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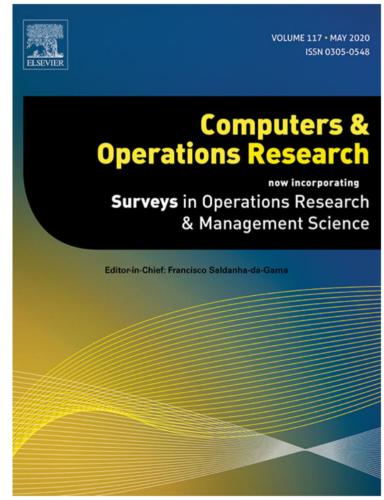

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A *Q*-learning-based multi-objective hyper-heuristic algorithm for energy-efficient integrated distributed hybrid flow-shop scheduling with preventive maintenance

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Author Contributions

The contributions of authors are as the follows.

1. Zi-Qi Zhang, Methodology, Funding acquisition, Investigation, Writing - original draft.

2. Xin-Yun Wu: Investigation, Methodology, Experiment, Software, Writing - original draft.

3. Bin Qian: Methodology, Funding acquisition, Investigation, Writing - review & editing.

4. Rong Hu: Methodology, Funding acquisition, Supervision, Writing - review & editing.

5. Jian-Bo Yang: Project administration.

ABSTRACT Driven by the dual engines of supply chain integration and low-carbon transformation in industrial Internet of Things (IIoT) manufacturing systems, energy-efficient integrated distributed scheduling has emerged as a pivotal component of industrial intelligence-driven smart manufacturing. This article investigates the energy-efficient integrated distributed hybrid flow shop scheduling problem with preventive maintenance (EE-IDHFSP-PM), which aims to minimize the dual objectives of makespan and total carbon emissions. In this study, a mixed-integer linear programming (MILP) model is established for the EE-IDHFSP-PM, making the first attempt to solve such NP-hard problem by using a *Q*-learning-based multi-objective hyper-heuristic algorithm (QLMHHA). First, a modified NEH-based initialization method is introduced to produce high-quality solutions that balance multiple optimization objectives, ensuring both the quality and diversity of initial populations. Second, a novel multi-stage collaborative energy-efficient strategy (MSC_EES) is developed to dynamically adjust the processing speeds of machines on non-critical paths, which reduces energy consumption across stages. Third, a new *Q*-learning-based high-level strategy (HLS) is devised to dynamically coordinate twelve low-level heuristics (LLHs) according to specific states, improving adaptive search efficiency through superior exploration-exploitation trade-offs. Fourth, a dual-criterion reward mechanism is proposed to evaluate population quality in terms of both convergence and diversity, which can deliver immediate feedback and effectively guide evolutionary processes. Fifth, comprehensive convergence and computational complexity analyses of critical components are conducted to confirm the stability, reliability, and efficiency of QLMHHA. Extensive experiments are carried out on 54 small-scale and 24 large-scale instances, which demonstrate that QLMHHA achieves promising performance in both effectiveness and efficacy against state-of-the-art multi-objective algorithms for addressing the EE-IDHFSP-PM. These findings validate the efficacy and superiority of QLMHHA in tackling complex scheduling challenges, providing valuable theoretical implications and practical insights for energy-efficient distributed manufacturing systems.

Keywords: hybrid flow shop scheduling; distributed scheduling; energy-efficient scheduling; hyper-heuristic algorithm; preventive maintenance.

1. Introduction

As economic globalization advances and market competition intensifies, traditional centralized manufacturing paradigms have become inadequate to address the dynamic demands of both markets and customers. Against the backdrop of the developmental trend of smart manufacturing propelled by industrial intelligence, distributed flexible manufacturing systems (FMSs), characterized by multi-factory collaboration and resource sharing, have garnered growing scholarly attention due to their strong strengths in rapidly responding to market dynamics, improving resource utilization, and enhancing production efficiency. Under such circumstances, distributed shop scheduling problems (DSPs) have garnered great interest within academic and industrial communities. Recent years have witnessed extensive emerging research efforts on various DSPs, including the distributed hybrid flow shop scheduling problem (DHFSP) (Wang et al.2020), distributed FSP (DFSP) (Pan et al.2023), and distributed assembly FSP (DAFSP) (Lin et al.2017). Due to the complexity of DSPs with strongly coupled and large-scale subproblems, these problems pose considerable challenges for developing effective scheduling strategies and techniques. The DHFSPs have been widely found in real-world manufacturing industries such as automated assembly systems, automotive manufacturing, and steel manufacturing industries (Shao et al.2020). However, practical production processes, particularly in automotive and electronics industries, necessitate the coordination and optimization of production, transportation, and assembly phases to ensure process coherence and resource efficiency throughout entire production lifecycles. Accordingly, recent research has expanded to investigate the integrated distributed HFSP (IDHFSP) (Jia et al.2023). While the majority of research on DSPs assumes that machines are always available and non-fatiguing over extended operational lifecycles, however, real-world scenarios inevitably involve progressive wear and stochastic breakdowns, resulting in periodic unavailability of machines or devices (Jia et al.2023). Such machine breakdowns and disruptions can severely disrupt production plans, potentially causing delivery delays, incurring additional costs, and leading to customer dissatisfaction. Hence, incorporating preventive maintenance (PM) strategies into production scheduling when solving the IDHFSP is essential for maintaining machine performance and increasing productivity throughout production processes.

Low-carbon manufacturing facilitates sustainable development through the efficient utilization of resources, environmentally friendly techniques, and comprehensive lifecycle management, which contributes to achieving carbon peak and carbon neutrality targets while driving green manufacturing transformation and high-quality development practices. Statistics indicate that industrial activities account for approximately 50% of global energy consumption and over 30% of carbon emissions (Lu et al.2021). Accordingly, it is urgent to adopt energy-efficient scheduling strategies for production processes to reduce emissions and carbon footprints. Nevertheless, modern enterprises often neglect sustainable manufacturing as well as energy conservation and emission reduction while pursuing economic efficiency. To strike a balance between economic benefits and environmental impacts, research focusing on the IDHFSP must incorporate both energy consumption and economic criteria to achieve synergistic optimization.

Motivated by the above insights, this work investigates a kind of DSP with practical significance, *i.e.*, the energy-efficient IDHFSP with preventive maintenance (EE-IDHFSP-PM). Table 1 provides an overview of related research on DHFSPs. Notably, the EE-IDHFSP-PM has not yet been studied, thereby highlighting the motivation of this study in bridging this research gap. The study of modeling and solving for the EE-IDHFSP-PM is closely related to the requirements of modern FMSs and has substantial theoretical and practical significance. Given that the HFSP has been proven NP-hard (Kis et al.2005) and the EE-IDHFSP-PM exhibits higher complexity than the HFSP, it follows that the EE-IDHFSP-PM is also NP-hard. Graham et al. (1979) introduced a tripartite notation representation $\alpha|\beta|\gamma$ to categorize scheduling problems, where α denotes production environments, β represents critical constraints, and γ indicates optimization objectives. Consequently, the IDHFSP-PM aiming at minimizing makespan is formulated as $DHF_m|PM,tspt,asmbly|C_{\max}$, where DHF_m indicates the distributed factories with m stages in the hybrid flow shop configuration; PM represents preventive maintenance; $tspt$ denotes transportation to assembly shops after processing; $asmbly$ represents the assembly stage; and $C_{\max}$ is the makespan criterion. Given that EE-IDHFSP-PM aims to minimize both makespan ($C_{\max}$) and total carbon emissions (TCE) with adjustable processing speeds (v_i), it is formulated as $DHF_m|PM,tspt,asmbly,v_i|C_{\max},TCE$. Since $DHF_m|PM,tspt,asmbly|C_{\max}$ has been demonstrated to be NP-hard (Jia et al.2023), its extended variant introduces additional complexity while retaining NP-hardness. So, obtaining robust and high-quality scheduling schemes within finite computational time is a significant challenge, which is of great importance for the study of IDHFSPs. Thus, developing effective algorithms for EE-IDHFSP-PM is a critical and impactful research efforts.

Due to the challenges in both engineering applications and theoretical investigations, research on IDHFSPs with complex constraints has emerged as a prominent research direction, leading to the development of various solution methods. However, for large-scale, strongly coupled IDHFSPs, exact mathematical methods inherent in the complexity of the decomposition and decoupling mechanisms often limit their practical application potential. As summarized in Table 1, recent studies have applied hybrid intelligent optimization algorithms (HIOAs) to address DHFSPs, including MPMA-QL (Jia et al.2023), QSFLA (Cai et al.2022), and QIGA (Qin et al.2023). These HIOAs employ specific search strategies to generate high-quality solutions within reasonable computational timeframes, showing significant strengths in solving strongly coupled complex DHFSPs (Zhang et al.2023). For addressing the EE-IDHFSP-PM, factory allocation of products, job sequencing, preventive maintenance, and machine speed selection are all subject to dynamic changes, resulting in highly complex search spaces. Hence, the development of efficient algorithms capable of yielding high-quality feasible solutions is crucial. Hyper-heuristic algorithms (HHAs), a subclass of HIOAs developed over recent decades, are designed to address such complex problems (Burke et al.2017).

HHAs employ a two-layer framework, where a set of low-level heuristics (LLHs) is coordinated by effective high-level strategies (HLSs). By dynamically selecting and sequencing LLHs, HHAs can produce high-quality solutions through the coordinated application of simple yet efficient heuristics.

Reinforcement learning (RL) enables agents to learn the best decision-making strategies through environmental interactions and select appropriate actions during iterative processes, similar to HLSs for LLHs in HHAs. Thus, employing RL-based methods as the HLSs presents promising perspectives. The RL-based HLSs can not only effectively evaluate the effectiveness of LLHs but also optimize heuristic sequences, while adapting to dynamic feedback, enhancing efficacy in tackling complex and dynamic scenarios. Among RL-based methods, the Q-learning algorithm is particularly noteworthy, as it can derive the best decisions by learning state-action relations through reward-driven iterations. Recent studies have demonstrated that HHAs incorporating Q-learning algorithms achieve remarkable success in solving various scheduling problems (Zhao et al.2023; Cheng et al.2022; Zhang et al.2023). Motivated by these advancements, this study introduces an effective and efficient Q-learning-based multi-objective hyper-heuristic algorithm (QLMHHA) to address the EE-IDHFSP-PM.

The main contributions of this study are summarized as follows:

- A mixed-integer linear programming (MILP) model is first established for the EE-IDHFSP-PM, which takes into account multi-stage preventive maintenance activities and aims to collaboratively optimize production efficiency and carbon emissions, thereby constructing an energy-efficient integrated distributed collaborative optimization framework suitable for sustainable distributed flexible manufacturing systems (FMSs).
- An MNEH-based initialization method is developed to generate high-quality and diverse initial populations, providing a stronger foundation for optimization processes. Furthermore, an effective multi-stage collaborative energy-efficient strategy (MSC_EES) based on non-critical paths is introduced to further reduce total carbon emissions.
- Twelve efficient low-level heuristics (LLHs) are designed to form a set of problem-specific search operators, enhancing the intensity of local search. Meanwhile, a novel Q-learning-based HLS incorporates a dual-criterion reward mechanism considering both diversity and convergence, enabling dynamic balancing between global exploration and local exploitation.
- An innovative sequential decision modeling method for the dynamic selection of LLHs as a finite Markov decision process is introduced. This design incorporates states, actions, and reward mechanisms based on problem characteristics. The rigorous convergence analysis demonstrates that QLMHHA can converge to the best Q-value functions under balanced

exploration-exploitation strategies, while providing computational complexity analyses of its critical components to clarify the efficiency of the algorithm.

- Comprehensive comparisons and statistical analysis with six state-of-the-art multi-objective optimization algorithms across different instance scales have validated the superiority of QLMHHA in addressing the EE-IDHFSP-PM, demonstrating that the proposed framework serves as a learning-driven paradigm with promising prospects.

The remainder of this paper is organized as follows: Section 2 reviews existing relevant literature. Section 3 describes the problem formulation of EE-IDHFSP-PM. Section 4 describes the framework of the proposed QLMHHA with comprehensive convergence analysis and computational complexity analysis. Section 5 provides experimental details and computational comparisons. Finally, Section 6 discusses conclusions and provides potential research directions.

Table 1 The relevant studies of DHFSPs.

Works	Problems	Objectives	Algorithms	PM	EE	TS
Jia et al. (2023)	DAHFSP	$C_{\max}$	Multi-population memetic algorithm with *Q*-learning (MPMA-QL)	√		√
Wang et al. (2020)	FDHFSP	$C_{\max}, TEC, TAI$	Shuffled frog leaping algorithm with collaboration of multiple search strategies (SFLA-CMSS)		√	
Yu et al. (2024)	DAHFSP	TT	Knowledge-based iterated greedy algorithm (KBIG)			
Shao et al. (2024)	DFHBFSP	$C_{\max}, TEC$	Meta-*Q*-learning-based multi-objective metaheuristic (MQL-MM)		√	
Shao et al. (2024)	DHHBFSP	$C_{\max}$	Feedback learning-based selection hyper-heuristic (FLS-HH)			
Wu et al. (2023)	DRHFSP	$C_{\max}, TC$	Improved multi-objective evolutionary algorithm based on decomposition with local search (IMOEA/D-LS)			
Shao et al. (2023)	DHHFLSP	$C_{\max}$	Iterated local search algorithm (ILS)			
Shao et al. (2023)	DHHBFSP	$C_{\max}$	Learning-based selection hyper-heuristic (LS-HH)			
Qin et al. (2023)	DHBFSP	TEC	*Q*-learning-based improved iterative greedy algorithm (QIGA)		√	
Lei et al. (2023)	DHFSP	$C_{\max}, TT$	Multi-class teaching-learning-based optimization (MTLBO)			
Shao et al. (2022)	DHHFSP	$C_{\max}, TES$	Ant colony optimization behavior-based multi-objective evolutionary algorithm based on decomposition (ACO_MOEA/D)		√	
Shao et al. (2022)	DHHFSP	TT, TPC, TCE	Network memetic algorithm (NMA)		√	
Qin et al. (2022)	DHHBFSP	$C_{\max}$	Collaborative iterative greedy algorithm (CIGA)			
Geng et al. (2022)	DHRHFSP	$C_{\max}, TEC$	Multi-objective artificial bee colony algorithm (MOABC)		√	
Cai et al. (2022)	DAHFSP	$C_{\max}$	*Q*-learning-based shuffled frog-learning algorithm (QSFLA)			√
Geng et al. (2021)	DRHFSP	$C_{\max}, TES$	Memetic algorithm (MA)		√	
Cai et al. (2021)	DHFSP	$C_{\max}, TEC, TAI$	Cooperated shuffled frog-leaping algorithm (CSFLA)		√	
Shao et al. (2020)	DHFSP	$C_{\max}$	DNEH with the multi-neighborhood iterated greedy algorithm			
Cai et al. (2020)	DHFSP	$C_{\max}$	Dynamic shuffled frog-leaping algorithm (DSFLA)			
Cai et al. (2020)	DHFSP	$C_{\max}, TT$	Shuffled frog-leaping algorithm with memeplex quality (MQSFLA)			

Constraints PM: Preventive maintenance; EE: Energy-efficient scheduling; TS: Three-stage production.

Objectives TT : Total tardiness; TEC : Total energy consumption; TES : Total energy cost; TCE : Total carbon emission; TPC : Total production cost; TAI : Total agreement index; TC : Transferring cost; $C_{\max}$: Makespan.

Problems Fuzzy distributed hybrid flow shop problem (FDHFSP); Distributed assembly hybrid flow shop scheduling problem (DAHFSP); Distributed fuzzy hybrid blocking flow-shop scheduling problem (DFHBFSP); Distributed heterogeneous hybrid blocking flow-shop scheduling problem (DHHBFSP); Distributed reentrant hybrid flow shop scheduling problem (DRHFSP); Distributed heterogeneous hybrid flow shop lot-streaming scheduling problem (DHHFLSP); Distributed hybrid blocking flow-shop scheduling problem (DHBFSP); Distributed heterogeneous hybrid flow shop scheduling problem (DHHFSP); Distributed heterogeneous reentrant hybrid flow shop scheduling problem (DHRHFSP).

2. Literature review

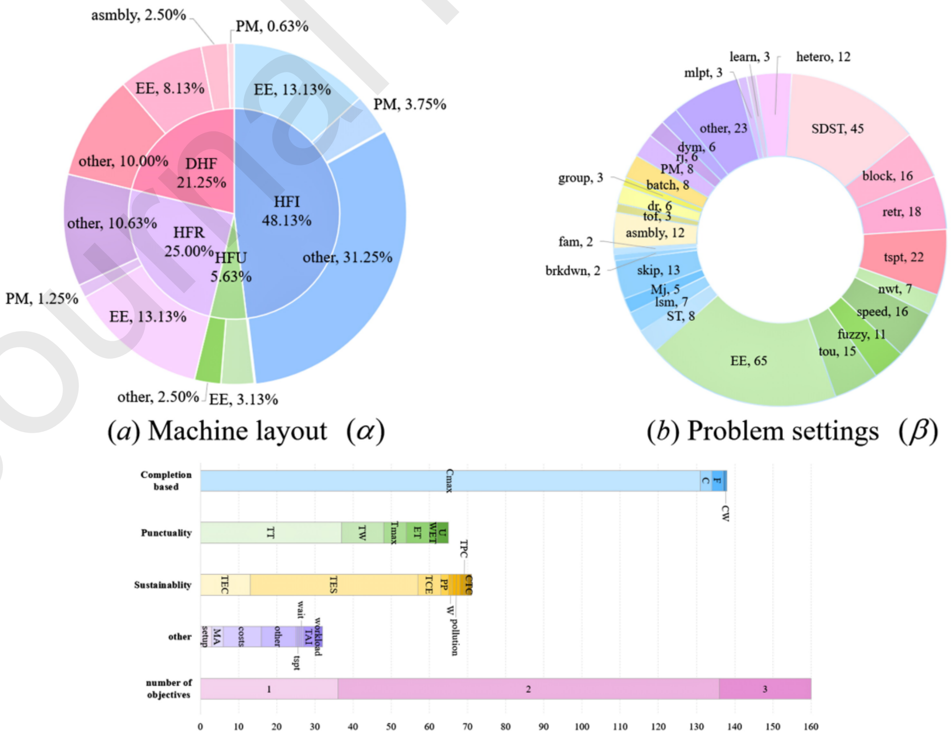
To the best of our knowledge, EE-IDHFSP-PM has not been studied in existing literature. Therefore, this section reviews the closely related research, including DHFSPs, energy-efficient scheduling problems, PM, and HHAs.

2.1. Related work on DHFSPs

Since the pioneering work of Arthanari et al. (1971) on the hybrid flow shop problem (HFSP), it has been extensively investigated for decades. Naderi et al. (2010) established the MILP model for the distributed extension of the HFSP, laying the foundation for subsequent research on the distributed HFSP (DHFSP). For DHFSP with the makespan criterion, Hao et al. (2019) formulated a mathematical model and employed a hybrid brain storm optimization (HBSO) algorithm. Shao et al. (2020) proposed the DNEH with the smallest-medium rule (DNEH_SMR) and a multi-neighborhood iterated greedy algorithm (IGA), which integrated multi-neighborhood local search to enhance exploitation capability. Cai et al. (2020) devised a dynamic shuffled frog-leaping algorithm (DSFLA) to efficiently obtain high-quality solutions. Recent years have witnessed emerging research efforts extending the DHFSP to diverse scenarios. To address production uncertainty, Cai et al. (2021) studied the energy-efficient DHFSP considering fuzzy processing times and devised a cooperative shuffled frog-leaping algorithm (CSFLA) to minimize makespan, *TAI*, and fuzzy *TEC*. Shao et al. (2022) investigated the heterogeneous DHFSP and proposed a network memetic algorithm (NMA) based on solution space and strategy space exploration. Qin et al. (2022) further focused on blocking constraints in distributed heterogeneous hybrid flow shop scheduling problem (DHHBFSP), and they presented a collaborative iterative greedy algorithm (CIGA) with neighborhood search strategy to reduce blocking times. Shao et al. (2023) developed a learning-based selection hyper-heuristic (LS-HH) algorithm for DHHBFSP. In LS-HH, the learned probabilistic model serves as the HLS to manage LLHs. Shao et al. (2024) extended this work by considering flexible assembly, setup time constraints, and proposed a feedback learning-based selection hyper-heuristic (FLS-HH) algorithm. For DHFSP with sequence-dependent setup times, Cai et al. (2020) introduced a new SFLA with memeplex quality (MQSFLA) to minimize the total tardiness and makespan. Lei et al. (2023) proposed a multi-class teaching-based optimization (MTLBO) algorithm to simultaneously optimize makespan and maximal tardiness. Shao et al. (2023) investigated the DHHFSP with lot streaming constraints for makespan minimization. Pan et al. (2023) developed an improved multi-objective evolutionary algorithm based on decomposition (IMOEA/D) with local search (IMOEA/D-LS) to minimize makespan and *TC* in DHFSP with reentrant constraints. More recently, Yu et al. (2024) presented a knowledge-based iterated greedy algorithm (KBIG) to minimize total tardiness for tackling DHFSP integrated with assembly lines and resource constraints.

Furthermore, Cai et al. (2022) studied a DHFSP integrating production, transportation, and assembly stages and developed a Q-learning-based SFLA (QSFLA) to minimize makespan. Jia et al. (2023) formulated a mixed-integer linear programming (MILP) model incorporating preventive maintenance (PM) and proposed a multi-population memetic algorithm with Q-learning (MPMA-QL). Existing studies on DHFSPs primarily focus on static production processes, assuming uninterrupted machine availability. However, such assumptions overlook three critical constraints in real industrial scenarios: (1) machine capability deteriorates progressively or experiences breakdowns over time, necessitating PM to ensure operational stability; (2) production operations require collaborative execution across production, transportation, and assembly stages, with dynamic resource coupling between stages; and (3) machine lifetime interacts with job processing, and neglecting PM activities may increase risks of production interruptions and total cost increases. Thus, it is essential to develop efficient scheduling methods for DHFSPs that integrate PM strategies and enable multi-stage collaborative optimization.

To examine emerging research trends and insights, we surveyed over 160 articles on HFSPs. These studies are categorized according to the machine layout criteria (α), namely, identical parallel machines (HFI), uniform parallel machines (HFU), unrelated parallel machines (HFR), and distributed hybrid flow shop (DHF). Fig.1. illustrates the classification of criteria across these articles. In addition, we also provide investigations on problem settings (β) and optimization objectives (γ). Table 2 summarizes details of the problem settings and objectives in these studies.



(a) Machine layout (α)

(b) Problem settings (β)

(c) Type of considered objectives (γ)

Fig. 1. Classification of literature related to HFSPs (160^+ articles).

Table 2 Glossary of abbreviations used for problem settings and objectives.

Problem settings		Objectives	
SDST	Sequence-dependent setup times	C_{max}	Maximum completion time
block	Limited/no buffers	TT	Total tardiness/average tardiness
retr	Reentrant	setup	Total setup time
tspt	Transportation/crane transportation	TW	Total weighted tardiness
nwt	No-wait/limited waiting times	T_{max}	Maximum tardiness
speed	Variable speed levels	ET	Total earliness and tardiness
fuzzy	Fuzzy/stochastic/interval numbers	TES	Total energy cost
tou	Time-of-use electricity prices	TEC	Total energy consumption
rj	Release times	WET	Weighted earliness and tardiness
ST	Setup times	C	Total (mean) completion time
lsm	Lot streaming/splitting/sizing	TCE	Total carbon dioxide emission
Mj	Machine/factory eligibility restrictions	PP	Peak power
skip	Skipping	MA	Machine availability/unavailability
brkdwn	Machine failures/breakdown	costs	Costs
fam	Family setups	other	Others
asmbly	Assembly stage	F	Total flow time/mean flow time
tof	Turn on and off control strategy	tspt	Total transportation time
dr	Dual resource constraints/worker constraints	U	Number of tardy jobs
group	Group scheduling	wait	Waiting time
batch	Batch scheduling	W	Material wastage
dym	Dynamic scheduling	pollution	Noise pollution/dust pollution
EE	Energy-efficient/energy awareness/green scheduling	TAI	Total agreement index
PM	Preventive maintenance	workload	Workload balance/total workload
other	Others	CW	Total weighted completion time
mlpt	Multiprocessor tasks	TPC	Total production cost
learn	Learning effects/skills	CTC	Carbon trading cost
hetero	Heterogeneous factories		

2.2. Related work on energy-efficient scheduling problems

With increasing global emphasis on green development and sustainability, the research focus in the field of shop scheduling has gradually shifted towards energy-efficient scheduling (Lu et al.2021; Wang et al.2021; Zhang et al.2022). For energy-efficient HFSP, Wang et al. (2019) considered two energy-saving strategies, *i.e.*, TOU electricity pricing and machine shutdown/restart, and developed bi-objective taboo search algorithm and ant colony optimization (ACO) algorithm to optimize both makespan and *TEC*. Extensive experiments showed the algorithms' efficacy in solving medium- and large-scale instances. Schulz et al. (2020) investigated a variable discrete production speed strategy to reduce *TEC*, while Lian et al. (2021) focused on *TEC* in steelmaking production and proposed an improved MOEA/D (IMOEA/D), which significantly reduced energy usage in steelmaking processes. Qin et al. (2022) proposed an improved IGA for energy-efficient HFSP with blocking constraints. They used a half-exchange-based global perturbation strategy to further reduce energy consumption. Regarding energy-efficient DHFSP, Hong et al. (2021) studied scheduling incorporating eligibility constraints and proposed an IMOEA/D to optimize *TEC*, total handling distance, and makespan. The IMOEA/D employed a novel decoding approach to reduce energy consumption while balancing machine load, with experimental results confirming its effectiveness. Shao et al. (2022) examined a DHHFSP under non-identical TOU electricity pricing (DHHFSP-NTOU) and also designed an ACO behavior-based MOEA/D (ACO_MOEA/D) to optimize both makespan and *TEC*. ACO_MOEA/D used a right-shift strategy to reduce electricity costs. Geng et al. (2022) proposed an energy-saving operator based on right-shifting, which avoids scheduling energy-intensive tasks during peak pricing periods without affecting production efficiency. Qin et al. (2023) formulated a mathematical model for DHFSP with blocking constraints and developed a *Q*-learning-based IGA (QIGA) to minimize makespan and *TEC*. For energy-efficient DHFSP in heterogeneous environments, Wang et al. (2022) proposed a cooperative memetic algorithm (CMA) to optimize makespan and *TEC*, incorporating two energy-saving strategies to improve non-dominated solution quality. Zhang et al. (2024) focused on heterogeneous shop constraints and proposed a multi-objective memetic algorithm featuring a global search strategy to accelerate Pareto front convergence. Under uncertain environments, Shao et al. (2024) investigated a DFHBFSP with uncertain processing and setup times and developed a meta-*Q*-learning-based multi-objective metaheuristic (MQL-MM) to minimize fuzzy makespan and *TEC*. In conclusion, research on energy-efficient DHFSPs remain limited, with many real-world production constraints not yet addressed. Notably, no existing study has integrated multi-stage coordination with preventive maintenance (PM) for energy-efficient DHFSP, presenting a significant gap in domains.

As enterprises increasingly emphasize the effect of PM, related studies of shop scheduling with PM are emerging. For FJSPs with PM, Wocker et al. (2023) incorporated PM activities and developed a local search method to optimize makespan and machine availability. Wang et al. (2024) established a failure rate model to determine machine maintenance requirements in high-frequency production switching environments. For FSPs with PM, Yu et al. (2016) introduced the concept of flexibility and diversity for PM, enabling machines to undergo maintenance at variable intervals with different PM types. Miyata et al. (2019) extended the PM to the *m*-machine no-wait FSP based on the concept of diversity of PM proposed by Yu et al. (2016). Zhang et al. (2021) integrated the PM into a two-stage AFSP where each machine was set initial maintenance level (ML) and used these ML thresholds to determine better PM periods. Regarding HFSPs with PM, Wang et al. (2013) proposed a multi-objective taboo search method for the two-stage HFSP, aiming to minimize makespan and machine unavailability in the first stage. Li et al. (2014) proposed a hybrid algorithm combining PSO and iterative local search (ILS). Zandieh et al. (2017) investigated the HFSP_SDST and performed PM on machines to reduce machine unavailability. Jia et al. (2023) considered PM into the DAHFSP by adding decision variables to determine whether to execute PM activities. Notably, Anunay Alexander et al. (2023) extended PM activities to multi-stage assembly processes via similar decision variables. Existing efforts demonstrate a predominant focus on PM in HFSPs, with limited attention to DHFSPs. Thus, this paper bridges this gap by extending PM integration to DHFSPs, inspired by Jia et al. (2023). Specifically, 1) each machine has an initial age and a uniform runtime upper bound; 2) machine age accumulates during operation but resets after PM; and 3) machine breakdowns occur upon reaching maximum runtime. Therefore, it is important to determine appropriate PM activities for machines.

2.4. Related Work on HHAs

Hyper-heuristic algorithms (HHAs) have emerged as a prominent paradigm for tackling complex optimization problems, gaining significant research interest from researchers (Dokeroglu et al.2024). HHAs can be classified into two categories: heuristic selection and heuristic generation (Burke et al.2019). While heuristic generation focuses on creating LLHs through modification and combination of existing ones, heuristic selection determines appropriate heuristics from a predefined pool through high-level strategies (HLSs). The HLS is critical in heuristic selection, enabling adaptive responses to different states by analyzing current conditions and guiding LLH selection, thus enhancing solution quality and computational efficiency. There are various approaches as HLS to manage and manipulate LLHs, such as backtracking search-based HHA (Lin et al.2017), feedback learning-based HHA (Shao et al.2024), genetic programming-based HHA (GP-HH) (Kieffer et al.2020), and RL-based HHA (Choong et al.2018). According to recent review by Zhang et al. (2023), the most prevalent HLSs are genetic programming (GP) and reinforcement learning (RL). GP-based HHAs have proven effective stochastic RCPSP (Zhang et al.2024). However, GP-based HHAs suffer from inherent limitations: its dependence on terminal sets for problem representation introduces redundant or irrelevant features, which unnecessarily expand the search scope and impede convergence (Zhang et al.2021). In contrast, RL-based HHAs circumvent these issues by directly learning optimal policies by reward mechanisms, thereby focusing on effective strategy design while maintaining controlled search spaces, improving search efficiency and convergence speed. These RL-based HHAs have demonstrated notable success in shop scheduling problems, including the energy-aware mixed shop scheduling problem (Cheng et al.2022), distributed assembly blocking FSP (Zhang et al.2023), distributed FJSP (Zhang et al.2023), energy-efficient distributed blocking FSP (Zhao et al.2023), and energy-efficient IDHFSP (Wu et al., 2025). Although existing studies have demonstrated the effectiveness of HHAs in shop scheduling problems, their application to the EE-IDHFSP-PM remains unexplored. Motivated by this research insight, this study introduces a novel *Q*-learning-based multi-objective HHA (QLMHHA) for the EE-IDHFSP-PM. The innovations between QLMHHA and existing HHAs are summarized in Table 3.

Table 3 Differences between this work and the previous literature.

Difference	The previous literature	This work
Learning mechanism	Most HHAs depend on offline strategies or fixed weights, which are inadequate for adapting to dynamic environments, show weak adjustment ability, and suffer from limited adaptability across scenarios (Kieffer et al.2020; Song et al.2021; Lin et al.2017; Fan et al.2021; Zhang et al.2024).	An online *Q*-learning mechanism is introduced, which has self-adaptation, self-learning, and real-time updating capabilities. It continuously adjusts the behavior of high-level search strategies, significantly enhancing robustness in uncertain scheduling environments.
Search behavior	Most HHAs executed LLHs independently and lack collaborative mechanism, which makes it difficult to form the hierarchical and the phased collaborative search paths (Shao et al.2024; Zhao et al.2023; Cheng et al.2022; Choong et al.2018; Wu et al., 2025).	LLHs are combined to form HLIs, and *Q*-learning is used to dynamically learn and optimize the orders of LLHs. A dual-criterion reward mechanism promotes the identification of optimal heuristic combinations, improving both search depth and algorithmic adaptability.
Reward function	Existing reward mechanisms primarily rely on numerical variations in objective functions (*e.g.*, *makespan*, *TEC*), which are insufficient for capturing overall solution quality or trade-offs among multiple objectives (Zhao et al.2023; Choong et al.2018; Zhang et al.2023; Zhang et al.2023).	A comprehensive feedback mechanism based on the convergence and diversity evaluation of the Pareto solution set is constructed to improve the recognition accuracy of overall solution quality and is particularly suited for complex multi-objective optimization problems such as EE-IDHFSP-PM.

3. Problem statement

3.1. Multi-objective optimization

Typically, a multi-objective optimization problem (MOP) can be formally defined as follows:

$$\begin{cases} \min_{z\in\Omega} \mathbf{F}(\mathbf{z}) = \left[f_1(\mathbf{z}), f_2(\mathbf{z}), \ldots, f_\eta(\mathbf{z})\right] \\ s.t. \quad g_j(\mathbf{z}) \le 0, j = 1,2,\ldots,p \\ \qquad h_k(\mathbf{z}) = 0, k = 1,2,\ldots,q \end{cases} . \tag{1}$$

where $\mathbf{z} = [z_1, z_2, \ldots, z_\eta]^T \in \mathbb{R}^\eta$ represents a candidate solution within the feasible domain $\mathbf{\Omega}$, and z_i is the ith decision variable, defined in $\mathbf{\Omega} \in \mathbb{R}^\eta$. $f_1(\mathbf{z}), f_2(\mathbf{z}), \ldots, f_\eta(\mathbf{z})$ denotes η conflicting objective functions. Key concepts of Pareto optimality are defined as follows:

- **Pareto dominance**: For two solutions $\mathbf{z} \in \mathbf{\Omega}$ and $\mathbf{z}' \in \mathbf{\Omega}$, $\mathbf{z}$ is said to dominate $\mathbf{z}'$ (denoted as $\mathbf{z} \prec \mathbf{z}'$) if and only if $f_i(\mathbf{z}) \le f_i(\mathbf{z}')$ for $\forall i \in \{1, \ldots, \eta\}$, and $f_j(\mathbf{z}) < f_j(\mathbf{z}')$ for $\exists j \in \{1, \ldots, \eta\}$.
- **Non-dominated solution**: A solution $\mathbf{z} \in \mathbf{\Omega}$ is a non-dominated solution if $\nexists \mathbf{z}' \in \mathbf{\Omega}$ satisfies $\mathbf{z}' \prec \mathbf{z}$.
- **Non-dominated set**: A set $A \in \mathbf{\Omega}$ is a non-dominated set if solutions $\mathbf{z} \in A$ are non-dominated, *i.e.*, $\nexists \mathbf{z}' \in \mathbf{\Omega}$ such that $\mathbf{z}' \prec \mathbf{z}$ for $\forall \mathbf{z} \in A$.
- **Pareto optimal solution:** If there is no solution $\mathbf{z}' \in \mathbf{\Omega}$ such that $\mathbf{z}' \prec \mathbf{z}^*$, $\mathbf{z}^*$ is Pareto optimal.
- **Pareto front (PF)**: The PF forms the collection of all non-dominated solutions in the objective space, representing the optimal trade-off surface between conflicting objectives.

3.2. Problem description

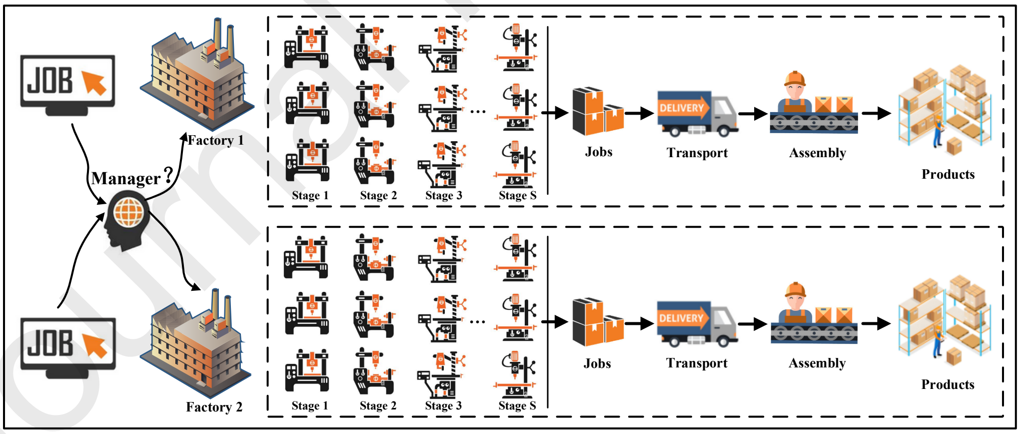


Fig. 2. Schematic diagram of the EE-IDHFSP-PM.

The schematic diagram of EE-IDHFSP-PM, illustrated in Fig. 2, is described as follows. There is a set of H products, denoted as $\boldsymbol{P} = \{P_1, P_2, \ldots, P_H\}$, are composed of N jobs $\boldsymbol{J} = \{J_1, J_2, \ldots, J_N\}$, where each product is assembled from a subset of jobs, and each job belongs only to one product. All jobs belonging to the same product are processed in the same factory layout as a three-stage hybrid flow shop: (1) production stage has S sub-production stages, each equipped with multiple unrelated parallel machines for flexible processing of jobs; (2) transportation stage allows the completed jobs with critical constraints including adherence to transportation stage processing order and resource allocation for both transportation and assembly per factory. Due to the processing time of jobs or products may be affected by machine wear, the linear degradation function is introduced to describe the degradation process of each machine, that is, the connection between the service life and the practical processing time of machines (Li et al.2020). To ensure the stable operation of machines, PM activities are conducted on these machines. Assume that each machine has an initial age a, and this age decreases as the machine runs. However, this age must not reach the upper limit of machine's cumulative operational threshold Γ (Jia et al.2023); Otherwise, the machine may breakdown. So, to avoid this situation, PM activities are required to restore the age to its original state. Furthermore, we consider machine energy consumption with machines' processing speeds at configurable speed levels $\boldsymbol{V}_{set} = \{V_1, V_2, \ldots, V_d\}$, where speeds are unchanged during processing and energy consumption accounts for both processing and standby states. EE-IDHFSP-PM meets the following conditions: Machines are available at initial time, and operations should be uninterrupted processing without preemption; Each operation should be processed on one machine, and each machine is limited to process one operation at any time; Once factory allocation for a product is determined, the product cannot be transferred between factories, and all operations contained in the product should be processed in the same factory; Machine startup delay is not considered and PM time for all machines is the same; Cumulative processing time of the machine does not exceed the upper limit Γ. Related notation definitions are summarized in Table 4.

Table 4 Notation and definition in the mathematical model of EE-IDHFSP-PM.

Notation	Description
Indices	
i, i'	Index of jobs, $i, i' = \{1, 2, ..., N\}$.
g, g'	Index of products, $g, g' = \{1, 2, ..., H\}$.
f	Index of factories, $f = \{1, 2, ..., \overline{f}\}$.
r	Index of speeds, $r = \{1, 2, ..., d\}$.
l	Index of operations of jobs, $l = \{1, 2, ..., S\}$.
m	Index of machines in the production stage, $m = \{1, 2, ..., m_{sub} \times S\}$.
k	Index of job sequences on the same machine in the production stage, $k = \{1, 2, ..., n\}$.
q	Index of product sequences in the transportation and assembly stages, $q = \{1, 2, ..., h\}$.
Sets	
$\boldsymbol{V}_{set}$	Set of speeds, $\boldsymbol{V}_{set} = \{V_1, V_2, \ldots, V_d\}$.
$\boldsymbol{M}$	Set of machines in the production stage, $\boldsymbol{M} = \{M_1, M_2, ..., M_{m_{sub} \times S}\}$.
$\boldsymbol{J}$	Set of jobs, $\boldsymbol{J} = \{J_1, J_2, ..., J_N\}$.

$\boldsymbol{F}$	Set of factories, $\boldsymbol{F}=\{F_1,F_2,...,F_{\bar{f}}\}$.
$\boldsymbol{P}$	Set of products, $\boldsymbol{P}=\{P_1,P_2,...,P_H\}$.
$\boldsymbol{\Omega_g}$	Set of jobs belonging to product P_g.
$\boldsymbol{\Omega_f}$	Set of jobs assigned to factory F_f.
Parameters	
H	Total number of products.
N	Total number of jobs.
d	Total number of speeds.
$\bar{f}$	Total number of factories.
S	Total number of production sub-stages.
m_{sub}	Total number of machines in each production sub-stage.
$O_{i,l}$	The operation l of J_i at the production stage.
M_T	The transportation machine.
M_A	The assembly machine.
$p_{i,l}^{P,m}$	The processing time for operation $O_{i,l}$ on machine M_m.
p_g^T	The transportation time for product P_g on machine M_T.
p_g^A	The assembly time for product P_g on machine M_A.
$p_{i,l}^{P,m,r}$	The processing time for operation $O_{i,l}$ on machine M_m at speed V_r, $p_{i,l}^{P,m,r}=p_{i,l}^{P,m}/V_r$.
$p_g^{T,r}$	Transportation time for product P_g on machine M_T at speed V_r, $p_g^{T,r}=p_g^T/V_r$.
$p_g^{A,r}$	The assembly time for product P_g on machine M_A at speed V_r, $p_g^{A,r}=p_g^A/V_r$.
$E_{m,r}$	Energy consumption per unit time for machine M_m operating at speed V_r.
SE_m	Energy consumption per unit time for machine M_m in standby mode.
$E_{T,r}$	Energy consumption per unit time for machine M_T operating at speed V_r.
SE_T	Energy consumption per unit time for machine M_T in standby mode.
$E_{A,r}$	Energy consumption per unit time for machine M_A operating at speed V_r.
SE_A	Energy consumption per unit time for machine M_A in standby mode.
ε_P	Machine deterioration factor at the production stage.
ε_T	Machine deterioration factor at the transportation stage.
ε_A	Machine deterioration factor at the assembly stage.
t_{PM}^P	Duration of PM in the production stage.
t_{PM}^T	Duration of PM in the transportation stage.
t_{PM}^A	Duration of PM in the assembly stage.
L	A large positive number.
Γ	Upper limit of cumulative operating time for machines.
β	Coefficient relating energy consumption to carbon emissions, where β =0.7559; β refers to the carbon emission per unit of energy consumption (kilogram CO_2 equivalent/kilowatt-hour).

A_g^T	The actual transportation time for product P_g .
A_g^A	The actual assembly time for product P_g .
$C_{i,l}^P$	The actual processing completion time for operation $O_{i,l}$.
C_g^T	The actual transportation completion time for product P_g .
C_g^A	The actual assembly completion time for product P_g .
$a_{i,l}^P$	The age of the machine prior to operation $O_{i,l}$.
a_g^T	The age of the machine prior to transporting product P_g .
a_g^A	The age of the machine prior to assembling product P_g .
$E_{i,l}^P$	The earliest processing start time for operation $O_{i,l}$.
E_g^T	The earliest transportation start time for product P_g .
E_g^A	The earliest assembly start time for product P_g .
Objective functions	
C_{max}	The maximum completion time of all factories, *i.e.*, makespan.
TCE	Total carbon emission.
Binary decision variables	
$\xi_{i,l}^P$	If PM is performed before processing operation $O_{i,l}$, value is 1; otherwise, 0.
ξ_g^T	If PM is performed before transporting product P_g ,value is 1; otherwise, 0.
ξ_g^A	If PM is performed before assembling product P_g , value is 1; otherwise, 0.
$X_{i,l,f,m,k}$	If operation $O_{i,l}$ is processed at the kth position on machine M_m in factory F_f , value is 1; otherwise, 0.
$Y_{g,f,q}$	If product P_g is transported at the qth position on machine M_T in factory F_f , value is 1; otherwise, 0.
$Z_{g,f,q}$	If product P_g is assembled at the qth position on machine M_A in factory F_f , value is 1; otherwise, 0.
$x_{m,r}^t$	If machine M_m operates at speed V_r at time t , value is 1; otherwise, 0.
y_m^t	If machine M_m is idle at time t , value is 1; otherwise, 0.
$x_{T,r}^t$	If machine M_T operates at speed V_r at time t , value is 1; otherwise, 0.
y_T^t	If machine M_T is idle at time t , value is 1; otherwise, 0.
$x_{A,r}^t$	If machine M_A operates at speed V_r at time t , value is 1; otherwise, 0.
y_A^t	If machine M_A is idle at time t , value is 1; otherwise, 0.

The MILP model for EE-IDHFSP-PM is as follows:

objective:

$$\min\{C_{\max}, TCE\} \tag{1}$$

subject to:

$$C_{\max} = \max(E_g^A + A_g^A), \forall g \tag{2}$$

$$E_g^A \ge E_g^T + A_g^T, \forall g \tag{3}$$

$$E_g^T \ge E_{i,l}^P + A_{i,l}^P, \forall g, l, i \in \boldsymbol{\Omega_g} \tag{4}$$

$$E_{i',l}^{P} \geq E_{i,l}^{P} + A_{i,l}^{P} + \xi_{i',l}^{P} t_{PM}^{P} - L\left(2 - X_{i,l,f,m,k-1} - X_{i',l,f,m,k}\right), \quad \forall i, i', l, f, m, k \geq 2 \tag{6}$$

$$E_{g'}^{T} \geq E_{g}^{T} + A_{g}^{T} + \xi_{g'}^{T} t_{PM}^{T} - L\left(2 - Y_{g,f,q-1} - Y_{g',f,q}\right), \quad \forall g, g', f, q \geq 2 \tag{7}$$

$$E_{g'}^{A} \geq E_{g}^{A} + A_{g}^{A} + \xi_{g'}^{A} t_{PM}^{A} - L\left(2 - Z_{g,f,q-1} - Z_{g',f,q}\right), \quad \forall g, g', f, q \geq 2 \tag{8}$$

$$E_{i,1}^{P} \geq 0, \forall i \tag{9}$$

$$a_{i,l}^{P} \geq 0, \forall i, l \tag{10}$$

$$a_{g}^{T} \geq 0, \forall g \tag{11}$$

$$a_{g}^{A} \geq 0, \forall g \tag{12}$$

$$A_{i,l}^{P} = p_{i,l}^{P,m,r} + \varepsilon_{P} a_{i,l}^{P}, \forall i, l, r, m \tag{13}$$

$$A_{g}^{T} = p_{g}^{T,r} + \varepsilon_{T} a_{g}^{T}, \forall g, r \tag{14}$$

$$A_{g}^{A} = p_{g}^{A,r} + \varepsilon_{A} a_{g}^{A}, \forall g, r \tag{15}$$

$$a_{i',l}^{P} \geq a_{i,l}^{P} + A_{i,l}^{P} - L\left(2 - X_{i,l,f,m,k-1} - X_{i',l,f,m,k} + \xi_{i',l}^{P}\right), \quad \forall i, i', l, f, m, k \geq 2 \tag{16}$$

$$a_{g'}^{T} \geq a_{g}^{T} + A_{g}^{T} - L\left(2 - Y_{g,f,q-1} - Y_{g',f,q} + \xi_{g'}^{T}\right), \quad \forall g, g', f, q \geq 2 \tag{17}$$

$$a_{g'}^{A} \geq a_{g}^{A} + A_{g}^{A} - L\left(2 - Z_{g,f,q-1} - Z_{g',f,q} + \xi_{g'}^{A}\right), \quad \forall g, g', f, q \geq 2 \tag{18}$$

$$a_{i,l}^{P} \geq -L\left(1 - \xi_{i,l}^{P}\right), \forall i, l \tag{19}$$

$$a_{g}^{T} \geq -L\left(1 - \xi_{g}^{T}\right), \forall g \tag{20}$$

$$a_{g}^{A} \geq -L\left(1 - \xi_{g}^{A}\right), \forall g \tag{21}$$

$$\xi_{i,l}^{P} \leq 1 - \sum_{f=1}^{\overline{f}} \sum_{m=1}^{m_{sub}} X_{i,l,f,m,1}, \forall i, l \tag{22}$$

$$\xi_{g}^{T} \leq 1 - \sum_{f=1}^{\overline{f}} Y_{g,f,1}, \forall g \tag{23}$$

$$\xi_{g}^{A} \leq 1 - \sum_{f=1}^{\overline{f}} Z_{g,f,1}, \forall g \tag{24}$$

$$\sum_{m=1}^{m_{sub}} \sum_{k=1}^{n} X_{i,l,f,m,k} = \sum_{m'=1}^{m_{sub}} \sum_{k'=1}^{n} X_{i',l,f,m',k'}, \quad \forall g, l, f; i, i' \in \boldsymbol{\Omega_g} \tag{25}$$

$$\sum_{q=1} Y_{g,f,q} = \sum_{m=1} \sum_{k=1} X_{i,l,f,m,k}, \quad \forall g,l,f; i \in \boldsymbol{\Omega_g} \tag{26}$$

$$\sum_{q=1}^{h} Z_{g,f,q} = \sum_{m=1}^{m_{sub}} \sum_{k=1}^{n} X_{i,l,f,m,k}, \quad \forall g,l,f; i \in \boldsymbol{\Omega_g} \tag{27}$$

$$\sum_{f=1}^{\overline{f}} \sum_{m=1}^{m_{sub}} \sum_{k=1}^{n} X_{i,l,f,m,k} = 1, \forall i,l \tag{28}$$

$$\sum_{i=1}^{N} X_{i,l,f,m,k} \le 1, \forall l,f,m,k \tag{29}$$

$$\sum_{g=1}^{H} Y_{g,f,q} \le 1, \forall f,q \tag{30}$$

$$\sum_{g=1}^{H} Z_{g,f,q} \le 1, \forall f,q \tag{31}$$

$$\sum_{i=1}^{N} X_{i,l,f,m,k} \le \sum_{i=1}^{N} X_{i,l,f,m,k-1}, \quad \forall l,f,m,k \ge 2 \tag{32}$$

$$\sum_{g=1}^{H} Y_{g,f,q} \le \sum_{g=1}^{H} Y_{g,f,q-1}, \forall f, q \ge 2 \tag{33}$$

$$\sum_{g=1}^{H} Z_{g,f,q} \le \sum_{g=1}^{H} Z_{g,f,q-1}, \forall f, q \ge 2 \tag{34}$$

$$\xi_{i',l}^{P} = \begin{cases} 1, \ if \ p_{i',l}^{P,m,r} + (1+\varepsilon_P)\left(a_{i,l}^{P} + A_{i,l}^{P}\right) > T \ and \\ \quad X_{i,l,f,m,k-1} + X_{i',l,f,m,k} = 2, \forall i,i',l,f,m,r,k \ge 2 \\ 0, \ otherwise \end{cases} \tag{35}$$

$$\xi_{g'}^{T} = \begin{cases} 1, \ if \ p_{g'}^{T,r} + (1+\varepsilon_T)\left(a_{g}^{T} + A_{g}^{T}\right) > T \ and \\ \quad Y_{g,f,k-1} + Y_{g',f,k} = 2, \forall g,g',f,r,k \ge 2 \\ 0, \ otherwise \end{cases} \tag{36}$$

$$\xi_{g'}^{A} = \begin{cases} 1, \ if \ p_{g'}^{A,r} + (1+\varepsilon_A)\left(a_{g}^{A} + A_{g}^{A}\right) > T \ and \\ \quad Z_{g,f,k-1} + Z_{g',f,k} = 2, \forall g,g',f,r,k \ge 2 \\ 0, \ otherwise \end{cases} \tag{37}$$

$$C_{i,l}^{P} = C_{i,l-1}^{P} + t_{PM}^{P} \xi_{i,l}^{P} + A_{i,l}^{P}, \forall i,l \tag{38}$$

$$C_{g}^{T} = C_{g-1}^{T} + t_{PM}^{T} \xi_{g}^{T} + A_{g}^{T}, \forall g \tag{39}$$

$$C_{g}^{A} = C_{g-1}^{A} + t_{PM}^{A} \xi_{g}^{A} + A_{g}^{A}, \forall g \tag{40}$$

$$TCE = \beta \sum_{f \in \boldsymbol{F}} \left(\begin{array}{l} \int_0^{\max_{\forall i} C_{i,S}} \sum_{m \in \{1,2,\mathrm{K}, m_{sub} \times S\}} \left(E_{m,r} x_{m,r}^t + SE_m y_m^t \right) dt + \\ \int_{\min_{\forall g} E_g^T}^{\max_{\forall g} C_g^T} \left(E_{T,r} x_{T,r}^t + SE_T y_T^t \right) dt + \\ \int_{\min_{\forall g} E_g^A}^{\max_{\forall g} C_g^A} \left(E_{A,r} x_{A,r}^t + SE_A y_A^t \right) dt \end{array} \right), i \in \boldsymbol{\Omega_g}, g \in \boldsymbol{\Omega_f} \tag{41}$$

The objective is to minimize both $C_{\max}$ and TCE, as formulated in Eq. (1). Eq. (2) defines the maximum completion time of products. Eqs. (3)-(4) define the earliest transportation times and the earliest assembly times of products. Eq. (5) determines the earliest production time of jobs. Eqs. (6)-(8) determine precedence relationships between jobs or products on machines across production, transportation, and assembly stages, accounting for machine maintenance by allocating PM time to preceding jobs or products. Eq. (9) determines the earliest start time for the first operation of jobs. Eqs. (10)-(12) ensure that the initial age of each machine in the three stages is zero. Eqs. (13)-(15) represent the actual completion times of the jobs or products. Eqs. (16)-(18) describe the updated formula for machine age in the absence of machine maintenance. Eqs. (19)-(21) ensure that the age of the machine will be reset after PM. Eqs. (22)-(24) restrict that PM activities are not required before the machine first processes an operation. Eqs. (25)-(27) ensure that jobs belonging to a product must be allocated to the same factory. Eq. (28) limits each job's operations to be processed only on specific machines in the assigned factory. Eqs. (29)-(31) dictate that each machine can only process one job or one product simultaneously. Eqs. (32)-(34) prevent any machine from having unoccupied positions before the scheduled operations. Eqs. (35)-(37) determine the positions at which machines undergo PM activities. Eqs. (38)-(40) determine the completion times of the jobs (or products) in three stages. Eq. (41) provides the formula for calculating *TCE*, considering the total emissions from three stages.

4. QLMHHA for EE-IDHFSP-PM

This section introduces the design of the QLMHHA for tackling the EE-IDHFSP-PM. Detailed descriptions of the encoding and decoding scheme, the MNEH-based initialization method, the multi-stage cooperative energy-efficient strategy, the destruction-construction mechanism, twelve problem-specific heuristics, and the *Q*-learning search mechanism are introduced in Sections 4.1-4.6. The framework of QLMHHA is presented in Section 4.7. Furthermore, convergence characteristics and computational complexities are analyzed in Sections 4.8 and 4.9, respectively.

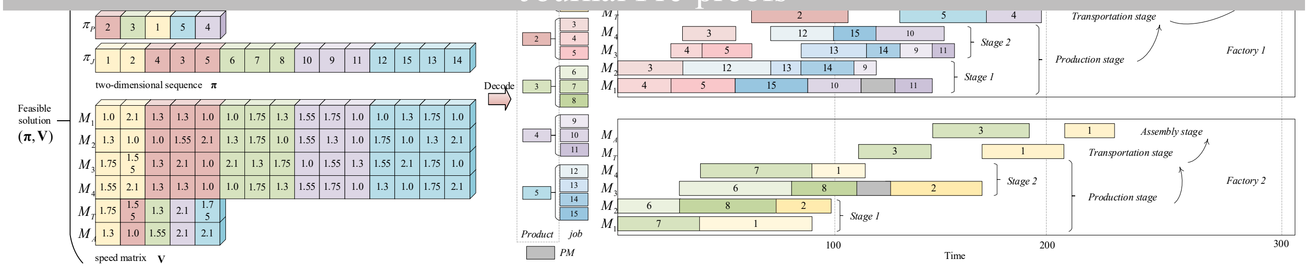


Fig. 3. Example of encoding and decoding schemes.

4.1. Encoding and decoding design

The encoding and decoding design acts as a bridge linking the problem and algorithm domains, and its pros and cons directly determine whether HIOAs can effectively capture critical characteristics of the problem and ensure computational efficiency. In this subsection, a three-segment sequence $\boldsymbol{\pi}=\{\pi_F,\pi_P,\pi_J\}$ with a processing speed matrix $\mathbf{V}$ (*i.e.*, $(\boldsymbol{\pi},\mathbf{V})$) is designed as an encoding scheme to represent feasible solutions of EE-IDHFSP-PM. The first segment π_F defines factory assignments for all products, the second segment π_P represents processing sequences for products, and the third segment π_J defines job orders for each product. $\mathbf{V}$ represents processing speeds of jobs on different machines. Jobs belonging to the same product must be assigned to the same factory and the order of these jobs must not be split. An example explanation of the coding scheme $(\boldsymbol{\pi},\mathbf{V})$ is illustrated in Fig. 3. There are 5 products with 15 jobs allocated across two factories, with factory 1 processing products 2, 4, and 5 in order 2-5-4 (job order: 4-3-5-12-15-13-14-10-9-11) and factory 2 processing products 3 and 1 in order 3-1 (job order: 6-7-8-1-2). The decoding strategy employs the earliest completion time (ECT) rule to assign jobs to machines that minimize completion times, sequentially progressing through production, transportation, and assembly stages. Jobs are assigned to machines in factories according to processing orders. Upon completion of all jobs for each product, these jobs subsequently proceed to transportation and assembly stages. Following the work of Jia et al. (2023), the decoding process must account for the service life of machines. If machine's age exceeds a predefined threshold, preventive maintenance (PM) strategies are executed immediately to reset the machine's service life to its initial age before processing jobs; otherwise, processing processes proceed directly.

4.2. Initialization method

The initialization method of HIOAs influences the population's initial distribution and diversity, directly determining algorithms' search capabilities and robustness. Existing studies have shown that NEH-based methods perform well in tackling HFSPs (Naderi et al.2010). Inspired by Li et al. (2023), this subsection proposes a modified NEH-based method (MNEH) for population initialization. Based on analysis of the encoding characteristics for EE-IDHFSP-PM, MNEH-based initialization method the factory assignment sequence (π_F) and the job processing sequence (π_J) are initialized randomly to maintain diversity. To balance solution quality and population diversity, some product sequences (π_P) are generated via Algorithm 1, with the rest randomly generated. The population initialization procedure is detailed in Algorithm 2.

Algorithm 1: Modified NEH-based method

Input: Set of products, $\boldsymbol{P} = \{P_1, P_2, ..., P_H\}$.
Output: π_P.

```
for g = 1 to H do
    Calculate the total processing time of all jobs belonging to the product P_g and
    add it into the product transporation and assembly time, define as HT_g, where
    HT_g = Σ_{i=1}^{|Ω_g|} Σ_{l=1}^{S} Σ_{m=1}^{m_sub} p_{i,l}^{P,m,r} + p_g^{T,r} + p_g^{A,r}.
Sort product P_g according to decreasing of HT_g, and obtain the product sequence
    π'_P = [P_1, P_2, ..., P_H].
Take the first two products and schedule them as π_P.
if rand() < 0.5 then
    for i = 3 to H do
        Take product π'_P(i) and insert it into all the possible places of π_P.
        Update π_P with the lowest C_max.
else
    for i = 3 to H do
        Take product π'_P(i) and insert it into all the possible places of π_P.
        Update π_P with the lowest TCE.
```

where $HT_g = \sum_{i=1}^{|\boldsymbol{\Omega}_g|} \sum_{l=1}^{S} \sum_{m=1}^{m_{sub}} p_{i,l}^{P,m,r} + p_g^{T,r} + p_g^{A,r}$ and $\pi'_P = [P_1, P_2, ..., P_H]$.

Algorithm 2: Initialization method

Input: Population size $popsize$.
Output: A population POP.

```
Initialize speed matrix V.
for i = 1 to popsize do
    Randomly generate π_F and π_J.
    if rand() < 0.5 then
        Generate π_P using MNEH via Algorithm 1.
    else
        Randomly generate π_P.
    Generate π_i ← {π_F, π_P, π_J}.
    Append (π_i, V) into POP.
```

4.3. Energy-efficient strategy

Increasing machine processing speed reduces job processing times, but it also increases machine energy consumption. To address this trade-off, this subsection introduces a multi-stage cooperative energy-efficient strategy (MSC_EES) that minimizes energy consumption by controlling the processing speeds of operations on non-critical paths without affecting overall completion times. Operations on critical paths directly determine the total completion time, and reducing their paths can be slowed down without impacting the overall schedule, thereby achieving energy savings while keeping efficiency. Based on the characteristics of EE-IDHFSP-PM, this section investigates how to reduce processing speeds for non-critical path operations during the production stage. Algorithm 3 provides a detailed description of MSC_EES, with detailed illustrations shown in Fig. 4. The specific steps are as follows:

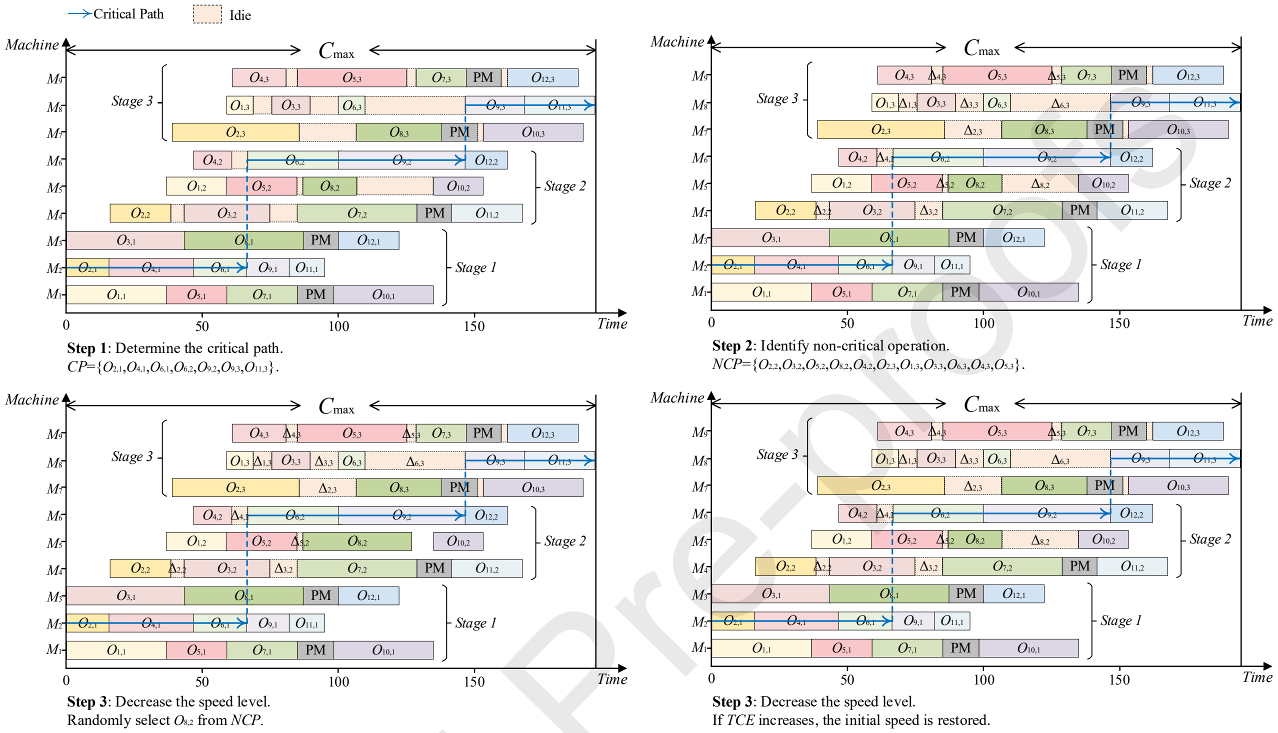


Fig. 4. Illustration of the multi-stage cooperative energy-efficient strategy.

Step 1: Determine critical paths.

Step 1.1: Identify the starting point at the last production sub-stage S of factory f. Locate the operation $O_{i,S}$ processed at position k on machine M_m with the maximum completion time in the current sub-stage. Add the corresponding operation $O_{i,S}$ to the critical path set *CP*.

Step 1.2: Trace critical jobs backward.

Step 1.2.1: Check the operation $O_{i',S}$ at position $k-1$ on machine M_m. If $C^P_{i',S} = E^P_{i,S}$, then $O_{i',S}$ is a critical operation and it is added to *CP*.

Step 1.2.2: Set $O_{i',S}$ as the current operation $O_{i,S}$ and continue checking operation at position $k-1$ on machine M_m. Repeat the above condition evaluation.

Step 1.3: Sub-stage backtracking mechanism: If the condition in **Step 1.2.1** is not satisfied (*i.e.*, $C^P_{i',S} \neq E^P_{i,S}$), switch to the previous sub-stage $S-1$. Locate operation $O_{i,S-1}$ processed at the k'-th position on machine $M_{m'}$ in this sub-stage, and repeat **Step 1.2.1** and **Step 1.2.2**.

have been checked). All critical jobs in factory f are added to *CP*.

Step 2: Identify non-critical operations. For operation $O_{i,l}$ processed at position k on machine M_m and operation $O_{i',l}$ processed at position $k+1$ on machine M_m: If $C_{i,l}^P < E_{i',l}^P$, operation $O_{i',l}$ is a non-critical operation and it is added to *NCP*. If $O_{i,l}$ is closely followed by PM activities, then $O_{i,l}$ does not make non-critical operation considerations.

Step 3: Decrease speed levels.

Step 3.1: Randomly select an operation $O_{i,l}$ from *NCP*.

Step 3.2: Determine that operation $O_{i,l}$ processes at the k th position of the processing machine M_m. Locate the operation $O_{i',l}$ for machining at the k -1th position of the machinie M_m. Calculate the left-side idle time $\Delta_{i,l}$ of $O_{i,l}$, *i.e.*, $\Delta_{i,l} = C_{i,l}^P - C_{i',l}^P$. Reduce its speed level from $V_{m,i}$ to $p_{i,l}^{P,m} \big/ (A_{i,l}^P + \Delta_{i,l})$, update the speed matrix $\mathbf{V}$ to the adjusted matrix $\mathbf{V}_{adj}$. Table 5 lists processing times for all jobs across production stages. Recompute fitness values for feasible solutions $(\boldsymbol{\pi}, \mathbf{V})$ and $(\boldsymbol{\pi}, \mathbf{V}_{adj})$. If $(\boldsymbol{\pi}, \mathbf{V}_{adj})$ dominates $(\boldsymbol{\pi}, \mathbf{V})$ (*i.e.*, speed reduction effectively lowers *TCE*), proceed to **Step 4**; otherwise, restore the machine's original speed level and remove $O_{i,l}$ from *NCP*.

Step 3.3: If *NCP* is non-empty, return to **Step 3.1**.

Step 4: Set $f = f + 1$. If $f \le \overline{f}$, jump to **Step 1**.

Table 5 The processing times of jobs.

	J_1	J_2	J_3	J_4	J_5	J_6	J_7	J_8	J_9	J_{10}	J_{11}	J_{12}
M_1	17	20	33	52	44	68	34	55	32	26	31	18
M_2	18	24	27	43	37	45	42	45	25	82	64	35
M_3	10	21	17	45	35	55	36	40	20	13	41	26
M_4	12	17	30	39	40	61	30	42	23	54	65	35
M_5	65	21	65	36	42	21	24	36	16	35	23	75
M_6	20	30	13	12	95	14	54	51	22	54	34	24
M_7	74	25	55	64	28	54	13	64	34	20	20	92
M_8	56	68	54	23	34	69	21	31	52	13	64	12
M_9	35	61	32	10	64	45	54	16	32	54	52	54

Input: Non-dominated set L, a feasible solution $(\boldsymbol{\pi}, \mathbf{V})$, factory number $\overline{f}$.
Output: L.
$\mathbf{V}_{adj} \leftarrow \mathbf{V}$.
for $f = 1$ *to* $\overline{f}$ **do**
  Define an empty set CP to store operations on the critical path under factory F_f.
  Define an empty set NCP to store non-critical operations not within CP.
  repeat
    for $k = 1$ *to* $|NCP|$ **do**
      Select the operation $O_{i,l}$ from NCP randomly.
      Locate operation $O_{i,l}$ at the k th position of machine M_m.
      Locate operation $O_{i',l}$ at the k-1th position of machinie M_m.
      And calculate the left- side idle time $\Delta_{i,l}$ of $O_{i,l}$. $\Delta_{i,l} = C_{i,l}^P - C_{i',l}^P$.
      // Calculate scalable time.
      Define the previous speed of machining operation $O_{i,l}$ on machine M_m as $V_{m,i}$.
      Calculate the changed processing speed $V'_{m,i}$ of M_m for processing $O_{i,l}$.
      $V'_{m,i} = p_{i,l}^{P,m} / (A_{i,l}^P + \Delta_{i,l})$.
      $\mathbf{V}_{adj}[M_m][J_i] \leftarrow$ next lower speed level $V'_{m,i}$.
      // Reduce the speed of the machine.
      Calculate the fitness values of $(\boldsymbol{\pi}, \mathbf{V}_{adj})$ and $(\boldsymbol{\pi}, \mathbf{V})$.
      if $(\boldsymbol{\pi}, \mathbf{V}_{adj}) \prec (\boldsymbol{\pi}, \mathbf{V})$ **then**
        Update the current solution $(\boldsymbol{\pi}, \mathbf{V}) \leftarrow (\boldsymbol{\pi}, \mathbf{V}_{adj})$.
        // Save energy.
        break;
      else
        Add $(\boldsymbol{\pi}, \mathbf{V}_{adj})$ to L.
        Restore the speed $\mathbf{V}_{adj}[M_m][J_i] \leftarrow$ previous speed $V_{m,i}$.
      $k \leftarrow k + 1$.
      Remove $O_{i,l}$ from NCP.
  until energy efficiency goal is met or $NCP = \phi$.
$L \leftarrow$ Update the non-dominated solutions from L.

4.4. Destruction and construction mechanism

The commonly employed destruction-construction operations are typically applied to sequences of jobs without precedence constraints, enabling d_{DC} jobs to be randomly removed from sequences and insertions performed. However, considering the characteristics and encoding features of the EE-IDHFSP-PM, the jobs and products exist correlation relationship in π_J, that is, the sequence of jobs belonging to the same product cannot be separated. Randomly removing d_{DC} jobs for reconstruction is not feasible. Therefore, this subsection proposes a problem-specific destruction-construction (DC) mechanism that operates at the product level. The process of DC is shown in Algorithm 4. Specifically, 1) destruction phase: Randomly select d_{DC} products from π_P and store them into π_P^{remove}; Remove all jobs belonging to these products from π_J, where $\pi_J^{P_g}$ is the sequence of jobs currently belonging to the product P_g. 2) construction phase: Employ a set l to track non-dominated partial solutions.

For each job in the removed job sequence π_J^{remove} of the product P_g, insert the job into all feasible positions of each partial solution in I. Each insertion generates a feasible solution stored in feasible solution set L. Update the non-dominated partial solutions I from L after each insertion. Once all jobs in π_J^{remove} have been reconstructed, merge non-dominated solutions from I into the global non-dominated solution set $\mathcal{L}$. Unlike traditional DC that yields a single solution, this problem-specific DC mechanism generates multiple local non-dominated solutions, which can enhance search diversity and strengthen the algorithm's performance to effectively explore local search spaces.

Algorithm 4: Destruction and construction mechanism

Input: A feasible solution $(\boldsymbol{\pi}, \mathbf{V})$, the number of removed products d_{DC}.
Output: Non-dominated set $\mathcal{L}$.

Define an empty set π_P^{remove} to store the reinserted products, $\boldsymbol{\pi}' \leftarrow \boldsymbol{\pi}$, $\boldsymbol{\pi}' = \{\pi_F, \pi_P, \pi_J\}$.
Select d_{DC} different products from π_P and insert into π_P^{remove}.
Define $\mathcal{L}$ to store non-dominated solutions from product reconstruction phase.
// Destruction phase 1.
for $g = 1$ *to* d_{DC} **do**
    Select a product $P_g \leftarrow \pi_P^{remove}(g)$.
    Define an empty set $\pi_J^{P_g}$ to store the sequence of jobs belonging to product P_g.
    Define an empty set π_J^{remove} and store the all reinserted jobs to belong to P_g.
    // Destruction phase 2.
    Define an empty set I to store non-dominated partial solutions.
    Remove π_J^{remove} from π_J to obtain new sequence $\boldsymbol{\pi}'$.
    $\mathrm{I} \leftarrow \mathrm{I} \cup (\boldsymbol{\pi}', \mathbf{V})$.
    for $i = 1$ to $\left|\pi_J^{remove}\right|$ **do**
        // Construction phase.
        Define an empty set L to store provisional feasible solutions after insertion operation.
        for $j = 1$ to $|\mathrm{I}|$ **do**
            $(\boldsymbol{\pi}_j, \mathbf{V}) \leftarrow \mathrm{I}(j)$.
            for $k = 1$ to $\left|\pi_J^{P_g}\right| + 1$ **do**
                // Perform insertion of jobs.
                Update $\boldsymbol{\pi}' \leftarrow$ Insert job $\pi_J^{remove}(i)$ into the kth possible position of the sequence $\pi_J^{P_g}$ in $\boldsymbol{\pi}_j$.
                $\mathrm{L} \leftarrow \mathrm{L} \cup (\boldsymbol{\pi}', \mathbf{V})$.
        Add $\pi_J^{remove}(i)$ into job sequence $\pi_J^{P_g}$.
        Update $\mathrm{I} \leftarrow$ Obtain the non-dominated solutions from L.
    $\mathcal{L} \leftarrow \mathrm{I} \cup \mathcal{L}$.

The LLHs are essential for HHAs, with the design and combination of simple and efficient LLHs enabling rapid finding of high-quality solutions while optimizing search behaviors to speed up search processes. This subsection develops twelve problem-specific heuristics to form a collection of LLHs (see Fig. 5). These LLHs can be divided into two types, conducting neighborhood searches on product and job sequences, respectively. For ease of explanations, pos_x denotes the x-th position of product P_g in π_P, and loc_y represents the y-th position of job J_i belonging to product P_g in $\pi_J^{P_g}$, where $len(\pi_J^{P_g}) = n$. The details are below:

LLH$_1$: **Swap critical product and non-critical product**. In critical factory f_c, select a critical product and a non-critical product randomly and swap their positions in π_P.

LLH$_2$: **Insert critical product after non-critical product**. Select a critical product and a non-critical product randomly from the critical factory f_c and insert the critical product after the non-critical product in π_P.

LLH$_3$: **Reverse subsequence between critical and non-critical products**. Select a critical product and a non-critical product randomly from the critical factory f_c and reverse the subsequence of products between them in π_P.

LLH$_4$: **Product exchange**. The products P_g and $P_{g'}$ ($g \neq g'$) are randomly selected from π_P at two different positions pos_x and $pos_{x'}$ ($pos_x \neq pos_{x'}$), and exchange the positions of P_g and $P_{g'}$ from $[\pi_P^{pos_1},...,\pi_P^{pos_x},...,\pi_P^{pos_{x'}},...,\pi_P^{pos_H}]$ to $[\pi_P^{pos_1},...,\pi_P^{pos_{x'}},...,\pi_P^{pos_x},...,\pi_P^{pos_H}]$.

LLH$_5$: **Product insertion (forward)**. The products P_g and $P_{g'}$ ($g \neq g'$) are randomly selected from π_P at two different positions pos_x and $pos_{x'}$ ($pos_x < pos_{x'}$). Insert $P_{g'}$ in front of P_g, change the sequence from $[\pi_P^{pos_1},...,\pi_P^{pos_x},...,\pi_P^{pos_{x'}},...,\pi_P^{pos_H}]$ to $[\pi_P^{pos_1},...,\pi_P^{pos_{x'}},\pi_P^{pos_x},...,\pi_P^{pos_H}]$.

LLH$_6$: **Product insertion (backward)**. The products P_g and $P_{g'}$ ($g \neq g'$) are randomly selected from π_P at two different positions pos_x and $pos_{x'}$ ($pos_x < pos_{x'}$). Insert $P_{g'}$ after P_g, change the sequence from $[\pi_P^{pos_1},...,\pi_P^{pos_x},...,\pi_P^{pos_{x'}},...,\pi_P^{pos_H}]$ to $[\pi_P^{pos_1},...,\pi_P^{pos_x},\pi_P^{pos_{x'}},...,\pi_P^{pos_H}]$.

LLH$_7$: **Product inversion (type 1)**. The product P_g is randomly selected from π_P with position pos_x. Invert subsequences $[\pi_P^{pos_1},\pi_P^{pos_2},...,\pi_P^{pos_{x-1}}]$ and $[\pi_P^{pos_{x+1}},\pi_P^{pos_{x+2}},...,\pi_P^{pos_H}]$ before and after product P_g, change the sequence from $[\pi_P,...,\pi_P,\pi_P,\pi_P,...,\pi_P]$ to $[\pi_P^{pos_{x-1}},...,\pi_P^{pos_1},\pi_P^{pos_x},\pi_P^{pos_H},...,\pi_P^{pos_{x+1}}]$.

LLH$_8$: **Product inversion (type 2)**. The products P_g and $P_{g'}$ ($g \neq g'$) are randomly selected from π_P at two different positions pos_x and $pos_{x'}$ ($pos_x < pos_{x'}$). Invert the subsequence of products between P_g and $P_{g'}$, change the sequence from $[\pi_P^{pos_1},...,\pi_P^{pos_x},\pi_P^{pos_x},...,\pi_P^{pos_{x'}},...,\pi_P^{pos_H}]$ to $[\pi_P^{pos_1},...,\pi_P^{pos_{x'}},...,\pi_P^{pos_{x+1}},\pi_P^{pos_x},...,\pi_P^{pos_H}]$.

LLH$_9$: **Product inversion (type 3)**. The products P_g and $P_{g'}$ ($g \neq g'$) are randomly selected from π_P at two different positions pos_x and $pos_{x'}$ ($pos_x < pos_{x'}$). Invert the subsequence of products $[\pi_P^{pos_1},\pi_P^{pos_2},...,\pi_P^{pos_{x-1}}]$ and $[\pi_P^{pos_{x'+1}},\pi_P^{pos_{x+2}},...,\pi_P^{pos_H}]$ outside the range of $P_{g'}$ and P_g, change the sequence from $[\pi_P^{pos_1},...,\pi_P^{pos_{x-1}},\pi_P^{pos_x},...,\pi_P^{pos_{x'}},\pi_P^{pos_{x'+1}},...,\pi_P^{pos_H}]$ to $[\pi_P^{pos_{x-1}},...,\pi_P^{pos_1},\pi_P^{pos_x},...,\pi_P^{pos_{x'}},\pi_P^{pos_H},...,\pi_P^{pos_{x'+1}}]$.

LLH$_{10}$: **Job swap**. Randomly select one product P_g and obtain its job sequence $\pi_J^{P_g}$. The jobs J_i and $J_{i'}$ ($i \neq i'$) are randomly selected from $\pi_J^{P_g}$ at two different positions loc_y and $loc_{y'}$ ($loc_y \neq loc_{y'}$). Swap the positions of jobs J_i and $J_{i'}$ from $[\pi_J^{P_g}(loc_1),...,\pi_J^{P_g}(loc_y),...,\pi_J^{P_g}(loc_{y'}),...,\pi_J^{P_g}(loc_n)]$ to $[\pi_J^{P_g}(loc_1),...,\pi_J^{P_g}(loc_{y'}),...,\pi_J^{P_g}(loc_y),...,\pi_J^{P_g}(loc_n)]$.

LLH$_{11}$: **Job insertion (forward)**. Randomly select one product P_g and obtain its job sequence $\pi_J^{P_g}$. The jobs J_i and $J_{i'}$ ($i \neq i'$) are randomly selected from $\pi_J^{P_g}$ at two different positions loc_y and $loc_{y'}$ ($loc_y < loc_{y'}$). Insert job $J_{i'}$ before job J_i, change the sequence from $[\pi_J^{P_g}(loc_1),...,\pi_J^{P_g}(loc_y),...,\pi_J^{P_g}(loc_{y'}),...,\pi_J^{P_g}(loc_n)]$ to $[\pi_J^{P_g}(loc_1),...,\pi_J^{P_g}(loc_{y'}),\pi_J^{P_g}(loc_y),...,\pi_J^{P_g}(loc_n)]$.

LLH$_{12}$: **Job insertion (backward)**. Randomly select one product P_g and obtain its job sequence $\pi_J^{P_g}$. The jobs J_i and $J_{i'}$ ($i \neq i'$) are randomly selected from $\pi_J^{P_g}$ at two different positions loc_y and $loc_{y'}$ ($loc_y < loc_{y'}$). Insert job $J_{i'}$ after job J_i, change the sequence from $[\pi_J^{P_g}(loc_1),...,\pi_J^{P_g}(loc_y),...,\pi_J^{P_g}(loc_{y'}),...,\pi_J^{P_g}(loc_n)]$ to $[\pi_J^{P_g}(loc_1),...,\pi_J^{P_g}(loc_y),\pi_J^{P_g}(loc_{y'}),...,\pi_J^{P_g}(loc_n)]$.

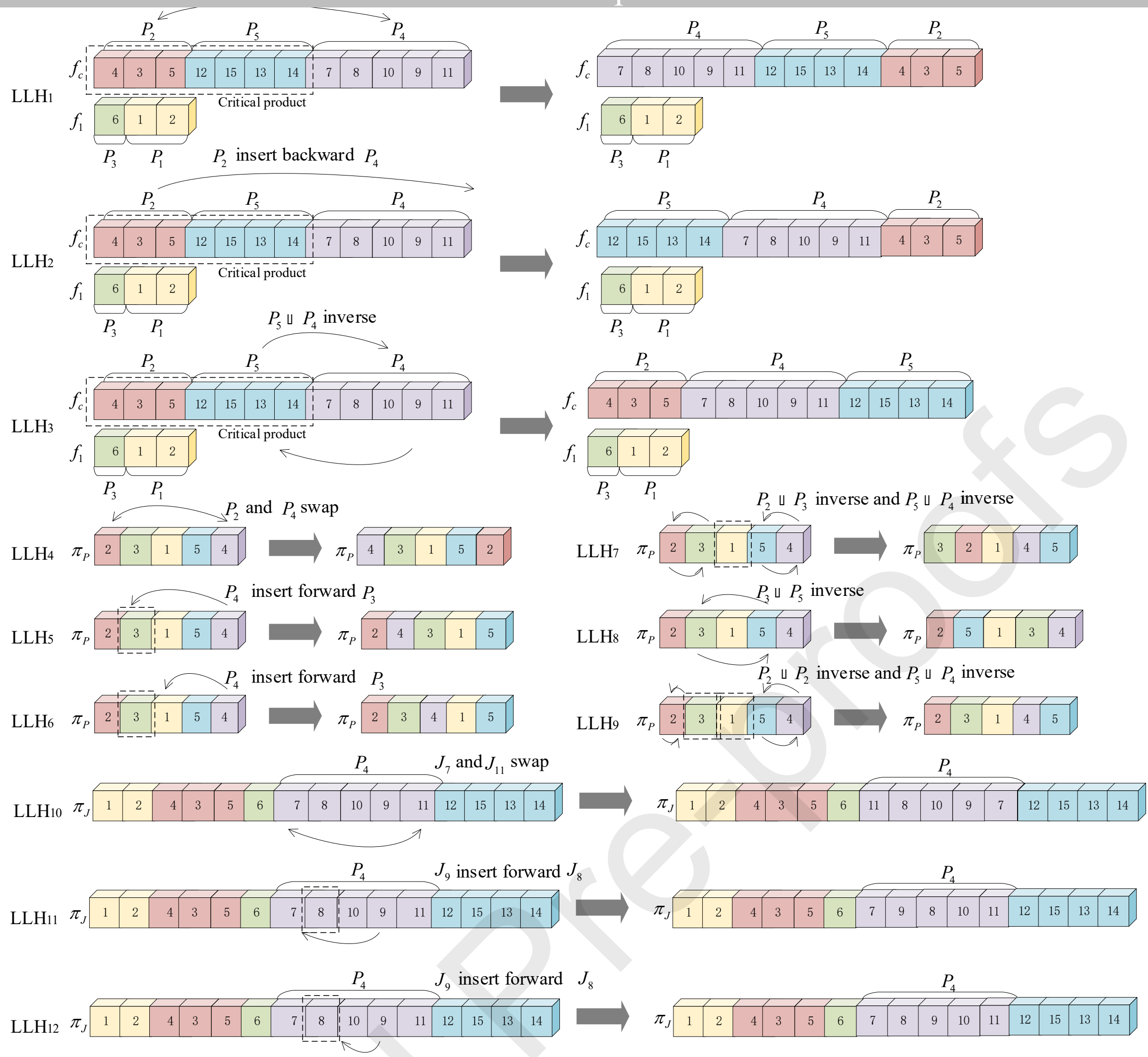


Fig. 5. Illustration of twelve low-level heuristics.

4.6. Q-learning search mechanism

The Q-learning algorithm, a classical RL-based method grounded in Markov Decision Processes (MDPs), enables agents to acquire the best behavioral policies in environments while maximizing long-term cumulative rewards. The MDP can be represented as $(\mathcal{S}, \mathcal{A}, P, r, \gamma)$, where $\mathcal{S}$ denotes the state space encompassing all possible environment states, $\mathcal{A}$ is the action space available to the agent ($\mathcal{A}(s)$ denotes actions available in state $s \in \mathcal{S}$), $P(s'|s,a)$ represents the transition probability to state s' when action a is executed in state s, r represents the immediate reward received after taking action a, and the discount factor $\gamma \in (0,1)$ governs the weight assigned to future rewards. As illustrated in Fig. 8, which depicts the interaction process between the agent and the environment in QLMHHA, the agent obtains an initial state s_0 based on the environment, then samples an action a_0 via probabilistic sampling method (see Section 4.6.1), executes the action a_0 using predefined LLHs, and receives a reward r_0 based on the reward mechanism (see Section 4.6.2). Subsequently, at each time step ($t \geq 1$), the agent observes the current state s_t and selects an action a_t for this state (see Section 4.6.1), executes LLHs in QLMHHA to gain reward r_t (see Section 4.6.2), and updates the Q-table (Q) (see Section 4.6.3). The learning process proceeds until the termination criteria are met.

Algorithm 5: Action selection

Input: $s_t, type, Q$.
Output: a_t.
1: **if** $type = 1$
2: $a_t \leftarrow$ Probability sampling(s_t, Q) via Algorithm 6.
3: **else**
4: Select an action a_t randomly.

4.6.1. State and action

In QLMHHA, the state represents the current environment observed by the agent, and an action corresponds to an executable behavior in that state. The state space S and action space A comprise the twelve LLHs defined in Section 4.5. The action space aligns with the state space, implying that for each state $s \in S$, the permissible actions are $A(s)=\{\text{LLH}_1, \text{LLH}_2, \ldots, \text{LLH}_{12}\}$. Fig. 6 illustrates the action transition mechanism. Appropriate action selection strategies can effectively refine search behaviors and must balance exploration (uncovering new actions) and exploitation (utilizing known high-reward actions). Excessive exploration wastes computational resources on suboptimal actions, while over-exploitation risks local optima stagnation. Unlike traditional ε-greedy approaches, probabilistic sampling provides a more robust balance by directly computing selection probabilities from Q-values, thereby reducing dependence on manually tuned parameters (*i.e.*, ε). Following Shao et al. (2024), we implement probabilistic sampling for action selection, as detailed in Algorithms 5 and 6. This probability is then used in a roulette wheel selection to choose the next action. The probability of selecting each action a_i in state s_t is computed by Eq. (42):

$$prob(a_i) = \frac{e^{Q(s_t, a_i)}}{\sum_{i \in \{1,2,\ldots,|A(s)|\}} e^{Q(s_t, a_i)}}. \tag{42}$$

In Algorithm 6, exponentiating Q-values amplifies the differences among actions, with priorities for actions with high Q values while preserving non-zero probabilities for actions with low Q values. This design enhances search diversity and mitigates premature convergence.

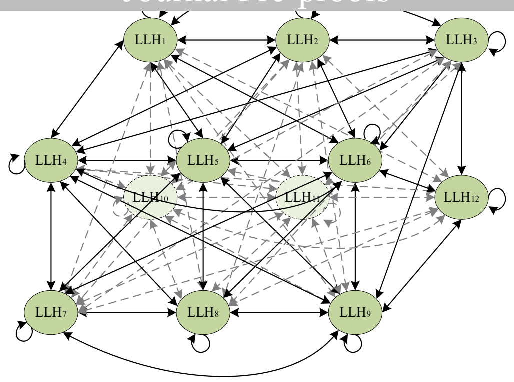


Fig. 6. Action transition relationship in state space S.

Algorithm 6: Probability sampling
Input: s_t, Q.
Output: a_t.
1: Define q to store the Q-values of all feasible actions in $Q(s_t, a)$ under state s_t.
2: Exponentiate the values in q.
3: Calculate the probability of each action being selected via Eq.(42). And store it in $prob(a)$.
4: Select action a_t from $prob(a)$ by roulette wheel selection.

4.6.2. Reward function

The design of the reward function is crucial in RL-based HLSs, as it provides feedback on the actions taken by the agent under different states and directly determines the agent's decision-making behaviors. In the initial stages, solutions generated are typically of poor quality, with significant gaps against superior solutions. To address this issue, agents must prefer to take actions that enable the best improvement in critical criteria. Since different actions contribute to solutions with different impact levels, higher rewards are assigned to those actions that yield significant improvements. During the execution of QLMHHA, the actions selected by Algorithm 5 are applied to each solution in the Pareto solution set, with the evaluation of actions determined by the quality of the obtained Pareto front (PF). Following the work of Li et al. (2023), we evaluate action performance via two metrics: convergence metric (CV) and diversity metric (DV), which are defined by Eqs. (43) and (44), respectively.

$$CV_t(P_t) = \frac{\sum_{y \in P_t} \min_{x \in P^*} d(x, y)}{|P_t|}, \tag{43}$$

$$DV_t(P_t) = \frac{d_f + d_l + \sum_{i=1}^{N-1} |d_i - \bar{d}|}{d_f + d_l + (N-1)\bar{d}}. \tag{44}$$

Here, P_t denotes the Pareto front (PF) obtained by executing action a_t in the state s_t, and P^* signifies the reference Pareto optimal front. d_i denotes the Euclidean distance between each pair of adjacent solutions in P_t and $\bar{d}$ is the average value of d_i. N is the number of non-dominated solutions in P_t. d_f and d_l are the Euclidean distances between the extremal solutions and boundary solutions in P_t. The lower value of CV_t indicates greater convergence of P_t toward P^*. Conversely, a higher value of DV_t suggests a richer diversity of solutions in P_t. Based on these metrics, we define four performance evaluation (PE) tiers: 1) **PE$_1$**: $DV_t < DV_{t+1}$ and $CV_t > CV_{t+1}$; 2) **PE$_2$**: $DV_t < DV_{t+1}$ and $CV_t \le CV_{t+1}$; 3) **PE$_3$**: $DV_t \ge DV_{t+1}$ and $CV_t > CV_{t+1}$; and 4) **PE$_4$**: $DV_t \ge DV_{t+1}$ and $CV_t \le CV_{t+1}$. The reward values for each PE's range are allocated according to Eq. (45).

$$r(s_{t+1}, a_{t+1}) = \begin{cases} 10,\ DV_t < DV_{t+1} \text{ and } CV_t > CV_{t+1}; \\ 5\ ,\ DV_t < DV_{t+1} \text{ and } CV_t \le CV_{t+1}; \\ 5\ ,\ DV_t \ge DV_{t+1} \text{ and } CV_t > CV_{t+1}; \\ 0\ ,\ DV_t \ge DV_{t+1} \text{ and } CV_t \le CV_{t+1}; \end{cases} \tag{45}$$

4.6.3. Update mechanism

In QLMHHA, the updating mechanism of the *Q*-table (Q) serves as a core component, which guides the agent to make optimal decisions through iterative learning and interactive feedback to update *Q*-values. The *Q*-table (Q) update formula is defined as follows:

$$Q_{t+1}(s_t, a_t) \leftarrow (1-\lambda) \cdot Q_t(s_t, a_t) + \lambda \cdot \left[r(s_t, a_t) + \gamma \max_{a \in A} Q_t(s_{t+1}, a_t) \right]. \tag{46}$$

Eq. (46) comprises key components: $Q_{t+1}(s_t, a_t)$ represents the expected cumulative reward for executing action a_t in specific state s_t; λ is the learning rate that determines the update weight of Q values with $\lambda \in (0,1)$; γ is the discount factor for future rewards with $\gamma \in (0,1)$; $r(s_t, a_t)$ denotes the immediate reward received after executing action a_t in state s_t, and s_{t+1} is the subsequent state after action execution.

4.7. The framework of QLMHHA

This subsection presents the framework of QLMHHA, wherein the high-level individuals (*HLIs*) are constructed by selecting 12 predefined LLHs from Section 4.5. The evaluation of the *HLIs* relies on the cumulative rewards obtained through the iterative application of each LLH to the global non-dominated solution set L. In this framework, LLHs simultaneously serve dual roles as both states and executable actions. The HLS employs the *Q*-learning algorithm to dynamically generate high-is illustrated in Fig. 8, and Fig. 7 describes the detailed implementation processes of QLMHHA. The detailed steps are given in Algorithm 8.

Step 1: Initialize key parameters of QLMHHA, *i.e.*, $popsize$, λ, γ, and φ. Set $T = 0$.

Step 2: Initialize the population POP. Set all values in Q to 0. Evaluate each feasible solution in POP. Extract the non-dominated solution set L from POP.

Step 3: Apply *DC* mechanism to feasible solutions in L and update L via Algorithm 4 in Section 4.4.

Step 4: Execute *MSC_EES* on each solution in L by Algorithm 3.

Step 5: Generate high-level individuals (*HLIs*) based on Q.

Step 5.1: Select action a_t by Algorithms 5 and 6 at the state s_t, and obtain the next state s_{t+1}.

Step 5.2: Apply action a_t to L in state s_t and update L. Calculate metrics DV_t and CV_t by Eqs. (43) and (44). Compute rewards $r(s_t, a_t)$ by Eq. (45) and update Q.

Step 5.3: If the complete HLI has not yet been generated, then skip to **Step 5.1**.

Step 5.4: If all $HLIs$ have been generated, go to **Step 6**; otherwise, jump to **Step 5**.

Step 6: Calculate the contribution rate (CR) for each HLI. Extract each LLH from these $HLIs$ and apply them to L to yield new non-dominated set L'. Calculate *CR* of each HLI by Algorithm 7.

Step 7: Select $\lceil popsize \times \varphi \rceil$ elite $HLIs$ based on *CR*. Apply all elite $HLIs$ to L, update Q via Algorithm 7.

Step 8: Set $T{++}$. Check if the termination condition is met. If not satisfied, skip to **Step 5**; otherwise, output L.

Input: Non-dominated solution set $\mathcal{L}$, a high-level individual HLI.
Output: New non-dominated solution set $\mathcal{L}'$, contribution rate CR.

Initialize $CR = 0$, $t = 1$.
$a_t \leftarrow HLI[1]$
Get new non-dominated set $\mathcal{L}'$ by executing a_t on $\mathcal{L}$.
Calculate $DV_t(\mathcal{L}')$ and $CV_t(\mathcal{L}')$ via Eqs.(43)-(44).
Perceive the current state s_{t+1}.
for $i = 2$ *to* $|HLI|$ **do**
  $a_{t+1} \leftarrow HLI[i]$.
  Get new non-dominated set $\mathcal{L}''$ by executing a_{t+1} on $\mathcal{L}'$.
  Calculate $DV_t(\mathcal{L}'')$ and $CV_t(\mathcal{L}'')$ via Eqs.(43)-(44).
  Determine the reward $r(s_{t+1}, a_{t+1})$ with Eq.(45).
  Update Q-table via Eq.(46).
  $CR \leftarrow CR + r(s_{t+1}, a_{t+1})$.
  $t = t + 1$.
  Perceive the next state s_{t+1}.
  $\mathcal{L}' \leftarrow \mathcal{L}''$.

Algorithm 8: QLMHHA

Input: population size $popsize$, learning rate λ, discount rate γ, elite high-level individual rate φ.
Output: Non-dominated solution set $\mathcal{L}$.

Initialize POP, Q. Evaluate each feasible solution $(\boldsymbol{\pi}, \mathbf{V})$ in POP.
Get non-dominated solution set $\mathcal{L}$ from POP.
Execute *DC* for all feasible solutions in $\mathcal{L}$ and update $\mathcal{L}$ by Algorithm 4.
Perform *MSC_EES* on each solution in $\mathcal{L}$ by Algorithm 3.
$T \leftarrow 0$.
While the termination conditions not satisfied **do**
  $T \leftarrow T + 1$.
  For $i = 1$ *to* $popsize$ **do**
    For $t = 1$ *to* 12 **do**
      If $t = 1$
        $s_t \leftarrow \text{Action selection}(0, 2, Q)$.
        $HLI[i,1] \leftarrow s_t$.
      Else
        $a_t \leftarrow \text{Action selection}(s_t, 1, Q)$.
        $HLI[i, t+1] \leftarrow a_t$.
        Get new non-dominated set $\mathcal{L}'$ by executing a_t on $\mathcal{L}$.
        Calculate $DV_t(\mathcal{L}')$ and $CV_t(\mathcal{L}')$ with Eqs.(43)-(44).
        Determine the reward $r(s_t, a_t)$ with Eq.(45).
        Update Q-table with Eq.(46).
        $\mathcal{L} \leftarrow \mathcal{L}'$.
        Perceive the next state s_{t+1}.
  Apply every high-level individuals to $\mathcal{L}$ to obtain *CR* via Algorithm 7. Select $popsize \times \varphi$ elite high-level individuals by *CR*.
  Apply elite high-level individuals to $\mathcal{L}$ by Algorithm 7. update Q and $\mathcal{L}$.

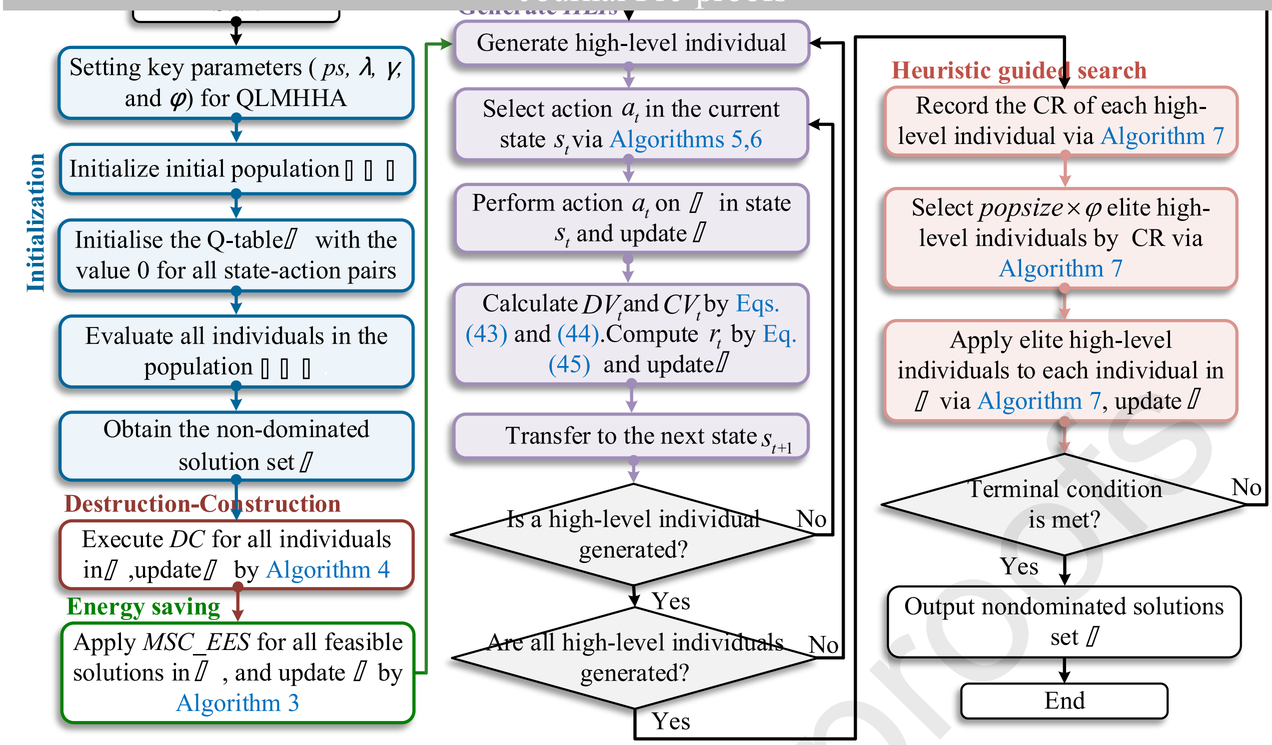


Fig. 7. The flowchart of QLMHHA for the EE-IDHFSP-PM.

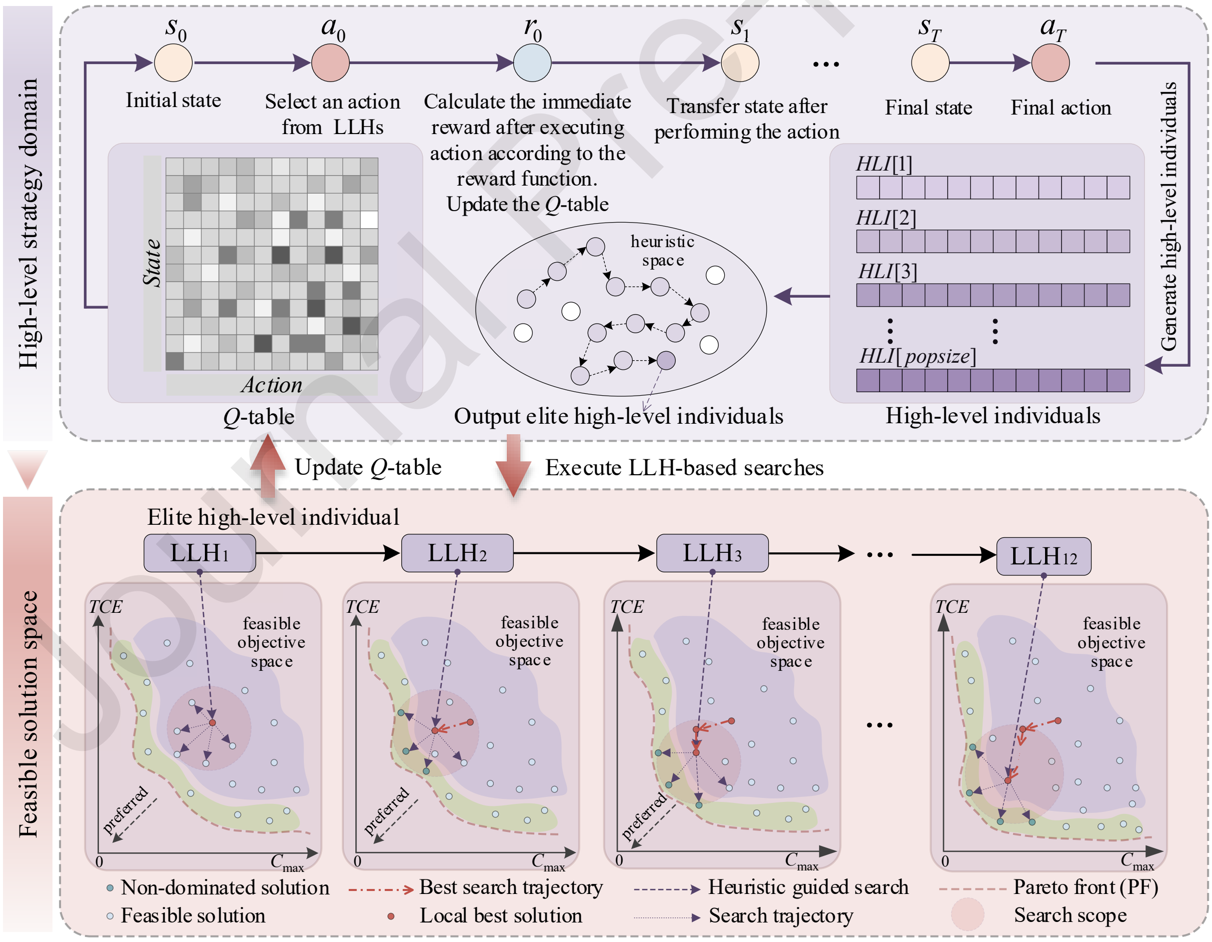


Fig. 8. The schematic diagram of QLMHHA.

This section systematically studies the convergence properties of QLMHHA, demonstrating the strict convergence of the *Q*-learning-based HHAs in the probabilistic sense and providing theoretical support for stability and superiority of the algorithm. The QLMHHA is modeled as a finite Markov Decision Process (MDP). The state space is defined as $\mathcal{S}=\{s_1,s_2,...,s_{12}\}$ (each state corresponds to an LLH); the action space is defined as $\mathcal{A}=\{a_1,a_2,...,a_{12}\}$ (each action maps to an LLH); the transition probability $P(s'|s,a)$ denotes the probability of reaching state s' after selecting action a in state s; the reward $r(s,a)$ is dynamically computed via Eq. (45) from convergence (*CV*) and diversity (*DV*) metrics.

Theorem 1. For a finite MDP $(\mathcal{S},\mathcal{A},P,r)$ with a discount factor $\gamma\in(0,1)$, the *Q*-value iteration operator T is a contraction mapping under the supremum norm $\|\cdot\|_\infty$. Thus, the iterative application of T converges to the unique optimal *Q*-function Q^*, satisfying the fixed-point equation:

$$\mathrm{T}(Q^*)(s,a)=r(s,a)+\gamma\sum_{s'\in\mathcal{S}}P(s'|s,a)\max_{a'\in\mathcal{A}}Q^*(s',a')=Q^*(s,a). \tag{47}$$

Proof. To verify the contraction property, let Q_1 and Q_2 be arbitrary *Q*-functions:

$$\|\mathrm{T}(Q_1)-\mathrm{T}(Q_2)\|_\infty\le\gamma\|Q_1-Q_2\|_\infty. \tag{48}$$

For any state-action pair (s,a), applying the triangle inequality and Lipschitz property as below:

$$\begin{aligned}
&|\mathrm{T}(Q_1)(s,a)-\mathrm{T}(Q_2)(s,a)|=\\
&\gamma\left|\sum_{s'\in\mathcal{S}}P(s'|s,a)\max_{a'}Q_1(s',a')-\sum_{s'\in\mathcal{S}}P(s'|s,a)\max_{a'}Q_2(s',a')\right|\\
&\le\gamma\sum_{s'\in\mathcal{S}}P(s'|s,a)\left|\max_{a'}Q_1(s',a')-\max_{a'}Q_2(s',a')\right| \text{ (Triangle Inequality)}\\
&\le\gamma\sum_{s'\in\mathcal{S}}P(s'|s,a)\max_{a'}|Q_1(s',a')-Q_2(s',a')| \text{ (Lipschitz Property max)}\\
&\le\gamma\sum_{s'\in\mathcal{S}}P(s'|s,a)\|Q_1-Q_2\|_\infty \text{ (Definition of } \|\cdot\|_\infty)\\
&=\gamma\|Q_1-Q_2\|_\infty \text{ (Since } \sum_{s'}P(s'|s,a)=1)
\end{aligned} \tag{49}$$

Taking the supremum over (s,a) completes the proof $\|\mathrm{T}(Q_1)-\mathrm{T}(Q_2)\|_\infty\le\gamma\|Q_1-Q_2\|_\infty$. Hence, T is a contraction mapping, guaranteeing convergence to Q^*.

Theorem 2. Consider the stochastic iterative process defined by the recursion:

$$V_{t+1}(s,a)=(1-\beta_t)V_t(s,a)+\beta_t[\mathrm{T}V_t(s,a)+\varepsilon_t(s,a)]. \tag{50}$$

function Q^* under the following conditions:

$$1)\ 0<\beta_t<1,\sum_{t=0}^{\infty}\beta_t=\infty,\sum_{t=0}^{\infty}\beta_t^2<\infty. \tag{51}$$

$$2)\ \left\|\mathsf{T}V-\mathsf{T}Q^*\right\|_\infty \le \gamma\left\|V-Q^*\right\|_\infty. \tag{52}$$

$$3)\ \operatorname{var}(\varepsilon_t\left|\mathsf{F}_t\right.)\le C(1+\left\|V_t-Q^*\right\|_\infty^2), C>0. \tag{53}$$

Proof. Define the deviation $D_t(s,a)=V_t(s,a)-Q^*(s,a)$ and substitute it into Eq. (45):

$$D_{t+1}(s,a)=(1-\beta_t)D_t(s,a)+\beta_t\left[\mathsf{T}V_t(s,a)-\mathsf{T}Q^*(s,a)+\varepsilon_t(s,a)\right]. \tag{54}$$

According to **Theorem 1**, the contraction property implies:

$$\left\|\mathsf{T}V_t-\mathsf{T}Q^*\right\|_\infty \le \gamma\left\|D_t\right\|_\infty. \tag{55}$$

Let $F_t(s,a)=\mathsf{T}V_t(s,a)-\mathsf{T}Q^*(s,a)+\varepsilon_t(s,a)$, and it holds $D_{t+1}(s,a)=(1-\beta_t)D_t(s,a)+\beta_t F_t(s,a)$. The expectation deviation is $\left\|\mathsf{E}[F_t(s,a)\left|\mathsf{F}_t\right.]\right\|_\infty=\left\|\mathsf{T}V_t-\mathsf{T}Q^*\right\|\le\gamma\left\|D_t\right\|_\infty$, and the variance of the noise term satisfies $\operatorname{var}(F_t\left|\mathsf{F}_t\right.)\le\operatorname{var}(\varepsilon_t\left|\mathsf{F}_t\right.)\le C(1+\left\|D_t\right\|_\infty^2)$. Based on the assumptions of the Kushner-Clark Theorem, $\left\|D_t\right\|_\infty\to 0$ almost surely.

Theorem 3. If the QLMHHA satisfies the following conditions: 1) Finite MDP: $|\mathcal{S}|<\infty,|\mathcal{A}|<\infty$; 2) Bounded rewards: $|r(s,a)|\le R_{\max}$; 3) Proper learning rate: $\beta_t=1/t$; and 4) Sufficient exploration: All (s,a) pairs are visited infinitely often under the probabilistic sampling strategy. Then the Q-value sequence $\{Q_t\}$ can converge almost surely to Q^*, and the optimal policy $\pi^*(s)=\arg\max_a Q^*(s,a)$ generates the optimal order of LLHs.

Proof. The state and action spaces, composed of 12 LLHs, ensure $|\mathcal{S}|<\infty,|\mathcal{A}|<\infty$. Normalized convergence (*CV*) and diversity (*DV*) metrics guarantee $r(s,a)\in[0,R_{\max}]$. The learning rate $\beta_t=1/t$ satisfies $\sum_{t=1}^{\infty}\beta_t=\infty$ and $\sum_{t=1}^{\infty}\beta_t^2=\pi^2/6<\infty$ and the action selection strategy defined by Eq. (42) ensures non-zero selection probabilities for all actions. Therefore, by **Theorems 1**-**2**, the Q-value update process can converge almost surely to Q^*. The optimal policy π^* derived from Q^* can yield a sequence of LLHs, *i.e.*, $\{a_0^*,a_1^*,...,a_T^*\}$, which holds global optimality.

Furthermore, QLMHHA formulates the LLH selection process as a finite MDP within the Q-learning framework. By using the contraction property of Q-value iteration, the stability of stochastic approximation, and sufficient exploration strategies, the algorithm is rigorously proven to converge with probability 1 to the globally optimal Q function Q^*. The policy π^* derived from the optimal Q function Q^* can generate optimal LLH sequences that effectively approximate the Pareto front and efficiently address various MOPs. This conclusion confirms both the theoretical convergence and the practical performance of QLMHHA.

4.9. Computational complexity analysis

Fig. 7 shows that the proposed QLMHHA comprises five key components, including the MNEH-based initialization method, the destruction-construction (DC) mechanism, the multi-stage cooperative energy-efficient strategy (MSC_EES), the action design and action selection, and the learning process. Accordingly, Table 6 presents the detailed computational complexity analysis of critical components of QLMHHA for solving EE-IDHFSP-PM. The total computational complexity (*TCC*) of QLMHHA per generation is denoted as TCC_{QLMHHA}, given by Eq. (56). The complexity of QLMHHA is about $O\left(popsize \times H^3\right)$, which is acceptable for dealing with the EE-IDHFSP-PM.

$$\begin{aligned} TCC_{\mathrm{QLMHHA}} = & O\left(popsize \times H^3\right) + O\left(\overline{f} \times \|NCP\| \times N \times S \times m_{sub}\right) \\ & + O\left(d_{\mathrm{DC}} \times \|\mathrm{I}\| \times \left\|\pi_{\mathrm{J}}^{\mathrm{P_g}}\right\|^3\right) + T \times \left[O\left(popsize \times H\right) + O\left(popsize^2\right)\right] \end{aligned} \quad (56)$$

Table 6 Computational complexity analysis of QLMHHA's components.

Critical components	Computational complexity	Complexity analysis
MNEH-based initialization method via Algorithms 1 and 2	$O\left(popsize \times H^3\right)$	The *CC* for the MNEH-based initialization method: $O\left(popsize \times H^3\right)$. The *CC* for the random selection method: $O\left(popsize \times H\right)$.
Multi-stage cooperative energy-efficient strategy (MSC_EES) via Algorithm 3	$O\left(\overline{f} \times \|NCP\| \times N \times S \times m_{sub}\right)$	The *CC* for the MSC_EES via Algorithm 3: $O\left(\overline{f} \times \|NCP\| \times N \times S \times m_{sub}\right)$.
Destruction-construction (DC) mechanism via Algorithm 4	$O\left(d_{\mathrm{DC}} \times \|\mathrm{I}\| \times \left\|\pi_{\mathrm{J}}^{\mathrm{P_g}}\right\|^3\right)$	The *CC* for the DC mechanism via Algorithm 4: $O\left(d_{\mathrm{DC}} \times \|\mathrm{I}\| \times \left\|\pi_{\mathrm{J}}^{\mathrm{P_g}}\right\|^2 \times \left\|\pi_{\mathrm{J}}^{\mathrm{remove}}\right\|\right)$.
Learning-driven hyper-heuristic framework via Algorithm 8	$T \times \begin{bmatrix} O(popsize \times H) \\ +O(popsize^2) \end{bmatrix}$	The *CC* for action selection strategies via Algorithms 5 and 6: $O(1)$. The *CC* for performing actions from HLIs on the population: $O\left(popsize \times H\right)$. The *CC* for non-dominated sorting method: $O\left(popsize^2\right)$. The *CC* for updating the *Q* table: $O(1)$.

This section provides a comprehensive evaluation of QLMHHA's performance through systematic studies and extensive experiments. Since EE-IDHFSP-PM is of significant research interest but lacks benchmark datasets, we provide simulation datasets combined with previous studies and based on problem characteristics to evaluate algorithms' performance. Section 5.1 provides the experimental setup and environment configuration. Section 5.2 determines QLMHHA's parameters through calibration experiments. Section 5.3 defines performance metrics for measuring algorithms' efficacy. Section 5.4 evaluates the effective contributions of core components in QLMHHA. Finally, Section 5.5 conducts comparative analyses between QLMHHA and state-of-the-art algorithms.

5.1. Experimental setup

To validate the effectiveness of the proposed QLMHHA, we randomly generated 78 instances (54 small-scale instances and 24 large-scale instances). Similar to previous studies, these instances are named according to the standard naming convention: "number of factories - number of products - number of jobs - number of production sub-stages - number of machines per sub-stage". Detailed experimental parameter settings are summarized in Table 7.

Table 7 Test instance parameter settings.

Parameters	Levels
Total number of factories ($\overline{f}$)	{2,3},{6,8}
Total number of products (H)	{10,15,20},{40,50,60}
Total number of jobs (N)	{34,57,68},{132,168,214}
Total number of production sub-stages (S)	{2,4,6},{10,15}
Total number of machines per each sub-stage (m_{sub})	{3,4,5},{6,8}
The set of speeds (V_{set})	{1.0,1.3,1.55,1.75,2.1}
The processing times ($p_{i,l}^{P}, p_{g}^{T}, p_{g}^{A}$)	$U[1,100]$
The energy consumption in processing ($E_{m,r}, E_{T,r}, E_{A,r}$)	$U[2,4]\times V_r$ (kW)
The energy consumption in standby mode (SE_m, SE_T, SE_A)	1 (kW)
The machine deterioration factors ($\varepsilon_P, \varepsilon_T, \varepsilon_A$)	0.1,0.05,0.15
The duration of PM ($t_{PM}^{P}, t_{PM}^{T}, t_{PM}^{A}$)	10
The number of jobs assigned to each product	$U[2,5]$
The number of removed products (d_{DC})	5

To ensure equitable comparisons and eliminate stochastic effects, all algorithms were executed under the same experimental conditions, including uniform experimental environment, programming implementation, and common termination criteria. All algorithms were written using Python 3.12 and experiments were conducted on the server with an Intel(R) Core(TM) i5-12400F@2.50 GHz processor and 16 GB RAM (Windows 11 OS). Each algorithm is independently executed 10 times per instance to ensure statistical reliability. The non-dominated solutions obtained from each run are collected into dominated solution set per instance is generated.

5.2. Parameter settings

Existing works reveal that the HIOAs are sensitive to parameter configurations, which directly influence the algorithm's search behavior, convergence speed, and the final optimization results. The QLMHHA (see Section 4.7) incorporates five critical parameters: population size $(popsize)$, elite HLI ratio (φ), learning rate (λ), discount rate (γ), and maximum runtime (Γ). To determine the optimal combinations for these parameters, the design of experiments (DOE) methods by Montgomery (2017) are employed for systematic calibration. The levels of these parameters were determined by referring to existing works and extensive pre-experiments (Zhang et al.2023), (Jia et al.2023). Each parameter was suggested four levels as shown in Table 8. To mitigate overfitting risks caused by using identical instances for parameter calibration and performance evaluation, we randomly generated a different set of 54 calibration instances following the generation method as in Section 5.1. These calibration instances involve the following ranges of parameters: $\bar{f}=[2,4,6]$, $H=[10,20,50]$, $m_{sub}=[2,6]$, and $S=[2,6,10]$. Given four levels per parameter, 4^5=1024 configurations resulted. Each configuration executed 10 independent runs and was evaluated across all 54 evaluation instances. Consequently, a total of $1024\times54\times10=552960$ experimental runs are conducted to implement a full-factorial DOE for all configurations. Additionally, the maximum CPU runtime of $H\times S\times0.5$ seconds was set as the termination criterion, resulting in approximately 3072 CPU days estimated to be required to complete all experimental tests. Since there were multiple PCs with the same configuration available to conduct these experiments in parallel, the actual time required to complete overall parameter tuning was about 38.4 days. By applying common data analysis techniques, all results of computational comparisons were analyzed through Analysis of Variance (ANOVA). As the robust method of statistical inference to determine whether multiple independent variables have significant statistical effects on the overall means under different conditions, ANOVA has been widely adopted in experimental analyses for shop scheduling problems (Zhang et al.2023). Three main assumptions: normality, homoscedasticity, and residual independence, were validated prior to analysis. After careful examination of the experimental results, no significant deviations from these assumptions were detected. Throughout all experimental results tables, the best-performing values appear in boldface. The response variable DI_R served as the performance metric (see Section 5.3) for the experiments. The ANOVA results for the QLMHHA parameters are presented in Table 9. For the results of ANOVA, when P-value tends to zero, the F-ratio is a clear indicator of statistical significance. This means that the higher the F-ratio, the greater the impact of calibration involves incremental adjustments rather than drastic modifications; thus, significant performance deviations are unlikely to occur after tuning, as this process primarily refines existing configurations within bounded operational thresholds.

Table 8 Levels of parameters.

Factors	Factor Levels			
	1	2	3	4
popsize	10	30	60	90
φ	0.1	0.2	0.3	0.4
λ	0.1	0.3	0.5	0.7
γ	0.3	0.5	0.7	0.9
Γ	50	100	150	200

The ANOVA results in Table 9 demonstrate that the key parameters $popsize$, φ, λ, γ, and Γ significantly influence QLMHHA's search behaviors. With *P*-values approaching zero for $popsize$, φ, λ, γ, and Γ, their *F*-ratios are 201.982, 100.853, 240.708, 143.700, and 137.990, respectively. Among these parameters, λ represents the most significant factor, exhibiting the highest *F*-ratio (*i.e.*, *F*-ratio=240.708, *P*-value<0.05), which directly influences the update rate during the learning process and determines the algorithm's responsiveness to new information versus the retention of historical experience. The remaining parameters were ranked in descending order according to the significance of their *F*-values: $popsize$, γ, Γ, and φ. Significant interaction effects exist for $popsize*\varphi$ (*i.e.*, *P*-value=0.0408) and $popsize*\lambda$ (*i.e.*, *P*-value=0.0193), highlighting the critical role of parameter interdependence. As illustrated in Fig. 9, $\lambda = 0.3$ achieves the best performance across all parameter levels. Both excessively high and low λ values degrade algorithm's performance. Lower λ reduces learning rates, impeding rapid adaptation to environmental feedback and hindering efficient strategy space exploration; conversely, higher λ excessively prioritizes new information, inducing instability through insufficient integration of historical experience. Hence, moderate λ value strikes a balance between learning agility and stability, facilitating effective exploration in both strategy and solution spaces. Regarding population size, $popsize$ is the second-most significant factor (*F*-ratio=201.982). Notably, configurations with $popsize = 30$ yield peak performance, whereas $popsize = 90$ results in marked performance deterioration. These findings indicate that population size affects exploration efficiency and computational cost. Smaller $popsize$ may inadequately explore the solution space due to limited diversity in high-level individuals. Conversely, larger $popsize$ increases the computational cost and the burden of evaluating solutions and weakens the efficiency of *Q*-learning-based strategies, thus impairing the learning effect for problem-specific knowledge acquisition to optimize decision-making behaviors. Hence, moderate $popsize$ enables balancing computational cost with evolutionary efficiency while achieving efficient search behavior across strategy and solution spaces. The third the local depth search while restricting global breadth exploration; however, higher γ may prioritize long-term rewards, impeding early convergence and reducing search efficiency. A moderate value of γ (*i.e.*, $\gamma = 0.7$) can balance short-term rewards and long-term gains, enabling rapid strategy learning and comprehensive solution space exploration. In addition, the upper limit of cumulative runtime threshold Γ is ranked fourth by *F*-ratio=137.990. Extreme values of Γ trigger frequent or delayed PM, both adversely affecting the completion times of factories and energy consumption for machines. As indicated in Table 9, the elite ratio φ exhibits the lowest statistical significance due to its minimal *F*-ratio=100.853, yet appropriate selection maintains the diversity of solution sets and accelerates the convergence of algorithms, and facilitates knowledge transfer from elite HLIs in the strategy space.

Table 9 ANOVA results for parameter calibration of QLMHHA.

Source	*Sum of Squares*	*Df*	*Mean Square*	*F - Ratio*	*P - Value*
Main effects					
popsize	0.0612220159	3	67.32730253	201.9819075752	**0.0000**
φ	0.0305693210	3	33.61780715	100.8534214418	**0.0000**
λ	0.0729600077	3	80.23585042	240.7075512582	**0.0000**
γ	0.0435564718	3	47.90008482	143.7002544656	**0.0000**
Γ	0.0418256667	3	45.99667733	137.9900319986	**0.0000**
interactions					
$popsize*\varphi$	0.0017829135	9	0.217856912	1.9607122124	**0.0408**
$popsize*\lambda$	0.0020129892	9	0.245970215	2.2137319318	**0.0193**
$popsize*\gamma$	0.0009483219	9	0.115876894	1.0428920452	0.4035
$popsize*\Gamma$	0.0006744114	9	0.082407352	0.7416661643	0.6708
$\varphi*\lambda$	0.0006803697	9	0.083135407	0.7482186585	0.6647
$\varphi*\gamma$	0.0004416923	9	0.053971061	0.4857395445	0.8848
$\varphi*\Gamma$	0.0009720027	9	0.118770496	1.0689344597	0.3832
$\lambda*\gamma$	0.0006927784	9	0.084651653	0.7618648801	0.6519
$\lambda*\Gamma$	0.0006505218	9	0.079488246	0.7153942122	0.6951
$\gamma*\Gamma$	0.0014107421	9	0.17238073	1.5514265732	0.1255
Residual	0.0927505691	918			
Total	0.353150795	1023			

Based on the ANOVA results in Table 9, the exceptionally high *F*-ratios (201.98-240.71) for all main effects confirm their statistically significant influence on algorithm's efficacy (*P*-value<<0.01). Notably, when significant interactions exist, the main effect interpretations may be unreliable (Fatih et al.2017). Hence, Table 9 reveals significant two-factor interactions alongside main effects and Fig. 10 presents interaction effect analyses for significant factor pairs. The results show that the interaction between the factors is weak for $popsize*\varphi$ and $popsize*\lambda$, which have significant effects in Table 9, not contradicting the conclusions drawn in Fig. 10. Consequently, by analyzing the above results, we provide the parameters that are suitable for following computational comparisons: $popsize = 30$, $\varphi = 0.2$, $\lambda = 0.3$, $\gamma = 0.7$, and $\Gamma = 100$.

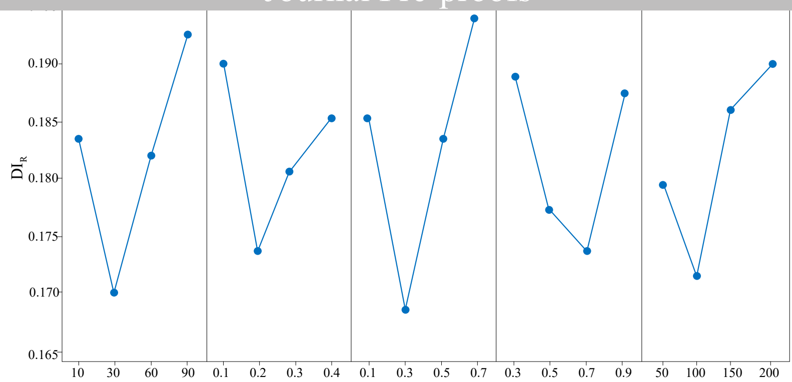


Fig. 9. The main effect plots of each parameter on metric DI_R .

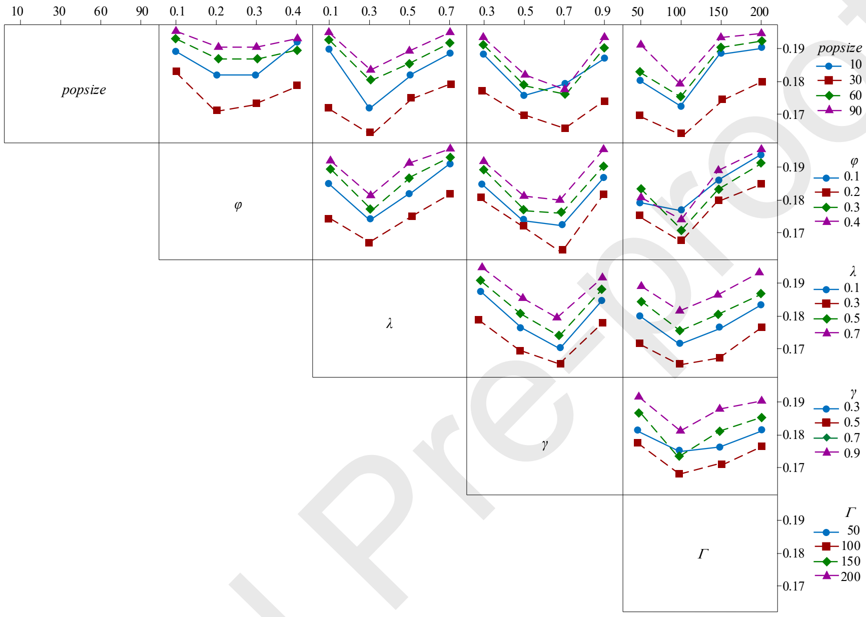


Fig. 10. The interaction effect plots of parameter pairs on metric DI_R .

5.3. Performance metrics

When it comes to tackling multi-objective optimization problems (MOPs), relying solely on a single performance metric may not sufficiently capture both the effectiveness and efficiency of multi-objective algorithms. Hence, to comprehensively evaluate multi-objective algorithms from multiple perspectives, this subsection employs four commonly used performance metrics to quantify the convergence, diversity, distribution uniformity, and coverage characteristics of algorithms, as below:

(a) **Reference Distance** (DI_R) : DI_R measures the difference between a non-dominated solution set $\boldsymbol{\Omega}'$ and a reference set $\boldsymbol{\Omega}^*$. It is computed as the average minimum distance from points in $\boldsymbol{\Omega}^*$ to their closest counterparts in $\boldsymbol{\Omega}'$, thereby evaluating how closely the solution set $\boldsymbol{\Omega}'$ approximates the reference set $\boldsymbol{\Omega}^*$. The formula for DI_R is given as:

$$DI_R(\boldsymbol{\Omega}') = \frac{1}{\left|\boldsymbol{\Omega}^*\right|} \sum_{y \in \boldsymbol{\Omega}^*} \min\left\{d(x,y) \middle| x \in \boldsymbol{\Omega}'\right\}. \quad (57)$$

distance $d(x, y)$. It is obvious that a smaller DI_R value corresponds to a closer approximation of $\mathbf{\Omega}'$ to the reference set $\mathbf{\Omega}^*$, which implies that the algorithm's iterations are converging toward the Pareto optimal front. Typically, $\mathbf{\Omega}^*$ is formed by aggregating all non-dominated solutions obtained from all algorithms under comparisons.

(b) **Non-dominance Ratio** (ρ_r): ρ_r quantifies the proportion of solutions in a given solution set $\mathbf{\Omega}_1$ that are not dominated by any solution in the reference set $\mathbf{\Omega}^*$. It is computed as follows:

$$\rho_r(\mathbf{\Omega_1} \left| \mathbf{\Omega}^* \right.) = \frac{\left| \left\{ a \in \mathbf{\Omega_1} \left| \nexists b \in \mathbf{\Omega}^* : b \succ a \right. \right\} \right|}{\left| \mathbf{\Omega_1} \right|}. \tag{58}$$

(c) **Hypervolume** (HV): The hypervolume metric calculates the volume in the objective space covered by the non-dominated solution set with respect to a predefined reference point R. It provides a comprehensive measure of both the convergence and diversity. For addressing MOPs, a higher HV value indicates that the solution set is closer to the true Pareto front and has more uniform distribution. The formula for calculating HV is as follows:

$$HV(A) = \sum_{i=1}^{|A|} \left((R_1 - x_{i,1}) \cdot (R_2 - x_{i,2}) \right). \tag{59}$$

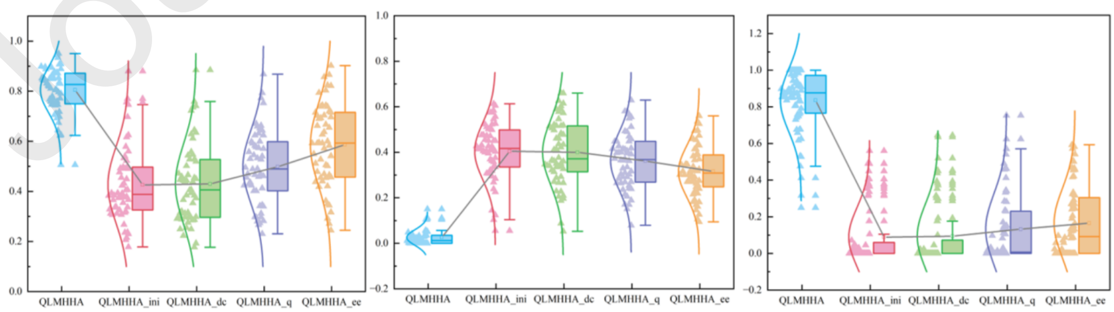
Here, A denotes the non-dominated solution set obtained by the algorithm; $x_{i,1}$ and $x_{i,2}$ are the two objective values of the i-th non-dominated solution; and $R = (R_1, R_2)$ is a reference point chosen such that it is dominated by all Pareto-optimal solutions. The objective values should be normalized, and the reference point R is usually set to (1.01, 1.01). An HV value closer to 1 suggests a higher quality Pareto set. This metric is widely used for evaluating multi-objective optimization algorithms, as it provides a unified measure of both quality and diversity for solutions in terms of their proximity to the ideal Pareto front.

5.4. Investigation of critical components

This section investigates the performance and contributions of QLMHHA's critical components, including the MNEH-based initialization method (see Algorithm 1), the destruction-construction (DC) mechanism (see Algorithm 3), the multi-stage cooperative energy-efficient strategy (MSC_EES) (see Algorithm 4), and *Q*-learning-based HLS (see Subsection 4.6). There are four variants designed for comparisons for the above strategies and methods: QLMHHA_ini, QLMHHA_dc, QLMHHA_ee, and QLMHHA_q. Specifically, QLMHHA_ini employs random initialization instead of the MNEH-based initialization method. QLMHHA_dc omits the DC mechanism and QLMHHA_ee excludes the MSC_EES. In QLMHHA_q, HLIs are generated by randomly sorting LLHs. To ensure comparability, same parameter settings and computational conditions. All algorithms were independently executed 5 times on 54 instances, with the stopping condition of $H \times S \times \rho$ $(\rho = 30)$ seconds. Table 10 presents the comparative analysis of performance metrics across different instances, where optimal values are highlighted in bold. Additionally, pairwise evaluations were conducted using the Wilcoxon rank-sum test. The notations "+/=/-" indicate whether each variant (QLMHHA_ini, QLMHHA_dc, QLMHHA_ee, or QLMHHA_q) is superior, equivalent, or inferior to QLMHHA, respectively.

As shown in Table 10, QLMHHA demonstrates significantly superior performance to its variants across most cases for the HV, DI_R and ρ_r metrics. For the HV metric, QLMHHA achieves the best results in 47 out of 54 instances, whereas its variants QLMHHA_q and QLMHHA_ee only achieve the best results in 7 instances. These HV results indicate that QLMHHA outperforms its variants in terms of both diversity and convergence. For the ρ_r metric, QLMHHA obtains the best results in 53 out of 54 instances, demonstrating its enhanced ability to generate non-dominated solutions. Similarly, QLMHHA achieves the best DI_R values across 53 instances, confirming that its generated solutions are significantly closer to the Pareto optimal set. These experimental results validate that all proposed strategies and methods collectively contribute to improving QLMHHA's performance. Specifically, the effectiveness of the MNEH-based initialization method is validated through comparisons between QLMHHA_ini and QLMHHA. QLMHHA outperforms QLMHHA_ini for all instances on both the HV and ρ_r metrics. As shown in Table 10, QLMHHA achieves superior DI_R values in 53 out of 54 instances. Experimental results demonstrate that QLMHHA significantly outperforms QLMHHA_ini in both convergence and diversity by adopting the proposed initialization method. Collectively, these findings indicate significant computational differences between the two algorithms, as attributable to their different initialization methods. MNEH-based initialization method can generate higher-quality initial populations and accelerate convergence to high-quality solutions; as instance scale increases, the advantages of QLMHHA become increasingly significant. Comparisons between QLMHHA_dc and QLMHHA demonstrate that QLMHHA significantly outperforms QLMHHA_dc in almost all instances. Based on the Wilcoxon rank-sum test results, QLMHHA achieves superior values for HV, DI_R and ρ_r values in 53, 53, and 54 instances, respectively, underscoring the critical role of the DC mechanism in enhancing the algorithm's performance. This local perturbation mechanism facilitates exploratory search near local optima, thereby avoiding trapping in local optima and exploring broader regions of the solution space. The efficacy of MSC_EES is validated through comparisons between QLMHHA_ee and QLMHHA. As shown in Table 10, for the HV metric, QLMHHA outperforms QLMHHA_ee in 49 consistently demonstrates better performance across all instances for the DI_R metric. For the ρ_r metric, QLMHHA outperforms QLMHHA_ee in 53 instances. Additionally, the Wilcoxon rank-sum test results demonstrate the statistically significant improvement in QLMHHA's performance compared to QLMHHA_ee. Collectively, these findings indicate that implementing the MSC_EES accelerates algorithm convergence and yields higher-quality non-dominated solution sets. Finally, the effectiveness of the *Q*-learning-based HLS is demonstrated through comparisons between QLMHHA_q and QLMHHA. For the HV metric, as shown in Table 10, QLMHHA yields better results in 51 out of 54 instances than QLMHHA_q. Statistical results for the DI_R metric indicate that QLMHHA exhibits superior performance in 53 instances; similarly, QLMHHA performs better in 53 instances for the ρ_r metric. The Wilcoxon rank-sum test results confirm the QLMHHA's statistically significant superiority, validating the optimization efficacy of *Q*-learning in generating HLIs. The *Q*-learning-based HLS can enhance QLMHHA's global search capability by dynamically selecting LLH operators, thereby improving the diversity and convergence of Pareto solution sets. The merit of this mechanism lies in adaptively selecting LLH operators based on environmental feedback, ensuring the generation of the best HLIs during iterative processes. Experimental results reveal that the *Q*-learning greatly improves the success rate of generating superior solutions compared to random methods. In summary, the *Q*-learning-driven HLS mechanism demonstrates significant technical advantages and applicability in multi-objective optimization with complex scenarios.

To visually validate the aforementioned findings, Fig. 11 presents box plots for QLMHHA_ini, QLMHHA_dc, QLMHHA_ee, QLMHHA_q, and QLMHHA on three metrics. Fig. 11 demonstrates that QLMHHA performs superior to the other four variants across all instances. Consequently, this analysis confirms that the designed MNEH-based initialization method, DC mechanism, MSC_EES, and *Q*-learning-based HLS collectively contribute to the significant performance improvement of the QLMHHA, validating the effective effects and critical contributions of these components.



(*a*) HV (*b*) DI_R (*c*) ρ_r

Fig. 11. The box plots of QLMHHA with its variants on three metrics HV, DI_R, and ρ_r.

Table 10 Statistical results of QLMHHA with its variants on three metrics HV, DI_R, and ρ_r.

Instance	QLMHHA			QLMHHA_ini			QLMHHA_dc			QLMHHA_q			QLMHHA_ee		
	HV	DI_R	ρ_r	HV	DI_R	ρ_r	HV	DI_R	ρ_r	HV	DI_R	ρ_r	HV	DI_R	ρ_r
3-10-34-2-3	**0.89**	**0.05**	**0.77**	0.49(+)	0.28(+)	0.00(+)	0.64(+)	0.27(+)	0.00(+)	0.46(+)	0.27(+)	0.23(+)	0.46(+)	0.33(+)	0.00(+)
3-10-34-2-4	**0.83**	**0.00**	**1.00**	0.41(+)	0.40(+)	0.00(+)	0.52(+)	0.36(+)	0.00(+)	0.57(+)	0.32(+)	0.00(+)	0.54(+)	0.32(+)	0.00(+)
3-10-34-2-5	**0.84**	**0.00**	**1.00**	0.45(+)	0.34(+)	0.00(+)	0.45(+)	0.33(+)	0.00(+)	0.41(+)	0.42(+)	0.00(+)	0.47(+)	0.32(+)	0.00(+)
3-10-34-4-3	**0.63**	**0.04**	**0.76**	0.37(+)	0.41(+)	0.00(+)	0.37(+)	0.35(+)	0.00(+)	0.47(+)	0.46(+)	0.15(+)	0.58(+)	0.37(+)	0.09(+)
3-10-34-4-4	**0.85**	**0.00**	**1.00**	0.34(+)	0.45(+)	0.00(+)	0.30(+)	0.48(+)	0.00(+)	0.28(+)	0.56(+)	0.00(+)	0.24(+)	0.56(+)	0.00(+)
3-10-34-4-5	**0.87**	**0.00**	**1.00**	0.38(+)	0.34(+)	0.00(+)	0.38(+)	0.34(+)	0.00(+)	0.23(+)	0.51(+)	0.00(+)	0.32(+)	0.43(+)	0.00(+)
3-10-34-6-3	**0.88**	**0.11**	**0.92**	0.32(+)	0.43(+)	0.00(+)	0.35(+)	0.38(+)	0.00(+)	0.46(+)	0.30(+)	0.00(+)	0.50(+)	0.28(+)	0.08(+)
3-10-34-6-4	**0.81**	**0.00**	**1.00**	0.23(+)	0.53(+)	0.00(+)	0.19(+)	0.57(+)	0.00(+)	0.34(+)	0.45(+)	0.00(+)	0.27(+)	0.47(+)	0.00(+)
3-10-34-6-5	**0.79**	**0.03**	**0.86**	0.41(+)	0.38(+)	0.00(+)	0.39(+)	0.43(+)	0.00(+)	0.46(+)	0.39(+)	0.14(+)	0.42(+)	0.39(+)	0.00(+)
3-15-57-2-3	**0.91**	**0.00**	**1.00**	0.32(+)	0.45(+)	0.00(+)	0.30(+)	0.42(+)	0.00(+)	0.49(+)	0.25(+)	0.00(+)	0.51(+)	0.26(+)	0.00(+)
3-15-57-2-4	**0.80**	**0.03**	**0.88**	0.49(+)	0.41(+)	0.05(+)	0.45(+)	0.48(+)	0.05(+)	0.54(+)	0.41(+)	0.02(+)	0.57(+)	0.26(+)	0.00(+)
3-15-57-2-5	**0.88**	**0.01**	**0.92**	0.56(+)	0.32(+)	0.00(+)	0.65(+)	0.21(+)	0.00(+)	0.75(+)	0.24(+)	0.08(+)	0.45(+)	0.32(+)	0.00(+)
3-15-57-4-3	**0.85**	**0.00**	**0.98**	0.38(+)	0.43(+)	0.00(+)	0.44(+)	0.34(+)	0.00(+)	0.62(+)	0.24(+)	0.02(+)	0.59(+)	0.29(+)	0.00(+)
3-15-57-4-4	**0.85**	**0.00**	**1.00**	0.35(+)	0.55(+)	0.00(+)	0.29(+)	0.62(+)	0.00(+)	0.35(+)	0.57(+)	0.00(+)	0.29(+)	0.53(+)	0.00(+)
3-15-57-4-5	**0.87**	**0.00**	**0.89**	0.50(+)	0.32(+)	0.00(+)	0.53(+)	0.34(+)	0.00(+)	0.64(+)	0.29(+)	0.11(+)	0.34(+)	0.50(+)	0.00(+)
3-15-57-6-3	**0.77**	**0.01**	**0.89**	0.39(+)	0.56(+)	0.00(+)	0.41(+)	0.52(+)	0.00(+)	0.53(+)	0.36(+)	0.11(+)	0.47(+)	0.43(+)	0.00(+)
3-15-57-6-4	**0.72**	**0.02**	**0.68**	0.31(+)	0.40(+)	0.00(+)	0.32(+)	0.44(+)	0.00(+)	0.40(+)	0.33(+)	0.00(+)	0.53(+)	0.34(+)	0.32(+)
3-15-57-6-5	**0.80**	**0.00**	**0.89**	0.38(+)	0.50(+)	0.00(+)	0.38(+)	0.52(+)	0.00(+)	0.47(+)	0.47(+)	0.11(+)	0.35(+)	0.45(+)	0.00(+)
3-20-68-2-3	**0.89**	**0.02**	**0.91**	0.24(+)	0.61(+)	0.00(+)	0.31(+)	0.55(+)	0.00(+)	0.60(+)	0.41(+)	0.00(+)	0.70(+)	0.30(+)	0.09(+)
3-20-68-2-4	**0.91**	**0.00**	**1.00**	0.43(+)	0.36(+)	0.00(+)	0.41(+)	0.41(+)	0.00(+)	0.40(+)	0.40(+)	0.00(+)	0.65(+)	0.24(+)	0.00(+)
3-20-68-2-5	**0.91**	**0.03**	**0.32**	0.88(+)	0.06(+)	0.10(+)	0.88(+)	0.05(+)	0.31(+)	0.66(+)	0.19(+)	0.00(+)	0.82(+)	0.09(+)	0.27(+)
3-20-68-4-3	**0.95**	**0.00**	**0.94**	0.33(+)	0.58(+)	0.00(+)	0.29(+)	0.60(+)	0.00(+)	0.65(+)	0.27(+)	0.00(+)	0.87(+)	0.13(+)	0.06(+)
3-20-68-4-4	**0.81**	**0.03**	**0.84**	0.37(+)	0.40(+)	0.00(+)	0.37(+)	0.46(+)	0.00(+)	0.49(+)	0.37(+)	0.00(+)	0.59(+)	0.32(+)	0.16(+)
3-20-68-4-5	**0.75**	**0.05**	**0.68**	0.39(+)	0.52(+)	0.00(+)	0.25(+)	0.66(+)	0.00(+)	0.30(+)	0.58(+)	0.00(+)	0.72(+)	0.41(+)	0.32(+)
3-20-68-6-3	**0.76**	**0.02**	**0.85**	0.31(+)	0.57(+)	0.00(+)	0.29(+)	0.59(+)	0.00(+)	0.42(+)	0.42(+)	0.02(+)	0.59(+)	0.30(+)	0.13(+)
3-20-68-6-4	**0.85**	**0.00**	**0.93**	0.50(+)	0.47(+)	0.00(+)	0.47(+)	0.52(+)	0.00(+)	0.44(+)	0.49(+)	0.00(+)	0.72(+)	0.35(+)	0.07(+)
3-20-68-6-5	**0.82**	**0.00**	**1.00**	0.34(+)	0.50(+)	0.00(+)	0.32(+)	0.52(+)	0.00(+)	0.27(+)	0.63(+)	0.00(+)	0.51(+)	0.54(+)	0.00(+)
2-10-34-2-3	**0.82**	**0.11**	**0.75**	0.34(+)	0.45(+)	0.00(+)	0.74(+)	0.20(+)	0.10(+)	0.63(+)	0.21(+)	0.00(+)	0.60(+)	0.18(+)	0.15(+)
2-10-34-2-4	0.71	**0.02**	**0.62**	0.63(+)	0.12(+)	0.03(+)	0.55(+)	0.19(+)	0.00(+)	0.66(+)	0.17(+)	0.15(+)	**0.74(-)**	0.11(+)	0.20(+)
2-10-34-2-5	**0.83**	**0.00**	**0.98**	0.38(+)	0.47(+)	0.00(+)	0.44(+)	0.31(+)	0.00(+)	0.43(+)	0.49(+)	0.00(+)	0.74(+)	0.39(+)	0.02(+)
2-10-34-4-3	0.62	**0.10**	**0.53**	0.52(+)	0.36(+)	0.03(+)	0.56(+)	0.29(+)	0.00(+)	0.57(+)	0.31(+)	0.11(+)	**0.70(-)**	0.28(+)	0.33(+)
2-10-34-4-4	0.51	**0.15**	0.25	0.42(+)	0.27(+)	0.00(+)	0.53(-)	0.23(+)	0.00(+)	0.53(-)	0.22(+)	0.33(-)	**0.67(-)**	0.16(+)	**0.42(-)**
2-10-34-4-5	**0.74**	**0.00**	**1.00**	0.18(+)	0.50(+)	0.00(+)	0.44(+)	0.21(+)	0.00(+)	0.27(+)	0.40(+)	0.00(+)	0.44(+)	0.22(+)	0.00(+)
2-10-34-6-3	**0.71**	**0.01**	**0.88**	0.22(+)	0.41(+)	0.00(+)	0.28(+)	0.35(+)	0.00(+)	0.28(+)	0.36(+)	0.00(+)	0.45(+)	0.25(+)	0.12(+)

2-10-34-6-4	**0.83**	**0.00**	**1.00**	0.37(+)	0.37(+)	0.00(+)	0.25(+)	0.51(+)	0.00(+)	0.47(+)	0.29(+)	0.00(+)	0.42(+)	0.27(+)	0.00(+)
2-10-34-6-5	**0.70**	**0.00**	**1.00**	0.21(+)	0.43(+)	0.00(+)	0.18(+)	0.46(+)	0.00(+)	0.31(+)	0.43(+)	0.00(+)	0.36(+)	0.35(+)	0.00(+)
2-15-57-2-3	**0.89**	**0.01**	**0.76**	0.58(+)	0.23(+)	0.00(+)	0.44(+)	0.32(+)	0.00(+)	0.67(+)	0.15(+)	0.00(+)	0.82(+)	0.29(+)	0.24(+)
2-15-57-2-4	**0.90**	0.11	**0.41**	0.75(+)	0.10(-)	0.18(+)	0.76(+)	0.09(-)	0.18(+)	0.79(+)	**0.08(-)**	0.23(+)	0.74(+)	0.27(+)	0.00(+)
2-15-57-2-5	0.75	**0.03**	**0.66**	0.70(+)	0.40(+)	0.00(+)	0.74(+)	0.34(+)	0.02(+)	**0.77(-)**	0.37(+)	0.17(+)	0.74(+)	0.49(+)	0.15(+)
2-15-57-4-3	0.84	**0.05**	**0.48**	0.77(+)	0.14(+)	0.04(+)	0.61(+)	0.27(+)	0.00(+)	**0.87(-)**	0.17(+)	0.29(+)	0.79(+)	0.17(+)	0.19(+)
2-15-57-4-4	**0.77**	**0.04**	**0.85**	0.46(+)	0.42(+)	0.05(+)	0.30(+)	0.41(+)	0.00(+)	0.58(+)	0.33(+)	0.10(+)	0.48(+)	0.30(+)	0.00(+)
2-15-57-4-5	**0.85**	**0.04**	**0.88**	0.54(+)	0.31(+)	0.00(+)	0.54(+)	0.29(+)	0.00(+)	0.58(+)	0.26(+)	0.00(+)	0.64(+)	0.27(+)	0.12(+)
2-15-57-6-3	**0.67**	**0.03**	**0.74**	0.53(+)	0.29(+)	0.03(+)	0.46(+)	0.34(+)	0.00(+)	0.62(+)	0.40(+)	0.13(+)	0.63(+)	0.15(+)	0.10(+)
2-15-57-6-4	**0.78**	**0.00**	**1.00**	0.24(+)	0.41(+)	0.00(+)	0.32(+)	0.35(+)	0.00(+)	0.34(+)	0.28(+)	0.00(+)	0.55(+)	0.23(+)	0.00(+)
2-15-57-6-5	**0.85**	**0.01**	**0.97**	0.49(+)	0.46(+)	0.00(+)	0.45(+)	0.50(+)	0.00(+)	0.55(+)	0.40(+)	0.03(+)	0.44(+)	0.37(+)	0.00(+)
2-20-68-2-3	**0.93**	**0.00**	**1.00**	0.27(+)	0.51(+)	0.00(+)	0.22(+)	0.58(+)	0.00(+)	0.59(+)	0.27(+)	0.00(+)	0.80(+)	0.22(+)	0.00(+)
2-20-68-2-4	**0.94**	**0.00**	**0.94**	0.62(+)	0.24(+)	0.00(+)	0.63(+)	0.23(+)	0.00(+)	0.46(+)	0.30(+)	0.00(+)	0.90(+)	0.19(+)	0.06(+)
2-20-68-2-5	**0.85**	**0.02**	**0.90**	0.76(+)	0.48(+)	0.07(+)	0.72(+)	0.36(+)	0.03(+)	0.60(+)	0.37(+)	0.00(+)	0.70(+)	0.38(+)	0.00(+)
2-20-68-4-3	**0.90**	**0.00**	**0.98**	0.31(+)	0.48(+)	0.00(+)	0.29(+)	0.52(+)	0.00(+)	0.50(+)	0.45(+)	0.00(+)	0.69(+)	0.32(+)	0.02(+)
2-20-68-4-4	0.64	**0.05**	**0.73**	0.48(+)	0.31(+)	0.00(+)	0.47(+)	0.37(+)	0.00(+)	0.53(+)	0.45(+)	0.00(+)	**0.69(-)**	0.35(+)	0.27(+)
2-20-68-4-5	**0.75**	**0.05**	**0.68**	0.39(+)	0.52(+)	0.00(+)	0.25(+)	0.66(+)	0.00(+)	0.30(+)	0.58(+)	0.00(+)	0.72(+)	0.41(+)	0.32(+)
2-20-68-6-3	**0.76**	**0.02**	**0.85**	0.31(+)	0.57(+)	0.00(+)	0.29(+)	0.59(+)	0.00(+)	0.42(+)	0.42(+)	0.02(+)	0.59(+)	0.30(+)	0.13(+)
2-20-68-6-4	**0.69**	**0.01**	**0.90**	0.33(+)	0.61(+)	0.00(+)	0.29(+)	0.63(+)	0.00(+)	0.30(+)	0.55(+)	0.00(+)	0.66(+)	0.46(+)	0.10(+)
2-20-68-6-5	0.77	**0.06**	**0.53**	0.38(+)	0.43(+)	0.00(+)	0.50(+)	0.27(+)	0.00(+)	0.56(+)	0.24(+)	0.00(+)	**0.86(-)**	0.18(+)	0.47(+)
+/ ≈ / −				54/0/0	53/0/1	54/0/0	53/0/1	53/0/1	54/0/0	51/0/3	53/0/1	53/0/1	49/0/5	54/0/0	53/0/1

5.5. Comparison with advanced algorithms

To further validate the effectiveness of the proposed QLMHHA, this subsection compares it with six advanced multi-objective optimization algorithms: the improved Jaya algorithm (Pan et al.2023), the modified MOEA/D (Wang et al.2021), the classical NSGA-II (Deb et al.2002), QVNS-NSGA-II (Li et al.2023), GP-HH (Song et al.2021), and FLS-HH (Shao et al.2024). These algorithms have demonstrated remarkable performance in solving various multi-objective shop scheduling problems. Specifically, the improved Jaya algorithm exhibits excellent effectiveness in bi-objective optimization; MOEA/D, based on the decomposition strategy, effectively balances conflicting objectives; NSGA-II remains a widely recognized baseline algorithm for multi-objective optimization; QVNS-NSGA-II enhances both global and local search capabilities of NSGA-II by incorporating *Q*-learning with variable neighborhood search methods. As the representative HHAs, FLS-HH and GP-HH have been successfully applied to DHFSPs and DAFSPs, respectively; consequently, they are reconstructed and adapted to address the EE-IDHFSP-PM. Parameter configurations for all algorithms are summarized in Table 11. These algorithms are fine-tuned based on parameter settings recommended in the original literature to achieve best performance for tackling EE-IDHFSP-PM. To ensure fairness, all algorithms adapt identical encoding and decoding methods, evaluation functions, and termination conditions. All experiments are conducted under the same hardware and computational resource conditions. Each algorithm is independently executed 10 times on each test instance, with the maximum CPU runtime set to $H \times S \times \rho \ \ (\rho = 30, 40, 50)$ seconds as the common termination criterion.

Table 11 Parameter settings for all compared algorithms.

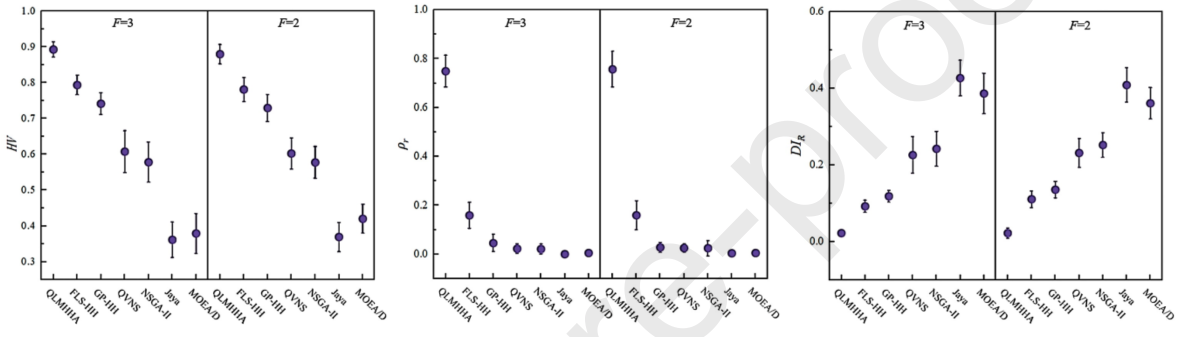
Parameter	Jaya	NSGA-II	QVNS-NSGA-II	MOEA/D	GP-HH	FLS-HH
Population size	20	120	120	100	40	-
Crossover rate	-	0.8	0.8	0.85	-	-
Mutation rate	-	0.3	0.3	0.2	0.15	-
Learning rate	-	-	0.1	-	-	0.4
Discount rate	-	-	0.1	-	-	-
$\varepsilon - greedy$	-	-	0.1	-	-	-
Neighborhood size	-	-	-	10	-	-
Destruction number	-	-	8	-	-	-
Update frequency	-	-	-	-	-	70
Maximal tree depth	-	-	-	-	4	-

Tables 12-23 present the statistical results of QLMHHA under different termination conditions $H \times S \times \rho \ \ (\rho = 30, 40, 50)$, along with comparisons against 6 outstanding multi-objective algorithms based on three performance metrics HV, ρ_r, DI_R across all instance sizes. These results indicate that QLMHHA consistently achieves the best results for both small-scale and large-scale cases under various runtime settings. These findings reveal that QLMHHA not only provides enhanced global search capabilities to generate high-quality sequences of HLIs in the strategy space but also efficiently significant advantages and stability in solving EE-IDHFSP-PM. Further analysis of statistical results in Tables 12-23 reveals that both FLS-HH and GP-HH exhibit strong performance, yet remain inferior to QLMHHA. The primary reasons may be that GP-HH lacks a real-time feedback mechanism during the search process, preventing dynamic adjustment of strategy combinations based on current search states, which can induce premature convergence. FLS-HH controls the selection of single-step LLHs solely through feedback probabilities; whereas QLMHHA dynamically updates selection strategies and yields high-quality sequences of HLIs, enabling learning-driven search behaviors. Additionally, QVNS-NSGA-II and NSGA-II performed comparably in most cases, yet still inferior to QLMHHA, FLS-HH, and GP-HH. This further indicates that HHAs surpass traditional evolutionary algorithms in search capability and strategy adaptability, as these HHAs effectively enhance solution diversity and quality through dynamic combinations of multiple LLHs, thereby demonstrating superior global optimization capabilities for complex scheduling problems. Finally, it is clear that the MOEA/D and Jaya algorithms perform poorly in nearly all cases. The underlying reasons may lie in the absence of problem-specific heuristic strategies and methods, coupled with inflexible and insufficiently adaptive search behaviors, which cause failure in effectively balancing solution set convergence and diversity. Consequently, they fail to obtain high-quality Pareto front solutions within finite computational time.

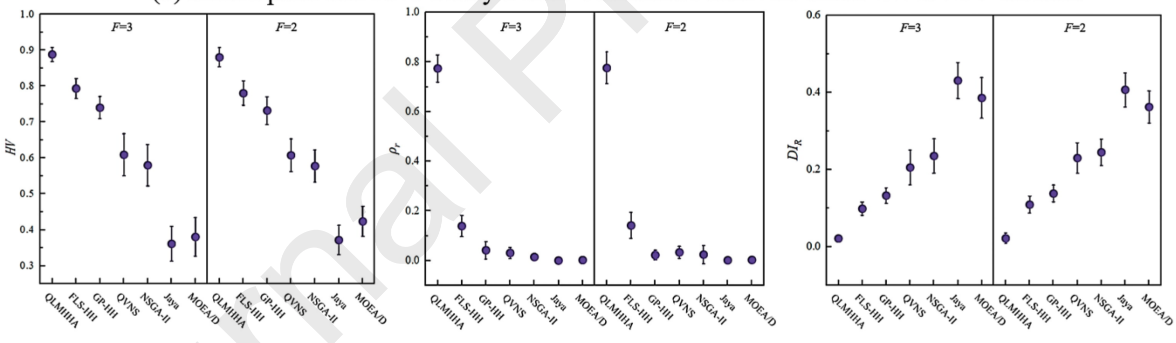
To strengthen the statistical stringency of these experimental findings, multi-factor ANOVA is employed to evaluate the statistical significance of observed differences. Figs. 12 (a)-(c) and Fig. 13 present mean plots with 95% Tukey HSD confidence intervals for HV, ρ_r, DI_R metrics, evaluated across different algorithms, CPU time expenses, and factory configurations, both for small-scale and large-scale instances. Moreover, Figs. 14 (a)-(c) and Fig. 15 provide the corresponding boxplots. The mean plots reveal that QLMHHA holds stable performance across different factory configurations, regardless of instance scales, thereby underlying its robustness and reliability in both small and large scenarios. This observation further validates QLMHHA's exceptional capability in handling instance groups with varying numbers of factories. Meanwhile, the boxplots indicate that the results obtained by QLMHHA are more tightly aggregated with reduced variance, reflecting higher solution quality, consistency, and stability. Overall, these results confirm that QLMHHA surpasses the other six multi-objective algorithms in terms of effectiveness and robustness.

Fig. 16 presents the Pareto fronts obtained by QLMHHA, FLS-HH, GP-HH, QVNS-NSGA-II, NSGA-II, Jaya, and MOEA/D for 12 test instances, under a total CPU runtime of $H \times S \times 30$ seconds. As observed in Fig. 16, the Pareto front produced by QLMHHA is positioned closer to the origin in both objective directions compared to those of other six multi-objective algorithms, indicating that it achieves a better balance between global exploration and local exploitation. Overall, these results

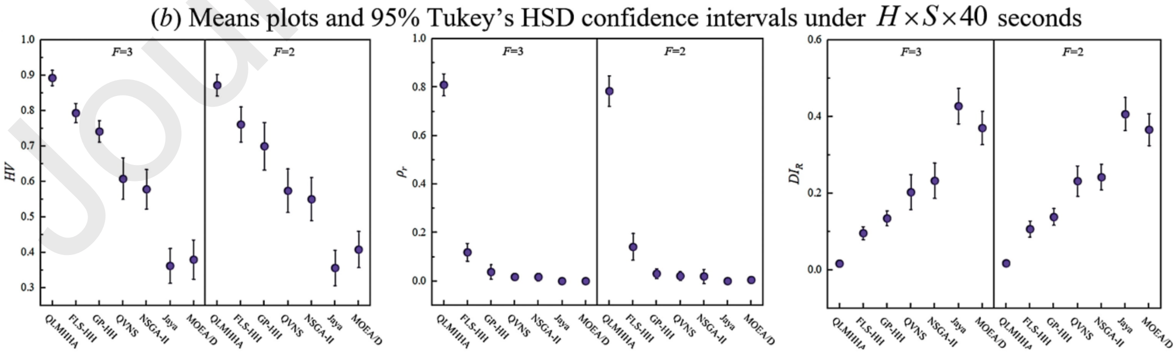
PM. To further illustrate scheduling results, Fig. 17 provides a Gantt chart corresponding to one non-dominated solution obtained by QLMHHA for instance 2-10-34-2-3 (*i.e.*, 2 factories, 10 products, 34 jobs, 2 production sub-stages, 3 machines per sub-stage). Products 3, 4, 6, 8, and 9 are assigned to factory 1, while products 1, 2, 5, 7, and 10 are processed in factory 2. The resulting schedule achieves the makespan of 285.702 and TEC of 37,083.776. This Gantt chart exhibits a compact job allocation with limited idle intervals, reflecting early start times that improve machine utilization. The solutions obtained by QLMHHA effectively reduces energy consumption, shortens the maximum completion time, and improves operational efficiency. These results confirm the efficacy of QLMHHA as a state-of-the-art algorithm for tackling the EE-IDHFSP-PM, demonstrating robust search capabilities and superior performance in energy-efficient distributed manufacturing systems.



(*a*) Means plots and 95% Tukey's HSD confidence intervals under $H \times S \times 30$ seconds



(*b*) Means plots and 95% Tukey's HSD confidence intervals under $H \times S \times 40$ seconds



(*c*) Means plots and 95% Tukey's HSD confidence intervals under $H \times S \times 50$ seconds

Fig. 12. Means plots and 95% Tukey's HSD confidence intervals for the interaction between the type of algorithms and number of factories for QLMHHA, FLS-HH, GP-HH, QVNS-NSGA-II, NSGA-II, Jaya, and MOEA/D on three metrics ρ_r, DI_R, HV for small-scale instances.

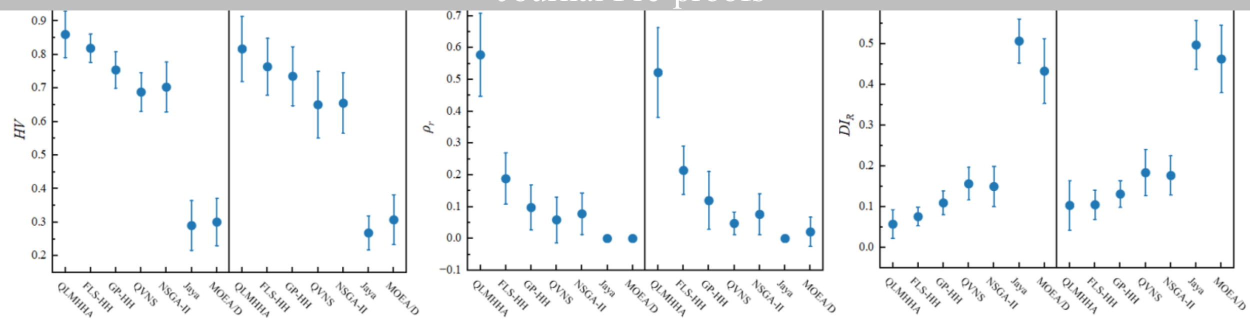
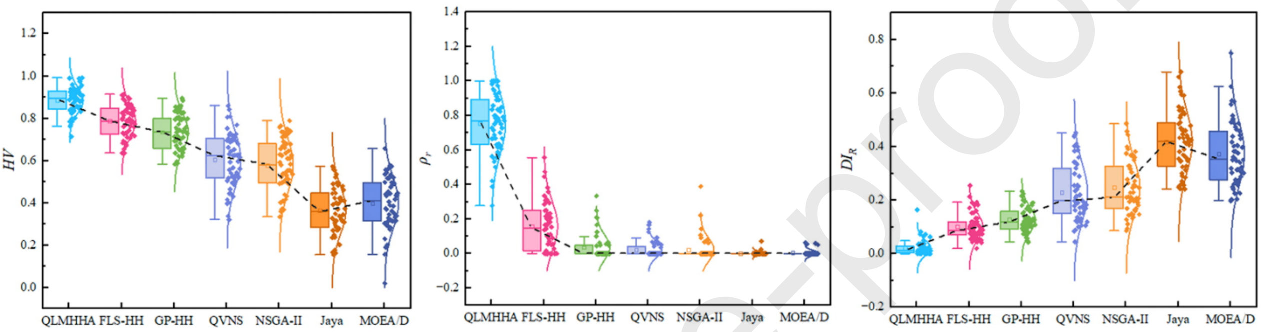
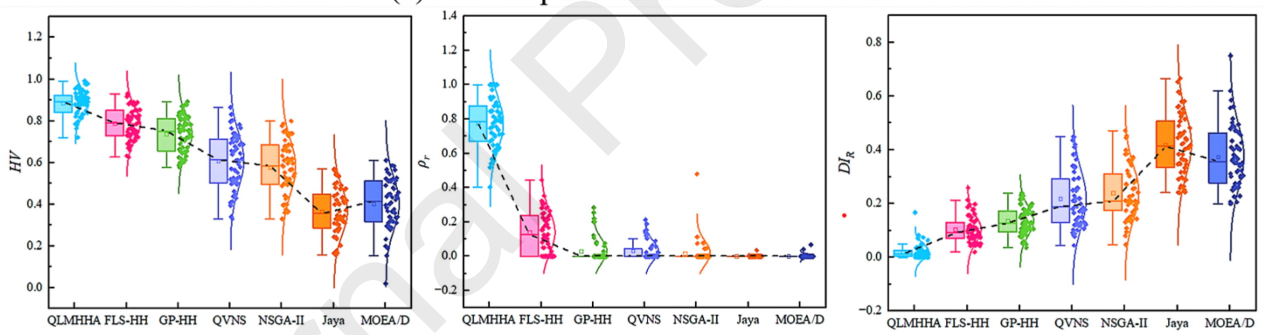


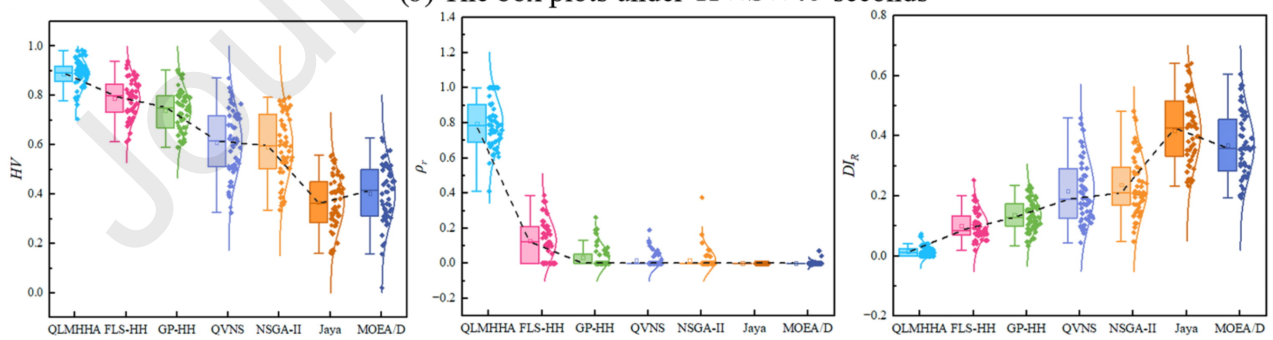
Fig. 13. Means plots and 95% Tukey's HSD confidence intervals for the interaction between the type of algorithms and number of factories for QLMHHA, FLS-HH, GP-HH, QVNS-NSGA-II, NSGA-II, Jaya, and MOEA/D on three metrics ρ_r, DI_R, HV for large-scale instances under $H \times S \times 30$ seconds.



(a) The box plots under $H \times S \times 30$ seconds



(b) The box plots under $H \times S \times 40$ seconds



(c) The box plots under $H \times S \times 50$ seconds

Fig. 14. The box plots of QLMHHA with other related algorithms on three metrics HV, ρ_r, DI_R for small-scale instances.

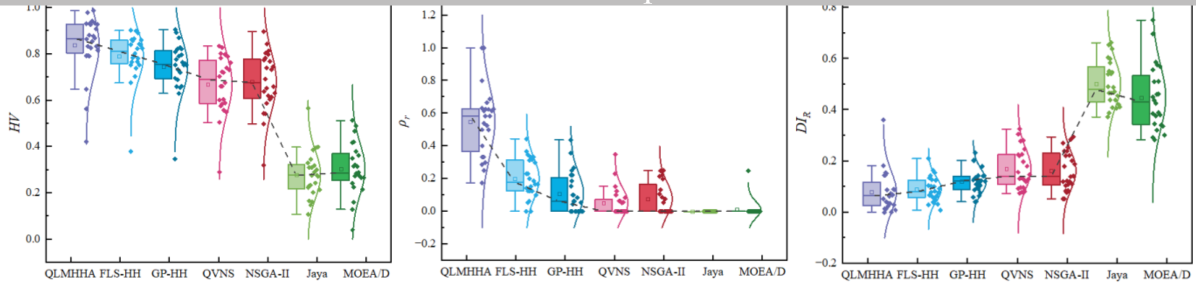


Fig. 15. The box plots of QLMHHA with other related algorithms on three metrics HV, ρ_r, DI_R for large-scale instances under $H \times S \times 30$ seconds.

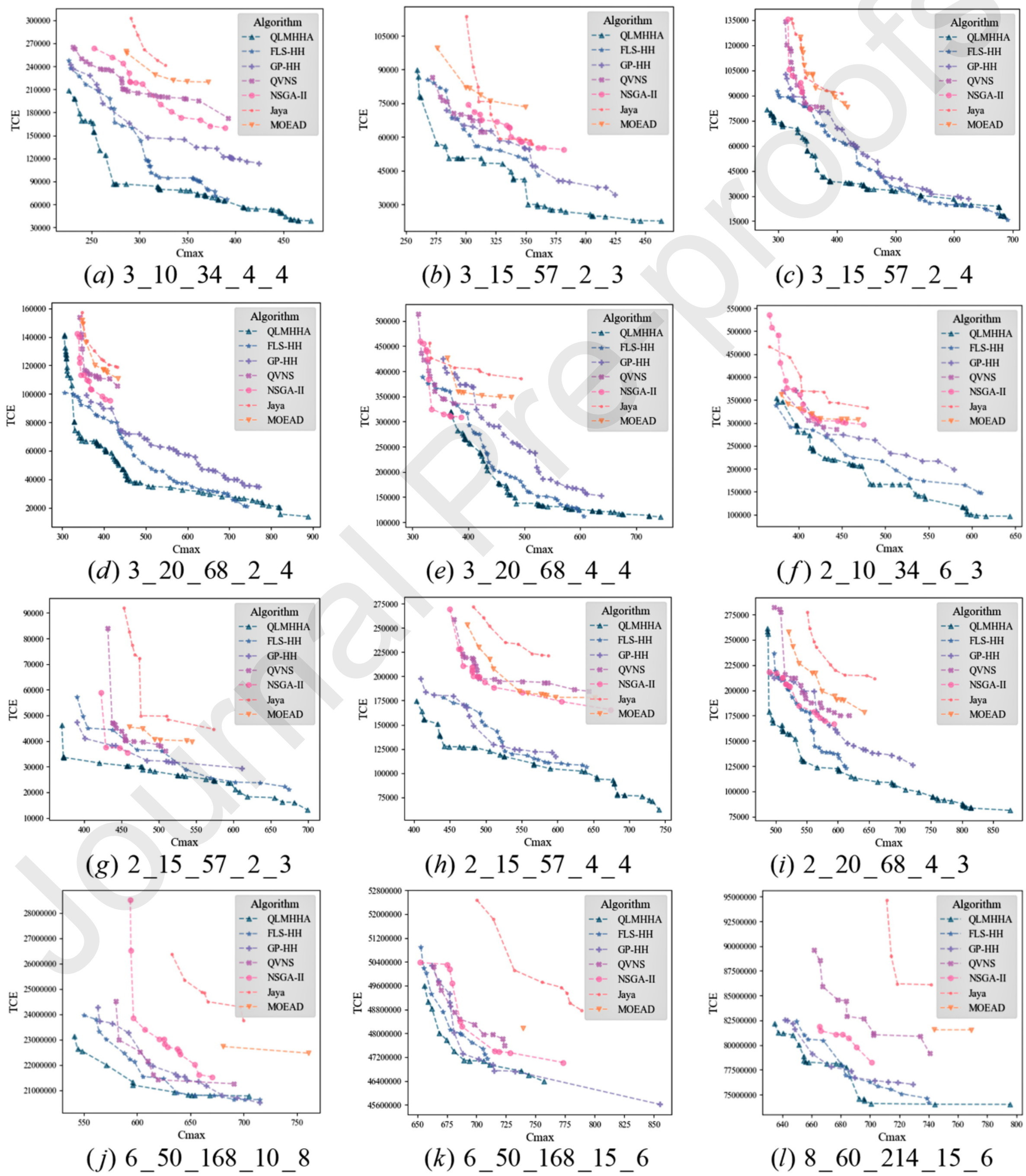


Fig. 16. Pareto fronts obtained by QLMHHA, FLS-HH, GP-HH, QVNS-NSGA-II, NSGA-II, Jaya, and MOEA/D for twelve instances under $H \times S \times 30$ seconds.

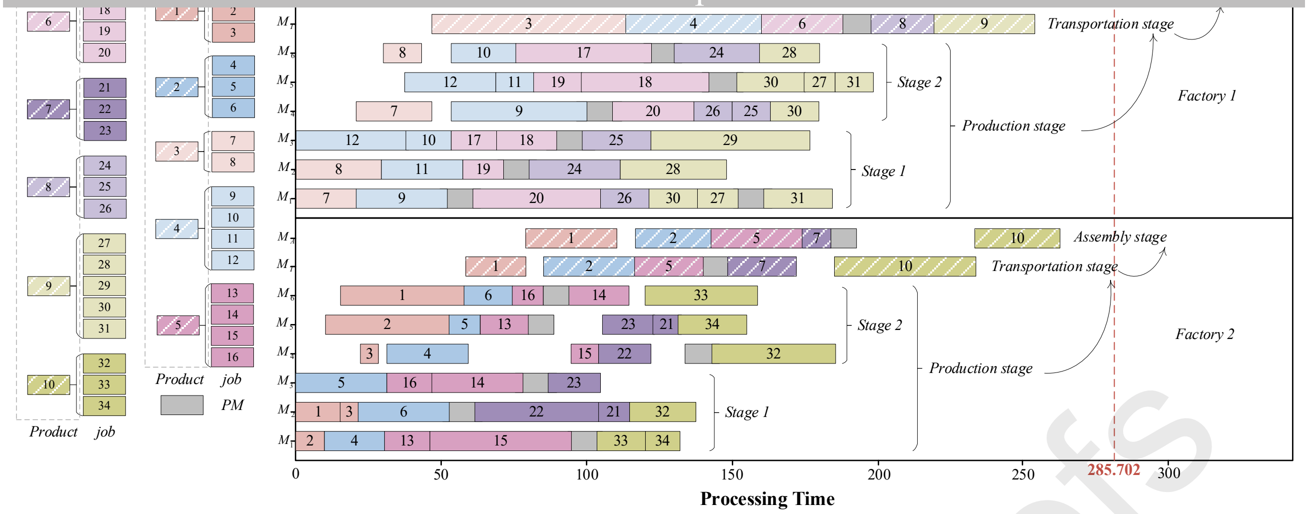


Fig. 17. The Gantt chart of the obtained optimal solution for instance 2-10-34-2-3.

s.

Instances	QLMHHA	FLS-HH	GP-HH	Jaya	NSGA-II	QVNS-NSGA-II	MOEA/D
3-10-34-2-3	**0.9894**	0.7939(+)	0.7139(+)	0.3278(+)	0.3350(+)	0.4157(+)	0.3045(+)
3-10-34-2-4	**0.9258**	0.8246(+)	0.8345(+)	0.5734(+)	0.7590(+)	0.6957(+)	0.6566(+)
3-10-34-2-5	**0.8890**	0.7314(+)	0.7953(+)	0.4446(+)	0.5930(+)	0.6624(+)	0.4977(+)
3-10-34-4-3	**0.9383**	0.7610(+)	0.7560(+)	0.3928(+)	0.4927(+)	0.5071(+)	0.4410(+)
3-10-34-4-4	**0.8424**	0.7264(+)	0.5841(+)	0.1673(+)	0.4019(+)	0.3967(+)	0.2300(+)
3-10-34-4-5	**0.9061**	0.8021(+)	0.7331(+)	0.5672(+)	0.6292(+)	0.6303(+)	0.5361(+)
3-10-34-6-3	**0.9136**	0.7676(+)	0.7483(+)	0.5581(+)	0.7657(+)	0.6876(+)	0.5558(+)
3-10-34-6-4	**0.8363**	0.7694(+)	0.6940(+)	0.4902(+)	0.6553(+)	0.6444(+)	0.4338(+)
3-10-34-6-5	**0.9610**	0.8923(+)	0.8551(+)	0.5550(+)	0.7370(+)	0.8057(+)	0.5429(+)
3-15-57-2-3	**0.8418**	0.6359(+)	0.6357(+)	0.4674(+)	0.4955(+)	0.5178(+)	0.3726(+)
3-15-57-2-4	**0.8240**	0.7321(+)	0.6721(+)	0.3210(+)	0.4086(+)	0.4026(+)	0.3694(+)
3-15-57-2-5	**0.9122**	0.8152(+)	0.6974(+)	0.2354(+)	0.3600(+)	0.4441(+)	0.3097(+)
3-15-57-4-3	**0.8999**	0.7870(+)	0.7159(+)	0.2706(+)	0.3719(+)	0.4662(+)	0.0205(+)
3-15-57-4-4	**0.9297**	0.8854(+)	0.8362(+)	0.4239(+)	0.6239(+)	0.7537(+)	0.5176(+)
3-15-57-4-5	**0.9424**	0.8353(+)	0.8008(+)	0.4467(+)	0.7121(+)	0.7050(+)	0.4507(+)
3-15-57-6-3	**0.9086**	0.8402(+)	0.7726(+)	0.4024(+)	0.7342(+)	0.7227(+)	0.3299(+)
3-15-57-6-4	**0.9373**	0.7106(+)	0.7445(+)	0.3047(+)	0.6700(+)	0.6941(+)	0.5615(+)
3-15-57-6-5	**0.8715**	0.7345(+)	0.6982(+)	0.1666(+)	0.5800(+)	0.6240(+)	0.2993(+)
3-20-68-2-3	**0.9175**	0.8664(+)	0.7942(+)	0.3488(+)	0.5239(+)	0.6527(+)	0.3465(+)
3-20-68-2-4	**0.8489**	0.7858(+)	0.6581(+)	0.2481(+)	0.4089(+)	0.3371(+)	0.2967(+)
3-20-68-2-5	**0.8357**	0.7432(+)	0.6219(+)	0.3167(+)	0.5389(+)	0.5566(+)	0.3892(+)
3-20-68-4-3	**0.8928**	0.8290(+)	0.8272(+)	0.2382(+)	0.6902(+)	0.7285(+)	0.3189(+)
3-20-68-4-4	0.7846	**0.7981(-)**	0.6508(+)	0.2947(+)	0.5004(+)	0.4432(+)	0.3621(+)
3-20-68-4-5	**0.9334**	0.8674(+)	0.8523(+)	0.2854(+)	0.7236(+)	0.8601(+)	0.2011(+)
3-20-68-6-3	**0.9289**	0.9108(+)	0.8055(+)	0.3958(+)	0.7149(+)	0.7231(+)	0.2921(+)
3-20-68-6-4	**0.7628**	0.6842(+)	0.6831(+)	0.1556(+)	0.4394(+)	0.4948(+)	0.1581(+)
3-20-68-6-5	**0.9301**	0.8996(+)	0.8389(+)	0.3707(+)	0.7492(+)	0.8415(+)	0.4523(+)
2-10-34-2-3	**0.7704**	0.7082(+)	0.5840(+)	0.1774(+)	0.4164(+)	0.4305(+)	0.3144(+)
2-10-34-2-4	**0.8019**	0.7627(+)	0.7031(+)	0.3973(+)	0.6619(+)	0.6253(+)	0.4065(+)
2-10-34-2-5	**0.9245**	0.9104(+)	0.8469(+)	0.4957(+)	0.7252(+)	0.7218(+)	0.4445(+)
2-10-34-4-3	**0.9291**	0.8526(+)	0.7941(+)	0.4835(+)	0.7460(+)	0.6936(+)	0.2867(+)
2-10-34-4-4	**0.8813**	0.7772(+)	0.7974(+)	0.4932(+)	0.6362(+)	0.5933(+)	0.4756(+)
2-10-34-4-5	**0.7145**	0.6932(+)	0.5901(+)	0.3384(+)	0.5414(+)	0.5396(+)	0.3855(+)
2-10-34-6-3	**0.7988**	0.7221(+)	0.5972(+)	0.4239(+)	0.5113(+)	0.5265(+)	0.4967(+)
2-10-34-6-4	**0.8551**	0.6558(+)	0.6535(+)	0.3340(+)	0.4339(+)	0.7053(+)	0.5177(+)
2-10-34-6-5	**0.9412**	0.7868(+)	0.7359(+)	0.4250(+)	0.6804(+)	0.7162(+)	0.5771(+)
2-15-57-2-3	**0.8573**	0.7217(+)	0.7105(+)	0.4129(+)	0.6099(+)	0.5663(+)	0.4862(+)
2-15-57-2-4	**0.9880**	0.8927(+)	0.8593(+)	0.3235(+)	0.4937(+)	0.5178(+)	0.3503(+)
2-15-57-2-5	**0.9735**	0.9139(+)	0.7988(+)	0.2715(+)	0.5315(+)	0.5054(+)	0.3317(+)
2-15-57-4-3	**0.8942**	0.7593(+)	0.7509(+)	0.5215(+)	0.5526(+)	0.6255(+)	0.5335(+)
2-15-57-4-4	**0.7847**	0.6370(+)	0.6541(+)	0.1626(+)	0.3702(+)	0.3214(+)	0.3160(+)
2-15-57-4-5	**0.8577**	0.6862(+)	0.6207(+)	0.3223(+)	0.5127(+)	0.5322(+)	0.2749(+)
2-15-57-6-3	**0.8884**	0.6769(+)	0.5877(+)	0.3144(+)	0.5558(+)	0.6807(+)	0.4965(+)
2-15-57-6-4	**0.8844**	0.8213(+)	0.7365(+)	0.5353(+)	0.4919(+)	0.5432(+)	0.4896(+)
2-15-57-6-5	**0.9102**	0.8480(+)	0.8250(+)	0.4119(+)	0.5825(+)	0.5978(+)	0.4175(+)
2-20-68-2-3	**0.9502**	0.9130(+)	0.8953(+)	0.4797(+)	0.7897(+)	0.8082(+)	0.4364(+)
2-20-68-2-4	**0.8452**	0.6715(+)	0.6378(+)	0.2736(+)	0.3676(+)	0.5233(+)	0.1911(+)
2-20-68-2-5	**0.9567**	0.8652(+)	0.8846(+)	0.4619(+)	0.6467(+)	0.7102(+)	0.5731(+)
2-20-68-4-3	**0.8783**	0.7167(+)	0.6626(+)	0.2802(+)	0.5497(+)	0.4877(+)	0.4365(+)
2-20-68-4-4	**0.8557**	0.8277(+)	0.7593(+)	0.4118(+)	0.6106(+)	0.6270(+)	0.4459(+)
2-20-68-4-5	**0.8319**	0.7734(+)	0.6570(+)	0.3454(+)	0.5363(+)	0.6150(+)	0.5235(+)
2-20-68-6-3	**0.8848**	0.8135(+)	0.7292(+)	0.3101(+)	0.6786(+)	0.7242(+)	0.5199(+)
2-20-68-6-4	**0.9923**	0.8501(+)	0.8267(+)	0.2027(+)	0.6265(+)	0.5626(+)	0.2840(+)
2-20-68-6-5	**0.9075**	0.8371(+)	0.7993(+)	0.3735(+)	0.7401(+)	0.7692(+)	0.3388(+)
+/ ≈ / −		53/0/1	54/0/0	54/0/0	54/0/0	54/0/0	54/0/0

Instances	QLMHHA	FLS-HH	GP-HH	Jaya	NSGA-II	QVNS-NSGA-II	MOEA/D
3-10-34-2-3	**1.0000**	0.0000(+)	0.0000(+)	0.0000(+)	0.0000(+)	0.0000(+)	0.0000(+)
3-10-34-2-4	**0.7273**	0.1364(+)	0.0455(+)	0.0000(+)	0.0908(+)	0.0000(+)	0.0000(+)
3-10-34-2-5	**0.5625**	0.0000(+)	0.2500(+)	0.0000(+)	0.0000(+)	0.1250(+)	0.0625(+)
3-10-34-4-3	**0.6667**	0.0000(+)	0.3333(+)	0.0000(+)	0.0000(+)	0.0000(+)	0.0000(+)
3-10-34-4-4	**0.9818**	0.0182(+)	0.0000(+)	0.0000(+)	0.0000(+)	0.0000(+)	0.0000(+)
3-10-34-4-5	**0.7143**	0.2857(+)	0.0000(+)	0.0000(+)	0.0000(+)	0.0000(+)	0.0000(+)
3-10-34-6-3	**0.8333**	0.1667(+)	0.0000(+)	0.0000(+)	0.0000(+)	0.0000(+)	0.0000(+)
3-10-34-6-4	**0.5556**	0.1667(+)	0.0000(+)	0.0000(+)	0.2221(+)	0.0000(+)	0.0556(+)
3-10-34-6-5	**0.7500**	0.2500(+)	0.0000(+)	0.0000(+)	0.0000(+)	0.0000(+)	0.0000(+)
3-15-57-2-3	**1.0000**	0.0000(+)	0.0000(+)	0.0000(+)	0.0000(+)	0.0000(+)	0.0000(+)
3-15-57-2-4	**0.7833**	0.2167(+)	0.0000(+)	0.0000(+)	0.0000(+)	0.0000(+)	0.0000(+)
3-15-57-2-5	**0.8571**	0.1429(+)	0.0000(+)	0.0000(+)	0.0000(+)	0.0000(+)	0.0000(+)
3-15-57-4-3	**0.6667**	0.3333(+)	0.0000(+)	0.0000(+)	0.0000(+)	0.0000(+)	0.0000(+)
3-15-57-4-4	**0.6875**	0.2500(+)	0.0625(+)	0.0000(+)	0.0000(+)	0.0000(+)	0.0000(+)
3-15-57-4-5	**0.8000**	0.1000(+)	0.1000(+)	0.0000(+)	0.0000(+)	0.0000(+)	0.0000(+)
3-15-57-6-3	**0.5333**	0.3333(+)	0.0000(+)	0.0000(+)	0.0667(+)	0.0667(+)	0.0000(+)
3-15-57-6-4	**0.9524**	0.0000(+)	0.0476(+)	0.0000(+)	0.0000(+)	0.0000(+)	0.0000(+)
3-15-57-6-5	**0.6667**	0.1667(+)	0.1666(+)	0.0000(+)	0.0000(+)	0.0000(+)	0.0000(+)
3-20-68-2-3	**0.8000**	0.2000(+)	0.0000(+)	0.0000(+)	0.0000(+)	0.0000(+)	0.0000(+)
3-20-68-2-4	**0.8909**	0.1091(+)	0.0000(+)	0.0000(+)	0.0000(+)	0.0000(+)	0.0000(+)
3-20-68-2-5	**0.9444**	0.0556(+)	0.0000(+)	0.0000(+)	0.0000(+)	0.0000(+)	0.0000(+)
3-20-68-4-3	**0.6316**	0.1053(+)	0.2105(+)	0.0000(+)	0.0000(+)	0.0526(+)	0.0000(+)
3-20-68-4-4	**0.7660**	0.0850(+)	0.0000(+)	0.0000(+)	0.1064(+)	0.0426(+)	0.0000(+)
3-20-68-4-5	**0.7857**	0.0714(+)	0.0000(+)	0.0000(+)	0.0000(+)	0.1429(+)	0.0000(+)
3-20-68-6-3	**0.7692**	0.2308(+)	0.0000(+)	0.0000(+)	0.0000(+)	0.0000(+)	0.0000(+)
3-20-68-6-4	**0.6154**	0.3077(+)	0.0000(+)	0.0000(+)	0.0769(+)	0.0000(+)	0.0000(+)
3-20-68-6-5	0.2778	**0.5556(-)**	0.0000(+)	0.0000(+)	0.0000(+)	0.1666(+)	0.0000(+)
2-10-34-2-3	**0.7000**	0.3000(+)	0.0000(+)	0.0000(+)	0.0000(+)	0.0000(+)	0.0000(+)
2-10-34-2-4	**0.3889**	0.2222(+)	0.0000(+)	0.0000(+)	**0.3889(≈)**	0.0000(+)	0.0000(+)
2-10-34-2-5	**0.6364**	0.1818(+)	0.0000(+)	0.0000(+)	0.0000(+)	0.1818(+)	0.0000(+)
2-10-34-4-3	**1.0000**	0.0000(+)	0.0000(+)	0.0000(+)	0.0000(+)	0.0000(+)	0.0000(+)
2-10-34-4-4	**0.7500**	0.0000(+)	0.1250(+)	0.0000(+)	0.0625(+)	0.0625(+)	0.0000(+)
2-10-34-4-5	**0.6250**	0.3125(+)	0.0000(+)	0.0000(+)	0.0625(+)	0.0000(+)	0.0000(+)
2-10-34-6-3	**0.9231**	0.0513(+)	0.0000(+)	0.0256(+)	0.0000(+)	0.0000(+)	0.0000(+)
2-10-34-6-4	**0.5000**	0.2143(+)	0.0000(+)	0.0714(+)	0.1429(+)	0.0714(+)	0.0000(+)
2-10-34-6-5	**0.7273**	0.1818(+)	0.0000(+)	0.0000(+)	0.0000(+)	0.0909(+)	0.0000(+)
2-15-57-2-3	**1.0000**	0.0000(+)	0.0000(+)	0.0000(+)	0.0000(+)	0.0000(+)	0.0000(+)
2-15-57-2-4	**0.9091**	0.0909(+)	0.0000(+)	0.0000(+)	0.0000(+)	0.0000(+)	0.0000(+)
2-15-57-2-5	**0.5556**	0.4444(+)	0.0000(+)	0.0000(+)	0.0000(+)	0.0000(+)	0.0000(+)
2-15-57-4-3	**0.8333**	0.0556(+)	0.0556(+)	0.0000(+)	0.0000(+)	0.0556(+)	0.0000(+)
2-15-57-4-4	**0.9722**	0.0278(+)	0.0000(+)	0.0000(+)	0.0000(+)	0.0000(+)	0.0000(+)
2-15-57-4-5	**0.8889**	0.1111(+)	0.0000(+)	0.0000(+)	0.0000(+)	0.0000(+)	0.0000(+)
2-15-57-6-3	**0.8235**	0.0000(+)	0.0588(+)	0.0000(+)	0.0000(+)	0.0589(+)	0.0588(+)
2-15-57-6-4	**0.8462**	0.1538(+)	0.0000(+)	0.0000(+)	0.0000(+)	0.0000(+)	0.0000(+)
2-15-57-6-5	**0.5882**	0.2941(+)	0.0589(+)	0.0000(+)	0.0000(+)	0.0588(+)	0.0000(+)
2-20-68-2-3	**0.7917**	0.0000(+)	0.2083(+)	0.0000(+)	0.0000(+)	0.0000(+)	0.0000(+)
2-20-68-2-4	**0.7692**	0.2308(+)	0.0000(+)	0.0000(+)	0.0000(+)	0.0000(+)	0.0000(+)
2-20-68-2-5	**0.9333**	0.0000(+)	0.0667(+)	0.0000(+)	0.0000(+)	0.0000(+)	0.0000(+)
2-20-68-4-3	**1.0000**	0.0000(+)	0.0000(+)	0.0000(+)	0.0000(+)	0.0000(+)	0.0000(+)
2-20-68-4-4	**0.5714**	0.3810(+)	0.0476(+)	0.0000(+)	0.0000(+)	0.0000(+)	0.0000(+)
2-20-68-4-5	**0.6071**	0.3571(+)	0.0000(+)	0.0000(+)	0.0000(+)	0.0358(+)	0.0000(+)
2-20-68-6-3	0.4211	**0.4737(-)**	0.0000(+)	0.0000(+)	0.0000(+)	0.0526(+)	0.0526(+)
2-20-68-6-4	**1.0000**	0.0000(+)	0.0000(+)	0.0000(+)	0.0000(+)	0.0000(+)	0.0000(+)
2-20-68-6-5	**0.6667**	0.2083(+)	0.1250(+)	0.0000(+)	0.0000(+)	0.0000(+)	0.0000(+)
+/ ≈ / –		52/0/2	54/0/0	54/0/0	53/1/0	54/0/0	54/0/0

Instances	QLMHHA	FLS-HH	GP-HH	Jaya	NSGA-II	QVNS-NSGA-II	MOEA/D
3-10-34-2-3	**0.0000**	0.1719(+)	0.1807(+)	0.4844(+)	0.4856(+)	0.4387(+)	0.4903(+)
3-10-34-2-4	**0.0090**	0.1002(+)	0.0948(+)	0.2480(+)	0.1278(+)	0.1784(+)	0.1975(+)
3-10-34-2-5	**0.0131**	0.1050(+)	0.1088(+)	0.3537(+)	0.2100(+)	0.1972(+)	0.3181(+)
3-10-34-4-3	**0.0349**	0.1085(+)	0.0960(+)	0.4329(+)	0.2771(+)	0.2538(+)	0.3021(+)
3-10-34-4-4	**0.0027**	0.1192(+)	0.2157(+)	0.6595(+)	0.3742(+)	0.4021(+)	0.5573(+)
3-10-34-4-5	**0.0372**	0.0925(+)	0.1499(+)	0.2537(+)	0.1692(+)	0.1617(+)	0.2022(+)
3-10-34-6-3	**0.0069**	0.0759(+)	0.0918(+)	0.2420(+)	0.1111(+)	0.1232(+)	0.2019(+)
3-10-34-6-4	**0.0332**	0.1524(+)	0.1042(+)	0.2597(+)	0.1430(+)	0.1874(+)	0.2495(+)
3-10-34-6-5	**0.0125**	0.0557(+)	0.0738(+)	0.2706(+)	0.1950(+)	0.1186(+)	0.2420(+)
3-15-57-2-3	**0.0000**	0.1916(+)	0.1551(+)	0.2901(+)	0.2636(+)	0.3425(+)	0.4167(+)
3-15-57-2-4	**0.0067**	0.0733(+)	0.1219(+)	0.4896(+)	0.4569(+)	0.4437(+)	0.4382(+)
3-15-57-2-5	0.0725	**0.0722(-)**	0.1949(+)	0.5493(+)	0.4441(+)	0.3840(+)	0.4960(+)
3-15-57-4-3	**0.0160**	0.0695(+)	0.1043(+)	0.4691(+)	0.3991(+)	0.2985(+)	0.7485(+)
3-15-57-4-4	**0.0362**	0.0434(+)	0.1346(+)	0.3813(+)	0.2125(+)	0.1360(+)	0.2610(+)
3-15-57-4-5	**0.0827**	0.1192(+)	0.0875(+)	0.3448(+)	0.1975(+)	0.1907(+)	0.3496(+)
3-15-57-6-3	**0.0237**	0.0483(+)	0.0890(+)	0.3472(+)	0.0859(+)	0.1003(+)	0.4021(+)
3-15-57-6-4	**0.0006**	0.1153(+)	0.0842(+)	0.4617(+)	0.1463(+)	0.1226(+)	0.2260(+)
3-15-57-6-5	**0.0124**	0.0866(+)	0.0887(+)	0.6216(+)	0.2097(+)	0.1721(+)	0.3613(+)
3-20-68-2-3	**0.0244**	0.0701(+)	0.1111(+)	0.4459(+)	0.2280(+)	0.1509(+)	0.3815(+)
3-20-68-2-4	**0.0035**	0.0821(+)	0.1404(+)	0.5347(+)	0.3911(+)	0.4522(+)	0.4848(+)
3-20-68-2-5	**0.0042**	0.0984(+)	0.1680(+)	0.4321(+)	0.2102(+)	0.2344(+)	0.3561(+)
3-20-68-4-3	**0.0226**	0.0594(+)	0.0723(+)	0.5094(+)	0.1504(+)	0.1176(+)	0.4724(+)
3-20-68-4-4	**0.0367**	0.0570(+)	0.1339(+)	0.4604(+)	0.3288(+)	0.3418(+)	0.3833(+)
3-20-68-4-5	**0.0112**	0.0769(+)	0.0868(+)	0.5165(+)	0.1328(+)	0.0849(+)	0.5717(+)
3-20-68-6-3	**0.0096**	0.0701(+)	0.1234(+)	0.4163(+)	0.2077(+)	0.2092(+)	0.4636(+)
3-20-68-6-4	**0.0489**	0.1526(+)	0.1250(+)	0.5938(+)	0.2460(+)	0.2145(+)	0.5156(+)
3-20-68-6-5	0.0382	**0.0197(-)**	0.0603(+)	0.4590(+)	0.1356(+)	0.0445(+)	0.3481(+)
2-10-34-2-3	**0.0276**	0.1135(+)	0.1775(+)	0.5428(+)	0.3273(+)	0.3192(+)	0.4053(+)
2-10-34-2-4	0.0688	**0.0568(-)**	0.1125(+)	0.3000(+)	0.1978(+)	0.1311(+)	0.3137(+)
2-10-34-2-5	**0.0258**	0.0448(+)	0.0789(+)	0.3244(+)	0.1637(+)	0.1057(+)	0.3759(+)
2-10-34-4-3	**0.0000**	0.1072(+)	0.1331(+)	0.3684(+)	0.2286(+)	0.2404(+)	0.6234(+)
2-10-34-4-4	**0.0081**	0.0738(+)	0.0576(+)	0.2587(+)	0.1598(+)	0.1988(+)	0.2749(+)
2-10-34-4-5	**0.0718**	0.1814(+)	0.1684(+)	0.4113(+)	0.2562(+)	0.2554(+)	0.3156(+)
2-10-34-6-3	**0.0078**	0.0793(+)	0.1583(+)	0.3900(+)	0.3177(+)	0.3386(+)	0.3433(+)
2-10-34-6-4	**0.1645**	0.2131(+)	0.2332(+)	0.4405(+)	0.3687(+)	0.1786(+)	0.2661(+)
2-10-34-6-5	**0.0138**	0.0827(+)	0.1038(+)	0.3634(+)	0.1688(+)	0.1523(+)	0.2037(+)
2-15-57-2-3	**0.0000**	0.0802(+)	0.1152(+)	0.3135(+)	0.3039(+)	0.2532(+)	0.2568(+)
2-15-57-2-4	**0.0016**	0.0808(+)	0.0948(+)	0.5290(+)	0.3643(+)	0.3336(+)	0.5036(+)
2-15-57-2-5	**0.0135**	0.0493(+)	0.1003(+)	0.5904(+)	0.3681(+)	0.3997(+)	0.5684(+)
2-15-57-4-3	**0.0045**	0.1152(+)	0.1029(+)	0.2559(+)	0.1959(+)	0.1677(+)	0.2654(+)
2-15-57-4-4	**0.0007**	0.1319(+)	0.1819(+)	0.6055(+)	0.3499(+)	0.4101(+)	0.4025(+)
2-15-57-4-5	**0.0064**	0.1595(+)	0.1901(+)	0.4327(+)	0.2714(+)	0.2519(+)	0.4871(+)
2-15-57-6-3	**0.0083**	0.1925(+)	0.2156(+)	0.4188(+)	0.2536(+)	0.1826(+)	0.2756(+)
2-15-57-6-4	**0.0112**	0.0827(+)	0.1310(+)	0.2414(+)	0.3603(+)	0.2893(+)	0.3403(+)
2-15-57-6-5	**0.0212**	0.0953(+)	0.1911(+)	0.4066(+)	0.2073(+)	0.1878(+)	0.3067(+)
2-20-68-2-3	**0.0103**	0.0516(+)	0.0485(+)	0.3275(+)	0.0945(+)	0.0889(+)	0.3227(+)
2-20-68-2-4	**0.0235**	0.1209(+)	0.1255(+)	0.4206(+)	0.2919(+)	0.2180(+)	0.4579(+)
2-20-68-2-5	**0.0012**	0.0415(+)	0.0441(+)	0.2846(+)	0.1909(+)	0.1052(+)	0.2258(+)
2-20-68-4-3	**0.0000**	0.2542(+)	0.2011(+)	0.5219(+)	0.3819(+)	0.3858(+)	0.3964(+)
2-20-68-4-4	**0.0181**	0.1319(+)	0.1579(+)	0.3566(+)	0.2031(+)	0.2199(+)	0.3502(+)
2-20-68-4-5	**0.0186**	0.0732(+)	0.1275(+)	0.3477(+)	0.2062(+)	0.1697(+)	0.2880(+)
2-20-68-6-3	**0.0634**	0.1823(+)	0.2239(+)	0.4783(+)	0.1819(+)	0.1826(+)	0.3006(+)
2-20-68-6-4	**0.0000**	0.1061(+)	0.1081(+)	0.6786(+)	0.2458(+)	0.3596(+)	0.4432(+)
2-20-68-6-5	**0.0105**	0.0892(+)	0.0798(+)	0.4255(+)	0.1470(+)	0.1200(+)	0.4301(+)
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Instances	QLMHHA	FLS-HH	GP-HH	Jaya	NSGA-II	QVNS-NSGA-II	MOEA/D
3-10-34-2-3	**0.9547**	0.8097(+)	0.7634(+)	0.3095(+)	0.3300(+)	0.4087(+)	0.3956(+)
3-10-34-2-4	**0.9310**	0.8020(+)	0.8258(+)	0.5693(+)	0.7764(+)	0.6937(+)	0.6104(+)
3-10-34-2-5	**0.8783**	0.7306(+)	0.7806(+)	0.4401(+)	0.5858(+)	0.6553(+)	0.4902(+)
3-10-34-4-3	**0.9181**	0.7470(+)	0.7487(+)	0.3986(+)	0.4884(+)	0.4980(+)	0.4383(+)
3-10-34-4-4	**0.8222**	0.7287(+)	0.5876(+)	0.1664(+)	0.3960(+)	0.3936(+)	0.2329(+)
3-10-34-4-5	**0.9066**	0.8231(+)	0.7178(+)	0.5616(+)	0.6319(+)	0.6385(+)	0.5222(+)
3-10-34-6-3	**0.9089**	0.7610(+)	0.7553(+)	0.5431(+)	0.7865(+)	0.6979(+)	0.5656(+)
3-10-34-6-4	**0.8347**	0.7910(+)	0.7055(+)	0.4977(+)	0.6596(+)	0.6536(+)	0.4421(+)
3-10-34-6-5	**0.9673**	0.8822(+)	0.8402(+)	0.5472(+)	0.7483(+)	0.7975(+)	0.5542(+)
3-15-57-2-3	**0.8348**	0.6311(+)	0.6242(+)	0.4799(+)	0.4929(+)	0.5183(+)	0.3828(+)
3-15-57-2-4	**0.8437**	0.7285(+)	0.6624(+)	0.3304(+)	0.3970(+)	0.4123(+)	0.3679(+)
3-15-57-2-5	**0.9093**	0.8104(+)	0.6886(+)	0.2423(+)	0.3599(+)	0.4383(+)	0.3100(+)
3-15-57-4-3	**0.8773**	0.8026(+)	0.7100(+)	0.2721(+)	0.3756(+)	0.4618(+)	0.0202(+)
3-15-57-4-4	**0.9555**	0.8900(+)	0.8151(+)	0.4204(+)	0.6108(+)	0.7642(+)	0.5114(+)
3-15-57-4-5	**0.9342**	0.8111(+)	0.8060(+)	0.4494(+)	0.7069(+)	0.7134(+)	0.4471(+)
3-15-57-6-3	**0.8954**	0.8418(+)	0.7730(+)	0.4047(+)	0.7532(+)	0.7406(+)	0.3394(+)
3-15-57-6-4	**0.9101**	0.7083(+)	0.7556(+)	0.3068(+)	0.6786(+)	0.6770(+)	0.5455(+)
3-15-57-6-5	**0.8714**	0.7350(+)	0.7056(+)	0.1693(+)	0.5826(+)	0.6101(+)	0.2953(+)
3-20-68-2-3	**0.9205**	0.8803(+)	0.7896(+)	0.3469(+)	0.5321(+)	0.6660(+)	0.3366(+)
3-20-68-2-4	**0.8612**	0.7694(+)	0.6533(+)	0.2425(+)	0.3982(+)	0.3396(+)	0.2898(+)
3-20-68-2-5	**0.8166**	0.7580(+)	0.6180(+)	0.3247(+)	0.5518(+)	0.5694(+)	0.4001(+)
3-20-68-4-3	**0.8825**	0.8241(+)	0.8402(+)	0.2345(+)	0.6731(+)	0.7230(+)	0.3236(+)
3-20-68-4-4	**0.7848**	0.7791(+)	0.6457(+)	0.3016(+)	0.4975(+)	0.4463(+)	0.3643(+)
3-20-68-4-5	**0.9482**	0.8901(+)	0.8695(+)	0.2790(+)	0.7440(+)	0.8642(+)	0.1991(+)
3-20-68-6-3	**0.9255**	0.9180(+)	0.7837(+)	0.4035(+)	0.7349(+)	0.7401(+)	0.2904(+)
3-20-68-6-4	**0.7857**	0.6911(+)	0.6726(+)	0.1566(+)	0.4428(+)	0.5018(+)	0.1534(+)
3-20-68-6-5	**0.9037**	0.8791(+)	0.8495(+)	0.3659(+)	0.7275(+)	0.8305(+)	0.4499(+)
2-10-34-2-3	**0.7640**	0.7014(+)	0.5772(+)	0.1784(+)	0.4268(+)	0.4298(+)	0.3175(+)
2-10-34-2-4	**0.7814**	0.7405(+)	0.7182(+)	0.4010(+)	0.6574(+)	0.6109(+)	0.4080(+)
2-10-34-2-5	**0.9452**	0.8759(+)	0.8342(+)	0.5045(+)	0.7400(+)	0.7360(+)	0.4341(+)
2-10-34-4-3	**0.9134**	0.8525(+)	0.7854(+)	0.4728(+)	0.7390(+)	0.7038(+)	0.2928(+)
2-10-34-4-4	**0.8876**	0.7557(+)	0.8074(+)	0.4912(+)	0.6194(+)	0.6020(+)	0.4793(+)
2-10-34-4-5	**0.7205**	0.7047(+)	0.5992(+)	0.3420(+)	0.5254(+)	0.5262(+)	0.3757(+)
2-10-34-6-3	**0.8138**	0.7176(+)	0.6024(+)	0.4314(+)	0.5154(+)	0.5192(+)	0.5113(+)
2-10-34-6-4	**0.8312**	0.6696(+)	0.6486(+)	0.3334(+)	0.4350(+)	0.6846(+)	0.5116(+)
2-10-34-6-5	**0.9172**	0.7859(+)	0.7534(+)	0.4316(+)	0.6840(+)	0.7159(+)	0.5870(+)
2-15-57-2-3	**0.8318**	0.7272(+)	0.7206(+)	0.4151(+)	0.5969(+)	0.5631(+)	0.4755(+)
2-15-57-2-4	**0.9915**	0.8845(+)	0.8763(+)	0.3230(+)	0.5013(+)	0.5032(+)	0.3589(+)
2-15-57-2-5	**0.9779**	0.9296(+)	0.8116(+)	0.2715(+)	0.5210(+)	0.4954(+)	0.3357(+)
2-15-57-4-3	**0.9107**	0.7464(+)	0.7547(+)	0.5365(+)	0.5654(+)	0.6441(+)	0.5218(+)
2-15-57-4-4	**0.7964**	0.6267(+)	0.6523(+)	0.1675(+)	0.3620(+)	0.3300(+)	0.3112(+)
2-15-57-4-5	**0.8797**	0.6994(+)	0.6122(+)	0.3192(+)	0.5001(+)	0.5232(+)	0.2791(+)
2-15-57-6-3	**0.8923**	0.6579(+)	0.5939(+)	0.3213(+)	0.5658(+)	0.6906(+)	0.5823(+)
2-15-57-6-4	**0.8989**	0.8186(+)	0.7353(+)	0.5344(+)	0.4939(+)	0.5514(+)	0.4993(+)
2-15-57-6-5	**0.8837**	0.8495(+)	0.8260(+)	0.4084(+)	0.5959(+)	0.5910(+)	0.4216(+)
2-20-68-2-3	**0.9753**	0.9276(+)	0.8768(+)	0.4723(+)	0.7998(+)	0.8007(+)	0.4404(+)
2-20-68-2-4	**0.8492**	0.6801(+)	0.6296(+)	0.2791(+)	0.3651(+)	0.5318(+)	0.1879(+)
2-20-68-2-5	**0.9378**	0.8879(+)	0.8923(+)	0.4736(+)	0.6547(+)	0.7287(+)	0.5647(+)
2-20-68-4-3	**0.8766**	0.7235(+)	0.6544(+)	0.2868(+)	0.5356(+)	0.4989(+)	0.4308(+)
2-20-68-4-4	**0.8764**	0.8114(+)	0.7811(+)	0.4177(+)	0.6130(+)	0.6176(+)	0.4566(+)
2-20-68-4-5	**0.8396**	0.7925(+)	0.6723(+)	0.3444(+)	0.5470(+)	0.6876(+)	0.5155(+)
2-20-68-6-3	**0.8903**	0.8247(+)	0.7233(+)	0.3112(+)	0.6851(+)	0.7806(+)	0.5264(+)
2-20-68-6-4	**0.9786**	0.8264(+)	0.8122(+)	0.2024(+)	0.6163(+)	0.5774(+)	0.2876(+)
2-20-68-6-5	**0.9090**	0.8531(+)	0.8196(+)	0.3726(+)	0.7402(+)	0.7660(+)	0.3385(+)
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Instances	QLMHHA	FLS-HH	GP-HH	Jaya	NSGA-II	QVNS-NSGA-II	MOEA/D
3-10-34-2-3	**1.0000**	0.0000(+)	0.0000(+)	0.0000(+)	0.0000(+)	0.0000(+)	0.0000(+)
3-10-34-2-4	**0.8441**	0.0816(+)	0.0000(+)	0.0000(+)	0.0743(+)	0.0000(+)	0.0000(+)
3-10-34-2-5	**0.5169**	0.0508(+)	0.2834(+)	0.0000(+)	0.0000(+)	0.1489(+)	0.0000(+)
3-10-34-4-3	**0.7462**	0.0000(+)	0.2538(+)	0.0000(+)	0.0000(+)	0.0000(+)	0.0000(+)
3-10-34-4-4	**0.9274**	0.0500(+)	0.0000(+)	0.0000(+)	0.0000(+)	0.0226(+)	0.0000(+)
3-10-34-4-5	**0.7194**	0.2806(+)	0.0000(+)	0.0000(+)	0.0000(+)	0.0000(+)	0.0000(+)
3-10-34-6-3	**0.8453**	0.1547(+)	0.0000(+)	0.0000(+)	0.0000(+)	0.0000(+)	0.0000(+)
3-10-34-6-4	**0.6693**	0.0000(+)	0.2185(+)	0.0000(+)	0.1122(+)	0.0000(+)	0.0000(+)
3-10-34-6-5	**0.7515**	0.2485(+)	0.0000(+)	0.0000(+)	0.0000(+)	0.0000(+)	0.0000(+)
3-15-57-2-3	**1.0000**	0.0000(+)	0.0000(+)	0.0000(+)	0.0000(+)	0.0000(+)	0.0000(+)
3-15-57-2-4	**0.7735**	0.2265(+)	0.0000(+)	0.0000(+)	0.0000(+)	0.0000(+)	0.0000(+)
3-15-57-2-5	**0.8457**	0.1543(+)	0.0000(+)	0.0000(+)	0.0000(+)	0.0000(+)	0.0000(+)
3-15-57-4-3	**0.7085**	0.2507(+)	0.0000(+)	0.0000(+)	0.0000(+)	0.0000(+)	0.0408(+)
3-15-57-4-4	**0.6742**	0.2527(+)	0.0731(+)	0.0000(+)	0.0000(+)	0.0000(+)	0.0000(+)
3-15-57-4-5	**0.8140**	0.1040(+)	0.0820(+)	0.0000(+)	0.0000(+)	0.0000(+)	0.0000(+)
3-15-57-6-3	**0.5400**	0.3233(+)	0.0000(+)	0.0000(+)	0.0000(+)	0.1367(+)	0.0000(+)
3-15-57-6-4	**1.0000**	0.0000(+)	0.0000(+)	0.0000(+)	0.0000(+)	0.0000(+)	0.0000(+)
3-15-57-6-5	**0.6782**	0.1703(+)	0.0000(+)	0.0000(+)	0.0000(+)	0.1515(+)	0.0000(+)
3-20-68-2-3	**0.7876**	0.2124(+)	0.0000(+)	0.0000(+)	0.0000(+)	0.0000(+)	0.0000(+)
3-20-68-2-4	**0.8752**	0.1248(+)	0.0000(+)	0.0000(+)	0.0000(+)	0.0000(+)	0.0000(+)
3-20-68-2-5	**1.0000**	0.0000(+)	0.0000(+)	0.0000(+)	0.0000(+)	0.0000(+)	0.0000(+)
3-20-68-4-3	**0.6477**	0.1011(+)	0.2032(+)	0.0000(+)	0.0000(+)	0.0480(+)	0.0000(+)
3-20-68-4-4	**0.7546**	0.0870(+)	0.0000(+)	0.0000(+)	0.1143(+)	0.0441(+)	0.0000(+)
3-20-68-4-5	**0.8058**	0.0734(+)	0.0000(+)	0.0000(+)	0.0000(+)	0.1208(+)	0.0000(+)
3-20-68-6-3	**0.7920**	0.2080(+)	0.0000(+)	0.0000(+)	0.0000(+)	0.0000(+)	0.0000(+)
3-20-68-6-4	**0.6129**	0.3101(+)	0.0000(+)	0.0000(+)	0.0770(+)	0.0000(+)	0.0000(+)
3-20-68-6-5	**0.5658**	0.2818(+)	0.0000(+)	0.0000(+)	0.0000(+)	0.1524(+)	0.0000(+)
2-10-34-2-3	**0.6981**	0.3019(+)	0.0000(+)	0.0000(+)	0.0000(+)	0.0000(+)	0.0000(+)
2-10-34-2-4	0.4021	0.1188(+)	0.0000(+)	0.0000(+)	**0.4791(-)**	0.0000(+)	0.0000(+)
2-10-34-2-5	**0.6246**	0.1871(+)	0.0000(+)	0.0000(+)	0.0000(+)	0.1883(+)	0.0000(+)
2-10-34-4-3	**1.0000**	0.0000(+)	0.0000(+)	0.0000(+)	0.0000(+)	0.0000(+)	0.0000(+)
2-10-34-4-4	**0.7358**	0.0000(+)	0.1217(+)	0.0000(+)	0.0780(+)	0.0645(+)	0.0000(+)
2-10-34-4-5	**0.6475**	0.3092(+)	0.0000(+)	0.0000(+)	0.0433(+)	0.0000(+)	0.0000(+)
2-10-34-6-3	**0.9142**	0.0509(+)	0.0000(+)	0.0349(+)	0.0000(+)	0.0000(+)	0.0000(+)
2-10-34-6-4	**0.5675**	0.2206(+)	0.0000(+)	0.0000(+)	0.0000(+)	0.2119(+)	0.0000(+)
2-10-34-6-5	**0.7093**	0.1865(+)	0.0000(+)	0.0000(+)	0.0000(+)	0.1042(+)	0.0000(+)
2-15-57-2-3	**1.0000**	0.0000(+)	0.0000(+)	0.0000(+)	0.0000(+)	0.0000(+)	0.0000(+)
2-15-57-2-4	**0.8963**	0.1037(+)	0.0000(+)	0.0000(+)	0.0000(+)	0.0000(+)	0.0000(+)
2-15-57-2-5	**0.5565**	0.4435(+)	0.0000(+)	0.0000(+)	0.0000(+)	0.0000(+)	0.0000(+)
2-15-57-4-3	**0.8453**	0.0000(+)	0.1077(+)	0.0000(+)	0.0470(+)	0.0000(+)	0.0000(+)
2-15-57-4-4	**0.9715**	0.0285(+)	0.0000(+)	0.0000(+)	0.0000(+)	0.0000(+)	0.0000(+)
2-15-57-4-5	**0.8729**	0.1271(+)	0.0000(+)	0.0000(+)	0.0000(+)	0.0000(+)	0.0000(+)
2-15-57-6-3	**0.8151**	0.0000(+)	0.0282(+)	0.0000(+)	0.0000(+)	0.0885(+)	0.0682(+)
2-15-57-6-4	**0.8322**	0.1678(+)	0.0000(+)	0.0000(+)	0.0000(+)	0.0000(+)	0.0000(+)
2-15-57-6-5	**0.5972**	0.2857(+)	0.0591(+)	0.0000(+)	0.0000(+)	0.0580(+)	0.0000(+)
2-20-68-2-3	**0.8011**	0.0000(+)	0.1989(+)	0.0000(+)	0.0000(+)	0.0000(+)	0.0000(+)
2-20-68-2-4	**0.7851**	0.2149(+)	0.0000(+)	0.0000(+)	0.0000(+)	0.0000(+)	0.0000(+)
2-20-68-2-5	**0.9255**	0.0000(+)	0.0745(+)	0.0000(+)	0.0000(+)	0.0000(+)	0.0000(+)
2-20-68-4-3	**1.0000**	0.0000(+)	0.0000(+)	0.0000(+)	0.0000(+)	0.0000(+)	0.0000(+)
2-20-68-4-4	**0.7852**	0.2148(+)	0.0000(+)	0.0000(+)	0.0000(+)	0.0000(+)	0.0000(+)
2-20-68-4-5	**0.6239**	0.3518(+)	0.0000(+)	0.0000(+)	0.0000(+)	0.0243(+)	0.0000(+)
2-20-68-6-3	**0.6329**	0.2373(+)	0.0000(+)	0.0000(+)	0.0000(+)	0.1298(+)	0.0000(+)
2-20-68-6-4	**1.0000**	0.0000(+)	0.0000(+)	0.0000(+)	0.0000(+)	0.0000(+)	0.0000(+)
2-20-68-6-5	**0.7130**	0.2644(+)	0.0000(+)	0.0000(+)	0.0000(+)	0.0226(+)	0.0000(+)
+/ ≈ / –		54/0/0	54/0/0	54/0/0	53/0/1	54/0/0	54/0/0

Instances	QLMHHA	FLS-HH	GP-HH	Jaya	NSGA-II	QVNS-NSGA-II	MOEA/D
3-10-34-2-3	**0.0000**	0.1873(+)	0.1478(+)	0.5590(+)	0.4712(+)	0.4350(+)	0.4787(+)
3-10-34-2-4	**0.0089**	0.0985(+)	0.1971(+)	0.2413(+)	0.1313(+)	0.1799(+)	0.1995(+)
3-10-34-2-5	**0.0133**	0.1568(+)	0.0781(+)	0.3456(+)	0.2110(+)	0.1023(+)	0.3110(+)
3-10-34-4-3	**0.0359**	0.1069(+)	0.0937(+)	0.4456(+)	0.2712(+)	0.2497(+)	0.3008(+)
3-10-34-4-4	**0.0027**	0.1167(+)	0.2139(+)	0.6529(+)	0.3837(+)	0.2106(+)	0.5559(+)
3-10-34-4-5	**0.0367**	0.0932(+)	0.1537(+)	0.2901(+)	0.1740(+)	0.1662(+)	0.2058(+)
3-10-34-6-3	**0.0069**	0.0764(+)	0.0898(+)	0.2400(+)	0.1082(+)	0.1198(+)	0.2000(+)
3-10-34-6-4	**0.0324**	0.1558(+)	0.1062(+)	0.2580(+)	0.1418(+)	0.1893(+)	0.2506(+)
3-10-34-6-5	**0.0122**	0.0554(+)	0.0756(+)	0.2662(+)	0.1951(+)	0.1210(+)	0.2491(+)
3-15-57-2-3	**0.0000**	0.1966(+)	0.1582(+)	0.2973(+)	0.2704(+)	0.3396(+)	0.4085(+)
3-15-57-2-4	**0.0066**	0.0718(+)	0.1246(+)	0.4891(+)	0.4450(+)	0.4487(+)	0.4326(+)
3-15-57-2-5	**0.0709**	0.0720(+)	0.2007(+)	0.5613(+)	0.4524(+)	0.3822(+)	0.4904(+)
3-15-57-4-3	**0.0162**	0.0701(+)	0.1071(+)	0.4597(+)	0.3982(+)	0.2904(+)	0.7497(+)
3-15-57-4-4	**0.0361**	0.0430(+)	0.1307(+)	0.3842(+)	0.2088(+)	0.1389(+)	0.2611(+)
3-15-57-4-5	**0.0816**	0.1184(+)	0.0892(+)	0.3513(+)	0.1977(+)	0.1882(+)	0.3597(+)
3-15-57-6-3	**0.0242**	0.0493(+)	0.1791(+)	0.3472(+)	0.2048(+)	0.1010(+)	0.4001(+)
3-15-57-6-4	**0.0000**	0.1168(+)	0.0818(+)	0.4596(+)	0.1480(+)	0.1250(+)	0.2256(+)
3-15-57-6-5	**0.0122**	0.0889(+)	0.1563(+)	0.6159(+)	0.2072(+)	0.0918(+)	0.3636(+)
3-20-68-2-3	**0.0239**	0.0688(+)	0.1141(+)	0.4548(+)	0.2248(+)	0.1483(+)	0.3811(+)
3-20-68-2-4	**0.0035**	0.0839(+)	0.1415(+)	0.5391(+)	0.3859(+)	0.4466(+)	0.4855(+)
3-20-68-2-5	**0.0000**	0.0993(+)	0.1667(+)	0.4206(+)	0.2094(+)	0.2285(+)	0.3516(+)
3-20-68-4-3	**0.0227**	0.0796(+)	0.0523(+)	0.5061(+)	0.1496(+)	0.1174(+)	0.4720(+)
3-20-68-4-4	**0.0376**	0.1383(+)	0.2312(+)	0.4594(+)	0.0799(+)	0.1720(+)	0.3766(+)
3-20-68-4-5	**0.0112**	0.0748(+)	0.0868(+)	0.5235(+)	0.1325(+)	0.0835(+)	0.5843(+)
3-20-68-6-3	**0.0098**	0.0694(+)	0.1262(+)	0.4244(+)	0.2097(+)	0.2084(+)	0.4715(+)
3-20-68-6-4	**0.0497**	0.1491(+)	0.2223(+)	0.5800(+)	0.2075(+)	0.2192(+)	0.5010(+)
3-20-68-6-5	**0.0173**	0.0195(+)	0.0588(+)	0.4669(+)	0.1375(+)	0.0442(+)	0.3526(+)
2-10-34-2-3	**0.0270**	0.1120(+)	0.1730(+)	0.5518(+)	0.3217(+)	0.3184(+)	0.4137(+)
2-10-34-2-4	0.0490	0.0561(+)	0.1119(+)	0.2937(+)	**0.0474(-)**	0.1286(+)	0.3184(+)
2-10-34-2-5	**0.0251**	0.0443(+)	0.0967(+)	0.3166(+)	0.1685(+)	0.0774(+)	0.3666(+)
2-10-34-4-3	**0.0000**	0.1051(+)	0.1316(+)	0.3667(+)	0.2299(+)	0.2475(+)	0.6199(+)
2-10-34-4-4	**0.0080**	0.1760(+)	0.0561(+)	0.2641(+)	0.1596(+)	0.1983(+)	0.2739(+)
2-10-34-4-5	**0.0739**	0.1790(+)	0.1728(+)	0.4018(+)	0.2550(+)	0.2570(+)	0.3232(+)
2-10-34-6-3	**0.0079**	0.0784(+)	0.1618(+)	0.3830(+)	0.3110(+)	0.3391(+)	0.3456(+)
2-10-34-6-4	**0.1671**	0.2126(+)	0.2386(+)	0.4429(+)	0.3609(+)	0.1758(+)	0.2584(+)
2-10-34-6-5	**0.0140**	0.0809(+)	0.1047(+)	0.3629(+)	0.1738(+)	0.1498(+)	0.2064(+)
2-15-57-2-3	**0.0000**	0.0817(+)	0.1139(+)	0.3177(+)	0.3024(+)	0.2577(+)	0.2603(+)
2-15-57-2-4	**0.0016**	0.0803(+)	0.0931(+)	0.5362(+)	0.3536(+)	0.3337(+)	0.5160(+)
2-15-57-2-5	**0.0137**	0.0495(+)	0.0978(+)	0.5807(+)	0.3578(+)	0.4100(+)	0.5604(+)
2-15-57-4-3	**0.0046**	0.1125(+)	0.1058(+)	0.2503(+)	0.1944(+)	0.1680(+)	0.2598(+)
2-15-57-4-4	**0.0007**	0.1303(+)	0.1843(+)	0.6104(+)	0.3418(+)	0.4208(+)	0.4059(+)
2-15-57-4-5	**0.0063**	0.1615(+)	0.1926(+)	0.4374(+)	0.2782(+)	0.2580(+)	0.4945(+)
2-15-57-6-3	**0.0084**	0.1973(+)	0.2186(+)	0.4070(+)	0.2562(+)	0.1835(+)	0.2756(+)
2-15-57-6-4	**0.0109**	0.0808(+)	0.1272(+)	0.2433(+)	0.3654(+)	0.2940(+)	0.3387(+)
2-15-57-6-5	**0.0212**	0.0944(+)	0.1889(+)	0.4073(+)	0.2095(+)	0.1846(+)	0.3025(+)
2-20-68-2-3	**0.0106**	0.0503(+)	0.0490(+)	0.3339(+)	0.0947(+)	0.0881(+)	0.3232(+)
2-20-68-2-4	**0.0235**	0.0808(+)	0.1282(+)	0.4092(+)	0.2857(+)	0.2191(+)	0.4630(+)
2-20-68-2-5	**0.0012**	0.0421(+)	0.0354(+)	0.2876(+)	0.1957(+)	0.1069(+)	0.2270(+)
2-20-68-4-3	**0.0000**	0.2585(+)	0.1982(+)	0.5143(+)	0.3712(+)	0.3841(+)	0.4004(+)
2-20-68-4-4	**0.0182**	0.1098(+)	0.1583(+)	0.3589(+)	0.2045(+)	0.2185(+)	0.3510(+)
2-20-68-4-5	**0.0184**	0.0735(+)	0.1662(+)	0.3467(+)	0.2037(+)	0.1283(+)	0.2952(+)
2-20-68-6-3	**0.0640**	0.1370(+)	0.2289(+)	0.4818(+)	0.1809(+)	0.1779(+)	0.3039(+)
2-20-68-6-4	**0.0000**	0.1064(+)	0.1061(+)	0.6649(+)	0.2417(+)	0.3642(+)	0.4550(+)
2-20-68-6-5	**0.0106**	0.0509(+)	0.0811(+)	0.4178(+)	0.1434(+)	0.1225(+)	0.4266(+)
+/ ≈ / –		54/0/0	54/0/0	54/0/0	53/0/1	54/0/0	54/0/0

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Instances	QLMHHA	FLS-HH	GP-HH	Jaya	NSGA-II	QVNS-NSGA-II	MOEA/D
3-10-34-2-3	**0.9565**	0.8236(+)	0.7714(+)	0.3005(+)	0.3358(+)	0.3980(+)	0.4013(+)
3-10-34-2-4	**0.9496**	0.8792(+)	0.8011(+)	0.5531(+)	0.7791(+)	0.6809(+)	0.6273(+)
3-10-34-2-5	**0.8536**	0.7375(+)	0.7994(+)	0.4426(+)	0.5981(+)	0.7506(+)	0.4979(+)
3-10-34-4-3	**0.9028**	0.7333(+)	0.7655(+)	0.4068(+)	0.4905(+)	0.5116(+)	0.4395(+)
3-10-34-4-4	**0.8906**	0.8095(+)	0.7924(+)	0.1619(+)	0.3859(+)	0.3905(+)	0.2338(+)
3-10-34-4-5	**0.8868**	0.8101(+)	0.7264(+)	0.5576(+)	0.6392(+)	0.6251(+)	0.5211(+)
3-10-34-6-3	**0.9331**	0.7418(+)	0.7621(+)	0.5589(+)	0.7637(+)	0.7178(+)	0.5494(+)
3-10-34-6-4	**0.8163**	0.8059(+)	0.6908(+)	0.4858(+)	0.7598(+)	0.6369(+)	0.4416(+)
3-10-34-6-5	**0.9823**	0.8827(+)	0.8430(+)	0.5368(+)	0.7349(+)	0.7826(+)	0.5518(+)
3-15-57-2-3	**0.8564**	0.6132(+)	0.6390(+)	0.4760(+)	0.5034(+)	0.5032(+)	0.3718(+)
3-15-57-2-4	**0.8578**	0.7384(+)	0.6751(+)	0.3225(+)	0.4066(+)	0.4071(+)	0.3582(+)
3-15-57-2-5	**0.8826**	0.8050(+)	0.7076(+)	0.2478(+)	0.3545(+)	0.4313(+)	0.3104(+)
3-15-57-4-3	**0.8996**	0.7867(+)	0.7249(+)	0.2743(+)	0.3659(+)	0.4501(+)	0.0205(+)
3-15-57-4-4	**0.9435**	0.9120(+)	0.8921(+)	0.4095(+)	0.6155(+)	0.7718(+)	0.5135(+)
3-15-57-4-5	**0.9134**	0.8078(+)	0.7256(+)	0.4500(+)	0.6987(+)	0.7989(+)	0.4457(+)
3-15-57-6-3	**0.8738**	0.8473(+)	0.7584(+)	0.4102(+)	0.7790(+)	0.7379(+)	0.3427(+)
3-15-57-6-4	**0.8832**	0.7074(+)	0.7777(+)	0.3133(+)	0.6831(+)	0.6792(+)	0.5559(+)
3-15-57-6-5	**0.8781**	0.7410(+)	0.7045(+)	0.1716(+)	0.5695(+)	0.6202(+)	0.2951(+)
3-20-68-2-3	**0.9185**	0.8600(+)	0.7949(+)	0.3402(+)	0.5305(+)	0.7633(+)	0.3373(+)
3-20-68-2-4	**0.8716**	0.7877(+)	0.6669(+)	0.2393(+)	0.3865(+)	0.3470(+)	0.2978(+)
3-20-68-2-5	**0.8342**	0.7714(+)	0.6137(+)	0.3225(+)	0.5501(+)	0.5753(+)	0.4104(+)
3-20-68-4-3	**0.8847**	0.8189(+)	0.8156(+)	0.2396(+)	0.7454(+)	0.7164(+)	0.3241(+)
3-20-68-4-4	**0.7915**	0.8009(+)	0.6650(+)	0.2954(+)	0.7845(+)	0.4489(+)	0.3609(+)
3-20-68-4-5	**0.9632**	0.8846(+)	0.8869(+)	0.2718(+)	0.7571(+)	0.8705(+)	0.2010(+)
3-20-68-6-3	**0.9044**	0.8952(+)	0.7886(+)	0.4064(+)	0.7495(+)	0.7246(+)	0.2979(+)
3-20-68-6-4	**0.7858**	0.6764(+)	0.6529(+)	0.1599(+)	0.4473(+)	0.5124(+)	0.1571(+)
3-20-68-6-5	**0.8994**	0.8652(+)	0.8068(+)	0.3755(+)	0.7397(+)	0.8309(+)	0.4532(+)
2-10-34-2-3	**0.7791**	0.7008(+)	0.5897(+)	0.1733(+)	0.4166(+)	0.4322(+)	0.3127(+)
2-10-34-2-4	**0.7634**	0.7408(+)	0.6992(+)	0.4098(+)	0.6757(+)	0.5977(+)	0.4089(+)
2-10-34-2-5	**0.9324**	0.9242(+)	0.8225(+)	0.5048(+)	0.7244(+)	0.7171(+)	0.4348(+)
2-10-34-4-3	**0.9149**	0.8451(+)	0.7765(+)	0.4807(+)	0.7492(+)	0.7056(+)	0.2911(+)
2-10-34-4-4	**0.8893**	0.7482(+)	0.7978(+)	0.4918(+)	0.6027(+)	0.5914(+)	0.4753(+)
2-10-34-4-5	**0.7052**	0.6919(+)	0.6069(+)	0.3483(+)	0.5316(+)	0.5193(+)	0.3799(+)
2-10-34-6-3	**0.8932**	0.7093(+)	0.6049(+)	0.4214(+)	0.5246(+)	0.5172(+)	0.5252(+)
2-10-34-6-4	**0.8439**	0.6873(+)	0.6435(+)	0.3363(+)	0.5432(+)	0.6697(+)	0.5193(+)
2-10-34-6-5	**0.9179**	0.7965(+)	0.7423(+)	0.4375(+)	0.6787(+)	0.7241(+)	0.5744(+)
2-15-57-2-3	**0.8335**	0.7308(+)	0.7195(+)	0.4163(+)	0.5963(+)	0.5469(+)	0.4836(+)
2-15-57-2-4	**0.9833**	0.8697(+)	0.9026(+)	0.3263(+)	0.4895(+)	0.5023(+)	0.3566(+)
2-15-57-2-5	**0.9667**	0.9394(+)	0.7986(+)	0.2728(+)	0.5282(+)	0.4905(+)	0.3299(+)
2-15-57-4-3	**0.9202**	0.7607(+)	0.7537(+)	0.5276(+)	0.5492(+)	0.6252(+)	0.5067(+)
2-15-57-4-4	**0.8582**	0.6219(+)	0.6697(+)	0.1717(+)	0.3596(+)	0.3251(+)	0.3033(+)
2-15-57-4-5	**0.9017**	0.6843(+)	0.6234(+)	0.3255(+)	0.5048(+)	0.5331(+)	0.2756(+)
2-15-57-6-3	**0.8935**	0.6471(+)	0.5903(+)	0.3122(+)	0.5742(+)	0.6699(+)	0.4722(+)
2-15-57-6-4	**0.9184**	0.8169(+)	0.7487(+)	0.5399(+)	0.5045(+)	0.5375(+)	0.5009(+)
2-15-57-6-5	**0.8779**	0.8391(+)	0.8076(+)	0.4059(+)	0.5938(+)	0.6045(+)	0.4219(+)
2-20-68-2-3	**0.9789**	0.9375(+)	0.8689(+)	0.4742(+)	0.7924(+)	0.8153(+)	0.4501(+)
2-20-68-2-4	**0.8744**	0.6987(+)	0.6128(+)	0.2732(+)	0.3701(+)	0.5247(+)	0.1931(+)
2-20-68-2-5	**0.9193**	0.8822(+)	0.8861(+)	0.4676(+)	0.6615(+)	0.7096(+)	0.5776(+)
2-20-68-4-3	**0.8986**	0.7322(+)	0.6463(+)	0.2853(+)	0.5373(+)	0.4978(+)	0.4231(+)
2-20-68-4-4	**0.8962**	0.7933(+)	0.7587(+)	0.4222(+)	0.6312(+)	0.6261(+)	0.4527(+)
2-20-68-4-5	**0.7948**	0.7675(+)	0.6702(+)	0.3507(+)	0.5504(+)	0.6144(+)	0.5143(+)
2-20-68-6-3	**0.8867**	0.8349(+)	0.7346(+)	0.3131(+)	0.7016(+)	0.7064(+)	0.6168(+)
2-20-68-6-4	**0.9626**	0.8452(+)	0.7917(+)	0.2009(+)	0.6118(+)	0.5883(+)	0.2805(+)
2-20-68-6-5	**0.9083**	0.8395(+)	0.8138(+)	0.3823(+)	0.7611(+)	0.7665(+)	0.3395(+)
+/ ≈ / −		54/0/0	54/0/0	54/0/0	54/0/0	54/0/0	54/0/0

Instances	QLMHHA	FLS-HH	GP-HH	Jaya	NSGA-II	QVNS-NSGA-II	MOEA/D
3-10-34-2-3	**1.0000**	0.0000(+)	0.0000(+)	0.0000(+)	0.0000(+)	0.0000(+)	0.0000(+)
3-10-34-2-4	**0.7885**	0.1436(+)	0.0679(+)	0.0000(+)	0.0000(+)	0.0000(+)	0.0000(+)
3-10-34-2-5	**0.6817**	0.0000(+)	0.1907(+)	0.0000(+)	0.0000(+)	0.1276(+)	0.0000(+)
3-10-34-4-3	**0.7382**	0.0000(+)	0.2618(+)	0.0000(+)	0.0000(+)	0.0000(+)	0.0000(+)
3-10-34-4-4	**1.0000**	0.0000(+)	0.0000(+)	0.0000(+)	0.0000(+)	0.0000(+)	0.0000(+)
3-10-34-4-5	**0.8293**	0.1707(+)	0.0000(+)	0.0000(+)	0.0000(+)	0.0000(+)	0.0000(+)
3-10-34-6-3	**0.8333**	0.1667(+)	0.0000(+)	0.0000(+)	0.0000(+)	0.0000(+)	0.0000(+)
3-10-34-6-4	**0.6641**	0.1711(+)	0.0000(+)	0.0000(+)	0.1648(+)	0.0000(+)	0.0000(+)
3-10-34-6-5	**0.7579**	0.2421(+)	0.0000(+)	0.0000(+)	0.0000(+)	0.0000(+)	0.0000(+)
3-15-57-2-3	**1.0000**	0.0000(+)	0.0000(+)	0.0000(+)	0.0000(+)	0.0000(+)	0.0000(+)
3-15-57-2-4	**0.7827**	0.2173(+)	0.0000(+)	0.0000(+)	0.0000(+)	0.0000(+)	0.0000(+)
3-15-57-2-5	**1.0000**	0.0000(+)	0.0000(+)	0.0000(+)	0.0000(+)	0.0000(+)	0.0000(+)
3-15-57-4-3	**0.7625**	0.2375(+)	0.0000(+)	0.0000(+)	0.0000(+)	0.0000(+)	0.0000(+)
3-15-57-4-4	**0.6724**	0.1545(+)	0.1731(+)	0.0000(+)	0.0000(+)	0.0000(+)	0.0000(+)
3-15-57-4-5	**0.8148**	0.1029(+)	0.0000(+)	0.0000(+)	0.0000(+)	0.0823(+)	0.0000(+)
3-15-57-6-3	**0.7457**	0.2108(+)	0.0000(+)	0.0000(+)	0.0435(+)	0.0000(+)	0.0000(+)
3-15-57-6-4	**0.9516**	0.0000(+)	0.0484(+)	0.0000(+)	0.0000(+)	0.0000(+)	0.0000(+)
3-15-57-6-5	**0.6823**	0.1478(+)	0.1699(+)	0.0000(+)	0.0000(+)	0.0000(+)	0.0000(+)
3-20-68-2-3	**0.7512**	0.2083(+)	0.0000(+)	0.0000(+)	0.0000(+)	0.0405(+)	0.0000(+)
3-20-68-2-4	**1.0000**	0.0000(+)	0.0000(+)	0.0000(+)	0.0000(+)	0.0000(+)	0.0000(+)
3-20-68-2-5	**0.9052**	0.0948(+)	0.0000(+)	0.0000(+)	0.0000(+)	0.0000(+)	0.0000(+)
3-20-68-4-3	**0.7485**	0.1035(+)	0.1018(+)	0.0000(+)	0.0462(+)	0.0000(+)	0.0000(+)
3-20-68-4-4	**0.7538**	0.0848(+)	0.0000(+)	0.0000(+)	0.1171(+)	0.0443(+)	0.0000(+)
3-20-68-4-5	**0.8115**	0.0725(+)	0.0000(+)	0.0000(+)	0.0000(+)	0.1160(+)	0.0000(+)
3-20-68-6-3	**0.7967**	0.2033(+)	0.0000(+)	0.0000(+)	0.0000(+)	0.0000(+)	0.0000(+)
3-20-68-6-4	**0.6167**	0.3068(+)	0.0000(+)	0.0000(+)	0.0765(+)	0.0000(+)	0.0000(+)
3-20-68-6-5	**0.7786**	0.1684(+)	0.0000(+)	0.0000(+)	0.0000(+)	0.0530(+)	0.0000(+)
2-10-34-2-3	**0.7013**	0.2987(+)	0.0000(+)	0.0000(+)	0.0000(+)	0.0000(+)	0.0000(+)
2-10-34-2-4	**0.4098**	0.2151(+)	0.0000(+)	0.0000(+)	0.3751(+)	0.0000(+)	0.0000(+)
2-10-34-2-5	**0.6205**	0.1892(+)	0.0000(+)	0.0000(+)	0.0000(+)	0.1903(+)	0.0000(+)
2-10-34-4-3	**1.0000**	0.0000(+)	0.0000(+)	0.0000(+)	0.0000(+)	0.0000(+)	0.0000(+)
2-10-34-4-4	**0.7364**	0.0000(+)	0.1227(+)	0.0000(+)	0.0768(+)	0.0641(+)	0.0000(+)
2-10-34-4-5	**0.6502**	0.3081(+)	0.0000(+)	0.0000(+)	0.0417(+)	0.0000(+)	0.0000(+)
2-10-34-6-3	**1.0000**	0.0000(+)	0.0000(+)	0.0000(+)	0.0000(+)	0.0000(+)	0.0000(+)
2-10-34-6-4	**0.6921**	0.1214(+)	0.0751(+)	0.0000(+)	0.0373(+)	0.0741(+)	0.0000(+)
2-10-34-6-5	**0.7142**	0.1858(+)	0.0000(+)	0.0000(+)	0.0000(+)	0.1000(+)	0.0000(+)
2-15-57-2-3	**1.0000**	0.0000(+)	0.0000(+)	0.0000(+)	0.0000(+)	0.0000(+)	0.0000(+)
2-15-57-2-4	**0.9017**	0.0983(+)	0.0000(+)	0.0000(+)	0.0000(+)	0.0000(+)	0.0000(+)
2-15-57-2-5	**0.6473**	0.3527(+)	0.0000(+)	0.0000(+)	0.0000(+)	0.0000(+)	0.0000(+)
2-15-57-4-3	**0.8435**	0.0551(+)	0.0531(+)	0.0000(+)	0.0000(+)	0.0483(+)	0.0000(+)
2-15-57-4-4	**1.0000**	0.0000(+)	0.0000(+)	0.0000(+)	0.0000(+)	0.0000(+)	0.0000(+)
2-15-57-4-5	**0.8779**	0.1221(+)	0.0000(+)	0.0000(+)	0.0000(+)	0.0000(+)	0.0000(+)
2-15-57-6-3	**0.8125**	0.0000(+)	0.0587(+)	0.0000(+)	0.0000(+)	0.0581(+)	0.0707(+)
2-15-57-6-4	**0.8368**	0.1632(+)	0.0000(+)	0.0000(+)	0.0000(+)	0.0000(+)	0.0000(+)
2-15-57-6-5	**0.6584**	0.2821(+)	0.0595(+)	0.0000(+)	0.0000(+)	0.0000(+)	0.0000(+)
2-20-68-2-3	**0.8053**	0.0000(+)	0.1947(+)	0.0000(+)	0.0000(+)	0.0000(+)	0.0000(+)
2-20-68-2-4	**0.7893**	0.2107(+)	0.0000(+)	0.0000(+)	0.0000(+)	0.0000(+)	0.0000(+)
2-20-68-2-5	**0.9093**	0.0000(+)	0.0907(+)	0.0000(+)	0.0000(+)	0.0000(+)	0.0000(+)
2-20-68-4-3	**1.0000**	0.0000(+)	0.0000(+)	0.0000(+)	0.0000(+)	0.0000(+)	0.0000(+)
2-20-68-4-4	**0.5698**	0.3873(+)	0.0429(+)	0.0000(+)	0.0000(+)	0.0000(+)	0.0000(+)
2-20-68-4-5	**0.6281**	0.3502(+)	0.0000(+)	0.0000(+)	0.0000(+)	0.0217(+)	0.0000(+)
2-20-68-6-3	**0.6065**	0.3525(+)	0.0000(+)	0.0000(+)	0.0000(+)	0.0000(+)	0.0410(+)
2-20-68-6-4	**1.0000**	0.0000(+)	0.0000(+)	0.0000(+)	0.0000(+)	0.0000(+)	0.0000(+)
2-20-68-6-5	**0.7582**	0.1121(+)	0.1297(+)	0.0000(+)	0.0000(+)	0.0000(+)	0.0000(+)
+/ ≈ / –		54/0/0	54/0/0	54/0/0	54/0/0	54/0/0	54/0/0

Instances	QLMHHA	FLS-HH	GP-HH	Jaya	NSGA-II	QVNS-NSGA-II	MOEA/D
3-10-34-2-3	**0.0000**	0.1849(+)	0.1537(+)	0.5674(+)	0.4815(+)	0.4392(+)	0.4869(+)
3-10-34-2-4	**0.0086**	0.0946(+)	0.1949(+)	0.2461(+)	0.1335(+)	0.1748(+)	0.1965(+)
3-10-34-2-5	**0.0132**	0.1520(+)	0.0764(+)	0.3525(+)	0.2103(+)	0.0992(+)	0.3075(+)
3-10-34-4-3	**0.0345**	0.1044(+)	0.0915(+)	0.4282(+)	0.2803(+)	0.2547(+)	0.2997(+)
3-10-34-4-4	**0.0000**	0.1124(+)	0.2200(+)	0.6323(+)	0.3777(+)	0.2136(+)	0.5560(+)
3-10-34-4-5	**0.0375**	0.0945(+)	0.1486(+)	0.2804(+)	0.1726(+)	0.1600(+)	0.2023(+)
3-10-34-6-3	**0.0071**	0.0749(+)	0.0916(+)	0.2327(+)	0.1093(+)	0.1156(+)	0.1933(+)
3-10-34-6-4	**0.0334**	0.1618(+)	0.1047(+)	0.2485(+)	0.1452(+)	0.1904(+)	0.2515(+)
3-10-34-6-5	**0.0123**	0.0533(+)	0.0741(+)	0.2561(+)	0.1960(+)	0.1238(+)	0.2541(+)
3-15-57-2-3	**0.0000**	0.1891(+)	0.1554(+)	0.2976(+)	0.2724(+)	0.3328(+)	0.4012(+)
3-15-57-2-4	**0.0065**	0.0722(+)	0.1248(+)	0.4722(+)	0.4608(+)	0.4591(+)	0.4408(+)
3-15-57-2-5	**0.0000**	0.0717(+)	0.1975(+)	0.5477(+)	0.4519(+)	0.3811(+)	0.4887(+)
3-15-57-4-3	**0.0167**	0.0701(+)	0.1112(+)	0.4714(+)	0.3922(+)	0.2927(+)	0.3545(+)
3-15-57-4-4	**0.0369**	0.0415(+)	0.1306(+)	0.3803(+)	0.2157(+)	0.1422(+)	0.2640(+)
3-15-57-4-5	**0.0312**	0.0792(+)	0.1281(+)	0.3373(+)	0.1716(+)	0.1023(+)	0.3559(+)
3-15-57-6-3	**0.0244**	0.0490(+)	0.1841(+)	0.3537(+)	0.1094(+)	0.1025(+)	0.3990(+)
3-15-57-6-4	**0.0000**	0.1145(+)	0.0841(+)	0.4734(+)	0.1498(+)	0.1249(+)	0.2336(+)
3-15-57-6-5	**0.0122**	0.0879(+)	0.1557(+)	0.6154(+)	0.2087(+)	0.0940(+)	0.3596(+)
3-20-68-2-3	**0.0245**	0.0677(+)	0.1156(+)	0.4368(+)	0.2325(+)	0.1483(+)	0.3696(+)
3-20-68-2-4	**0.0000**	0.0828(+)	0.1435(+)	0.5290(+)	0.3806(+)	0.4383(+)	0.4683(+)
3-20-68-2-5	**0.0000**	0.0987(+)	0.1691(+)	0.4368(+)	0.2077(+)	0.2335(+)	0.3579(+)
3-20-68-4-3	**0.0223**	0.0776(+)	0.0520(+)	0.5160(+)	0.1528(+)	0.1207(+)	0.4790(+)
3-20-68-4-4	**0.0377**	0.1434(+)	0.2231(+)	0.4765(+)	0.0786(+)	0.1733(+)	0.3790(+)
3-20-68-4-5	**0.0109**	0.0744(+)	0.0891(+)	0.5158(+)	0.1318(+)	0.0851(+)	0.5694(+)
3-20-68-6-3	**0.0099**	0.0704(+)	0.1303(+)	0.4319(+)	0.2123(+)	0.2090(+)	0.4545(+)
3-20-68-6-4	**0.0413**	0.1481(+)	0.2234(+)	0.5576(+)	0.2059(+)	0.2229(+)	0.5188(+)
3-20-68-6-5	**0.0178**	0.0197(+)	0.0597(+)	0.4496(+)	0.1430(+)	0.0451(+)	0.3597(+)
2-10-34-2-3	**0.0265**	0.1116(+)	0.1739(+)	0.5614(+)	0.3268(+)	0.3086(+)	0.4254(+)
2-10-34-2-4	**0.0403**	0.0543(+)	0.1162(+)	0.3012(+)	0.0485(+)	0.1333(+)	0.3244(+)
2-10-34-2-5	**0.0256**	0.0442(+)	0.0993(+)	0.3170(+)	0.1706(+)	0.0769(+)	0.3627(+)
2-10-34-4-3	**0.0000**	0.1049(+)	0.1287(+)	0.3641(+)	0.2309(+)	0.2532(+)	0.6053(+)
2-10-34-4-4	**0.0080**	0.1815(+)	0.0578(+)	0.2565(+)	0.1624(+)	0.1957(+)	0.2826(+)
2-10-34-4-5	**0.0656**	0.1845(+)	0.1691(+)	0.3895(+)	0.2468(+)	0.2671(+)	0.3570(+)
2-10-34-6-3	**0.0000**	0.0763(+)	0.1639(+)	0.3719(+)	0.3133(+)	0.3285(+)	0.3530(+)
2-10-34-6-4	**0.0734**	0.1542(+)	0.2348(+)	0.4330(+)	0.2749(+)	0.1710(+)	0.2658(+)
2-10-34-6-5	**0.0145**	0.0784(+)	0.1033(+)	0.3532(+)	0.1780(+)	0.1480(+)	0.2066(+)
2-15-57-2-3	**0.0000**	0.0832(+)	0.1157(+)	0.3193(+)	0.2959(+)	0.2677(+)	0.2615(+)
2-15-57-2-4	**0.0016**	0.0775(+)	0.0900(+)	0.5220(+)	0.3555(+)	0.3445(+)	0.5305(+)
2-15-57-2-5	**0.0138**	0.0484(+)	0.1015(+)	0.5717(+)	0.3513(+)	0.4211(+)	0.5672(+)
2-15-57-4-3	**0.0045**	0.1100(+)	0.1022(+)	0.2524(+)	0.1946(+)	0.1712(+)	0.2626(+)
2-15-57-4-4	**0.0000**	0.1331(+)	0.1883(+)	0.6100(+)	0.3499(+)	0.4183(+)	0.3924(+)
2-15-57-4-5	**0.0061**	0.1592(+)	0.1983(+)	0.4400(+)	0.2759(+)	0.2660(+)	0.5130(+)
2-15-57-6-3	**0.0084**	0.2013(+)	0.2135(+)	0.3980(+)	0.2503(+)	0.1871(+)	0.2759(+)
2-15-57-6-4	**0.0109**	0.0782(+)	0.1289(+)	0.2451(+)	0.3702(+)	0.2917(+)	0.3397(+)
2-15-57-6-5	**0.0216**	0.0927(+)	0.1935(+)	0.3978(+)	0.2125(+)	0.1776(+)	0.3072(+)
2-20-68-2-3	**0.0105**	0.0520(+)	0.0479(+)	0.3328(+)	0.0960(+)	0.0884(+)	0.3108(+)
2-20-68-2-4	**0.0234**	0.0794(+)	0.1264(+)	0.4244(+)	0.2903(+)	0.2274(+)	0.4793(+)
2-20-68-2-5	**0.0012**	0.0417(+)	0.0341(+)	0.2895(+)	0.1907(+)	0.1108(+)	0.2237(+)
2-20-68-4-3	**0.0000**	0.2522(+)	0.2005(+)	0.5168(+)	0.3609(+)	0.3783(+)	0.3908(+)
2-20-68-4-4	**0.0178**	0.1093(+)	0.1557(+)	0.3964(+)	0.2046(+)	0.2203(+)	0.3622(+)
2-20-68-4-5	**0.0186**	0.0731(+)	0.1676(+)	0.3441(+)	0.2066(+)	0.1331(+)	0.3013(+)
2-20-68-6-3	**0.0651**	0.1384(+)	0.2264(+)	0.4656(+)	0.1866(+)	0.1825(+)	0.2966(+)
2-20-68-6-4	**0.0000**	0.1070(+)	0.1085(+)	0.6404(+)	0.2424(+)	0.3647(+)	0.4650(+)
2-20-68-6-5	**0.0104**	0.0526(+)	0.0805(+)	0.4633(+)	0.1479(+)	0.1267(+)	0.4225(+)
+/ ≈ / –		54/0/0	54/0/0	54/0/0	54/0/0	54/0/0	54/0/0

s.

Instances	QLMHHA	FLS-HH	GP-HH	Jaya	NSGA-II	QVNS-NSGA-II	MOEA/D
6-40-132-10-6	**0.9302**	0.9018(+)	0.7524(+)	0.2234(+)	0.5442(+)	0.5043(+)	0.2425(+)
6-40-132-10-8	**0.9787**	0.8393(+)	0.7757(+)	0.3291(+)	0.6379(+)	0.6019(+)	0.4168(+)
6-40-132-15-6	**0.9309**	0.8613(+)	0.7327(+)	0.3103(+)	0.6919(+)	0.6614(+)	0.3505(+)
6-40-132-15-8	0.6827	**0.7520(-)**	0.6953(-)	0.2311(+)	0.6520(+)	0.7027(-)	0.2250(+)
6-50-168-10-6	0.7919	0.8571(-)	**0.9047(-)**	0.3453(+)	0.8964(-)	0.7812(+)	0.0400(+)
6-50-168-10-8	**0.9258**	0.8650(+)	0.7896(+)	0.3087(+)	0.6197(+)	0.7225(+)	0.2740(+)
6-50-168-15-6	**0.8642**	0.8007(+)	0.8098(+)	0.2342(+)	0.7737(+)	0.7196(+)	0.4207(+)
6-50-168-15-8	**0.8337**	0.6759(+)	0.6594(+)	0.5664(+)	0.7822(+)	0.6600(+)	0.2840(+)
6-60-214-10-6	**0.9410**	0.8999(+)	0.8926(+)	0.3849(+)	0.8034(+)	0.7983(+)	0.4669(+)
6-60-214-10-8	**0.8818**	0.8232(+)	0.6889(+)	0.2770(+)	0.7397(+)	0.7397(+)	0.2818(+)
6-60-214-15-6	**0.8267**	0.7662(+)	0.6302(+)	0.1072(+)	0.7980(+)	0.7896(+)	0.2894(+)
6-60-214-15-8	**0.8540**	0.7834(+)	0.7203(+)	0.1652(+)	0.4976(+)	0.5768(+)	0.3146(+)
8-40-132-10-6	**0.8763**	0.8716(+)	0.6541(+)	0.2024(+)	0.5418(+)	0.6037(+)	0.3205(+)
8-40-132-10-8	**0.8692**	0.8423(+)	0.8213(+)	0.2906(+)	0.6041(+)	0.5849(+)	0.1289(+)
8-40-132-15-6	**0.9881**	0.7892(+)	0.7968(+)	0.2430(+)	0.5868(+)	0.5577(+)	0.2711(+)
8-40-132-15-8	0.6484	**0.7023(-)**	0.6603(-)	0.2759(+)	0.6141(+)	0.5877(+)	0.3788(+)
8-50-168-10-6	**0.7928**	0.7385(+)	0.7635(+)	0.3172(+)	0.7686(+)	0.7318(+)	0.2660(+)
8-50-168-10-8	0.8327	**0.8387(-)**	0.7520(+)	0.1474(+)	0.6591(+)	0.6767(+)	0.1605(+)
8-50-168-15-6	**0.8167**	0.7668(+)	0.7590(+)	0.2725(+)	0.7329(+)	0.7633(+)	0.3146(+)
8-50-168-15-8	0.7903	0.7810(+)	**0.8710(-)**	0.3943(+)	0.7444(+)	0.8343(-)	0.3626(+)
8-60-214-10-6	**0.9325**	0.8598(+)	0.7517(+)	0.3005(+)	0.8428(+)	0.8010(+)	0.4897(+)
8-60-214-10-8	**0.9078**	0.7525(+)	0.8284(+)	0.3980(+)	0.8140(+)	0.8284(+)	0.5127(+)
8-60-214-15-6	**0.9281**	0.8241(+)	0.8212(+)	0.2136(+)	0.6315(+)	0.5488(+)	0.2144(+)
8-60-214-15-8	**0.4214**	0.3793(+)	0.3463(+)	0.1614(+)	0.3201(+)	0.2891(+)	0.2666(+)
+/ ≈ / −		20/0/4	20/0/4	24/0/0	23/0/1	22/0/2	24/0/0

Table 22 Statistical results of all algorithms on metric ρ_r for large-scale instances under $H \times S \times 30$ seconds.

Instances	QLMHHA	FLS-HH	GP-HH	Jaya	NSGA-II	QVNS-NSGA-II	MOEA/D
6-40-132-10-6	**0.6316**	0.3684(+)	0.0000(+)	0.0000(+)	0.0000(+)	0.0000(+)	0.0000(+)
6-40-132-10-8	**1.0000**	0.0000(+)	0.0000(+)	0.0000(+)	0.0000(+)	0.0000(+)	0.0000(+)
6-40-132-15-6	**0.6154**	0.2308(+)	0.1538(+)	0.0000(+)	0.0000(+)	0.0000(+)	0.0000(+)
6-40-132-15-8	0.1739	**0.3478(-)**	0.0435(+)	0.0000(+)	0.0870(+)	**0.3478(-)**	0.0000(+)
6-50-168-10-6	0.2857	0.1429(+)	**0.3571(-)**	0.0000(+)	0.2143(+)	0.0000(+)	0.0000(+)
6-50-168-10-8	**0.6250**	0.3125(+)	0.0625(+)	0.0000(+)	0.0000(+)	0.0000(+)	0.0000(+)
6-50-168-15-6	**0.6667**	0.0556(+)	0.2221(+)	0.0000(+)	0.0556(+)	0.0000(+)	0.0000(+)
6-50-168-15-8	**0.5000**	0.1250(+)	0.0625(+)	0.0000(+)	0.2500(+)	0.0625(+)	0.0000(+)
6-60-214-10-6	**0.6250**	0.1250(+)	0.1875(+)	0.0000(+)	0.0000(+)	0.0625(+)	0.0000(+)
6-60-214-10-8	**0.6875**	0.0625(+)	0.0000(+)	0.0000(+)	0.2500(+)	0.0000(+)	0.0000(+)
6-60-214-15-6	**0.5385**	0.1538(+)	0.0000(+)	0.0000(+)	0.0769(+)	0.2308(+)	0.0000(+)
6-60-214-15-8	**0.5833**	0.3334(+)	0.0833(+)	0.0000(+)	0.0000(+)	0.0000(+)	0.0000(+)
8-40-132-10-6	**0.6154**	0.2308(+)	0.0000(+)	0.0000(+)	0.0000(+)	0.1538(+)	0.0000(+)
8-40-132-10-8	**0.5263**	0.3158(+)	0.0000(+)	0.0000(+)	0.1053(+)	0.0526(+)	0.0000(+)
8-40-132-15-6	**1.0000**	0.0000(+)	0.0000(+)	0.0000(+)	0.0000(+)	0.0000(+)	0.0000(+)
8-40-132-15-8	**0.3333**	**0.3333(≈)**	0.0556(+)	0.0000(+)	0.2222(+)	0.0556(+)	0.0000(+)
8-50-168-10-6	**0.6250**	0.1250(+)	0.2500(+)	0.0000(+)	0.0000(+)	0.0000(+)	0.0000(+)
8-50-168-10-8	0.3334	**0.4444(-)**	0.2222(+)	0.0000(+)	0.0000(+)	0.0000(+)	0.0000(+)
8-50-168-15-6	**0.3000**	**0.3000(≈)**	0.1000(+)	0.0000(+)	0.2000(+)	0.1000(+)	0.0000(+)
8-50-168-15-8	0.2500	0.1875(+)	**0.4375(-)**	0.0000(+)	0.0000(+)	0.1250(+)	0.0000(+)
8-60-214-10-6	**0.5000**	0.1667(+)	0.0000(+)	0.0000(+)	0.2500(+)	0.0833(+)	0.0000(+)
8-60-214-10-8	**0.4000**	0.2000(+)	0.2667(+)	0.0000(+)	0.1333(+)	0.0000(+)	0.0000(+)
8-60-214-15-6	**0.8000**	0.1000(+)	0.1000(+)	0.0000(+)	0.0000(+)	0.0000(+)	0.0000(+)
8-60-214-15-8	**0.5833**	0.1667(+)	0.0000(+)	0.0000(+)	0.0000(+)	0.0000(+)	0.2500(+)
+/ ≈ / −		20/2/2	22/0/2	24/0/0	24/0/0	23/0/1	24/0/0

 s.

Instances	QLMHHA	FLS-HH	GP-HH	Jaya	NSGA-II	QVNS-NSGA-II	MOEA/D
6-40-132-10-6	0.0526	**0.0284(-)**	0.1258(+)	0.6057(+)	0.2710(+)	0.3045(+)	0.5565(+)
6-40-132-10-8	**0.0000**	0.0658(+)	0.1309(+)	0.5396(+)	0.2402(+)	0.2252(+)	0.3850(+)
6-40-132-15-6	**0.0134**	0.0687(+)	0.1351(+)	0.4901(+)	0.2221(+)	0.1564(+)	0.3453(+)
6-40-132-15-8	0.1121	**0.0789(-)**	0.1094(-)	0.4657(+)	0.1198(+)	0.0934(-)	0.4515(+)
6-50-168-10-6	0.0852	0.0613(-)	**0.0590(-)**	0.4400(+)	0.0863(-)	0.1308(-)	0.7511(+)
6-50-168-10-8	**0.0115**	0.0394(+)	0.0875(+)	0.4654(+)	0.1793(+)	0.1341(+)	0.3772(+)
6-50-168-15-6	**0.0403**	0.1052(+)	0.0408(+)	0.5522(+)	0.1044(+)	0.1253(+)	0.3207(+)
6-50-168-15-8	**0.0880**	0.1347(+)	0.1251(+)	0.3728(+)	0.1226(+)	0.1984(+)	0.3903(+)
6-60-214-10-6	**0.0235**	0.0547(+)	0.0423(+)	0.4088(+)	0.1077(+)	0.1240(+)	0.2919(+)
6-60-214-10-8	**0.0360**	0.0827(+)	0.1424(+)	0.4887(+)	0.0525(+)	0.1208(+)	0.4378(+)
6-60-214-15-6	**0.0620**	0.1436(+)	0.2032(+)	0.6537(+)	0.0828(+)	0.0741(+)	0.5104(+)
6-60-214-15-8	**0.0342**	0.0470(+)	0.1111(+)	0.6001(+)	0.2401(+)	0.1806(+)	0.3804(+)
8-40-132-10-6	0.0737	**0.0645(-)**	0.2024(+)	0.5845(+)	0.2532(+)	0.2212(+)	0.4264(+)
8-40-132-10-8	0.0942	0.1016(+)	0.1237(+)	0.4720(+)	0.2229(+)	0.2778(+)	0.6965(+)
8-40-132-15-6	**0.0000**	0.1277(+)	0.1227(+)	0.6629(+)	0.2850(+)	0.3260(+)	0.5870(+)
8-40-132-15-8	**0.1556**	0.1634(+)	0.1801(+)	0.4388(+)	0.2022(+)	0.2269(+)	0.3402(+)
8-50-168-10-6	**0.0666**	0.1004(+)	0.0964(+)	0.4073(+)	0.1201(+)	0.1287(+)	0.4612(+)
8-50-168-10-8	0.0683	**0.0320(-)**	0.0901(+)	0.6385(+)	0.1400(+)	0.1470(+)	0.5773(+)
8-50-168-15-6	**0.0810**	0.0958(+)	0.1008(+)	0.4552(+)	0.1099(+)	0.0969(+)	0.4630(+)
8-50-168-15-8	0.0905	0.1250(+)	**0.0782(-)**	0.4211(+)	0.1421(+)	0.1054(+)	0.3361(+)
8-60-214-10-6	**0.0295**	0.0772(+)	0.1221(+)	0.5413(+)	0.0772(+)	0.0965(+)	0.2832(+)
8-60-214-10-8	0.0885	0.0927(+)	**0.0641(-)**	0.3882(+)	0.0852(-)	0.0901(+)	0.3016(+)
8-60-214-15-6	**0.0086**	**0.0086(≈)**	0.0758(+)	0.5470(+)	0.1879(+)	0.2801(+)	0.5119(+)
8-60-214-15-8	**0.1529**	0.2103(+)	0.2320(+)	0.4107(+)	0.2947(+)	0.2462(+)	0.5693(+)
+/ ≈ / −		18/1/5	20/0/4	24/0/0	22/0/2	22/0/2	24/0/0

6. Conclusions

Industrial intelligence enabled modern manufacturing systems are transitioning toward energy-efficient distributed flexible manufacturing systems (FMSs). Preventive maintenance enables early detection of potential breakdowns, effectively preventing production disruptions, while prolonging equipment service life and stabilizing production efficiency, serving as the core support for ensuring the sustained and efficient operation of FMSs. To address the energy-efficient integrated distributed hybrid flow shop scheduling problem with preventive maintenance (EE-IDHFSP-PM), a *Q*-learning-based multi-objective hyper-heuristic algorithm (QLMHHA) is developed, aiming to simultaneously minimize makespan and total carbon emission (TCE). Based on prior works, this work introduces a reinforcement learning-based hyper-heuristic framework specifically designed for tackling the multi-objective energy-efficient distributed scheduling problem. Experimental results demonstrate that the proposed QLMHHA exhibits excellent efficacy in balancing global exploration and local exploitation capabilities while optimizing production efficiency and energy efficiency metrics. Given the findings and insights from the statistical results, the main contributions of this study are summarized as follows: 1) An MNEH-based initialization method is designed to generate initial populations with high quality and diversity. 2) A multi-stage cooperative energy-efficient strategy (MSC_EES) is developed based on non-critical paths to reduce machine energy consumption. 3) The search behavior of the algorithm is enhanced via 12 problem-specific low-level heuristics (LLHs), along with an effective *Q*-learning-driven high-level strategy (HLS) to control the generation of high-level individuals (HLIs) composed of low-level heuristics (LLHs). 4) A novel reward mechanism is introduced to dynamically balance exploration and exploitation for *Q*-learning, thereby accelerating the convergence rate and improving the quality of the Pareto solution sets. 5) Both convergence analysis and computational complexity analysis are provided to confirm the stability, reliability, and efficiency of QLMHHA in a theoretical sense, thereby laying theoretical foundations for future improvement and implementation of learning-driven methods.

There are three crucial aspects on which future research will focus. 1) Developing efficient and strongly exploratory low-level heuristics (LLHs) adapted to problem characteristics and incorporating experience-based domain knowledge to enhance the efficacy of QLMHHA. 2) Considering critical scheduling challenges in distributed FMSs, deeply investigate critical constraint handling techniques and develop scheduling strategies and optimization methods to balance between production efficiency and energy efficiency across various metrics, such as total tardiness and electricity costs. 3) Analyzing the features of energy-efficient distributed scheduling for cloud-edge collaboration architectures in IIoT, considering dynamic disturbances, preventive maintenance, and human-machine collaboration frameworks to provide superior scheduling schemes for decision-makers.

CRediT authorship contribution statement

Zi-Qi Zhang: Methodology, Funding acquisition, Investigation, Experiment, Writing - original draft. **Xin-Yun Wu:** Investigation, Methodology, Experiment, Software, Writing - original draft. **Bin Qian:** Methodology, Funding acquisition, Investigation, Writing - review & editing. **Rong Hu:** Methodology, Funding acquisition, Supervision, Writing - review & editing. **Jian-Bo Yang:** Project administration.

Declaration of competing interest

The authors declared that they have no competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Data availability

Data will be made available on request.

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