Expert Systems with Applications 38 (2011) 3937–3943

Contents lists available at ScienceDirect

Expert Systems with Applications

journal homepage: www.elsevier.com/locate/eswa

Bayesian reasoning approach based recursive algorithm for online updating belief rule based expert system of pipeline leak detection

Zhi-Jie Zhou [a,b,c], Chang-Hua Hu [a], Dong-Ling Xu [c], Jian-Bo Yang [c], Dong-Hua Zhou [b,*]

[a] High-Tech Institute of Xi'an, Xi'an, Shaanxi 710025, PR China
[b] Department of Automation, TNList, Tsinghua University, Beijing 100084, PR China
[c] Manchester Business School, The University of Manchester, Manchester M15 6PB, UK

ARTICLE INFO

Keywords:
Belief rule base
Expert system
Bayesian reasoning
Recursive algorithm
Leak detection

ABSTRACT

In this paper a recursive algorithm based on the Bayesian reasoning approach is proposed to update a belief rule based (BRB) expert system for pipeline leak detection and leak size estimation. In addition to using available real time data, expert knowledge on the relationships of the parameters among different rules is incorporated into the updating process so that the performance of the expert system can be improved. Experiments are carried out to compare the newly proposed algorithm with the previously published algorithms, and results show that the proposed algorithm can update the BRB expert system faster and more accurately, which is important for real-time applications. The BRB expert systems can be automatically tuned to represent complex real world systems, and applied widely in engineering.

1. Introduction

Leaks from pipelines may cause immeasurable damage to the environment and losses to the pipeline operating companies. To minimize the damages and improve the reliability, many methods and types of systems for pipeline leak detection are developed, such as those based on mass balance (Rougier, 2005), real time transit models (Abhulimen & Susu, 2004; Yang, Li, & Lai, 2007), probabilistic models (Cagno, Caron, Mancini, & Ruggeri, 2000; Davis, Burn, Moglia, & Gould, 2007), statistical analysis (Buchberger & Nadimpalli, 2004) and acoustic emission detection (Gao, Brennan, Joseph, Muggleton, & Hunaidi, 2005). Real time systems based on mass balance corrected with pressure are among the very popular ones. The belief rule based (BRB) expert system for leak detection which has been studied is also based on the mass balance principle (Xu et al., 2007; Zhou, Hu, Yang, Xu, & Zhou, 2009).

In the previous studies, based on a generic rule-base inference methodology using the evidential reasoning (RIMER) approach in which the belief rule base and the evidential reasoning (ER) approach are used as knowledge formulation and inference tool respectively (Yang, Liu, Wang, Sii, & Wang, 2006a), the algorithms for training the BRB expert system for pipeline leak detection (Xu et al., 2007; Zhou et al., 2009) include three steps. Firstly, human experts provide a set of belief rules as an initial BRB expert system to represent the relationships of pipeline flow and pressure changes under various normal and leak conditions. Secondly, based on the ER approach (Yang & Singh, 1994; Yang et al., 2006a, Yang, Wang, Xu, & Chin, 2006b; Yang, 2001), the dedicated learning algorithms which are named as ER based algorithms are used to train the BRB expert system (Xu et al., 2007; Zhou et al., 2009). Finally, the trained BRB expert system is used for leak detection and leak size estimation. Compared with traditional rule based expert systems, the BRB expert system can detect smaller leaks and provide more accurate information on leak sizes. Generally, by using belief rules, a BRB expert system can capture relationships between system inputs and outputs that could be discrete or continuous, complete or incomplete, linear, nonlinear or non-smooth, or their mixture (Yang et al., 2006a).

Compared with other methods, such as probability based methods where it is very difficult to decide the prior and conditional probabilities (Cagno et al., 2000; Davis et al., 2007; Pearl, 1988; Spiegelhalter, Dawld, Lauritzen, & Cowell, 1993), the BRB expert system has an important characteristic to allow the direct expert intervention (Xu et al., 2007; Yang et al., 2006a) which can be used to guide the training of the parameters such as belief degrees in the BRB expert system.

However, the RIMER approach is a complex nonlinear mapping between the inputs and outputs of a system. It involves complicated calculations (Xu et al., 2007; Zhou et al., 2009). In order to overcome this weakness, a fuzzy rule-based Bayesian reasoning algorithm was proposed and used in failure mode and effects analysis (Yang, Bonsall, & Wang, 2008).

In the study given by Yang et al. (2008), the belief degrees in the rules were given by the experts. But in engineering, it is difficult for experts to provide belief degrees accurately, especially for large

* Corresponding author. Tel.: +86 010 62783125; fax: +86 010 62786911.
E-mail address: zdh@mail.tsinghua.edu.cn (D.-H. Zhou).

0957-4174/$ - see front matter © 2010 Elsevier Ltd. All rights reserved.
doi:10.1016/j.eswa.2010.09.055

scale rule bases with hundreds or thousands of rules. Therefore, it is necessary to update these parameters when new information becomes available.

Based on the relationship between belief distribution and probability distribution (Dempster, 1967, 1968; Halpern & Fagin, 1992; Pearl, 1988; Shafer, 1976; Simon, Weber, & Evsukoff, 2008; Spiegelhalter et al., 1993), in this paper a recursive learning algorithm is proposed which is based on the Bayesian reasoning approach and can online update the parameters of a BRB expert system. The new algorithm is applied to the pipeline leak detection problem studied in Xu et al. (2007) and Zhou et al. (2009). Compared with the ER based algorithms which include offline learning algorithm (Xu et al., 2007) and online updating algorithm (Zhou et al., 2009), the proposed algorithm can update the BRB expert system faster, which is important when there is higher real-time requirement.

This paper is organized as follows. In Section 2, based on the relationship between belief distribution and probability distribution, the inference method of the BRB expert system using the Bayesian reasoning approach is briefly introduced. Section 3 proposes a Bayesian reasoning based recursive algorithm for online updating the BRB expert system. The proposed algorithm is used for oil pipeline leak detection in Section 4. The paper is concluded in Section 5.

2. Inference method of BRB expert system using Bayesian reasoning approach

2.1. The structure and representation of BRB expert system

In order to capture the dynamics of a system, a belief rule based (BRB) expert system consisting of a collection of belief rules is defined as follows (Yang et al., 2006a):

$$R_k: \text{If } x_1 \text{ is } A_1^k \wedge x_2 \text{ is } A_2^k \cdots \wedge x_M \text{ is } A_M^k, \\ \text{Then } \{(D_1, \beta_{1,k}), \ldots, (D_N, \beta_{N,k})\} \text{ with } \sum_{j=1}^{N} \beta_{j,k} = 1, \tag{1}$$

where R_k denotes the kth belief rule. $x_m(m = 1,\ldots,M)$ is the mth antecedent attribute. $A_m^k(m = 1,\ldots,M,\ k = 1,\ldots,L)$ is the referential value of the mth antecedent attribute in the kth rule and $A_m^k \in A_m$. $A_m = \{A_{m,j}, j_m = 1,\ldots,J_m\}$ is a set of referential values for the mth antecedent attribute and J_m is the number of the referential values. $\beta_{j,k}(j = 1,\ldots,N, k = 1,\ldots,L)$ is the belief degree assessed to D_j which denotes the jth consequence where $D_j \in \mathbf{D}$ and $\mathbf{D} = \{D_1,\ldots,D_N\}$. Note that "$\wedge$" is a logical connective to represent the "AND" relationship.

2.2. Relationship between belief distribution and probability distribution

In order to use the Bayesian reasoning approach on the BRB expert system as represented in Eq. (1), the relationship between belief distribution and probability distribution is given as follows:

Halpern and Fagin (1992) proposed that there were two useful and quite different ways of interpreting belief functions. The first is that a belief function is interpreted as a generalized probability function and the second is that a belief function is used as a way for representing evidence (Dempster, 1967, 1968; Halpern & Fagin, 1992; Pearl, 1988; Shafer, 1976; Simon et al., 2008; Spiegelhalter et al., 1993). To facilitate the discussion, the following concepts are defined.

Let H be the frame of discernment defined by $H = \{H_1, H_2, \ldots, H_N\}$ and Θ as the power set of H, consisting of all the subsets of H, or

$$\Theta = \{H_1, \ldots, H_N, \{H_1, H_2\}, \ldots, \{H_1, H_N\}, \ldots, \{H_1 \cdots H_{N-1}\}, H\}. \tag{2}$$

A piece of evidence is represented as a belief distribution, defined as follows in general:

$$S(Z) = \{(\Psi_s, \beta_s), \Psi_s \subseteq \Theta\}, \tag{3}$$

where Ψ_s is any subset of Θ and β_s is a belief degree assigned to Ψ_s with $\sum_{\Psi_s \subseteq \Theta} \beta_s = 1$.

A belief distribution defined on the power set Θ reduces to a conventional probability distribution defined on H if the following two assumptions are satisfied:

(1) $\beta_n \geqslant 0$ holds only for the singleton evaluation grade H_n for $n = 1,\ldots,N$, $\beta_s = 0$ for any other $\Psi_s \subseteq \Theta$, i.e., these N evaluation grades are mutually exclusive and collectively exhaustive (Pearl, 1988; Yang et al., 2006a).
(2) There is $\sum_{n=1}^{N} \beta_n = 1$.

The above analysis means that a conventional probability distribution is a special case of a belief distribution. In other words, a belief distribution is a generalized probability distribution (Pearl, 1988; Yang et al., 2006a). Thus, it is possible to use the Bayesian reasoning approach as the inference tool in the belief distribution when it is a probability distribution.

2.3. Belief rule inference using Bayesian reasoning approach

In Eq. (1), it is assumed that the linguistic set $\mathbf{D}$ is mutually exclusive and collectively exhaustive. In addition, the belief degrees should satisfy the equality constraint as given in Eq. (1). Therefore, the belief degrees can be treated as the conventional probabilities according to the two assumptions given in sub Section 2.2. Thus, the belief rule in Eq. (1) can be further expressed in the form of conditional probability as follows (Yang et al., 2008; Pearl, 1988):

$$\text{Given } A_1^k \wedge \cdots \wedge A_M^k, \quad \text{Then } p(D_1) \text{ is } \beta_{1,k}, \ldots, p(D_N) \text{ is } \beta_{N,k}, \tag{4}$$

where there are $A_m^k \in A_m, A_m = \{A_{m,j_m}, m = 1,\ldots,M, j_m = 1,\ldots,J_m\}$, $k = 1,\ldots,L$, and $L = \prod_{m=1}^{M} J_m$. $p(\cdot)$ denotes the probability.

Eq. (4) can also be written as the following conditional probability.

$$p\left(D_s | A_1^k, \ldots, A_M^k\right) = \beta_{s,k}, \quad s = 1,\ldots,N. \tag{5}$$

Based on the probability expression of a belief rule, the Bayesian reasoning approach can be used to combine rules and generate final conclusions, i.e., the marginal probabilities of $D_s(s = 1,\ldots,N)$ which can be calculated as:

$$p(D_s) = \sum_{k=1}^{L} p\left(D_s | A_1^k, \ldots, A_M^k\right) \prod_{m=1}^{M} p\left(A_m^k\right). \tag{6}$$

In Eq. (6), the prior probability $p\left(A_m^k\right)(m = 1,\ldots,M, k = 1,\ldots,L)$ and conditional probability $p\left(D_s | A_1^k, \ldots, A_M^k\right)$ are needed. In the probability based methods, it is very difficult to decide these probabilities (Cagno et al., 2000; Davis et al., 2007; Pearl, 1988; Spiegelhalter et al., 1993). It is noted however due to the introduction of the BRB expert system, the direct expert intervention is allowed (Xu et al., 2007; Yang et al., 2006a) and can be used to determine the probabilities as follows:

(1) In Eq. (5), the belief degree $\beta_{s,k}$ is given by experts, so the conditional probability $p\left(D_s | A_1^k, \ldots, A_M^k\right)$ in Eq. (6) can be obtained (Yang et al., 2008).
(2) In this study, it is also assumed that the linguistic set $A_m = \{A_{m,j_m}, m = 1,\ldots,M, j_m = 1,\ldots,J_m\}$ is mutually exclusive and collectively exhaustive, so the prior probability $p\left(A_m^k\right)$ is equal to the belief degree to the referential value

A_m^k from Eq. (4), where $A_m^k \in A_m$. On the other hand, the belief degree can be calculated using the rule based information transformation technique when the antecedent attributes are available (Yang, 2001). Thus, $p\left(A_m^k\right)$ can be obtained. The detail example about how to calculate the belief degree was introduced by Xu et al. (2007).

3. Bayesian reasoning approach based recursive algorithm for online updating the BRB expert system

As mentioned above, the conditional probabilities in Eq. (6) are given by experts in the current study in order to calculate the marginal probability. However, it is difficult to accurately determine these parameters entirely subjectively in engineering. As such, based on the recursive expectation maximization (EM) algorithm, a Bayesian reasoning approach based recursive algorithm for online estimating the belief degrees will be given in this Section.

In the proposed recursive algorithm, the observations on the system inputs and outputs are required. We assume that a set of observation pairs ($\mathbf{x}$,$\mathbf{y}$) is available, where $\mathbf{x}$ is a given input vector and $\mathbf{y}$ is the corresponding observed output vector. From Eq. (1), there is $\mathbf{x} = [x_1,\ldots,x_M]^T$.

3.1. Bayesian reasoning based recursive algorithm

Assume that the output $\mathbf{y}$ is a random variable. Furthermore, it is assumed that $\mathbf{y}(1),\ldots,\mathbf{y}(n)$ are independent. Therefore, there is

$$f(\mathbf{y}(1),\ldots,\mathbf{y}(n)|\mathbf{x}(1),\ldots,\mathbf{x}(n),\mathbf{Q}) = \prod_{\tau=1}^{n} f(\mathbf{y}(\tau)|\mathbf{x}(\tau),\mathbf{Q}), \tag{7}$$

where $f(\mathbf{y}(\tau)|\mathbf{x}(\tau),\mathbf{Q})$ denotes the conditional probability density function (pdf) of $\mathbf{y}$ at time instant τ and $\mathbf{Q}$ is the unknown parameter vector.

The recursive formulation of the expectation of the log-likelihood of Eq. (7) can be written as:

$$\mathrm{L}_{n+1}(\mathbf{Q}) = \mathrm{L}_n(\mathbf{Q}) + E\{\log f(\mathbf{y}(n)|\mathbf{x}(n),\mathbf{Q})|\mathbf{Q}(n)\}, \tag{8}$$

where $E\{\cdot|\cdot\}$ denotes the conditional expectation at $\mathbf{Q} = \mathbf{Q}(n)$.

Define

$$\Gamma(\mathbf{Q}(n)) \triangleq \nabla_{\mathbf{Q}} \log f(\mathbf{y}(n)|\mathbf{x}(n),\mathbf{Q}(n)), \tag{9}$$

$$\Xi(\mathbf{Q}(n)) \triangleq E\left\{-\nabla_{\mathbf{Q}}\nabla_{\mathbf{Q}}^T \log f(\mathbf{y}(n)|\mathbf{x}(n),\mathbf{Q})|\mathbf{Q}(n)\right\}, \tag{10}$$

where $\nabla_{\mathbf{Q}}\log f(\mathbf{y}(n)|\mathbf{x}(n),\mathbf{Q}(n))$ denotes the gradient vector at $\mathbf{Q}(n)$.

Based on the recursive EM algorithm (Chung & Bohme, 2005; Titterington, 1984), the maximizing parameter vector $\mathbf{Q}(n+1)$ is given by:

$$\mathbf{Q}(n+1) = \mathbf{Q}(n) + \frac{1}{n}[\Xi(\mathbf{Q}(n))]^{-1}\Gamma(\mathbf{Q}(n)). \tag{11}$$

Due to the fact that the recursive EM algorithm is indeed a maximum likelihood (ML) algorithm, the proposed recursive algorithm as given in Eq. (11) is also a ML algorithm.

3.2. Recursive algorithm under normal distribution of observation

Since the belief degrees are treated as a generalized probability in this paper, in Eq. (6) let

$$\beta_s \triangleq p(D_s), \tag{12}$$

where $\beta_s (s = 1,\ldots,N)$ is one of the final conclusions generated by the BRB expert system using the Bayesian reasoning approach and denotes the belief degree to an individual consequence D_s.

Substituting Eqs. (5) and (12) into Eq. (6), we can obtain:

$$\beta_s = \sum_{k=1}^{L} \beta_{s,k} \prod_{m=1}^{M} p\left(A_m^k\right). \tag{13}$$

The expected output $\hat{\mathbf{y}}(n)$ of the BRB expert system can be calculated by (Yang, 2001):

$$\hat{\mathbf{y}}(n) = \sum_{s=1}^{N} \mu_s \beta_s(n), \tag{14}$$

where μ_s denotes the utility (or score) of an individual consequence D_s which can be either given using a scale or estimated using the decision maker's preferences (Yang, 2001; Yang et al., 2006a). $\beta_s(n)$ denotes the value of β_s at time instant n.

It is hoped that given the input $\mathbf{x}(n)$, the BRB expert system can generate an output as represented in Eq. (14), which can be close to $\mathbf{y}(n)$ as possible. It is noted however in this paper, $\mathbf{y}(n)$ is referred to the observation of a system obtained by a sensor at time instant n and it is random. In the ML algorithm, it is usually assumed that $\mathbf{y}(n)$ obeys the following normal distribution.

$$f(\mathbf{y}(n)|\mathbf{x}(n),\mathbf{Q}) = \frac{1}{\sqrt{2\pi\sigma}} \exp\left\{-\frac{(\mathbf{y}(n)-\hat{\mathbf{y}}(n))^2}{2\sigma}\right\}, \tag{15}$$

where $\hat{\mathbf{y}}(n)$ is the expectation of $\mathbf{y}(n)$. $\mathbf{Q} = [\mathbf{V}^T,\sigma]^T$ denotes parameter vector. $\mathbf{V} = [\beta_{1,1},\ldots,\beta_{N,L}]^T$ denotes parameter vector of the BRB expert system and σ denotes variance.

Due to the independence between the elements of $\mathbf{V}$ and σ, $\Gamma(\mathbf{Q}(n))$ and $\Xi(\mathbf{Q}(n))$ in Eq. (11) can be written as:

$$\Gamma(\mathbf{Q}(n)) = \left[\Gamma'(\mathbf{Q}(n))^T, \Gamma''(\mathbf{Q}(n))^T\right]^T, \tag{16}$$

$$\Xi(\mathbf{Q}(n)) = \begin{bmatrix} \Xi'(\mathbf{Q}(n)) & \mathbf{0} \\ \mathbf{0} & \Xi''(\mathbf{Q}(n)) \end{bmatrix}, \tag{17}$$

where $\Gamma'(\mathbf{Q}(n))$ and $\Xi'(\mathbf{Q}(n))$ are the derivates with respect to the entries of $\mathbf{V}$. $\Gamma''(\mathbf{Q}(n))$ and $\Xi''(\mathbf{Q}(n))$ are the derivates with respect to σ.

When we consider only parameter vector $\mathbf{V}$, from Eqs. (16) and (17), the recursive algorithm in Eq. (11) changes into the following form:

$$\mathbf{V}(n+1) = \mathbf{V}(n) + \frac{1}{n}[\Xi'(\mathbf{Q}(n))]^{-1}\Gamma'(\mathbf{Q}(n)). \tag{18}$$

In Eq. (18), $\mathbf{V}(n)$ is known. From Eqs. (9), (10), (13)–(15), the ath element of the gradient vector of $\Gamma'(\mathbf{Q}(n))$ and the entries of $\Xi'(\mathbf{Q}(n))$ at time instant n are:

$$[\Gamma'(\mathbf{Q}(n))]_a = \frac{(\mathbf{y}(n)-\hat{\mathbf{y}}(n))}{\sigma(n)} \sum_{s=1}^{N} \mu_s \left.\frac{\partial\beta_s(n)}{\partial Q_a}\right|_{\mathbf{V}=\mathbf{V}(n)}, \tag{19}$$

$$[\Xi'(\mathbf{Q}(n))]_{a,b} = \frac{1}{\sigma(n)} \left[\sum_{s=1}^{N} \mu_s \frac{\partial\beta_s(n)}{\partial Q_a}\right]\left.\left[\sum_{s=1}^{N} \mu_s \frac{\partial\beta_s(n)}{\partial Q_b}\right]\right|_{\mathbf{V}=\mathbf{V}(n)}, \tag{20}$$

where there are $a = 1,\ldots,L \times N$ and $b = 1,\ldots,L \times N$. $\partial\beta_s(n)/\partial Q_a$ denotes the value of $\partial\beta_s/\partial Q_a$ at time instant n. From Eq. (13), $\partial\beta_s/\partial Q_a$ can be determined by:

$$\frac{\partial\beta_s}{\partial Q_a} = \begin{cases} \prod_{m=1}^{M} p\left(A_m^k\right), & a = (k-1)\times N + s, \\ 0, & \text{others}, \end{cases} \tag{21}$$

where $s = 1,\ldots,N,\ k = 1,\ldots,L,\ A_m^k \in \left\{A_{m j_m, m=1,\ldots,M, j_m=1,\ldots,J_m}\right\}$

As shown in Eq. (1), the belief degrees should satisfy the equality constraint $\sum_{j=1}^{N}\beta_{j,k} = 1$ and the inequality constraints $0 \leqslant \beta_{j,k} \leqslant 1$, where $j = 1,\ldots,N$ and $k = 1,\ldots,L$. In order to deal with these constraints, the recursive algorithm as given in Eq. (18) can be written as follows:

$$\mathbf{V}(n+1) = \prod_{H} \left\{ \mathbf{V}(n) + \frac{1}{n}\left[\Xi'(\mathbf{Q}(n))\right]^{-1}\Gamma'(\mathbf{Q}(n)) \right\}, \tag{22}$$

where $\prod_H\{\cdot\}$ is the projection onto the constraint set H which is composed of the equality and inequality constraints. The detailed projection algorithm for dealing with these constraints has been given by Zhou et al. (2009).

In Eqs. (19) and (20), $\sigma(n)$ is required. If $\mathbf{x}(n)$, $\mathbf{y}(n)$ and $\mathbf{V}(n)$ are available, it can be estimated by:

$$\begin{aligned}\sigma(n) &= \arg\max_{\sigma} \log f(\mathbf{y}(n)|\mathbf{x}(n), \mathbf{Q})|_{\mathbf{V}=\mathbf{V}(n)} \\ &= (\mathbf{y}(n) - \hat{\mathbf{y}}(n))^2|_{\mathbf{V}=\mathbf{V}(n)}.\end{aligned} \tag{23}$$

As a result of the above discussion, the procedure of the Bayesian reasoning approach based recursive algorithm for online updating the BRB expert system may be summarized as the following steps:

Step 1: Let $n = 0$. Assign initial values to the parameter vector $\mathbf{V}(n)$. The elements of $\mathbf{V}(n)$ should satisfy the equality and inequality constraints.

Step 2: Since the observations $\mathbf{x}(n)$, $\mathbf{y}(n)$ and $\mathbf{V}(n)$ are available, $\sigma(n)$ can be estimated by using Eq. (23). Then the recursive algorithm as given in Eq. (22) is used to estimate $\mathbf{V}(n+1)$.

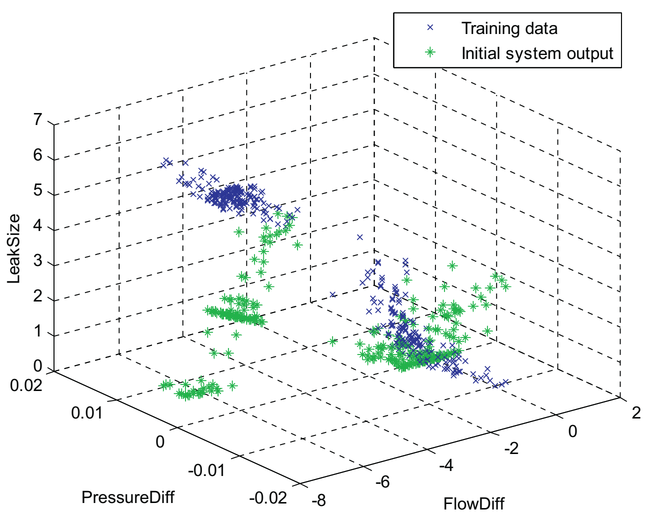

Fig. 1. Training data and output generated by the initial BRB expert system.

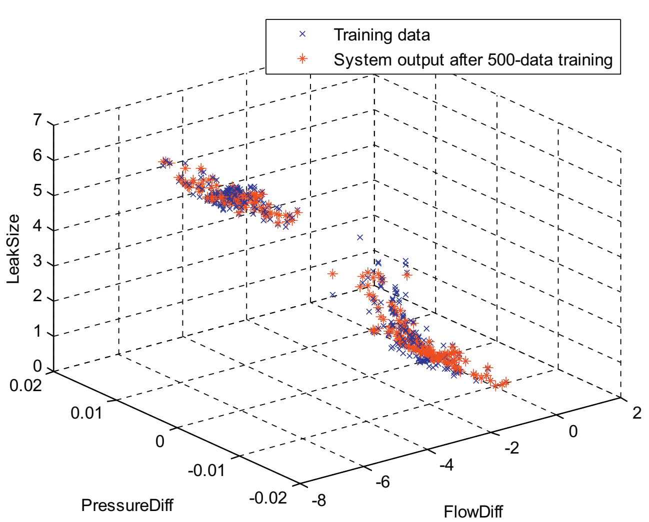

Fig. 2. Training data and the output generated by the updated BRB expert system.

Step 3: After $\mathbf{x}(n+1)$, $\mathbf{y}(n+1)$ and $\mathbf{V}(n+1)$ are available, let $n = n + 1$ and go to *Step 2*. Otherwise, go to *Step 4*.

Step 4: Once the BRB expert system is updated, its knowledge can be used to perform inference from the given inputs.

4. Online updating BRB expert system for pipeline leak detection

4.1. Problem formulation

In this Section, we will also consider a pipeline studied in Xu et al. (2007) and Zhou et al. (2009) and the pipeline leak data will be used to demonstrate the validity of the proposed algorithm.

We also choose the difference between inlet flow and outlet flow, the average pipeline pressure change over time and the leak rate, denoted by *FlowDiff*, *PressureDiff* and *LeakSize* respectively, as the leak data (Xu et al., 2007). During the leak trial, 2008 samples of 25% leak data were collected at the rate of 10s per sample,

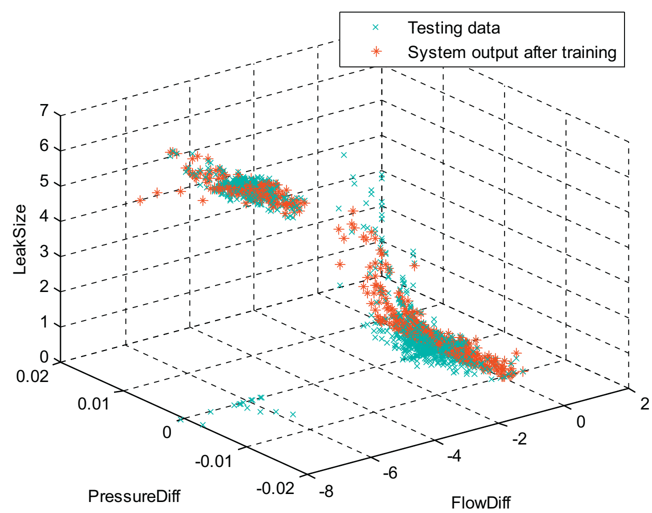

Fig. 3. Testing data of no leak and 25% leak and the output generated by the updated BRB expert system.

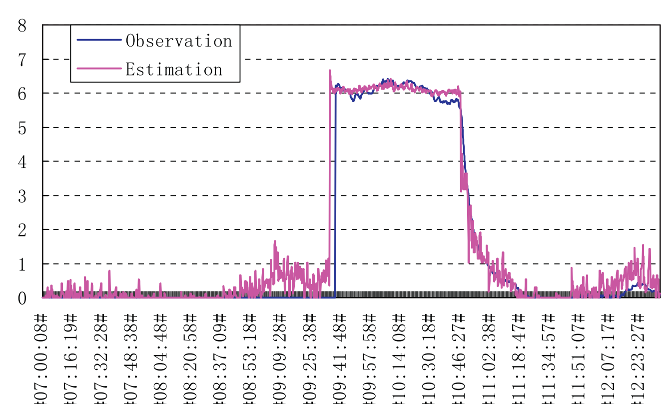

Fig. 4. Testing data of no leak and 25% leak and the output generated by the updated BRB expert system.

Table 1
Calculation time of three algorithms.

Calculation time	ER based offline algorithm	ER based online algorithm	Proposed algorithm
Minute	300	20	2

Table 2
MAE of three algorithms.

Calculation time	ER based offline algorithm	ER based online algorithm	Proposed algorithm
Minute	0.2245	0.4227	0.1732

respectively. The leak data is completely same to the one used by Xu et al. (2007).

4.2. Construction of the initial BRB expert system for pipeline leak detection

In this study, *FlowDiff* and *PressureDiff* are also chosen as the antecedent attributes of the rule base. And *LeakSize* is the consequent attribute of the rule base. Therefore, the input vector $\mathbf{x}$ is composed of *FlowDiff* and *PressureDiff*, and the output vector $\mathbf{y}$ only includes *LeakSize*. The antecedents and consequence in the rule base should be given some referential points. Here *FlowDiff*, *PressureDiff* and *LeakSize* are given eight, seven and five referential points, respectively. The linguistic terms and quantified results of these points are the same as the ones used by Xu et al. (2007).

Thus, based on the running patterns of pipeline leak, a BRB expert system for the pipeline leak detection can be constructed. A belief rule is represented as:

$$R_k: \text{ If } \mathit{FlowDiff} \text{ is } A_1^k \wedge \mathit{PressureDiff} \text{ is } A_2^k \quad \text{Then } \mathit{LeakSize} \text{ is } \{(\mathrm{Z},\beta_{1,k}), (\mathrm{VS},\beta_{2,k}), (\mathrm{M},\beta_{3,k}), (\mathrm{H},\beta_{4,k}), (\mathrm{VH},\beta_{5,k})\}, \tag{24}$$

where A_1^k and $A_2^k (k = 1, \ldots, 56)$ are the referential values.

Table A-1
Updated belief rules for pipeline oil leak detection.

Rule number	*FlowDiff* AND *PressureDiff*	*LeakSize* distribution $\{D_1,D_2,D_3,D_4,D_5\} = \{0,2,4,6,8\}$
1	NL AND NL	$\{(D_1,0.1425), (D_2,0.0559), (D_3,0), (D_4,0.0054), (D_5,0.7961)\}$
2	NL AND NM	$\{(D_1,0.0561), (D_2,0.0281), (D_3,0), (D_4,0.2173), (D_5,0.6985)\}$
3	NL AND NS	$\{(D_1,0), (D_2,0), (D_3,0), (D_4,0.667), (D_5,0.333)\}$
4	NL AND Z	$\{(D_1,0.1027), (D_2,0.0329), (D_3,0.0366), (D_4,0.0056), (D_5,0.8223)\}$
5	NL AND PS	$\{(D_1,0.0419), (D_2,0.0446), (D_3,0), (D_4,0.3043), (D_5,0.6092)\}$
6	NL AND PM	$\{(D_1,0.0724), (D_2,0.005), (D_3,0), (D_4,0.1484), (D_5,0.7741)\}$
7	NL AND PL	$\{(D_1,0.016), (D_2,0), (D_3,0), (D_4,0.2656), (D_5,0.7184)\}$
8	NM AND NL	$\{(D_1,0.0024), (D_2,0.0021), (D_3,0), (D_4,0.8844), (D_5,0.1111)\}$
9	NM AND NM	$\{(D_1,0.0347), (D_2,0.0174), (D_3,0.3098), (D_4,0.2826), (D_5,0.3555)\}$
10	NM AND NS	$\{(D_1,0.0952), (D_2,0.1067), (D_3,0.0864), (D_4,0.166), (D_5,0.5457)\}$
11	NM AND Z	$\{(D_1,0.246), (D_2,0.0158), (D_3,0.0004), (D_4,0.0066), (D_5,0.7312)\}$
12	NM AND PS	$\{(D_1,0.1397), (D_2,0.1005), (D_3,0), (D_4,0.2175), (D_5,0.5422)\}$
13	NM AND PM	$\{(D_1,0.1843), (D_2,0.1109), (D_3,0), (D_4,0.0132), (D_5,0.6916)\}$
14	NM AND PL	$\{(D_1,0.1722), (D_2,0.0624), (D_3,0), (D_4,0.1217), (D_5,0.6437)\}$
15	NS AND NL	$\{(D_1,0), (D_2,0), (D_3,0.4), (D_4,0.6), (D_5,0)\}$
16	NS AND NM	$\{(D_1,0), (D_2,0), (D_3,0), (D_4,0.1621), (D_5,0.8379)\}$
17	NS AND NS	$\{(D_1,0.0012), (D_2,0.0049), (D_3,0.4782), (D_4,0.1965), (D_5,0.3193)\}$
18	NS AND Z	$\{(D_1,0.1714), (D_2,0.0601), (D_3,0.0204), (D_4,0.1719), (D_5,0.5762)\}$
19	NS AND PS	$\{(D_1,0.0478), (D_2,0.0687), (D_3,0), (D_4,0.1926), (D_5,0.6909)\}$
20	NS AND PM	$\{(D_1,0.1718), (D_2,0.0049), (D_3,0), (D_4,0.2488), (D_5,0.5745)\}$
21	NS AND PL	$\{(D_1,1), (D_2,0), (D_3,0), (D_4,0), (D_5,0)\}$
22	NVS AND NL	$\{(D_1,0.6018), (D_2,0.1038), (D_3,0.1079), (D_4,0.1533), (D_5,0.0332)\}$
23	NVS AND NM	$\{(D_1,0.5749), (D_2,0.425), (D_3,0.0001), (D_4,0), (D_5,0)\}$
24	NVS AND NS	$\{(D_1,0.4621), (D_2,0.5327), (D_3,0.0034), (D_4,0.0005), (D_5, 0.0013)\}$
25	NVS AND Z	$\{(D_1,0.8206), (D_2,0.0256), (D_3,0), (D_4,0.0385), (D_5, 0.1152)\}$
26	NVS AND PS	$\{(D_1,0.8644), (D_2,0.0579), (D_3,0), (D_4,0.0139), (D_5, 0.0637)\}$
27	NVS AND PM	$\{(D_1,0.742), (D_2,0.0401), (D_3,0), (D_4,0.0565), (D_5, 0.1614)\}$
28	NVS AND PL	$\{(D_1,0.668), (D_2,0.0552), (D_3,0), (D_4,0.0712), (D_5, 0.2056)\}$
29	Z AND NL	$\{(D_1,1), (D_2,0), (D_3,0), (D_4,0), (D_5,0)\}$
30	Z AND NM	$\{(D_1,1), (D_2,0), (D_3,0), (D_4,0), (D_5,0)\}$
31	Z AND NS	$\{(D_1,1), (D_2,0), (D_3,0), (D_4,0), (D_5,0)\}$
32	Z AND Z	$\{(D_1,1), (D_2,0), (D_3,0), (D_4,0), (D_5,0)\}$
33	Z AND PS	$\{(D_1,1), (D_2,0), (D_3,0), (D_4,0), (D_5,0)\}$
34	Z AND PM	$\{(D_1,1), (D_2,0), (D_3,0), (D_4,0), (D_5,0)\}$
35	Z AND PL	$\{(D_1,0.9957), (D_2,0.0018), (D_3,0), (D_4,0.0009), (D_5, 0.0016)\}$
36	PS AND NL	$\{(D_1,1), (D_2,0), (D_3,0), (D_4,0), (D_5,0)\}$
37	PS AND NM	$\{(D_1,0.9935), (D_2,0.0065), (D_3,0), (D_4,0), (D_5,0)\}$
38	PS AND NS	$\{(D_1,0.9993), (D_2,0.0007), (D_3,0), (D_4,0), (D_5,0)\}$
39	PS AND Z	$\{(D_1,1), (D_2,0), (D_3,0), (D_4,0), (D_5,0)\}$
40	PS AND PS	$\{(D_1,1), (D_2,0), (D_3,0), (D_4,0), (D_5,0)\}$
41	PS AND PM	$\{(D_1,1), (D_2,0), (D_3,0), (D_4,0), (D_5,0)\}$
42	PS AND PL	$\{(D_1,1), (D_2,0), (D_3,0), (D_4,0), (D_5,0)\}$
43	PM AND NL	$\{(D_1,1), (D_2,0), (D_3,0), (D_4,0), (D_5,0)\}$
44	PM AND NM	$\{(D_1,0.3), (D_2,0.7), (D_3,0), (D_4,0), (D_5,0)\}$
45	PM AND NS	$\{(D_1,0.85), (D_2,0.15), (D_3,0), (D_4,0), (D_5,0)\}$
46	PM AND Z	$\{(D_1,0.98), (D_2,0.02), (D_3,0), (D_4,0), (D_5,0)\}$
47	PM AND PS	$\{(D_1,1), (D_2,0), (D_3,0), (D_4,0), (D_5,0)\}$
48	PM AND PM	$\{(D_1,1), (D_2,0), (D_3,0), (D_4,0), (D_5,0)\}$
49	PM AND PL	$\{(D_1,1), (D_2,0), (D_3,0), (D_4,0), (D_5,0)\}$
50	PL AND NL	$\{(D_1,0.9), (D_2,0.1), (D_3,0), (D_4,0), (D_5,0)\}$
51	PL AND NM	$\{(D_1,0.99), (D_2,0.01), (D_3,0), (D_4,0), (D_5,0)\}$
52	PL AND NS	$\{(D_1,1), (D_2,0), (D_3,0), (D_4,0), (D_5,0)\}$
53	PL AND Z	$\{(D_1,1), (D_2,0), (D_3,0), (D_4,0), (D_5,0)\}$
54	PL AND PS	$\{(D_1,1), (D_2,0), (D_3,0), (D_4,0), (D_5,0)\}$
55	PL AND PM	$\{(D_1,1), (D_2,0), (D_3,0), (D_4,0), (D_5,0)\}$
56	PL AND PL	$\{(D_1,1), (D_2,0), (D_3,0), (D_4,0), (D_5,0)\}$

In Eq. (24), the experts can give the values of the belief degrees, thus an initial BRB expert system is constructed. Here the initial BRB expert system is the same as the one used by Xu et al. (2007). As shown in Fig. 1, it is obvious that the values of the estimated *LeakSize* calculated by the initial BRB expert system using Eqs. (6) and (14) do not match the training data. This means the initial BRB expert system provided by an expert is not good. So it is necessary to update the initial BRB expert system.

4.3. Online updating the initial BRB expert system and pipeline leak detection

In order to update the initial BRB expert system for pipeline leak detection, 500 data are selected as the training data, which include 300 from no leak and 200 from 25% leak. The process of updating and testing the BRB is implemented using MATLAB.

Step 1: Update the BRB expert system
Using the training data, the proposed recursive algorithm is used to update the BRB expert system. The updated belief degrees are listed in Table A-1 of Appendix A. Fig. 2 shows that the updated BRB expert system can closely replicate the relationship among *FlowDiff*, *PressureDiff* and *LeakSize* in the training data. Furthermore, the calculation speed of the recursive algorithm is very high.

Step 2: Test
For testing the updated BRB expert system, all the 2008 data are used. Fig. 3 shows that the estimated outcomes match the observed ones very closely. Fig. 4 displays the observed and estimated *LeakSize* on the time scale. It shows that rule base can detect the leak which happened at around 9:38 a.m. and ended at around 10:53 a.m.

4.4. Comparative studies

Under the same experimental conditions, the initial BRB expert system is also trained by the ER based offline learning algorithm (Xu et al., 2007) and the ER based online updating algorithm (Zhou et al., 2009), respectively. The experimental results show that the trained BRB expert systems by these two ER based algorithms can both accomplish the leak detection and leak size estimation.

In order to further demonstrate the validity of the proposed algorithm, the following two cases are considered.

Case 1. When the initial BRB expert system is trained, the calculation time of the proposed algorithm in this paper, the ER based offline algorithm and the ER based online algorithm is recorded, respectively. Table 1 gives the calculation time of three algorithms. From Table 1, it can be seen that compared with the other two algorithms, the calculation time used by the proposed algorithm is least.

Case 2. When the BRB expert systems trained by the three learning algorithms are tested, a quantitative index – Mean Absolute Error (MAE) is chosen to compare the prediction accuracy of the proposed algorithm with the ER based algorithms. Table 2 gives the MAE of the three algorithms. It shows that the prediction accuracy of the BRB expert system updated by the proposed algorithm is highest.

Therefore, it can be concluded that although three algorithms can accomplish the leak detection, the proposed algorithm can train the BRB expert system more quickly and accurately, which is important when there is higher requirement in engineering.

5. Conclusions

In this paper, a feasible Bayesian reasoning approach based recursive algorithm to online update the belief rule based (BRB) expert system is proposed on the basis of the relationship between belief distribution and probability distribution. Then the recursive algorithm is used to online update the BRB expert system for pipeline leak detection. The study demonstrates that the updated BRB expert system can learn the relationship between leak sizes and the pipeline flow and pressure readings from pipeline operating data, and can be applied for pipeline leak detection and leak size estimation. Compared with the evidential reasoning (ER) based offline and online optimal algorithms for pipeline leak detection (Xu et al., 2007; Zhou et al., 2009), the proposed algorithm can reduce the parameter tuning time significantly. Except for oil pipeline leak detection, the BRB expert system and the proposed algorithm may be widely applied in the other fields of engineering such as real-time reliability and safety analysis of the complex systems.

From this study, we see that based on the belief rule base, the Bayesian reasoning approach and the ER based approach can both be used as the inference tools. Therefore, the difference and the relation between these two tools should be studied in the future. Furthermore, it is necessary to develop a more generalized reasoning approach for the BRB expert system and also the optimal methods to train the parameters of the BRB expert system in the future.

Acknowledgements

Zhou thanks the partial support by the NSFC under Grant 61004069 and the Foundation of Department of Education of JiLin Province of China under Grant 2009109. Hu thanks the partial support by National Natural Science Funds for Distinguished Young Scholar and the NCET under Grant 07144. Xu and Yang thank the partial support by the UK Engineering and Physical Science Research Council under Grant No.: EP/F024606/1 and by the NSFC under Grant 60736026; Zhou thanks the partial support by the National 973 Project under Grants 2010CB731800 and 2009CB32602, and the NSFC under Grants 60721003 and 60736026.

Appendix A

See Table A-1.

References

Abhulimen, K. E., & Susu, A. A. (2004). Liquid pipeline leak detection system: Model development and numerical simulation. *Chemical Engineering Journal, 97*(1), 47–67.

Buchberger, S., & Nadimpalli, G. (2004). Leak estimation in water distribution systems by statistical analysis of flow readings. *Journal of Water Resources Planning and Management – ASCE, 130*(4), 321–329.

Cagno, E., Caron, F., Mancini, M., & Ruggeri, F. (2000). Using AHP in determining the prior distribution on gas pipeline failures in a robust Bayesian approach. *Reliability Engineering and System Safety, 67*(3), 275–284.

Chung, P. J., & Bohme, J. F. (2005). Recursive EM and SAGE-inspired algorithms with application to DOA estimation. *IEEE Transactions on Signal Processing, 53*(8), 2664–2677.

Davis, P., Burn, S., Moglia, M., & Gould, S. (2007). A physical probabilistic model to predict failure rates in buried PVC pipelines. *Reliability Engineering and System Safety, 92*(9), 1258–1266.

Dempster, A. P. (1967). Upper and lower probabilities induced by a multi-valued mapping. *Annals Mathematical Statistics, 38*(2), 325–339.

Dempster, A. P. (1968). A generalization of Bayesian inference. *Journal of the Royal Statistical Society, Series B, 30*(2), 205–247.

Gao, Y., Brennan, M. J., Joseph, P. F., Muggleton, J. M., & Hunaidi, O. (2005). On the selection of acoustic/vibration sensors for leak detection in plastic water pipes. *Journal of Sound and Vibration, 283*(3–5), 927–941.

Halpern, J. Y., & Fagin, R. (1992). Two views of belief: Belief as generalized probability and belief as evidence. *Artificial Intelligence, 54*(3), 275–317.

Pearl, J. (1988). *Probabilistic reasoning in intelligence systems*. San Mateo, CA: Morgan Kaufman.

Rougier, J. C. (2005). Probabilistic leak detection in pipelines using the mass imbalance approach. *Journal of Hydraulic Research, 43*(5), 556–566.

Shafer, G. (1976). *A mathematical theory of evidence*. Princeton, NJ: Princeton University Press.

Simon, C., Weber, P., & Evsukoff, A. (2008). Bayesian networks inference algorithm to implement Dempster Shafer theory in reliability analysis. *Reliability Engineering and System Safety, 93*(7), 950–963.

Spiegelhalter, D. J., Dawld, A. P., Lauritzen, S. L., & Cowell, R. G. (1993). Bayesian analysis in expert systems. *Statistical Science, 8*(8), 219–247.

Titterington, D. M. (1984). Recursive parameter estimation using incomplete data. *Journal of the Royal Statistical Society, Series B, 46*(2), 257–267.

Xu, D. L., Liu, J., Yang, J. B., Liu, G. P., Wang, J., Jenkinson, I., et al. (2007). Inference and learning methodology of belief-rule-based expert system for pipeline leak detection. *Expert Systems with Applications, 32*(1), 103–113.

Yang, J. B. (2001). Rule and utility based evidential reasoning approach for multi-attribute decision analysis under uncertainties. *European Journal of Operational Research, 131*(1), 31–61.

Yang, J. B., & Singh, M. G. (1994). An evidential reasoning approach for multiple attribute decision making with uncertainty. *IEEE Transactions on Systems, Man, and Cybernetics – Part A: Systems and Humans, 24*(1), 1–18.

Yang, J. B., Liu, J., Wang, J., Sii, H. S., & Wang, H. W. (2006a). Belief rule-base inference methodology using the evidential reasoning approach – RIMER. *IEEE Transactions on Systems, Man, and Cybernetics – Part A: Systems and Humans, 36*(2), 266–285.

Yang, J. B., Wang, Y. M., Xu, D. L., & Chin, K. S. (2006b). The evidential reasoning approach for MADA under both probabilistic and fuzzy uncertainties. *European Journal of Operational Research, 171*(1), 309–343.

Yang, Z., Li, X. H., & Lai, J. B. (2007). Analysis on the diffusion hazards of dynamic leakage of gas pipeline. *Reliability Engineering and System Safety, 92*(1), 47–53.

Yang, Z. L., Bonsall, S., & Wang, J. (2008). Fuzzy rule-based Bayesian reasoning approach for prioritization of failures in FMEA. *IEEE Transactions on Reliability, 57*(3), 517–528.

Zhou, Z. J., Hu, C. H., Yang, J. B., Xu, D. L., & Zhou, D. H. (2009). Online updating belief-rule-based system for pipeline leak detection under expert intervention. *Expert Systems with Applications, 36*(3), 7700–7709.