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ScienceDirect
European Journal of Operational Research 195 (2009) 205–222
EUROPEAN JOURNAL OF OPERATIONAL RESEARCH
www.elsevier.com/locate/ejor

Decision Support

Integrating *DEA*-oriented performance assessment and target setting using interactive *MOLP* methods

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Received 8 October 2006; accepted 8 January 2008
Available online 20 January 2008

Abstract

Data envelopment analysis (*DEA*) and multiple objective linear programming (*MOLP*) are tools that can be used in management control and planning. Whilst these two types of model are similar in structure, *DEA* is directed to assessing past performances as part of management control function and *MOLP* to planning future performance targets. This paper is devoted to investigating equivalence models and interactive tradeoff analysis procedures in *MOLP*, such that *DEA*-oriented performance assessment and target setting can be integrated in a way that the decision makers' preferences can be taken into account in an interactive fashion. Three equivalence models are investigated between the output-oriented dual *DEA* model and the minimax reference point formulations, namely the super-ideal point model, the ideal point model and the shortest distance model. These models can be used to support efficiency analysis in the same way as the conventional *DEA* model does and also support tradeoff analysis for setting target values by individuals or groups. A case study is conducted to illustrate how *DEA*-oriented efficiency analysis can be conducted using the *MOLP* methods and how such performance assessment can be integrated into an interactive procedure for setting realistic target values.

Keywords: Data envelopment analysis; Multiple objective linear programming; Minimax method; Tradeoff analysis; Performance assessment

1. Introduction

DEA and *MOLP* can be used as tools in management control and planning. The structures of these two types of model have much in common but *DEA* is directed to assessing past performances as part of management control function and *MOLP* to planning future performances (Cooper, 2004). *DEA* has evolved tremendously over the years and emerged as a body of concepts and methodologies, which consist of a collection of models and extensions to the original work of Charnes et al. (1978). Its popularity is reflected by a large number of successful applications.

As a performance measurement and analysis technique, *DEA* is a non-parametric frontier estimation methodology based on linear programming for evaluating the relative efficiency of a set of comparable decision making units (*DMU*s) that share common functional goals. The usefulness of *DEA* extends to assessment and benchmarking against efficient units, target setting and resource allocation between inputs and outputs. The concept of *DEA* was based on the generalisation of the framework of Farrell's (1957) single output over input measurement of productive efficiency to include

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0377-2217/$ - see front matter
doi:10.1016/j.ejor.2008.01.013

multiple, incommensurate inputs and outputs that are considered at the same time to evaluate the efficiency of comparable *DMU*s.

In a broader picture, there have been various studies highlighting the similarities between *DEA* and *multiple criteria decision analysis* (*MCDA*) generally and *MOLP* in particular, though it is said that they retain their own distinctive traits (Belton and Stewart, 2001; Agrell and Tind, 2001; Joro et al., 1998; Stewart, 1994, 1996). Taking a step further, Doyle and Green (1993) suggested that *DEA* is an *MCDA* method itself. Belton and Vickers (1993) and Stewart (1996) described the equivalence between the formulations of the basic *DEA* models and the classic linear multi-attribute value function of *MCDA*. More specifically, Belton and Stewart (2001) pointed out that the mechanism of *DEA* involves comparison of *DMU*s on the basis of multiple criteria of both inputs and outputs, but the emphasis of *DEA* is put on evaluating *DMU*s against the best practice units and on setting targets to improve efficiency, whilst *MCDA* focuses on ranking and assessing alternatives. Likewise, Stewart (1996) argued that the fundamental connection between the two schools of thought is the objective function that is either for maximising outputs or for minimising inputs. While *DEA* permits determining the productive efficiency frontier of a *DMU* by optimising weighted outputs over weighted inputs, *MCDA* models allow assessing and ranking alternatives based on a set of criteria that may be conflicting and involve subjective judgments. Arguably, *DEA* could be seen as a 'lazy' *DM*s methodology to *MCDA* and Sarkis (2000) termed it as a reactive approach to *MCDA* where different alternatives are evaluated objectively.

The original *DEA* models do not include the *DM*s preference structures or value judgments in measuring relative efficiency and setting target values, so there is minimal input from the *DM*. Allen et al. (1997) defined value judgments as "logical constructs, incorporated within an efficiency assessment study, reflecting the *DM*s' preferences in the process of assessing efficiency". To incorporate the *DM*s preference information in *DEA*, various techniques have been proposed such as the goal and target setting models of Golany (1988), Thanassoulis and Dyson (1992) and Athanassopoulis (1995, 1998), and weight restriction models including imposing bounds on individual weights (Dyson and Thanassoulis, 1988), assurance region (Thompson et al., 1990), restricting composite inputs and outputs, weight ratios and proportions (Wong and Beasley, 1990), and the cone ratio concept by adjusting the observed input–output levels or weights to capture value judgments to belong to a given closed cone (Charnes et al., 1990, 1994). Alternative approaches include Thanassoulis and Simpson's model (2000) which adopts unobserved *DMU*s, derived from observed Pareto-optimal *DMU*s. Zhu (1996) also integrated preference information into a modified *DEA* formulation, while Golany (1995) used hypothetical *DMU*s to represent preference information. However, the above-mentioned techniques require *prior* preferences from the *DM*, which are often subjective. Nevertheless, preferences required for setting future target values may not be provided *a priori* but need to be generated on the basis of what are realistically achievable.

An appealing method to incorporate preference information into both efficiency analysis and target setting, without necessarily requiring *prior* judgments, is the use of interactive *MOLP* techniques. Golany (1988) first proposed an interactive model combining both *DEA* and *MOLP* approaches where the *DM* is assumed to be able to allocate a set of input levels as resources and to select the most preferred set of output levels from a set of alternative points on the efficient frontier. Post and Spronk (1999) also proposed combining the use of *DEA* and interactive multiple goal programming where preference information is incorporated interactively by the *DM* by adjusting the upper and lower feasible boundaries of the input and output levels. Joro et al. (1998) showed the synergies between *DEA* and *MOLP* and proved that the *DEA* formulations are structurally similar to *MOLP* models based on the reference point approach. In summary, the effective integration of assessing past performances and planning future targets with the *DM*s preferences taken into account is of increasing interests to support both management control and planning (Cooper, 2004). This has indeed motivated the research as reported in this paper.

To facilitate performance assessment and target value setting in the domain of *MOLP* in an integrated way, three equivalence models in *MOLP* are investigated in this paper, including the super-ideal point model, the ideal point model and the shortest distance model. The super-ideal point model is proven identical to the output oriented *DEA* dual model and thus can be used to generate, in the context of *MOLP*, the same efficiency scores and corresponding composite inputs and outputs just as the output-oriented *DEA* dual model does. In other words, an efficient solution generated using the *DEA* model is in fact the one on the efficient frontier that is the closest to the super ideal point in the objective space expanded by the composite outputs, as investigated in detail later in this paper. On the other hand, the generic *MOLP* formulation, in which the super-ideal point model is built, provides a platform for exploring efficiency measures and efficient frontiers using the concepts and techniques in *MOLP* and also for design of solution schemas in which to conduct interactive tradeoff analysis for setting realistic target values for an *DMU* within its existing production possibility set expanded by the original *DMU*s in question.

The second ideal point model is designed from the generic *MOLP* formulation, hence sharing the same decision and objective spaces with the super-ideal point model. It is used to construct an interactive tradeoff analysis procedure based on the gradient projection and local region search method (Li and Yang, 1996; Yang and Sen, 1996; Yang, 1999; Yang and Li, 2002) to locate a most preferred solution (*MPS*) that can maximize the *DM*s implicit utility function using the *DM*s local preference information. Such a *MPS* is then set as a target for the observed *DMU* to benchmark against. In this interactive process, the DM can explore what could be technically achieved and therefore gets in a better position to decide what should be planned as future targets. The gradient projection is conducted through the identification of normal vectors on the efficient frontier based on the ideal point model. A normal vector itself provides information about the optimal indifference tradeoff that the MPS must satisfy, so it can also be used as a criterion to terminate the interactive process. On the other hand, the projection of the gradient of an implicit utility function onto the tangent plane of the efficient frontier using the normal vector provides a direction closest to the efficient frontier, along which a better efficient solution can be sought.

The third shortest distance model is designed also from the generic $MOLP$ formulation and can be used to facilitate group negotiation and discussion in deciding overall and localised performance targets with both individual and group preferences taken into account. It uses a group MPS ($GMPS$) as a reference point, which could be provided either by a leading DM with overall responsibility for the performance management of an organisation or generated by aggregating solutions locally preferred by individual DMUs. A case study for analysing the efficiencies of seven UK retail banks and setting their business targets is conducted to illustrate and validate the proposed models and procedure.

The remainder of the paper is organised as follows. Section 2 provides a brief description of the typical DEA models and basic multiple objective optimisation techniques, followed by an investigation into the incorporation of DMs preferences into management planning. Section 3 reports the investigation into the integrated performance assessment and target value setting. Section 4 shows a case study on bank performance analysis using the integrated approach. The paper is concluded in Section 5.

2. *DEA*-oriented performance assessment and target setting

DEA was initially developed by Charnes et al. (1978) for measuring and analyzing the relative efficiencies of comparable DMUs with incommensurate inputs and outputs. In DEA, an efficient frontier is formed, where efficient DMUs lie. An efficient DMU means that no other DMU can either produce the same outputs by consuming fewer inputs, known as the input-orientated approach, or produce more outputs by consuming the same inputs, known as output-orientated approach. The mechanism behind the methodology of the conventional DEA models is that it works on maintaining the appropriate input–output mix so as to project inefficient DMUs radially onto the efficient frontier. The DEA models can provide efficiency scores scaled to a maximum value of 1 for efficient DMUs and can inform the DM of the amount of percentage by which an inefficient DMU should decrease its inputs and/or increase its outputs in order to become efficient. It also provides reference units known as composite or virtual units which lie on the efficient frontier and are used as target units for inefficient DMUs to benchmark against.

2.1. Typical output-orientated DEA models for performance assessment

The original DEA model proposed by Charnes et al. (1978) is a fractional non-linear programming model, known as the CCR model. The objective function in the model is to maximise the single ratio of the weighted outputs over weighted inputs for a particular DMU, referred to as an observed DMU and denoted by DMU_o.

Suppose an organisation has n DMUs ($j = 1,\dots,n$), produces s outputs denoted by y_{rj} (the rth output of DMU j for $r = 1,\dots,s$) and consumes m inputs denoted by x_{ij} (the ith input of DMU j for $i = 1,\dots,m$). The fractional formulation of DEA is then defined as follows:

$$\begin{aligned} &\text{Max} \quad e_o = \sum_{r=1}^{s} u_r y_{rj_o} \Big/ \sum_{i=1}^{m} v_i x_{ij_o} \\ &\text{s.t.} \quad \sum_{r=1}^{s} u_r y_{rj} \Big/ \sum_{i=1}^{m} v_i x_{ij} \leqslant 1, \quad \forall\, j = 1,\dots,n;\ u_r, v_i \geqslant 0 \quad \text{for} \quad r = 1,\dots,s;\ i = 1,\dots,m \end{aligned} \tag{1}$$

u_r is the weight parameter for output r and v_i the weight parameter for input i. e_o denotes the optimal efficiency score with a possible range of $0 \leqslant e_o \leqslant 1$. The score of $e_o = 1$ represents full efficiency and $0 < e_o < 1$ reveals the presence of inefficiency. Each DMU can be evaluated by setting itself as the observed DMU_o and is allowed freedom in the DEA model to assign the set of its own output weights u_r and input weights v_i, which will render the observed DMU as efficient as possible. In other words, the efficiency measure e_o is maximised within the production possibility set formulated by the n DMUs (Cooper et al., 2000). It is noted that the fractional program model is a computationally complex non-linear and non-convex problem, making calculations extremely difficult for large scale problems (Charnes et al., 1978). As such, Charnes introduced the transformation of the fractional programming problem into equivalent linear programming problems. In this paper, we investigate the equivalence between the output-oriented DEA dual models and the minimax formulations in $MOLP$, so only the typical output-oriented DEA models are briefly discussed in this section.

The following *LP* models are the conventional *CCR* models for efficiency analysis

Output-orientated *CCR* primal model.

$$\begin{aligned} \text{Min} \quad & h_o = \sum_{i=1}^{m} v_i x_{ij_o} \\ \text{s.t.} \quad & \sum_{i=1}^{m} v_i x_{ij} - \sum_{r=1}^{s} u_r y_{rj} \geqslant 0 \quad j = 1, \ldots, n, \\ & \sum_{r=1}^{s} u_r y_{rj_o} = 1, u_r, v_i \geqslant 0 \quad \text{for all } r, i. \end{aligned} \tag{2}$$

Output-Orientated *CCR* Dual Model

$$\begin{aligned} \text{Max} \quad & h_o = \theta_{j_o} \\ \text{s.t.} \quad & \theta_{j_o} y_{rj_o} - \sum_{j=1}^{n} \lambda_j y_{rj} \leqslant 0 \quad r = 1, \ldots, s, \\ & \sum_{j=1}^{n} \lambda_j x_{ij} \leqslant x_{ij_o}, \quad i = 1, \ldots, m; \ \lambda_j \geqslant 0 \quad \text{for all } j. \end{aligned} \tag{3}$$

In the output-orientated *CCR* primal model (2), the weighted outputs are fixed to unity and the weighted inputs minimised. The output weights u_r and input weights v_i are adjusted accordingly to generate an efficiency score which is given by $e_o = 1/h_o$. In the output-orientated *CCR* dual model (3), for each observed DMU_o an imaginary composite unit is constructed that outperforms DMU_o. λ_j represents the proportion to which *DMU j* is used to construct the composite unit for DMU_o ($j = 1, \ldots, n$). The composite unit consumes at most the same levels of inputs as DMU_o and produces outputs that are at least equal to a proportion θ_{j_o} of the outputs of DMU_o with $\theta_{j_o} \geqslant 1$. The inverse of θ_{j_o} is the efficiency score of DMU_o. If $\theta_{j_o} > 1$, DMU_o is not efficient and the parameter θ_{j_o} indicates the extent by which DMU_o has to increase its outputs to become efficient. The increase is employed concurrently to all outputs and results in a radial movement towards the envelopment surface (Charnes et al., 1994). Note that such a radial improvement strategy is imbedded in the *DEA* modelling mechanism *a priori* and does not necessarily take account of management preferences, so it is technical rather than preferential. In the following sections, we will explore how *DM*s preferences can be incorporated into improvement strategies using interactive multiple objective optimisation techniques. Note that the above models are based on constant returns to scale (*CRS*). This, however, disregards economies of scale. Variable returns to scale in efficiency analysis were taken into account in another version of *DEA* model developed by Banker et al.*BCC* (1984), called the *BCC* model which is different from the *CCR* model in that the former has an additional convexity constraint of all λ_j restricted to sum to 1 in the dual case.

2.2. Basic concepts and minimax formulations in multiple objective optimization

In a *DEA* model, an efficiency score is generated for a *DMU* by maximizing outputs with limited inputs, or minimizing inputs with desired or fixed outputs, or simultaneously maximizing outputs and minimizing inputs. Either way, this can be regarded as a kind of multiple objective optimization problem. In this section, we briefly describe basic concepts and models in multiple objective optimization, in particular the minimax formulations as a basis for the investigation to be reported in the following sections.

Suppose an optimization problem has *s* objectives reflecting the different purposes and desires of the *DM*. Such a problem can be represented in a general form as follows:

$$\begin{aligned} \text{Max} \quad & f(\lambda) = [f_1(\lambda), \ldots, f_r(\lambda), \ldots, f_s(\lambda)] \\ \text{s.t.} \quad & \lambda \in \Omega = \{\lambda \mid g_j(\lambda) \leqslant 0, \quad h_l(\lambda) = 0; \ j = 1, \ldots, k_1, \ l = 1, \ldots, k_2\}, \end{aligned} \tag{4}$$

where Ω is the feasible decision space, $f_r(\lambda)(r = 1, \ldots, s)$ are continuously differentiable objective functions, and $g_j\ (\lambda)(j = 1, \ldots, k_1)$ and $h_l(\lambda)\ (l = 1, \ldots, k_2)$ are continuously differentiable inequality and equality constraint functions, respectively. In this paper, $f_r(\lambda)$, $g_j(\lambda)$ and $h_l(\lambda)$ are all assumed to be linear functions of λ, so formulation (4) is referred to as multiple objective linear programming or *MOLP* in short.

Due to conflict among objectives in general, a *MOLP* problem does not normally have a single solution that could optimize all objectives simultaneously. What can be generated are efficient or non-dominated solutions. Conceptually, a feasible solution λ^* is said to be efficient or non-dominated if there exists no other feasible solution which is better than λ^* at least on one objective and as good as λ^* on all other objectives. An efficient or non-dominated solution is also referred to as a Pareto-optimal solution, which can be formally defined as follows. For simplicity, we use the term "efficient solution" in this paper.

Definition 1. In formulation (4), a feasible solution $\lambda^* \in \Omega$ is an efficient solution if and only if there does not exist any other feasible solution $\lambda \in \Omega$ such that $f_r(\lambda) \geqslant f_r(\lambda^*)$ for all $r = 1, \ldots, s$ and $f_\tau(\lambda) > f_\tau(\lambda^*)$ for at least one $\tau \in \{1, \ldots, s\}$.

Any efficient solutions of a *MOLP* problem can be generated using the weighted minimax formulation (Steuer and Choo, 1983; Yang, 1999; Yang and Li, 2002). Suppose λ is an efficient solution of (4) and f_r^* the maximum feasible value of objective r. There exists a weighting vector w satisfying $w_1 = 1$ and $w_r > 0$ for $r = 2, \ldots, s$ such that λ can be generated by solving the following weighted minimax problem

$$\begin{aligned} &\underset{\lambda}{\text{Min}} \ \underset{1 \leqslant r \leqslant s}{\text{Max}} \quad \left\{ w_r \left| (f_r^* - f_r(\lambda)) \right| \right\} \\ &\text{s.t.} \qquad\qquad \lambda \in \Omega. \end{aligned} \tag{5}$$

The weighted minimax formulation will be called the ideal point model, a special case of the reference point formulation, if $f^* = \left[f_1^* \cdots f_r^* \cdots f_s^* \right]^{\mathrm{T}}$ is the ideal point and used as the reference point. In the minimax formulation, for a given weight vector the *DM* is assumed to be satisfied with an efficient solution $\lambda \in \Omega$ at which $f(\lambda)$ has the shortest weighted distance from f^* measured in ∞-norm in the objective space.

If f^* is an ideal point, then the non-smooth weighted minimax formulation given in (5) can be equivalently transformed into the following smooth form by introducing an auxiliary variable θ (Lightner and Director, 1981; Yang and Li, 2002)

$$\begin{aligned} &\text{Min} \quad \theta \\ &\text{s.t.} \quad w_r(f_r^* - f_r(\lambda)) \leqslant \theta \quad r = 1, \ldots, s; \ \lambda \in \Omega. \end{aligned} \tag{6}$$

Note that the above minimax formulation (6) is still equivalent to formulation (5) if f^* is replaced by a better reference (super-ideal) point f^+ with $f^+ \geqslant f^*$. It will be shown that the minimax formulation using a particular super-ideal point as the reference point can be used to generate *DEA* scores and corresponding composite inputs and outputs in the same way as the output-oriented dual *DEA* model does, and also to design an interactive procedure to support the *DM* to search for the *MPS* on the efficient frontier by systematically changing the weighting parameters $w_r(r = 1, \ldots, s)$.

2.3. Incorporation of DMs preferences into target setting

DEA models can be used to measure and assess how efficiently an organisation utilises its resources (inputs) to generate desirable outcomes (outputs) in its business activities in comparison with its peers (*DMU*s). In this sense, *DEA* is a management control tool and can be used to identify whether a business is run efficiently and where it currently stands in the market. If a *DMU* is found to be inefficient, *DEA* can provide suggestions as to where and by how much it should be improved in order to achieve full efficiency in comparison to its peers. As such, *DEA* does provide certain degrees of support to both management control and planning. However, in supporting management planning, conventional *DEA* does not take appropriate account of the *DM*s preferences. This is explained in some detail as follows.

In Fig. 1, there are five *DMU*s: A, B, C, D and E with the first four *DMU*s being fully efficient and E inefficient which is the observed *DMU*. $\overline{AB}, \overline{BC}$ and $\overline{CD}$ constitute the efficient frontier. In *DEA*, the efficiency score of the observed *DMU* E is measured by $\overline{OE}/\overline{OE_1}$ with E_1 being the intersection point of the efficient segment $\overline{AB}$ and a line emitting from the origin O through the point E. Since E is inefficient in comparison with A, B, C and D, *DEA* technically suggests the radial improvement of *DMU* E from point E to point E_1 along the line. Whilst this is a valid suggestion, there are many other alternatives.

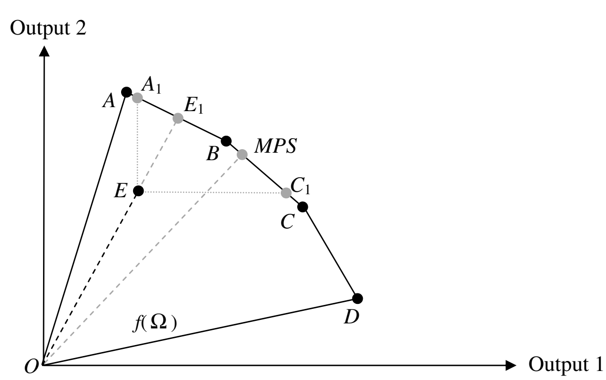

Fig. 1. Illustration of efficiency analysis and tradeoffs.

In fact, in Fig. 1 any efficient solution on the segments $\overline{A_1B}$ and $\overline{BC_1}$ of the efficient frontier dominates $DMU\ E$ and could potentially be used as its target, or its MPS. In general, if tradeoffs between outputs are allowed in management planning, other efficient solutions could also be candidates as the MPS for $DMU\ E$, depending on the DMs preferences. Even if a DMU is already technically efficient, the DMs may prefer to achieve a better balance between outputs than its current output levels without necessarily consuming more resources, in other words to find a new MPS along the efficient frontier as future targets. Although there are existing DEA models that support target setting by allowing restrictions on input weights and output weights or by imposing input and output targets, they require *prior* knowledge about what are achievable, which, however, depends on the selection of reference DMUs and their performances. Thus, such knowledge in general is not available *a priori* but needs to be explored.

The above discussions show that a few questions remain to be answered after the performance of each DMU is assessed by generating a DEA score and a technical target such as E_1 for $DMU\ E$ in Fig. 1. The first question is how to find different realistic targets from E_1 that $DMU\ E$ could benchmark against based on its current performances and within the existing production possibility set. The second question is how to support the DMs to identify a new target that is not only technically achievable but also most preferred by the DMs.

Multiobjective optimization is a tool for management planning and can be used to support the search for efficient solutions and the location of the MPS. To answer these questions, a super-ideal point model will be first investigated in next section which is proven identical to the output-orientated CCR dual model. The super-ideal point model can thus be used for conventional efficiency analysis. Furthermore, it leads to the identification of a generic $MOLP$ formulation, which defines the production possibility set of the observed DMU and in which solution schemas can be developed to explore all efficient solutions of a DMU.

An interactive tradeoff analysis procedure based on the ideal point model and the gradient projection method will then be investigated to support the search for the MPS with the DMs preferences taken into account. In this procedure, the implicit utility function of the DM can be maximised by using local tradeoff information. A group procedure is also suggested on the basis of the shortest distance model to support the negotiations and discussions in setting targets with both organizational and individuals preferences taken into account.

3. Interactive *MOLP* methods for integrating efficiency analysis and target setting

In this section, we first investigate the equivalence between the output-oriented DEA dual model (3) and the minimax reference point formulations. A super-ideal point model will first be investigated for conducting efficiency analysis in the same way as the DEA output-oriented dual model does. An equivalent ideal point model will then be investigated to design an interactive tradeoff analysis procedure for locating the MPS by systematically adjusting weights. Finally, a group decision making process is proposed to support the determination and mapping of group most preferred solution ($GMPS$), which is used as a new reference point to construct the shortest distance model to identify locally most preferred solutions ($LMPS$s) or to set or update target values for individual DMUs.

3.1. Conducting DEA-oriented performance assessment using a MOLP method

The output-orientated CCR dual model, as shown in formulation (3), can be equivalently re-written as follows:

$$\begin{aligned}
&\text{Max} \quad \theta_{j_o} \\
&\text{s.t.} \quad \theta_{j_o} y_{rj_o} - \sum_{j=1}^{n} \lambda_j y_{rj} \leqslant 0, \quad r = 1, \ldots, s, \\
&\qquad \lambda \in \Omega_{j_o} = \left\{ \lambda \,\middle|\, \sum_{j=1}^{n} \lambda_j x_{ij} \leqslant x_{ij_o}, \quad i = 1, \ldots, m;\ \lambda_j \geqslant 0,\ j = 1, \ldots, n \right\}.
\end{aligned} \tag{7}$$

In this subsection, we first show that formulation (7) is the same as formulation (6) under certain conditions. The purpose for establishing this equivalence is to use formulation (6) to conduct efficiency analysis, so that interactive $MOLP$ techniques can be used to locate the MPS or set target values for the observed DMU_o. Note that in formulation (6) the weight w_r is subject to change in an interactive process of locating the MPS.

Suppose in formulation (7) the rth composite output is denoted by $f_r(\lambda)$ as follows:

$$f_r(\lambda) = \sum_{j=1}^{n} \lambda_j y_{rj} (r = 1, \ldots, s) \quad \text{and} \quad \lambda = (\lambda_1, \ldots, \lambda_n)^{\mathrm{T}}. \tag{8}$$

In this equivalence analysis, the rth composite output is defined as an objective for maximisation, so there are s objectives in total. The maximum feasible value of the rth composite output for the observed DMU_o is denoted by $\bar{f}_{rj_o} = f_r(\lambda^*)$, where λ^* can be found by solving the following single objective optimisation problem:

$$\begin{aligned} &\text{Max} \quad f_r(\lambda) = \sum_{j=1}^{n} \lambda_j y_{rj} \\ &\text{s.t.} \quad \lambda \in \Omega_{j_o}. \end{aligned} \tag{9}$$

Note that $\bar{f}_{j_o} = [\bar{f}_{1j_o}, \ldots, \bar{f}_{sj_o}]^{\mathrm{T}}$ is the ideal point in the objective space spanned by (8) on Ω_{j_o} for the observed DMU_o, as shown in Fig. 2.

Suppose the feasible decision space Ω in formulation (6) is set to be the same as defined in formulation (7), or $\Omega = \Omega_{j_o}$. The equivalence relationship between the output-oriented CCR dual model (3) or (7) and the minimax formulation (6) can be established as follows.

Suppose $y_{rj_o} > 0$ for all $r = 1, \ldots, s$ and $j_o = 1, \ldots, n$. The output-oriented CCR dual model (7) can be equivalently transformed to the minimax formulation (6) using formulations (8) and (9) and the following equations:

$$w_r = 1/y_{rj_o}, \tag{10}$$

$$f_r^* = \frac{F^{\max}}{w_r} = y_{rj_o} F^{\max}, \tag{11}$$

$$\theta = F^{\max} - \theta_{j_o} \tag{12}$$

with

$$F^{\max} = \max_{1 \leqslant r \leqslant s} \{ w_r \bar{f}_{rj_o} \} = \max_{1 \leqslant r \leqslant s} \left\{ \frac{\bar{f}_{rj_o}}{y_{rj_o}} \right\}. \tag{13}$$

The above equivalence relation can be explained as follows. Using Eqs. (8) and (10), the output-orientated CCR dual model can be equivalently rewritten as follows:

$$\begin{aligned} &\text{Max} \quad \theta_{j_o} \\ &\text{s.t.} \quad \theta_{j_o} \frac{1}{w_r} - f_r(\lambda) \leqslant 0, \quad r = 1, \ldots, s; \ \lambda \in \Omega_{j_o}. \end{aligned} \tag{14}$$

The first s objective constraints in (14) can be equivalently transformed as follows, where "$\Leftrightarrow$" means "is equivalent to". For any $r = 1, \ldots, s$, we have

$$\theta_{j_o} \frac{1}{w_r} - f_r(\lambda) \leqslant 0 \Longleftrightarrow -w_r f_r(\lambda) \leqslant -\theta_{j_o} \Longleftrightarrow F^{\max} - w_r f_r(\lambda) \leqslant F^{\max} - \theta_{j_o} \Longleftrightarrow w_r(f_r^* - f_r(\lambda)) \leqslant \theta. \tag{15}$$

Moreover, the objective function of model (14) becomes

$$\text{Max } \theta_{j_o} = \text{Min}(-\theta_{j_o}) = \text{Min}(F^{\max} - \theta_{j_o}) = \text{Min } \theta. \tag{16}$$

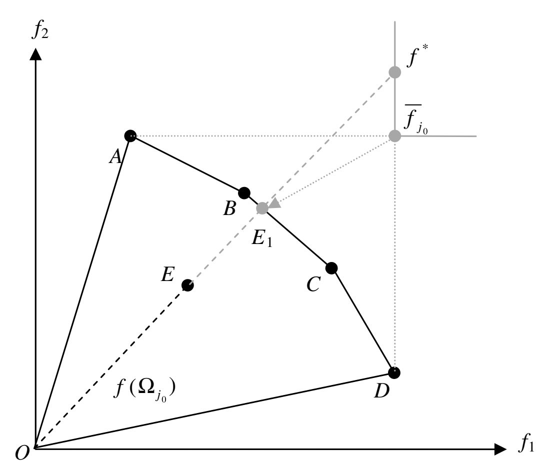

Fig. 2. Illustration of super-ideal point and ideal point.

Note that for any $\lambda \in \Omega_{j_o}$

$$\theta = F^{\max} - \theta_{j_o} \geqslant w_r \bar{f}_{rj_o} - \theta_{j_o} \geqslant w_r f_r(\lambda) - \theta_{j_o} \geqslant 0 \quad \text{for any} \quad r = 1, \ldots, s, \tag{17}$$

$$f_r^* = \frac{F^{\max}}{w_r} \geqslant \frac{w_r \bar{f}_{rj_o}}{w_r} = \bar{f}_{rj_o} = \max_{\lambda \in \Omega_{j_o}} f_r(\lambda) \quad \text{for any} \quad r = 1, \ldots, s. \tag{18}$$

The above analyses show that if the reference point in model (6) is set by $f^* = [f_1^*, \ldots, f_s^*]^{\mathrm{T}} = F^{\max}[y_{1j_o}, \ldots, y_{sj_o}]^{\mathrm{T}}$ and w_r by Eq. (10), then the output-oriented *CCR* dual model will be identical to the following minimax reference point formulation:

$$\begin{aligned} &\text{Min} \quad \theta \\ &\text{s.t.} \quad w_r\left(f_r^* - \sum_{j=1}^n \lambda_j y_{rj}\right) \leqslant \theta \quad r = 1, \ldots, s, \\ &\qquad \sum_{j=1}^n \lambda_j x_{ij} \leqslant x_{ij_o}, i = 1, \ldots, m, \quad \lambda_j \geqslant 0, \ j = 1, \ldots, n. \end{aligned} \tag{19}$$

By identical we mean that they share the same decision and objective spaces and have the same optimal solution. Since $f^* = F^{\max}[y_{1j_o}, \ldots, y_{sj_o}]^{\mathrm{T}} \geqslant \bar{f}_{j_o} = [\bar{f}_{1j_o}, \ldots, \bar{f}_{sj_o}]^{\mathrm{T}}$, such a f^* is called super-ideal point and the minimax reference point formulation established using the super-ideal point is therefore referred to as the super-ideal point model in this paper. The output-oriented *BCC* dual model can also be transformed to an identical minimax formulation similar to the above super-ideal point model with an extra convexity constraint of $\sum_{j=1}^n \lambda_j = 1$.

From equivalence Eqs. (10)–(13), the following three remarks can be drawn.

Remark 1. The super-ideal point model (19) can be used to generate the *CCR* efficiency score and the efficient composite inputs and outputs of DMU_o if in model (19) w_r is calculated by using Eq. (10) and f_r^* by Eqs. (11) and (13).

The above analyses show that the *CCR* dual model is actually constructed to locate a specific efficient solution, termed as *DEA* efficient solution on the efficient frontier of the following generic *MOLP* formulation for the observed DMU_o

$$\begin{aligned} &\text{Max} \quad \left[\sum_{j=1}^n \lambda_j y_{1j}, \ldots, \sum_{j=1}^n \lambda_j y_{rj}, \ldots, \sum_{j=1}^n \lambda_j y_{sj}\right] \\ &\text{s.t.} \quad \sum_{j=1}^n \lambda_j x_{ij} \leqslant x_{ij_o}, i = 1, \ldots, m, \quad \lambda_j \geqslant 0, \end{aligned} \tag{20}$$

from which the super-ideal point model was constructed. The generic *MOLP* problem (20) defines the production possibility set for the observed DMU_o, in which there may be more preferred efficient solutions than the *DEA* efficient solution.

Remark 2. In the minimax reference point formulation (19), if the reference point is set as the ideal point, or $f^* = \bar{f}_{j_o} = [\bar{f}_{1j_o}, \ldots, \bar{f}_{sj_o}]^{\mathrm{T}}$, and w_r is allowed to change, then we get the conventional ideal point model equivalent to the *CCR* dual model in the sense that they share the same decision and objective spaces. Changing w_r in R^+ leads to the generation of any efficient solutions for the observed DMU_o, as defined in formulation (20). For the purpose of initialising an interactive procedure, an initial efficient solution can be generated by assigning normalised equal weights to the first s constraints in formulation (19). Alternatively, let $f_r^t = f_r(\lambda^t)$ be the value of the rth composite output at the solution λ^t, defined in problem (9). If $f^t = [f_1^t, \ldots, f_s^t]^{\mathrm{T}}$ happens to be on the efficient frontier, then setting $w_r = 1/(\bar{f}_{rj_o} - f_r^t)$ for the rth output and solving the ideal point model (19) will lead to the identification of $\lambda^t = [\lambda_1^t, \ldots, \lambda_s^t]$ with $f_r^t = \sum_{j=1}^n \lambda_j^t y_{rj}$ (Lightner and Director, 1981; Yang and Li, 2002).

Remark 3. If $f^t = [f_1^t, \ldots, f_s^t]^{\mathrm{T}}$ is not on the efficient frontier, then setting $w_r = 1/(\bar{f}_{rj_o} - f_r^t)$ and solving problem (19) will lead to the identification of λ^t with $f_r^t \neq \sum_{j=1}^n \lambda_j^t y_{rj}$ and $\left[\sum_{j=1}^n \lambda_j^t y_{1j}, \ldots, \sum_{j=1}^n \lambda_j^t y_{sj}\right]^{\mathrm{T}}$ lying at the crossing point on the efficient frontier by a line from the ideal point $\bar{f}_{j_o} = [\bar{f}_{1j_o}, \ldots, \bar{f}_{sj_o}]^{\mathrm{T}}$ towards $[f_1^t, \ldots, f_s^t]^{\mathrm{T}}$. In this case, $\sum_{j=1}^n \lambda_j^t y_{rj}$ might not be the target value of the rth output preferred by the *DM*. To help the *DM* set realistic target values, a gradient projection based interactive tradeoff procedure can be employed by calculating the normal vector of the efficient frontier at $\left[\sum_{j=1}^n \lambda_j^t y_{1j}, \ldots, \sum_{j=1}^n \lambda_j^t y_{sj}\right]^{\mathrm{T}}$. This projection can be used to assist the *DM* to provide a realistic tradeoff for finding better efficient solutions than λ^t. At each interaction, a new weighting vector generated from the tradeoff can be determined and used to solve formulation (19), leading to a new efficient solution λ^{t+1}.

Graphically, the super-ideal point f^* and the ideal point $\bar{f}_{j_o}$ are shown in Fig. 2 in the objective space spanned by f_1 and f_2 defined in Eq. (8). The observed DMU E is given by $E = [y_{1j_o}, y_{2j_o}]^T$ and assumed to be inefficient in Fig. 2. E_1 is the intersection of the efficient frontier and a line emitting from the origin O through the point E, given by $E_1 = \theta_{j_o}[y_{1j_o}, y_{2j_o}]^T$. The efficiency score can be calculated by $e_o = \overline{OE}/\overline{OE}_1 = 1/\theta_{j_o}$, irrespective of the number of inputs. The ideal point is given by $\bar{f}_{j_o} = [\bar{f}_{1j_o}, \bar{f}_{2j_o}]^T$. The super-ideal point is defined by $f^* = F^{\max}[y_{1j_o}, y_{2j_o}]^T$, which is proportional to point E. So, f^*, E and E_1 are all on the above same line. In Fig. 2, $(y_{2j_o}/y_{1j_o}) > (\bar{f}_{2j_o}/\bar{f}_{1j_o})$, or $(\bar{f}_{1j_o}/y_{1j_o}) > (\bar{f}_{2j_o}/y_{2j_o})$. So, $F^{\max} = \bar{f}_{1j_o}/y_{1j_o}$ and $f^* = \bar{f}_{1j_o}[1, (y_{2j_o}/y_{1j_o})]^T$, which means that f^* and $\bar{f}_{j_o}$ are on the same vertical line with f^* above $\bar{f}_{j_o}$. From Remark 1, the solution of the super-ideal point model leads to $f^* - E_1 = \theta[y_{1j_o}, y_{2j_o}]^T$, or $F^{\max}[y_{1j_o}, y_{2j_o}]^T - \theta_{j_o}[y_{1j_o}, y_{2j_o}]^T = \theta[y_{1j_o}, y_{2j_o}]^T$, so $\theta = F^{\max} - \theta_{j_o}$. On the other hand, changing weights in R^+ and solving the ideal-point model can identify any efficient solutions from point A to point D.

3.2. An interactive tradeoff analysis procedure for setting target values

The gradient projection method can be used to identify a normal vector at a given efficient solution on the efficient frontier and to design an interactive procedure for searching for the MPS that maximizes the DMs implicit utility function. We first describe the method and then develop the tradeoff procedure.

Suppose for a given positive weight vector $w^t = \{w_1^t \cdots w_r^t \cdots w_s^t\}$ the optimal solution of the minimax model (6) is given by $\lambda^t = \{\lambda_1^t \cdots \lambda_r^t \cdots \lambda_s^t\}$, which must be an efficient solution. The optimal value of the dual variable (simplex multiplier in linear case) of the rth objective constraint $w_r(f_r^* - f_r(\lambda)) \leqslant \theta$ is given by β_r^t. Let $f(\lambda^t) = [f_1(\lambda^t) \cdots f_r(\lambda^t) \cdots f_s(\lambda^t)]^T$ represent the efficient solution in the corresponding objective space. It is proved that the normal vector N^t at $f(\lambda^t)$ on the efficient frontier is given by (Yang and Li, 2002):

$$N^t = [w_1^t\beta_1^t, w_2^t\beta_2^t, \ldots, w_r^t\beta_r^t, \ldots, w_s^t\beta_s^t]^T. \tag{21}$$

In linear programming, β_r^t can be generated using many existing software packages at no extra cost. Suppose the implicit utility function of the DM is denoted by $u\,(f(\lambda))$. The gradient of $u\,(f(\lambda))$ at the solution $f(\lambda^t)$ is given by

$$\nabla u^t = \frac{\partial u}{\partial f(\lambda^t)} = \left[\frac{\partial u}{\partial f_1^t} \cdots \frac{\partial u}{\partial f_r^t} \cdots \frac{\partial u}{\partial f_s^t}\right]^T. \tag{22}$$

Although $u\,(f(\lambda))$ is unknown in general, the utility gradient ∇u^t may be estimated using the local preference information of the DM, for example marginal rates of substitution.

If $f(\lambda^t)$ is the MPS, it is necessary as well as sufficient in a convex case (e.g. linear case) that the normal vector N^t is proportional to the utility gradient ∇u^t at $f(\lambda^t)$, or

$$[w_1^t\beta_1^t \cdots w_r^t\beta_r^t \cdots w_s^t\beta_s^t]^T \propto \left[\frac{\partial u}{\partial f_1^t} \cdots \frac{\partial u}{\partial f_r^t} \cdots \frac{\partial u}{\partial f_s^t}\right]^T. \tag{23}$$

If the optimal condition were met, then the gradient $\vec{G}$, as shown in Fig. 3, would be proportional to the normal vector $\vec{N}$; consequently, the dashed line would overlap the dotted line in Fig. 3, or point E_3 would overlap E_2. If the optimal condition is not met, as is the case in Fig. 3, then the gradient $\vec{G}$ can be projected onto the tangent plane (dotted line) of the efficient frontier at $f(\lambda^t)$. Such projection provides a direction on or closest to the efficient frontier, along which the DMs utility can be improved. The projection, denoted by Δu^t as shown in Fig. 3, is given by

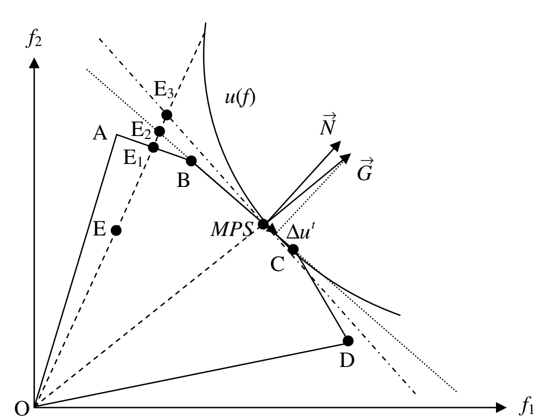

Fig. 3. Projection of utility gradient onto tangent plane of efficient frontier.

$$\Delta u^t = [\Delta f_1^t \cdots \Delta f_r^t \cdots \Delta f_s^t]^{\mathrm{T}} = -\nabla u^t + \frac{[(\nabla u^t)^{\mathrm{T}} N^t]}{[(N^t)^{\mathrm{T}} N^t]} N^t. \tag{24}$$

However, since the utility function is not known explicitly, a utility gradient needs to be estimated by for example indifference tradeoffs or marginal rates of substitution, M, which may be provided by the DM. Without loss of generality, set the first objective f_1 as the reference objective. Then the indifference tradeoff M_{1r}^t between the first and the rth objectives and the marginal rate of substitution M^t at $f(\lambda^t)$ are given by:

$$M_{1r}^t = -\frac{\mathrm{d}f_1^t}{\mathrm{d}f_r^t} \quad \text{and} \quad M^t = [1, M_{12}^t \cdots M_{1r}^t \cdots M_{1s}^t]^{\mathrm{T}}, \tag{25}$$

where $\mathrm{d}f_1^t$ is a change in $f_1(\lambda)$ that is assumed to be exactly offset by a change $\mathrm{d}f_r^t$ in $f_r(\lambda)$ with the overall utility kept constant, given that all other objectives remain unchanged. It can be shown that the gradient of the utility function ∇u^t given in (22) is proportional to the marginal rate of substitution M^t at $f(\lambda^t)$, or

$$M^t = \frac{\nabla u^t}{\partial u / \partial f_1^t}. \tag{26}$$

At the MPS, the following optimal indifference tradeoff between $f_1(\lambda)$ and the rth objective can be calculated using the following equation:

$$\mathrm{d}f_r^t = -\mathrm{d}f_1^t \frac{w_1^t \beta_1^t}{w_r^t \beta_r^t}. \tag{27}$$

On the other hand, the optimal indifference tradeoff can be used to check whether the MPS is achieved.

If the MPS is not achieved, then the projection Δu^t can be calculated using M^t as follows and will not be zero, denoted by $\Delta \bar{u}^t$, which provides a new tradeoff direction to improve the DMs utility:

$$\Delta \bar{u}^t = [\Delta \bar{f}_1^t \cdots \Delta \bar{f}_r^t \cdots \Delta \bar{f}_s^t]^{\mathrm{T}} = -M^t + \frac{(M^t)^{\mathrm{T}} N^t}{(N^t)^{\mathrm{T}} N^t} N^t. \tag{28}$$

Suppose $\bar{\alpha}$ is a tradeoff step. Update the weighting parameters w_r as follows:

$$w_r^{t+1} = \frac{f_1(\lambda^t) + \bar{\alpha} \Delta \bar{f}_1^t - f_1(\lambda^*)}{f_r(\lambda^t) + \bar{\alpha} \Delta \bar{f}_r^t - f_r(\lambda^*)} \quad r = 1, \ldots, s. \tag{29}$$

Replacing w_r by w_r^{t+1} in formulation (6) and solving it leads to a new solution λ^{t+1} which should have a higher utility than λ^t for a sufficiently small $\bar{\alpha}$, or $u(\lambda^{t+1}) > u(\lambda^t)$. In the following, we will design an interactive procedure to use the above results to support the DM to search for the MPS, summarised as follows:

Step 1: *Generate an output payoff table*.
Optimise each of the composite outputs of the observed DMU using formulation (9) to generate $\bar{f}_{rj_o} (r = 1, \ldots, s)$ and collect the results in a payoff table. For each composite output, elicit a target output value $\overline{Y}_{rj_o}^*$ from the DM as a starting point.

Step 2: *Generate initial efficient solution*
Set the initial weighting parameters as $w_r^0 = 1/(\bar{f}_{rj_o} - \overline{Y}_{rj_o}^*) (r = 1, \ldots, s)$. For the observed DMU, solve model (19) and obtain the initial solution of the decision variables $\lambda^0 = [\lambda_1^0 \cdots \lambda_n^0]^{\mathrm{T}}$, the initial objective values $f(\lambda^0) = [f_1(\lambda^0) \cdots f_s(\lambda^0)]^{\mathrm{T}}$, and the initial dual variable values $\beta^t = [\beta_1^t \ldots \beta_s^t]^{\mathrm{T}}$ for the first s constraints on the outputs. Set $t = 0$.

Step 3: *Compute the normal vector and check optimality condition*
At interaction t, calculate the normal vector N^t using Eq. (21). Choose a reference composite output, for example the first composite output $f_1(\lambda)$. Then check whether a given small change $\mathrm{d}f_1^t$ in $f_1(\lambda)$ can be exactly offset by the amount of change $\mathrm{d}f_r^t$ in the rth composite output $f_r(\lambda)$ with $\mathrm{d}f_r^t$ generated using Eq. (27). If the DM agrees with such optimal indifference tradeoffs between $f_1(\lambda)$ and each of the other composite outputs, then the current solution $f(\lambda^t)$ is already the MPS and the interactive process is terminated. Otherwise, the DM would provide new indifference tradeoffs, or $M^t = [1, M_{12}^t \cdots M_{1r}^t \cdots M_{1s}^t]^{\mathrm{T}}$.

Step 4: *Determine the tradeoff direction*
Use Eq. (28) to calculate the projection of the DMs indifference tradeoffs M^t onto the tangent plane of the efficient frontier, or $\Delta \bar{u}^t = [\Delta \bar{f}_1^t \cdots \Delta \bar{f}_r^t \cdots \Delta \bar{f}_s^t]^{\mathrm{T}}$, which determines the new tradeoff direction.

Step 5: *Determine the tradeoff step size and calculate the new weighting vector*
The tradeoff step size $\bar{\alpha}$ can be estimated by $\bar{\alpha} = \alpha^t_{\max}\alpha$, where $\alpha^t_{\max}$ is the largest permissible step size and α a regulating factor with $0 \leqslant \alpha \leqslant 1$. Suppose I^1 is the set of the subscripts of objectives for increase. The maximum step size can be determined as follows:

$$\alpha^t_{\max} = \max_{r \in I^1} \frac{\bar{f}_{rj_o} - f^t_r}{|\Delta \bar{f}^t_r|}. \tag{30}$$

An explicit tradeoff table as illustrated below and detailed in the case study can be used to determine the tradeoff step size $\bar{\alpha}$ using the following general formula:

$$f_r(\bar{\alpha}) = f_r(\lambda^t) + \bar{\alpha}\Delta\bar{f}^t_r. \tag{31}$$

In Table 1, the *DM* may determine the step size by analysing the tradeoffs among the outputs along the tangent plane of the efficient frontier. Some heuristics about the selection of the step size are discussed in the case study. Once the step size is decided, the weighting parameters can be updated using Eq. (29).

The above interactive process continues until the optimal indifference tradeoffs are achieved and thus the *MPS* is found maximising the implicit utility function of the *DM*. The decision variables at the *MPS* are represented by $\lambda^* = [\lambda^*_1 \cdots \lambda^*_j \cdots \lambda^*_n]^{\mathrm{T}}$.

3.3. Taking group preferences into account in setting target values

The above *MPS* located for each *DMU* relative to its peers only represents the preferences of the *DM* of the *DMU* at a divisional or local level. In order to set a performance benchmark with the organisational or group preferences taken into account, a group most preferred solution (*GMPS*) would need to be determined first. It is possible that a *GMPS* is assigned by a single leading *DM* having the overall responsibility for an organisation or group, by choosing a convex combination of the individual *MPS*s generated in the last section, or by simply picking up an existing efficient *DMU* (Korhonen et al., 2002) or a convex combination of certain existing efficient *DMU*s as a *GMPS* for the whole organisation. Alternatively, group decision making techniques such as voting theory or the Delphi technique could be used, especially for the purpose of negotiation and finding a compromised *GMPS*. Nevertheless, a group decision making process has to be participative and flexible so as to reflect the opinions of group members, in particular the evolving discussions between the group members.

A *GMPS*, as defined by m *GMPS* inputs $x^{GMPS}_i (i = 1, \ldots, m)$ and s *GMPS* outputs $y^{GMPS}_r (i = 1, \ldots, m)$ may lie within, on or outside the efficient frontier of a specific *DMU*. Although a *GMPS* point may represent the preferences of a group as a whole, it will not be practical to a *DMU* if it is not attainable by the *DMU*. In the rest of this section, a procedure will be proposed where a *GMPS* is mapped back to the feasible space of each *DMU* to generate a locally most preferred solution (*LMPS*) for each *DMU*. The new local input and output targets could then be used as benchmark to align towards the organisation's or group's targets with both group and individual *DM*s preferences taken into account.

By constructing a minimax reference point model with the *GMPS* as the reference point, a *LMPS* for each *DMU* could be generated as the one closest to the *GMPS* in the composite output space of the *DMU*. The following minimax reference point model equivalent to the minimax formulation (5) is constructed for this purpose (Yang, 2001a).

$$\begin{aligned} \text{Min} \quad & d \\ \text{s.t.} \quad & w_r(f^{GMPS}_r - f_r(\lambda)) \leqslant d, \\ & -w_r(f^{GMPS}_r - f_r(\lambda)) \leqslant d \quad r = 1, \ldots, s, \\ & \lambda \in \Omega_{j_o} = \left\{ \lambda \left| \sum_{j=1}^{n} \lambda_j x_{ij} \leqslant x_{ij_o}, \quad i = 1, \ldots, m; \ \lambda_j \geqslant 0, \ j = 1, \ldots, n \right. \right\}, \end{aligned} \tag{32}$$

where f^{GMPS}_r is given by $f^{GMPS}_r = \bar{\lambda} y^{GMPS}_r$ with $\bar{\lambda} = \min_{1 \leqslant i \leqslant m} \{x_{ij_o}/x^{GMPS}_i\}$.

Table 1
Tradeoff step size table

α	$f_1(\bar{\alpha})$	$f_2(\bar{\alpha})$	$\ldots$	$f_s(\bar{\alpha})$
0.1	$f_1(\lambda^t) + 0.1\alpha_{\max}\Delta\bar{f}^t_1$	$f_2(\lambda^t) + 0.1\alpha_{\max}\Delta\bar{f}^t_2$	$\ldots$	$f_s(\lambda^t) + 0.1\alpha_{\max}\Delta\bar{f}^t_s$
0.2	$f_1(\lambda^t) + 0.2\alpha_{\max}\Delta\bar{f}^t_1$	$f_2(\lambda^t) + 0.2\alpha_{\max}\Delta\bar{f}^t_2$	$\ldots$	$f_s(\lambda^t) + 0.2\alpha_{\max}\Delta\bar{f}^t_s$
$\vdots$	$\vdots$	$\vdots$	$\ddots$	$\vdots$
1.0	$f_1(\lambda^t) + 1.0\alpha_{\max}\Delta\bar{f}^t_1$	$f_2(\lambda^t) + 1.0\alpha_{\max}\Delta\bar{f}^t_2$	$\ldots$	$f_s(\lambda^t) + 1.0\alpha_{\max}\Delta\bar{f}^t_s$

Note that it is not always the case that $f_r^{GMPS} \geqslant f_r(\lambda)$ for all r in Ω_{j_o}. The above reference point model is referred to as the shortest distance model to differentiate it from the previous super-ideal and ideal point models, though they are all derived from formulation (20), so sharing the same decision and objective spaces. The main advantage of the above $LMPS$ procedure is that the weight w_r for an objective $f_r(\lambda)$ in formulation (32) can be set individually for each DMU, which can thus be used to represent local preferences. The rationale is that each DMU may be different from others, and based on the same $GMPS$, each DMU may have different preferences and relative weights for the objectives or outputs. This is relevant since different DMUs have different capabilities and specialities and may perform well in certain areas. On the other hand, there is also flexibility where a compromised common set of weights could be decided by the group members through negotiation and discussion. Alternatively, the same set of relative weights could be explicitly assigned by a group leader with an authoritative role such as the chief executive of a company representing the views of the organisation. This happens when the organisation wants to focus on improving only certain aspects of the business or output levels.

In the $LMPS$ procedure, a $GMPS$ is mapped back to the feasible space of each DMU to match its capabilities, size and scale of operations. However, it is possible that a $LMPS$ generated by the mapping procedure may lie within the efficient frontier, thus an inefficient solution with $d = 0$. In order to alleviate this problem, the super-ideal point model (3) can be constructed and solved with the $LMPS$ of the observed DMU added as a new DMU in the reference set. The corresponding composite outputs generated by the model for the $LMPS$ are realistic and achievable and thus could be used as the performance benchmark for the observed DMU, which will be referred to as the efficient $LMPS$ that takes into account the preferences of both individual DMs and group members.

4. A case study for performance analysis of UK retail banks

4.1. Problem description and efficiency analysis

A case study is carried out to demonstrate how performance assessment and target setting can be conducted in an integrated way using the interactive $MOLP$ methods investigated in the previous sections. The UK retail bank industry, specifically seven major retail banks, is examined to show the equivalence models, demonstrate the interactive procedure to search for MPSs along the efficient frontier, and illustrate the group negotiation and discussion process. The data set is obtained from Wong and Yang (2004) through a study on data envelopment analysis and multiple criteria decision making based on the evidential reasoning approach – performance measurement of UK retail banks (Yang, 2001b; Yang and Xu, 2002), as shown in Table 2.

For the DEA formulation, the reference set consists of seven DMUs (retail banks), and four inputs and two outputs are considered. The DMUs are comparable major banks in the UK including Abbey National, Barclays, Halifax, HSBC, Lloyds TSB, NatWest and RBS (Royal Bank of Scotland). The four inputs are namely number of branches, number of ATMs, number of staff and asset size. The two outputs are customer satisfaction and total revenue. It should be noted that although both bank staff and customers were interviewed in person or through questionnaires at certain stages of this research it is the researchers who acted as the DMs. Also note that this was not a full scale performance analysis and only limited data were collected. As such, the conclusions of the paper are for the purpose of illustrating the new approach rather than for providing an authentic performance assessment of these retail banks.

The output-oriented CCR dual model (3) is run to find the respective efficiency scores. As shown in Table 3, only Barclays and NatWest are found to be inefficient within the reference set of the seven banks. For instance, NatWest has an efficiency score of 74.7%, and its composite point on the efficient frontier can be represented as a linear combination of

Table 2
Original data set

DMU	Bank	Inputs			Outputs		
		No. of branches ('000)	No. of ATMs ('000)	No. of Staff ('0,000)	Asset size (£'00 bn)	Customer satisfaction*	Total revenue (£ m)
1	Abbey national	0.77	2.18	2.35	2.96	6.79	10.57
2	Barclays	1.95	3.19	8.43	3.53	2.55	13.35
3	Halifax	0.80	2.10	3.21	2.41	9.17	8.14
4	HSBC	1.75	4.00	13.30	4.85	5.82	23.67
5	Lloyds TSB	2.50	4.30	9.27	2.40	6.57	14.01
6	NatWest	1.73	3.30	7.70	3.09	4.86	12.04
7	RBS	0.65	1.73	2.67	1.34	7.28	7.36

* Customer satisfaction values are converted scores based on the average expected utilities of the survey respondents. Source: bank brochures, banks and financial advice websites, interviews with bank staff and customers.

Table 3
DEA results and efficiency scores

DMU	Bank	*DEA* score	Observed *DMU*s composite unit						
			1	2	3	4	5	6	7
1	Abbey national	1.000	1.000						
2	Barclays	0.778	0.135			0.512			0.489
3	Halifax	1.000			1.000				
4	HSBC	1.000				1.000			
5	Lloyds TSB	1.000					1.000		
6	NatWest	0.747				0.310			1.192
7	RBS	1.000							1.000

Table 4
Equivalence between *CCR* dual model and super-ideal point model

DMU	*DEA* dual model		Minimax model								$\theta = F^{\max} - \theta_{j_o}$
	DEA Score	θ_{j_o}	$F^{\max}$	θ	Composite inputs and outputs						
					x_1	x_2	x_3	x_4	y_1	y_2	
1	1.000	1.000	1.000	0.000	0.77	2.18	2.35	2.96	6.79	10.57	0.000
2	0.778	1.285	5.449	4.164	1.32	3.19	8.43	3.53	7.45	17.15	4.164
3	1.000	1.000	1.238	0.238	0.80	2.10	3.21	2.41	9.17	8.14	0.238
4	1.000	1.000	3.001	2.001	1.75	4.00	13.30	4.85	5.82	23.67	2.001
5	1.000	1.000	1.989	0.989	2.50	4.30	9.27	2.40	6.57	14.01	0.989
6	0.747	1.338	2.905	1.567	1.32	3.30	7.30	3.09	10.48	16.11	1.567
7	1.000	1.000	1.000	0.000	0.65	1.73	2.67	1.34	7.28	7.36	0.000

0.310 of HSBC and 1.192 of RBS. Note that the decision variables do not add up to 1 as the constant returns to scale is assumed.

Before conducting interactive tradeoff analysis, let us first validate the equivalence between the *CCR* dual model and the minimax formulations developed in the previous sections. Using Remark 1, the super-ideal point model shown in formulation (19) is run for each *DMU*, with w_r assigned by (10) and the reference point as the super-ideal point $f^* = F^{\max}[y_{1j_o}, \ldots, y_{sj_o}]^{\mathrm{T}}$. The results are shown in Table 4, which shows that the equivalence $\theta = F^{\max} - \theta_{j_o}$ holds for each *DMU* with θ generated using the super-ideal point model, $F^{\max}$ assigned using Eq. (13) and θ_{j_o} being the same as the *DEA* score given in Table 3 for each *DMU*. The composite inputs and outputs for each *DMU* shown in Table 4 are also the same as the results of Table 3 generated by the output-oriented *CCR* dual model.

4.2. Interactive tradeoff analysis to find MPSs for management planning

In the proposed interactive tradeoff analysis procedure, the first step is to solve model (9) for each composite output of the observed *DMU* to generate a payoff table for the *DMU*, in which the feasible ranges of tradeoffs are defined. Table 5 shows the ranges of possible output values when each composite output of every *DMU* is maximized.

From Table 5, it is clear that without increasing the consumption of resources (inputs) there is no further improvement or possible tradeoffs between the outputs of Abbey National and RBS as for each of them maximising y_1 and y_2 leads to the same set of solutions. The other efficient *DMU*s such as Halifax, Lloyds TSB and HSBC can sacrifice one of the outputs to increase the other output. For the inefficient *DMU*s of Barclays and NatWest, both their outputs can be further improved without consuming extra inputs. For illustration purpose, the interactive tradeoff analysis procedure will be demonstrated for the sixth *DMU* NatWest, which is an inefficient *DMU*. The maximum feasible value of the first composite output of NatWest is generated as $\bar{f}_1 = 14.12$, while the maximum feasible value of its second composite output is given by $\bar{f}_2 = 16.11$. The ideal point of the composite outputs is thus given by $\bar{f} = [14.12, 16.11]^{\mathrm{T}}$, which is then used in the next step of the procedure.

The starting solution of the interactive procedure for NatWest is generated by solving model (19) with the initial weights w_r assigned as described in Remarks 2 and 3. In fact, the starting solution could be the same point on the efficient frontier as that of the *DEA* composite solution for NatWest. In this way, the *DM* will be able to decide whether to accept or reject the initial composite solution or the *DEA* composite input and output values as the *MPS* for NatWest. However, for NatWest, the maximum composite output 2 from the payoff table is the same as the *DEA* composite output 2, or $\bar{f}_2 = f_2^0 = 16.11$. This means that the *DEA* composite unit is an extreme point on the efficient frontier. In this case, w_r can be assigned by $w_2 = 1$ and $w_1 = 0$ in formulation (19), and the starting solution is equal to the *DEA* composite unit. At this extreme point, there may be multiple normal vectors. One of the normal vectors can be calculated as $N^0 = [0, 1]^{\mathrm{T}}$.

Table 5
Payoff table for all DMUs

DMU	Bank	Max y_1		Max y_2		Maximum values	
		y_1	y_2	y_1	y_2	y_1	y_2
1	Abbey national	6.79	10.57	6.79	10.57	6.79	10.57
2	Barclays	13.87	12.47	7.45	17.15	13.87	17.15
3	Halifax	9.17	8.14	6.85	10.08	9.17	10.08
4	HSBC	17.47	15.50	5.82	23.67	17.47	23.67
5	Lloyds TSB	13.07	13.22	6.57	14.01	13.07	14.01
6	NatWest	14.12	13.49	10.48	16.11	14.12	16.11
7	RBS	7.28	7.36	7.28	7.36	7.28	7.36

Alternatively, if f_2^0 is perturbed or decreased by a very small amount so that $\bar{f}_2 > f_2^0$, for example $f_2^0 = 16.10(< \bar{f}_2 = 16.11)$, then a starting solution with the maximum objective values of $f(\lambda^0) = [10.49,\ 16.10]^{\mathrm{T}}$ can be found by solving formulation (19) that is very close to but not exactly the same as the DEA composite outputs of $f(\lambda^t) = [10.48,\ 16.11]^{\mathrm{T}}$. This perturbation technique is used to start the interactive process, resulting in the initial decision variables $\lambda^0 = [0, 0, 0, 0.31, 0, 0, 1.20]^{\mathrm{T}}$ and the normal vectors $N^0 = [0.27,\ 0.45]^{\mathrm{T}}$ calculated using formulation (21). Thus, the initial efficient solution of NatWest can be characterised as a linear combination of 0.31 of HSBC and 1.20 of RBS.

For the first interaction, suppose f_2 is treated as the reference objective. The optimal indifference tradeoff vector at the solution $f(\lambda^0)$ for a unit change of f_2 can be calculated using Eq. (27) as $\mathrm{d}f^0 = [1.66,\ 1.00]^{\mathrm{T}}$, leading to the initial optimal indifference tradeoff of $(10.49,\ 16.10) \Leftrightarrow (10.49 + 1.66,\ 16.10 - 1.00)$. If the DM does not agree with this initial optimal indifference tradeoff, which means that the initial target values are not most preferred, then a new set of indifference tradeoffs may be proposed by the DM, for example $(10.49,\ 16.10) \Leftrightarrow (10.49 + 2.00,\ 16.10 - 1.00)$, resulting in the marginal rate of substitution $M^0 = [0.50,\ 1.00]^{\mathrm{T}}$. Note that the tradeoff of $(10.49 + 2.00)$ for f_1 is less than its maximum feasible value of 14.12.

The gradient projection is calculated using Eq. (24) to find the tradeoff direction with $\Delta\bar{f}^0 = [0.076, -0.046]^{\mathrm{T}}$, which means that the DM prefers to improve f_1 at the expense of f_2. As for the tradeoff size, the maximum permissible step size is calculated by Eq. (30), resulting in $\alpha^0_{\max} = 48.00$, which is used to construct the step size table. Table 6 shows that f_1 increases and f_2 decreases for every incremental step size when 10 equal incremental steps are used between the current value of 10.49 for f_1 and its maximum feasible value of 14.12. Suppose the DM sets the target level for f_1 at 11.00. This new target value of 11.00 is exceeded when $\alpha \geqslant 0.2$, so the regulating parameter is set to $\alpha = 0.1$.

For a more precise size-step assignment, 100 equal incremental steps could be used between the current value of 10.49 for f_1 and its maximum feasible value of 14.12. Using the same maximum upper bound for f_1 at 11.00, the target value of 11.00 is exceeded when $\alpha \geqslant 0.15$ and hence the regulating parameter of the step size is set to $\alpha = 0.14$.

The weighting vector can be updated using Eq. (29) with $w^1 = [1,\ 9.907]^{\mathrm{T}}$. Solving formulation (19) again with the new weight vector w^1 in the second interaction, a new efficient solution is generated with $f(\lambda^1) = [11.00,\ 15.79]^{\mathrm{T}}$ and $\lambda^1 = [0, 0, 0, 0.26, 0, 0, 1.30]^{\mathrm{T}}$. The new normal vectors at $f(\lambda^1)$ is calculated as $N^1 = 0.32[0.27,\ 0.45]^{\mathrm{T}}$. Note that the new normal vector N^1 is in parallel with the previously identified normal vector N^0, which means that the interactive tradeoff analysis is done in the same facet of the efficient frontier as in the last interaction. The optimal indifference tradeoff vector at $f(\lambda^1)$ for a unit change of f_2 is given by $\mathrm{d}f^1 = [1.66,\ 1.00]^{\mathrm{T}}$.

If DM still does not agree with the optimal indifference tradeoff of $(11.00,\ 15.79) \Leftrightarrow (11.00 + 1.66,\ 15.79 - 1.00)$, a new set of indifference tradeoffs may be provided by the DM, say $(11.00,\ 15.79) \Leftrightarrow (11.00 + 1.80,\ 15.79 - 1.00)$, leading to the

Table 6
Determination of tradeoff step size

$C@ = 10$		
@	y_1	y_2
0	10.491	16.100
0.1	10.853	15.881
0.2	11.216	15.663
0.3	11.579	15.444
0.4	11.941	15.225
0.5	12.304	15.007
0.6	12.667	14.788
0.7	13.029	14.569
0.8	13.392	14.351
0.9	13.755	14.132
1.0	14.117	13.913

marginal rate of substitution $M^1 = [0.556, 1.000]^T$. The new gradient projection is given by $\Delta\bar{f}^1 = [0.035, -0.021]^T$ and the maximum step size by $\alpha^1_{max} = 89.60$. Suppose the *DM* re-sets the target level of f_1 as 11.50. The tradeoff step size $\alpha = 0.16$ can then be calculated to provide a new weighting vector $w^2 = [1, 4.225]^T$ to be used for the third interaction. Note that the new 'learnt' target level of 11.50 for f_1 is different from the previous target level of 11.00 for f_1 set in the first interaction. The change in the target level made by the *DM* represents part of the learning process about what could be achieved, which is the main feature of the interactive tradeoff analysis procedure and can help the *DM* to set realistic target values.

In the third interaction, the new efficient solution is given by $f(\lambda^2) = [11.50, 15.49]^T$ and $\lambda^2 = [0, 0, 0, 0.22, 0.0, 0, 1.41]^T$. The normal vector at $f(\lambda^2)$ is calculated as $N^2 = 2.63[0.27, 0.45]^T$. Again, the new normal vector N^2 is still in parallel to N^1, which means that the interactive tradeoff process is searching for the *MPS* along the same facet of the efficient frontier as the last interaction. If the *DM* agrees with the optimal indifference tradeoff of $(11.50, 15.49) \Leftrightarrow (11.50 + 1.66, 15.49 - 1.00)$, then the interactive process will be terminated and $f(\lambda^2) = [11.50, 15.49]^T$ will be the *MPS* maximising the *DM*s implicit utility function for NatWest. Otherwise, the interactive process continues.

The *DEA* composite outputs for NatWest are given by $[10.48, 16.11]^T$ for customer satisfaction and total revenue. Further analysis on the areas of improvement that NatWest needs to focus upon and the amount of improvement needed for each input and output are shown in Table 7 if the *DEA* composite *DMU* is benchmarked against. In fact, number of branches should be reduced from 1730 to 1320, a decrease of 31%, and number of staff should be reduced from 7700 to 7300, a drop of 5% for NatWest to become efficient. Also, customer satisfaction and total revenue could be increased by 116% and 34%, respectively. So, instead of the current total revenue of £12.04 m, the target total revenue that could be achieved is £16.11 m.

On the other hand, the *MPS* maximising the *DM*s utility function for NatWest as determined by the interactive process has target composite outputs of $[11.50, 15.49]^T$ for customer satisfaction and total revenue as shown in Table 7. The new target values show that NatWest should in future achieve an increase of 137% for customer satisfaction and 29% for total revenue from its current value. Likewise, inputs should be better allocated in comparison with the original data, and number of branches, number of staff and asset size should be reduced by about 34%, 16% and 6%, respectively.

The *MPS* target output values for NatWest are $[11.50, 15.49]^T$ when the *DM*s value judgements are taken into account. In contrast, the *DEA* target output values are $[10.48, 16.11]^T$. Hence, when *MPS* is compared to the target values generated by *DEA*, NatWest should sacrifice total revenue by 3.8% from £16.11 to £15.49 m, and aim to increase the output level of customer satisfaction by 9.8% from the score of 10.48 to 11.50. Subsequently, NatWest should better allocate or utilise its resources by decreasing its number of branches, number of staff and asset size by about 1.8%, 9.9% and 5.6%, respectively. It is evident that during the interactive tradeoff analysis process the *DM* placed more emphasis on generating high customer satisfaction rating rather than high total revenue.

The individual *MPS* for all the banks could be found by solving model (19) for each *DMU* using the interactive procedure. For illustration purpose, the target output and input levels for all *DMU*s are shown in Table 8. It can be observed that the inefficient *DMU*s: Barclays and NatWest have different *MPS* and *DEA* composite inputs and outputs. Likewise, three of the efficient *DMU*s: Halifax, HSBC and Lloyds TSB also have different *MPS* from their *DEA* composite units. The other two efficient *DMU*s of Abbey National and RBS have the composite inputs and outputs that are the same as the original data values, as there are no tradeoffs between their outputs.

Table 9 shows the decision variables of the individual *MPS*s of the banks. One way to set group performance benchmark is to choose a bank whose decision variable occurs most frequently in construction of the imaginary composite units for other banks. So, in this case, the efficient RBS could be set as the performance benchmark or the group *MPS* for all banks to follow. However, such an approach lacks coherence and reliability, and the individual *DM*s of the *DMU*s may

Table 7
NatWest – target setting and resource allocation

	Inputs				Outputs	
	No. of branches ('000)	No. of ATMs ('000)	No. of staff ('0,000)	Asset size (£'00 bn)	Customer satisfaction	Total revenue (£ m)
Actual value	1.73	3.30	7.70	3.09	4.86	12.04
DEA composite unit	1.32	3.30	7.30	3.09	10.48	16.11
Improvement %	31.38	0.00	5.48	0.00	115.57	33.80
Actual value	1.73	3.30	7.70	3.09	4.86	12.04
MPS composite unit	1.29	3.30	6.64	2.93	11.50	15.49
Improvement %	33.69	0.00	15.97	5.55	136.57	28.69
DEA composite unit	1.32	3.30	7.30	3.09	10.48	16.11
MPS composite unit	1.29	3.30	6.64	2.93	11.50	15.49
Improvement %	1.76	0.00	9.94	5.55	9.75	−3.82

Table 8
Individual *MPS* for all *DMU*s

DMU	Bank	Inputs				Outputs	
		No. of branches ('000)	No. of ATMs ('000)	No. of staff ('0,000)	Asset size (£ bn)	Customer satisfaction	Total revenue (£ m)
		x_1	x_2	x_3	x_4	y_1	y_2
1	Abbey national	0.77	2.18	2.35	2.96	6.79	10.57
2	*Barclays*	*1.29*	*3.19*	*7.45*	*3.09*	*9.50*	*15.93*
3	**Halifax**	**0.78**	**2.10**	**3.21**	**2.08**	**7.69**	**9.60**
4	**HSBC**	**1.69**	**4.00**	**11.45**	**4.39**	**8.67**	**21.95**
5	**Lloyds TSB**	**2.13**	**3.97**	**8.04**	**2.40**	**8.35**	**13.80**
6	*NatWest*	*1.29*	*3.30*	*6.64*	*2.93*	*11.50*	*15.49*
7	RBS	0.65	1.73	2.67	1.34	7.28	7.36

Note. Data in italics shows inefficient *DMU*s and data in bold shows different target values as compared to *DEA* calculations.

Table 9
MPS-decision variables of all *DMU*s

DMU	Bank	Decision variables						
		1	2	3	4	5	6	7
1	Abbey national	1.000						
2	Barclays				0.355			1.021
3	Halifax	0.305			0.039			0.741
4	HSBC				0.741			0.599
5	Lloyds TSB					0.726		0.492
6	NatWest				0.217			1.407
7	RBS							1.000

disagree with such group target levels. An alternative approach is to aggregate all the individual *MPS*s to provide a group *MPS*, as discussed in the next section, which in essence reflects the preferences of the group members or all the individual *DM*s.

4.3. Target setting with group preferences taken into account

The *GMPS* reference point in terms of composite outputs for customer satisfaction and total revenue are given as a convex combination of the generated *MPS*s shown in Table 8 by $f^{GMPS} = [8.62, 14.98]^{\mathrm{T}}$, from which the *LMPS* can be determined. Suppose the relative weights for customer satisfaction and total revenue are assumed to be $[0.6, 0.4]^{\mathrm{T}}$ for NatWest. Solving formulation (32) and using f^{GMPS} as the reference point, the *LMPS* in terms of outputs are generated for NatWest as $y_6^{LMPS} = [8.62, 14.98]^{\mathrm{T}}$ and the corresponding inputs are given by $x_6^{LMPS} = [1.45, 3.30, 7.53, 3.09]^{\mathrm{T}}$ with $d = 0$ at the optimal solution of formulation (32), which indicates that the *LMPS* may be an inefficient solution. To test whether the *LMPS* is an efficient solution, the super-ideal model (19) is run where the observed *DMU* (or NatWest in this case) has its *LMPS* added as an additional *DMU* in the reference set. The results are as shown in Table 10.

If the same set of relative weights of $[0.6, 0.4]^{\mathrm{T}}$ is used to represent the preferences of the *DM*s for customer satisfaction and total revenue, the *LMPS*s can be generated for all the*DMUs* using the minimax reference point model (32), as shown in

Table 10
Efficient *LMPS* for all *DMU*s

DMU	Bank	Test of efficiency (%)	Composite inputs				Composite outputs	
			No. of branches ('000)	No. of ATMs ('000)	No. of staff ('0,000)	Asset size (£ bn)	Customer satisfaction	Total revenue (£ m)
			x_1	x_2	x_3	x_4	y_1	y_2
1	Abbey national	100.0	0.77	2.18	2.35	2.96	6.79	10.57
2	Barclays	95.2	1.26	3.12	7.44	3.06	9.05	15.73
3	Halifax	100.0	0.78	2.10	3.21	2.41	6.85	10.08
4	HSBC	98.0	1.23	3.03	7.24	2.97	8.80	15.29
5	Lloyds TSB	100.0	2.24	4.07	8.39	2.40	7.85	13.86
6	NatWest	93.0	1.32	3.30	7.30	3.09	10.48	16.11
7	RBS	100.0	0.65	1.73	2.67	1.34	7.28	7.36

Table 10. For instance, NatWest's *LMPS* has an efficiency score of 93.0%, the corresponding composite inputs are $[1.32, 3.30, 7.30, 3.09]^T$ and the corresponding composite outputs are $[10.48, 16.11]^T$.

The implications of the efficient *LMPS* for each *DMU* can be further analysed. For example, the efficient *LMPS* is different from the *MPS* composite unit for Halifax as shown in Tables 8 and 10. From the original *DEA* test, Halifax is an efficient *DMU*. Based on the individual *DM*s preferences and tradeoff analysis process, Halifax should sacrifice the output level of customer satisfaction by 16.2% from the score 9.17 to 7.69, and aim to increase its total revenue by 17.9% from £8.14 to £9.60 m. Subsequently, Halifax should better allocate or utilise its resources by decreasing its number of branches and reducing its asset size by 2.4% and 13.5%, respectively.

However, after taking account of the group preferences, it is evident that the *DM* has placed more emphasis on generating more revenue than a high customer satisfaction rating for Halifax. The *LMPS* shows that Halifax should preferably increase its total revenue by 23.8% from £8.14 to £10.08 m, while its customer satisfaction should be reduced by 25.3% from the score of 9.17 to 6.85. Only the number of branches should be reduced by 2.4% while the levels of the other inputs should be maintained. This implies that although the *DM* for Halifax correctly targets the area of improvements to be total revenue at the expense of customer satisfaction rating, based on the collective view of the group, generating more total revenue is of greater importance for Halifax. Note that the above analysis for Hilfax is for illustration purpose. It is intended to show that tradeoffs even for efficient *DMU*s are possible if the decision makers so wish. However, tradeoffs along the efficient frontiers will not be possible if the decision makers are not prepared to sacrifice any output.

5. Conclusion

In this paper, interactive *MOLP* methods were investigated to conduct efficiency analysis and set realistic target values in an integrated way with the *DM*s preferences taken into account and with the *DM* supported to explore what could be technically achievable. The equivalence relationship established between the output-oriented *DEA* dual models and the minimax formulations led to the construction of the three equivalence models: namely the super-ideal point model, the ideal point model and the shortest distance model. These models share the same decision and objective spaces and are different from each other in terms of reference points and weighting schemas. They provide a basis to apply interactive tradeoff analysis methods and other techniques in *MOLP* to support integrated *DEA*-oriented performance assessment and target setting.

In this paper, the use of the interactive gradient projection approach for target setting was explored. The features of such a procedure include that the identification of normal vectors on the efficient frontier provides a vigorous measure to check whether the *MPS* is achieved that maximises the *DM*s implicit utility function. On the other hand, the projection of the utility gradient onto the tangent plane of the efficient frontier using the normal vector leads to a direction along which the *DM*s utility can be further improved. The *MPS* generated using this procedure provides feasible target values that can also maximise the *DM*s implicit utility function. The case study illustrated how the equivalence models and the interactive procedure can be implemented to support integrated efficiency analysis and target setting. This case study is relatively small in terms of sample size, as only 7 *DMU*s were included. For large scale problems with many *DMU*s, the equivalent minmax models will increase their size in terms of the number of decision variables. However, this would not create a problem to apply the methods proposed in this paper, as the tradeoff analyses are conducted in the objective space whose complexity is decided by the number of outputs.

Acknowledgements

This work forms part of the Projects supported by the UK Engineering and Physical Science Research Council (EPSRC) under the Grant No: *GR/S85498/01* and Grant No: *EP/C014723/1* and the Natural Science Foundation of China under the Grant No: 60736026 and the Grant No: 70631003. Part of the work reported in this paper was conducted during the first authors sabbatical visit to the University of Cape Town, South Africa, which was made possible by Grants from both the UK EPSRC and The University of Manchester. The constructive comments provided by the anonymous referees are greatly appreciated.

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