Decision Support

Integrating DEA-oriented performance assessment and target setting using interactive MOLP methods

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Abstract

Data envelopment analysis (DEA) and multiple objective linear programming (MOLP) are tools that can be used in management control and planning. Whilst these two types of model are similar in structure, DEA is directed to assessing past performances as part of management control function and MOLP to planning future performance targets. This paper is devoted to investigating equivalence models and interactive tradeoff analysis procedures in MOLP, such that DEA-oriented performance assessment and target setting can be integrated in a way that the decision makers’ preferences can be taken into account in an interactive fashion. Three equivalence models are investigated between the output-oriented dual DEA model and the minimax reference point formulations, namely the super-ideal point model, the ideal point model and the shortest distance model. These models can be used to support efficiency analysis in the same way as the conventional DEA model does and also support tradeoff analysis for setting target values by individuals or groups. A case study is conducted to illustrate how DEA-oriented efficiency analysis can be conducted using the MOLP methods and how such performance assessment can be integrated into an interactive procedure for setting realistic target values.

1. Introduction

DEA and MOLP can be used as tools in management control and planning. The structures of these two types of model have much in common but DEA is directed to assessing past performances as part of management control function and MOLP to planning future performances (Cooper, 2004). DEA has evolved tremendously over the years and emerged as a body of concepts and methodologies, which consist of a collection of models and extensions to the original work of Charnes et al. (1978). Its popularity is reflected by a large number of successful applications.

As a performance measurement and analysis technique, DEA is a non-parametric frontier estimation methodology based on linear programming for evaluating the relative efficiency of a set of comparable decision making units (DMUs) that share common functional goals. The usefulness of DEA extends to assessment and benchmarking against efficient units, target setting and resource allocation between inputs and outputs. The concept of DEA was based on the generalisation of the framework of Farrell’s (1957) single output over input measurement of productive efficiency to include
multiple, incommensurate inputs and outputs that are considered at the same time to evaluate the efficiency of comparable DMUs.

In a broader picture, there have been various studies highlighting the similarities between DEA and multiple criteria decision analysis (MCDA) generally and MOLP in particular, though it is said that they retain their own distinctive traits (Belton and Stewart, 2001; Agrell and Tind, 2001; Joro et al., 1998; Stewart, 1994, 1996). Taking a step further, Doyle and Green (1993) suggested that DEA is an MCDA method itself. Belton and Vickers (1993) and Stewart (1996) described the equivalence between the formulations of the basic DEA models and the classic linear multi-attribute value function of MCDA. More specifically, Belton and Stewart (2001) pointed out that the mechanism of DEA involves comparison of DMUs on the basis of multiple criteria of both inputs and outputs, but the emphasis of DEA is put on evaluating DMUs against the best practice units and on setting targets to improve efficiency, whilst MCDA focuses on ranking and assessing alternatives. Likewise, Stewart (1996) argued that the fundamental connection between the two schools of thought is the objective function that is either for maximising outputs or for minimising inputs. While DEA permits determining the productivity efficiency frontier of a DMU by optimising weighted outputs over weighted inputs, MCDA models allow assessing and ranking alternatives based on a set of criteria that may be conflicting and involve subjective judgments. Arguably, DEA could be seen as a ‘lazy’ DMs methodology to MCDA and Sarkis (2000) termed it as a reactive approach to MCDA where different alternatives are evaluated objectively.

The original DEA models do not include the DMs preference structures or value judgments in measuring relative efficiency and setting target values, so there is minimal input from the DM. Allen et al. (1997) defined value judgments as “logical constructs, incorporated within an efficiency assessment study, reflecting the DMs’ preferences in the process of assessing efficiency.” To incorporate the DMs preference information in DEA, various techniques have been proposed such as the goal and target setting models of Golany (1988), Thansassoulis and Dyson (1992) and Athanassopoulos (1995, 1998), and weight restriction models including imposing bounds on individual weights (Dyson and Thansassoulis, 1988), assurance region (Thompson et al., 1990), restricting composite inputs and outputs, weight ratios and proportions (Wong and Beasley, 1990), and the cone ratio concept by adjusting the observed input–output levels or weights to capture value judgments to belong to a given closed cone (Charnes et al., 1990, 1994). Alternative approaches include Thansassoulis and Simpson’s model (2000) which adopts unobserved DMUs, derived from observed Pareto-optimal DMUs. Zhu (1996) also integrated preference information into a modified DEA formulation, while Golany (1995) used hypothetical DMUs to represent preference information. However, the above-mentioned techniques require prior preferences from the DM, which are often subjective. Nevertheless, preferences required for setting future target values may not be provided a priori but need to be generated on the basis of what are realistically achievable.

An appealing method to incorporate preference information into both efficiency analysis and target setting, without necessarily requiring prior judgments, is the use of interactive MOLP techniques. Golany (1988) first proposed an interactive model combining both DEA and MOLP approaches where the DM is assumed to be able to allocate a set of input levels as resources and to select the most preferred set of output levels from a set of alternative points on the efficient frontier. Post and Spronk (1999) also proposed combining the use of DEA and interactive multiple goal programming where preference information is incorporated interactively by the DM by adjusting the upper and lower feasible boundaries of the input and output levels. Joro et al. (1998) showed the synergies between DEA and MOLP and proved that the DEA formulations are structurally similar to MOLP models based on the reference point approach. In summary, the effective integration of assessing past performances and planning future targets with the DMs preferences taken into account is of increasing interest to support both management control and planning (Cooper, 2004). This has indeed motivated the research as reported in this paper.

To facilitate performance assessment and target value setting in the domain of MOLP in an integrated way, three equivalence models in MOLP are investigated in this paper, including the super-ideal point model, the ideal point model and the shortest distance model. The super-ideal point model is proven identical to the output oriented DEA dual model and thus can be used to generate, in the context of MOLP, the same efficiency scores and corresponding composite inputs and outputs just as the output-oriented DEA dual model does. In other words, an efficient solution generated using the DEA model is in fact the one on the efficient frontier that is the closest to the super ideal point in the objective space expanded by the composite outputs, as investigated in detail later in this paper. On the other hand, the generic MOLP formulation, in which the super-ideal point model is built, provides a platform for exploring efficiency measures and efficient frontiers using the concepts and techniques in MOLP and also for design of solution schemas in which to conduct interactive tradeoff analysis for setting realistic target values for a DM within its existing production possibility set expanded by the original DMUs in question.

The second ideal point model is designed from the generic MOLP formulation, hence sharing the same decision and objective spaces with the super-ideal point model. It is used to construct an interactive tradeoff analysis procedure based on the gradient projection and local region search method (Li and Yang, 1996; Yang and Sen, 1996; Yang, 1999; Yang and Li, 2002) to locate a most preferred solution (MPS) that can maximize the DMs implicit utility function using the DMs local preference information. Such a MPS is then set as a target for the observed DMU to benchmark against. In this inter-
active process, the DM can explore what could be technically achieved and therefore gets in a better position to decide what should be planned as future targets. The gradient projection is conducted through the identification of normal vectors on the efficient frontier based on the ideal point model. A normal vector itself provides information about the optimal indifference tradeoff that the MPS must satisfy, so it can also be used as a criterion to terminate the interactive process. On the other hand, the projection of the gradient as an implicit utility function onto the tangent plane of the efficient frontier using the normal vector provides a direction closest to the efficient frontier, along which a better efficient solution can be sought.

The third shortest distance model is designed also from the generic MOLP formulation and can be used to facilitate group negotiation and discussion in deciding overall and localised performance targets with both individual and group preferences taken into account. It uses a group MPS (GMPS) as a reference point, which could be provided either by a leading DM with overall responsibility for the performance management of an organisation or generated by aggregating solutions locally preferred by individual DMUs. A case study for analysing the efficiencies of seven UK retail banks and setting their business targets is conducted to illustrate and validate the proposed models and procedure.

The remainder of the paper is organised as follows. Section 2 provides a brief description of the typical DEA models and basic multiple objective optimisation techniques, followed by an investigation into the incorporation of DMs preferences into management planning. Section 3 reports the investigation into the integrated performance assessment and target value setting. Section 4 shows a case study on bank performance analysis using the integrated approach. The paper is concluded in Section 5.

2. DEA-oriented performance assessment and target setting

DEA was initially developed by Charnes et al. (1978) for measuring and analyzing the relative efficiencies of comparable DMUs with incommensurate inputs and outputs. In DEA, an efficient frontier is formed, where efficient DMUs lie. An efficient DMU means that no other DMU can either produce the same outputs by consuming fewer inputs, known as the input-orientated approach, or produce more outputs by consuming the same inputs, known as output-orientated approach. The mechanism behind the methodology of the conventional DEA models is that it works on maintaining the appropriate input–output mix so as to project inefficient DMUs radially onto the efficient frontier. The DEA models can provide efficiency scores scaled to a maximum value of 1 for efficient DMUs and can inform the DM of the amount of percentage by which an inefficient DMU should decrease its inputs and/or increase its outputs in order to become efficient. It also provides reference units known as composite or virtual units which lie on the efficient frontier and are used as target units for inefficient DMUs to benchmark against.

2.1. Typical output-orientated DEA models for performance assessment

The original DEA model proposed by Charnes et al. (1978) is a fractional non-linear programming model, known as the CCR model. The objective function in the model is to maximise the single ratio of the weighted outputs over weighted inputs for a particular DMU, referred to as an observed DMU and denoted by DMU_r.

Suppose an organisation has n DMUs (j = 1, ..., n), produces s outputs denoted by y_{rj} (the rth output of DMU j for r = 1, ..., s) and consumes m inputs denoted by x_{ij} (the ith input of DMU j for i = 1, ..., m). The fractional formulation of DEA is then defined as follows:

\[
\text{Max } e_o = \frac{\sum_{r=1}^{s} u_r y_{r1}}{\sum_{i=1}^{m} v_i x_{i1}}
\]
\[\text{s.t. } \sum_{r=1}^{s} u_r y_{rj} / \sum_{i=1}^{m} v_i x_{ij} \leq 1, \quad \forall j = 1, \ldots, n; \quad u_r, v_i \geq 0 \quad \text{for } r = 1, \ldots, s; \quad i = 1, \ldots, m \]

u_r is the weight parameter for output r and v_i the weight parameter for input i. e_o denotes the optimal efficiency score with a possible range of 0 \leq e_o \leq 1. The score of e_o = 1 represents full efficiency and 0 < e_o < 1 reveals the presence of inefficiency. Each DMU can be evaluated by setting itself as the observed DMU_o and is allowed freedom in the DEA model to assign the set of its own output weights u_r and input weights v_i, which will render the observed DMU as efficient as possible. In other words, the efficiency measure e_o is maximised within the production possibility set formulated by the n DMUs (Cooper et al., 2000). It is noted that the fractional program model is a computationally complex non-linear and non-convex problem, making calculations extremely difficult for large scale problems (Charnes et al., 1978). As such, Charnes introduced the transformation of the fractional programming problem into equivalent linear programming problems. In this paper, we investigate the equivalence between the output-oriented DEA dual models and the minimax formulations in MOLP, so only the typical output-oriented DEA models are briefly discussed in this section.
The following LP models are the conventional CCR models for efficiency analysis

**Output-orientated CCR primal model.**

\[
\begin{align*}
\text{Min} & \quad h_0 = \sum_{j=1}^{m} v_j x_{ij} \\
\text{s.t.} & \quad \sum_{i=1}^{v} v_i x_{ij} - \sum_{r=1}^{s} u_r y_{rg} \geq 0 \quad j = 1, \ldots, n, \\
& \quad \sum_{r=1}^{s} u_r y_{rg} = 1, u_r, v_j \geq 0 \quad \text{for all } r, i.
\end{align*}
\]

**Output-Orientated CCR Dual Model**

\[
\begin{align*}
\text{Max} & \quad h_0 = \theta_{ho} \\
\text{s.t.} & \quad \theta_{ho} y_{r0} - \sum_{j=1}^{n} \lambda_j y_{rj} \leq 0 \quad r = 1, \ldots, s, \\
& \quad \sum_{j=1}^{n} \lambda_j x_{ij} \leq x_{ito}, \quad i = 1, \ldots, m; \quad \lambda_j \geq 0 \quad \text{for all } j.
\end{align*}
\]

In the output-orientated CCR primal model (2), the weighted outputs are fixed to unity and the weighted inputs minimized. The output weights \(u_r\) and input weights \(v_j\) are adjusted accordingly to generate an efficiency score which is given by \(e_o = 1/h_0\). In the output-orientated CCR dual model (3), for each observed DMU, an imaginary composite unit is constructed that outperforms DMU. \(\lambda_j\) represents the proportion to which DMU \(j\) is used to construct the composite unit for DMU \(o\) \((j = 1, \ldots, n)\). The composite unit consumes at most the same levels of inputs as DMU \(o\) and produces outputs that are at least equal to a proportion \(\theta_{ho}\) of the outputs of DMU \(o\) with \(\theta_{ho} \geq 1\). The inverse of \(\theta_{ho}\) is the efficiency score of DMU \(o\). If \(\theta_{ho} > 1\), DMU \(o\) is not efficient and the parameter \(\theta_{ho}\) indicates the extent by which DMU \(o\) has to increase its outputs to become efficient. The increase is employed concurrently to all outputs and results in a radial movement towards the envelopment surface (Charnes et al., 1994). Note that such a radial improvement strategy is imbedded in the DEA modelling mechanism a priori and does not necessarily take account of management preferences, so it is technical rather than preferential. In the following sections, we will explore how DMs preferences can be incorporated into improvement strategies using interactive multiple objective optimisation techniques. Note that the above models are based on constant returns to scale (CRS). This, however, disregards economies of scale. Variable returns to scale in efficiency analysis were taken into account in another version of DEA model developed by Banker et al. BCC (1984), called the BCC model which is different from the CCR model in that the former has an additional convexity constraint of all \(\lambda_j\) restricted to sum to 1 in the dual case.

### 2.2. Basic concepts and minimax formulations in multiple objective optimization

In a DEA model, an efficiency score is generated for a DMU by maximizing outputs with limited inputs, or minimizing inputs with desired or fixed outputs, or simultaneously maximizing outputs and minimizing inputs. Either way, this can be regarded as a kind of multiple objective optimization problem. In this section, we briefly describe basic concepts and models in multiple objective optimization, in particular the minimax formulations as a basis for the investigation to be reported in the following sections.

Suppose an optimization problem has \(s\) objectives reflecting the different purposes and desires of the DM. Such a problem can be represented in a general form as follows:

\[
\begin{align*}
\text{Max} & \quad f(\lambda) = [f_1(\lambda), \ldots, f_s(\lambda), \ldots, f_s(\lambda)] \\
\text{s.t.} & \quad \lambda \in \Omega = \{ \lambda \mid g_j(\lambda) \leq 0, \quad h_l(\lambda) = 0; \quad f = 1, \ldots, k_1, \quad l = 1, \ldots, k_2\},
\end{align*}
\]

where \(\Omega\) is the feasible decision space, \(f_r(\lambda)(r = 1, \ldots, s)\) are continuously differentiable objective functions, and \(g_j(\lambda)(j = 1, \ldots, k_1)\) and \(h_l(\lambda)(l = 1, \ldots, k_2)\) are continuously differentiable inequality and equality constraint functions, respectively. In this paper, \(f_1(\lambda), g_j(\lambda)\) and \(h_l(\lambda)\) are all assumed to be linear functions of \(\lambda\), so formulation (4) is referred to as multiple objective linear programming or MOLP in short.

Due to conflict among objectives in general, a MOLP problem does not normally have a single solution that could optimize all objectives simultaneously. What can be generated are efficient or non-dominated solutions. Conceptually, a feasible solution \(\lambda^*\) is said to be efficient or non-dominated if there exists no other feasible solution which is better than \(\lambda^*\) at least on one objective and as good as \(\lambda^*\) on all other objectives. An efficient or non-dominated solution is also referred to as
a Pareto-optimal solution, which can be formally defined as follows. For simplicity, we use the term “efficient solution” in this paper.

**Definition 1.** In formulation (4), a feasible solution \( \lambda^* \in \Omega \) is an efficient solution if and only if there does not exist any other feasible solution \( \lambda \in \Omega \) such that \( f_r(\lambda) \geq f_r(\lambda^*) \) for all \( r = 1, \ldots, s \) and \( f_\tau(\lambda) > f_\tau(\lambda^*) \) for at least one \( \tau \in \{1, \ldots, s\} \).

Any efficient solutions of a MOLP problem can be generated using the weighted minimax formulation (Steuer and Choo, 1983; Yang, 1999; Yang and Li, 2002). Suppose \( \lambda \) is an efficient solution of (4) and \( f^*_r \) the maximum feasible value of objective \( r \). There exists a weighting vector \( w \) satisfying \( w_1 = 1 \) and \( w_r > 0 \) for \( r = 2, \ldots, s \) such that \( \lambda \) can be generated by solving the following weighted minimax problem

\[
\begin{align*}
\text{Min} & \quad \text{Max}_{s \in \{1, \ldots, s\}} \{w_r | (f^*_r - f_r(\lambda))| \} \\
\text{s.t.} & \quad \lambda \in \Omega. 
\end{align*}
\]

The weighted minimax formulation will be called the ideal point model, a special case of the reference point formulation, if \( f^* = [f^*_1 \cdots f^*_s]^{T} \) is the ideal point and used as the reference point. In the minimax formulation, for a given weight vector the DM is assumed to be satisfied with an efficient solution \( \lambda \in \Omega \) at which \( f(\lambda) \) has the shortest weighted distance from \( f^* \) measured in \( \infty \)-norm in the objective space.

If \( f^* \) is an ideal point, then the non-smooth weighted minimax formulation given in (5) can be equivalently transformed into the following smooth form by introducing an auxiliary variable \( \theta \) (Lightner and Director, 1981; Yang and Li, 2002)

\[
\begin{align*}
\text{Min} & \quad \theta \\
\text{s.t.} & \quad w_r(f^*_r - f_r(\lambda)) - \theta \leq 0, \quad r = 1, \ldots, s; \quad \lambda \in \Omega. 
\end{align*}
\]

Note that the above minimax formulation (6) is still equivalent to formulation (5) if \( f^* \) is replaced by a better reference (super-ideal) point \( f^+ \) with \( f^+ \geq f^* \). It will be shown that the minimax formulation using a particular super-ideal point as the reference point can be used to generate DEA scores and corresponding composite inputs and outputs in the same way as the output-oriented dual DEA model does, and also to design an interactive procedure to support the DM to search for the MPS on the efficient frontier by systematically changing the weighting parameters \( w_r(r = 1, \ldots, s) \).

### 2.3. Incorporation of DMs’ preferences into target setting

DEA models can be used to measure and assess how efficiently an organisation utilises its resources (inputs) to generate desirable outcomes (outputs) in its business activities in comparison with its peers (DMUs). In this sense, DEA is a management control tool and can be used to identify whether a business is run efficiently and where it currently stands in the market. If a DMU is found to be inefficient, DEA can provide suggestions as to where and by how much it should be improved in order to achieve full efficiency in comparison to its peers. As such, DEA provides certain degrees of support to both management control and planning. However, in supporting management planning, conventional DEA does not take appropriate account of the DMs’ preferences. This is explained in some detail as follows.

In Fig. 1, there are five DMUs: A, B, C, D and E with the first four DMUs being fully efficient and E inefficient which is the observed DMU. AB, BC and CD constitute the efficient frontier. In DEA, the efficiency score of the observed DMU E is measured by \( \frac{OE}{OE_1} \) with \( E_1 \) being the intersection point of the efficient segment AB and a line emitting from the origin O through the point E. Since E is inefficient in comparison with A, B, C and D, DEA technically suggests the radial improvement of DMU E from point E to point \( E_1 \) along the line. Whilst this is a valid suggestion, there are many other alternatives.

![Fig. 1. Illustration of efficiency analysis and tradeoffs.](image-url)
In fact, in Fig. 1 any efficient solution on the segments $AB$ and $BC_1$ of the efficient frontier dominates $DMU E$ and could potentially be used as its target, or its $MPS$. In general, if tradeoffs between outputs are allowed in management planning, other efficient solutions could also be candidates as the $MPS$ for $DMU E$, depending on the $DMs$ preferences. Even if a $DMU$ is already technically efficient, the $DMs$ may prefer to achieve a better balance between outputs than its current output levels without necessarily consuming more resources, in other words to find a new $MPS$ along the efficient frontier as future targets. Although there are existing $DEA$ models that support target setting by allowing restrictions on input weights and output weights or by imposing input and output targets, they require prior knowledge about what are achievable, which, however, depends on the selection of reference $DMUs$ and their performances. Thus, such knowledge in general is not available a priori but needs to be explored.

The above discussions show that a few questions remain to be answered after the performance of each $DMU$ is assessed by generating a $DEA$ score and a technical target such as $E_1$ for $DMU E$ in Fig. 1. The first question is how to find different realistic targets from $E_1$ that $DMU E$ could benchmark against based on its current performances and within the existing production possibility set. The second question is how to support the $DMs$ to identify a new target that is not only technically achievable but also most preferred by the $DMs$.

Multiobjective optimization is a tool for management planning and can be used to support the search for efficient solutions and the location of the $MPS$. To answer these questions, a super-ideal point model will be first investigated in next section which is proven identical to the output-orientated $CCR$ dual model. The super-ideal point model can thus be used for conventional efficiency analysis. Furthermore, it leads to the identification of a generic $MOLP$ formulation, which defines the production possibility set of the observed $DMU$ and in which solution schemas can be developed to explore all efficient solutions of a $DMU$.

An interactive tradeoff analysis procedure based on the ideal point model and the gradient projection method will then be investigated to support the search for the $MPS$ with the $DMs$ preferences taken into account. In this procedure, the implicit utility function of the $DM$ can be maximised by using local tradeoff information. A group procedure is also suggested on the basis of the shortest distance model to support the negotiations and discussions in setting targets with both organizational and individuals preferences taken into account.

### 3. Interactive MOLP methods for integrating efficiency analysis and target setting

In this section, we first investigate the equivalence between the output-oriented $DEA$ dual model (3) and the minimax reference point formulations. A super-ideal point model will first be investigated for conducting efficiency analysis in the same way as the $DEA$ output-oriented dual model does. An equivalent ideal point model will then be investigated to design an interactive tradeoff analysis procedure for locating the $MPS$ by systematically adjusting weights. Finally, a group decision making process is proposed to support the determination and mapping of group most preferred solution ($GMPS$), which is used as a new reference point to construct the shortest distance model to identify locally most preferred solutions ($LMPS$s) or to set or update target values for individual $DMUs$.

#### 3.1. Conducting DEA-oriented performance assessment using a MOLP method

The output-orientated $CCR$ dual model, as shown in formulation (3), can be equivalently re-written as follows:

\[
\begin{align*}
\text{Max} \quad & \theta_c \\
\text{s.t.} \quad & \theta_j y_{rj} - \sum_{j=1}^{n} \lambda_j y_{rj} \leqslant 0, \quad r = 1, \ldots, s, \\
& \lambda \in \Omega_{y_{rj}} = \left\{ \lambda : \sum_{j=1}^{n} \lambda_j x_{ij} \leqslant x_{ij}, \quad i = 1, \ldots, m; \quad \lambda_j \geqslant 0, \quad j = 1, \ldots, n \right\}. \\
\end{align*}
\]

In this subsection, we first show that formulation (7) is the same as formulation (6) under certain conditions. The purpose for establishing this equivalence is to use formulation (6) to conduct efficiency analysis, so that interactive $MOLP$ techniques can be used to locate the $MPS$ or set target values for the observed $DMU_o$. Note that in formulation (6) the weight $w_r$ is subject to change in an interactive process of locating the $MPS$.

Suppose in formulation (7) the $r$th composite output is denoted by $f_r(\lambda)$ as follows:

\[
f_r(\lambda) = \sum_{j=1}^{n} \lambda_j y_{rj} (r = 1, \ldots, s) \quad \text{and} \quad \lambda = (\lambda_1, \ldots, \lambda_n)^T.
\]
In this equivalence analysis, the \( r \)th composite output is defined as an objective for maximisation, so there are \( s \) objectives in total. The maximum feasible value of the \( r \)th composite output for the observed DMU\( \alpha \) is denoted by \( \bar{f}_{rj\alpha} = f_r(\lambda^*) \), where \( \lambda^* \) can be found by solving the following single objective optimisation problem:

\[
\begin{align*}
\text{Max} & \quad f_r(\lambda) = \sum_{j=1}^{n} \lambda_{ij}y_{rj} \\
\text{s.t.} & \quad \lambda \in \Omega_{h\alpha}.
\end{align*}
\]

(9)

Note that \( \bar{f}_{j\alpha} = [\bar{f}_{1\alpha}, \ldots, \bar{f}_{s\alpha}]^T \) is the ideal point in the objective space spanned by (8) on \( \Omega_{h\alpha} \) for the observed DMU\( \alpha \), as shown in Fig. 2.

Suppose the feasible decision space \( \Omega \) in formulation (6) is set to be the same as defined in formulation (7), or \( \Omega = \Omega_{h\alpha} \). The equivalence relationship between the output-oriented CCR dual model (3) or (7) and the minimax formulation (6) can be established as follows.

Suppose \( y_{rj\alpha} > 0 \) for all \( r = 1, \ldots, s \) and \( j_\alpha = 1, \ldots, n \). The output-oriented CCR dual model (7) can be equivalently transformed to the minimax formulation (6) using formulations (8) and (9) and the following equations:

\[
\begin{align*}
w_r &= 1/y_{rj_{\alpha}}, \\
f_r^* &= \frac{F_{\text{max}}}{w_r} = y_{rj_{\alpha}} F_{\text{max}}, \\
\theta &= F_{\text{max}} - \theta_{j_{\alpha}}
\end{align*}
\]

(10) (11) (12)

with

\[
F_{\text{max}} = \max_{i \leq r \leq s} \left\{ w_r \bar{f}_{rj_{\alpha}} \right\} = \max_{i \leq r \leq s} \left\{ \frac{f_{rj_{\alpha}}}{y_{rj_{\alpha}}} \right\}.
\]

(13)

The above equivalence relation can be explained as follows. Using Eqs. (8) and (10), the output-orientated CCR dual model can be equivalently rewritten as follows:

\[
\begin{align*}
\text{Max} & \quad \theta_{j_{\alpha}} \\
\text{s.t.} & \quad \theta_{j_{\alpha}} \frac{1}{w_r} - f_r(\lambda) \leq 0, \quad r = 1, \ldots, s; \quad \lambda \in \Omega_{h\alpha}.
\end{align*}
\]

(14)

The first \( s \) objective constraints in (14) can be equivalently transformed as follows, where “\( \Leftrightarrow \)” means “is equivalent to”. For any \( r = 1, \ldots, s \), we have

\[
\theta_{j_{\alpha}} \frac{1}{w_r} - f_r(\lambda) \leq 0 \iff -w_rf_r(\lambda) \leq -\theta_{j_{\alpha}} \iff F_{\text{max}} - w_rf_r(\lambda) \leq F_{\text{max}} - \theta_{j_{\alpha}} \iff w_r(f_r^* - f_r(\lambda)) \leq \theta.
\]

(15)

Moreover, the objective function of model (14) becomes

\[
\begin{align*}
\text{Max} & \quad \theta_{j_{\alpha}} = \text{Min}(-\theta_{j_{\alpha}}) = \text{Min}(F_{\text{max}} - \theta_{j_{\alpha}}) = \text{Min} \theta.
\end{align*}
\]

(16)

Fig. 2. Illustration of super-ideal point and ideal point.
Note that for any \( \lambda \in \Omega_{i_0} \),
\[
\theta = F^\text{max} - \theta_{i_0} \geq w^\text{T} f_{i_0} - \theta_{i_0} \geq w^\text{T} f_{i_0}(\lambda) - \theta_{i_0} \geq 0 \quad \text{for any} \quad r = 1, \ldots, s,
\]
\[
f^*_r = \frac{F^\text{max}}{w^r} \geq w^\text{T} f_{i_0} = f_{i_0} = \max_{\lambda \in \Omega_{i_0}} f_{i_0}(\lambda) \quad \text{for any} \quad r = 1, \ldots, s.
\]

The above analyses show that if the reference point in model (6) is set by \( f^* = [f^*_1, \ldots, f^*_n]^\text{T} = F^\text{max}[y_{j_{i_0}}, \ldots, y_{j_{k_0}}]^\text{T} \) and \( w_r \) by Eq. (10), then the output-oriented CCR dual model will be identical to the following minmax reference point formulation:

\[
\begin{align*}
\text{Min} & \quad \theta \\
\text{s.t.} & \quad w^\text{T} \left(f^*_r - \sum_{j=1}^{n} \lambda_j y_{jr}\right) \leq \theta \quad r = 1, \ldots, s, \\
& \quad \sum_{j=1}^{n} \lambda_j x_{ij} \leq x_{ij}, i = 1, \ldots, m, \quad \lambda_j \geq 0, \quad j = 1, \ldots, n.
\end{align*}
\]

By identical we mean that they share the same decision and objective spaces and have the same optimal solution. Since \( f^* = F^\text{max}[y_{j_{i_0}}, \ldots, y_{j_{k_0}}]^\text{T} \geq \hat{f}_{i_0} = [\hat{f}_{j_{i_0}}, \ldots, \hat{f}_{j_{k_0}}]^\text{T} \), such a \( f^* \) is called super-ideal point and the minmax reference point formulation established using the super-ideal point is therefore referred to as the super-ideal point model in this paper. The output-oriented BCC dual model can also be transformed to an identical minmax formulation similar to the above super-ideal point model with an extra convexity constraint of \( \sum_{j=1}^{n} \lambda_j = 1 \).

From equivalence Eqs. (10)-(13), the following three remarks can be drawn.

**Remark 1.** The super-ideal point model (19) can be used to generate the CCR efficiency score and the efficient composite inputs and outputs of \( DMU_0 \) if in model (19) \( w_r \) is calculated by using Eq. (10) and \( f^*_r \) by Eqs. (11) and (13).

The above analyses show that the CCR dual model is actually constructed to locate a specific efficient solution, termed as DEA efficient solution on the efficient frontier of the following generic MOLP formulation for the observed \( DMU_0 \):

\[
\begin{align*}
\text{Max} & \quad \left[ \sum_{j=1}^{n} \lambda_j y_{j1}, \ldots, \sum_{j=1}^{n} \lambda_j y_{jr}, \ldots, \sum_{j=1}^{n} \lambda_j y_{jT} \right] \\
\text{s.t.} & \quad \sum_{j=1}^{n} \lambda_j x_{ij} \leq x_{ij}, i = 1, \ldots, m, \quad \lambda_j \geq 0,
\end{align*}
\]

from which the super-ideal point model was constructed. The generic MOLP problem (20) defines the production possibility set for the observed \( DMU_0 \), in which there may be more preferred efficient solutions than the DEA efficient solution.

**Remark 2.** In the minmax reference point formulation (19), if the reference point is set as the ideal point, or \( f^* = \hat{f}_{i_0} = [\hat{f}_{j_{i_0}}, \ldots, \hat{f}_{j_{k_0}}]^\text{T} \) , and \( w_r \) is allowed to change, then we get the conventional ideal point model equivalent to the CCR dual model in the sense that they share the same decision and objective spaces. Changing \( w_r \) in \( R^s \) leads to the generation of any efficient solutions for the observed \( DMU_0 \), as defined in formulation (20). For the purpose of initialising an interactive procedure, an initial efficient solution can be generated by assigning normalised equal weights to the first \( \kappa \) constraints in formulation (19). Alternatively, let \( f^*_j = f_j(\lambda^*) \) be the value of the \( r \)th composite output at the solution \( \lambda^* \), defined in problem (9). If \( f^* = [f^*_1, \ldots, f^*_r]^\text{T} \) happens to be on the efficient frontier, then setting \( w_r = 1/(\hat{f}_{j_{i_0}} + f^*_r) \) for the \( r \)th output and solving the ideal point model (19) will lead to the identification of \( \lambda^* = [\lambda^*_1, \ldots, \lambda^*_r] \) with \( f^*_r = \sum_{j=1}^{n} \lambda^*_j y_{jr} \) (Lightner and Director, 1981; Yang and Li, 2002).

**Remark 3.** If \( f^* = [f^*_1, \ldots, f^*_r]^\text{T} \) is not on the efficient frontier, then setting \( w_r = 1/(\hat{f}_{j_{i_0}} - f^*_r) \) and solving problem (19) will lead to the identification of \( \lambda^* \) with \( f^*_j \neq \sum_{j=1}^{n} \lambda^*_j y_{jr} \) and \( \left[ \sum_{j=1}^{n} \lambda^*_j y_{j1}, \ldots, \sum_{j=1}^{n} \lambda^*_j y_{jr}, \ldots, \sum_{j=1}^{n} \lambda^*_j y_{jT} \right]^\text{T} \) lying at the crossing point on the efficient frontier by a line from the ideal point \( \hat{f}_{i_0} = [\hat{f}_{j_{i_0}}, \ldots, \hat{f}_{j_{k_0}}]^\text{T} \) towards \( [f^*_1, \ldots, f^*_r]^\text{T} \). In this case, \( \sum_{j=1}^{n} \lambda^*_j y_{jT} \) might not be the target value of the \( r \)th output preferred by the DM. To help the DM set realistic target values, a gradient projection based interactive tradeoff procedure can be employed by calculating the normal vector of the efficient frontier at \( \left[ \sum_{j=1}^{n} \lambda^*_j y_{j1}, \ldots, \sum_{j=1}^{n} \lambda^*_j y_{jT} \right]^\text{T} \). This projection can be used to assist the DM to provide a realistic tradeoff for finding better efficient solutions than \( \lambda^* \). At each interaction, a new vector generated from the tradeoff can be determined and used to solve formulation (19), leading to a new efficient solution \( \lambda^{t+1} \).
Graphically, the super-ideal point \( f^* \) and the ideal point \( \tilde{f}_j \) are shown in Fig. 2 in the objective space spanned by \( f_1 \) and \( f_2 \) defined in Eq. (8). The observed DMU \( E \) is given by \( E = [y_{1j}, y_{2j}]^T \) and assumed to be inefficient in Fig. 2. \( E_1 \) is the intersection of the efficient frontier and a line emitting from the origin \( O \) through the point \( E \), given by \( E_1 = \theta_{j_o}[y_{1j}, y_{2j}]^T \). The efficiency score can be calculated by \( e_o = OE/OE_1 = 1/\theta_{j_o} \), irrespective of the number of inputs. The ideal point is given by \( \tilde{f}_j = [\tilde{f}_{1j}, \tilde{f}_{2j}]^T \). The super-ideal point is defined by \( f^* = F_{\text{max}}[y_{1j}, y_{2j}]^T \), which is proportional to point \( E \). So, \( f^*, E \) and \( E_1 \) are all on the above same line. In Fig. 2, \((y_{2j_e}/y_{1j_e}) > (\tilde{f}_{2j_e}/\tilde{f}_{1j_e}) \) or \((\tilde{f}_{1j_e}/y_{1j_e}) > (\tilde{f}_{2j_e}/y_{2j_e}) \). So, \( F^\text{max} = \tilde{f}_{1j_e}/y_{1j_e} \) and \( f^* = \tilde{f}_{1j_e}[1, (y_{2j_e}/y_{1j_e})]^T \), which means that \( f^* \) and \( \tilde{f}_j \) are on the same vertical line with \( f^* \) above \( \tilde{f}_j \). From Remark 1, the solution of the super-ideal point model leads to \( f^* - E_1 = \theta[y_{1j_e}, y_{2j_e}]^T \), or \( F^\text{max}[y_{1j}, y_{2j}]^T - \theta[y_{1j}, y_{2j}]^T = \theta[y_{1j}, y_{2j}]^T \), so \( \theta = F^\text{max} - \theta_{j_o} \). On the other hand, changing weights in \( R^t \) and solving the ideal-point model can identify any efficient solutions from point \( A \) to point \( D \).

### 3.2. An interactive tradeoff analysis procedure for setting target values

The gradient projection method can be used to identify a normal vector at a given efficient solution on the efficient frontier and to design an interactive procedure for searching for the MPS that maximizes the DMUs implicit utility function. We first describe the method and then develop the tradeoff procedure.

Suppose for a given positive weight vector \( w^e = \{w_1^e, \ldots, w_r^e\} \) the optimal solution of the minimax model (6) is given by \( x = \{x_1^e, \ldots, x_r^e\} \), which must be an efficient solution. The optimal value of the dual variable (simplex multiplier in linear case) of the \( r \)th objective constraint \( w_i(f_i^e - f_i(x)) \leq \theta \) is given by \( \beta_i^e \). Let \( f(x') = [f_1(x'), \ldots, f_k(x')] \) represent the efficient solution in the corresponding objective space. It is proved that the normal vector \( N^r \) at \( f(x') \) on the efficient frontier is given by (Yang and Li, 2002):

\[
N^r = [w_1^e \beta_1^e, w_2^e \beta_2^e, \ldots, w_r^e \beta_r^e]^T.
\]

In linear programming, \( \beta_i^e \) can be generated using many existing software packages at no extra cost. Suppose the implicit utility function of the DM is denoted by \( u(f(x)) \). The gradient of \( u(f(x)) \) at the solution \( f(x') \) is given by

\[
\nabla u' = \frac{\partial u}{\partial f(x')} = \left[ \frac{\partial u}{\partial f_1}, \ldots, \frac{\partial u}{\partial f_r} \right]^T.
\]

Although \( u(f(x')) \) is unknown in general, the utility gradient \( \nabla u' \) may be estimated using the local preference information of the DM for example marginal rates of substitution.

If \( f(x') \) is the MPS, it is necessary as well as sufficient in a convex case (e.g. linear case) that the normal vector \( N^r \) is proportional to the utility gradient \( \nabla u' \) at \( f(x') \), or

\[
[w_1^e \beta_1^e, \ldots, w_r^e \beta_r^e]^T \propto \left[ \frac{\partial u}{\partial f_1}, \ldots, \frac{\partial u}{\partial f_r} \right]^T.
\]

If the optimal condition were met, then the gradient \( \tilde{G} \), as shown in Fig. 3, would be proportional to the normal vector \( \tilde{N} \); consequently, the dashed line would overlap the dotted line in Fig. 3, or point \( E_2 \) would overlap \( E_3 \). If the optimal condition is not met, as is the case in Fig. 3, then the gradient \( \tilde{G} \) can be projected onto the tangent plane (dotted line) of the efficient frontier at \( f(x') \). Such projection provides a direction on or closest to the efficient frontier, along which the DMs utility can be improved. The projection, denoted by \( \Delta u' \) as shown in Fig. 3, is given by

![Fig. 3. Projection of utility gradient onto tangent plane of efficient frontier.](image-url)
\[ \Delta u' = [\Delta f'_1 \cdots \Delta f'_r \cdots \Delta f'_s]^T = -\nabla u' + \left[ \left( \nabla u' \right)^T N' \right] N', \] 

However, since the utility function is not known explicitly, a utility gradient needs to be estimated by for example indifference tradeoffs or marginal rates of substitution, \( M \), which may be provided by the DM. Without loss of generality, set the first objective \( f_1 \) as the reference objective. Then the indifference tradeoff \( M'_{ir} \) between the first and the \( r \)th objectives and the marginal rate of substitution \( M' \) at \( f(\lambda') \) are given by:

\[ M'_{ir} = -\frac{d f'_i}{d f'_r} \quad \text{and} \quad M' = [1, M'_{12} \cdots M'_{ir} \cdots M'_{is}]^T, \]

where \( d f'_i \) is a change in \( f_i(\lambda) \) that is assumed to be exactly offset by a change \( d f'_r \) in \( f_r(\lambda) \) with the overall utility kept constant, given that all other objectives remain unchanged. It can be shown that the gradient of the utility function \( \nabla u' \) given in (22) is proportional to the marginal rate of substitution \( M' \) at \( f(\lambda') \), or

\[ M' = \frac{\nabla u'}{\partial u' / \partial f'_r}. \]

At the MPS, the following optimal indifference tradeoff between \( f_i(\lambda) \) and the \( r \)th objective can be calculated using the following equation:

\[ df'_r = -d f'_i \frac{w_r f_r'}{w_r f'_r} \]

On the other hand, the optimal indifference tradeoff can be used to check whether the MPS is achieved.

If the MPS is not achieved, then the projection \( \Delta u' \) can be calculated using \( M' \) as follows and will not be zero, denoted by \( \Delta u' \), which provides a new tradeoff direction to improve the DM’s utility:

\[ \Delta u' = [\Delta f'_1 \cdots \Delta f'_r \cdots \Delta f'_s]^T = -M' + \frac{(M')^T N'}{(N')^T N'}. \]

Suppose \( \bar{z} \) is a tradeoff step. Update the weighting parameters \( w_r \), as follows:

\[ w_{r+1} = \frac{f_i(\lambda') + \bar{z} \Delta f'_i - f_i(\lambda^*)}{f_i(\lambda') + \bar{z} \Delta f'_i - f_i(\lambda^*)}, \quad r = 1, \ldots, s. \]

Replacing \( w_r \) by \( w_{r+1} \) in formulation (6) and solving it leads to a new solution \( \bar{\lambda}^{t+1} \) which should have a higher utility than \( \lambda' \) for a sufficiently small \( \bar{z} \), or \( u(\bar{\lambda}^{t+1}) > u(\lambda') \). In the following, we will design an interactive procedure to use the above results to support the DM to search for the MPS, summarised as follows:

**Step 1:** Generate an output payoff table.

Optimise each of the composite outputs of the observed DMU using formulation (9) to generate \( \bar{f}_{r/o}(r = 1, \ldots, s) \) and collect the results in a payoff table. For each composite output, elicit a target output value \( \bar{Y}_{r/o} \) from the DM as a starting point.

**Step 2:** Generate initial efficient solution.

Set the initial weighting parameters as \( w_r^0 = 1/(\bar{f}_{r/o} - \bar{Y}_{r/o})(r = 1, \ldots, s) \). For the observed DMU, solve model (19) and obtain the initial solution of the decision variables \( \bar{z}^0 = [z^0_1 \cdots z^0_s]^T \), the initial objective values \( f(\bar{z}^0) = [f_1(\bar{z}^0) \cdots f_s(\bar{z}^0)]^T \), and the initial dual variable values \( \beta^0 = [\beta^0_1 \cdots \beta^0_s]^T \) for the first \( s \) constraints on the outputs. Set \( t = 0 \).

**Step 3:** Compute the normal vector and check optimality condition.

At interaction \( t \), calculate the normal vector \( N' \) using Eq. (21). Choose a reference composite output, for example the first composite output \( f_1(\lambda) \). Then check whether a given small change \( d f'_i \) in \( f_i(\lambda) \) can be exactly offset by the amount of change \( d f'_r \) in the \( r \)th composite output \( f_r(\lambda) \) with \( d f'_r \) generated using Eq. (27). If the DM agrees with such optimal indifference tradeoffs between \( f_1(\lambda) \) and each of the other composite outputs, then the current solution \( f(\lambda') \) is already the MPS and the interactive process is terminated. Otherwise, the DM would provide new indifference tradeoffs, or \( M' = [1, M'_{12} \cdots M'_{ir} \cdots M'_{is}]^T \).

**Step 4:** Determine the tradeoff direction.

Use Eq. (28) to calculate the projection of the DMs indifference tradeoffs \( M' \) onto the tangent plane of the efficient frontier, or \( \Delta u' = [\Delta f'_1 \cdots \Delta f'_r \cdots \Delta f'_s]^T \), which determines the new tradeoff direction.
Step 5: Determine the tradeoff step size and calculate the new weighting vector

The tradeoff step size \( \bar{\alpha} \) can be estimated by \( \bar{\alpha} = \bar{\alpha}^{\text{max}} \alpha \), where \( \bar{\alpha}^{\text{max}} \) is the largest permissible step size and \( \alpha \) a regulating factor with \( 0 \leq \alpha \leq 1 \). Suppose \( l^1 \) is the set of the subscripts of objectives for increase. The maximum step size can be determined as follows:

\[
\bar{\alpha}^{\text{max}} = \max_{i \in l^1} \frac{f_{r_i}^i - f_{r_j}^i}{\Delta f_{r_i}^i}.
\]

An explicit tradeoff table as illustrated below and detailed in the case study can be used to determine the tradeoff step size \( \bar{\alpha} \) using the following general formula:

\[
f_r(\bar{\alpha}) = f_r(\bar{\alpha}') + \bar{\alpha}\Delta f_r.
\]

In Table 1, the DM may determine the step size by analysing the tradeoffs among the outputs along the tangent plane of the efficient frontier. Some heuristics about the selection of the step size are discussed in the case study. Once the step size is decided, the weighting parameters can be updated using Eq. (29).

The above interactive process continues until the optimal indifference tradeoffs are achieved and thus the MPS is found maximising the implicit utility function of the DM. The decision variables at the MPS are represented by \( \bar{x} = [\bar{x}_1^* \cdots \bar{x}_n^*]^T \).

### 3.3. Taking group preferences into account in setting target values

The above MPS located for each DMU relative to its peers only represents the preferences of the DM of the DMU at a divisional or local level. In order to set a performance benchmark with the organisational or group preferences taken into account, a group most preferred solution (GMPS) would need to be determined first. It is possible that a GMPS is assigned by a single leading DM having the overall responsibility for an organisation or group, by choosing a convex combination of the individual MPSs generated in the last section, or by simply picking up an existing efficient DMU (Korhonen et al., 2002) or a convex combination of certain existing efficient DMUs as a GMPS for the whole organisation. Alternatively, group decision making techniques such as voting theory or the Delphi technique could be used, especially for the purpose of negotiation and finding a compromised GMPS. Nevertheless, a group decision making process has to be participative and flexible so as to reflect the opinions of group members, in particular the evolving discussions between the group members.

A GMPS, as defined by \( m \) GMPS inputs \( x_{i\text{GMPS}}^i (i = 1, \ldots, m) \) and \( s \) GMPS outputs \( y_{r\text{GMPS}}^r (i = 1, \ldots, m) \) may lie within, on or outside the efficient frontier of a specific DMU. Although a GMPS point may represent the preferences of a group as a whole, it will not be practical to a DMU if it is not attainable by the DMU. In the rest of this section, a procedure will be proposed where a GMPS is mapped back to the feasible space of each DMU to generate a locally most preferred solution (LMPS) for each DMU. The new local input and output targets could then be used as benchmark to align towards the organisation’s or group’s targets with both group and individual DMUs preferences taken into account.

By constructing a minimax reference point model with the GMPS as the reference point, a LMPS for each DMU could be generated as the one closest to the GMPS in the composite output space of the DMU. The following minimax reference point model equivalent to the minimax formulation (5) is constructed for this purpose (Yang, 2001a).

\[
\begin{align*}
\text{Min} & \quad d \\
\text{s.t.} & \quad w_r(f_{r\text{GMPS}}^i - f_r(\bar{x})) \leq d, \\
& \quad -w_r(f_{r\text{GMPS}}^i - f_r(\bar{x})) \leq d \quad r = 1, \ldots, s, \\
& \quad \bar{x} \in \Omega_\bar{x} = \left\{ \bar{x} \bigg| \sum_{j=1}^m \bar{\lambda}_j x_{ij} \leq \bar{x}_{ij}, \quad i = 1, \ldots, m; \quad \bar{\lambda}_j \geq 0, \quad j = 1, \ldots, n \right\},
\end{align*}
\]

where \( f_{r\text{GMPS}}^i \) is given by \( f_{r\text{GMPS}}^i = \lambda_{r\text{GMPS}}^i \) with \( \bar{x} = \min_{1 \leq i \leq m} \{ x_{ij}/x_{i\text{GMPS}}^i \} \).

Table 1

<table>
<thead>
<tr>
<th>( \bar{x} )</th>
<th>( f_1(\bar{x}) )</th>
<th>( f_2(\bar{x}) )</th>
<th>\ldots</th>
<th>( f_s(\bar{x}) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.1</td>
<td>( f_1(\bar{x}') + 0.1x_\text{max}\Delta f_1^1 )</td>
<td>( f_2(\bar{x}') + 0.1x_\text{max}\Delta f_2^1 )</td>
<td>\ldots</td>
<td>( f_s(\bar{x}') + 0.1x_\text{max}\Delta f_s^1 )</td>
</tr>
<tr>
<td>0.2</td>
<td>( f_1(\bar{x}') + 0.2x_\text{max}\Delta f_1^1 )</td>
<td>( f_2(\bar{x}') + 0.2x_\text{max}\Delta f_2^1 )</td>
<td>\ldots</td>
<td>( f_s(\bar{x}') + 0.2x_\text{max}\Delta f_s^1 )</td>
</tr>
<tr>
<td>\vdots</td>
<td>\vdots</td>
<td>\vdots</td>
<td>\ldots</td>
<td>\vdots</td>
</tr>
<tr>
<td>1.0</td>
<td>( f_1(\bar{x}') + 1.0x_\text{max}\Delta f_1^1 )</td>
<td>( f_2(\bar{x}') + 1.0x_\text{max}\Delta f_2^1 )</td>
<td>\ldots</td>
<td>( f_s(\bar{x}') + 1.0x_\text{max}\Delta f_s^1 )</td>
</tr>
</tbody>
</table>
Note that it is not always the case that \( g_r^{GMPS} \geq f_r(\lambda) \) for all \( r \in Q_{j_r} \). The above reference point model is referred to as the shortest distance model to differentiate it from the previous super-ideal and ideal point models, though they are all derived from formulation (20), so sharing the same decision and objective spaces. The main advantage of the above \textit{LMPS} procedure is that the weight \( w_r \) for an objective \( f_r(\lambda) \) in formulation (32) can be set individually for each \textit{DMU}, which can thus be used to represent local preferences. The rationale is that each \textit{DMU} may be different from others, and based on the same \textit{GMPS}, each \textit{DMU} may have different preferences and relative weights for the objectives or outputs. This is relevant since different \textit{DMUs} have different capabilities and specialties and may perform well in certain areas. On the other hand, there is also flexibility where a compromised common set of weights could be decided by the group members through negotiation and discussion. Alternatively, the same set of relative weights could be explicitly assigned by a group leader with an authoritative role such as the chief executive of a company representing the views of the organisation. This happens when the organisation wants to focus on improving only certain aspects of the business or output levels.

In the \textit{LMPS} procedure, a \textit{GMPS} is mapped back to the feasible space of each \textit{DMU} to match its capabilities, size and scale of operations. However, it is possible that a \textit{LMPS} generated by the mapping procedure may lie within the efficient frontier, thus an inefficient solution with \( d = 0 \). In order to alleviate this problem, the super-ideal point model (3) can be constructed and solved with the \textit{LMPS} of the observed \textit{DMU} added as a new \textit{DMU} in the reference set. The corresponding composite outputs generated by the model for the \textit{LMPS} are realistic and achievable and thus could be used as the performance benchmark for the observed \textit{DMU}, which will be referred to as the efficient \textit{LMPS} that takes into account the preferences of both individual \textit{DMs} and group members.

4. A case study for performance analysis of UK retail banks

4.1. Problem description and efficiency analysis

A case study is carried out to demonstrate how performance assessment and target setting can be conducted in an integrated way using the interactive \textit{MOLP} methods investigated in the previous sections. The UK retail bank industry, specifically seven major retail banks, is examined to show the equivalence models, demonstrate the interactive procedure to search for \textit{MPSs} along the efficient frontier, and illustrate the group negotiation and discussion process. The data set is obtained from \textit{Wong and Yang (2004)} through a study on data envelopment analysis and multiple criteria decision making based on the evidential reasoning approach – performance measurement of UK retail banks (Yang, 2001b; Yang and Xu, 2002), as shown in Table 2.

For the \textit{DEA} formulation, the reference set consists of seven \textit{DMUs} (retail banks), and four inputs and two outputs are considered. The \textit{DMUs} are comparable major banks in the UK including Abbey National, Barclays, Halifax, HSBC, Lloyds TSB, NatWest and RBS (Royal Bank of Scotland). The four inputs are namely number of branches, number of ATMs, number of staff and asset size. The two outputs are customer satisfaction and total revenue. It should be noted that although both bank staff and customers were interviewed in person or through questionnaires at certain stages of this research it is the researchers who acted as the \textit{DMs}. Also note that this was not a full scale performance analysis and only limited data were collected. As such, the conclusions of the paper are for the purpose of illustrating the new approach rather than for providing an authentic performance assessment of these retail banks.

The output-oriented \textit{CCR} dual model (3) is run to find the respective efficiency scores. As shown in Table 3, only Barclays and NatWest are found to be inefficient within the reference set of the seven banks. For instance, NatWest has an efficiency score of 74.7%, and its composite point on the efficient frontier can be represented as a linear combination of

<table>
<thead>
<tr>
<th>DMU</th>
<th>Bank</th>
<th>Inputs</th>
<th>Outputs</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>No. of branches (’000)</td>
<td>No. of ATMs (’000)</td>
<td>No. of Staff (’000)</td>
</tr>
<tr>
<td>1</td>
<td>Abbey national</td>
<td>0.77</td>
<td>2.18</td>
</tr>
<tr>
<td>2</td>
<td>Barclays</td>
<td>1.95</td>
<td>3.19</td>
</tr>
<tr>
<td>3</td>
<td>Halifax</td>
<td>0.80</td>
<td>2.10</td>
</tr>
<tr>
<td>4</td>
<td>HSBC</td>
<td>1.75</td>
<td>4.00</td>
</tr>
<tr>
<td>5</td>
<td>Lloyds TSB</td>
<td>2.50</td>
<td>4.30</td>
</tr>
<tr>
<td>6</td>
<td>NatWest</td>
<td>1.73</td>
<td>3.30</td>
</tr>
<tr>
<td>7</td>
<td>RBS</td>
<td>0.65</td>
<td>1.73</td>
</tr>
</tbody>
</table>

Customer satisfaction values are converted scores based on the average expected utilities of the survey respondents. Source: bank brochures, banks and financial advice websites, interviews with bank staff and customers.
Before conducting interactive tradeoff analysis, let us first validate the equivalence between the CCR dual model and the minimax formulations developed in the previous sections. Using Remark 1, the super-ideal point model shown in formulation (19) is run for each DMU, with \( w_r \) assigned by (10) and the reference point as the super-ideal point \( f^* = F_{max}^{\frac{1}{T}}[y_{1,0},\ldots,y_{m,0}]^T \). The results are shown in Table 4, which shows that the equivalence \( \theta = F_{max} - \theta_o \) holds for each DMU with \( \theta \) generated using the super-ideal point model, \( F_{max} \) assigned using Eq. (13) and \( \theta_o \) being the same as the DEA score given in Table 3 for each DMU. The composite inputs and outputs for each DMU shown in Table 4 are also the same as the results of Table 3 generated by the output-oriented CCR dual model.

### 4.2. Interactive tradeoff analysis to find MPSs for management planning

In the proposed interactive tradeoff analysis procedure, the first step is to solve model (9) for each composite output of the observed DMU to generate a payoff table for the DMU, in which the feasible ranges of tradeoffs are defined. Table 5 shows the ranges of possible output values when each composite output of every DMU is maximized.

From Table 5, it is clear that without increasing the consumption of resources (inputs) there is no further improvement or possible tradeoffs between the outputs of Abbey National and RBS as for each of them maximizing \( y_1 \) and \( y_2 \) leads to the same set of solutions. The other efficient DMUs such as Halifax, Lloyds TSB and HSBC can sacrifice one of the outputs to increase the other output. For the inefficient DMUs of Barclays and NatWest, both their outputs can be further improved without consuming extra inputs. For illustration purpose, the interactive tradeoff analysis procedure will be demonstrated for the sixth DMU NatWest, which is an inefficient DMU. The maximum feasible value of the first composite output of NatWest is generated as \( f_1 = 14.12 \), while the maximum feasible value of its second composite output is given by \( f_2 = 16.11 \). The ideal point of the composite outputs is thus given by \( f = [14.12, 16.11]^T \), which is then used in the next step of the procedure.

The starting solution of the interactive procedure for NatWest is generated by solving model (19) with the initial weights \( w_r \) assigned as described in Remarks 2 and 3. In fact, the starting solution could be the same point on the efficient frontier \( D_j \) as that of the initial composite solution or the DEA composite unit.

### Table 3
DEA results and efficiency scores

<table>
<thead>
<tr>
<th>DMU</th>
<th>Bank</th>
<th>DEA score</th>
<th>Observed DMU's composite unit</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td>1</td>
</tr>
<tr>
<td>1</td>
<td>Abbey national</td>
<td>1.000</td>
<td>1.000</td>
</tr>
<tr>
<td>2</td>
<td>Barclays</td>
<td>0.778</td>
<td>0.135</td>
</tr>
<tr>
<td>3</td>
<td>Halifax</td>
<td>1.000</td>
<td>1.000</td>
</tr>
<tr>
<td>4</td>
<td>HSBC</td>
<td>1.000</td>
<td>1.000</td>
</tr>
<tr>
<td>5</td>
<td>Lloyds TSB</td>
<td>1.000</td>
<td>1.000</td>
</tr>
<tr>
<td>6</td>
<td>NatWest</td>
<td>0.747</td>
<td>0.310</td>
</tr>
<tr>
<td>7</td>
<td>RBS</td>
<td>1.000</td>
<td>1.000</td>
</tr>
</tbody>
</table>

### Table 4
Equivalence between CCR dual model and super-ideal point model

<table>
<thead>
<tr>
<th>DMU</th>
<th>DEA dual model</th>
<th>Minimax model</th>
<th>Composite inputs and outputs</th>
<th>( \theta = F_{max} - \theta_o )</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>DEA Score ( \theta_o )</td>
<td>( F_{max} )</td>
<td>( \theta )</td>
<td>( x_1 )</td>
</tr>
<tr>
<td>1</td>
<td>1.000</td>
<td>1.000</td>
<td>1.000</td>
<td>0.000</td>
</tr>
<tr>
<td>2</td>
<td>0.778</td>
<td>1.285</td>
<td>5.449</td>
<td>4.164</td>
</tr>
<tr>
<td>3</td>
<td>1.000</td>
<td>1.000</td>
<td>1.238</td>
<td>0.238</td>
</tr>
<tr>
<td>4</td>
<td>1.000</td>
<td>1.000</td>
<td>3.001</td>
<td>2.001</td>
</tr>
<tr>
<td>5</td>
<td>1.000</td>
<td>1.000</td>
<td>1.989</td>
<td>0.989</td>
</tr>
<tr>
<td>6</td>
<td>0.747</td>
<td>1.338</td>
<td>2.905</td>
<td>1.567</td>
</tr>
<tr>
<td>7</td>
<td>1.000</td>
<td>1.000</td>
<td>1.000</td>
<td>0.000</td>
</tr>
</tbody>
</table>

0.310 of HSBC and 1.192 of RBS. Note that the decision variables do not add up to 1 as the constant returns to scale is assumed.
Table 5
Payoff table for all DMUs

<table>
<thead>
<tr>
<th>DMU</th>
<th>Bank</th>
<th>Max $y_1$</th>
<th>Max $y_2$</th>
<th>Maximum values</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>$y_1$</td>
<td>$y_2$</td>
<td>$y_1$</td>
</tr>
<tr>
<td>1</td>
<td>Abbey national</td>
<td>6.79</td>
<td>10.57</td>
<td>6.79</td>
</tr>
<tr>
<td>2</td>
<td>Barclays</td>
<td>13.87</td>
<td>12.47</td>
<td>7.45</td>
</tr>
<tr>
<td>3</td>
<td>Halifax</td>
<td>9.17</td>
<td>8.14</td>
<td>6.85</td>
</tr>
<tr>
<td>4</td>
<td>HSBC</td>
<td>17.47</td>
<td>15.50</td>
<td>5.82</td>
</tr>
<tr>
<td>5</td>
<td>Lloyds TSB</td>
<td>13.07</td>
<td>13.22</td>
<td>6.57</td>
</tr>
<tr>
<td>6</td>
<td>NatWest</td>
<td>14.12</td>
<td>13.49</td>
<td>10.48</td>
</tr>
<tr>
<td>7</td>
<td>RBS</td>
<td>7.28</td>
<td>7.36</td>
<td>7.28</td>
</tr>
</tbody>
</table>

Alternatively, if $f^0_2$ is perturbed or decreased by a very small amount so that $\bar{f}_2 > f^0_2$, for example $f^0_2 = 16.10 (< \bar{f}_2 = 16.11)$, then a starting solution with the maximum objective values of $f(\lambda^0) = [10.49, 16.10]^T$ can be found by solving formulation (19) that is very close to but not exactly the same as the DEA composite outputs of $f(\lambda^1) = [10.48, 16.11]^T$. This perturbation technique is used to start the interactive process, resulting in the initial decision variables $z^0 = [0, 0, 0, 0.31, 0, 0.120]^T$ and the normal vectors $N^0 = [0.27, 0.45]^T$ calculated using formulation (21). Thus, the initial efficient solution of NatWest can be characterized as a linear combination of 0.31 of HSBC and 1.20 of RBS.

For the first interaction, suppose $f_2$ is treated as the reference objective. The optimal indifference tradeoff vector at the solution $f(z^0)$ for a unit change of $f_2$ can be calculated using Eq. (27) as $d^{01} = [1.66, 1.00]^T$, leading to the initial optimal indifference tradeoff of (10.49, 16.10) for a unit change of $f_2$. If the $DM$ does not agree with this initial optimal indifference tradeoff, which means that the initial target values are not most preferred, then a new set of indifference tradeoffs may be proposed by the $DM$, for example (10.49, 16.10) $\Leftrightarrow$ (10.49 + 2.00, 16.10 – 1.00), resulting in the marginal rate of substitution $M^{0} = [0.5, 1.00]^T$. Note that the tradeoff of (10.49 + 2.00) for $f_1$ is less than its maximum feasible value of 14.12.

The gradient projection is calculated using Eq. (24) to find the tradeoff direction with $\Delta \bar{f}^0 = [0.076, -0.046]^T$, which means that the $DM$ prefers to improve $f_1$ at the expense of $f_2$. As for the tradeoff size, the maximum permissible step size is calculated by Eq. (30), resulting in $\delta^0_{\text{max}} = 48.00$, which is used to construct the step size table. Table 6 shows that $f_1$ increases and $f_2$ decreases for every incremental step size when 10 equal incremental steps are used between the current value of 10.49 for $f_1$ and its maximum feasible value of 14.12. Suppose the $DM$ sets the target level for $f_1$ at 11.00. This new target value of 11.00 is exceeded when $\alpha > 0.2$, so the regulating parameter is set to $\alpha = 0.1$.

For a more precise size-step assignment, 100 equal incremental steps could be used between the current value of 10.49 for $f_1$ and its maximum feasible value of 14.12. Using the same maximum upper bound for $f_1$ at 11.00, the value of 11.00 is exceeded when $\alpha > 0.15$ and hence the regulating parameter of the step size is set to $\alpha = 0.14$.

The weighting vector can be updated using Eq. (29) with $w^1 = [1, 9.907]^T$. Solving formulation (19) again with the new weight vector $w^1$ in the second interaction, a new efficient solution is generated with $f(\lambda^1) = [11.00, 15.79]^T$ and $\lambda^1 = [0, 0, 0, 0.26, 0, 0, 1.30]^T$. The new normal vectors at $f(\lambda^1)$ is calculated as $N^1 = 0.32[0.27, 0.45]^T$. Note that the new normal vector $N^1$ is in parallel with the previously identified normal vector $N^0$, which means that the interactive tradeoff analysis is done in the same facet of the efficient frontier as in the last interaction. The optimal indifference tradeoff vector at $f(\lambda^1)$ for a unit change of $f_2$ is given by $df = [1.66, 1.00]^T$.

If $DM$ still does not agree with the optimal indifference tradeoff of (11.00, 15.79) $\Leftrightarrow$ (11.00 + 1.66, 15.79 – 1.00), a new set of indifference tradeoffs may be provided by the $DM$, say (11.00, 15.79) $\Leftrightarrow$ (11.00 + 1.80, 15.79 – 1.00), leading to the

Table 6
Determination of tradeoff step size

<table>
<thead>
<tr>
<th>$\alpha$</th>
<th>$y_1$</th>
<th>$y_2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>10.491</td>
<td>16.100</td>
</tr>
<tr>
<td>0.1</td>
<td>10.853</td>
<td>15.881</td>
</tr>
<tr>
<td>0.2</td>
<td>11.216</td>
<td>15.663</td>
</tr>
<tr>
<td>0.3</td>
<td>11.579</td>
<td>15.444</td>
</tr>
<tr>
<td>0.4</td>
<td>11.941</td>
<td>15.225</td>
</tr>
<tr>
<td>0.5</td>
<td>12.304</td>
<td>15.007</td>
</tr>
<tr>
<td>0.6</td>
<td>12.667</td>
<td>14.788</td>
</tr>
<tr>
<td>0.7</td>
<td>13.029</td>
<td>14.569</td>
</tr>
<tr>
<td>0.8</td>
<td>13.392</td>
<td>14.351</td>
</tr>
<tr>
<td>0.9</td>
<td>13.755</td>
<td>14.132</td>
</tr>
<tr>
<td>1.0</td>
<td>14.117</td>
<td>13.913</td>
</tr>
</tbody>
</table>
marginal rate of substitution $M^i = [0.556, 1.000]^T$. The new gradient projection is given by $\Delta f^i = [0.035, -0.021]^T$ and the maximum step size by $s^i_{\text{max}} = 89.60$. Suppose the $DM$ re-sets the target level of $f_1$ as 11.50. The tradeoff step size $\alpha = 0.16$ can then be calculated to provide a new weighting vector $w = [1, 4.225]^T$ to be used for the third interaction. Note that the new 'learnt' target level of 11.50 for $f_1$ is different from the previous target level of 11.00 for $f_1$ set in the first interaction. The change in the target level made by the $DM$ represents part of the learning process about what could be achieved, which is the main feature of the interactive tradeoff analysis procedure and can help the $DM$ to set realistic target values.

In the third interaction, the new efficient solution is given by $f(x^3) = [11.50, 15.49]^T$ and $x^2 = [0, 0, 0, 0.22, 0.0, 0, 1.41]^T$. The new normal vector $N^3$ at $f(x^3)$ is calculated as $N^3 = 2.63[0.27, 0.45]^T$. Again, the new normal vector $N^2$ is still in parallel to $N^1$, which means that the interactive tradeoff process is searching for the $MPS$ along the same facet of the efficient frontier as the last interaction. If the $DM$ agrees with the optimal indifference tradeoff of $(11.50, 15.49)\Rightarrow(11.50 + 1.66, 15.49 − 1.00)$, then the interactive process will be terminated and $f(x^2) = [11.50, 15.49]^T$ will be the $MPS$ maximising the $DM$s implicit utility function for NatWest. Otherwise, the interactive process continues.

The $DEA$ composite outputs for NatWest are given by $[10.48, 16.11]^T$ for customer satisfaction and total revenue. Further analysis on the areas of improvement that NatWest needs to focus upon and the amount of improvement needed for each input and output are shown in Table 7 if the $DEA$ composite $DMU$ is benchmarked against. In fact, number of branches should be reduced from 1730 to 1320, a decrease of 31%, and number of staff should be reduced from 7700 to 7300, a drop of 5% for NatWest to become efficient. Also, customer satisfaction and total revenue could be increased by 116% and 34%, respectively. So, instead of the current total revenue of £12.04 m, the target total revenue that could be achieved is £16.11 m.

On the other hand, the $MPS$ maximising the $DM$s utility function for NatWest as determined by the interactive process has target composite outputs of $[11.50, 15.49]^T$ for customer satisfaction and total revenue as shown in Table 7. The new target values show that NatWest should in future achieve an increase of 137% for customer satisfaction and 29% for total revenue from its current value. Likewise, inputs should be better allocated in comparison with the original data, and number of branches, number of staff and asset size should be reduced by about 34%, 16% and 6%, respectively.

The $MPS$ target output values for NatWest are $[11.50, 15.49]^T$ when the $DM$s value judgements are taken into account. In contrast, the $DEA$ target output values are $[10.48, 16.11]^T$. Hence, when $MPS$ is compared to the target values generated by $DEA$, NatWest should sacrifice total revenue by 3.8% from £16.11 to £15.49 m, and aim to increase the output level of customer satisfaction by 9.8% from the score of 10.48 to 11.50. Subsequently, NatWest should better allocate or utilise its resources by decreasing its number of branches, number of staff and asset size by about 1.8%, 9.9% and 5.6%, respectively. It is evident that during the interactive tradeoff analysis process the $DM$ placed more emphasis on generating high customer satisfaction rating rather than high total revenue.

The individual $MPS$ for all the banks could be found by solving model (19) for each $DMU$ using the interactive procedure. For illustration purpose, the target output and input levels for all $DMUs$ are shown in Table 8. It can be observed that the inefficient $DMUs$: Barclays and NatWest have different $MPS$s and $DEA$ composite inputs and outputs. Likewise, three of the efficient $DMUs$: Halifax, HSBC and Lloyds TSB also have different $MPS$s from their $DEA$ composite units. The other two efficient $DMUs$ of Abbey National and RBS have the composite inputs and outputs that are the same as the original data values, as there are no tradeoffs between their outputs.

Table 9 shows the decision variables of the individual $MPS$s of the banks. One way to set group performance benchmark is to choose a bank whose decision variable occurs most frequently in construction of the imaginary composite units for other banks. So, in this case, the efficient RBS could be set as the performance benchmark or the group $MPS$ for all banks to follow. However, such an approach lacks coherence and reliability, and the individual $DM$s of the $DMUs$ may

<table>
<thead>
<tr>
<th>Inputs</th>
<th>Outputs</th>
</tr>
</thead>
<tbody>
<tr>
<td>No. of branches ('000)</td>
<td>No. of ATMs ('000)</td>
</tr>
<tr>
<td>Actual value</td>
<td>1.73</td>
</tr>
<tr>
<td>DEA composite unit</td>
<td>1.32</td>
</tr>
<tr>
<td>Improvement %</td>
<td>31.38</td>
</tr>
<tr>
<td>Actual value</td>
<td>1.73</td>
</tr>
<tr>
<td>MPS composite unit</td>
<td>1.29</td>
</tr>
<tr>
<td>Improvement %</td>
<td>33.69</td>
</tr>
<tr>
<td>DEA composite unit</td>
<td>1.32</td>
</tr>
<tr>
<td>MPS composite unit</td>
<td>1.29</td>
</tr>
<tr>
<td>Improvement %</td>
<td>31.38</td>
</tr>
</tbody>
</table>
disagree with such group target levels. An alternative approach is to aggregate all the individual MPSs to provide a group MPS, as discussed in the next section, which in essence reflects the preferences of the group members or all the individual DMs.

### 4.3. Target setting with group preferences taken into account

The GMPS reference point in terms of composite outputs for customer satisfaction and total revenue are given as a convex combination of the generated MPSs shown in Table 8 by \( f^{\text{GMPS}} = [8.62, 14.98] \text{T} \), from which the LMPS can be determined. Suppose the relative weights for customer satisfaction and total revenue are assumed to be \([0.6, 0.4] \text{T}\) for NatWest. Solving formulation (32) and using \( f^{\text{GMPS}} \) as the reference point, the LMPS in terms of outputs are generated for NatWest as \( y^{\text{LMPS}}_6 = [8.62, 14.98] / C_138 \text{T} \) and the corresponding inputs are given by \( x^{\text{LMPS}}_6 = [1.45, 3.30, 7.53, 3.09] \text{T} \) with \( d = 0 \) at the optimal solution of formulation (32), which indicates that the LMPS may be an inefficient solution. To test whether the LMPS is an efficient solution, the super-ideal model (19) is run where the observed DMU (or NatWest in this case) has its LMPS added as an additional DMU in the reference set. The results are as shown in Table 10.

If the same set of relative weights of \([0.6, 0.4] \text{T}\) is used to represent the preferences of the DMs for customer satisfaction and total revenue, the LMPS can be generated for all the DMUs using the minimax reference point model (32), as shown in

---

### Table 8

Individual MPSs for all DMUs

<table>
<thead>
<tr>
<th>DMU</th>
<th>Bank</th>
<th>Inputs</th>
<th>Outputs</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>No. of branches ('000)</td>
<td>No. of ATMs ('000)</td>
</tr>
<tr>
<td>1</td>
<td>Abbey national</td>
<td>0.77</td>
<td>2.18</td>
</tr>
<tr>
<td>2</td>
<td>Barclays</td>
<td>1.29</td>
<td>3.19</td>
</tr>
<tr>
<td>3</td>
<td>Halifax</td>
<td>0.78</td>
<td>2.10</td>
</tr>
<tr>
<td>4</td>
<td>HSBC</td>
<td>1.69</td>
<td>4.00</td>
</tr>
<tr>
<td>5</td>
<td>Lloyds TSB</td>
<td>2.13</td>
<td>3.97</td>
</tr>
<tr>
<td>6</td>
<td>NatWest</td>
<td>1.29</td>
<td>3.30</td>
</tr>
<tr>
<td>7</td>
<td>RBS</td>
<td>0.65</td>
<td>1.73</td>
</tr>
</tbody>
</table>

**Note:** Data in italics shows inefficient DMUs and data in bold shows different target values as compared to DEA calculations.

### Table 9

MPS-decision variables of all DMUs

<table>
<thead>
<tr>
<th>DMU</th>
<th>Bank</th>
<th>Decision variables</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Abbey national</td>
<td>1.000</td>
</tr>
<tr>
<td>2</td>
<td>Barclays</td>
<td>0.355</td>
</tr>
<tr>
<td>3</td>
<td>Halifax</td>
<td>0.039</td>
</tr>
<tr>
<td>4</td>
<td>HSBC</td>
<td>0.741</td>
</tr>
<tr>
<td>5</td>
<td>Lloyds TSB</td>
<td>0.726</td>
</tr>
<tr>
<td>6</td>
<td>NatWest</td>
<td>0.217</td>
</tr>
<tr>
<td>7</td>
<td>RBS</td>
<td>1.000</td>
</tr>
</tbody>
</table>

### Table 10

Efficient LMPS for all DMUs

<table>
<thead>
<tr>
<th>DMU</th>
<th>Bank</th>
<th>Test of efficiency (%)</th>
<th>Composite inputs</th>
<th>Composite outputs</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>No. of branches ('000)</td>
<td>No. of ATMs ('000)</td>
<td>No. of staff ('000)</td>
</tr>
<tr>
<td>1</td>
<td>Abbey national</td>
<td>100.0</td>
<td>0.77</td>
<td>2.18</td>
</tr>
<tr>
<td>2</td>
<td>Barclays</td>
<td>95.2</td>
<td>1.26</td>
<td>3.12</td>
</tr>
<tr>
<td>3</td>
<td>Halifax</td>
<td>100.0</td>
<td>0.78</td>
<td>2.10</td>
</tr>
<tr>
<td>4</td>
<td>HSBC</td>
<td>98.0</td>
<td>1.23</td>
<td>3.03</td>
</tr>
<tr>
<td>5</td>
<td>Lloyds TSB</td>
<td>100.0</td>
<td>2.24</td>
<td>4.07</td>
</tr>
<tr>
<td>6</td>
<td>NatWest</td>
<td>93.0</td>
<td>1.32</td>
<td>3.30</td>
</tr>
<tr>
<td>7</td>
<td>RBS</td>
<td>100.0</td>
<td>0.65</td>
<td>1.73</td>
</tr>
</tbody>
</table>
The implications of the efficient LMPS for each DMU can be further analysed. For example, the efficient LMPS is different from the MPS composite unit for Halifax as shown in Tables 8 and 10. From the original DEA test, Halifax is an efficient DMU. Based on the individual DMs preferences and tradeoff analysis process, Halifax should sacrifice the output level of customer satisfaction by 16.2% from the score 9.17 to 7.69, and aim to increase its total revenue by 17.9% from £8.14 to £9.60 m. Subsequently, Halifax should better allocate or utilise its resources by decreasing its number of branches and reducing its asset size by 2.4% and 13.5%, respectively.

However, after taking account of the group preferences, it is evident that the DM has placed more emphasis on generating more revenue than a high customer satisfaction rating for Halifax. The LMPS shows that Halifax should preferably increase its total revenue by 23.8% from £8.14 to £10.08 m, while its customer satisfaction should be reduced by 25.3% from the score of 9.17 to 6.85. Only the number of branches should be reduced by 2.4% while the levels of the other inputs should be maintained. This implies that although the DM for Halifax correctly targets the area of improvements to be total revenue at the expense of customer satisfaction rating, based on the collective view of the group, generating more total revenue is of greater importance for Halifax. Note that the above analysis for Halifax is for illustration purpose. It is intended to show that tradeoffs even for efficient DMUs are possible if the decision makers so wish. However, tradeoffs along the efficient frontiers will not be possible if the decision makers are not prepared to sacrifice any output.

5. Conclusion

In this paper, interactive MOLP methods were investigated to conduct efficiency analysis and set realistic target values in an integrated way with the DMs preferences taken into account and with the DM supported to explore what could be technically achievable. The equivalence relationship established between the output-oriented DEA dual models and the minimax formulations led to the construction of the three equivalence models: namely the super-ideal point model, the ideal point model and the shortest distance model. These models share the same decision and objective spaces and are different from each other in terms of reference points and weighting schemas. They provide a basis to apply interactive tradeoff analysis methods and other techniques in MOLP to support integrated DEA-oriented performance assessment and target setting.

In this paper, the use of the interactive gradient projection approach for target setting was explored. The features of such a procedure include that the identification of normal vectors on the efficient frontier provides a vigorous measure to check whether the MPS is achieved that maximises the DMs implicit utility function. On the other hand, the projection of the utility gradient onto the tangent plane of the efficient frontier using the normal vector leads to a direction along which the DMs utility can be further improved. The MPS generated using this procedure provides feasible target values that can also maximise the DMs implicit utility function. The case study illustrated how the equivalence models and the interactive procedure can be implemented to support integrated efficiency analysis and target setting. This case study is relatively small in terms of sample size, as only 7 DMUs were included. For large scale problems with many DMUs, the equivalent minimax models will increase their size in terms of the number of decision variables. However, this would not create a problem to apply the methods proposed in this paper, as the tradeoff analyses are conducted in the objective space whose complexity is decided by the number of outputs.

Acknowledgements

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References


