Expert Systems with Applications 39 (2012) 4749–4759

Contents lists available at SciVerse ScienceDirect

Expert Systems with Applications

journal homepage: www.elsevier.com/locate/eswa

Belief rule-based methodology for mapping consumer preferences and setting product targets

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ARTICLE INFO

Keywords:
Consumer preference modeling
Preference mapping
Belief rule-based system
Optimization
Product design
Quality assessment
Target setting

ABSTRACT

Rapid and accurate identification of consumer demands and systematic assessment of product quality are essential to success for new product development, in particular for fast moving consumer goods such as food and drink products. This paper reports an investigation into a belief rule-based (BRB) methodology for quality assessment, target setting and consumer preference prediction in retro-fit design of food and drink products. The BRB methodology can be used to represent the relationships between consumer preferences and product attributes, which are complicated and nonlinear. A BRB system can initially be established using expert knowledge and then optimally trained and validated using data generated from consumer or expert panel assessments or from tests and experiments. The established BRBs can then be used to predict the consumer acceptance of new products or set product target values in retro-fit design. The proposed BRB methodology is applied to the design of a lemonade drink product using real data provided by a sensory product manufacturer in the UK. The results show that the BRB methodology can be used to predict consumer preferences with high accuracy and to set optimal target values for product quality improvement.

1. Introduction

Consumer preferences for a food and drink product are closely related to its sensory attributes, which are those such as appearance, aroma, flavor, and the like (Meilgaard, Civille, & Carr, 1999). It is very important to understand and establish such relationships for predicting consumer acceptance or liking in the design and testing of new food and drink product designs.

To investigate the impacts of sensory attributes of food and drink products on consumer preferences, models that can relate consumer preferences to sensory attributes need to be developed. Several methodologies have been suggested in the literature that can be used to model relationships between consumer preferences and sensory attributes. Multiple linear regression (MLR) analysis is perhaps the simplest methodology that could be used for this purpose, but it is proven to be not adequate in capturing consumer preferences from sensory attributes. This is largely because the relationships between consumer preferences and sensory attributes are complicated and highly nonlinear and hence cannot be adequately interpreted by linear models. Another reason is that the number of sensory attributes is often larger than the number of products, which makes MLR model parameters estimated from a small number of samples highly unstable, but in some cases, MLR can still provide some useful models.

External preference mapping (EPM) (Arditti, 1997; Faber, Mojet, & Poelman, 2003; Geel, Kinear, & de Kock, 2005; Guinard, Uotani, & Schlich, 2001; Heyd & Danzart, 1998; Martínez, Cruz, Hough, & Vega, 2002; van Kleef, van Trijp, & Luning, 2006) turns out to be the most extensively used methodology for sensory analysis. EPM models consumer preferences as a polynomial function of the first two principal components (PCs) that are extracted from the sensory data of food and drink products using the principal component analysis (PCA). The polynomial function could be quadratic, elliptical, circular, or vector models, as shown below (Faber et al., 2003):

$$\begin{aligned}
&\text{Vector}: \ y = a + b_1x_1 + b_2x_2 + \varepsilon,\\
&\text{Circular}: \ y = a + b_1x_1 + b_2x_2 + c(x_1^2 + x_2^2) + \varepsilon,\\
&\text{Elliptical}: \ y = a + b_1x_1 + b_2x_2 + c_1x_1^2 + c_2x_2^2 + \varepsilon,\\
&\text{Quadratic}: \ y = a + b_1x_1 + b_2x_2 + c_1x_1^2 + c_2x_2^2 + dx_1x_2 + \varepsilon,
\end{aligned}$$

where y is consumer preferences, x_1 and x_2 are the first two PCs, and ε is residual error.

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doi:10.1016/j.eswa.2011.09.105

A vector model is often used, but the choice for a more complex model is possible, depending upon a goodness-of-fit test. It is well documented that not all consumer preference data can be well fitted and accounted for by EPM (Faber et al., 2003). In particular, vector models are essentially linear, which makes their explanatory power very limited. Furthermore, the aggregation of all sensory attributes into two PCs without proper classification makes the PCs difficult to explain and understand.

Artificial neural networks (ANNs) (Boccorh & Paterson, 2002; Bomio, 1998; Krishnamurthy, Srivastava, Paton, Bell, & Levy, 2007; Tan, Gao, & Gerrard, 1999; Zhang & Chen, 1997) are another popular methodology for modeling the relationships between consumer preferences and sensory attributes, which model consumer preferences as a complicated nonlinear function of sensory attributes. The nonlinear function is defined by a multilayer network, including one or more hidden layers, with sensory attributes as inputs and consumer preferences as output. It is believed that a three-layer neural network with a sufficiently large number of hidden neurons can model any nonlinear relationships between inputs and outputs (Haykin, 1994). So, three-layer neural networks (NNs) are most widely used in practice. Although ANNs have very strong nonlinear modeling capabilities, they suffer from some drawbacks, two of which are black-box problem and over-fitting problem. The black-box problem means that ANNs are basically a black box simulator. The relationships between inputs and output are not explicit or transparent, so it is difficult to understand or predict its behaviors. The over-fitting problem means that ANNs may sometimes fit training data set perfectly well, whereas perform poorly on testing data set. In other words, ANNs may sometimes have a poor generalization capability. Another significant drawback of ANNs is their inability to be trained to set targets for sensory attributes.

Support vector machines (SVMs) as an Artificial Intelligence method have gained increasing popularity in recent years and also been used for learning consumer preferences from the analysis of sensory data by Bahamonde, Díez, Quevedo, Luaces, and del Coz (2007). SVM is a machine learning algorithm developed by Vapnik (1995) and has been shown to be very effective for both classification and regression analyses. In classification analysis, SVMs attempt to find an optimal hyperplane to separate two classes. For regression analysis, SVMs are aimed at estimating an unknown continuously-valued function based on a finite number of noisy samples. In SVMs for regression analysis, input data are mapped into a high dimensional feature space where a linear model is constructed and linear regression is performed by using ε-insensitive loss. Mathematically, SVM regression is formulated as a convex quadratic programming problem (QPP) with inequality constraints, which produces a global optimal solution. SVMs offer new possibilities to learn consumer preferences from sensory data with a high order polynomial kernel function or even a Gaussian radial basis function (RBF), but they need to pre-specify kernel functional forms and predetermine SVM parameters, which prove to be highly subjective and may have significant impact on the performances of SVM regression models. Since the true relationships between consumer preferences and sensory attributes are not known, it is not appropriate, if not entirely impossible, to specify the functional forms of the relationships in advance. Besides, why an unknown relationship could be described by a particularly chosen kernel function is also difficult to explain in theory.

Based on the above analyses, in this paper we propose a belief rule-based (BRB) methodology for quality assessment, target setting and consumer preference prediction in retro-fit design of food and drink products. The BRB methodology does not need to specify any functional form and characterizes the causal relationships between consumer preferences and product attributes using belief rule bases (BRBs). Each BRB is a collection of belief rules which are the generalization of traditional IF-THEN rules. So, a BRB system can represent functional relationships transparently, where expert knowledge can be explicitly embedded. Several types of parameters as well as the structure of a BRB system, such as belief degrees, rule weights and attribute weights, can be trained using collected data for food and drink products. The trained BRB system can then be used for predicting consumer preferences for new products or setting target values for product attributes to support product retro-design. In this paper, a practical retro-fit design case for a lemonade drink product is examined using the BRB methodology to show its potential in wide applications.

The rest of the paper is organized as follows. In Section 2, we investigate the BRB methodology and the models for learning and target setting. In Section 3, we examine a case study using the BRB methodology to illustrate its applications and potentials in sensory analysis and the retro design of food and drink products. The paper is concluded in Section 4.

2. BRB methodology

The BRB methodology for preference mapping and target setting of food and drink products includes preference mapping, inference, learning, prediction and target setting, which are elaborated in this section in detail.

2.1. Preference mapping by BRB

2.1.1. Model structure

In sensory analysis, food and drink products are usually characterized by a large number of sensory attributes. To simplify the structure of a BRB, the BRB methodology requires sensory attributes of food and drink products to be grouped by their characteristics such as appearance, taste, aroma, texture, flavor, and so on. Let A_j ($j = 1,\ldots,K$) be the jth group attribute, related to m_j sensory attributes as shown in Fig. 1. For such a BRB model structure, there are $(K + 1)$ BRBs to be developed, one of which is for total goal, i.e. quality evaluation of food and drink products, with the other being for the assessment of the K group attributes.

2.1.2. PCA

Some of the group attributes may be related to many sensory attributes. In this situation, it is impractical to build a BRB using the relevant sensory attributes directly because of the large number of rules that could be generated. In this case, instead of using the original sensory attributes, a BRB system can be built using the first two

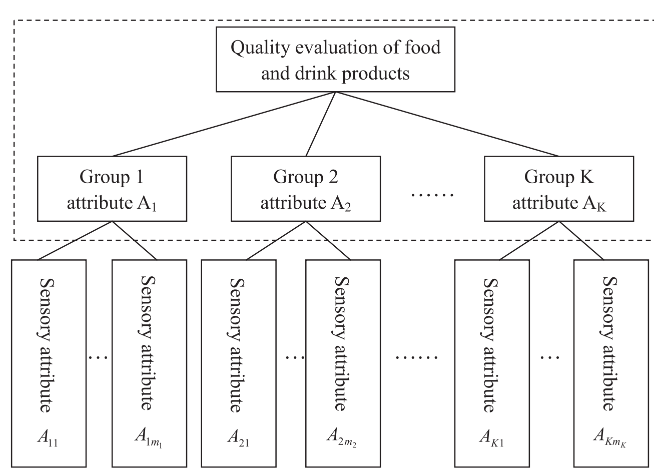

Fig. 1. BRB model structure for preference mapping of food and drink products.

or three PCs of the relevant sensory attributes. This idea is somehow similar to what is used in the external preference mapping method. In this section, we briefly describe some of the basic equations for calculation of PCs in the framework of a BRB model as shown in Fig. 1, which will be used in the case study.

Suppose A_h is a group attribute related to many sensory attributes $A_{h1},\ldots,A_{hm_h}$ (for example $m_h > 5$), whose values for the product P_i $(i = 1,\ldots,n)$ are denoted by $\mathbf{x}_{ih} = (x_{ih1},\ldots,x_{ihm_h})$. Let $R_h = (r_{ij})_{m_h \times m_h}$ be their correlation matrix, whose elements are computed by

$$r_{ij} = \frac{\sum_{k=1}^{n}(x_{khi} - \bar{x}_{hi})(x_{khj} - \bar{x}_{hj})}{\sqrt{\sum_{k=1}^{n}(x_{khi} - \bar{x}_{hi})^2 \cdot \sum_{k=1}^{n}(x_{khj} - \bar{x}_{hj})^2}}, \quad i,j = 1,\ldots,m_h, \tag{1}$$

where $\bar{x}_{hj} = \frac{1}{n}\sum_{k=1}^{n} x_{khj}$ from $j = 1$ to m_h are the average values of x_{hj} over the n products. Denote by λ_j $(j = 1,\ldots,m_h)$ the eigenvalues of the characteristic equation $|R_h - \lambda I| = 0$ and by $\mathbf{u}_j = (u_{j1},\ldots,u_{jm_h})^T$ the corresponding eigenvectors derived from $R_h\mathbf{u}_j = \lambda_j\mathbf{u}_j$, where λ_j $(j = 1,\ldots,m_h)$ are sorted in descending order, i.e. $\lambda_1 \geqslant \lambda_2 \geqslant \cdots \geqslant \lambda_{m_h}$, and $\mathbf{u}_j$ $(j = 1,\ldots,m_h)$ satisfy the condition $\mathbf{u}_j^T\mathbf{u}_j = 1$. Then, the m_h PCs are obtained as

$$F_{hj} = u_{j1}\left(\frac{x_{h1} - \bar{x}_{h1}}{\sqrt{\mathrm{var}(x_{h1})}}\right) + \cdots + u_{jm_h}\left(\frac{x_{hm_h} - \bar{x}_{hm_h}}{\sqrt{\mathrm{var}(x_{hm_h})}}\right), \quad j = 1,\ldots,m_h, \tag{2}$$

where F_{hj} $(j = 1,\ldots,m_h)$ are the j^{th} PC for the group attribute A_h and $\sqrt{\mathrm{var}(x_{hj})}$ the standard deviation of x_{hj} over the n products. That is

$$\sigma_{hj} = \sqrt{\mathrm{var}(x_{hj})} = \sqrt{\frac{1}{n}\sum_{i=1}^{n}(x_{ihj} - \bar{x}_{hj})^2}, \quad j = 1,\ldots,m_h. \tag{3}$$

The PCs explain as much of the variation as possible in the original sensory data. In particular, the first PC explains the maximal amount of variance in the original sensory data, the second PC explains the maximal remaining variance in the data, and so on. The percentage of the total variation of the original sensory data explained by each PC is defined as $100 \times \lambda_j / \sum_{i=1}^{m_h}\lambda_i$. Accordingly, the accumulative contribution ratio (ACR) of the first L_h $(L_h \leqslant m_h)$ principal components is computed by

$$ACR = \left(\sum_{i=1}^{L}\lambda_i \Big/ \sum_{j=1}^{m_h}\lambda_j\right) \times 100\%. \tag{4}$$

As a rule of thumb, it is required that $ACR \geqslant 80\%$ in determining how many PCs should be used for constructing a BRB for a group attribute. Once the PCs for building a BRB are determined, the original sensory data will be transformed into the corresponding PC scores by the following equation:

$$F_{hj}(i) = u_{j1}\left(\frac{x_{ih1} - \bar{x}_{h1}}{\sqrt{\mathrm{var}(x_{h1})}}\right) + \cdots + u_{jm_h}\left(\frac{x_{ihm_h} - \bar{x}_{hm_h}}{\sqrt{\mathrm{var}(x_{hm_h})}}\right),$$
$$i = 1,\ldots,n;\ j = 1,\ldots,L_h, \tag{5}$$

where L_h is the number of PCs used for establishing a BRB.

2.1.3. BRBs

A belief rule base is a collection of belief rules which are the generalization of traditional IF-THEN rules. In traditional IF-THEN rules, the consequence of each rule is certain such as *High*, *Medium* or *Low*; while the consequences of belief rules can be certain or uncertain, characterized by distributions such as {(*High*, 100%)}, which represents a certainly (100%) *High* consequence, and {(*High*,20%),(*Medium*,50%),(*Low*,30%)}, which represents that the consequence could be *High* to the extent of 20%, *Medium* to the extent of 50% and *Low* to the extent of 30%, where 20%, 50% and 30% are referred to as belief degrees. If the belief degrees for the consequence of a belief rule are summed to one, then the belief rule is said to be complete; otherwise it is said to be incomplete and the remaining belief degree which is not assigned to any possible consequences is referred to as ignorance. For example, the following belief rule is incomplete: IF Taste is *Good* and Aroma is *Average* THEN Quality is {(*Good*,50%), (*Average*, 30%), (*Poor*,15%)}, in which the missing belief degree 5% is called ignorance. Each belief rule plays a different role in the corresponding belief rule base and is therefore given a different rule weight. Likewise, each antecedent attribute in the "IF" part of a rule also plays a different role and is thus given a different attribute weight.

Mathematically, a belief rule can be defined as (Yang, Liu, Wang, Sii, & Wang, 2006): IF $A_1^k \wedge A_2^k \wedge \cdots \wedge A_T^k$ THEN $\{(C_1,\beta_{k1}),(C_2,\beta_{k2}),\ldots,(C_N,\beta_{kN})\}$ with $\sum_{l=1}^{N}\beta_{kl} \leqslant 1$, a rule weight θ_k, and attribute weights $\delta_1,\ldots,\delta_T$, where $\{(C_1,\beta_{k1}),(C_2,\beta_{k2}),\ldots,(C_N,\beta_{kN})\}$ is referred to as a belief structure. This belief rule can be understood as IF (A_1 is A_1^k, A_2 is $A_2^k,\ldots$, and A_T is A_T^k) THEN the consequence is $\{(C_1,\beta_{k1}),(C_2,\beta_{k2}),\ldots,(C_N,\beta_{kN})\}$, where $A_1,\ldots,A_T$ are the antecedent attributes of the belief rule, $C_1,\ldots,C_N$ are assessment grades used in the consequence, $A_j^k \in \{C_{j1},\ldots,C_{jT_j}\}$ is an assessment grade for the antecedent attribute $A_j, j \in \{1,\ldots,T\}$, and β_{kl} is the belief degree to which C_l is believed to be the consequence, $l \in \{1,\ldots,N\}$. If $\sum_{l=1}^{N}\beta_{kl} = 1$, the belief rule is said to be complete; otherwise it is said to be incomplete. Incomplete belief rules occur when sensory data include imprecise or missing data, which will not be discussed in this paper.

In order to build a BRB, discrete assessment grades such as *High*, *Medium*, and *Low* need to be defined for each antecedent attribute and consequence. For the hierarchical structure in Fig. 1, discrete assessment grades need to be defined for each group attribute, each sensory attribute that will be directly used for building a BRB, and each PC that has been chosen for building a BRB for a group attribute. The number of grades for each attribute and PC can be either the same or different, depending upon the need to solve real problems. Consumer preferences for food and drink products are usually assessed by consumers or sensory experts using the 9-point or 5-point hedonic ratings as defined in Table 1, which can be directly viewed as assessment grades for consumer preferences. Once the discrete assessment grades are defined, the BRBs for the goal and for each group attribute shown in Fig. 1 can be built using expert knowledge initially and then trained by the BRB learning algorithm using collected preference and sensory data.

Tables 2 and 3 show the generic forms of a BRB for preference mapping of food and drink products, where $C_1,\ldots,C_N$ are assessment grades for consumer preferences and can be hedonic ratings as defined in Table 1, $C_{h1},\ldots,C_{hT_h}$ are assessment grades for group attribute A_h $(h = 1,\ldots,K)$, $A_{h1},\ldots,A_{hm_h}$ are sensory attributes or PCs used for building the BRB for the group attribute A_h, $A_{hj}^k \in \{C_{hj1},\ldots,C_{hjT_{hj}}\}$ $(k = 1,\ldots,M_h)$ are assessment grades for A_{hj} $(j = 1,\ldots,m_h)$, M and M_h are the numbers of belief rules and equal to the numbers of all possible combinations of different

Table 1
The 9-point and 5-point hedonic scales used for assessing consumer preferences.

Rating	9-Point hedonic scale	5-Point hedonic scale
1	Dislike extremely	Very dissatisfied
2	Dislike very much	Somewhat dissatisfied
3	Dislike moderately	Neither satisfied nor dissatisfied
4	Dislike slightly	Somewhat satisfied
5	Neither like nor dislike	Very satisfied
6	Like slightly	
7	Like moderately	
8	Like very much	
9	Like extremely	

Table 2
Belief rule base for consumer preferences of food and drink products.

Rule	Rule weight	Antecedent attributes (weight)				Consequence			
		$A_1(\delta_1)$	$A_2(\delta_2)$	$\cdots$	$A_K(\delta_K)$	C_1	C_2	$\cdots$	C_N
1	θ_1	A_1^1	A_2^1	$\cdots$	A_K^1	β_{11}	β_{12}	$\cdots$	β_{1N}
2	θ_2	A_1^2	A_2^2	$\cdots$	A_K^2	β_{21}	β_{22}	$\cdots$	β_{2N}
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\cdots$	$\vdots$	$\vdots$	$\vdots$	$\cdots$	$\vdots$
M	θ_M	A_1^M	A_2^M	$\cdots$	A_K^M	β_{M1}	β_{M2}	$\cdots$	β_{MN}

Table 3
Belief rule bases for group attributes A_h (h = 1,...,K) in Fig. 1.

Rule	Rule weight	Antecedent attributes (weight)				Consequence (A_h)			
		$A_{h1}(\delta_{h1})$	$A_{h2}(\delta_{h2})$	$\cdots$	$A_{hm_h}(\delta_{hm_h})$	C_{h1}	C_{h2}	$\cdots$	C_{hT_h}
1	θ_{h1}	A_{h1}^1	A_{h2}^1	$\cdots$	$A_{hm_h}^1$	$\beta_{11}^{(h)}$	$\beta_{12}^{(h)}$	$\cdots$	$\beta_{1T_h}^{(h)}$
2	θ_{h2}	A_{h1}^2	A_{h2}^2	$\cdots$	$A_{hm_h}^2$	$\beta_{21}^{(h)}$	$\beta_{22}^{(h)}$	$\cdots$	$\beta_{2T_h}^{(h)}$
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\cdots$	$\vdots$	$\vdots$	$\vdots$	$\cdots$	$\vdots$
M_h	θ_{hM_h}	$A_{h1}^{M_h}$	$A_{h2}^{M_j}$	$\cdots$	$A_{hm_h}^{M_h}$	$\beta_{M_h1}^{(h)}$	$\beta_{M_h2}^{(h)}$	$\cdots$	$\beta_{M_hT_h}^{(h)}$

assessment grades for antecedent attributes. The outputs (consequences) of the BRBs in Table 3 serve as the inputs (antecedent attributes) of the BRBs in Table 2.

2.2. BRB inference by the evidential reasoning (ER) approach

2.2.1. Transformation of sensory data and PC scores

Let $C_{hj1},\ldots,C_{hjT_{hj}}$ be a set of assessment grades defined for assessing sensory attribute A_{hj} or principal component F_{hj} and x_{hj} be a numerical value of A_{hj} or PC score of F_{hj}, which lies between two adjacent assessment grade values, say $u(C_{hjl})$ and $u(C_{hjl+1})$, $1 \leqslant l \leqslant T_{hj}-1$. Then, the numerical value (or PC score) x_{hj} can be transformed into the belief structure $\{(C_{hjl},\alpha_{hjl}), (C_{hjl+1},\alpha_{hjl+1})\}$ by using the following piece-wise linear function (Yang, 2001):

$$\alpha_{hjl} = \frac{u(C_{hjl+1}) - x_{hj}}{u(C_{hjl+1}) - u(C_{hjl})} \quad \text{and} \quad \alpha_{hjl+1} = \frac{x_{hj} - u(C_{hjl})}{u(C_{hjl+1}) - u(C_{hjl})}, \tag{6}$$

where $u(C_{hjl+1})$ and $u(C_{hjl})$ are the grade values of C_{hjl+1} and C_{hjl}, respectively. The belief structure can be rewritten as a distributed assessment in full by $\{(C_{hj1},0),\ldots,(C_{hjl-1},0),(C_{hjl},\alpha_{hjl}),(C_{hjl+1},\alpha_{hjl+1}),(C_{hjl+2},0),\ldots,(C_{hjT_{hj}},0)\}$, which means the numerical value x_{hj} is equivalently assessed to assessment grades C_{hjl} and C_{hjl+1}. All sensory data and PC scores that are chosen for building BRBs can be transformed into belief structures in terms of their numerical values and assessment grades in this way.

2.2.2. Data matching and rule activation

Consider a set of sensory data or PC scores, say $\mathbf{x}_h = (x_{h1},\ldots,x_{hm_h})$, each of which has been transformed into the belief structure $\{(C_{hjl},\alpha_{hjl})|l=1,\ldots,T_{hj},\ \sum_{l=1}^{T_{hj}}\alpha_{hjl}=1\}$ (j = 1,...,m_h) by Eq. (6). What we need to do now is to match the sensory data or PC scores with the antecedent attribute values (i.e. grade values) of each belief rule in Table 3 to see what belief rules can be activated and to what degrees they are activated so that an appropriate activation weight can be created for each of the activated belief rules.

Without loss of generality, consider the kth belief rule in Table 3:

$$\text{IF} \quad A_{h1}^k \wedge A_{h2}^k \wedge \cdots \wedge A_{hm_h}^k \quad \text{THEN} \quad \left\{\left(C_{h1},\beta_{k1}^{(h)}\right),\left(C_{h2},\beta_{k2}^{(h)}\right),\ldots,\left(C_{hT_h},\beta_{kT_h}^{(h)}\right)\right\}.$$

From the belief structures $\left\{(C_{hjl},\alpha_{hjl})|l=1,\ldots,T_{hj},\ \sum_{l=1}^{T_{hj}}\alpha_{hjl}=1\right\}$ (j = 1,...,m_h) transformed from $\mathbf{x}_h=(x_{h1},\ldots,x_{hm_h})$, it is easy to find the belief degree $\alpha_{h1}^{(k)} \in \{\alpha_{h11},\ldots,\alpha_{h1T_{h1}}\}$ to which x_{h1} is assessed to grade $A_{h1}^k \in \{C_{h11},\ldots,C_{h1T_{h1}}\}$, the belief degree $\alpha_{h2}^{(k)} \in \{\alpha_{h21},\ldots,\alpha_{h2T_{h2}}\}$ to which x_{h2} is assessed to grade $A_{h2}^k \in \{C_{h21},\ldots,C_{h2T_{h2}}\}$, and all the other degrees including $\alpha_{hm_h}^{(k)} \in \{\alpha_{hm_h1},\ldots,\alpha_{hm_hT_{hm_h}}\}$ to which x_{hm_h} is assessed to grade $A_{hm_h}^k \in \{C_{hm_h1},\ldots,C_{hm_hT_{hm_h}}\}$. Obviously, $\alpha_{hj}^{(k)}$ (j = 1,...,m_h) represent the extents to which each antecedent attribute of the kth belief rule is matched by the input data $\mathbf{x}_h=(x_{h1},\ldots,x_{hm_h})$. Considering the fact that each antecedent attribute plays a different role in the kth belief rule, the overall degree γ_{hk} to which the kth belief rule is matched can be defined as (Yang et al., 2006)

$$\gamma_{hk} = \prod_{j=1}^{m_h}\left(\alpha_{hj}^{(k)}\right)^{\bar{\delta}_{hj}}, \quad k=1,\ldots,M_h, \tag{7}$$

where $\bar{\delta}_{hj} = \delta_{hj}/\max_{l=1,\ldots,m_h}\{\delta_{hl}\}$ so that $0 < \bar{\delta}_{hj} \leqslant 1$ and $\delta_{hj} > 0$ is the relative weight of A_{hj} which is either a sensory attribute or a PC. The overall degree γ_{hk} is referred to as matching degree.

It is easy to find that $0 \leqslant \gamma_{hk} \leqslant 1$, $\gamma_{hk} = 1$ if $\alpha_{hj}^{(k)} = 1$ for all j = 1,...,m_h, and $\gamma_{hk} = 0$ if there exists some $\alpha_{hj}^{(k)} = 0$ for $j \in \{1,\ldots,m_h\}$. The contribution of an antecedent attribute towards γ_{hk} is also positively related to its relative weight, which means an important antecedent attribute plays a greater role in the determination of γ_{hk}.

Once a belief rule is matched by the input data with a nonzero matching degree, it will be activated. Accordingly, an activation weight needs to be generated to show to what degree the belief rule is activated. According to Yang et al. (2006), the activation weight for an activated belief rule can be defined as

$$w_{hk} = \frac{\theta_{hk}\gamma_{hk}}{\sum_{l=1}^{M_h}\theta_{hl}\gamma_{hl}} = \frac{\theta_{hk}\left[\prod_{j=1}^{m_h}\left(\alpha_{hj}^{(k)}\right)^{\bar{\delta}_{hj}}\right]}{\sum_{l=1}^{M_h}\theta_{hl}\left[\prod_{j=1}^{m_h}\left(\alpha_{hj}^{(l)}\right)^{\bar{\delta}_{hj}}\right]}, \quad k=1,\ldots,M_h, \tag{8}$$

where θ_{hk} is the rule weight of the kth belief rule. The larger the matching degree γ_{hk}, the larger the activation weight w_{hk}. If a belief rule is not matched (i.e. $\gamma_{hk} = 0$) or has a zero weight ($\theta_{hk} = 0$), then it will not be activated (i.e. $w_{hk} = 0$). After the examination of the belief rules in Table 3 one by one, all the belief rules that have been assigned a nonzero activation weight will be activated simultaneously.

2.2.3. Aggregation of the consequences of the activated belief rules

For each activated belief rule, it will produce a consequence characterized by a belief structure. Such a belief structure can be viewed as a piece of evidence and needs to be combined with the other consequences (evidence) produced by other activated belief rules to provide an aggregated conclusion. The evidential reasoning (ER) approach (Yang, 2001) developed for multiple attribute decision analysis provides a useful analytical algorithm for combining multiple pieces of evidence and is thus adopted by the BRB methodology.

The ER approach treats activation weights as the relative importance of each consequence and aggregates the consequences produced by all the activated belief rules in Table 3 using the equations below (Wang, Yang, & Xu, 2006):

$$\alpha_{hj} = \frac{\mu_h * \left[\prod_{k=1}^{M_h}(w_{hk}\beta_{kj}^{(h)} + 1 - w_{hk}) - \prod_{k=1}^{M_h}(1-w_{hk})\right]}{1-\mu_h * \left[\prod_{k=1}^{M_h}(1-w_{hk})\right]}, \quad j = 1,\ldots,T_h, \tag{9}$$

$$\mu_h = \left[\sum_{l=1}^{T_h}\prod_{k=1}^{M_h}(w_{hk}\beta_{kl}^{(h)} + 1 - w_{hk}) - (T_h - 1)\prod_{k=1}^{M_h}(1-w_{hk})\right]^{-1}, \tag{10}$$

where w_{hk} is determined by Eq. (8) and α_{hj}, which satisfies $\alpha_{hj} \geqslant 0$ and $\sum_{j=1}^{T_h}\alpha_{hj} = 1$, is the belief degree to which the conclusion is assessed to grade $C_{hj} \in \{C_{h1},\ldots,C_{hT_h}\}$. The conclusion provided by the ER approach is a belief structure: $\{(C_{h1},\alpha_{h1}),\ldots,(C_{hT_h},\alpha_{hT_h})\}$, which serves as the assessment of the group attribute A_h and also the input to the BRB defined in Table 2.

2.2.4. Final output of the BRB model

For the belief rules shown in Table 2, similar rule inference can be performed once the inputs of all the group attributes become available. Since the inputs of the group attributers are all distributions, the rule inference at this stage can directly start from rule activation. By Eq. (8), the activation weight of each belief rule defined in Table 2 can be computed as

$$w_k = \frac{\theta_k\left[\prod_{h=1}^{K}\left(\alpha_h^{(k)}\right)^{\bar{\delta}_h}\right]}{\sum_{l=1}^{M}\theta_l\left[\prod_{h=1}^{K}\left(\alpha_h^{(l)}\right)^{\bar{\delta}_h}\right]}, \quad k = 1,\ldots,M, \tag{11}$$

where $\bar{\delta}_h = \delta_h/\max_{l=1,\ldots,K}\{\delta_l\}$, $\delta_h > 0$ is the relative weight of the group attribute A_h, θ_k is the rule weight of the kth belief rule, and $\alpha_h^{(k)} \in \{\alpha_{h1},\ldots,\alpha_{hT_h}\}$ is the belief degree to which the group attribute A_h is assessed to the grade $A_h^k \in \{C_{h1},\ldots,C_{hT_h}\}$ ($h = 1,\ldots,K$). Note that $\alpha_h^{(k)}$ directly comes from the input of $A_h : \{(C_{h1},\alpha_{h1}),\ldots,(C_{hT_h},\alpha_{hT_h})\}$.

Let $O = f(\mathbf{x})$ be the final output of the BRB model shown in Fig. 1. It is also the output of the BRB model defined in Table 2 and can be obtained by using the ER analytical algorithm once again. That is

$$\alpha_j = \frac{\mu^*\left[\prod_{k=1}^{M}(w_k\beta_{kj} + 1 - w_k) - \prod_{k=1}^{M}(1-w_k)\right]}{1-\mu^*\left[\prod_{k=1}^{M}(1-w_k)\right]}, \quad j = 1,\ldots,N, \tag{12}$$

$$\mu = \left[\sum_{l=1}^{N}\prod_{k=1}^{M}(w_k\beta_{kl} + 1 - w_k) - (N-1)\prod_{k=1}^{M}(1-w_k)\right]^{-1}, \tag{13}$$

where w_k is the activation weight determined by Eq. (11) and α_j is the belief degree to which the final output is assessed to grade $C_j \in \{C_1,\ldots,C_N\}$. The final output obtained through the ER algorithm is a distribution characterized by the following belief structure:

$$O = f(\mathbf{x}) = \{(C_1,\alpha_1),\ldots,(C_N,\alpha_N)\}, \tag{14}$$

which satisfies $\alpha_j \geqslant 0$ and $\sum_{j=1}^{N}\alpha_j = 1$. Through the grade values of $C_1,\ldots,C_N$, the belief structure can be characterized by an expected numerical value using the following equation:

$$O = f(\mathbf{x}) = \sum_{j=1}^{N}\alpha_j u(C_i), \tag{15}$$

where $u(C_j)$ for $j = 1,\ldots,N$ are the grade values of $C_1,\ldots,C_N$, respectively. The numerical output value of the BRB model provides a basis for training or estimating BRB parameters.

2.3. BRB learning

Consider n food or drink products P_i ($i = 1,\ldots,n$), whose sensory data $\mathbf{x}_i$ and consumer preference $\mathbf{b}_i = (b_{i1},\ldots,b_{iN})$ or y_i are assumed to be known, where $\mathbf{b}_i$ represents the consumer preferences for product P_i characterized by the belief degrees and y_i the consumer preferences characterized by an average score or rating. From Eq. (14) or (15), n outputs can be generated from the BRB preference mapping and BRB inference introduced above, as shown in the right-hand side of Table 4, where α_j^i ($j = 1,\ldots,N$) are the belief degrees obtained from the sensory data ($\mathbf{x}_i$) from Eq. (12) and O_i ($i = 1,\ldots,n$) the expected values computed using Eq. (15).

Obviously, once the belief rules in Tables 2 and 3 are given or known, the output of BRB model will be determined. It may be different from the real consumer preferences. Let $\boldsymbol{\varepsilon}_i = (\varepsilon_{i1},\ldots,\varepsilon_{iN})$ and η_i be deviation vectors and deviation variables, which are defined by the following equations:

$$\varepsilon_{ij} = b_{ij} - \alpha_j^i, \quad i = 1,\ldots,n;\ j = 1,\ldots,N, \tag{16}$$

$$\eta_i = y_i - O_i, \quad i = 1,\ldots,n. \tag{17}$$

It is desirable that the mode of the above deviation vector and the value of the deviation variable be kept as close to zero as possible, which enables us to build the following objective functions and optimization models:

$$\text{Minimize} \quad J_1 = \sum_{i=1}^{n}\|\boldsymbol{\varepsilon}_i\|^2 = \sum_{i=1}^{n}\sum_{j=1}^{N}\varepsilon_{ij}^2 = \sum_{i=1}^{n}\sum_{j=1}^{N}(b_{ij} - \alpha_j^i)^2 \tag{18}$$

$$\text{Subject to} \quad \sum_{j=1}^{N}\beta_{kj} = 1, k = 1,\ldots,M, \tag{18a}$$

$$\sum_{j=1}^{T_h}\beta_{kj}^{(h)} = 1, \quad h = 1,\ldots,K;\ k = 1,\ldots,M_h, \tag{18b}$$

$$\sum_{j=1}^{K}\delta_j = 1, \tag{18c}$$

$$\sum_{j=1}^{m_h}\delta_{hj} = 1, \quad h = 1,\ldots,K, \tag{18d}$$

$$0 \leqslant \theta_k \leqslant 1, \quad k = 1,\ldots,M, \tag{18e}$$

$$0 \leqslant \theta_{hk} \leqslant 1, \quad h = 1,\ldots,K;\ k = 1,\ldots,M_h, \tag{18f}$$

$$\beta_{kj} \geqslant 0, \quad k = 1,\ldots,M;\ j = 1,\ldots,N, \tag{18g}$$

$$\beta_{kj}^{(h)} \geqslant 0, \quad h = 1,\ldots,K;\ k = 1,\ldots,M_h;\ j = 1,\ldots,T_h, \tag{18h}$$

$$\delta_j \geqslant 0, \quad j = 1,\ldots,K, \tag{18i}$$

$$\delta_{hj} \geqslant 0, \quad h = 1,\ldots,K;\ j = 1,\ldots,m_h, \tag{18j}$$

$$\text{Minimize} \quad J_2 = \sum_{i=1}^{n}\eta_i^2 = \sum_{i=1}^{n}(y_i - O_i)^2$$
$$\text{Subject to} \quad 18a,\ 18b,\ 18c,\ 18d,\ 18e,\ 18f,\ 18g,\ 18h,\ 18i,\ 18j \tag{19}$$

In the above model, (18a) and (18g) are the constraints on the belief degrees in the consequence of the kth belief rule in Table 2, (18b) and (18h) on the belief degrees in the consequence of the kth belief rule in Table 3, (18c) and (18i) on the relative weights of the group attributes in Fig. 1, (18d) and (18j) on the relative weights of the sensory attributes or PCs belonging to the same group attribute, and (18e) and (18f) are the constraints on the rule weights of the

Table 4
Consumer preferences and their predictions by BRB methodology.

Product	Consumer preference					Output of BRB model				
	C_1	C_2	$\cdots$	C_N	Average score	C_1	C_2	$\cdots$	C_N	Expected value
1	b_{11}	b_{12}	$\cdots$	b_{1N}	y_1	α_1^1	α_2^1	$\cdots$	α_N^1	O_1
2	b_{21}	b_{22}	$\cdots$	b_{2N}	y_2	α_1^2	α_2^2	$\cdots$	α_N^2	O_2
$\vdots$	$\vdots$	$\vdots$	$\cdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\cdots$	$\vdots$	$\vdots$
n	b_{n1}	b_{n2}	$\cdots$	b_{nN}	y_n	α_1^n	α_2^n	$\cdots$	α_N^n	O_n

belief rules in Tables 2 and 3. When PCs are used for building a BRB, it is required that the relative weight of the first PC be no less than that of the second PC, which is in turn no less than the relative weight of the third PC. So, additional constrains need to be added when PCs are utilized for building the BRBs in Table 3. Besides, other forms of objective functions are also possible. Interested readers may refer to Liu, Yang, Ruan, Martinez, and Wang (2008), Xu et al. (2007) and Yang, Liu, Xu, Wang, and Wang (2007) for different objective functions and applications of BRBs in other areas.

The above two models (18) and (19) provide two options for training a BRB system for preference mapping of food and drink products. It is desirable that preferences provided by a group of consumers be characterized by belief degrees. In this case, model (18) should be used because it provides more information and more accurate parameter estimation than model (19). In the case that consumer preferences are described only by average ratings or scores, model (19) can be used.

The parameters of all the BRBs in Tables 2 and 3 are trained by solving the optimization model (18) or (19). The trained models can be used to predict consumer preferences for design of new product and to set target values for the sensory attributes of new products.

2.4. Prediction of consumer preference and target setting of sensory attributes

Let $\mathbf{x}_F$ be the sensory data of a new product. Consumer preference for this new product can be predicted by Eq. (14) or (15). The former provides a distribution as the prediction of consumer preference, while the latter provides an expected value as the average rating or score of consumer preference for the new product.

In order to design a product preferred by consumers, it is essential to set a target value for each sensory attribute. Target values should be determined in a systematic and optimal manner rather than purely subjectively or experientially. This is important to the success of a new product.

Suppose there are the m $\left(m = \sum_{h=1}^{K} m_h\right)$ sensory attributes in Fig. 1. Each attribute may vary within an interval $\left[x_{hj}^{\min}, x_{hj}^{\max}\right]$, where $x_{hj}^{\min}$ and $x_{hj}^{\max}$ are the minimum and maximum values that a sensory attribute A_{hj} ($h = 1,\ldots,K$; $j = 1,\ldots,m_h$) can take. To set a target value for each sensory attribute, the following two optimization models are constructed:

$$\begin{aligned} &\text{Minimize} \quad J_3 = \sum_{j=1}^{N} \left(b_{kj} - \alpha_j^{new}\right)^2 \\ &\text{Subject to} \quad x_{hj}^{\min} \leqslant x_{hj} \leqslant x_{hj}^{\max}, \quad h = 1,\ldots,K;\ j = 1,\ldots,m_h, \end{aligned} \tag{20}$$

$$\begin{aligned} &\text{Maximize} \quad J_4 = \sum_{j=1}^{N} u(C_j)\alpha_j^{new} \\ &\text{Subject to} \quad x_{hj}^{\min} \leqslant x_{hj} \leqslant x_{hj}^{\max}, \quad h = 1,\ldots,K;\ j = 1,\ldots,m_h, \end{aligned} \tag{21}$$

where b_{kj} ($j = 1,\ldots,N$) are the consumer preferences for a targeted (or market leader) food and drink product which is characterized by belief degrees and α_j^{new} ($j = 1,\ldots,N$) represent consumer

Table 5
Sensory profiles of 27 lemonade products.

Product	Sweet	Acidic	Aftertaste	Carbonation	Peely	Zesty/citrally	Fruity	Juicy	Limey	Floral	Soapy/aldehydic	Oxidized
1	39.76	48.86	20.33	37.95	31.33	32.93	18.74	15.39	23.03	24.44	41.23	46.54
2	58.77	39.63	19.81	38.4	26.51	44.44	25.3	28.92	23.69	33.94	15.36	29.85
3	69.61	41.37	13.52	45.92	42.32	49.4	50.74	43.11	37.48	16.72	24.5	21.15
4	34.14	48.06	18.98	50.87	35.74	45.78	34.54	34.14	23.29	11.38	18.21	10.67
5	54.48	44.58	17.27	42.44	27.71	49	28.78	37.48	13.25	13.6	15.66	18.37
6	68.41	50.47	6.02	43.11	28.92	48.19	35.34	44.85	44.71	18.42	16.27	14.01
7	38.82	37.08	17.21	34	26.37	33.73	17	20.21	31.99	40.29	48.06	39.76
8	29.05	35.48	34	29.45	34.79	30.92	18.88	14.73	18.67	37.08	49.8	43.64
9	74.7	46.99	14.86	39.31	26.24	50.6	40.83	45.38	32.4	13.65	11.75	15.66
10	56.49	46.18	21.99	37.35	29.45	39.22	30.52	22.59	24.5	11.14	25.3	35.48
11	63.99	60.78	17.73	49.93	40.56	55.15	46.72	51.81	38.15	11.6	16.6	23.96
12	18.21	29.59	33.22	29.59	13.52	11.3	12.58	4.99	7.23	7.06	21.82	24.1
13	57.03	48.06	16.87	43.37	30.79	38.82	41.5	32.66	30.79	10.04	12.99	31.93
14	40.43	44.44	27.44	30.12	30.92	27.04	31.59	29.32	24.77	11.91	24.63	23.43
15	65.86	63.72	24.23	46.32	44.31	65.19	50.33	51.94	45.63	18.74	17.4	11.11
16	43.11	39.76	33.22	35.48	28.78	35.88	21.55	26.24	42.3	43.57	37.08	47.29
17	39.09	43.11	24.63	30.52	31.59	36.81	29.05	34.4	19.79	21.17	16.27	11.88
18	23.29	29.85	37.95	24.9	21.02	18.07	14.59	12.45	16.57	16.11	13.43	27.54
19	29.72	36.68	31.63	40.16	30.12	25.9	10.69	18.76	14.99	13.94	26.64	34.49
20	71.35	45.25	16.01	27.58	37.62	48.46	44.85	44.44	41.63	19.28	34.14	37.88
21	67.34	33.47	16.73	40.03	36.28	41.5	39.09	35.21	21.42	12.5	31.59	33.05
22	49.8	65.33	31.93	44.58	38.42	49.67	32.4	39.16	40.43	11.7	9.64	38.21
23	61.85	51	21.69	31.73	20.48	33.07	25.97	30.92	22.09	23.41	28.4	37.88
24	72.82	35.21	16.2	34.4	33.43	52.74	32.8	36.41	18.07	6.33	24.63	29.78
25	59.3	48.19	15.39	44.58	32.13	43.78	48.19	46.32	35.88	21.34	16.18	25.82
26	49.8	53.28	19.14	41.63	39.49	55.56	34	42.84	23.03	13.79	30.52	37.75
27	50.33	53.92	23.92	43.11	34.40	40.29	20.62	19.28	25.44	11.60	9.29	34.81

Table 6
Consumer preferences for the 27 lemonade products.

Product	Belief degrees for hedonic ratings										Average rating
	0	1	2	3	4	5	6	7	8	9	
1	0	0.222	0	0.222	0.333	0.111	0.111	0	0	0	3.44
2	0.222	0	0	0	0.111	0.333	0.222	0	0.111	0	4.33
3	0.222	0	0	0	0.111	0	0.444	0.222	0	0	4.67
4	0.222	0	0	0.111	0.222	0	0.444	0	0	0	3.89
5	0	0	0.222	0	0.222	0.111	0.444	0	0	0	4.56
6	0	0	0.111	0.222	0	0.222	0.333	0.111	0	0	4.78
7	0.111	0.111	0.222	0.222	0.111	0.222	0	0	0	0	2.78
8	0.111	0.333	0.111	0.333	0	0.111	0	0	0	0	2.11
9	0.222	0	0	0	0	0.556	0	0	0.222	0	4.56
10	0.111	0.111	0.111	0.111	0.333	0	0.111	0.111	0	0	3.44
11	0.111	0	0	0	0.111	0.111	0.111	0.222	0.333	0	5.89
12	0.222	0.222	0.111	0.222	0.222	0	0	0	0	0	2.00
13	0.111	0	0.111	0.111	0	0.222	0.222	0.111	0	0.111	4.78
14	0.222	0	0.444	0	0	0	0.111	0.222	0	0	3.11
15	0.111	0	0	0	0.167	0.056	0	0.111	0.333	0.222	6.39
16	0.111	0.222	0.444	0	0	0.222	0	0	0	0	2.22
17	0.222	0.111	0	0	0.222	0	0.444	0	0	0	3.67
18	0.111	0.111	0.111	0.111	0.333	0.111	0.111	0	0	0	3.22
19	0.111	0.222	0.333	0	0.222	0	0.111	0	0	0	2.44
20	0.111	0	0.111	0	0.222	0	0.222	0.111	0.111	0.111	5.11
21	0.111	0	0	0	0.222	0.111	0.111	0.333	0.111	0	5.33
22	0.222	0	0	0	0.111	0.111	0.222	0.333	0	0	4.67
23	0	0	0.111	0.111	0.222	0.333	0.111	0	0.111	0	4.67
24	0.111	0	0.056	0.056	0.222	0.222	0.111	0.111	0.111	0	4.61
25	0.111	0.111	0	0	0	0.056	0.167	0.222	0.333	0	5.61
26	0	0.111	0	0	0	0.111	0.333	0.222	0.111	0.111	6.11
27	0.111	0.111	0.111	0.222	0.111	0.333	0	0	0	0	3.11

preferences predicted for a new product. The purpose of model (20) is to design a new product which is as close to a targeted product as possible, while the purpose of model (21) is to design a new product that can maximize consumer preference. Which model should be used in practice depends upon the manufacturer's desire and also its production conditions. This is because although the target values determined by model (21) can maximize consumer preferences, they will not be practical if they cannot be realized by the product process.

3. Application to the retro design of a lemonade product

3.1. Problem description and assessment data

A company designing and manufacturing sensory products in Northwest England wishes to design a lemonade drink product. Tables 5 and 6 show the sensory profiles of 27 lemonade drink products and consumer preferences for these products provided by the company, where the first 26 products are the popular products available in the market and the last one is the product of the company. The quality of the 27 lemonades is evaluated by a panel of nine experts using the 1–9 hedonic rating as shown in Table 1 and is described by distributions. In Table 6, for example, the distribution assessment of product 1 by the nine experts is given by {(1,2/9),(3,2/9),(4,3/9),(5,1/9),(6,1/9)}, which reads that two experts rate the product to grade 1, two to grade 3, three to grade 4, one to grade 5 and one to grade 6. When a sensory expert fails to rate a product to any of the 9 grades, his/her preference for this product is uncertain and is treated as zero by the company.

3.2. The BRB system for product quality evaluation

Fig. 2 shows the BRB model structure for quality evaluation of the lemonade products, in which 12 sensory attributes are grouped into two categories (taste and aroma) as suggested by the sensory manager of the company. Since aroma is related to nine sensory attributes, the direct modeling of them would require a large number of belief rules. So, PCA is performed to extract principal components from the nine attributes. The results are shown in Table 7, where the first three PCs account for 83% of the total variance of the data for the nine sensory attributes. So, the first three PCs are considered to be sufficiently representative of the aroma-type of nine sensory attributes.

The BRB system consists of three BRB sub-systems: the first one for the overall consumer preference assessment, the second one for assessing taste and the third one for assessing aroma. To build the BRB system, the taste-type of three sensory attributes (sweet, acidic, and aftertaste) and the aroma-type of three PCs are all assessed using the grades *High*, *Medium*, and *Low*, which are defined in Table 8. The maximum values of the sensory attributes and PCs are defined as *High*, the minimum values as *Low*, and their averages as *Medium*. The maximum and minimum values of the three PCs are computed by the following linear programming models:

$$\text{Max/Min}\quad F_{2j} = u_{j1}\left(\frac{x_{21}-\bar{x}_{21}}{\sqrt{var(x_{21})}}\right)+\cdots+u_{j9}\left(\frac{x_{29}-\bar{x}_{29}}{\sqrt{var(x_{29})}}\right),\quad j=1,2,3 \tag{22}$$

$$\text{Subject to}\quad x_{2j}^{\min}\leqslant x_{2j}\leqslant x_{2j}^{\max},\quad j=1,\ldots,9.$$

The two group attributes (taste and aroma) are intermediate variables and are also assessed by *High*, *Medium*, and *Low*. Based on these definitions of grades, 63 belief rules are built in total, 9 of which are for the overall quality assessment as shown in Table 9, 27 for the assessment of taste as shown in Table 10, and also 27 for the assessment of aroma as shown in Table 11. In Table 9, the input attributes (taste and aroma) are assessed by *High*, *Medium*, and *Low* and the output (Consumer preferences or overall quality) is assessed by the 1–9 hedonic. In Table 10, the input attributes (sweet, acidic, aftertaste) and the output (taste) are all

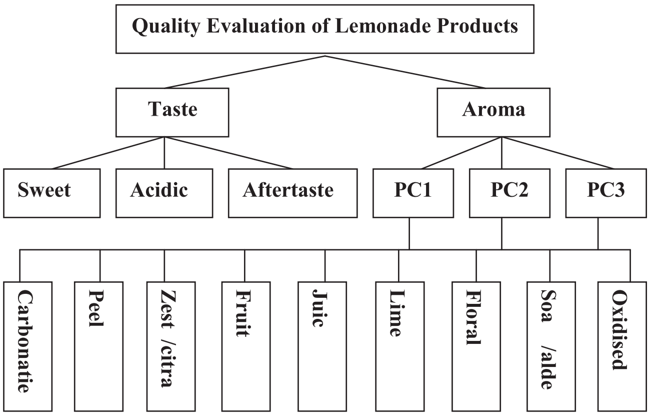

Fig. 2. BRB model structure for quality evaluation of lemonade products.

Table 7
Principal components for nine sensory attributes.

Item	Principal components		
	PC1	PC2	PC3
Eigenvalue	4.725	1.994	0.740
% Variance	52.502	22.161	8.226
Cumulative %	52.502	74.663	82.889
Eigenvector			
Carbonation	0.349	−0.017	0.202
Peely	0.338	0.297	0.463
Zesty/citrally	0.411	0.164	0.083
Fruity	0.421	0.044	−0.010
Juicy	0.435	0.059	−0.115
Limey	0.301	0.360	−0.474
Floral	−0.161	0.520	−0.543
Soapy/aldehydic	−0.232	0.519	0.309
Oxidized	−0.241	0.459	0.332

Table 8
Assessment grades for sensory attributes and three PCs.

Grade	Sweet	Acidic	Aftertaste	PC1	PC2	PC3
High	74.70	65.33	37.95	4.99	5.059	4.752
Medium	46.46	47.46	21.96	0	0	0
Low	18.21	29.59	6.02	−6.25	−4.428	−5.199

assessed by *High*, *Medium*, and *Low*. In Table 11, the input attributes (PC1, PC2, PC3) and the output (aroma) are also assessed by *High*, *Medium*, and *Low*.

3.3. The BRB system training

In all the three BRB sub-systems, the input attributes are weighted equally. The belief degrees and the rule weights shown in Tables 9–11 are generated by integrating the three BRB sub-systems to a single BRB system and training them together using the sensory data of the first 26 products and the consumer preferences provided by the nine experts based on model (18).

It can be observed from the trained BRB sub-system shown in Table 9 that this particular group of nine experts seemingly prefers the lemonade products with *Low* taste and *Low* aroma. They tend to be neural (rating 5, neither like nor dislike) to the lemonade products with *Low* taste but *Medium* aroma or *High* taste but *Low* aroma. If a lemonade product is of High taste and Medium aroma, then their preference for it is more likely to be uncertain (rating 0). This group of expert seemingly dislikes lemonade products with medium taste and medium aroma. Note that the above observations are based on the trained BRB sub-system and can be communicated to the experts. If they disagree with the trained conclusions, they can modify the beliefs, which will be used as the starting point for re-train the BRB system. Note that the optimization problem (18) may have multiple solutions. Providing the well thought initial values of the parameters for the BRB system by the experts can lead to a better BRB system in terms of not only the minimal residual value in model (18) but the closeness to what the expert thought as well. If it proves that the experts' intuitive conclusions for the BRB rules are in conflict with the sensory and rating data provided, this would provide a basis for further discussions to improve the consistency of the information.

As revealed in Table 10, there may be several ways to design a lemonade product with *Low* taste. For example, if sweet, acidic, and aftertaste are all *Low*, then taste will be *Low* as shown by Rule 1 in Table 10; if sweet and aftertaste are *Low*, but acidic is *Medium*, then taste will still be *Low* as shown by Rule 4. Similarly, as revealed in Table 11, there may be several ways to design a lemonade product with *Low* aroma. For instance, if PC1 for aroma is *High* and PC2 for aroma is *Medium*, then aroma is *Low*. These belief rules provide clear explanations about sensory formation and consumer preferences for design of lemonade products. Such conclusions may or may not be what the experts expect. If there is conflict, the BRB system can be re-trained or updated using new expert knowledge. This forms a learning cycle which can lead to an improved system and better performances.

Table 12 shows the consumer preferences calculated by the trained BRB system for the 26 lemonade products on the same 1–9 hedonic scale plus the zero grade used by the company. The average ratings are computed using Eq. (15), where the grade scores are defined as $u(0) = 0, u(1) = 1, \ldots, u(9) = 9$. The fitting performance of the BRB system to the sensory data is measured by the mean absolute error (MAE) and mean absolute percentage error (MAPE), which are defined as follows:

$$\mathrm{MAE} = \frac{1}{n \times N} \sum_{i=1}^{n} \sum_{j=1}^{N} \left| b_{ij} - \alpha_j^i \right| \times 100, \tag{23}$$

$$\mathrm{MAPE} = \frac{1}{n} \sum_{i=1}^{n} \left| \frac{y_i - O_i}{y_i} \right| \times 100. \tag{24}$$

Table 9
Belief rule base for overall quality assessment of lemonade products.

Rule	Rule weight	Group attributes		Consumer preferences									
		Taste (0.5)	Aroma (0.5)	0	1	2	3	4	5	6	7	8	9
1	0.101	Low	Low	0.103	0.005	0.007	0.002	0.107	0.043	0.005	**0.254**	**0.365**	**0.107**
2	0.731	Low	Medium	0.111	0.005	0.005	0.060	0.007	**0.782**	0.026	0.001	0.003	0.000
3	0.023	Low	High	0.000	0.095	0.132	0.240	0.458	0.013	0.050	0.003	0.004	0.005
4	0.635	Medium	Low	0.171	0.008	0.007	0.033	0.039	0.003	0.614	0.124	0.001	0.002
5	0.318	Medium	Medium	0.009	0.423	0.007	0.529	0.000	0.007	0.009	0.005	0.005	0.005
6	0.546	Medium	High	0.008	0.018	0.000	0.013	0.891	0.000	0.042	0.010	0.007	0.011
7	0.079	High	Low	0.000	0.006	0.027	0.045	0.002	**0.913**	0.000	0.003	0.001	0.003
8	0.142	High	Medium	**0.752**	0.153	0.041	0.047	0.002	0.002	0.004	0.000	0.000	0.000
9	1.000	High	High	0.164	0.166	0.600	0.000	0.000	0.000	0.015	0.055	0.000	0.000

Table 10
Belief rule base for taste.

Rule	Rule weight	Sensory attributes			Taste		
		Sweet (1/3)	Acidic (1/3)	Aftertaste (1/3)	Low	Medium	High
1	1.000	Low	Low	Low	**0.932**	0.000	0.068
2	0.830	Low	Low	Medium	0.000	0.039	0.961
3	1.000	Low	Low	High	0.083	0.693	0.224
4	0.872	Low	Medium	Low	**0.994**	0.000	0.006
5	1.000	Low	Medium	Medium	0.002	0.998	0.000
6	0.202	Low	Medium	High	0.022	0.909	0.069
7	1.000	Low	High	Low	0.349	0.332	0.319
8	0.899	Low	High	Medium	0.452	0.319	0.229
9	1.000	Low	High	High	0.333	0.333	0.333
10	0.250	Medium	Low	Low	0.067	0.933	0.000
11	0.169	Medium	Low	Medium	0.000	0.564	0.436
12	0.070	Medium	Low	High	0.000	0.000	1.000
13	0.120	Medium	Medium	Low	0.841	0.000	0.159
14	0.026	Medium	Medium	Medium	0.000	0.779	0.221
15	1.000	Medium	Medium	High	0.000	0.000	1.000
16	0.342	Medium	High	Low	0.758	0.040	0.202
17	0.797	Medium	High	Medium	0.820	0.169	0.011
18	0.923	Medium	High	High	0.488	0.117	0.395
19	0.290	High	Low	Low	0.000	1.000	0.000
20	0.648	High	Low	Medium	**0.979**	0.008	0.013
21	1.000	High	Low	High	0.333	0.333	0.333
22	0.088	High	Medium	Low	0.867	0.000	0.133
23	0.001	High	Medium	Medium	0.301	0.000	0.699
24	1.000	High	Medium	High	0.367	0.304	0.329
25	1.000	High	High	Low	0.000	0.200	0.800
26	1.000	High	High	Medium	**0.972**	0.000	0.028
27	1.000	High	High	High	0.574	0.136	0.291

Table 11
Belief rule base for aroma.

Rule	Rule weight	Principal components for aroma			Aroma		
		PC1 (1/3)	PC2 (1/3)	PC3 (1/3)	Low	Medium	High
1	1.000	Low	Low	Low	0.020	0.000	0.980
2	1.000	Low	Low	Medium	0.000	0.772	0.228
3	1.000	Low	Low	High	0.354	0.339	0.306
4	0.483	Low	Medium	Low	0.000	0.030	0.970
5	0.097	Low	Medium	Medium	0.000	0.011	0.988
6	0.775	Low	Medium	High	0.000	0.379	0.621
7	1.000	Low	High	Low	0.036	0.305	0.659
8	0.800	Low	High	Medium	0.000	1.000	0.000
9	0.318	Low	High	High	0.000	1.000	0.000
10	0.483	Medium	Low	Low	0.837	0.163	0.000
11	0.576	Medium	Low	Medium	0.674	0.000	0.326
12	1.000	Medium	Low	High	0.000	0.113	0.887
13	0.217	Medium	Medium	Low	0.848	0.152	0.000
14	0.000	Medium	Medium	Medium	0.300	0.000	0.700
15	0.143	Medium	Medium	High	0.304	0.023	0.673
16	1.000	Medium	High	Low	0.000	0.007	0.993
17	0.048	Medium	High	Medium	0.029	0.000	0.971
18	0.603	Medium	High	High	0.867	0.133	0.000
19	1.000	High	Low	Low	0.907	0.058	0.035
20	1.000	High	Low	Medium	0.058	0.715	0.227
21	1.000	High	Low	High	0.000	0.150	0.850
22	1.000	High	Medium	Low	**1.000**	0.000	0.000
23	0.251	High	Medium	Medium	**1.000**	0.000	0.000
24	1.000	High	Medium	High	**0.959**	0.041	0.000
25	1.000	High	High	Low	0.400	0.234	0.365
26	0.648	High	High	Medium	0.461	0.000	0.539
27	1.000	High	High	High	0.748	0.146	0.106

It can be computed that MAE and MAPE for the consumer preferences characterized by the belief degrees on the grades 0 and 1–9 is 0.041 (i.e. 4.1%) and 8.82%, respectively. So, the average fitting accuracy for the 26 lemonade products is 91.18%. The fitting results are visualized in Fig. 3, which gives a clear picture of the fitting performance of the BRB methodology.

3.4. Prediction of consumer preferences and target setting

Based on the developed BRB system, the prediction of consumer preferences for the lemonade product of the company is given by 3.23, while the nine experts' actual average rating for this product is 3.11. The prediction accuracy for this test product is about

Table 12
Fitted consumer preferences for the 26 competitive lemonade products.

Product	Belief degrees										Average rating
	0	1	2	3	4	5	6	7	8	9	
1	0.000	0.222	0.000	0.222	0.333	0.111	0.111	0.000	0.000	0.000	3.37
2	0.222	0.000	0.000	0.000	0.111	0.333	0.222	0.000	0.111	0.000	4.68
3	0.222	0.000	0.000	0.000	0.111	0.000	0.444	0.222	0.000	0.000	5.11
4	0.222	0.000	0.000	0.111	0.222	0.000	0.444	0.000	0.000	0.000	4.52
5	0.000	0.000	0.222	0.000	0.222	0.111	0.444	0.000	0.000	0.000	4.39
6	0.000	0.000	0.111	0.222	0.000	0.222	0.333	0.111	0.000	0.000	4.63
7	0.111	0.111	0.222	0.222	0.111	0.222	0.000	0.000	0.000	0.000	2.89
8	0.111	0.333	0.111	0.333	0.000	0.111	0.000	0.000	0.000	0.000	2.29
9	0.222	0.000	0.000	0.000	0.000	0.556	0.000	0.000	0.222	0.000	4.71
10	0.111	0.111	0.111	0.111	0.333	0.000	0.111	0.111	0.000	0.000	3.67
11	0.111	0.000	0.000	0.000	0.111	0.111	0.111	0.222	0.333	0.000	6.00
12	0.222	0.222	0.111	0.222	0.222	0.000	0.000	0.000	0.000	0.000	2.65
13	0.111	0.000	0.111	0.111	0.000	0.222	0.222	0.111	0.000	0.111	4.48
14	0.222	0.000	0.444	0.000	0.000	0.000	0.111	0.222	0.000	0.000	2.57
15	0.111	0.000	0.000	0.000	0.167	0.056	0.000	0.111	0.333	0.222	6.20
16	0.111	0.222	0.444	0.000	0.000	0.222	0.000	0.000	0.000	0.000	2.62
17	0.222	0.111	0.000	0.000	0.222	0.000	0.444	0.000	0.000	0.000	4.31
18	0.111	0.111	0.111	0.111	0.333	0.111	0.111	0.000	0.000	0.000	3.31
19	0.111	0.222	0.333	0.000	0.222	0.000	0.111	0.000	0.000	0.000	2.63
20	0.111	0.000	0.111	0.000	0.222	0.000	0.222	0.111	0.111	0.111	4.86
21	0.111	0.000	0.000	0.000	0.222	0.111	0.111	0.333	0.111	0.000	5.19
22	0.222	0.000	0.000	0.000	0.111	0.111	0.222	0.333	0.000	0.000	5.33
23	0.000	0.000	0.111	0.111	0.222	0.333	0.111	0.000	0.111	0.000	4.05
24	0.111	0.000	0.056	0.056	0.222	0.222	0.111	0.111	0.111	0.000	4.90
25	0.111	0.111	0.000	0.000	0.000	0.056	0.167	0.222	0.333	0.000	5.70
26	0.000	0.111	0.000	0.000	0.000	0.111	0.333	0.222	0.111	0.111	5.24

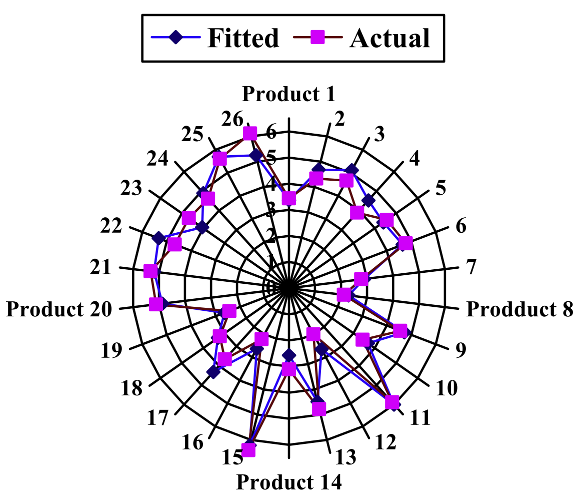

Fig. 3. Fitting performance of the BRB methodology for the 26 competitive lemonade products.

Table 13
Compare predictions of consumer preference for the lemonade product.

Method	Actual mean score	Predicted mean score	Accuracy (%)
BRB methodology	3.11	3.23	96.32
ANN (12-10-1)	3.11	3.19	97.46
Multiple linear regression	3.11	2.62	84.09

96.32%, which shows the BRB system performed quite well. Table 13 provides a comparison about the predictions of consumer preferences for the same product generated by artificial neural networks (ANN) and multiple linear regression (MLR) analysis, where the structure of neural networks was optimized with 10 hidden neurons. It is observed that MLR is apparently not as good as the BRB system and ANN in terms of prediction accuracy. Although ANN performs slightly better than the BRB system, it cannot be used for supporting the retro design of food and drink products. What is a more of a concern to the practitioners is that ANN provides the prediction through black box simulation, so it is not possible to scrutinize how such prediction is achieved, whilst the reasoning process of the BRB system is transparent and its reasoning logic can be directly examined, as discussed above.

Table 14
Target values for 12 sensory attributes of lemonade products.

Sensory attribute	Minimum	Maximum	Target values			
			Solution 1	Solution 2	Solution 3	Solution 4
Sweet	18.21	74.7	21.11	21.90	29.85	18.21
Acidic	29.59	65.33	46.44	46.45	46.45	45.40
Aftertaste	6.02	37.95	6.02	6.02	6.02	6.02
Carbonation	24.90	50.87	43.03	50.49	50.22	50.14
Peely	13.52	44.31	41.33	44.22	31.97	27.88
Zesty/citrally	11.30	65.19	47.52	49.44	50.33	55.56
Fruity	10.69	50.74	20.07	14.53	47.86	27.89
Juicy	4.99	51.94	45.97	22.37	14.75	21.94
Limey	7.23	45.63	24.31	45.51	18.23	40.39
Floral	3.27	43.57	24.76	16.33	31.19	11.72
Soapy/ aldehydic	9.29	49.8	24.92	15.07	24.15	37.02
Oxidized	10.67	47.29	10.87	15.17	18.44	13.54

The trained BRB system shown in Tables 9–11 can then be used for setting target values for the 12 sensory attributes of lemonade products. The results are shown in Table 14, where the target values are obtained by solving model (21). It can be found that there are multiple optimal solutions to model (21). This is consistent with the above-mentioned analysis that there are several ways to design lemonade products with *Low* taste and aroma. Multiple solutions provide flexibility for the retro design of lemonade products and allow the company to design different new products. Through setting target values, consumer preferences for new lemonade products can be improved, for example from 3.11 to 6.37 for the lemonade product of the company. This shows the important feature of the BRB methodology in supporting the retro design of food and drink products.

4. Conclusions

Rapid and accurate identification of consumer demands and systematic assessment of product quality are very important to manufacturers. In this paper we investigated a novel belief rule-based (BRB) methodology for predicting consumer preferences in the retro-fit design of food and drink products. The BRB methodology links consumer preferences for food and drink products with their sensory attributes through a group of BRB models, which can be trained and validated using sensory data and consumer preferences data. The trained belief rule bases can then be used for predicting consumer preferences for a new product or setting target values for the product attributes to support product retro-design. A case study was conducted for the applications of the BRB methodology to supporting the retro design of lemonade products. The study shows that satisfactory consumer preference accuracy can be achieved and the consumer preferences for new product can also be predicted accurately. Also, significant improvement in consumer preferences can be made by setting target values for the sensory attributes of lemonade products.

In comparison with some existing methodologies for preference mapping in design of food and drink products, the BRB methodology has the following unique features:

- BRB models are transparent and their input–output relationships characterized by belief rules are interpretable and easy to understand, whilst ANN models are black boxes in nature and their input–output relationships cannot be interpreted or verified explicitly.
- BRB models can accommodate expert knowledge explicitly, while ANN models do not have such capability. In other words, expert knowledge can be incorporated into BRB models (18) and (19) as constraints, or the initial values of the training parameters in these BRB models can be set by experts intuitively whenever possible, leading to an expert-data driven BRB system.
- BRB models allow their outputs to be either average values or distributions, while ANN or MLR models can only use average values as outputs and have no way to model distributions. Theoretically, ANN could model multiple outputs, but there is no guarantee that the outputs of an ANN model can meet some required conditions such as the normalized belief degrees, rule weights and attribute weights.
- BRB models can be used to set target values for product attributes and therefore to support product retro-fit design, while ANN models do not have such capability.
- BRB models can handle uncertainty such as incomplete information although this has not been discussed in this paper, while ANN or MLR models cannot.
- BRB models can identify what belief rules cannot be learned by the given data sets and what samples or products cannot be predicted with confidence, but ANN or MLR models cannot provide such information and may generate false prediction that cannot be learned from the training samples.

In summary, the BRB methodology provides a novel and systematic way for modeling and predicting consumer preferences and setting target values for food and drink products, thereby providing useful decision support to the retro design of food and drink products. It should be noted, however, that the structure of a BRB system needs to be designed carefully to avoid creating too complex a system, although, as discussed in this paper, grouping sensory attributes using domain specific or expert knowledge and conducting principal component analysis provide useful ways to simplify a BRB system.

Acknowledgements

This project was supported by DEFRA under the Grant No. AFM222 and by EPSRC under the Grant No. EP/F024606/1. The work was also partially supported by the Natural Science Foundation of China (NSFC) under the Grant Nos. 60736026 and 70925004 and by City University of Hong Kong under the Grant No. 7002571.

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