute are then aggregated to generate the pairwise assessments at alternative level, which are shown in Table 12. Step 10 is completed.

Subsequently, the results in Table 12 are transformed into interval score matrix by Def. 10. Then, the complete interval score matrix of aggregated pairwise assessments at alternative level can be obtained by using function $g(y,z)$ which satisfies the transitivity of consistency in Def. 4 with the parameter b determined in Step 9. The results are shown in Table 13.

According to the interval scores in Table 13, possibility degree matrix can be generated as shown in Table 14.

From the results in Table 14, we can directly view the relative importance when comparing each pair of vehicle models. For example, when comparing vehicle model 2 and other vehicle models, the possibilities are all more than 0.5 which means that vehicle model 2 is superior to other vehicle models. With respect to vehicle model 1, it is better than others except vehicle model 2. Consequently, the ranking order of these six vehicle models with possibility degree is generated as$A_2 \succ^{0.99965} A_1 \succ^{0.7687} A_3 \succ^{0.7146} A_5 \succ^{0.6854} A_4 \succ^{0.9213} A_6$. Step 11 is completed.

5.4. Discussion and comparative analysis

In order to demonstrate the efficiency of the feedback mechanism proposed in Section 4, comparative analysis is conducted here. There are few CRP strategies that particularly specified to attribute level. Furthermore, the GC adjustment of each level in this paper is constructed on the foundation of consensus measure at attribute level. Therefore, a selection of the feedback mechanism with similar GC reaching principle is essential in order to acquire a relatively valid and reasonable comparison. Taking the above into consideration, an attribute weight based feedback model (AWFM) (Fu & Yang, 2011) for MAGDA problem is selected to compare with the proposed feedback mechanism. The results of the comparison are shown in Fig. 8.

Obviously, compared with the process based on AWFM, GC reaching based on the proposed feedback mechanism is slightly faster. Therefore, the feedback mechanism including identification and direction rules proposed in this paper is relatively effective.

Several representative concepts and methods are selected to conduct a comparative analysis against the proposed method. The details are shown in Table 15 and below.

(1) Zhang et al. (2016) proposed the concept of PLPR based on probabilistic linguistic term set. PLPR and DPR have a high degree of similarity from the perspective of information expression style. Both PLPR and DPR can be used to construct uncertain and complex decision-making problems in which preference information are modeled in the form of pairwise comparisons between alternatives. They employ a set of exhaustive and exclusive discrete linguistic variables (or called evaluation grades) with different probabilities (or called belief degrees) to express preference information. But there are some discrepancies between them from several aspects. Firstly, the theoretical foundation of PLPR and DPR are linguistic computation model and probability theory respectively. Secondly, consistency measure and adjustment are the main tasks in Zhang et al. (2016). Comparatively, the function satisfying the consistency of score value is defined and used in DPR to reflect the consistency. Finally, SNA and consensus measure which are the main components of the proposed method are neither considered in Zhang et al. (2016) nor Fu et al. (2016).

(2) Fu & Yang (2011) proposed an attribute weight based feedback model (AWFM) with GC for MAGDA. GC at three levels is analyzed in the proposed method and AWFM. Compatibility

measure among the assessments of experts is defined to deduce the three levels of GC measure, including attribute level, alternative level and global level. Correspondingly, feedback model in the proposed method which contains identification and direction rules is also constructed from three aspects. The most significant distinction between the proposed method and AWFM is the style of the preference expression. The concept of DPR is used in the proposed method, which means the preference information is based on pairwise comparisons. Comparatively, the preference in AWFM is expressed as assessments of alternatives directly on each attribute. This requires more sufficient high-quality information than the pairwise comparison model. Moreover, considering the concept of OL which is based on TCA proposed in this paper, the consensus measure, identification rules and direction rules are different from AWFM. In addition, AWFM is neither designed for GDM with large scale of experts nor SNA.

(3) Zhang et al. (2018) proposed consensus reaching and feedback adjustment mechanism for heterogeneous LSGDA. Four schemes of preference representation structure are involved, including preference ordering (PO), utility function (UF), multiplicative preference relation (MPR) and additive preference relation (APR). Individual preference vector can be obtained by using individual selection method according to the format of preference representation structure. As for the clustering method, experts are classified into different clusters according to the obtained preference vectors. The difference between the proposed method and Zhang et al. (2018) lies in that the latter divides experts only by their preference information, while the former is a two-dimensional method that classifies experts by trust relationship and self-confidence degree under SNA. The clustering based on preference information may cause the changing of category when GC reaching is conducted.

(4) Zhang et al. (2020b) proposed a CRP with leadership and bounded confidence. Interval fuzzy preference relation (IFPR) is used to construct experts' preference scheme on alternatives. The social network of experts is divided into several sub-networks with corresponding leadership by network partition algorithm. Following this, consensus status on individual and group levels are measured by calculating the distances between different opinions. Feedback adjustment is implemented by considering the bounded confidence level which combines the willingness to adjust opinions and leadership of expert. Compared with Zhang et al (2020b)'s method which uses trust relationship as the classification basis, we propose the TCA based trust relationship and self-confidence level as the clustering dimension. To some extent, leadership plays similar role in feedback mechanism with OL in the proposed method.

In conclusion, the advantages of dimension reduction (i.e. expert clustering) and CRP proposed in Sections 3 and 4 compared with some typical methods are specified above. To address the problem of excessive CRP iterations that probably resulted from expert clustering based on assessments, we creatively take expert clustering based on SNA and CRP as two separate processes. Furthermore, TCA is proposed based on trust relationship and the self-confidence of experts under social network, which is suitable for K-means algorithm and provides the basis for expert clustering. OL can also be obtained from TCA that plays an important role in the subsequent CRP. As for the CRP, three levels of consensus measure including the attribute level, alternative level and global level can be processed in sequence in order to satisfy different requirements. Consensus measure that is oriented to the attribute level under pairwise comparisons has not been fully discussed in previous studies. However, it is highly consistent with the characteristics and structure of LS-MAGDA. Therefore, the three levels of GC measure and corresponding identification and direction rules are essential.

6. Conclusions

In this paper, a trust-confidence analysis (TCA) framework which takes into consideration both trust relationship and self-confidence based on social network analysis (SNA) is proposed. It overcomes the disadvantage of clustering through distance among DMs' assessments or trust relationship. Under the proposed clustering method, the clusters are stable when conducting the GC reaching procedure. Authority, confident, ordinary and abnormal regions are classified according to the trust-confidence analysis. Each region contains a type of individuals with certain trust and self-confidence degrees, representing different kinds of social status and personality traits. Based on the scheme of collective DPR (CDPR), the measurement of GC and consensus feedback mechanism are proposed at three levels, i.e. the attribute, alternative and global levels, which are applicable for specific LS-MAGDA problems. The mechanism is designed for each consensus level considering the lower and upper consensus thresholds and importance of attributes. The final DPRs are generated by aggregating the CDPRs of multiple attributes and different experts via the ER rule, which includes both the reliability and weight of expert computed based on historical data and trust relationships. Future research will be conducted on how to determine the consensus threshold from experts' subjective judgments or other available information. Moreover, how to measure and adjust consistency based on complete DPR where each pair of alternatives are compared in LS-MAGDA seems an interesting topic.

CRediT authorship contribution statement

Mi Zhou: Conceptualization, Methodology, Writing – review & editing, Funding acquisition, Funding acquisition. **Yong-Kang Qiao:** Methodology, Writing – original draft, Validation, Investigation, Software, Data curation. **Jian-Bo Yang:** Writing – review & editing, Validation. **Ya-Jing Zhou:** Formal analysis, Data curation. **Xin-Bao Liu:** Formal analysis, Supervision, Funding acquisition. **Jian Wu:** Supervision.

Declaration of Competing Interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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Appendix

The abbreviations used in this paper is summarized in Table A.1.

Table A1
Abbreviations and corresponding descriptions.

Abbreviation	Description
ALS	set of assessment which needs to be adjusted at alternative level
APR	additive preference relation
ATS	set of assessment which needs to be adjusted at attribute level
AWAF	attribute weight based feedback model
BD	belief distribution
CC	cluster consensus, representing similarity of opinions between a certain cluster and other clusters
CDPR	collective distributed preference relation
CRP	consensus reaching process
DLPR	distribution linguistic preference relation
DM	decision maker
DPR	distributed preference relation
ER	evidential reasoning
GC	group consensus
GDM	group decision making
GLS	set of assessment which needs to be adjusted at global level
IFPR	interval fuzzy preference relation
LCC	life cycle cost
LS-MAGDA	large-scale multi-attribute group decision analysis
MPR	multiplicative preference relation
OL	opinion leader who provides direction of assessment adjustment
PLPR	probabilistic linguistic preference relation
PLTS	probabilistic linguistic term set
PO	preference ordering
R&D	research and development
SNA	social network analysis
SSE	sum of the squared errors
TC	trust-confidence relationship
TCA	trust-confidence analysis based on trust relationship and self-confidence
TS	comprehensive trust score to an expert
TTP	trust third partner to propagate trust relationship
UF	utility function
WAO	weighted average operator

Appendix A. Supplementary data

Supplementary data to this article can be found online at https://doi.org/10.1016/j.eswa.2022.117603.

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