Multiple Criteria Decision Analysis Applied to Safety and Cost Synthesis[1]

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Abstract

Following a brief introduction to offshore safety assessment, safety based decision making is discussed. This is followed by a description of a new safety and cost modelling approach which is demonstrated by an example. Three typical multiple criteria decision analysis methods for safety and cost synthesis are described. Their potential for use in safety based decision making is discussed.

Keywords: Multiple Criteria Decision Analysis, Health and Safety, Cost

1. Introduction

The responsibilities for offshore safety regulations were transferred from the Department of Energy to the Health and Safety Commission (HSC) acting through the Health and Safety Executive (HSE) as the single regulatory body for offshore safety in 1990 after Lord Cullen's Report on the public inquiry into the Piper Alpha accident was published. In response to the findings of the Piper Alpha accident inquiry the HSE launched a review of all the offshore safety legislation and implemented changes, which sought to substitute the prescriptive legislation with a goal-setting risk based regime. A safety case approach was suggested.

A safety case covers all aspects of the safety of the process in question and specifies how risks involved are to be managed and minimised [Department of Energy, 1990]. A safety case should be prepared to demonstrate safety by design to describe operational requirements, to provide for continuing safety assurance of regular review and also to set out the contingency plans and arrangements for emergency response. A safety case must demonstrate that all hazards with potential to generate major accidents should be managed and be reduced to a level as low as reasonably practicable (ALARP). The safety case regulations came into force in the UK offshore industry in 1993.

After several years' field experience on the application of the offshore safety case approach in the UK, the safety case regulations were amended in 1996 to include the verification of safety-critical elements. The offshore installations and wells (design and construction, etc) regulations 1996 were introduced to deal with various stages of the life cycle of the installation. An appropriate combination of inherent safety design, prevention, detection, control and mitigation measures is required to be implemented, maintained and verified throughout the life-cycle of the installation. A verification scheme is also introduced where an independent and competent person is involved.

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The main feature of the UK new offshore safety regulations is the absence of a prescriptive regime and defining specific duties of the operator and definition as regard to what are adequate means. The regulations set forth a high level safety objective while placing responsibilities in the industry to set out and justify their basis for managing the risks of offshore operations. This is in recognition of the fact that hazards related to an individual installation are specific to its function and site conditions.

Recently, the UK Offshore Operators Association (UKOOA) has produced "a framework for risk related decision support" [UKOOA, 1999]. The proposed generic framework could be useful for a wide range of applications under various conditions. The framework is not intended to be a prescriptive method to be followed each time a decision is made. Its purpose is to assist with the major or difficult decisions relating to major accident hazard management and can be adapted to suit the situation at hand [UKOOA, 1999]. It can help decision-makers to define the context of the decision and give some indication of the relative weightings of different contributors in the decision making process, assisting the assessment of decision alternatives. The proposed framework provides a means to assess the relative importance of codes and standards, good practice, engineering judgement, risk analysis, cost benefit analysis, company values and societal values when making decisions. It can also be combined with other formal decision-making aids such as Multiple-Attribute Utility Analysis (MAUA), Analytical Hierarchy Process (AHP) or decision trees if a more detailed or quantitative analysis of the various decision alternative is desired [UKOOA, 1999]. The framework can help define the key decision factors and weightings for the various decision bases; techniques such as MAUA can then be applied.

Safety and cost are among the most important objectives that need to be considered in the design and operational processes of a large offshore engineering product. Formal multiple criteria decision making techniques including MAUA may be used to generate the best compromise designs. To facilitate safety based decision making under the UKOOA framework, many multiple criteria decision making techniques need to be investigated in detail for their appropriate application in a practical environment. The theme of this paper is to examine several multiple criteria decision analysis methods using examples for demonstrating their application in safety and cost synthesis.

2 Safety and cost modelling

2.1 Safety modelling

Safety synthesis of an engineering system is usually conducted by aggregating safety assessments for its sub-systems, components and failure modes. A safety assessment framework may constitute a hierarchical structure with failure modes at the bottom level [Wang et al., 1996]. A failure mode could be described in several ways, for example in terms of failure likelihood, consequence severity and failure consequence probability using linguistic variables. This is a natural and sensible way for capturing ambiguity and uncertainty inherent in safety assessment.

Fuzzy sets are well suited to characterizing linguistic variables by fuzzy memberships to the defined categories for a particular situation. Failure likelihood, consequence severity and failure consequence probability could all be characterised using the same set of categories but different membership functions, resulting to the fuzzy sets of failure likelihood, consequence severity and failure consequence probability. In this way, the safety associated with a failure mode may also be modelled using fuzzy sets.

For example, the fuzzy safety description (S) associated with a failure mode can be defined as the following product of the fuzzy sets of the related failure likelihood (L), consequence severity (C) and failure consequence probability (E) [Wang et al., 1995 & 1996]:

$$S = C \circ E \times L$$

where the symbol "$\circ$" represents the composition operation and "$\times$" the Cartesian product operation in fuzzy set theory. If seven categories are used to describe fuzzy sets, the above product could generate the safety description of the failure mode as a fuzzy set as follows:

$$S = [\mu_1/1,\ \mu_2/2,\ \mu_3/3,\ \mu_4/4,\ \mu_5/5,\ \mu_6/6,\ \mu_7/7]$$

where μ_i denotes the membership degree of the failure mode to the *i*th category.

Similar fuzzy sets could be generated for describing the safety of other failure modes, which could be aggregated using conventional fuzzy operations to generate safety descriptions for the components, the subsystems and the whole system of the assessment hierarchy. However, this process may lead to information loss.

Alternatively, safety could be more clearly expressed and communicated using linguistic variables (or assessment grades) such as "*Poor*", "*Average*", "*Good*" and "*Excellent*". Such assessment grades could be defined as distinctive safety standards on the basis of safety guidelines, regulations, laws and other situations specific to the engineering system in question. If the above four linguistic variables are used, then the safety of a failure mode could be described using the following expectation or distribution:

$$S = \{(\beta_1, Poor), (\beta_2, Average),\ (\beta_3, Good), (\beta_4, Excellent)\}$$

where β_j denotes the degree of belief that the safety of the failure mode should be assessed to the *j*th assessment grade. A safety distribution provides a panoramic view about the safety status of a failure mode, component, subsystem or the whole system. It can be used to identify areas for improvement and to simulate action plans to improve safety. β_j could be generated using various ways, for example by analysing historical data using statistical approaches if such data is available; otherwise expert judgements could be used to estimate β_j. If assessment grades are initially defined as fuzzy sets, then β_j could be generated from the fuzzy safety description using the best-fit method as described by [Wang et al., 1995].

There are other ways to describe safety. The simplest approach would be to use a scale for scoring the safety of a failure mode. While this may be easy for safety aggregation to product an average indicator about system safety, it could not capture uncertainty inherent in safety assessment and thereby the credibility of such assessment may become questionable. Unfortunately, several well known multiple criteria decision analysis methods, which could be used for safety synthesis, can only be implemented using certain types of scores. This will be discussed in detail in the next section.

2.2 Cost modelling

Safety is closely related to cost. Although safety must have paramount importance over cost in most situations, there are cases where safety standards are already achieved and cost effectiveness needs to be given more attention. In such cases, cost should be analysed in conjunction with safety. Costs can be modelled using the methods discussed in the previous subsection. Costs related to safety improvement are usually affected by a number of factors [Wang et al. 1996], including

a. Costs for the provision of redundancies of critical components, the provision of protection systems and alarm systems to reduce or eliminate the probabilities of occurrence of undesirable system to events, and the use of more reliable components.
b. Costs of labour incurred in redesign of the system.

c. Benefits resulting from the likelihood reduction of system failure events and the improvement of system efficiency as a result of the improvement of system safety.

Ideally, costs could be estimated using precise numerical figures so that conventional methods could be applied to analyse costs together with safety. However, this is often not achievable due to the high uncertainty in estimation of safety related costs. Fuzzy sets provide an alternative way to model costs. For example, costs could be described using linguistic variables such as "*Very low*", "*Low*", "*Moderately low*", "*Average*", "*Moderately high*", "*High*" and "*Very high*". If the seven categories are used to describe fuzzy sets, a fuzzy cost description can be represented as follows:

$$C = [\gamma_1/1,\ \gamma_2/2,\ \gamma_3/3,\ \gamma_4/4,\ \gamma_5/5,\ \gamma_6/6,\ \gamma_7/7]$$

where γ_i is the membership degree of the cost to the *i*th category.

Alternatively, costs could be clearly described using expectations or distributions to indicate to what degrees costs are preferred, for example using linguistic variables (or assessment grades) such as "*Slightly preferred*", "*Moderately preferred*", "*Preferred*" and "*Greatly preferred*". Such an assessment grade could be defined as a clear cost threshold that an organisation determines for a specific situation. If the above four linguistic variables are used, then the cost of a design option could be described using the following expectation or distribution

$$S = \{(\beta_1, \textit{Slightly preferred}), (\beta_2, \textit{Moderately preferred}),$$
$$(\beta_3,\ \textit{preferred}), (\beta_4, \textit{Greatly preferred})\}$$

where β_j denotes the degree of belief that the cost of the design option should be assessed to the *j*th assessment grade. A cost distribution provides a range of possible financial consequences with different probabilities, which may be incurred in order to develop and adopt the design option. As discussed before, β_j could be generated using various ways, either statistically or subjectively.

2.3 Safety and cost modelling – an example

In safety modelling, the safety associated with a failure mode of a component may be judged by multiple designers. A diagram for synthesis of the safety for a failure mode is shown in Fig. 1. Suppose there are *e* designers, each of whom is given a relative weight in the design selection process. The designers' judgements can be aggregated to generate assessments on the safety of failure modes, which can in turn be aggregated to produce assessments for component safety. Assessments for component safety can eventually be aggregated to generate an assessment for system safety using various methods such as those to be investigated in the next section.

In cost modelling, the cost incurred for each design option can also be judged by *e* designers. These judgements can be aggregated to generate an assessment for a design option using various methods such as those to be investigated in the next section. A diagram for synthesis of costs incurred for design options by multiple designers is shown in Fig. 2.

In this section, safety and cost modelling is discussed for an engineering system in order to demonstrate the multiple criteria decision analysis methods in the next section. Consider a hydraulic hoist transmission system of a marine crane [Wang et al., 1995 & 1996], which is used to control the crane motions such as hoisting down loads as required by the operator. It consists of five subsystems: the hydraulic oil tank, the auxiliary system, the control system, the protection system and the hydraulic servo transmission system. Suppose there are four options for selection by four designers. The safety modelling and cost modelling of the four design options are described as follows using expectations or distributions. To simplify discussion and without loss of generality, the same set of evaluation grades are used to model both safety and cost, that is "*Slightly preferred*", "*Moderately preferred*", "*Preferred*" and "*Greatly preferred*". More detailed discussions about safety and cost modelling can be found in [Wang et al. 1996].

Option 1: No failure mode is eliminated in the design review process.

For this first design option, suppose the safety assessments provided by the four designers are the same and are represented as the following expectation:

$$\begin{aligned} S_1^1 &= S_1^2 = S_1^3 = S_1^4 \\ &= \{(0.122425,\ \textit{Slightly preferred}), \\ &\quad (0.180205,\ \textit{Modertaely preferred}), \\ &\quad (0.463370,\ \textit{Preferred}), \\ &\quad (0.233999,\ \textit{Greatly preferred})\} \end{aligned}$$

For this option, there is no additional cost for eliminating failure modes. Suppose the four designers judge the cost incurred for this option as follows:

$$\begin{aligned} C_1^1 &= C_1^2 = C_1^3 = C_1^4 \\ &= \{(0,\ \textit{Slightly preferred}), \\ &\quad (0,\ \textit{Modertaely preferred}), \\ &\quad (0,\ \textit{Preferred}), \\ &\quad (1,\ \textit{Greatly preferred})\} \end{aligned}$$

Option 2: Eliminate "hoist up limit failure" and "hoist down limit failure" associated with the protection system.

For this second design option, suppose the safety assessments provided by the four designers are represented as follows:

$$\begin{aligned} S_2^1 &= S_2^2 = S_2^3 = S_2^4 \\ &= \{(0.102676,\ \textit{Slightly preferred}), \\ &\quad (0.156934,\ \textit{Modertaely preferred}), \\ &\quad (0.38486,\ \textit{Preferred}), \\ &\quad (0.355531,\ \textit{Greatly preferred})\} \end{aligned}$$

Suppose the four designers have different opinions about the costs incurred to eliminate the failure modes and their individual assessments are given as follows:

$$\begin{aligned} C_2^1 = \{&(0.054309,\ \textit{Slightly preferred}), \\ &(0.066442,\ \textit{Modertaely preferred}), \\ &(0.821848,\ \textit{Preferred}), \\ &(0.057400,\ \textit{Greatly preferred})\} \end{aligned}$$

$$C_2^2 = \{(0.102638,\ \textit{Slightly preferred}),\\ (0.134831,\ \textit{Modertaely preferred}),\\ (0.657202,\ \textit{Preferred}),\\ (0.105330,\ \textit{Greatly preferred})\}$$

$$C_2^3 = \{(0,\ \textit{Slightly preferred}),\\ (0,\ \textit{Modertaely preferred}),\\ (1,\ \textit{Preferred}),\\ (0,\ \textit{Greatly preferred})\}$$

$$C_2^4 = \{(0.067060,\ \textit{Slightly preferred}),\\ (0.083011,\ \textit{Modertaely preferred}),\\ (0.777240,\ \textit{Preferred}),\\ (0.072689,\ \textit{Greatly preferred})\}$$

Option 3: Eliminate the failure modes involving “major leak” and “no output from the package motor” associated with the hydraulic servo transmission system.

For the third design option, suppose the safety assessments provided by the four designers are represented as follows:

$$S_3^1 = S_3^2 = S_3^3 = S_3^4\\ = \{(0.022722,\ \textit{Slightly preferred}),\\ (0.033659,\ \textit{Modertaely preferred}),\\ (0.073367,\ \textit{Preferred}),\\ (0.870253,\ \textit{Greatly preferred})\}$$

Suppose the four designers’ individual cost assessments are given as follows:

$$C_3^1 = \{(0.067604,\ \textit{Slightly preferred}),\\ (0.084062,\ \textit{Modertaely preferred}),\\ (0.777037,\ \textit{Preferred}),\\ (0.071297,\ \textit{Greatly preferred})\}$$

$$C_3^2 = \{(0.102638,\ \textit{Slightly preferred}),\\ (0.134831,\ \textit{Modertaely preferred}),\\ (0.657202,\ \textit{Preferred}),\\ (0.105330,\ \textit{Greatly preferred})\}$$

$$
\begin{aligned}
C_3^3 = \{&(0.067060,\ \textit{Slightly preferred}),\\
&(0.083011,\ \textit{Modertaely preferred}),\\
&(0.777240,\ \textit{Preferred}),\\
&(0.072689,\ \textit{Greatly preferred})\}
\end{aligned}
$$

$$
\begin{aligned}
C_3^4 = \{&(0.067060,\ \textit{Slightly preferred}),\\
&(0.083011,\ \textit{Modertaely preferred}),\\
&(0.777240,\ \textit{Preferred}),\\
&(0.072689,\ \textit{Greatly preferred})\}
\end{aligned}
$$

Option 4: Eliminate the failure modes associated with the protection system in design option 2 and the two failure modes associated with the hydraulic servo transmission system in design option 3.

For the fourth design option, the safety assessments provided by the four designers are given by:

$$
\begin{aligned}
&S_4^1 = S_4^2 = S_4^3 = S_4^4\\
&\quad = \{(0.013049,\ \textit{Slightly preferred}),\\
&\qquad (0.019045,\ \textit{Modertaely preferred}),\\
&\qquad (0.035897,\ \textit{Preferred}),\\
&\qquad (0.932027,\ \textit{Greatly preferred})\}
\end{aligned}
$$

The four designers' individual cost assessments are given as follows:

$$
\begin{aligned}
C_4^1 = \{&(0.059846,\ \textit{Slightly preferred}),\\
&(0.822751,\ \textit{Modertaely preferred}),\\
&(0.062553,\ \textit{Preferred}),\\
&(0.054850,\ \textit{Greatly preferred})\}
\end{aligned}
$$

$$
\begin{aligned}
C_4^2 = \{&(0.028571,\ \textit{Slightly preferred}),\\
&(0.912923,\ \textit{Modertaely preferred}),\\
&(0.031480,\ \textit{Preferred}),\\
&(0.027027,\ \textit{Greatly preferred})\}
\end{aligned}
$$

$$
\begin{aligned}
C_4^3 = \{&(0.057708,\ \textit{Slightly preferred}),\\
&(0.826250,\ \textit{Modertaely preferred}),\\
&(0.062819,\ \textit{Preferred}),\\
&(0.053223,\ \textit{Greatly preferred})\}
\end{aligned}
$$

$$
\begin{aligned}
C_4^4 = \{&(0,\ \textit{Slightly preferred}),\\
&(1,\ \textit{Modertaely preferred}),\\
&(0,\ \textit{Preferred}),\\
&(0,\ \textit{Greatly preferred})\}
\end{aligned}
$$

3. Safety and cost synthesis using typical multiple criteria decision analysis (MCDA) methods

Once safety and cost are assessed for a design option, there is a need to combine the assessments to provide an overall assessment for the option and eventually rank it against other design options. Several methods could be used in such a synthesis process. In this section, three methods are discussed and compared in dealing with the example presented in the previous section, including the additive utility function approach, the analytic hierarchy process (AHP) approach and the evidential reasoning approach. The assessment hierarchy for the example is shown in Fig. 3.

Let ω_S and ω_C denote the relative weights of safety and cost, and ω_1, ω_2, ω_3, ω_4 the relative weights of the opinions of designers 1, 2, 3 and 4, respectively. For demonstration purpose, suppose safety is twice as important as cost and the opinions of designers 2 and 3 are twice as important as those given by designers 1 and 4. So, ω_S = 2ω_C, and $\omega_2 = \omega_3$=2ω_1=2ω_4. Suppose the relative weights of the same group of criteria are normalised so that they are added to one. Then, we have ω_S=0.6667, ω_C=0.3333; and $\omega_2 = \omega_3$=0.3333, $\omega_1 = \omega_4$=0.1667. It should be noted that a range of weights could be assigned to test the robustness of the assessments generated.

3.1 Additive utility function approach

Before this method can be applied, each assessment of a design option on either safety or cost given by a designer must be quantified using for example a score. Since an assessment in the example is represented as an expectation using the four evaluation grades, we need to quantify the grades first for example by using a scale or estimating the utilities of the grades [Winston, 1994]. Suppose the utilities of the four evaluation grades are given as follows [Wang et al., 1996]:

$$
\begin{aligned}
&u(\textit{Slightly preferred}) = 0.217\\
&u(\textit{Moderately preferred}) = 0.478\\
&u(\textit{preferred}) = 0.739\\
&u(\textit{Greatly preferred}) = 1
\end{aligned}
$$

Then the scores of the four options on both safety and cost for each designer can be calculated as the following weighted average scores of the expectations with the degrees of belief used as weights:

3.1.1 Option 1

$$
\begin{aligned}
u_1(\textit{safety/designer } 1) &= u_1(\textit{safety/designer } 2)\\
&= u_1(\textit{safety/designer } 3) = u_1(\textit{safety/designer } 4)\\
&= 0.122425 \times 0.217+0.180205 \times 0.478+0.46337 \times 0.739+0.233999 \times 1\\
&= 0.6891
\end{aligned}
$$

$u_1(cost/designer\ 1) = u_1(cost/designer\ 2) =$
$u_1(cost/designer\ 3) = u_1(cost/designer\ 4)$
$= 0\times 0.217+0\ \times 0.478+0\times 0.739+1\times 1 = 1$

3.1.2 Option 2

$u_2(safety/designer\ 1) = u_2(safety/designer\ 2)$
$= u_2(safety/designer\ 3) = u_2(safety/designer\ 4)$
$= 0.102676\times 0.217+0.156934\times 0.478+0.38486\times 0.739+0.355531\times 1$
$= 0.7372$
$u_2(cost/designer\ 1)$
$= 0.054309\times 0.217+0.066442\times 0.478+0.821848\times 0.739+0.0574\times 1$
$= 0.7083$
$u_2(cost/designer\ 2) = 0.6777$
$u_2(cost/designer\ 3) = 0.7390$
$u_2(cost/designer\ 4) = 0.7013$

3.1.3 Option 3

$u_3(safety/designer\ 1) = u_3(safety/designer\ 2)$
$= u_3(safety/designer\ 3) = u_3(safety/designer\ 4)$
$= 0.022722\times 0.217+0.033659\times 0.478+0.073367\times 0.739+0.870253\times 1$
$= 0.9455$
$u_3(cost/designer\ 1)$
$= 0.067604\times 0.217+0.084062\times 0.478+0.777037\times 0.739+0.071297\times 1$
$= 0.7004$
$u_3(cost/designer\ 2) = 0.6777$
$u_3(cost/designer\ 3) = 0.7013$
$u_3(cost/designer\ 4) = 0.7013$

3.1.4 Option 4

$u_4(safety/designer\ 1) = u_4(safety/designer\ 2)$
$= u_4(safety/designer\ 3) = u_4(safety/designer\ 4)$
$= 0.013049\times 0.217+0.019045\times 0.478+0.035897\times 0.739+0.932027\times 1$
$= 0.9705$
$u_4(cost/designer\ 1)$
$= 0.059846\times 0.217+0.822751\times 0.478+0.062553\times 0.739+0.05485\times 1$
$= 0.5073$
$u_3(cost/designer\ 2) = 0.4929$
$u_3(cost/designer\ 3) = 0.5071$
$u_3(cost/designer\ 4) = 0.4780$

3.1.5 Assessment of design options

The above scores show the average assessments of the four design options on both safety and cost provided by the four designers. Note that the four designers provided the same average assessment for each design option on safety. The additive utility function approach operates on average scores, as summarised in a decision matrix shown in Table 1.

One way to synthesize the assessments is to generate an overall weight for the cost provided by every designer. For example, the overall weight for the cost provided by designer 1 can be calculated as $0.3333 \times 0.1667 = 0.0556$. The overall weight multiplied by a score results in a weighted score. For example, the weighted score for the safety of design option 1 is given by $0.6667 \times 0.6891 = 0.4594$ and that for the cost of design option 1 provided by designer 1 is given by $0.3333 \times 0.1667 \times 1 = 0.0556$. All the other weighted scores are shown in Table 2.

Table 1 Decision Matrix for Design Selection

Attribute		Alternative design			
		Option 1	Option 2	Option 3	Option 4
Safety (0.6667)		0.6891	0.7372	0.9455	0.9705
Cost (0.3333)	Designer 1 (0.1667)	1	0.7083	0.7004	0.5073
	Designer 2 (0.3333)	1	0.6777	0.6777	0.4929
	Designer 3 (0.3333)	1	0.7390	0.7013	0.5071
	Designer 4 (0.1667)	1	0.7013	0.7013	0.4780

Table 2 Weighted Decision Matrix for Design Selection

	Option 1	Option 2	Option 3	Option 4
Safety	0.4594	0.4915	0.6304	0.6470
Cost by Designer 1	0.0556	0.0394	0.0389	0.0282
Cost by Designer 2	0.1111	0.0753	0.0753	0.0548
Cost by Designer 3	0.1111	0.0821	0.0779	0.0563
Cost by Designer 4	0.0556	0.0390	0.0390	0.0266

In the additive utility (value in this case) function approach, the weighted scores on the safety and cost attributes are added up for an option, resulting in an overall score for the option. For example, the overall score for option 1 is given by:

$$u(\text{option 1}) = 0.4594 + 0.0556 + 0.1111 + 0.1111 + 0.0556 = 0.7928.$$

Similarly, the overall scores of the other three options are given by

$$u(\text{option 2}) = 0.7273,\ u(\text{option 3}) = 0.8615,\ u(\text{option 4}) = 0.8129.$$

The ranking of the four design options is then given on the basis of the magnitude of their overall scores as follows

$$\text{option 3} \succ \text{option 4} \succ \text{option 1} \succ \text{option 2}$$

The additive utility (value) function approach provides a simple process for criteria aggregation. To use the method properly, however, one should be aware of its limits and drawbacks. Despite the loss of the original features and diversity of the distributed assessments given in section 2.3, this approach assumes preference independence, a linear utility function for each criterion, and direct and proportional compensation among criteria. These assumptions are not always acceptable. For example, a linear utility function implies that the decision maker is neutral to risk. In many decision situations, however, decision makers are often averse to risk. This is particularly the case when safety is assessed. Preference independence means that tradeoffs between two criteria are independent of other criteria. While this is not easy to test, it is not appropriate to assume this is always satisfied.

3.2 AHP

The Analytic Hierarchy Process (AHP) is another method that can be used to deal with MCDA problems. AHP is based on the eigenvector method that is usually applied to estimating relative weights of criteria by means of pairwise comparisons. To solve the above design selection problem, AHP may be summarised as follows.

Since it is already assumed that safety is twice as important as cost in selection of design options, a pairwise comparison matrix can be constructed as in Table 3, where the element "2" in the second row of the last column means that safety is twice as important as cost.

Table 3 Pairwise Comparison 1

	Safety	Cost
Safety	1	2
Cost	½	1

In AHP, the normalised right eigenvector of the pairwise comparison matrix with respect to its largest eigenvalue is employed as the weights of safety and cost. Suppose A represents the pairwise comparison matrix, or

$$A = \begin{bmatrix} 1 & 2 \\ 1/2 & 1 \end{bmatrix}$$

W a weight vector or $W = [\omega_s \quad \omega_c]^{\mathrm{T}}$, and $\lambda_{\max}$ the maximum eigenvalue of the matrix A. Then, W is calculated using the following equation:

$$AW = W\lambda_{\max}$$

There are software packages that can be used to solve the above vector equation to find W [Saaty, 1988]. An approximate solution procedure can be found in [Sen & Yang, 1998], as summarised below.

Step 1: Provide an initially normalised vector $W^0 = [1 \ 0 \ \cdots \ 0]^T$ and let $t = 0$.

Step 2: Calculate a new eigenvector as follows:

$$W^{t+1} = AW^t$$

Step 3: Calculate the maximum eigenvalue by:

$$\lambda_{\max} = \sum_{i=1}^{n} w^{t+1}$$

Step 4: Normalise and update the eigenvector as follows:

$$\overline{w}^{t+1} = \frac{w^{t+1}}{\lambda_{\max}}, \text{ and let } w^{t+1} = \overline{w}^{t+1} \text{ for all } i=1, \ldots, n$$

Step 5: Calculate the error between the old and new eigenvectors and then check if

$$\left| w^{t+1} - w^t \right| \le \delta \text{ for all } i=1, \ldots, n$$

where δ is a small non-negative real number (say $\delta = 1.0 \times 10^{-6}$). If the condition is satisfied, go to step 6. Otherwise, let t=t+1 and go to step 2.

Step 6: Calculate the consistency index (CI) as follows:

$$CI = \frac{\lambda_{\max} - n}{n-1}$$

If $CI \leq 0.1$, the pairwise comparisons provided in the matrix A are satisfactorily consistent. Otherwise, the comparisons need to be revised.

Applying the above procedure to solve the eigenvector equation (n=2) leads to the following results:

$$W = [\omega_s \quad \omega_{\mathrm{c}}] = [0.6667 \quad 0.3333]$$

In the above pairwise comparison matrix, the weights between safety and cost are already made clear. Generally, if pairwise comparisons are provided for three or more criteria, they may not be completely consistent and as such it is not straightforward to obtain relative weights of criteria from the comparisons. The AHP method and several other methods can be used to generate weights using pairwise comparisons. For example, suppose the pairwise comparison matrix is provided for the importance of the opinions of the four designers, as shown in Table 4

Table 4 Pairwise Comparisons between Designers

	Designer 1	Designer 2	Designer 3	Designer 4
Designer 1	1	½	½	1
Designer 2	2	1	1	2
Designer 3	2	1	1	2
Designer 4	1	½	½	1

A pairwise comparison matrix $A = (a_{ij})_{n \times n}$ is completely consistent if $a_{ij} = a_{ik} a_{kj}$ for all i, j, k =1, ..., n where n is the dimension of the matrix. It is easy to show that the comparison matrix of Table 4 is completely consistent. Using the above procedure, the normalised eigenvector of the matrix is given by

$$W = [\omega_1 \quad \omega_2 \quad \omega_3 \quad \omega_4] = [0.1667 \quad 0.3333 \quad 0.3333 \quad 0.1667]$$

To use AHP for ranking design options, one also needs to compare them in a pairwise fashion with respect to each criterion. This may not be an easy task in general. Given the assessment data as in Table 1, however, a pairwise comparison matrix can be constructed for each criterion [Huang and Yoon, 1981]. With respect to safety, for example, the four design options can be compared as in Table 5.

Table 5 Pairwise Comparisons of Designs on Safety

	Option 1	Option 2	Option 3	Option 4
Option 1	1	0.9348	0.7288	0.7100
Option 2	1.0697	1	0.7797	0.7596
Option 3	1.3721	1.2826	1	0.9742
Option 4	1.4085	1.3165	1.0264	1

A number in Table 5 denotes the extent to which one option is more attractive than another. For example, the number "1.4085" in the second column of the last row means that option 4 is 1.4085 times as attractive as option 1 in terms of safety. It can be seen from Table 5 that the differences between the four design options are quite small and this would make it difficult to provide direct pairwise comparisons between the options without reference to the assessment data shown in Table 1.

In AHP, the scores of the four design options in terms of safety are generated, using the above solution procedure, as the eigenvector of the pairwise comparison matrix with respect to its largest eigenvalue, as shown in Table 6.

Table 6 Scores of Design Options on Safety

	Option 1	Option 2	Option 3	Option 4
Safety	0.2062	0.2206	0.2829	0.2904

In a similar way, the pairwise comparison matrices of the cost criteria for the four designers can be generated, as shown in Tables 7 to 10.

Table 7 Pairwise Comparisons of Designs on Cost by Designer 1

	Option 1	Option 2	Option 3	Option 4
Option 1	1	1.4118	1.4278	1.9712
Option 2	0.7083	1	1.0113	1.3962
Option 3	0.7004	0.9888	1	1.3806
Option 4	0.5073	0.7162	0.7243	1

Table 8 Pairwise Comparisons of Designs on Cost by Designer 2

	Option 1	Option 2	Option 3	Option 4
Option 1	1	1.4756	1.4756	2.0288
Option 2	0.6777	1	1	1.3749
Option 3	0.6777	1	1	1.3749
Option 4	0.4929	0.7273	0.7273	1

Table 9 Pairwise Comparisons of Designs on Cost by Designer 3

	Option 1	Option 2	Option 3	Option 4
Option 1	1	1.3532	1.4259	1.9720
Option 2	0.7390	1	1.0538	1.4573
Option 3	0.7013	0.9489	1	1.3830
Option 4	0.5071	0.6862	0.7231	1

Table 10 Pairwise Comparisons of Designs on Cost by Designer 4

	Option 1	Option 2	Option 3	Option 4
Option 1	1	1.4259	1.4259	2.0921
Option 2	0.7013	1	1	1.4672
Option 3	0.7013	1	1	1.4672
Option 4	0.4780	0.6816	0.6816	1

The scores of the four design options in terms of cost for the four designers can also be generated by identifying the eigenvectors of the matrices in Tables 7 to 10 with respect to their respective largest eigenvalues, as shown in Table 11.

In AHP, different ways are suggested to aggregate the scores generated from the pairwise comparisons [Saaty, 1988]. One way is to use the simple weighting approach for aggregation from one criteria level to another [Huang & Yoon, 1981]. Firstly, aggregate the cost scores of the four designers by multiplying each score with the relevant weight and then adding up the weighted scores for each option, which leads to an aggregated cost score for each option, as shown in Table 12. Finally, the safety score and the cost score for an option are multiplied by their weights and then added up to generate an overall score for the option, as shown in Table 13.

Table 11 Scores of Options on Safety and Cost

		Option 1	Option 2	Option 3	Option 4
Safety (0.6667)		0.2062	0.2206	0.2829	0.2904
Cost (0.3333)	Designer 1 (0.1667)	0.3429	0.2429	0.2402	0.1740
	Designer 2 (0.3333)	0.3511	0.2379	0.2379	0.1731
	Designer 3 (0.3333)	0.3393	0.2507	0.2379	0.1720
	Designer 4 (0.1667)	0.3471	0.2435	0.2435	0.1659

Table 12 Aggregated Assessment of Options on Safety and Cost

	Option 1	Option 2	Option 3	Option 4
Safety (0.6667)	0.2062	0.2206	0.2829	0.2904
Cost (0.3333)	0.3451	0.2439	0.2392	0.1717

Table 13 Overall Assessment of Options on Safety and Cost

	Option 1	Option 2	Option 3	Option 4
Safety & cost	0.2525	0.2284	0.2683	0.2508

Based on the overall scores of Table 13, the ranking of the four design options are given as follows:

option 3 $\succ$ option 1 $\succ$ option 4 $\succ$ option 2

This ranking is different from that generated using the additive utility function approach in that the positions of option 1 and option 4 are swapped. In fact, the AHP method does not significantly differentiate the two options, as the difference between the overall scores of the two options is very small. AHP is usually applied to generating relative weights. However, the use of AHP to assess design options may lead to problems like rank reversal [Belton, 1986][Islei & Lockett, 1988][Stewart, 1992][Barzilai, 1997], that is, the introduction of new options for assessment may cause the unexpected and irrational change of the ranking of the current options.

3.3 The evidential reasoning approach

The evidential reasoning (ER) approach can be used to deal with multiple criteria decision analysis problems of both a quantitative and qualitative nature with uncertainty [Yang & Singh, 1994][Yang & Sen, 1994][Yang, 2001]. It can process several types of information within an ER framework. The ER framework is different from most conventional MCDA modelling frameworks in that it employs a belief structure to represent an assessment as a distribution. In Section 2.3, four evaluation grades were defined as follows:

$$H = \{H_1,\ H_2,\ H_3,\ H_4\}$$
$$= \{\textit{Slightly preferred},\ \ \textit{Modertaely preferred},$$
$$\textit{Preferred},\ \textit{Greatly preferred}\}$$

Using the four evaluation grades, the assessment of an attribute A_1 on an option O_1, denoted by $S(A_1(O_1))$, can be represented using the following belief structure:

$$S(A_1(O_1)) = \{(\beta_{1,1}, H_1), (\beta_{2,1}, H_2), (\beta_{3,1}, H_3), (\beta_{4,1}, H_4)\}$$

where $1 \geq \beta_{n,1} \geq 0$ (n=1, …, 4) denotes the degree of belief that the attribute A_1 is assessed to the evaluation grade H_n. $S(A_1(O_1))$ reads that the attribute A_1 is assessed to the grade H_n to a degree of $\beta_{n,1} \times 100\%$ (n=1, …, 4) for the option O_1.

There must not be $\sum_{n=1}^{4} \beta_{n,1} > 1$. $S(A_1(O_1))$ can be considered to be a complete distributed assessment if $\sum_{n=1}^{4} \beta_{n,1} = 1$ and an incomplete assessment if $\sum_{n=1}^{4} \beta_{n,1} < 1$. In the ER framework, both complete and incomplete assessments can be accommodated (Yang (2001)).

In the ER framework, a MCDA problem with M attributes A_i (i=1, …, M), K options O_j (j=1, …, K) and N evaluation grades H_n (n=1, …, N) for each attribute is represented using an extended decision matrix with $S(A_i(O_j))$ as its element at the ith row and jth column where $S(A_i(O_j))$ is given as follows:

$$S(A_i(O_j)) = \{(H_n, \beta_{n,i}(O_j)),\ \ n = 1, \cdots, N\} \quad i=1, \ldots, M,\ j=1, \ldots, K$$

It should be noted that an attribute can have its own set of evaluation grades that may be different from those of other attributes [Yang, 2000].

Instead of aggregating average scores, the ER approach employs an evidential reasoning algorithm developed on the basis of the evidence combination rule of the Dempster-Shafer theory to aggregate belief degrees [Yang & Singh, 1994)], [Yang & Sen, 1994] [Yang, 2001]. Thus, scaling grades are not necessary for aggregating attributes in the ER approach and in this way it is different from traditional MCDA approaches, most of which aggregate average scores.

Suppose ω_i is the relative weight of the attribute A_i and is normalised so that $1 \geq \omega_i \geq 0$ and $\sum_{i=1}^{L} \omega_i = 1$ where L is the total number of attributes in the same group for aggregation. To simplify the discussion, only the combination of complete assessments is examined. The description of the recursive ER algorithm capable of aggregating both complete and incomplete assessments is detailed in [Yang & Sen, 1994][Yang, 2001]. Without loss of generality and for illustration purpose, the ER algorithm is presented below for combining two assessments only.

Suppose the second assessment $S(A_2(O_1))$ is given by

$$S(A_2(O_1)) = \{(H_1, \beta_{1,2}), (H_2, \beta_{2,2}), (H_3, \beta_{3,2}), (H_4, \beta_{4,2})\}$$

The problem is to aggregate the two assessments $S(A_1(O_1))$ and $S(A_2(O_1))$ to generate a combined assessment $S(A_1(O_1)) \oplus S(A_2(O_1))$. Suppose $S(A_1(O_1))$ and $S(A_2(O_1))$ are both complete. Let

$$m_{n,1} = \omega_1 \beta_{n,1} \text{ } (n=1, \ldots, 4) \text{ and } m_{H,1} = 1 - \omega_1 \sum_{n=1}^{4} \beta_{n,1} = 1 - \omega_1$$

$$m_{n,2} = \omega_2 \beta_{n,2} \text{ } (n=1, \ldots, 4) \text{ and } m_{H,2} = 1 - \omega_2 \sum_{n=1}^{4} \beta_{n,2} = 1 - \omega_2$$

where each $m_{n,j}$ (j = 1, 2) is referred to as basic probability mass and each $m_{H,j}$ is the remaining belief unassigned to H_j (j = 1, 2, 3, 4).

The ER algorithm is used to aggregate the basic probability masses to generate combined probability masses, denoted by m_n (n=1, …, 4) and m_H using the following equations:

$$m_n = k(m_{n,1} m_{n,2} + m_{H,1} m_{n,2} + m_{n,1} m_{H,2}), \quad (n=1, \ldots, 4)$$

$$m_H = k(m_{H,1} m_{H,2})$$

where

$$k = \left(1 - \sum_{t=1}^{4} \sum_{\substack{n=1 \\ n \neq t}}^{4} m_{t,1} m_{n,2}\right)^{-1}$$

The combined probability masses can then be aggregated with the third assessment in the same fashion. The process is repeated until all assessments are aggregated. The final combined probability masses are independent of the order in which individual assessments are aggregated.

If there are only two assessments, the combined degrees of belief β_n (n=1, …, 4) are generated by:

$$\beta_n = \frac{m_n}{1 - m_H} \quad (n=1, \ldots, 4)$$

The combined assessment for the option O_1 can then be represented as follows:

$$S(O_1) = \{(H_1, \beta_1), (H_2, \beta_2), (H_3, \beta_3), (H_4, \beta_4)\}$$

An average score for O_1, denoted by $u(O_1)$, can also be provided as the weighted average of the scores (utilities) of the evaluation grades with the belief degrees as weights, or

$$u(O_1) = \sum_{i=1}^{4} u(H_i) \beta_i$$

where $u(H_i)$ is the utility of the ith evaluation grade H_i. For i=1, for example, we have $u(H_1) = u(\mathit{Slightly\ preferred}) = 0.217$.

An intelligent decision system (IDS[2]) has been developed on the basis of the ER approach [Yang & Xu, 2000]. The IDS software is designed to transform the lengthy and tedious model building and result analysis process into an easy window-based click and design activity. The rest of this sub-section is devoted to demonstrating the solution process of the above safety & cost-based design selection problem using the IDS software.

The main window of IDS for solving the design selection problem is shown in Fig. 4, which has a menu bar, a tool bar and a model display window. The hierarchy of the assessment criteria can be readily constructed using the modelling menu or the related short cuts on the tool bar. IDS also provides an assistant model builder for building large-scale models that may have hundreds of criteria and options.

In the model display window, each criterion object is coloured in blue and has three boxes for displaying the criterion name, its weight and average score. For example, the criterion "1. Safety" has a weight of "0.6667" and its average score for "Design option 1" is "0.6956". Each alternative object is coloured in yellow and also has three boxes for displaying the alternative name, its ranking and overall average score. For example, "Design option 1" is ranked the third and has an overall average score of "0.776495". Apart from an average score, IDS is capable of generating a distributed assessment for each option on any criterion. Fig. 5 shows the overall distributed assessment of Design option 1. In Fig. 5, the degrees of belief to the evaluation grades clearly show the merits and drawbacks of the design option.

In IDS, a number of dialog windows are designed to support model building, data input, result analysis, reporting and sensitivity analysis. For example, Fig. 5 is generated using an IDS dialog window for reporting results graphically. Fig. 6 shows an IDS dialog window for data input for "Design option 1" on a cost criterion for "2.1 Designer 1". All data can be entered using similar dialog windows, whether they are precise numbers, random numbers with probabilities, or subjective assessments. Fig. 7 shows the visual cross comparison of the four design options on both safety and cost generated using the IDS visual comparison dialog window.

In IDS, AHP and other methods are used for generating relative weights of criteria and the evidential reasoning approach is used to aggregate criteria from the bottom level of criteria to the top level criterion "Design selection". The overall assessment for each option can be characterised as shown for option 1 in Fig. 5. In IDS, dialog window are designed to support visually scaling the evaluation grades or estimating the utilities of the grades. For example, Fig. 8 shows a utility curve for the four evaluation grades, given at the beginning of Section 3.1. The curve can be changed onscreen to suit the requirements of individual designers. For the given utility curve, the average scores for the four design options are generated as shown in Table 14.

Table 14 Overall Assessment of Options on Safety and Cost

	Option 1	Option 2	Option 3	Option 4
Safety & cost	0.7765	0.7407	0.9085	0.8818

Based on the overall scores of Table 14, the ranking of the four design options are given as follows:

option 3 $\succ$ option 4 $\succ$ option 1 $\succ$ option 2

[2] A free demo version of IDS can be obtained from Dr J B Yang via email: jian-bo.yang@umist.ac.uk

The above ranking is the same as that generated using the additive utility function approach. Apart from the average scores and the related ranking for the design options, however, the ER approach can provide much richer information for analysis. This provides a panoramic view on each design option so that the benefits and risks involved in selecting an option are made clear to the designers.

4. Discussion

When designing a large offshore engineering product, especially at the initial design stages, there are usually several design options. It should be noted that such options are produced at the top level where only non-numerical data may be available. The options may only be associated with some parameters such as the dimensions of the installation, the type of the platform, the number of wells, etc. The information available for making decisions on which option to select at this stage may be incomplete. As a design proceeds to a more detailed stage, the selection of design options at lower levels is required and again a similar process for selecting a particular design option may be required. It should be noted that the decision making process at all levels needs to deal with multiple objectives and may involve uncertain or incomplete information. The MCDA methods described in this paper may prove useful to select the best design option by taking into account safety and other design objectives in a rational manner.

As the best design option is chosen, the design can further proceed. More and more information becomes available for more detailed safety analysis. Decision making may need to be carried out at the next level. At this stage, it may be the case that only part of the information is complete for quantitative safety estimate. This may also be true for modelling of other design objectives. In such cases, MCDA techniques may be required to combine safety estimate with other design objectives to arrive at the best designs within both technical and economic constraints.

As the design further proceeds, it reaches a stage where there is enough information for carrying out design optimisation based on quantitative safety assessment. At this stage, safety may be assessed using various safety assessment techniques in terms of likelihood of occurrence and magnitude of consequences. A mathematical model can be formulated and then again MCDA techniques can be used to process the model in order to optimise the design.

5. Conclusion

The safety culture in the UK offshore industry has changed significantly over the last several years. The offshore industry has been moving towards a “goal setting” risk-based regime. This safety culture change gives designers/safety analysts more flexibility and also encourages them to develop and apply more advanced risk modelling and decision making tools to deal with their safety issues. There is a great potential for MCDA methods to be applied in the design selection and optimisation processes. Appropriate application of MCDA tools can facilitate decision making in offshore engineering design and operations to improve efficiency.

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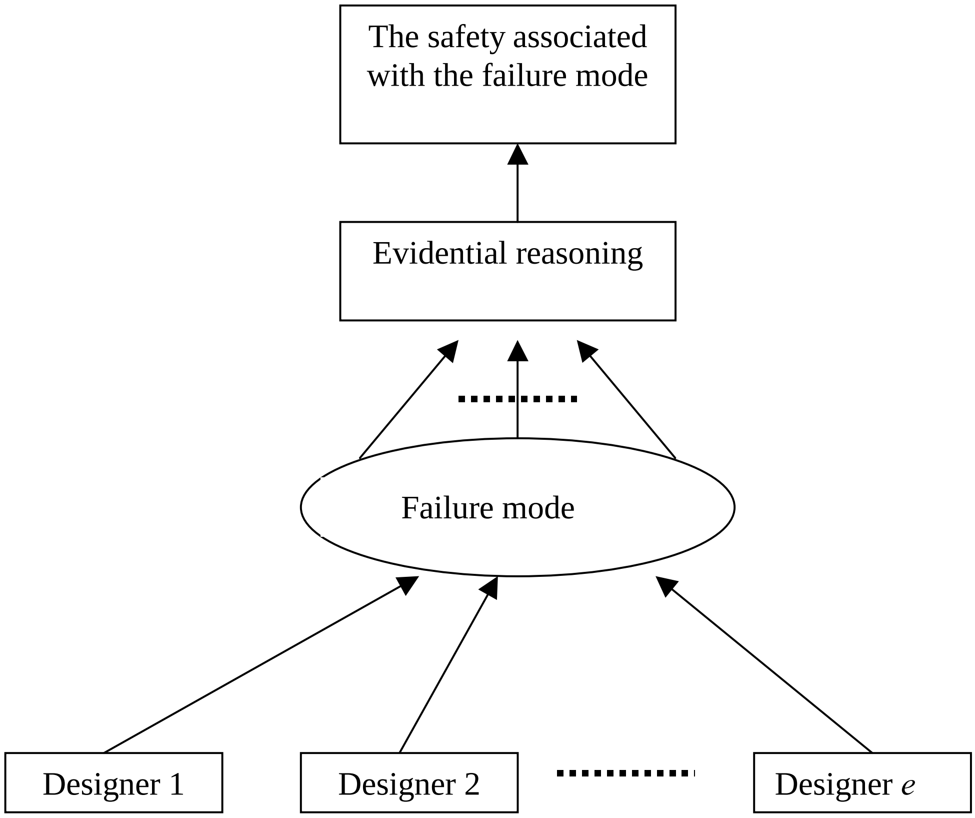

Fig. 1 A diagram for synthesising the safety associated with a failure mode

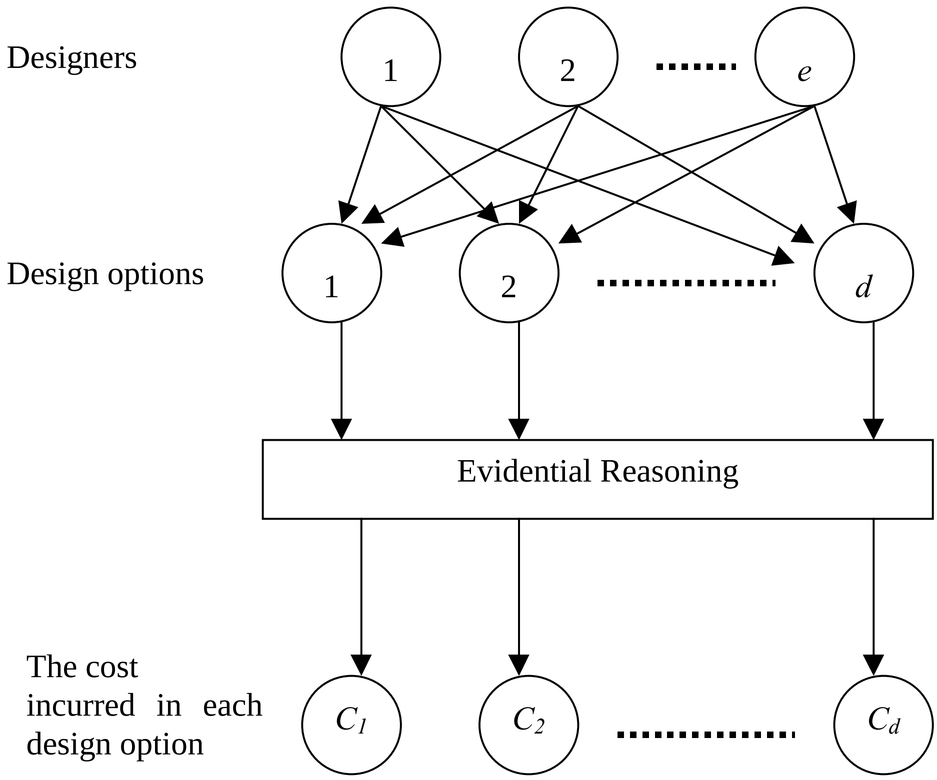

Fig. 2 A hierarchical diagram of cost modelling

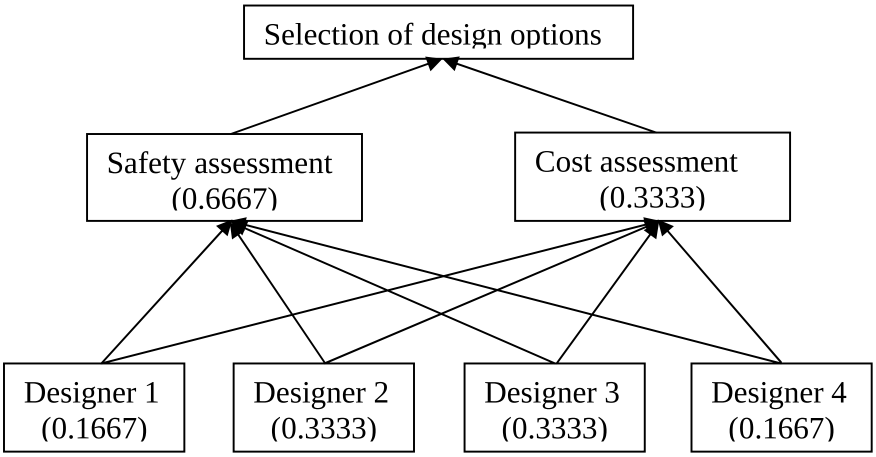

Fig. 3 Safety and cost assessment hierarchy

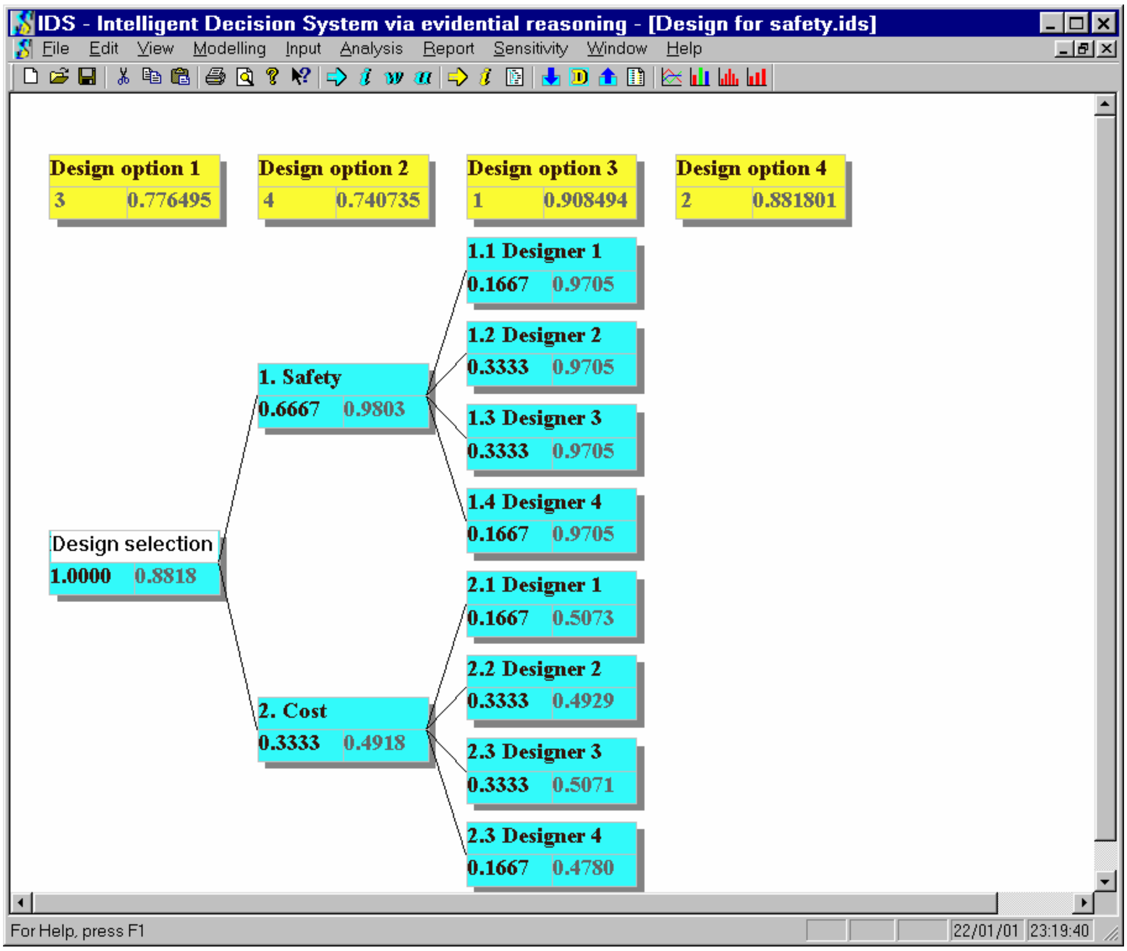

Fig. 4 IDS Main Window for Safety & Cost Based Design Selection

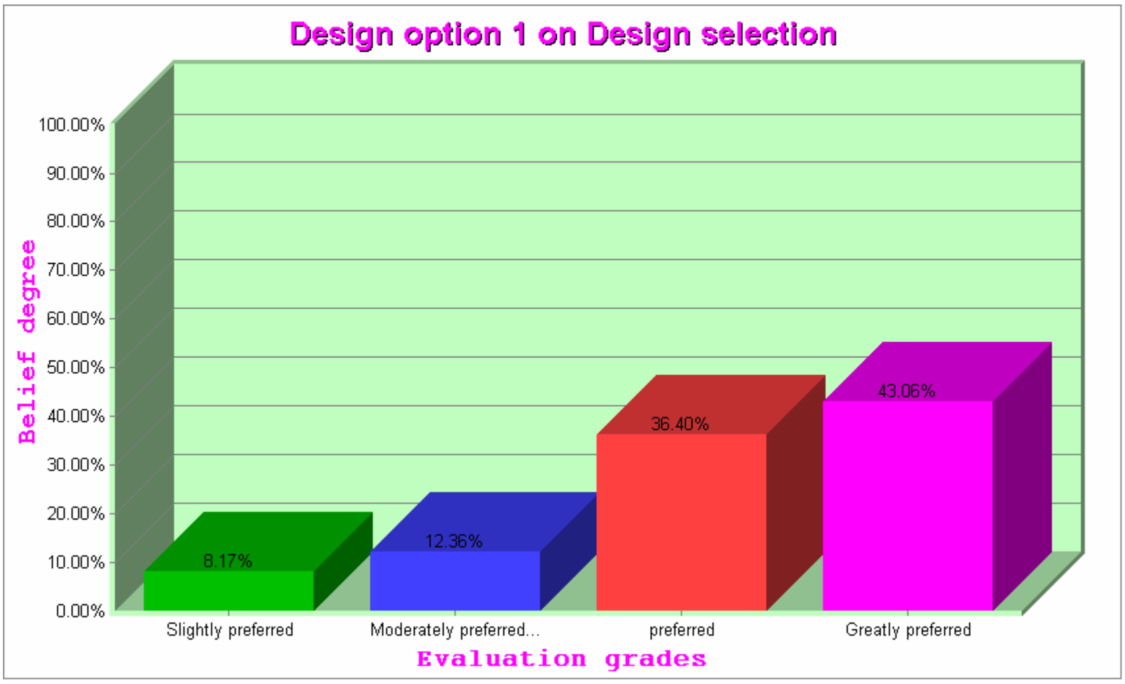

Fig. 5 The overall distributed assessment of design option 1 generated by IDS

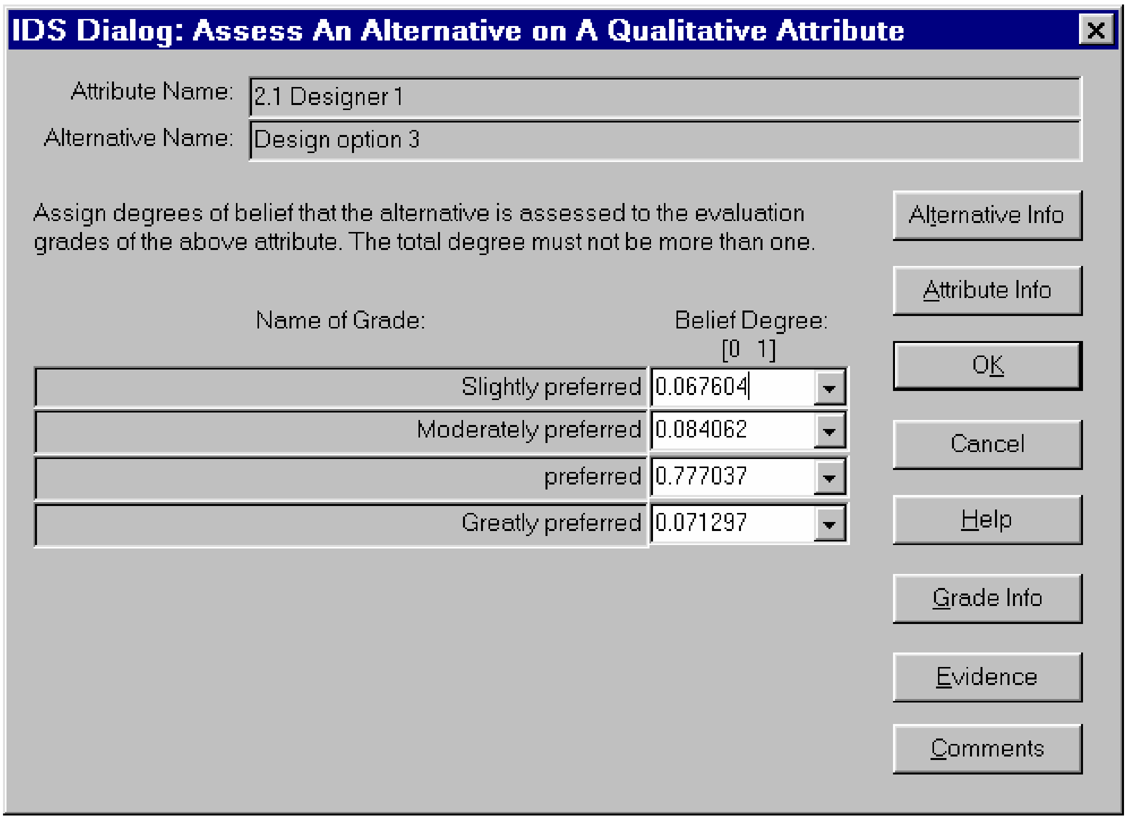

Fig. 6 IDS Data input dialog window

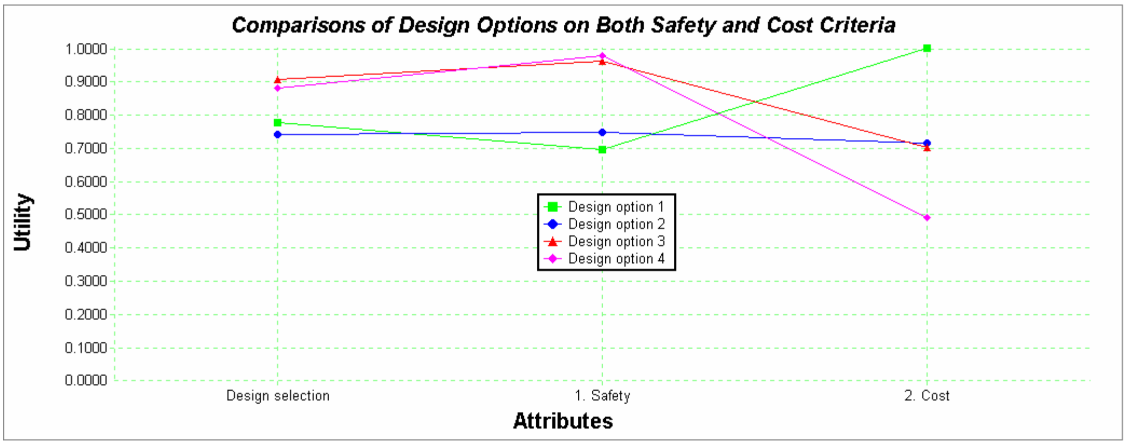

Fig. 7 Comparison of design options on both safety and cost generated by IDS

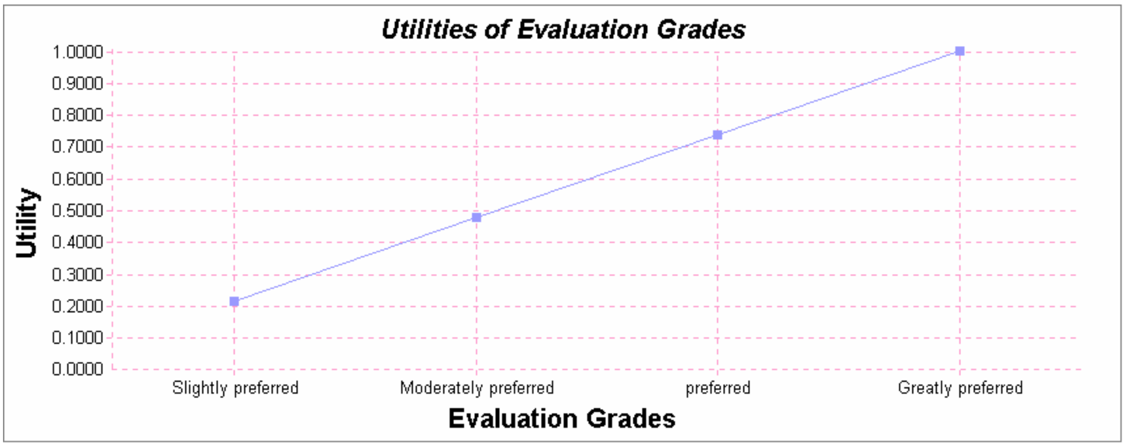

Fig. 8 Utility curve of evaluation grades generated by IDS