Omega 71 (2017) 93–105

Contents lists available at ScienceDirect

Omega

journal homepage: www.elsevier.com/locate/omega

A three-stage hybrid approach for weight assignment in MADM ☆

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ARTICLE INFO

Article history:
Received 6 February 2016
Accepted 26 September 2016
Available online 3 October 2016

Keywords:
Weights
Attribute
Multiple attribute decision making
Data envelopment analysis
Minimax reference point optimisation

ABSTRACT

How to determine weights for attributes is one of the key issues in multiple attribute decision making (MADM). This paper aims to investigate a new approach for determining attribute weights based on a data envelopment analysis (DEA) model without explicit inputs (DEA-WEI) and minimax reference point optimisation. This new approach first considers a set of preliminary weights and the most favourite set of weights for each alternative or decision making unit (DMU) and then aggregates these weight sets to find the best compromise weights for attributes with the interests of all DMUs taken into account fairly and simultaneously. This approach is intended to support the solution of such MADM problems as performance assessment and policy analysis where (a) the preferences of decision makers (DMs) are either unclear and partial or difficult to acquire and (b) there is a need to consider the best "will" of each DMU. Two case studies are conducted to show the property of this new proposed approach and how to use it to determine weights for attributes in practice. The first case is about the assessment of research strengths of EU-28 member countries under certain measures and the second is for analysing the performances of Chinese Project 985 universities, where the weights of the attributes need to be assigned in a fair and unbiased manner.

1. Introduction

Many methods have been proposed to determine the weights of attributes in multiple attribute decision making (MADM). Eckenrode [25] suggested six techniques for collecting the judgments of decision makers (DMs) concerning the relative values of attributes. Hwang and Yoon [32] summarised four techniques: eigenvector method, weighted least square method, entropy method, and LINMAP (The linear programming technique for multidimensional analysis of preference).

In general, according to the extent of dependence on the preferences or subjective judgments of DMs, there can be three types of approaches for weight assignment, depending on information provided and used to identify the weights of attributes. The first type of approaches may be named as subjective approaches, such as Simple Multi-Attribute Rating Technique (SMART, [26,27]) and Analytic Hierarchy Process (AHP, [30,44,45]). Subjective approaches rely entirely on the subjective judgments or intuition of DMs, while the quality of the analytical results or rankings of decision making units (DMUs) based on such weights can be compromised by the lack of experience or knowledge. In contrast, objective approaches, such as Entropy methods [32,59], Principal Components Analysis [36], and weighted least square method [14], determine weights by making use of mathematical models or statistical methods. They focus on differences among attributes and generate attributes' weights from data alone without requiring any preference information from the DMs. As the name implies, hybrid approaches take the advantages of both subjective and objective approaches, such as the integrated subjective and objective approach [42], which integrates the subjective consideration of DMs and the objective measures of attribute importance to form a two-objective programming model to integrate the subjective consideration of DMs and the objective measures of attribute importance. Shirland et al. [46] proposed a goal programming model for determining attribute weights based on pairwise comparisons between attributes. However these hybrid approaches also need clear or complete preferences from DMs.

In many MADM situations where there are no obvious DMs, subjective approaches are not applicable because no subjective judgments or intuition can be provided on the relative importance of attributes, for example, the assessment of academic impact of national research institutes in the world and the assessment of academic capacity of countries. Although a hybrid approach can incorporate subjective information into objective techniques to obtain attribute weights, it also depends on the prerequisite that

☆This manuscript was processed by Associate Editor Triantaphyllou.

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http://dx.doi.org/10.1016/j.omega.2016.09.011
0305-0483/

the preferences of DMs are clear or easy to acquire. The existing objective approaches can determine weights based on decision matrix and do not depend on the subjective judgments of DMs; however, they produce weight vectors according to data purely and are not designed to take into account the indisputable "will" of DMUs, that is, a set of the most favourable weights for each DMU.

This paper aims to investigate a new three-stage approach for determining the weights of attributes based on a linear programming (LP) model and minimax reference point optimisation, especially for the special cases of performance assessment or policy analysis where (a) the preferences of DMs are either unclear and partial or difficult to acquire and (b) there is a need to consider each DMU's best "will". The purpose of the assessment or analysis relates mostly to the ranking of DMUs instead of a more generic MADM context in which a specific choice has to be made. In this approach, we combine preliminary weights, LP model and minimax reference point model together to determine attribute weights for decision support where limited or no decision makers' preferences are available and fairness among DMUs being assessed is paramount. The first stage of this approach is to obtain the preliminary weights as the initial inputs to the third stage. In the second stage, a set of most favourite weights is derived for each DMU to maximise its utility. This weight scheme respects the best "will" of each DMU. In the third stage, we attempt to aggregate the information from the first two stages using the minimax reference point approach to find the most compromise solution as the common weights of attributes recommended for all DMUs so that the interests of all DMUs are taken care of fairly and simultaneously.

The remainder of the paper is organised as follows: Section 2 summarises the existing literatures and classifies them into three groups. Section 3 introduces the preliminaries and notations for this paper first. Second, we propose the three-stage hybrid LP model and minimax reference point approach for compromise weights, which regards the fairness as the basic principle to reflect the will of different stakeholders. Section 4 provides two case studies to demonstrate the features of the new approach, including the assessment of research strengths for EU-28 member countries and the performance analysis of selected Chinese universities. Section 5 concludes this paper.

2. Literature review

In existing literature on how to determine the attributes' weights in MADM problems, there are different types of approaches for weight assignment, depending on the information provided and used to identify the weights of attributes. Wang and Luo [51] and Chin et al. [13] argued that there could be three categories for determining attribute weights: subjective, objective, and hybrid approaches. In this paper we follow this classification to organize the literature review.

2.1. Subjective approach

Edwards [26] proposed the SMART technique to determine attribute weights in MADM. Subsequently, Edwards and Barron [27] further presented two advanced approximate methods for multi-attribute utility measurement (SMARTS and SMARTER) based on an elicitation procedure for weights. Doyle et al. [23] investigated two commonly used methods for assigning numerical values (i.e. decision weights) to attributes in order to signify their perceived relative importance, which are to ask people to directly rate each of the attributes in turn, or to allocate a budget of points to the attributes. The Analytic Hierarchy Process (AHP) is a widely used method for generating weights in MADM based on pair-wise comparison matrix [2,30,44,45]. In this approach, attribute weights can be derived from the eigenvector of a pair-wise comparison matrix. Roberts and Goodwin [43] showed how to obtain rank order distribution weights. Bottomley and Doyle [4] compared three weight elicitation methods including direct rating method, which rates each attribute in turn on a scale of 0–100. Other techniques in this category include Delphi method (e.g. [32]), etc.

2.2. Objective approach

In the existing literature, there are several popular objective approaches for determining attribute weights, *e.g.* Entropy method (see, *e.g.* [11,12,19,32,59]). Entropy, a useful concept in information theory, can be used as a tool to determine weight for each attribute. In particular, the degree of diversification of information provided by the values of each attribute can be used to determine the weight for this attribute. An attribute does not play a significant role to differentiate DMUs when all DMUs have similar values on that attribute. However, using an entropy method to assign weight to such an attribute can disproportionally decrease its role in overall ranking in some MADM methods such as additive value function (AVF) approach. This is because the same information as given in a decision matrix is double counted to get both values and weights that are multiplied in the AVF approach. Shannon's entropy has also been applied to calculate objective weights in group decision making problems. In this category, Principal Components Analysis (PCA, [36]) is also a widely used objective method for generating weights [36]. Chu et al. [14] proposed a weighted least-square method to obtain attribute weights, which includes the solution of a set of simultaneous linear algebraic equations and is thus conceptually easy to understand. Yue [56] proposed an approach to group decision making based on determining the weights of experts by using projection method. Deng et al. [19] used the standard deviation method to calculate objective weights for attributes. Wang and Luo [51] proposed a correlation coefficient and standard deviation integrated approach for determining the weights of attributes in MADM. Diakoulaki et al. [22] proposed a method for generating objective weights, which is based on the quantification of two fundamental notions of MCDM: the contrast intensity and the conflicting character of evaluation criteria.

2.3. Hybrid approach

Choo and Wedley (1985) used an integer linear goal-programming technique based on a training set of choices (value judgments) from DMs to determine optimal criterion weights which minimize the number of misclassification of decisions. The UTA (UTilités Additives) method [35,47] uses a weak-order preference structure on a set of actions, together with the performances of DMUs on all attributes, to produce a set of additive value functions based on multiple criteria, so that the resulting structure would be as consistent as possible with the initial structure given by DMs. In essence, UTA method uses LP techniques to infer additive value/utility functions optimally, so that these functions are as consistent as possible with the DMs' preference structure. In order to remove arbitrariness in this process, the robust ordinal regression has been proposed, aiming to take into account all the sets of parameters compatible with the DM's preference information [31]. Ma et al. [42] proposed a subjective and objective integrated approach to determine attribute weights. Their approach uses subjective information provided by a DM and objective information extracted from a decision matrix to form a two-objective programming model to integrate the subjective considerations of DMs and the objective measures of attribute importance. However, Xu [52] noted that the weights obtained from Ma *et al.* (2009)'s approach can be quite different from other objective weights, *e.g.*, entropy weights. Wang and Lee [50]

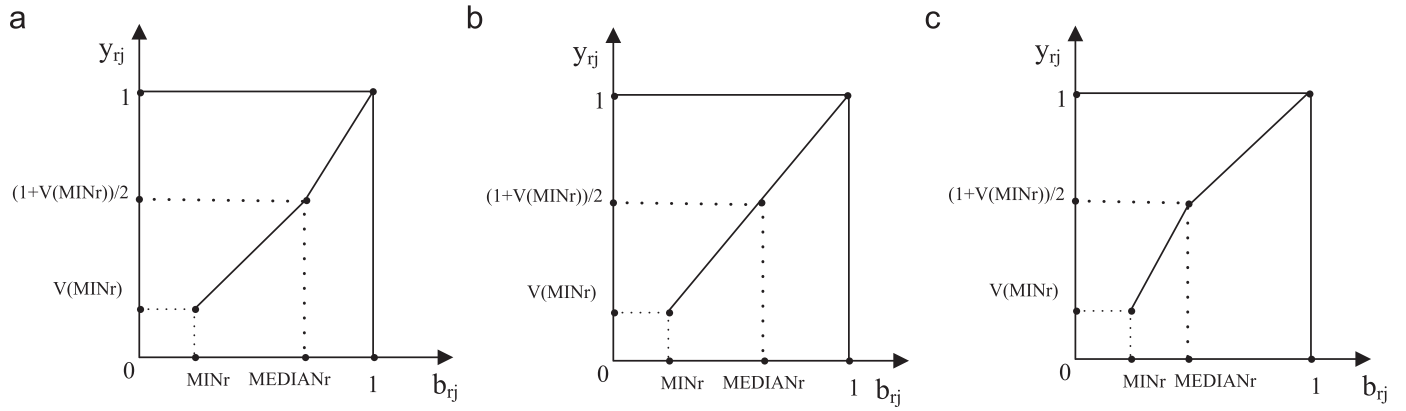

Fig. 1. Illustration of piecewise linear value function.

proposed an extension of TOPSIS approach that integrates subjective and objective weights. Their approach is based on DMs' judgements and also calculates objective weights based on Shannon's entropy. The advantage of the approach is that it not only benefits from DMs' expertise but also involves end-users in the whole decision making process. Fan et al. [28] integrated DM's fuzzy preference on DMUs and an objective matrix into one model. Wang and Parkan [52] integrated DM's fuzzy preference relations on DMUs, DM's multiplicative preference relations on attribute weights, and an objective matrix into a general framework. Horsky and Rao [33] formulated a linear program (LINPAC) for estimation of attribute weights from paired preference comparisons. Horowitz and Zappe [34] proposed a linear programming model for eliciting criteria weights in a performance appraisal process. Deng et al. [21] estimated attribute weights through evidential reasoning and mathematical programming based on preferences given by DM, and represented as intervals. Shirland et al. [46] proposed a goal programming model for determining constrained regression estimates of attribute weights using pair-wise comparisons by using triads of attributes.

As discussed in the previous section, there are real MADM issues where no obvious or explicit DMs can provide clear subjective judgments or intuition on the relative importance of attributes or DMs can only provide partial preference information. In such cases, subjective and hybrid approaches are not applicable because of the lack of clear or complete information from DMs. Although objective approaches can produce weights based on existing data, they fail to consider the indisputable "will" of DMUs, *i.e.*, a set of the most favourable weights for each DMU and, therefore, may generate "rude" and unreasonable results which can undermine the reliability of decision support due to the use of such reckless weights.

3. Hybrid linear programming model and minimax reference point approach

3.1. Preliminaries and notations

The following notations are used to represent a MADM problem:

$S=\{DMU_j, j=1,2,\ldots,n\}$: a discrete set of n possible DMUs.
$Y=\{Y_1,Y_2,\ldots,Y_s\}$: a set of s attributes. In this paper, we assume that the attributes are additively independent.
$W^T=(w_1,w_2,\ldots,w_s)^T$: the weight vector of attributes (or weights thereafter), where $\sum_{r=1}^{s} w_r=1, w_r\geq 0$.
$A=[a_{jr}]_{n\times s}$: the decision matrix where a_{rj} is the value of DMU j on attribute r which is non-negative for $r=1,2,\ldots,s, j=1,2,\ldots,n$.

We first resort to normalising the decision matrix A by transforming every element in A into a corresponding element in the normalised (value) matrix $D=[b_{jr}]_{n\times s}$ using the following linear formulas:

$$\begin{cases} b_{jr}=\frac{a_{jr}}{a_r^{max}}, \textit{for benefit attribute} \\ b_{jr}=\frac{a_r^{min}}{a_{jr}} \textit{or } 1-\frac{a_{jr}}{a_j^{max}}, \textit{ for cost attribute} \end{cases} \tag{1}$$

Remark 1. *It should be noted that there are several data normalisation methods. For example*, Zavadskas and Turskis [58] *summarized the widely known and logarithmic normalization methods. Interested readers can refer to* Zavadskas and Turskis [58] *for more details.*

Second, without obvious DMs to provide value functions for attributes, we define a piecewise linear value function for each attribute (see e.g. [20]). Let $MIN_r$and $MEDIAN_r$ be the minimal and median values of attribute $r(r=1,\ldots,s)$, respectively. Then, without loss of generality, let $V(MIN_r)=MIN_r$, $V(MEDIAN_r)=(1+V(MIN_r))/2$, and $V(1)=1$. Fig. 1 illustrates the piecewise linear value function $V(b_{jr})$, where b_{jr} denotes the attribute's value after the normalisation by formula (1). In Fig. 1(a), (b), and (c), we illustrate risk seeking value faction, risk neutral value function, and risk averse value function, respectively.

Mathematically, the value function shown in Fig. 1 is given by $y_{jr}=V(b_{jr})$ as follows:

$$y_{jr}=V(b_{jr})=\begin{cases} \frac{1-MIN_r}{2\times(MEDIAN_r-MIN_r)}b_{jr}+\frac{2\times MEDIAN_r-MIN_r-1}{2\times(MEDIAN_r-MIN_r)}\times MIN_r, b_{jr}\in[MIN_r, MEDIAN_r] \\ \frac{1-MIN_r}{2\times(1-MEDIAN_r)}b_{jr}+\frac{1-2\times MEDIAN_r+MIN_r}{2\times(1-MEDIAN_r)}, b_{jr}\in[MEDIAN_r,1] \end{cases} \tag{2}$$

Based on the above value function (2), we can have the corresponding matrix $DV=[y_{jr}]_{n\times s}$after the transformation via value functions.

Remark 2. *In* Fig. 1 *we illustrate piecewise value functions. There are also different approaches to define value function. In this paper, without loss of generality, the above value functions are proposed because they are relatively simple yet consistent across criteria. Also, it is not irrational to assume that the median point of a criterion attains the middle value between the highest value and the lowest* *value available. In the Section 4, we will show that the value function defined in formula (2) is risk averse in our two illustrative examples.*

One of the best known and most widely used MADM methods is the additive value function method, which requires that attributes be additively independent. Using this method, we can obtain the overall value of a DMU as follows, where the marginal value y_{rj} is given by Eq. (2),

$$U(DMU_j) = \sum_{r=1}^{s} w_r y_{rj}, j = 1, 2, \ldots, n. \tag{3}$$

Remark 3. *The theoretical foundation of this method is utility theory [38]. It should be noted that Eq. (3) is a much simplified form of utility function because y_{rj} in Eq. (3) results from the normalisation process using Eqs. (1) and (2). A possible way to improve the precision of formula (3) is to identify value functions for each attribute for stakeholders using empirical studies. In this paper we use this simple additive value function to demonstrate how weights can be generated using the proposed three stage approach.*

3.2. Hybrid linear programming model and minimax reference point approach

In Eq. (3), one question is how to assign weight w_r fairly and consistently, which poses a challenge when (a) there is no obvious DM who can provide his preferences, and (b) there is a need to fully consider the "will" of both DM (if it exists) and each DMU to reflect fairness. To achieve this goal, in this section, we propose a new three-stage approach to combine preliminary weights, LP model and the reference point model together to determine the attribute weights for decision making. The first stage of this approach is to obtain the preliminary weights as input to the second stage. If there is no obvious DM, we can use data-driven method (*e.g.* Entropy method) to obtain the preliminary weights. On the other hand, if there is an explicit DM, the DM can set attribute weights according to the DM's own will. The weights from the first stage are called preliminary weights in this paper. In the second stage, we use LP models to generate the most favourite attribute weights for each DMU. In this stage, each DMU can propose its most favourite weights to maximise its utility. This weight scheme respects the best "will" of each DMU. In the third stage, we attempt to aggregate the best weights using the minimax reference point approach for each DMU to find the best compromise solution as the common weights of attributes recommended for all DMUs.

3.2.1. Stage 1: preliminary weights assignment

In real MADM problems we can also set preliminary weights as inputs to reflect (1) the characteristics of the data using some data-driven approaches, *e.g.*, Entropy method (*e.g.* [32,59]), or (2) the subjective wishes or needs of explicit DMs. Here we briefly give the procedure of Entropy method as follows.

Procedure 1.

Step 1: Obtain the matrix after transformation $DV = \left[y_{jr}\right]_{n\times s}$ via value function (2) based on normalised decision matrix $D = [b_{jr}]_{n\times s}$.

Step 2: Calculate the entropy value (E_r) of each attribute r using the following formula:

$$E_r = -ln\,(n)^{-1} \sum_{j=1}^{n} p_{jr} ln\left(p_{jr}\right),\ r = 1, 2\ldots, s \tag{4}$$

where $p_{jr} = y_{jr} / \sum_{j=1}^{n} y_{jr}$ and variable $-ln\,(n)^{-1}$ is the normalisation factor. If $p_{jr} = 0$, we define $p_{jr} ln(p_{jr}) = 0$ to ensure the validity of Eq. (4).

Step 3: Determine the preliminary weights (ω_r) for attributes as follows:

$$\omega_r = \frac{1 - E_r}{s - \sum_{r=1}^{s} E_r}, r = 1, 2, \ldots, s. \tag{5}$$

Note that $\sum_{r=1}^{s} \omega_r = 1$.

It should be noted that the above entropy method is not without its potential drawbacks, as discussed in Section 2.2, and there are other alternative approaches for determining preliminary weights.

3.2.2. Stage 2: LP models for most favourite weights for each DMU

In this subsection, we use LP models to generate the most favourite weights for each DMU. The idea of those LP models originates from data envelopment analysis (DEA). DEA, first put forward by Charnes, Cooper and Rhodes [7], is a set of mathematical programming models for performance analysis. There are many DEA models, the most well-known of which include the CCR model [7], the BCC model [3], the Additive model [6], and the Cone Ratio model [8]. Excellent reviews on DEA theory and applications can be found in several recent studies, *e.g.* Cooper et al. [16–18], Cook and Seiford [15]. The interested readers can also refer to newly published papers for research trends in this field, e.g. Chen [9], Kao [37], Lee and Worthington [39], etc. These classic DEA models are formulated for desired inputs and outputs to measure the technical efficiency of decision-making units (DMUs). A DEA model provides the weighting coefficients that each DMU can allocate to each input or output for its best benefit in terms of maximising its efficiency. The principle that each DMU is allowed to freely generate its flexible weights for its inputs/outputs is the very essence of DEA.

In many applications, however, there are no explicit input data available (see, *e.g.*, [40,54]). For example, it is difficult or sometimes impossible to reformulate data into original inputs and outputs and then to apply the classic DEA models to measure the performance of DMUs. In MADM problems, attributes are not normally divided into input and output, with the higher value of an attribute being preferred. As such, the values of attributes can be regarded as outputs in the context of DEA. Adolphson et al. [1] first proposed DEA models without inputs. Lovell and Pastor [41] studied these DEA models without inputs systematically. Liu et al. [40] conducted systematic studies on this group of DEA models, which are called DEA models without explicit inputs (DEA-WEI model). Toloo [49] proposed a new approach for the most efficient unit without explicit inputs.

Yang et al. [54] studied a DEA-WEI model with quadratic terms from the perspective of extended utility. In general, DEA-WEI does not reflect the technical efficiency of the input-output system of DMUs, which is what the classic DEA does. Instead, they investigate the link between a DEA-WEI model and multi-attribute utility theory (MAUT). DEA-WEI is generally used for evaluation purposes. However, the functional form of its objective function shares familiar resemblance with classic utility function. Unlike the classic utility theory, however, only the functional form of the objective function is determined by DMs to reflect subjective emphasis on evaluation, whereas it is a DMU that determines the coefficients of the function by DEA-style programming to ensure the most favourable evaluation for the DMU.

Suppose that the “additive independence” condition holds [10,29]. We then have the following DEA-WEI model for evaluating DMU_0 [40,54]:

$$\theta_0 = \max_{u_r} \sum_{r=1}^{s} u_r y_{r0}$$

$$s.t.\begin{cases} \sum_{r=1}^{s} u_r y_{rj} \le 1, j=1,2,\dots,n \\ u_r \ge 0, r=1,2,\dots,s \end{cases} \tag{6}$$

where the subscript “0” refers to the DMU of interest.

The equivalent form of the above model (6) appears also in Caporaletti et al. [5], which proposes a framework based on non-parametric frontiers to rate and classify.

Remark 4. *In model (6), it is obvious that there could be zero elements in the set of weights $u_r (r=1,\dots,s)$. According to Toloo [49], to ensure $u_r \ge \varepsilon$, it is necessary to keep the elements in the set of weights away from zero in DEA-WEI models. Therefore, we rewrite model (6) as the following model with non-Archimedean construct ε: $\theta_0 = \max_{u_r} \sum_{r=1}^{s} u_r y_{r0}$ $s.t.\begin{cases} \sum_{r=1}^{s} u_r y_{rj} \le 1, j=1,2,\dots,n \\ u_r \ge \varepsilon, r=1,2,\dots,s \end{cases}$.*

Toloo [48] *showed how to choose non-Archimedean construct ε in DEA-WEI models as follows: $\varepsilon \in (0, \max\left\{1/\sum_{r=1}^{s} y_{rj}: j=1,\dots,n\right\}]$. It should be noted that the epsilon in the above model only represents a very small positive number to ensure the non-zero coefficients of outputs. It does not represent a two-stage way of calculation as normally being done in traditional DEA with slack consideration.*

As we mentioned above, it is the very essence of DEA or DEA-WEI that each DMU provides flexible weights for its indicators. So far, we have showed that DEA-WEI model can produce the optimal weight scheme for each DMU. This weight scheme reflects the best "will" of the DMU to be evaluated. However, it should be noted that $u_r \ge 0, r=1,2,\dots,s$ do not necessarily satisfy $\sum_{r=1}^{s} u_r = 1$ in model (6). In this paper, we use a DEA-WEI-like LP model to obtain the most favourite weights to reflect the best "will" of each DMU as follows (see [49]):

$$\theta_0 = \max_{u_r} \sum_{r=1}^{s} u_r y_{r0}$$
$$s.t.\begin{cases} \sum_{r=1}^{s} u_r y_{rj} \le 1, j=1,2,\dots,n \\ u_r \ge \varepsilon, r=1,2,\dots,s \end{cases} \tag{7}$$

In model (7), we can see that each DMU can use its most favourite weights to reflect its best "will" to achieve its maximised utility.

Second, in MADM, however, preferential or cognitive constraints may be imposed on the relative importance of each attribute. For example, the ratio range of weights of one attribute A to another B can be set to $\left[\frac{1}{9}, 9\right]$ as in the AHP method (see, *e.g.* [44,45]). Thus we can reformulate model (7) more generally as follows:

$$\theta_0 = \max_{u_r} \sum_{r=1}^{s} u_r y_{r0}$$
$$s.t.\begin{cases} \sum_{r=1}^{s} u_r y_{rj} \le 1, j=1,2,\dots,n \\ u_r \in \Omega, r=1,2,\dots,s \end{cases} \tag{8}$$

where Ω denotes the set of preferential and cognitive constraints on attribute weights. For example, when there are no explicit DMs, we can setΩ as follows:

$$\Omega = \left\{ \frac{u_i}{u_j} \in \left[\frac{1}{9}, 9\right],\ i,j=1,2,\dots,s, i \ne j; u_r \ge 0, r=1,2,\dots,s \right\}. \tag{9}$$

When there are explicit DMs, we can let Ω be the set reflecting the prior information or value judgements of the DMs.

Thus, based on model (8), we can obtain the optimal weights for DMU_0 as $\left(u_1^{0*}, u_2^{0*}, \dots, u_r^{0*}\right)^T$ individually, which is the optimal solution of model (8). So far, we have shown that the DEA-WEI-like LP model (8) can produce the optimal weight scheme for each DMU. This weight scheme reflects the best "will" of the DMU to be evaluated in a fair way because each DMU has the same right to maximise its utility. We need to emphasize that stage 2 is always intended to estimate the best "interests" of DMU with or without the intervention of DM. It should also be noted that alternative optimal solutions may exist in the DEA-WEI-like LP model (8). However, we will show in the next Subsection 3.2.3 that this non-uniqueness of optimal solutions will not affect our results.

3.2.3. Stage 3: minimax reference point optimisation for weight assignment

In this subsection, we will aggregate the preliminary weights from Subsection 3.2.1 and the most favourable weight vectors generated for individual DMUs from Subsection 3.2.2 to find the best compromise solution as the common weights of attributes recommended for all DMUs.

Suppose the optimal solution of the above model (8) is$(u_1^*, u_2^*, \dots, u_s^*)^T$. Then, the optimal objective value for DMU_j in model (8) is given by $\theta_j^* = \sum_{r=1}^{s} u_r^* y_{rj}$. Since each DMU_j seeks to find its best set of weights to achieve its maximum utility$\theta_j^* (j=1,2,\dots,n)$, for the sake of finding the common weights of attributes, in this paper we use the following minimax reference point method [55].

We first define the ideal point in the objective space as follows:

$$\Theta = \left[\theta_1^*, \dots, \theta_j^*, \dots, \theta_n^*\right]$$

where θ_j^* denotes the maximal utility of DMU_j in the DEA-WEI-like model (8). In general, the ideal point Θ is not a feasible solution. Otherwise, the objectives would not be in conflict with another one. The rationale behind is that a feasible solution or a set of weights should be selected such that the combined deviation between the feasible solution and the ideal solution Θ is minimised. In other words, the best compromise solution is the one that is the closest to the ideal point Θ. The minimax optimisation problem is given as follows:

$$\min \tau = \max \begin{Bmatrix} t_1 = \theta_1^* - \sum_{r=1}^{s} u_r y_{r1} \\ t_2 = \theta_2^* - \sum_{r=1}^{s} u_r y_{r2} \\ \cdots \\ t_n = \theta_n^* - \sum_{r=1}^{s} u_r y_{rn} \end{Bmatrix}$$
$$s.t.\begin{cases} \sum_{r=1}^{s} u_r y_{rj} \le 1, & j=1,2,\dots,n \\ u_r \in \Omega, & r=1,2,\dots,s \end{cases} \tag{10}$$

The above model (10) is referred to as the minimax reference point optimisation model in this paper. It is worth noting that model (10) provides a fair way to generate the best compromise weights of attributes for all DMUs. This is because the model works like an equalizer [55,56] so that the maximum difference between the best compromise utility vector and the reference or ideal utility vector for all attributes is minimised for every DMU with no weight unfairly given any special treatment for any DMU. This fairness principle is regarded to be appropriate for weight assignment in MADM in particular where the preferences of DMs are either unclear and partial or difficult to acquire.

Fig. 2 shows the rationale of the above minimax optimisation approach (model 10). We assume in Fig. 2 that there are only two attributes whose weights are denoted by u_1 and u_2, and there are three DMUs, say DMU_1, DMU_2 and DMU_3. Thus, based on formula (10), we define the objective space as follows:

$$\begin{bmatrix} f_1 = \sum_{r=1}^{2} u_r y_{r1} \\ f_2 = \sum_{r=1}^{2} u_r y_{r2} \end{bmatrix} s.t. \begin{cases} \sum_{r=1}^{2} u_r y_{r1} \le 1 \\ \sum_{r=1}^{2} u_r y_{r2} \le 1 \\ \sum_{r=1}^{2} u_r y_{r3} \le 1 \\ u_r \in \Omega, r=1,2 \end{cases}$$

Therefore Fig. 2 illustrates the minimax optimisation approach in model (10) intuitively.

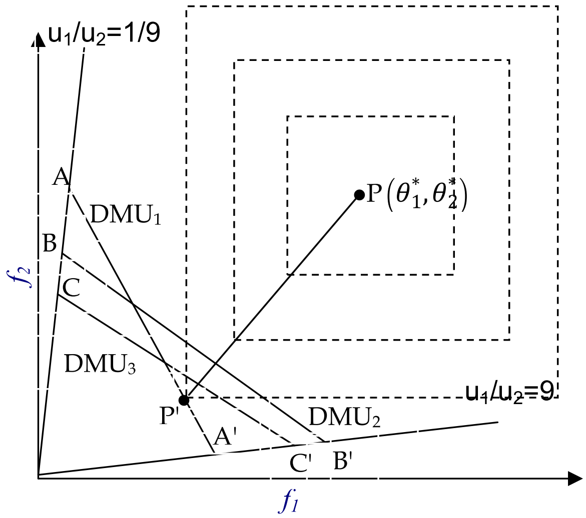

Fig. 2. Illustration of the minimax optimisation approach.

In Fig. 2, the variables f_1 and f_2 represent the horizontal and vertical coordinates respectively. Thus the points on and below line segments AA′, BB′ and CC′ represent the feasible utility values for the three DMUs. Thus the minimax optimisation can be reflected as the distance from the ideal point $P(\theta_1^*, \theta_2^*)$ to point P'. Because for every DMU the maximum difference between the best compromise weight vector and the reference weight vector for all attributes is minimised and there is no special treatment for any DMU, this approach is fair for every DMU. The rationale behind model (10) is to treat each DMU using a fair way to reduce its utility from its ideal point without bias to any DMU. Model (10) can be easily transformed into the following model (11):

$$s.t.\begin{cases} \min \tau \\ \theta_j^* - \sum_{r=1}^{s} u_r y_{rj} \le \tau,\ j=1,\dots,n \\ \sum_{r=1}^{s} u_r y_{rj} \le 1, j=1,2,\dots,n \\ u_r \in \Omega, r=1,2,\dots,s \end{cases} \tag{11}$$

Theorem 1. *Model (10) is equivalent to the above model (11).*

Proof. (a) If (u_r^*, t_j^*, τ^*) is the optimal solution of model (10), then solution (u_r^*, τ^*) must also be the optimal solution of model (11). Otherwise, we assume that there would be another solution $(u_r^{*\prime}, \tau^{*\prime})$ of model (11) satisfying $\tau^{*\prime} < \tau^*$. It is easy to see that $(u_r^{*\prime}, \tau^{*\prime})$ is also a feasible solution of model (10) when $t_j = \tau^{*\prime}$, which contradicts our first assumption. **(b)** If (u_r^*, τ^*) is the optimal solution of model (11), then it must also be the optimal solution of model (10). Otherwise, we assume that $\left(u_r^{*\prime}, t_j^{*\prime}, \tau^{*\prime}\right)$ is the optimal solution of model (10) satisfying $\tau^{*\prime} < \tau^*$. Because $\tau^{*\prime} = max\left\{t_j^{*\prime}\right\}$, we know $\tau^{*\prime} = max\left\{t_j^{*\prime}\right\} < \tau^*$. Therefore if we let $\tau = \tau^{*\prime}$, it is easy to verify that solution $(u_r^{*\prime}, \tau^{*\prime})$ will be a feasible solution of model (11), which contradicts our second assumption. **Q.E.D.**

Stage 3 is based on the ∞-norm formulation and the ideal solution taking the best values of all DMUs. As discussed in the literature [55], the min-max formulation works like an equalizer and is capable of locating a solution on the efficient frontier as a result of trade-offs among the needs of all DMUs with the normalized weight (or "right of speak") of each DUM taken into account in a fair manner without bias towards any DMU. It is also argued in the literature [55] that the minimax reference point approach is capable of handling general preference information in both convex and non-convex cases and generating any efficient solution by setting appropriate reference points and weights.

Next, we incorporate the preliminary weights into the above models (10) and (11) through the following procedure:

Procedure 2.

Step 1: We first calculate the utility for each DMU_j using the preliminary weights and denote it as $\theta_j^p (j=1,2,\dots,n)$. The preliminary weights can be from the data-driven approaches (*e.g.* Entropy method) or subjective approach by DM.

Step 2: We calculate the gap between $\theta_j^p (j=1,2,\dots,n)$ and $\theta_j^* (j=1,2,\dots,n)$ for each DMU. We define the gap $g_j = \left(\theta_j^* - \theta_j^p\right)$ as "right to speak" of each DMU. If the gap g_j is large, it means that this DMU has more "right" than other DMUs with lower gap to argue for reassignment of weights. We normalise g_j into g_j^* to satisfy $\sum_{j=1}^{n} g_j^* = 1$ as the normalised weights ("right to speak") for each DMU.

Step 3: We incorporate the weights for each DMU into model (10) and have the following model (12):

$$\min \tau = max\begin{Bmatrix} t_1 = g_1^*\left(\theta_1^* - \sum_{r=1}^{s} u_r y_{r1}\right) \\ t_2 = g_2^*\left(\theta_2^* - \sum_{r=1}^{s} u_r y_{r2}\right) \\ \dots \\ t_n = g_n^*\left(\theta_n^* - \sum_{r=1}^{s} u_r y_{rn}\right) \end{Bmatrix}$$
$$s.t.\begin{cases} \sum_{r=1}^{s} u_r y_{rj} \le 1, & j=1,2,\dots,n \\ u_r \in \Omega, & r=1,2,\dots,s \end{cases} \tag{12}$$

Similarly, the above model (12) can be transformed into the following model (13):

$$s.t.\begin{cases} \min \tau & \\ g_j^*\left(\theta_j^* - \sum_{r=1}^{s} u_r y_{rj}\right) \le \tau, & j=1,\dots,n \\ \sum_{r=1}^{s} u_r y_{rj} \le 1, & j=1,2,\dots,n \\ u_r \in \Omega, & r=1,2,\dots,s \end{cases} \tag{13}$$

Using model (13) we can have the final compromised solution as the assignment for attributes in MADM problems.

Similarly, we can also have the following Theorem 2:

Theorem 2. *Model (12) is equivalent to the above model (13). The proof for this theorem is similar to that of Theorem 1 and omitted here.*

We can still incorporate other information of preference into the above model (13). For example, if there is a need to stress that one attribute should be more important than another attribute. We denote this information as $u_r \in \Phi, r=1,2,\dots,s$. Thus the above model (13) can be transformed further into the following model (14):

$$s.t.\begin{cases} \min \tau & \\ g_j^*\left(\theta_j^* - \sum_{r=1}^{s} u_r y_{rj}\right) \le \tau, & j=1,\dots,n \\ \sum_{r=1}^{s} u_r y_{rj} \le 1, & j=1,2,\dots,n \\ u_r \in \Omega \cup \Phi, & r=1,2,\dots,s \end{cases} \tag{14}$$

where symbol $\cup$ denotes the union operator of two sets.

The optimal solution of model (14) $(u_1^*, u_2^*, \dots, u_s^*)^T$ after normalisation using the following formula (15) is the final optimised weight vector $W^T = (w_1, w_2, \ldots, w_s)^T$ of the attributes:

$$w_r = u_r^* / \sum_{r=1}^{s} u_r^* \tag{15}$$

where variable $r = 1, 2, \ldots, s$.

4. Illustrative examples

In this section two cases are investigated to show the proposed approach for determining attribute weights in MADM. The first one is the assessment of the research strength of EU-28 member countries to compare the comparative advantages of these countries. The second case is the assessment of a number of selected universities under the direct management of Ministry of Education (MOE) of China. In the second case, there is the explicit DM, which is the MOE, whose prior preference information as preliminary weights could be incorporated into our new approach for attribute weights.

4.1. Assessment of research strength of EU-28 member countries

In this section, a case study is conducted to apply the new weight assignment approach to assess the research strengths of EU-28 member countries on 9 disciplines[1] related to medical science of Essential Scientific Indicators (ESI) from Thomson Reuters. See Table 1 for details on these disciplines and their abbreviations in this paper.

We use the numbers of SCI papers in these 9 disciplines to denote their capacities of publications, so the 9 indicators refer to the number of SCI papers of each discipline respectively. To achieve a reasonable level of discrimination, according to Dyson et al. [24] we select EU-28 countries as our sample. We further use the population of each country to reduce those 9 indicators on a common measure to ensure the comparability of those 28 countries. The SCI papers in these 9 disciplines used are from Thomson Reuters' research evaluation tool InCites[2] and the time window is the year 2015 (from Jan., 2015 to Dec. 2015). The detailed data is shown in Table A-1.

In this case study, there is no obvious DM who can provide the weight information on these disciplines and this kind of study could be for general policy analysis. In such cases it is hard or impossible to have prior information about attribute weights from potential DMs. In order to consider fully the information from the existing data and the fairness of the evaluation, it is suitable to use our proposed approach to generate weights for each attribute (discipline) to reflect the objectiveness and rationality of the whole evaluation process.

Firstly, we assume that these 9 attributes are all benefit attributes. We use normalisation function defined in formula (1) to normalise the 9 indicators (per capita). Then we have the descriptive statistics on 9 attributes as follows:

From Table 2 we can see that in this example, the value function in formula (2) is risk averse (see Fig. 1(c)), which means the publication will have more value when its number is relatively low. However, when the number of publications reaches a large number, the increase of value it creates becomes smaller gradually. We use formula (2) to have the corresponding matrix after the transformation via value functions, which is shown in Table A-2.

Secondly, we use the Entropy method to obtain the preliminary weights for attributes as follows using formulae (4), (5) (Table 3):

Table 1
9 ESI disciplines and their abbreviations.

Disciplines	Abbreviations
Biology & Biochemistry	BB
Clinical Medicine	CM
Immunology	Immu.
Microbiology	Mic.
Molecular Biology & Genetics	MG
Neuroscience & Behavior	NB
Pharmacology & Toxicology	PT
Plant & Animal Science	PA
Psychiatry/Psychology	PP

Thirdly, we use model (8) on each DMU to obtain its best utility to form the ideal point in the objective space. We can have normalised gap g_j^* as the weight for DMUj using Step 2 in Procedure 2. See Table 4 for details.

In this case there is no other information, so the set Φ is null. Using model (14) and formula (15), we can have the optimal normalised weights for the 9 ESI disciplines as shown in the following Table 5.

Thus the final utilities and rankings of EU-28 member countries are shown in Table 6. We take United Kingdom, Denmark, Sweden, Latvia, and Bulgaria as examples. From the results, we find that United Kingdom ranks the first due to its excellent performance in almost all those 9 disciplines. Denmark, Sweden, Bulgaria, and Latvia rank the second, the third, the 27$^{\text{th}}$, and the 28$^{\text{th}}$, respectively. Denmark's utility grows significantly due to the maximum normalised gap ($g_j^* = 0.0758$). However, the utility of United Kingdom drops slightly because of its normalised gap is only 0.0079. Similarly we can analyse the changes of utilities of other countries.

We can use the following Fig. 3 to show the publication proportions of the above five selected countries. In this figure, we find that United Kingdom develops balanced and excellent performance at almost all 9 disciplines. The proportions of publications of United Kingdom to all publications of sample countries all exceed 15%. The figure shapes of Denmark and Sweden are similar, and there is an obvious shortcoming for both countries. However Denmark has less population than Sweden. Therefore, it performs better than Sweden in term of publication per capita. The number of publications of Latvia is less than 0.2% in all those 9 disciplines. Bulgaria performs relatively better in the fields of Plant & Animal Science than other disciplines.

4.2. Assessment of Project 985 universities in China

In 1998, the Ministry of Education of China (MOE) implemented the "Educational Revitalisation Action Plan for the 21st Century", which focuses on creating world-class high-level universities, *i.e.*, the 985 project. As of March 2011, the 985 project included 39 universities, *e.g.*, Tsinghua University, Peking University, Xiamen University, Nanjing University, Fudan University, etc. In this paper, we denote these universities as Project 985 universities, with the exception of the National University of Defence Science and Technology. Thus, in this paper 38 Project 985 universities are analysed.

The Project 985 universities, as the main bearers of research activities, represent the highest level of university research in China. There is a need to rank those universities in terms of selected key outputs. In this case, there are obvious DMs who are the high education committee in MOE. The DMs in this case select the key outputs to reflect the policy of MOE and intend to develop

[1] Note: There are 22 disciplines in Essential Scientific Indicators (ESI). Among them, there are 9 disciplines which relate to medical science.

[2] http://incites.isiknowledge.com/Home.action (Note that this URL can be accessed only when having a subscription to InCites).

Table 2
Descriptive statistics of normalised data using formula (1) on 9 attributes.

Disciplines	BB	CM	Immu.	Mic.	MG	NB	PT	PA	PP
Median	0.2584	0.0416	0.2630	0.0486	0.0154	0.0451	0.0775	0.0397	0.0321
Min	0.0599	0.0073	0.0355	0.0058	0.0019	0.0061	0.0199	0.0052	0.0022
Max	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
(Min+Max)/2	0.5300	0.5037	0.5178	0.5029	0.5009	0.5030	0.5099	0.5026	0.5011

Table 3
Entropy weights for 9 ESI disciplines.

Discipline	BB	CM	Immu.	Mic.	MG	NB	PT	PA	PP
Entropy weights	0.0890	0.1151	0.1229	0.1166	0.1173	0.1182	0.1102	0.0995	0.1113

Table 4
The utilities and their gap.

Countries	Best utility θ_j^* from model (8)	Utility θ_j^p using Entropy weights in Table 2.	The gap $g_j = \left(\theta_j^* - \theta_j^p\right)$	Normalised gap g_j^*
Austria	0.7418	0.5439	0.1979	0.0419
Belgium	0.7318	0.5549	0.1769	0.0374
Bulgaria	0.1230	0.0533	0.0697	0.0148
Croatia	0.3937	0.2523	0.1414	0.0299
Cyprus	0.5440	0.3567	0.1873	0.0397
Czech Republic	0.5854	0.3716	0.2138	0.0453
Denmark	1.0000	0.6421	0.3579	0.0758
Estonia	0.6196	0.4621	0.1575	0.0333
Finland	0.7441	0.5462	0.1979	0.0419
France	0.5811	0.4483	0.1328	0.0281
Germany	0.6809	0.5063	0.1746	0.0370
Greece	0.4651	0.3376	0.1275	0.0270
Hungary	0.4291	0.3100	0.1191	0.0252
Ireland	0.6986	0.5335	0.1651	0.0350
Italy	0.5888	0.4543	0.1345	0.0285
Latvia	0.1412	0.0783	0.0629	0.0133
Lithuania	0.3477	0.1684	0.1793	0.0380
Luxembourg	0.8277	0.5736	0.2541	0.0538
Malta	0.4807	0.3226	0.1581	0.0335
The Netherlands	0.8213	0.5889	0.2324	0.0492
Poland	0.3522	0.2036	0.1486	0.0315
Portugal	0.6622	0.4976	0.1646	0.0349
Romania	0.1500	0.0597	0.0903	0.0191
Slovakia	0.3781	0.1773	0.2008	0.0425
Slovenia	0.6736	0.4671	0.2065	0.0437
Spain	0.6281	0.4871	0.1410	0.0299
Sweden	0.8995	0.6058	0.2937	0.0622
United Kingdom	1.0000	0.9628	0.0372	0.0079

democracy by allowing the universities to decide the common weights for the key outputs based on their best wishes. In such a case, it is appropriate to use the proposed method to determine the weights for the outputs because the method can find the best compromise weights based on minimax reference point optimisation and the initial weights set by the DMs.

In this paper, we use the indicators in “Science & Technology (S&T) statistics compilation in 2011”, which is published by the Ministry of Education (MOE) in China, to analyse the performance of the S&T activities of the Project 985 universities. The attributes used in this paper include monograph per capita, papers per capita, technology transfer income (TT INCOME) per capita and awards per capita. Among these attributes, monograph denotes the academic works of a more comprehensive specialised topic of a subject. Papers denote publications in important international and domestic journals in a statistical year. TT INCOME denotes the total income from the process of technology transfer in a university in the statistical year. Award refers to something given to a person or a group of people in the university to recognise their excellence in a certain field in the statistical year. We further use STAFF as the denominator which refers to the employees registered in the universities in the statistical year who are engaged in teaching, research and development, applications of research and development results, and scientific and technological services as well as those serving the above works, such as the foreign and domestic experts from outside the higher education system whose accumulated work time is more than one month.

Based on the above dataset, because the values of attributes differ greatly, we first normalise the data and transform them through value function using Eqs. (1)–(3) to avoid the distortion of the results. The original dataset and standard decision matrix after normalisation and transformation of value function is shown in Tables A-3 and A-4.

Firstly, we assume that these 4 attributes are all benefit attributes and normalise the dataset using formulae (1), (2) to form a normalised decision matrix and the corresponding matrix after the transformation via value functions. The following Table 7 shows that the value function in formula (2) in this case is also risk averse (see Fig. 1(c)), which is similar to our first illustrative example on EU-28 member countries.

Secondly the DMs use subjective way to set the preliminary weights for attributes as the average value, i.e., each of the four indicators are 0.25 (Table 8).

Thirdly, we use model (8) for each DMU to obtain its best utility to form the ideal point in the objective space. We can have normalised gap g_j^* as the weight for each university using Step 2 in Procedure 2, as shown in Table 9.

In this case study, we first assume that there is no other preference information in model (14), i.e., the set Φ is null. Therefore, using model (14), we can have the optimal weights for 4 attributes as shown in the second row of the following Table 10. Second, there is a need to incorporate the DMs' partial preference that Technology transfer should be no less important than Paper. Thus we need to incorporate this information $\Phi = \{u_2 \le u_3\}$ into model (14) and have another weight set for the attributes, as shown in the fourth row. After that, we use formula (15) to normalise those two weight sets for the final weight vectors of the four attributes, respectively, which are shown in the third and the fifth rows of the Table 10.

From Table 10 we can see that if we need to incorporate this information $\Phi = \{u_2 \le u_3\}$ into model (14), our three-stage hybrid

Table 5
Compromised weights for 9 ESI disciplines.

Discipline	BB	CM	Immu.	Mic.	MG	NB	PT	PA	PP
Compromised weights	0.4141	0.0460	0.0460	0.0460	0.0460	0.0460	0.2637	0.0460	0.0460

Table 6
The final utilities and rankings of EU-28 member countries.

Countries	Utility	Ranking	Utility θ_j^p using Entropy weights in Table 3.	Normalised gap g_j^*
United Kingdom	0.8400	1	0.9628	0.0079
Denmark	0.7625	2	0.6421	0.0758
Sweden	0.6564	3	0.6058	0.0622
Luxembourg	0.6082	4	0.5736	0.0538
The Netherlands	0.5983	5	0.5889	0.0622
Austria	0.5831	6	0.5439	0.0419
Finland	0.5831	7	0.5462	0.0419
Belgium	0.5801	8	0.5549	0.0374
Ireland	0.5533	9	0.5335	0.0350
Slovenia	0.5395	10	0.4671	0.0437
Portugal	0.5295	11	0.4976	0.0349
Germany	0.5275	12	0.5063	0.0370
Estonia	0.5016	13	0.4621	0.0333
Spain	0.4880	14	0.4871	0.0299
Italy	0.4806	15	0.4543	0.0285
France	0.4505	16	0.4483	0.0281
Czech Republic	0.4462	17	0.3716	0.0453
Cyprus	0.3707	18	0.3567	0.0397
Greece	0.3631	19	0.3376	0.0270
Hungary	0.3578	20	0.3100	0.0252
Croatia	0.3032	21	0.2523	0.0299
Malta	0.2863	22	0.3226	0.0335
Slovakia	0.2619	23	0.1773	0.0425
Poland	0.2585	24	0.2036	0.0315
Lithuania	0.2308	25	0.1684	0.0380
Romania	0.1190	26	0.0597	0.0191
Bulgaria	0.0907	27	0.0533	0.0148
Latvia	0.0789	28	0.0783	0.0133

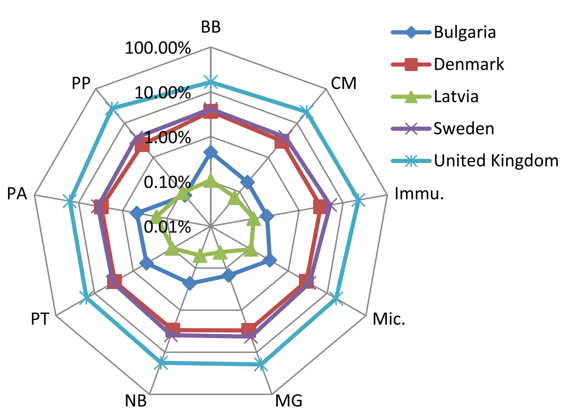

Fig. 3. Illustration of the publication proportions of five selected countries.

Table 7
Descriptive statistics of normalised data using formula (1) on 4 attributes.

Attributes	**Monograph per capita (Number)**	**Paper per capita (Number)**	**Technology transfer income per capita (RMB in thousands)**	**Award per capita (Number)**
Median	0.1166	0.5820	0.0059	0.4719
Min	0.0000	0.2304	0.0000	0.0000
Max	1.0000	1.0000	1.0000	1.0000
(Min + Max)/2	0.5000	0.6152	0.5000	0.5000

Table 8
Preliminary subjective weights for 4 attributes.

	Monograph per capita (Number)	Paper per capita (Number)	Technology transfer income per capita (RMB in thousands)	Award per capita (Number)
Weights	0.25	0.25	0.25	0.25

Table 9
Ideal point in the objective space.

Universities	Best utility θ_j^* from model (8)	Utility θ_j^p using average weights in Table 6.	The gap $g_j = \left(\theta_j^* - \theta_j^p\right)$	Normalised gap g_j^*
DMU_1	0.5380	0.3480	0.1900	0.0174
DMU_2	0.9697	0.4021	0.5677	0.0519
DMU_3	1.0000	0.8354	0.1646	0.0151
DMU_4	0.7577	0.4488	0.3089	0.0283
DMU_5	0.9034	0.5791	0.3244	0.0297
DMU_6	0.9942	0.7030	0.2912	0.0266
DMU_7	0.7994	0.3937	0.4057	0.0371
DMU_8	1.0000	0.3975	0.6025	0.0551
DMU_9	0.7552	0.4707	0.2844	0.0260
DMU_{10}	0.7980	0.4312	0.3669	0.0336
DMU_{11}	0.8491	0.6107	0.2384	0.0218
DMU_{12}	0.8369	0.5038	0.3332	0.0305
DMU_{13}	0.5179	0.2857	0.2322	0.0212
DMU_{14}	0.9902	0.6594	0.3308	0.0303
DMU_{15}	0.7030	0.4410	0.2620	0.0240
DMU_{16}	0.5559	0.4085	0.1474	0.0135
DMU_{17}	0.7792	0.4836	0.2957	0.0271
DMU_{18}	0.7249	0.4673	0.2576	0.0236
DMU_{19}	0.9869	0.6587	0.3281	0.0300
DMU_{20}	0.7900	0.5708	0.2192	0.0201
DMU_{21}	0.6882	0.5153	0.1729	0.0158
DMU_{22}	0.7835	0.3345	0.4491	0.0411
DMU_{23}	1.0000	0.6203	0.3798	0.0347
DMU_{24}	0.7247	0.4976	0.2271	0.0208
DMU_{25}	0.5604	0.2395	0.3209	0.0294
DMU_{26}	0.6726	0.4250	0.2476	0.0227
DMU_{27}	0.7810	0.5388	0.2422	0.0222
DMU_{28}	0.7098	0.4788	0.2310	0.0211
DMU_{29}	0.6549	0.4553	0.1996	0.0183
DMU_{30}	0.5750	0.2626	0.3124	0.0286
DMU_{31}	0.8696	0.6281	0.2415	0.0221
DMU_{32}	0.8670	0.5909	0.2761	0.0253
DMU_{33}	0.6402	0.4410	0.1992	0.0182
DMU_{34}	0.8454	0.5684	0.2770	0.0253
DMU_{35}	0.5279	0.3847	0.1432	0.0131
DMU_{36}	1.0000	0.6362	0.3639	0.0333
DMU_{37}	0.5404	0.3705	0.1699	0.0155
DMU_{38}	0.7300	0.4050	0.3250	0.0297

approach assigns more weights on Monograph per capita, Award per capita and Technology transfer income per capita (RMB in thousands), and less weight on Paper per capita, which means that a university which has better performance on paper than other three indicators will have its utility decreased. Based on the above normalised weights in Table 10 (Row 3 and Row 5), we can have the results of performance assessment of those universities as follows (See Table 11):

The following Fig. 4 shows the changes of rankings of those 38 universities using model (14) with Φ = null and with $\Phi = \{u_2 \le u_3\}$ respectively.

Table 10
Compromised weights for 4 attributes.

Weights	Monograph per capita (Number)	Paper per capita (Number)	Technology transfer income per capita (RMB in thousands)	Award per capita (Number)
Weights (Φ=null)	0.5809	0.5809	0.0645	0.0645
Normalised weights (Φ=null)	0.4500	0.4500	0.0500	0.0500
Weights ($\Phi=\{u_2 \le u_3\}$)	0.9385	0.1043	0.1043	0.2941
Normalised weights ($\Phi=\{u_2 \le u_3\}$)	0.6512	0.0724	0.0724	0.2041

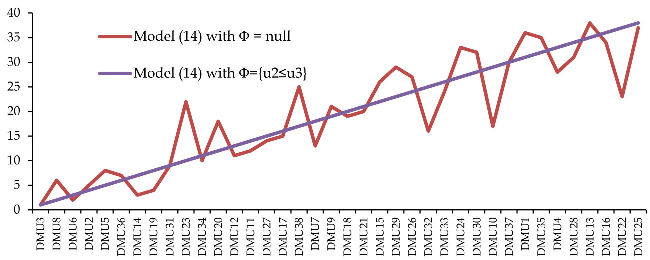

Fig. 4. The comparison of ranks of those 38 universities.

Table 11
The final utilities and rankings of 38 universities.

Universities	Using normalised weights from model (14) with Φ=null and formula (15)		Using normalised weights from model (14) with $\Phi=\{u_2 \le u_3\}$ and formula (15)	
	Utilities	Rank	Utilities	Rank
DMU_3	0.7747	1	0.6940	1
DMU_8	0.7155	6	0.6939	2
DMU_6	0.7500	2	0.6197	3
DMU_2	0.7237	5	0.6182	4
DMU_5	0.6923	8	0.6004	5
DMU_{36}	0.6996	7	0.5991	6
DMU_{14}	0.7322	3	0.5657	7
DMU_{19}	0.7305	4	0.5628	8
DMU_{31}	0.6702	9	0.5479	9
DMU_{23}	0.5094	22	0.5413	10
DMU_{34}	0.6549	10	0.5342	11
DMU_{20}	0.5528	18	0.5331	12
DMU_{12}	0.6484	11	0.5277	13
DMU_{11}	0.6371	12	0.5226	14
DMU_{27}	0.6050	14	0.4999	15
DMU_{17}	0.6034	15	0.4997	16
DMU_{38}	0.4820	25	0.4990	17
DMU_7	0.6193	13	0.4945	18
DMU_9	0.5148	21	0.4820	19
DMU_{18}	0.5477	19	0.4791	20
DMU_{21}	0.5154	20	0.4776	21
DMU_{15}	0.4772	26	0.4628	22
DMU_{29}	0.4145	29	0.4544	23
DMU_{26}	0.4482	27	0.4543	24
DMU_{32}	0.5993	16	0.4474	25
DMU_{33}	0.4842	24	0.4255	26
DMU_{24}	0.3645	33	0.4193	27
DMU_{30}	0.3738	32	0.3990	28
DMU_{10}	0.5604	17	0.3865	29
DMU_{37}	0.3924	30	0.3750	30
DMU_1	0.3101	36	0.3492	31
DMU_{35}	0.3313	35	0.3368	32
DMU_4	0.4370	28	0.3282	33
DMU_{28}	0.3738	31	0.3149	34
DMU_{13}	0.2721	38	0.3073	35
DMU_{16}	0.3360	34	0.2922	36
DMU_{22}	0.4892	23	0.2265	37
DMU_{25}	0.3010	37	0.1101	38

In Fig. 4 we can see that the ranking of some universities changes in the two cases Φ=null and $\Phi=\{u_2 \le u_3\}$. As we discussed before, our three-stage hybrid approach for weight assignment lead to the fact that a university which has better relative performance on paper than other three indicators will decrease its utility. For example, DMU_{14} performs the best on Paper per capita than its performance on other indicators and its ranking drops from the third to the seventh when the $\Phi=\{u_2 \le u_3\}$ and Φ=null are taken into account, respectively. This means that the comparative strength of DMU14 is mainly on publication rather than other three indicators. Similarly we can analyze the rank changes of other universities. See Tables A-3 and A-4 for detailed data for those 38 universities.

5. Conclusions and discussions

This paper investigated a new three-stage approach which uses linear programming (LP) model and minimax reference point optimisation to determine attribute weights. In the first stage, this approach considers preliminary weights for attributes. In the second stage, the most favourite set of weights for each alternative or DMU is found through the DEA-WEI-like LP model. In the third stage, the weight sets are aggregated to find the best compromise weights for attributes with the interests of all DMUs taken into account fairly and simultaneously. This three-stage approach is intended to support the analysis of such MADM problems as performance assessment and policy analysis where (a) the preferences of DMs are either unclear and partial or difficult to acquire and (b) there is a need to consider the best "will" of each DMU. Based on the empirical results, we find that the proposed approach has at least three advantages in the sense that (1) it can assign weights to attributes in a fair and unbiased manner (2) it is a data-driven approach and needs no clear information on the preferences of DMs as prerequisite, while it also has the flexibility to incorporate DMs' prior information if necessary, and (3) it can reflect the best "will" of each DMU to ensure the most favourable evaluation for the DMU through the DEA-WEI-like LP model.

Acknowledgement

We would like to acknowledge the supports from National Natural Science Foundation of China (NSFC No. 71201158, 71671181) and China Scholarship Council (CSC No. 201304910099). The second author is grateful for the support from the US Air Force Office of Scientific Research under Grant no.: FA2386-15-1-5004. The authors thank the editors of Omega, The International Journal of Management Science, and the anonymous referees for valuable comments which helped improve the quality of the paper.

Appendix

See Tables A-1 to A-4 here.

Table A1
Original data of publications in 9 ESI disciplines and population of EU-28 countries in 2015.

EU-28 Countries	Population (thousand)	BB	CM	Imm-u.	Mic.	MG	NB	PT	PA	PP
Austria	8576.261	922	5364	379	237	618	809	457	1014	500
Belgium	11258.434	1089	6554	613	423	802	1225	780	1433	993
Bulgaria	7202.198	161	375	34	34	39	83	87	165	20
Croatia	4225.316	191	839	46	32	89	123	99	230	89
Cyprus	847.008	50	230	7	8	39	52	14	37	47
Czech Republic	10538.275	789	2125	174	261	364	386	308	1222	148
Denmark	5659.715	1320	5898	561	298	795	1073	579	1053	609
Estonia	1313.271	80	222	30	38	97	61	61	185	58
Finland	5471.753	590	2801	239	199	505	596	262	757	468
France	66415.161	3722	18332	2247	1418	3039	3569	2059	3479	1487
Germany	81197.537	6835	30571	2637	1671	4711	6887	2774	4937	4315
Greece	10858.018	477	3912	236	115	287	427	345	454	221
Hungary	9855.571	443	1620	165	121	243	381	315	556	214
Ireland	4628.949	387	3795	174	118	376	437	225	376	356
Italy	60795.612	3437	23025	1781	765	2679	4625	2464	3246	1912
Latvia	1986.096	38	134	17	11	11	18	19	59	24
Lithuania	2921.262	139	354	42	32	30	36	33	108	34
Luxembourg	562.958	68	158	47	15	47	48	31	59	50
Malta	429.344	6	129	5	2	9	16	52	28	17
The Netherlands	16900.726	1724	14230	1440	619	1768	2826	1166	1529	2711
Poland	38005.614	1519	5029	402	347	599	765	839	2136	474
Portugal	10374.822	773	3095	204	266	551	622	472	960	599
Romania	19870.647	525	1463	70	50	235	152	294	184	172
Slovakia	5421.349	270	470	30	90	78	87	87	224	53
Slovenia	2062.874	194	605	66	55	72	95	117	188	43
Spain	46449.565	2769	15134	1415	846	1881	2790	1523	3502	1914
Sweden	9747.355	1489	7763	924	357	1119	1422	644	1261	978
United Kingdom	64875.165	5985	42548	4056	1749	5099	6434	3099	5489	6902

Data source: (1) Thomson Reuters' research evaluation tool InCites[a], and (2) http://europa.eu/about-eu/countries/member-countries/.

Table A2
Data of EU-28 countries in 2015 after normalization and transformation using value function (2).

EU-28 Countries	BB	CM	Immu.	Mic.	MG	NB	PT	PA	PP
Austria	0.6583	0.5277	0.6374	0.5110	0.5056	0.5185	0.5275	0.5166	0.5084
Belgium	0.6291	0.5245	0.7051	0.5230	0.5055	0.5245	0.5452	0.5193	0.5207
Bulgaria	0.1450	0.0073	0.0612	0.0652	0.0019	0.0452	0.0642	0.1164	0.0022
Croatia	0.3770	0.3057	0.1930	0.1421	0.1996	0.2229	0.2644	0.3733	0.2447
Cyprus	0.5174	0.4543	0.1370	0.1925	0.5011	0.5049	0.1427	0.2857	0.5073
Czech Republic	0.5697	0.3119	0.3133	0.5075	0.3699	0.2988	0.3667	0.5160	0.1516
Denmark	1.0000	0.5580	1.0000	0.5413	0.5175	0.5578	0.5816	0.5365	0.5287
Estonia	0.5317	0.2455	0.4488	0.5126	0.5060	0.3980	0.5200	0.5232	0.5026
Finland	0.6592	0.5194	0.6340	0.5216	0.5092	0.5245	0.5216	0.5225	0.5196
France	0.4871	0.4634	0.5690	0.5034	0.5011	0.4716	0.3980	0.3566	0.2623
Germany	0.5949	0.5095	0.5601	0.4925	0.5032	0.5146	0.4538	0.4252	0.5063
Greece	0.3641	0.5084	0.4251	0.2234	0.2674	0.3261	0.4116	0.2705	0.2353
Hungary	0.3744	0.2360	0.3183	0.2688	0.2450	0.3193	0.4149	0.3894	0.2533
Ireland	0.5934	0.5418	0.5938	0.5084	0.5072	0.5185	0.5224	0.5058	0.5161
Italy	0.4921	0.5097	0.5391	0.2771	0.4902	0.5110	0.5134	0.3648	0.3824
Latvia	0.1123	0.0387	0.1433	0.0873	0.0034	0.0203	0.0199	0.1718	0.1256
Lithuania	0.4012	0.1481	0.2677	0.2332	0.0632	0.0533	0.0504	0.2310	0.1197
Luxembourg	0.6944	0.4728	0.8968	0.5098	0.5076	0.5148	0.5295	0.5127	0.5210
Malta	0.0599	0.5040	0.2093	0.0635	0.1983	0.3052	0.6023	0.4612	0.4904
The Netherlands	0.6434	0.5435	0.9081	0.5219	0.5113	0.5485	0.5448	0.5085	0.5503
Poland	0.3239	0.1708	0.1864	0.1840	0.1326	0.1321	0.2405	0.3877	0.1308
Portugal	0.5687	0.5038	0.3808	0.5086	0.5024	0.5043	0.5189	0.5091	0.5082
Romania	0.1863	0.0512	0.0355	0.0058	0.0829	0.0061	0.1121	0.0052	0.0802
Slovakia	0.4237	0.0778	0.0785	0.3853	0.1152	0.0909	0.1342	0.2664	0.0950
Slovenia	0.6218	0.4985	0.5569	0.5098	0.3745	0.3940	0.5313	0.5087	0.2418
Spain	0.5233	0.5058	0.5468	0.4288	0.4451	0.5044	0.4295	0.5041	0.5014
Sweden	0.7813	0.5401	0.9714	0.5219	0.5131	0.5398	0.5416	0.5199	0.5257
United Kingdom	0.6169	1.0000	0.9750	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000

http://incites.isiknowledge.com/Home.action (Note that this URL can be accessed only when having a subscription to InCites).

Table A3
Original data of 38 Chinese universities.

Universities	Research STAFFs (Number)	Monograph (Number)	Paper (Number)	TT income (RMB in thousands)	Award (Number)
DMU1	12284	46	6751	54900	62
DMU2	151	5	259	0	0
DMU3	5071	62	11114	306567	86
DMU4	1743	3	2792	212	27
DMU5	2240	24	4572	180	35
DMU6	1854	15	4344	891	30
DMU7	1364	18	2352	0	6
DMU8	231	10	308	0	0
DMU9	1777	16	2021	28662	7
DMU10	2707	9	5448	0	29
DMU11	2408	11	4714	6135	31
DMU12	2401	22	4578	309	22
DMU13	9196	25	5811	547	95
DMU14	3912	20	9332	1370	51
DMU15	7027	47	7245	35053	35
DMU16	5257	10	5375	5471	51
DMU17	12465	108	20669	3032	80
DMU18	1097	9	1473	680	5
DMU19	1753	9	4163	3845	22
DMU20	3761	27	4841	98141	35
DMU21	12709	58	16638	40672	127
DMU22	1739	3	3630	60	8
DMU23	1246	5	1596	3092	26
DMU24	5777	19	4515	120120	77
DMU25	1804	0	2587	20	11
DMU26	7802	41	7263	2700	47
DMU27	7941	39	13937	2480	74
DMU28	1934	3	2375	13890	26
DMU29	8988	40	7633	3250	91
DMU30	10131	66	6749	555	34
DMU31	2840	16	5699	36047	31
DMU32	2670	9	5462	10519	34
DMU33	9186	42	11195	6502	49
DMU34	1659	11	3237	475	17
DMU35	5420	17	4101	38453	37
DMU36	2006	10	4459	190	41
DMU37	3044	12	2834	800	19
DMU38	1689	11	1860	0	21

Table A4
Data of 38 Chinese universities after normalization and transformation using value function (2).

Universities	**Attributes**			
	Monograph per capita (Number)	Paper per capita (Number)	Technology transfer income per capita (RMB in thousands)	Award per capita (Number)
DMU1	0.3709	0.2304	0.5342	0.2563
DMU2	0.8669	0.7413	0.0000	0.0000
DMU3	0.5938	0.9252	1.0000	0.8227
DMU4	0.1705	0.6976	0.1709	0.7561
DMU5	0.5741	0.8671	0.1129	0.7622
DMU6	0.5398	0.9836	0.5010	0.7874
DMU7	0.6065	0.7448	0.0000	0.2234
DMU8	1.0000	0.5899	0.0000	0.0000
DMU9	0.5517	0.5000	0.6312	0.2000
DMU10	0.3293	0.8561	0.0000	0.5393
DMU11	0.4524	0.8349	0.5182	0.6373
DMU12	0.5538	0.8152	0.1808	0.4653
DMU13	0.2692	0.2682	0.0836	0.5219
DMU14	0.5008	1.0000	0.4920	0.6447
DMU15	0.5214	0.4512	0.5385	0.2529
DMU16	0.1884	0.4473	0.5057	0.4926
DMU17	0.5473	0.7193	0.3417	0.3259
DMU18	0.5413	0.5943	0.5022	0.2314
DMU19	0.5011	0.9959	0.5153	0.6226
DMU20	0.5279	0.5687	0.7141	0.4725

Table A4 (*continued*)

Universities	**Attributes**			
	Monograph per capita (Number)	Paper per capita (Number)	Technology transfer income per capita (RMB in thousands)	Award per capita (Number)
DMU21	0.4520	0.5788	0.5237	0.5066
DMU22	0.1708	0.8850	0.0485	0.2336
DMU23	0.3974	0.5659	0.5177	1.0000
DMU24	0.3257	0.3368	0.6700	0.6580
DMU25	0.0000	0.6328	0.0156	0.3096
DMU26	0.5027	0.4053	0.4862	0.3059
DMU27	0.4864	0.7567	0.4388	0.4732
DMU28	0.1536	0.5416	0.5568	0.6632
DMU29	0.4407	0.3679	0.5000	0.5126
DMU30	0.5192	0.2839	0.0770	0.1704
DMU31	0.5076	0.8538	0.6026	0.5485
DMU32	0.3338	0.8689	0.5298	0.6310
DMU33	0.4528	0.5373	0.5029	0.2709
DMU34	0.5207	0.8324	0.4022	0.5181
DMU35	0.3106	0.3254	0.5561	0.3466
DMU36	0.4937	0.9372	0.1331	0.9806
DMU37	0.3904	0.4054	0.3692	0.3169
DMU38	0.5191	0.4835	0.0000	0.6173

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