Overall Efficiency and its Decomposition in Two-Stage Network DEA Model

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Abstract. This paper aims to propose a new approach to decompose an overall DEA model into equivalent two-stage models. In this approach we use a minimax reference point method to set the weights and reliabilities of the two stage models so that the combined efficiency of the two stages is equal to the overall efficiency. The equivalent multi-stage models are useful to support planning for performance improvement. An illustrative example is first explored to compare the results from the new approach with those of 4 other existing approaches. The main finding from the comparisons is that the new decomposition approach of this paper satisfies the proposed several assumptions. A case study is then conducted on a two-stage process of steel manufacturing to illustrate the validity and applicability of the proposed approach.

Keywords. Two-stage network DEA, overall efficiency, decomposition, evidential reasoning, performance improvement

1. Introduction

Data envelopment analysis (DEA), first proposed by Charnes *et al.* (1978), can be used to measure the relative efficiency or performance of a group of decision making units (DMUs) regarding multiple inputs and multiple outputs. While most DEA related research treats the operation of a DMU as a "black box" (Färe & Grosskopf 2000), recent research intends to go inside the "black box" and the internal structure of the DMUs (Tone & Tsutsui 2009). For instance, network DEA models have been developed to take into account the process within a DMU which can include several sub-processes or stages. Every stage is characterized by its own inputs and outputs and related to other stages through intermediate flows (Färe & Grosskopf 2000). A simple network DEA model can have a two-stage structure where the first stage uses inputs to generate outputs that become the inputs to the second stage (e.g. Chen & Zhu 2004, Liang *et al.* 2008, Chen *et al.* 2009a, Chen *et al.* 2009b, Wade *et al.* 2010, Premachandra *et al.* 2012, Sahoo *et al.* 2014, Ma 2015). This avenue of research goes inside the "black box" to investigate the internal structure of a DMU through the decomposition of the overall efficiency into two sub-stages linked by intermediate variables. The composition is meaningful in the sense that it provides a novel way to analyze not only the efficiency for the overall process but also the efficiencies of two internal stages so that the decision makers (DMs) can gain a clearer insight to find the key issues hindering the DMU's performances and the sensible ways for performance improvement.

From the literature review, the existing research related to the decomposition of overall efficiency is mostly based on two-stage network DEA models using either a multiplicative or an additive approach. However several concerns on existing approaches can be raised. First both multiplicative and additive approaches need to satisfy rather strong assumptions. Secondly multiplicative approach may cause irrational results in real applications and additive approach needs to assign unique weights for two stages. Also, neither the multiplicative efficiency decomposition approach nor the additive efficiency decomposition approach can generate the overall efficiency that is equivalent to what is generated by a traditional DEA model treating DMU as "black box". The detailed discussions on those concerns can be found in the literature review in Section 2. Therefore there is a need to reinvestigate the efficiency decomposition in a two-stage network DEA model.

This paper aims to investigate the efficiency decomposition in a two-stage network DEA model. Different from previous research, this paper proposes to use the Evidential-Reasoning (ER) approach to combine the efficiencies from two separated stages to generate overall efficiency. The link of two stages is reflected in the aggregation process. The ER approach can be used for multiple criteria decision making (MCDM) problems where the assessment of an alternative on each attribute is described by a belief structure. It provides a new procedure for aggregating multiple attributes based on the belief assessment framework and an evidence combination rule. It has been used in many real world MCDM problems, such as engineering design assessment (Yang & Singh 1994), environmental impact assessment (Wang *et al.* 2006), the Quality Function Deployment assessment (Chin *et al.* 2009), etc. The rationale for this proposed approach is two folds. First, the ER approach is capable of meeting the requirement that the combined efficiency generated through a two-stage network DEA model be equivalent to the overall efficiency generated by a traditional DEA model treating each DMU as a "black box", so as to resolve the afore-mentioned dilemma encountered by the additive approach or the multiplicative approach. Secondly, the equivalence provides a sound basis for setting performance targets at each of the two stages with the relationship kept intact between the two stage DEA model and the overall DEA model. This ensures that the target setting process is rigorous and the targets set are feasible and rational.

The remainder of this paper is organized as follows. Section 2 summarises the related literatures for two-stage network DEA models and applications. We discuss the aggregation of separated efficiencies in two-stage network DEA models in Section 3, including efficiency decomposition and how to conduct resource planning to improve the performance of both overall process and two sub-stages. In Section 4, two illustrative examples are examined regarding combining efficiencies from two stages in two-stage network DEA model using the ER rule. The first example is to illustrate how to use the ER rule to obtain the aggregated overall efficiency and the corresponding weights for efficiencies from two stages separately. In the case study, we use the ER rule to combine the efficiencies on the performance evaluation in a two-stage short process of steel manufacturing. Finally, some conclusions are presented in Section 5.

2. Literature review on two-stage network DEA model

We first consider the semi-positive[1] input, intermediate, and output vectors $(x_j, z_j, y_j)(j = 1,2, \ldots n)$ of n DMUs where $x_j \geq 0, x_j \neq 0$, $z_j \geq 0, z_j \neq 0$, and $y_j \geq 0, y_j \neq 0$ for $j = 1,2, \ldots n$ denote the inputs to the first stage, the outputs from the first stage (or intermediate measures as they become the inputs to the second stage), and the outputs from the second stage. We call such non-negative input $x \in \mathbb{R}_m^+$ andintermediate $z \in \mathbb{R}_D^+$, and output $y \in \mathbb{R}_s^+$ *an activity* and represent it by the notation (x, z, y). The above two-stage network structure is shown in Figure 1.

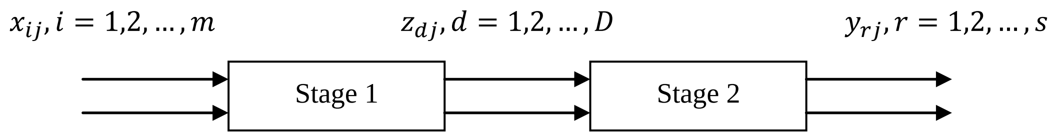

Figure 1. Simple two-stage process.

We assume that the individual efficiency measures are denoted as θ_j^1 and θ_j^2 for the first and the second stage in Figure 1 respectively. If we use DEA model under constant returns to scale (CRS) assumption, we have

$$\theta_j^1 = \frac{\sum_{d=1}^{D} \omega_d z_{dj}}{\sum_{i=1}^{m} v_i x_{ij}} \text{ and } \theta_j^2 = \frac{\sum_{r=1}^{s} u_r y_{rj}}{\sum_{d=1}^{D} \widetilde{\omega}_d z_{dj}} \qquad (1)$$

where variables v_i, ω_d, $\widetilde{\omega}_d$, and u_r are unknown non-negative weights.

It should be noted that the above Figure 1 shows the case where the intermediate measures are the only and all inputs to the second stage and there are no additional independent inputs to the second stage. Cook *et al.* (2010) pointed out that there are other types of two-stage processes where there are other inputs to the second stage in addition to the intermediate measures, such as the one shown in Figure 2 (Ma 2015).

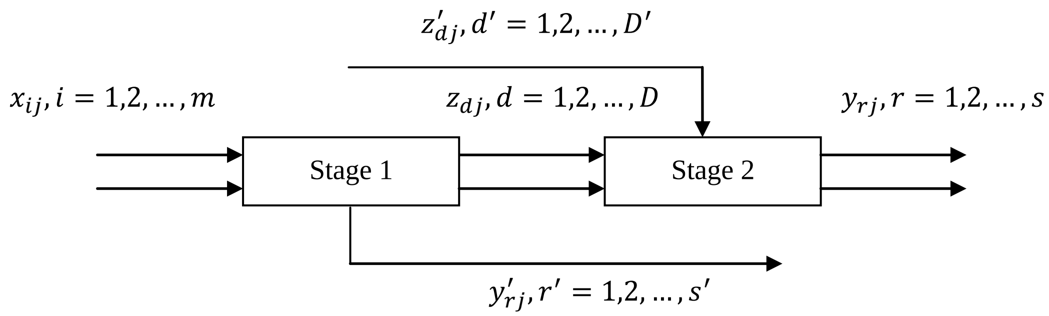

Figure 2. Two-stage network process.

In Figure 2 variables $z'_{dj}(d = 1,2, \ldots, D')$ denote the other inputs to the second stage in addition to the intermediate measures. In this example, some of the outputs of the first stage become the final outputs directly.

In the past few years two-stage DEA models as one type of network DEA have attracted researchers' attentions. Many DEA studies argued that DMUs can have a

[1]Note: A vector x is called semi-positive if $x \geq 0$ and $x \neq 0$.

two-stage structure where the first stage uses inputs to generate outputs which become the inputs to the second stage subsequently. Seiford and Zhu (1999) presented a two-stage process to measure the profitability and marketability of US commercial banks. Zhu (2000) proposed a multi-factor financial performance model based on a two-stage DEA model, which inherently recognizes tradeoffs among various financial measures, and applied it to the performance analysis of the Fortune Global 500 companies. Chen and Zhu (2004) showed that banks use assets and labor to generate deposits which are in turn used to generate loan income. They use profitability measured in terms of assets and labor as inputs and use profits and revenue as outputs in the first stage. In the second stage of marketability, they use the profits and revenue as inputs, and market value, returns and earnings per share as outputs. Yang (2006) used a two-stage DEA model to provide valuable managerial insights when assessing the dual impacts of operating and business strategies for the Canadian life and health insurance industry. Chen *et al.* (2009a) developed an additive efficiency decomposition approach wherein the overall efficiency is expressed as a weighted sum of the efficiencies of the individual stages with application to the case of Taiwanese non-life insurance companies. Kao and Huang (2008) used two-stage DEA to measure the efficiencies of Taiwanese non-life insurance companies. In that paper, they take into account the serial relationship between the two sub-processes and the whole process, and measure the efficiencies of the whole process as well as each stage independently. Liang *et al.* (2008) examined and extended the two-stage DEA model using game theory concepts, where all the outputs from the first stage are the only inputs to the second stage. Chen *et al.* (2009a) examined relations and equivalence between two existing DEA approaches that address the question of measuring the performance of two-stage processes. Chen *et al.* (2010) developed an approach for determining the frontier points for inefficient DMUs within the framework of two-stage DEA. Wang and Chin (2010) proposed some alternative DEA models for two-stage process and showed that (a) the overall efficiency of a two-stage process can be modeled as a weighted harmonic mean of the efficiencies of the two individual stages, (b) the two-stage DEA model of Kao and Hwang (2008) can be extended to variable returns to scale (VRS) assumption, and (c)the additive efficiency decomposition model of Chen *et al.* (2009a) can be generalized to taking into account the relative importance weights of two individual stages. Kao and Hwang (2011) combined the input-oriented DEA model in the first stage and the output-oriented model in the second process and then express the system efficiency as the product of the overall technical and scale efficiencies, where the overall technical and scale efficiencies are the products of the corresponding efficiencies of the two processes, respectively. Ma (2015) proposed a two-stage DEA model considering simultaneously the structure of inputs and intermediate measures in efficiency evaluation and decomposition.

Kao and Hwang (2008) and Chen *et al.* (2009) pointed out that a common approach to the two-stage problem is to use a standard DEA model separately in each stage, which treat the two stages as operating independently of one another. Therefore Kao and Hwang (2008) considered the serial relationship of the two stages and decomposed the overall process's efficiency into the product of the two stages' efficiencies. On the contrary, Chen *et al.* (2009a) developed an additive efficiency decomposition approach wherein the overall efficiency is calculated as a weighted sum of individual stages' efficiencies.

As summarized in the above discussion, our literature review shows that there are mainly three efficiency decomposition approaches as follows.

(1) Multiplicative efficiency decomposition. Kao and Hwang (2008) decomposed the overall process's efficiency into the product of the two stages' efficiencies, *i.e.* the overall efficiency θ_j^* of DMU_j is denoted as $\theta_j^* = \theta_j^{1*} \times \theta_j^{2*}$, under the assumption $\omega_d = \widetilde{\omega}_d$. Therefore their model for measuring the overall efficiency of a DMU is given as follows:

$$\theta_j^* = max\left(\theta_j^1 \times \theta_j^2 = \frac{\sum_{d=1}^{D}\omega_d z_{dj}}{\sum_{i=1}^{m} v_i x_{ij}} \times \frac{\sum_{r=1}^{s} u_r y_{rj}}{\sum_{d=1}^{D}\widetilde{\omega}_d z_{dj}} = \frac{\sum_{r=1}^{s} u_r y_{rj}}{\sum_{i=1}^{m} v_i x_{ij}}\right)$$
$$s.t.\begin{cases} \frac{\sum_{d=1}^{D}\omega_d z_{dj}}{\sum_{i=1}^{m} v_i x_{ij}} \le 1, j = 1, \dots, n \\ \frac{\sum_{r=1}^{s} u_r y_{rj}}{\sum_{d=1}^{D}\widetilde{\omega}_d z_{dj}} \le 1, j = 1, \dots, n \\ \omega_d = \widetilde{\omega}_d, d = 1, \dots, D \\ v_i, \omega_d, \widetilde{\omega}_d, u_r \ge 0 \end{cases} \tag{2}$$

(2) Additive efficiency decomposition. Chen *et al.* (2009a) proposed to define the overall efficiency of the two-stage process as $\theta_j^* = w_1\theta_j^{1*} + w_2\theta_j^{2*}$ where w_1 and w_2 are user-specified weights such that $w_1 + w_2 = 1$. They pointed out that these weights are the functions of optimization variables. Thus their model for measuring the overall efficiency of a DMU is given as follows:

$$\theta_j^* = max\left(w_1\theta_j^1 + w_2\theta_j^2 = w_1\frac{\sum_{d=1}^{D}\omega_d z_{dj}}{\sum_{i=1}^{m} v_i x_{ij}} + w_2\frac{\sum_{r=1}^{s} u_r y_{rj}}{\sum_{d=1}^{D}\widetilde{\omega}_d z_{dj}}\right)$$
$$s.t.\begin{cases} \frac{\sum_{d=1}^{D}\omega_d z_{dj}}{\sum_{i=1}^{m} v_i x_{ij}} \le 1, j = 1, \dots, n \\ \frac{\sum_{r=1}^{s} u_r y_{rj}}{\sum_{d=1}^{D}\widetilde{\omega}_d z_{dj}} \le 1, j = 1, \dots, n \\ \omega_d = \widetilde{\omega}_d, d = 1, \dots, D \\ v_i, \omega_d, \widetilde{\omega}_d, u_r \ge 0 \end{cases} \tag{3}$$

If the two weights w_1 and w_2 are defined as $w_1 = \frac{\sum_{i=1}^{m} v_i x_{ij}}{\sum_{i=1}^{m} v_i x_{ij} + \sum_{d=1}^{D}\omega_d z_{dj}}$ and $w_2 = \frac{\sum_{d=1}^{D}\omega_d z_{dj}}{\sum_{i=1}^{m} v_i x_{ij} + \sum_{d=1}^{D}\omega_d z_{dj}}$, then model (3) can be transformed into a linear form using Charnes-Cooper transformation (Charnes & Cooper 1962). Furthermore they have also discussed the additive efficiency decomposition of the overall efficiency of the two-stage process under variable returns to scale (VRS) assumption.

(3) Game approach. Liang *et al.* (2006) proposed that a two-stage process can be viewed as one where the stages jointly determine a set of optimal weights on the intermediate factors to maximize their efficiencies based on the cooperative game theory. In their work, several DEA-based approaches are developed for measuring efficiency of supply chain when intermediate measures are incorporated into the performance evaluation. They used a seller-buyer supply chain to build the models and treated the seller and buyer as ones of leader-follower and cooperation respectively. Liang *et al.* (2008) proposed the two-stage non-cooperative game DEA model. Du *et al.* (2011) developed a Nash bargaining game model to measure the performance of DMUs that have a two-stage structure. Li *et al.* (2012) extended the two-stage non-cooperative game DEA (Liang *et al.* 2008) by assuming that the inputs to the second stage include both the outputs from the first stage and additional inputs to the second stage. Two models are proposed to evaluate the performance of this type of general two-stage network structures.

In the multiplicative efficiency decomposition approach, the assumption $\omega_d = \widetilde{\omega}_d$ is rather strong and difficult to be satisfied in real applications because the two sub-stages should have the flexibility to select weights separately. The weights for intermediates in different stages need not be the same all the time. Furthermore, multiplicative efficiency decomposition approach may cause irrational results. For example if the efficiencies in stage one and stage two are both 0.2, the multiplicative overall efficiency will be 0.04, which is very low and not reasonable. In the additive efficiency decomposition approach, the assumption $\omega_d = \widetilde{\omega}_d$ is also required. This approach needs the "additive independence" condition *(Fishburn 1970)* to be satisfied too. Neither the multiplicative efficiency decomposition approach nor the additive efficiency decomposition approach can generate the overall efficiency that is equivalent to that generated by traditional DEA models treating DMU as "black box". The game approach can be similar to either additive or multiplicative efficiency decomposition approach which assumes $\omega_d = \widetilde{\omega}_d$ and the optimisation can be based upon maximising the average of θ_j^1 and θ_j^2 (Liang *et al.* 2006) or maximising the product of θ_j^1 and θ_j^2 (Liang *et al.* 2008, Li *et al.* 2012). Intuitively, the aggregated overall efficiency from two-stage DEA models should be equal to the efficiency generated using classic DEA models which treat the DMU as "black box", as far as performance assessment is concerned. It becomes even more important to meet this equivalence condition if performance improvement targets need to be set in a consistent manner at each stage. From our literature review, however, all the existing approaches for efficiency decomposition in two-stage network DEA models fail to take this notion into consideration. Therefore there is a need to reinvestigate the relationships between the overall efficiency and its decomposition in two-stage network DEA model.

3. The aggregation of separated efficiencies in two-stage network DEA

In this section, we first propose to apply the ER rule to transform the efficiencies of two stages in two-stage network DEA into two pieces of evidence. The ER rule is then explored to establish the equivalence between an overall DEA model and its two-stage decomposition models. At last we will investigate further how to make resource planning to improve the performance of DMUs in two-stage network DEA models.

3.1 Transforming DEA efficiency into a piece of evidence

On the basis of Dempster's original work on multi-valued mapping (Dempster 1967) and the belief distribution to represent evidence with ambiguity (Shafer 1976), the ER rule is established for conjunctive combination of independent evidence with weights and reliabilities (Yang and Xu 2013). The ER rule generalises the ER algorithm developed for multiple criteria decision analysis (Yang and Singh 1994, Yang and Sen 1994, Yang and Xu 2002a, 2002b).In this paper the generalised ER rule (Yang and Xu, 2014; Yang et al. 2016) is used to combine the efficiencies generated from two stages. The description of the generalised ER rule with two pieces of evidence is provided in subsection 3.2.

In DEA models, a DMU is DEA efficient if it has the best possible relative efficiency of unity; otherwise, it is DEA inefficient. The efficiency measured by DEA is an estimation of true efficiency. Banker (1993) provided a formal statistical basis for the efficiency evaluation techniques of DEA. He argued that, if the deviation of actual output from the efficient output is regarded as a stochastic variable with a monotonically decreasing probability density function, the DEA estimators of the best practice monotonically increasing and concave production function were shown to be the *maximum likelihood* estimators under certain assumptions (Banker 1993) with a known rate of convergence (Korostelev *et al.* 1995).Inspired by this work, Yang *et al.* (2013) proposed the idea of transforming the efficiency in DEA into pieces of evidence. Cao *et al.* (2016) used this idea to combine efficient and anti-efficient frontiers and constructed a performance indicator. The core of the idea is to define two mutually exclusive states named "Efficient" and "Not efficient", and then the DEA efficiency is considered as the expected value of the belief distribution on these two states. It should be noted that in this paper, "Not efficient" is a different concept from "inefficient" defined above. "Not efficient" denotes the case where the efficiency score is zero, which is the worst possible relative efficiency of a DMU to be evaluated. It is easy to see that the combination of these two evaluation grades can cover all the possible state of DEA efficiency.

The efficiency scores of DMUs in two stages can be viewed as the expected scores of the belief distributions on both "Efficient" and "Not efficient" states. For example, if the efficiency score of one DMU in the first stage is 0.75, then the belief distribution is $\{(Efficient, 0.75), (Notefficient, 0.25)\}$.This distribution means that the DMU evaluated is efficient with 0.75degree of belief and is not efficient with 0.25 degree of belief in the first stage. Similarly, we can address the efficiency from the second stage. *For example*, if the efficiency score of one DMU in the second stage is 0.6, then the distribution is $\{(Efficient, 0.6), (Notefficient, 0.4)\}$. In other words, it is efficient with 60% degree of belief and is not efficient with 40% degree of belief. Thus, we have two pieces of evidence generated in an independent way for each DMU_j from two stages within the whole process, respectively.

It should be noted that an assumption is implied in the process of transforming efficiency into a piece of evidence. That is, any efficiency score from DEA model can be expressed by a convex combination of two mutually exclusive states “Efficient” and “Not efficient”. We can justify this assumption as follows. Because of the fact that the DEA estimators of the best practice monotonically increasing and concave production function are *maximum likelihood* estimators under certain assumptions (Banker 1993) with a known rate of convergence (Korostelev *et al.* 1995), the DEA efficiencies are also *maximum likelihood* estimations of true efficiency under certain assumptions. See the following Figure 3 for the relationship between DEA efficiency and the best practice production function. Moreover, the range of DEA efficiency score is the interval $(0, 1]$, which is exactly the range of the linear combination of the two states “Efficient” and “Not efficient”. Furthermore DEA frontier is an estimation of true production frontier. The efficiency score of one DMU means an estimation of its real efficiency. From statistical point of view, we can regard the efficiency score as the degree of belief that this DMU is efficient.

The case of single input and single output is used as example to show the process of transformation as in Figure 3. CCR frontier and BCC frontier are both the estimations of true efficiency frontier under the constant returns to scale (RTS) assumption and the variable RTS assumption respectively. DEA models (including CCR and BCC) use distance to frontier to measure the inefficiency of DMUs. The CCR efficiency and BCC efficiency of Point A are $E'E_0/AE_0$ and EE_0/AE_0 respectively. However, the true efficiency of point A is $E''E_0/AE_0$. Thus CCR efficiency and BCC efficiency are both the estimations of true efficiency under different assumptions.

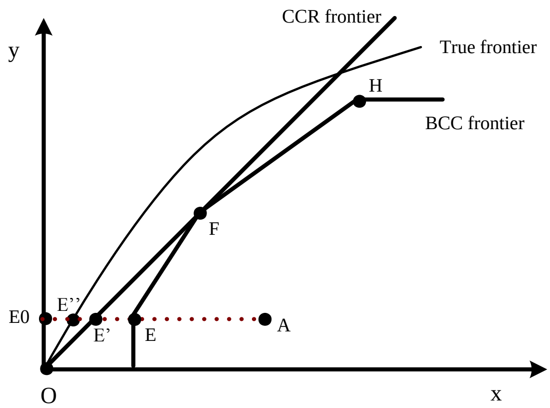

Figure 3. Transform DEA efficiency into a piece of evidence.

3.2 Aggregation of efficiencies of two stages using the ER rule

In this subsection we will use the ER rule (Yang & Xu 2013) to combine the efficiencies generated from the first stage and the second stage respectively. Yang and Xu (2013) proved that the ER approach (see, *e.g.* Yang & Singh 1994, Yang & Sen 1994, and Yang & Xu 2002a,b) is the result of implementing the orthogonal sum operations with the sum of weights being unity. In this paper, we will use the general form of ER rule (Yang and Xu 2013, 2014b; Yang *et al.* 2016) to provide both rigor and flexibility for our results. The generalised ER rule is briefly introduced in the case of combining two pieces of evidence.

Step 1: Let $E_j = \{e_{j1}, e_{j2}\}$ denote two pieces of evidence for $DMU_j (j = 1, \dots, n)$, where e_{j1} and e_{j2} represent the evidence from the first stage θ_j^{1*} and the second stage θ_j^{2*} of $DMU_j (j = 1, \dots, n)$, respectively.

Step 2: Suppose there are two distinctive evaluation grades H_1 and H_2, which provide a complete set of grades for evaluation $\Theta = \{H_1, H_2\}$, where H_1 and H_2 represent the grades of "Efficient" and "Not efficient", respectively. Mathematically, we can represent the given assessments for e_{j1} and e_{j2} of $DMU_j (j = 1, \dots, n)$ as the following belief distribution:

$$S(e_{j1}) = \{(H_1, \beta_{j,1,1}), (H_2, \beta_{j,2,1})\} \tag{4}$$

$$S(e_{j2}) = \{(H_1, \beta_{j,1,2}), (H_2, \beta_{j,2,2})\} \tag{5}$$

where $\beta_{j,1,1} = \theta_j^{1*}$, $\beta_{j,2,1} = 1 - \theta_j^{1*}$, and $\beta_{j,1,2} = \theta_j^{2*}$, $\beta_{j,2,2} = 1 - \theta_j^{2*}$. Namely, we consider that the first stage efficiency θ_j^{1*} of $DMU_j (j = 1, \dots, n)$ represents the degree of belief in "Efficient", and $1 - \theta_j^{1*}$ represents the degree of belief in "Not Efficient". Similarly, the efficiency score $\theta_j^{2*}$of the second stage of $DMU_j (j = 1, \dots, n)$represents the degree of belief in "Efficient", and $1 - \theta_j^{2*}$ represents the degree of belief in "Not Efficient". From the above definitions, there are $\beta_{j,1,1} \geq 0$, $\beta_{j,2,1} \geq 0$, $\beta_{j,1,1} + \beta_{j,2,1} = 1$ and $\beta_{j,1,2} \geq 0$, $\beta_{j,2,2} \geq 0$, $\beta_{j,1,2} + \beta_{j,2,2} = 1$.

Step 3: Suppose the weights of two pieces of evidence with the two states ("Efficient" and "Not Efficient") are given by $W = (w_{1E}, w_{1N}, w_{2E}, w_{2N})^T$, where w_{1E}, w_{1N} and $w_{2E}, w_{2N}$are the relative weights of the two different states of two pieces of evidence, respectively, with $0 < w_{1E}, w_{1N}, w_{2E}, w_{2N} < 1$. Define the reliabilities of two pieces of evidence with the two states as r_{1E}, r_{1N} and r_{2E}, r_{2N}, with$0 < r_{1E}, r_{1N}, r_{2E}, r_{2N} < 1$.

Remark 1.*In this paper we set* $w_{1E} = r_{1E}, w_{1E} = r_{1E}$ *and* $r_{1E} = r_{1N}, r_{2E} = r_{2N}$ *to denote the weights and reliabilities are the same respectively because the same dataset is used for efficiency analysis in this paper.*

Step 4: The probability masses are calculated as follows:

$$m_{j,1,1} = w_{1E}\beta_{j,1,1};\ m_{j,2,1} = w_{1N}\beta_{j,2,1}$$

$$m_{j,1,2} = w_{2E}\beta_{j,1,2};\ m_{j,2,2} = w_{2N}\beta_{j,2,2}.$$

where $j = 1, \dots, n$. Variables $m_{j,1,t}$ and $m_{j,2,t}$ $(t = 1\ or\ 2)$ are the probability masses that evidence e_{jt} supports H_1(Efficient) and H_2(Not Efficient) respectively.

Step 5: The efficiency scores from two stages can be combined through the following ER rule. We define $m_{j,1,T(2)}$ and $m_{j,2,T(2)}$ as the combined probability masses for H_1 and H_2 from two pieces of evidence. Then, the ER rule is given as follows:

$$\{H_1\}: m_{j,1,T(2)} = \left[(1 - r_{2E})m_{j,1,1} + (1 - r_{1E})m_{j,1,2}\right] + m_{j,1,1}m_{j,1,2} \tag{6}$$

$$\{H_2\}: m_{j,2,T(2)} = \left[(1 - r_{2N})m_{j,2,1} + (1 - r_{1N})m_{j,2,2}\right] + m_{j,2,1}m_{j,2,2} \tag{7}$$

where $j = 1, \dots, n$.

Thus, after combining two pieces of evidence, the overall belief distribution for the $DMU_j(j = 1, \dots, n)$ on the frame of discernment Θ is given as follows:

$$\{H_1\}: \beta_{j,1} = m_{j,1,T(2)}/\left(m_{j,1,T(2)} + m_{j,2,T(2)}\right) \tag{8}$$

$$\{H_2\}: \beta_{j,2} = m_{j,2,T(2)}/\left(m_{j,1,T(2)} + m_{j,2,T(2)}\right) \tag{9}$$

Therefore, for $DMU_j(j = 1, \dots, n)$, the overall support function after aggregating these two pieces of evidence on the frame of discernment Θ is given as follows:

$$S_j(E) = \{(H_1, \beta_{j,1}), (H_2, \beta_{j,2})\} \tag{10}$$

which reads that $DMU_j(j = 1, \dots, n)$ is assessed to the grade H_1 and grade H_2 with the degree of belief of $\beta_{j,1}$ and $\beta_{j,2}$, respectively.

Based on formula (10), we can use the degree of belief $\beta_{j,1}$ for H_1 as the overall efficiency of $DMU_j(j = 1, \dots, n)$ in the two-stage process. It is easy to see that $\beta_{j,1} + \beta_{j,2} = 1$, where symbol $\beta_{j,1}$ can be transformed into the aggregated overall efficiency of $DMU_j(j = 1, \dots, n)$, which is denoted by θ_j^a. Therefore we have $\theta_j^a = \beta_{j,1} = 1 - \beta_{j,2}$.

3.3 Formulations for overall efficiency and its decomposition with weights and reliabilities selection

In this section we analyse the more complex two-stage network process, which is shown in **Figure 2**. Firstly, we recall the conception of the production possibility set (PPS) in traditional DEA models, which is defined as

$$PPS = \{(Inputs, Outputs) |\, Inputs\ can\ produce\ Outputs\}.$$

The boundary of PPS is referred to as production frontier. Technically efficient DMUs operate on the frontier, while those technically inefficient DMUs operate at points in the interior of PPS. In this paper we deal with two-stage network process in Figure 2, so we denote the PPSs for the overall process and two sub-stages as $PPS_{Overall}$, PPS_{Stage1}, and PPS_{Stage2}, respectively, as follows:

$$PPS_{Overall} = \left\{\{(x, z'), (y, y')\} \middle| \sum_{j=1}^{n} \lambda_j x_j \leq x, \sum_{j=1}^{n} \lambda_j z'_j \leq z', \sum_{j=1}^{n} \lambda_j y_j \geq y, \sum_{j=1}^{n} \lambda_j y'_j \geq y', \lambda_j \geq 0\right\} \tag{13}$$

$$PPS_{Stage1} = \left\{\{x, (z, y')\} \middle| \sum_{j=1}^{n} \lambda_j x_j \leq x, \sum_{j=1}^{n} \lambda_j z_j \geq z, \sum_{j=1}^{n} \lambda_j y'_j \geq y', \lambda_j \geq 0\right\} \tag{14}$$

$$PPS_{Stage2} = \left\{\{(z, z'), y\} \middle| \sum_{j=1}^{n} \lambda_j z_j \leq z, \sum_{j=1}^{n} \lambda_j z'_j \leq z', \sum_{j=1}^{n} \lambda_j y_j \geq y, \lambda_j \geq 0\right\} \tag{15}$$

Secondly, the DEA models and their duals to evaluate DMU_{j0} for the whole process and two separated stages are as follows, respectively:

For the whole process:

$$max\ \theta_{j0}=\frac{\sum_{r=1}^{s}u_r y_{rj0}+\sum_{r'=1}^{s'}u_r'\left(y_{r'j0}'\right)}{\sum_{i=1}^{m}v_i x_{ij0}+\sum_{d'=1}^{D'}\omega_{d'}'\left(z_{d'j0}'\right)}$$

$$s.t.\begin{cases}\dfrac{\sum_{r=1}^{s}u_r y_{rj}+\sum_{r'=1}^{s'}u_r'\left(y_{r'j}'\right)}{\sum_{i=1}^{m}v_i x_{ij}+\sum_{d'=1}^{D'}\omega_{d'}'\left(z_{d'j}'\right)}\le 1, j=1,\dots,n\\ v_i,u_r,u_r',\omega_{d'}'\ge 0\end{cases}\tag{16}$$

For Stage 1:

$$max\ \theta_{j0}^{1}=\frac{\sum_{d=1}^{D}\omega_d z_{dj0}+\sum_{r'=1}^{s'}u_r'\left(y_{r'j0}'\right)}{\sum_{i=1}^{m}v_i x_{ij0}}$$

$$s.t.\begin{cases}\dfrac{\sum_{d=1}^{D}\omega_d z_{dj}+\sum_{r'=1}^{s'}u_r'\left(y_{r'j}'\right)}{\sum_{i=1}^{m}v_i x_{ij}}\le 1, j=1,\dots,n\\ u_r',v_i,\omega_d\ge 0\end{cases}\tag{17}$$

For Stage 2:

$$max\ \theta_{j0}^{2}=\frac{\sum_{r=1}^{s}u_r y_{rj0}}{\sum_{d=1}^{D}\widetilde{\omega}_d z_{dj0}+\sum_{d'=1}^{D'}\omega_{d'}'\left(z_{d'j0}'\right)}$$

$$s.t.\begin{cases}\dfrac{\sum_{r=1}^{s}u_r y_{rj}}{\sum_{d=1}^{D}\widetilde{\omega}_d z_{dj}+\sum_{d'=1}^{D'}\omega_{d'}'\left(z_{d'j}'\right)}\le 1, j=1,\dots,n\\ u_r,\omega_{d'}',\widetilde{\omega}_d\ge 0\end{cases}\tag{18}$$

Remark 2. *As discussed above, either multiplicative efficiency decomposition or additive efficiency decomposition requires the assumption $\omega_d=\widetilde{\omega}_d$ on the weights for interlinked intermediates. However this assumption is not a necessity for the two stage process and is difficult to be satisfied in real applications. Thus in this paper we drop this assumption in our following formulations and use the ER rule to reflect the generic relation between two sub-stages.*

From the systematic point of view, we consider as an axiom that the overall efficiency and its decompositions should satisfy the following basic conditions:

Condition 1.*The efficiency for one stage is generated in a way independent of whether the efficiency for the other stage is known or not. However those two efficiencies are linked with each other through intermediate outputs. Thus those two efficiencies do not satisfy the "additive independence" condition (Fishburn 1970).*

Condition 2.*The aggregated efficiency from the two stages should be equal to the overall efficiency produced from the conventional DEA model.*

Condition 3.*There could be overlaps of the weights of two stages' efficiencies, or the weights do not have to be added to unity.*

Based on the above considerations, we can have the following nonlinear programming for DMU_{j0}:

$$\min_{w_{1E},w_{2E},r_{1E},r_{2E},w_{1N},w_{2N},r_{1N},r_{2N}} \left|\theta_{j0}^{a} - \theta_{j0}^{*}\right|$$
$$s.t.\begin{cases} \theta_{j0}^{a} = m_{j0,1,T(2)}/\left(m_{j0,1,T(2)} + m_{j0,2,T(2)}\right) \\ w_{1E} = r_{1E}, w_{2E} = r_{2E}, w_{1N} = r_{1N}, w_{2N} = r_{2N} \\ 0 < w_{1E}, w_{1N}, w_{2E}, w_{2N} < 1; 0 < r_{1E}, r_{1N}, r_{2E}, r_{2N} < 1 \end{cases} \tag{19}$$

where θ_j^a denotes the aggregated overall efficiency based on two stages' separated efficiencies θ_j^{1*} and θ_j^{2*} .Variables $w_{1E}, w_{1N}, w_{2E}, w_{2N}$ are the weights for the efficiencies (with two states H_1 and H_2) of two stages respectively and $r_{1E}, r_{1N}, r_{2E}, r_{2N}$ are the reliabilities of two states of two stages. Variables $m_{j0,1,T(2)}$ and $m_{j0,2,T(2)}$ are defined based on formulae (6) and (7) as follows:

$$m_{j0,1,T(2)} = \left[(1 - r_{2E})w_{1E}\theta_{j0}^{1*} + (1 - r_{1E})w_{2E}\theta_{j0}^{2*}\right] + w_{1E}\theta_{j0}^{1*}w_{2E}\theta_{j0}^{2*} \tag{19a}$$

$$m_{j0,2,T(2)} = \left[(1 - r_{2N})w_{1N}(1 - \theta_{j0}^{1*}) + (1 - r_{1N})w_{2N}(1 - \theta_{j0}^{2*})\right] + w_{1N}(1 - \theta_{j0}^{1*})w_{2N}(1 - \theta_{j0}^{2*}) \tag{19b}$$

Model (19) is a nonlinear program and can be solved easily using conventional software, *e.g.* Lingo 10.0. It should be noted that the nonlinear program can also be solved by commonly available software package such as EXCEL.

If model (19) has multiple optimal solutions, we can use the following models to deal with this issue. Given the aggregated overall efficiency obtained from model (19) as θ_{j0}^{a*}, we can use the following minimax model as the equalizer to set the equal priority on two stages. Thus we have

$$min_{w_{iE},w_{iN}}(max_i\{(1 - w_{iE}), i = 1,2\} + max_i\{(1 - w_{iN}), i = 1,2\})$$
$$s.t.\begin{cases} m_{j0,1,T(2)}/\left(m_{j0,1,T(2)} + m_{j0,2,T(2)}\right) = \theta_{j0}^{a*} \\ w_{1E} = r_{1E}, w_{2E} = r_{2E}, w_{1N} = r_{1N}, w_{2N} = r_{2N} \\ 0 < w_{1E}, w_{1N}, w_{2E}, w_{2N} < 1; 0 < r_{1E}, r_{1N}, r_{2E}, r_{2N} < 1 \end{cases} \tag{20}$$

In model (20) we minimize the sum of maximal values of $1 - w_{iE}$ and $1 - w_{iN}$ and maximal values of $1 - r_{kE}$ and $1 - r_{kN}$ to obtain the equalized weights and reliabilities for those two stages to emphasize that the contributions to the combined efficiency from the two stages are equally treated. If the contributions are unequal in specific applications, this can be taken into account by adding different preferential weights for the two stages in the above model (20). The above process forms a new approach to characterize the relationship between the overall efficiency and its decomposition. This approach is called the ER decomposition in this paper.

Remark 3. *In traditional two-stage DEA model (e.g. Premachandra et al. 2008) DMs can have the choice of allocating more weights on certain stage if he/she wishes. This is also true in our approach because the feasible region of model (20) provide countless combinations of weights and reliabilities. The DMs also can incorporate their preference into model (20) if they wish.*

It should be noted that this proposed approach in this paper is not restrictive in terms of orientation, i.e. the approach works in DEA models with either constant RTS or variable RTS. Furthermore we can easily see that this model is translation invariant because the classic DEA models run in two-stages separately. In this paper we focus on the aggregation and decomposition approach using the ER rule (Yang and Xu 2013, 2014b; Yang *et al.* 2016).

3.4 Resource planning for performance improvement for two-stage process

Given the decomposition of overall efficiency into two stages based on model (20), we are now in a position to investigate how to deal with the resource planning for performance improvement at each stage in a two-stage process. We consider the more complex case shown in **Figure 2**. First, the following assumptions are made for performance improvement.

Assumption 1. *The objective of performance improvement is to achieve the status that the overall efficiency and the efficiencies of two stages are all maximized.*

This assumption makes clear the objective of the performance improvement which aims to improve the overall efficiency of a two-stage process after adjustment to be maximized first. Second, the efficiencies of two stages are also to be maximized as much as possible.

Thus, this assumption leads to the following constraints (21a)-(21d):

$$m_{j0',1,T(2)}/\left(m_{j0',1,T(2)} + m_{j0',2,T(2)}\right) = \theta_{j0} \tag{21a}$$

where $m_{j0',1,T(2)} = \left[(1-r_{2E})w_{1E}\theta_{j0}^{1} + (1-r_{1E})w_{2E}\theta_{j0}^{2}\right] + w_{1E}\theta_{j0}^{1}w_{2E}\theta_{j0}^{2}$, $m_{j0',2,T(2)} = \left[(1-r_{2N})w_{1N}(1-\theta_{j0}^{1}) + (1-r_{1N})w_{2N}(1-\theta_{j0}^{2})\right] + w_{1N}(1-\theta_{j0}^{1})w_{2N}(1-\theta_{j0}^{2})$, and

$$\max \theta_{j0} = \frac{\sum_{r=1}^{s} a_r(y_{rj0}+\Delta y_{rj0}) + \sum_{r'=1}^{s'} u'_r\left(y'_{r'j0}+\Delta y'_{r'j0}\right)}{\sum_{i=1}^{m} b_i(x_{ij0}+\Delta x_{ij0}) + \sum_{d'=1}^{D'} \omega'_{d'}\left(z'_{d'j0}+\Delta z'_{d'j0}\right)}$$
$$s.t.\begin{cases} \dfrac{\sum_{r=1}^{s} a_r(y_{rj0}+\Delta y_{rj0}) + \sum_{r'=1}^{s'} u'_r\left(y'_{r'j0}+\Delta y'_{r'j0}\right)}{\sum_{i=1}^{m} b_i(x_{ij0}+\Delta x_{ij0}) + \sum_{d'=1}^{D'} \omega'_{d'}\left(z'_{d'j0}+\Delta z'_{d'j0}\right)} \le 1 \\ \dfrac{\sum_{r=1}^{s} a_r(y_{rj}) + \sum_{r'=1}^{s'} u'_r\left(y'_{r'j0}\right)}{\sum_{i=1}^{m} b_i(x_{ij0}) + \sum_{d'=1}^{D'} \omega'_{d'}\left(z'_{d'j0}\right)} \le 1, j = 1, \dots, n \\ y_{rj0} + \Delta y_{rj0} \ge 0; x_{ij0} + \Delta x_{ij0} \ge 0; y'_{r'j0} + \Delta y'_{r'j0} \ge 0; z'_{d'j0} + \Delta z'_{d'j0} \ge 0 \\ a_r, b_i \ge 0;\ \Delta x_{ij0}, \Delta y_{rj0}, \Delta y'_{r'j0}, \Delta z'_{d'j0}\ free \end{cases} \tag{21b}$$

$$\max \theta_{j0}^{1} = \frac{\sum_{d=1}^{D} \omega_d(z_{dj0}+\Delta z_{dj0}) + \sum_{r'=1}^{s'} u'_r\left(y'_{r'j0}+\Delta y'_{r'j0}\right)}{\sum_{i=1}^{m} v_i(x_{ij0}+\Delta x_{ij0})}$$
$$s.t.\begin{cases} \dfrac{\sum_{d=1}^{D} \omega_d(z_{dj0}+\Delta z_{dj0}) + \sum_{r'=1}^{s'} u'_r\left(y'_{r'j0}+\Delta y'_{r'j0}\right)}{\sum_{i=1}^{m} v_i(x_{ij0}+\Delta x_{ij0})} \le 1 \\ \dfrac{\sum_{d=1}^{D} \omega_d(z_{dj}) + \sum_{r'=1}^{s'} u'_r\left(y'_{r'j}\right)}{\sum_{i=1}^{m} v_i(x_{ij})} \le 1, j = 1, \dots, n \\ z_{dj0} + \Delta z_{dj0} \ge 0; y'_{r'j0} + \Delta y'_{r'j0} \ge 0; x_{ij0} + \Delta x_{ij0} \ge 0 \\ \omega_d, u'_r, v_i \ge 0; \Delta x_{ij0}, \Delta z_{dj0}, \Delta y'_{r'j0}\ free \end{cases} \tag{21c}$$

$$\max \theta_{j0}^{2} = \frac{\sum_{r=1}^{s} u_r(y_{rj0}+\Delta y_{rj0})}{\sum_{d=1}^{D} \tilde{\omega}_d(z_{dj0}+\Delta z_{dj0}) + \sum_{d'=1}^{D'} \omega'_{d'}\left(z'_{d'j0}+\Delta z'_{d'j0}\right)}$$
$$s.t.\begin{cases} \dfrac{\sum_{r=1}^{s} u_r(y_{rj0}+\Delta y_{rj0})}{\sum_{d=1}^{D} \tilde{\omega}_d(z_{dj0}+\Delta z_{dj0}) + \sum_{d'=1}^{D'} \omega'_{d'}\left(z'_{d'j0}+\Delta z'_{d'j0}\right)} \le 1 \\ \dfrac{\sum_{r=1}^{s} u_r(y_{rj})}{\sum_{d=1}^{D} \tilde{\omega}_d(z_{dj}) + \sum_{d'=1}^{D'} \omega'_{d'}\left(z'_{d'j}\right)} \le 1, j = 1, \dots, n \\ y_{rj0} + \Delta y_{rj0} \ge 0; z_{dj0} + \Delta z_{dj0} \ge 0; z'_{d'j0} + \Delta z'_{d'j0} \ge 0 \\ \tilde{\omega}_d, u_r, \omega'_{d'} \ge 0; \Delta z_{dj0}, \Delta z'_{d'j0}, \Delta y_{rj0}\ free \end{cases} \tag{21d}$$

However the above (21a)-(21d) may change the frontier of the overall process or those of two sub-stages, which may not be attainable or possible under the existing technology. Thus we make the following Assumption 2 to ensure that the frontiers of the overall process and two sub-stages are all kept intact in the process of both performance assessment and improvement.

Assumption 2. The frontiers of the overall process and two sub-stages are all fixed in the process of performance improvement.

This assumption leads to the following additional constraints:

$$\{(x_{j0}+\Delta x_{j0}, z'_{j0}+\Delta z'_{j0}), (y_{j0}+\Delta y_{j0}, y'_{j0}+\Delta y'_{j0})\} \in PPS_{Overall} \tag{22a}$$

$$\{(x_{j0}+\Delta x_{j0}), (z_{j0}+\Delta z_{j0}, y'_{j0}+\Delta y'_{j0})\} \in PPS_{Stage1} \tag{22b}$$

$$\{(z_{j0}+\Delta z_{j0}, z'_{j0}+\Delta z'_{j0}), (y_{j0}+\Delta y_{j0})\} \in PPS_{Stage2} \tag{22c}$$

Assumption 3. *The performance improvement should be made at the lowest costs of organizational adjustments.*

We first denote the "price" for the cost of changes of variables as follows:

$$\begin{cases} C_{x_i}: unit\ price\ for\ adjusting\ x_{ij}, j=1,\dots,n \\ C_{z_d}: unit\ price\ for\ adjusting\ z_{dj}, j=1,\dots,n \\ C_{z'_{d'}}: unit\ price\ for\ adjusting\ z'_{d'j}, j=1,\dots,n \\ C_{y'_{r'}}: unit\ price\ for\ adjusting\ y'_{r'j}, j=1,\dots,n \\ C_{y_r}: unit\ price\ for\ adjusting\ y_{rj}, j=1,\dots,n \end{cases} \tag{23a}$$

Following this assumption, we have the following additional objective for performance improvement

$$min\,\Delta = \sum_{i=1}^{m} C_{x_i}|\Delta x_{ij0}| + \sum_{d=1}^{D} C_{z_d}|\Delta z_{dj0}| + \sum_{d'=1}^{D'} C_{z'_{d'}}|\Delta z'_{d'j0}| + \sum_{r'=1}^{s'} C_{y'_{r'}}|\Delta y'_{r'j0}| + \sum_{r=1}^{s} C_{y_r}|\Delta y_{rj0}| \tag{23b}$$

where variables Δx_{ij0}, Δz_{dj0}, $\Delta z'_{d'j0}$, $\Delta y'_{r'j0}$, and $\Delta y_{rj0}$are the amounts of changes and free of sign.

Without loss of generality and for purpose of illustration, in this paper we assume that the "unit prices" of changes invariables for DMU_{j0} are given as follows:

$$C_{x_i} = \frac{1}{x_{ij0}}, C_{z_d} = \frac{1}{z_{dj0}}, C_{z'_{d'}} = \frac{1}{z'_{d'j0}}, C_{y'_{r'}} = \frac{1}{y'_{r'j0}}, C_{y_r} = \frac{1}{y_{rj0}} \tag{23c}$$

This assumption is meaningful in the sense that the relative changes of all variables are regarded to have the same "prices".

Therefore, we can have our objective of minimizing the total costs as follows:

$$min\,\Delta = \sum_{i=1}^{m} \frac{|\Delta x_{ij0}|}{x_{ij0}} + \sum_{d=1}^{D} \frac{|\Delta z_{dj0}|}{z_{dj0}} + \sum_{d'=1}^{D'} \frac{|\Delta z'_{d'j0}|}{z'_{d'j0}} + \sum_{r'=1}^{S'} \frac{|\Delta y'_{r'j0}|}{y'_{r'j0}} + \sum_{r=1}^{S} \frac{|\Delta y_{rj0}|}{y_{rj0}} \tag{23d}$$

where $|*|$ denotes the absolute value function.

Assumption 4. *The weights and reliabilities of two stages can only be changed in the feasible space of model (19).*

The above Assumption 4 leads to the following feasible space for weights in model (19) and the equalizer for weights:

$$\begin{cases} m_{j0,1,T(2)}/\left(m_{j0,1,T(2)}+m_{j0,2,T(2)}\right) = \theta_{j0}^{\alpha*} \\ min_{w_{iE},r_{kE},w_{iE},r_{kE}} \begin{pmatrix} max_i\{(1-w_{iE}), i=1,2\} + max_i\{(1-w_{iN}), i=1,2\} \\ + max_k\{(1-r_{kE}), k=1,2\} + max_k\{(1-r_{kN}), k=1,2\} \end{pmatrix} \end{cases} \tag{24}$$

where $0 < w_{1E}, w_{1N}, w_{2E}, w_{2N} < 1; 0 < r_{1E}, r_{1N}, r_{2E}, r_{2N} < 1, w_{1E} = r_{1E}, w_{2E} = r_{2E}, w_{1N} = r_{1N}, w_{2N} = r_{2N}$, and $\theta_{j0}^{\alpha*}$ denotes the aggregated overall efficiency obtained from model (19).

Based on the above Assumptions 1-4, a feasible space Ω is defined as follows:

$$\Omega = \begin{bmatrix} m_{j0',1,T(2)}/\left(m_{j0',1,T(2)}+m_{j0',2,T(2)}\right) = \theta_{j0} \\ \dfrac{\sum_{r=1}^{s} a_r\left(y_{rj0}+\Delta y_{rj0}\right)+\sum_{r'=1}^{s'} u_r'\left(y_{r'j0}'+\Delta y_{r'j0}'\right)}{\sum_{i=1}^{m} b_i\left(x_{ij0}+\Delta x_{ij0}\right)+\sum_{d'=1}^{D'} \omega_{d'}'\left(z_{d'j0}'+\Delta z_{d'j0}'\right)} = \theta_{j0} \le 1 \\ \dfrac{\sum_{r=1}^{s} a_r\left(y_{rj}\right)+\sum_{r'=1}^{s'} u_r'\left(y_{r'j}'\right)}{\sum_{i=1}^{m} b_i\left(x_{ij}\right)+\sum_{d'=1}^{D'} \omega_{d'}'\left(z_{d'j}'\right)} \le 1, j=1,\dots,n \\ \theta_{j0}^{1} = \dfrac{\sum_{d=1}^{D} \omega_d\left(z_{dj0}+\Delta z_{dj0}\right)+\sum_{r'=1}^{s'} u_r'\left(y_{r'j0}'+\Delta y_{r'j0}'\right)}{\sum_{i=1}^{m} v_i\left(x_{ij0}+\Delta x_{ij0}\right)} \le 1 \\ \dfrac{\sum_{d=1}^{D} \omega_d\left(z_{dj}\right)+\sum_{r'=1}^{s'} u_r'\left(y_{r'j}'\right)}{\sum_{i=1}^{m} v_i\left(x_{ij}\right)} \le 1, j=1,\dots,n \\ \theta_{j0}^{2} = \dfrac{\sum_{r=1}^{s} u_r\left(y_{rj0}+\Delta y_{rj0}\right)}{\sum_{d=1}^{D} \widetilde{\omega}_d\left(z_{dj0}+\Delta z_{dj0}\right)+\sum_{d'=1}^{D'} \omega_{d'}'\left(z_{d'j0}'+\Delta z_{d'j0}'\right)} \le 1 \\ \dfrac{\sum_{r=1}^{s} u_r\left(y_{rj}\right)}{\sum_{d=1}^{D} \widetilde{\omega}_d\left(z_{dj}\right)+\sum_{d'=1}^{D'} \omega_{d'}'\left(z_{d'j}'\right)} \le 1, j=1,\dots,n \\ m_{j0,1,T(2)}/\left(m_{j0,1,T(2)}+m_{j0,2,T(2)}\right) = \theta_{j0}^{\alpha*} \\ \left\{\left(x_{j0}+\Delta x_{j0}, z'_{j0}+\Delta z'_{j0}\right), \left(y_{j0}+\Delta y_{j0}, y'_{j0}+\Delta y'_{j0}\right)\right\} \in PPS_{Overall} \\ \left\{\left(x_{j0}+\Delta x_{j0}\right), \left(z_{j0}+\Delta z_{j0}, y'_{j0}+\Delta y'_{j0}\right)\right\} \in PPS_{Stage1} \\ \left\{\left(z_{j0}+\Delta z_{j0}, z'_{j0}+\Delta z'_{j0}\right), \left(y_{j0}+\Delta y_{j0}\right)\right\} \in PPS_{Stage2} \\ 1-w_{1E} \le t_1;\ 1-w_{2E} \le t_1; 1-r_{1E} \le t_1;\ 1-r_{2E} \le t_1; 1-w_{1N} \le t_2;\ 1-w_{2N} \le t_2; 1-r_{1N} \le t_2;\ 1-r_{2N} \le t_2; \\ y_{rj0}+\Delta y_{rj0} \ge 0; y_{r'j0}'+\Delta y_{r'j0}' \ge 0; x_{ij0}+\Delta x_{ij0} \ge 0; z_{d'j0}'+\Delta z_{d'j0}' \ge 0; z_{dj0}+\Delta z_{dj0} \ge 0; x_{ij0}+\Delta x_{ij0} \ge 0 \\ w_{1E} = r_{1E}, w_{2E} = r_{2E}, w_{1N} = r_{1N}, w_{2N} = r_{2N} \\ 0 < w_{1E}, w_{1N}, w_{2E}, w_{2N} < 1; 0 < r_{1E}, r_{1N}, r_{2E}, r_{2N} < 1; \Delta x_{ij}, \Delta y_{rj}', \Delta y_{rj}, \Delta z_{dj}', \Delta z_{dj}\ free \\ \omega_d, u_r', v_i, u_r, \widetilde{\omega}_d, b_i, a_r, t_1, t_2 \ge 0 \end{bmatrix}$$

Therefore, we have the following multi-stage preemptive nonlinear minimax reference point model for performance improvement.

$$\begin{gathered} max = P_1\theta_{j0} + P_2\left(\theta_{j0}^1+\theta_{j0}^2\right) - P_3\left(\sum_{i=1}^{m}\frac{|\Delta x_{ij0}|}{x_{ij0}} + \sum_{d=1}^{D}\frac{|\Delta z_{dj0}|}{z_{dj0}} + \sum_{d'=1}^{D'}\frac{|\Delta z_{d'j0}'|}{z_{d'j0}'} + \sum_{r'=1}^{s'}\frac{|\Delta y_{r'j0}'|}{y_{r'j0}'} + \sum_{r=1}^{s}\frac{|\Delta y_{rj0}|}{y_{rj0}}\right) - P_4(t_1+t_2) \\ s.t.\left(\theta_{j0}, \theta_{j0}^1, \theta_{j0}^2, t_1, t_2, w_1, w_2, r_1, r_2, \Delta x_{ij}, \Delta y_{rj}', \Delta y_{rj}, \Delta z_{dj}', \Delta z_{dj}, \omega_d, u_r', v_i, u_r, \widetilde{\omega}_d, b_i, a_r\right) \in \Omega \end{gathered} \tag{25}$$

where P_1, P_2, P_3, and P_4 denote four priority objectives in a lexicographical order from high to low.

The above model (25) can be solved in a multi-stage way. The first stage is given by:

$$\begin{gathered} max\left(\theta_{j0}\right) \\ s.t.\left(\theta_{j0}, \theta_{j0}^1, \theta_{j0}^2, t_1, t_2, w_1, w_2, \Delta x_{ij}, \Delta y_{rj}', \Delta y_{rj}, \Delta z_{dj}', \Delta z_{dj}, \omega_d, u_r', v_i, u_r, \widetilde{\omega}_d, b_i, a_r\right) \in \Omega \end{gathered} \tag{25a}$$

The second stage of model (25) is given by:

$$\begin{gathered} max(\theta_{j0}^{1} + \theta_{j0}^{2}) \\ s.t.\left(\theta_{j0}^{*}, \theta_{j0}^{1}, \theta_{j0}^{2}, t_1, t_2, w_1, w_2, \Delta x_{ij}, \Delta y'_{rj}, \Delta y_{rj}, \Delta z'_{dj}, \Delta z_{dj}, \omega_d, u'_r, v_i, u_r, \widetilde{\omega}_d, b_i, a_r\right) \in \Omega \end{gathered} \tag{25b}$$

where θ_{j0}^{*} isthe optimal objective function value of the first stage model (25a).

The third stage of model (25) is given by:

$$\begin{gathered} min\left(\left(\sum_{i=1}^{m}\frac{|\Delta x_{ij0}|}{x_{ij0}} + \sum_{d=1}^{D}\frac{|\Delta z_{dj0}|}{z_{dj0}} + \sum_{d'=1}^{D'}\frac{|\Delta z'_{d'j0}|}{z'_{d'j0}} + \sum_{r'=1}^{s'}\frac{|\Delta y'_{r'j0}|}{y'_{r'j0}} + \sum_{r=1}^{s}\frac{|\Delta y_{rj0}|}{y_{rj0}}\right) + t\right) \\ s.t.\left(\theta_{j0}^{*}, \theta_{j0}^{1*}, \theta_{j0}^{2*}, t_1, t_2, w_1, w_2, r_1, r_2, \Delta x_{ij}, \Delta y'_{rj}, \Delta y_{rj}, \Delta z'_{dj}, \Delta z_{dj}, \omega_d, u'_r, v_i, u_r, \widetilde{\omega}_d, b_i, a_r\right) \in \Omega \end{gathered} \tag{25c}$$

where θ_{j0}^{1*} and θ_{j0}^{2*} are optimal objective function value of the second stage model (25b). The third stage model (25c) can be easily transformed into the nonlinear programming model (25e) using the deviation variables in formulae (25d) to tackle the absolute sign in the objective function.

$$\Omega' = \begin{cases} dx_{ij0}^{+} = \frac{1}{2}\{|\Delta x_{ij0}| - \Delta x_{ij0}\}, i = 1, \dots, m \\ dx_{ij0}^{-} = \frac{1}{2}\{|\Delta x_{ij0}| + \Delta x_{ij0}\}, i = 1, \dots, m \\ dz_{dj0}^{+} = \frac{1}{2}\{|\Delta z_{dj0}| - \Delta z_{dj0}\}, d = 1, \dots, D \\ dz_{dj0}^{-} = \frac{1}{2}\{|\Delta z_{dj0}| + \Delta z_{dj0}\}, d = 1, \dots, D \\ dz'^{+}_{d'j0} = \frac{1}{2}\{|\Delta z'_{d'j0}| - \Delta z'_{d'j0}\}, d' = 1, \dots, D' \\ dz'^{-}_{d'j0} = \frac{1}{2}\{|\Delta z'_{d'j0}| + \Delta z'_{d'j0}\}, d' = 1, \dots, D' \\ dy'^{+}_{r'j0} = \frac{1}{2}\{|\Delta y'_{r'j0}| - \Delta y'_{r'j0}\}, r' = 1, \dots, s' \\ dy'^{-}_{r'j0} = \frac{1}{2}\{|\Delta y'_{r'j0}| + \Delta y'_{r'j0}\}, r' = 1, \dots, s' \\ dy_{rj0}^{+} = \frac{1}{2}\{|\Delta y_{rj0}| - \Delta y_{rj0}\}, r = 1, \dots, s \\ dy_{rj0}^{-} = \frac{1}{2}\{|\Delta y_{rj0}| + \Delta y_{rj0}\}, r = 1, \dots, s \end{cases} \tag{25d}$$

Based on formula (25d), we have the following model (25e) as the third stage of model (25):

$$\begin{gathered} min\left(\begin{gathered}\sum_{i=1}^{m}\frac{dx_{ij0}^{+} + dx_{ij0}^{-}}{x_{ij0}} + \sum_{d=1}^{D}\frac{dz_{dj0}^{+} + dz_{dj0}^{-}}{z_{dj0}} + \sum_{d'=1}^{D'}\frac{dz'^{+}_{d'j0} + dz'^{-}_{d'j0}}{z'_{d'j0}} \\ + \sum_{r'=1}^{s'}\frac{dy'^{+}_{r'j0} + dy'^{-}_{r'j0}}{y'_{r'j0}} + \sum_{r=1}^{s}\frac{dy_{rj0}^{+} + dy_{rj0}^{-}}{y_{rj0}}\end{gathered}\right) \\ s.t.\begin{cases}\left(\theta_{j0}^{*}, \theta_{j0}^{1*}, \theta_{j0}^{2*}, t_1, t_2, w_1, w_2, r_1, r_2, \Delta x_{ij}, \Delta y'_{rj}, \Delta y_{rj}, \Delta z'_{dj}, \Delta z_{dj}, \omega_d, u'_r, v_i, u_r, \widetilde{\omega}_d, b_i, a_r\right) \in \Omega \\ \left(dx_{ij0}^{+}, dx_{ij0}^{-}, dz_{dj0}^{+}, dz_{dj0}^{-}, dz'^{+}_{d'j0}, dz'^{-}_{d'j0}, y'^{+}_{r'j0}, dy'^{-}_{r'j0}, dy_{rj0}^{+}, dy_{rj0}^{-}\right) \in \Omega'\end{cases} \end{gathered} \tag{25e}$$

After the optimal solutions for the first three stages for model (25) are generated, the following model (25f) for the fourth stage is solved to obtain the final optimal solutions to find a unique optimal solution for two-stage performance improvement:

$$\begin{gathered} min(t_1 + t_2) \\ s.t.\begin{cases}\left(\theta_{j0}^{*}, \theta_{j0}^{1*}, \theta_{j0}^{2*}, t_1, t_2, w_1, w_2, r_1, r_2, \Delta x_{ij}, \Delta y'_{rj}, \Delta y_{rj}, \Delta z'_{dj}, \Delta z_{dj}, \omega_d, u'_r, v_i, u_r, \widetilde{\omega}_d, b_i, a_r\right) \in \Omega \\ \left(dx_{ij0}^{+*}, dx_{ij0}^{-*}, dz_{dj0}^{+*}, dz_{dj0}^{-*}, dz'^{+*}_{d'j0}, dz'^{-*}_{d'j0}, y'^{+*}_{r'j0}, dy'^{-*}_{r'j0}, dy_{rj0}^{+*}, dy_{rj0}^{-*}\right) \in \Omega'\end{cases} \end{gathered} \tag{25f}$$

where $\left(dx_{ij0}^{+*}, dx_{ij0}^{-*}, dz_{dj0}^{+*}, dz_{dj0}^{-*}, dz'^{+*}_{d'j0}, dz'^{-*}_{d'j0}, y'^{+*}_{r'j0}, dy'^{-*}_{r'j0}, dy^{+*}_{rj0}, dy^{-*}_{rj0}\right)$are optimal solutions from the third stage model (25e).

Based on model (25) or models (25a)-(25f), the optimal solution to improve the performance of the whole process and two sub-stages is generated.

4. Empirical examples

In this section one numerical example and one case study regarding combining efficiencies from two stages in two-stage network DEA model using the ER rule are examined. First the example given in Kao and Huang (2008) is revisited to illustrate how to use the ER rule to obtain the aggregated overall efficiency with the optimal weights for the two stages efficiencies, so that it is equal to the overall efficiency generated from a "black box" DEA model. Second, we will use the ER rule to combine the efficiencies in a case study on the performance evaluation and improvement in a two-stage process of steel manufacturing.

4.1 An illustrative example

Kao and Huang (2008) proposed the two-stage DEA modelsand analysed them for a dataset of 24 non-life insurance companies in Taiwan. The indicators used in Kao and Huang (2008) are shown in Table A-1 in Appendix A. The data of the 24 non-life insurance companies is listed in Table A-2.

Based on the data in Table A-2, the separated CCR efficiencies of stage 1 and stage 2 are first calculated respectively, as shown in column 3 and 4 of Table 3. Second we calculate the CCR efficiency of each company as a "black box" and list them in column 5 of Table 1. The overall efficiency of the corresponding two-stage DEA models is then generated using the multiplicative decomposition and additive decomposition approaches respectively, which are listed in columns 6 and 7 of Table 1. The minimax model (20) is used to generate a set of unique weights and reliabilities. The results are shown in Columns 8-12 of Table 1. It is worth noting that w_1 is equal to w_2 for all 24 DMUs and the reliabilities of two pieces of evidence (r_1 and r_2) are both unity. Thus we can see we use the ER rule to build the bridge between the whole process and two sub-stages, which satisfies the intuition of the proper relationship between the whole process and two components.

Table 1. CCR results and the aggregation using the ER rule[2].

No.	Company	Stage1-Separated	Stage2-Separated	Overall efficiency	Multiplicative efficiency	Additive efficiency	The ER decomposition and the weights for two stages				
							Overall efficiency	w_{1E}	w_{1N}	w_{2E}	w_{2N}

[2]Note: The weights and reliabilities for two stages in this paper can't be zero or unity precisely due to the constraints in Step 2 in subsection 3.3. We use zero or unity as the replacements of a very small positive real number or a real number very close to unity due to the computational accuracy. Similarly hereafter.

1	Taiwan Fire	0.993	0.713	0.984	0.708	0.849	0.984	1	1	0.967	1
2	Chung Kuo	0.998	0.627	1.000	0.626	0.812	1.000	1	1	0	0
3	Tai Ping	0.690	1.000	0.988	0.690	0.817	0.988	1	1	1	0.973
4	China Mariners	0.724	0.432	0.488	0.313	0.596	0.488	1	1	0.583	1
5	Fubon	0.838	1.000	1.000	0.838	0.873	1.000	1	1	1	1
6	Zurich	0.964	0.406	0.594	0.391	0.689	0.594	1	1	0.570	1
7	Taian	0.752	0.538	0.470	0.405	0.580	0.470	0.084	0.084	1	1
8	Ming Tai	0.726	0.511	0.415	0.371	0.579	0.415	0.082	0.082	1	1
9	Central	1.000	0.292	0.327	0.292	0.612	0.327	1	1	0.151	1
10	The First	0.862	0.674	0.781	0.581	0.713	0.781	1	1	0.580	1
11	Kuo Hua	0.741	0.327	0.283	0.242	0.509	0.283	1	1	0.119	1
12	Union	1.000	0.760	1.000	0.760	0.880	1.000	1	1	1	1
13	Shingkong	0.811	0.543	0.353	0.440	0.557	0.353	0.036	0.036	1	1
14	South China	0.725	0.518	0.470	0.376	0.577	0.470	1	1	0.167	1
15	Cathay Century	1.000	0.705	0.979	0.705	0.807	0.979	1	1	0.949	1
16	Alianz President	0.907	0.385	0.472	0.349	0.639	0.472	1	1	0.402	1
17	Newa	0.723	1.000	0.635	0.723	0.613	0.635	0	0	1	1
18	AIU	0.794	0.374	0.427	0.297	0.587	0.427	1	1	0.458	1
19	North America	1.000	0.416	0.822	0.416	0.706	0.822	1	1	0.846	1
20	Federal	0.933	0.901	0.935	0.841	0.765	0.935	1	1	0.439	1
21	Royal &Sunalliance	0.751	0.280	0.333	0.210	0.541	0.333	1	1	0.553	1
22	Asia	0.590	1.000	1.000	0.590	0.742	1.000	1	1	1	1
23	AXA	0.851	0.560	0.599	0.477	0.685	0.599	1	1	0.323	1
24	Mitsui Sumitomo	1.000	0.335	0.257	0.335	0.544	0.257	0	0	1	1

In the following Table 2 the existing four approaches and the proposed ER decomposition approach are compared for overall efficiency and decompositions for two DMUs: DMU_3 and DMU_{17}, which are selected without loss of generality to save the space. The features of those five approaches are also shown in this table. We find that only the ER decomposition approach satisfies the proposed Conditions 1-3.

Table 2. Comparison of five approaches for overall efficiency and decompositions.

Approaches	Efficiencies	DMU_3 (Tai Ping)	DMU_{17} (Newa)
Separated models	Efficiency in Stage 1 (weight)	0.690 (N/A)	0.723 (N/A)
	Efficiency in Stage 2 (weight)	1 (N/A)	1 (N/A)
	Overall efficiency	0.988	0.635
Multiplicative decomposition (Kao	Efficiency in Stage 1 (weight)	0.690 (N/A)	0.574 (N/A)
	Efficiency in Stage 2 (weight)	1 (N/A)	0.628 (N/A)

& Hwang 2008)	Overall efficiency	0.690	0.360
Additive decomposition (Chen *et al.* 2009)	Efficiency in Stage 1 (weight)	0.690 (0.592)	0.723 (0.580)
	Efficiency in Stage 2 (weight)	1 (0.408)	0.460 (0.420)
	Overall efficiency	0.817	0.613
Game decomposition[3] (Liang *et al.* 2006)	Efficiency in Stage 1 (weight)	0.690 (N/A)	0.628 (N/A)
	Efficiency in Stage 2 (weight)	1 (N/A)	0.574 (N/A)
	Overall efficiency	0.690	0.360
The ER decomposition	Efficiency in Stage 1 (weight)	0.690 (0.539)	0.723 (0.385)
	Efficiency in Stage 2 (weight)	1 (0.539)	1 (0.385)
	Overall efficiency	0.988	0.635

Note: In this table (N/A) denotes there is no weight for the efficiency.

Next we use models (25a)-(25f) to obtain the possible performance improvement through the adjustments in the two-stages as shown in Table 3. DMU_3 (Tai Ping) and DMU_{17} (Newa) are taken for example. In Table 3, the overall efficiency of DMU_3 will be improved to unity if ①the second input Insurance expenses (X2) is increased by 18490.25 (1.57%), ② the first intermediate Direct written premiums (Z1) is increased by 2992819 (504.87%), ③ the second intermediate Reinsurance premiums (Z2) is decreased by -255585 (-5.35%), and ④the second final output Investment profit (Y2) is increased by 23447.62 (7.99%). By doing so, the efficiencies of the overall process and the first and the second stages of DMU_3all reach 1, which are 0.988, 1, and 0.690before adjustment, respectively. For DMU_{17}, the maximal overall efficiency after adjustment is also unity, which can be achieved by decreasing the first input Operations expenses (X1) by -6509.422 (-0.45%), decreasing the second input Insurance expenses (X2) by –557971.2 (-51.43%), and increasing the second intermediate Reinsurance premiums (Z2) by 12694.9 (3.71%). After the above adjustments, the performance of DMU_3 (Tai Ping) and DMU_{17} (Newa) achieve the status that the overall efficiency and the efficiencies of the two stages are all unity. Furthermore, the above improvement strategy is generated on the basis of minimizing relative changes for all inputs and outputs, as shown in Table 3.

Table 3. Resource planning for performance improvement.

DMUs	DMU_3	DMU_{17}
Operations expenses (X1)	0	6509.422
Insurance expenses (X2)	18490.25	-557971.2
Direct written premiums (Z1)	2992819	0
Reinsurance premiums (Z2)	-255585	12694.9
Underwriting profit (Y1)	0	0
Investment profit (Y2)	23447.62	0
Overall efficiency	1	1
Efficiency of stage 1	1	1
Efficiency of stage 2	1	1
t_1(Equalizer for weights of H1 (Efficient))	0	1

[3]Note: Here we list the results of centralized model used in Liang *et al.* (2006).

t_2(Equalizer for weights of H2 (Not Efficient))	0.0270	0
w_{1E}(Weight of H1 (Efficient) of stage 1)	1	0
w_{2E}(Weight of H1 (Efficient) of stage 2)	1	0
w_{1N}(Weight of H2(Not Efficient) of stage 1)	1	1
w_{2N}(Weight of H2 (Not Efficient) of stage 2)	0.9730	1
r_{1E}(Weight of H1 (Efficient) of stage 1)	1	0
r_{2E}(Weight of H1 (Efficient) of stage 2)	1	0
r_{1N}(Weight of H2(Not Efficient) of stage 1)	1	1
r_{2N}(Weight of H2 (Not Efficient) of stage 2)	0.9730	1

In order to illustrate each DMU's efficiencies in two stages intuitively so that we can show the gap between the current status of inefficient DMU or two stages and the efficient frontiers, we use the DEA-IMRP (DEA-oriented interactive minimax reference point) approach proposed by Yang *et al.* (2012) and Yang and Xu (2014a) to project each DMU from decision space onto objective space. Yang *et al.* (2012) and Yang and Xu (2014a) proved the equivalence between the DEA models and the multiple objective optimisation (MOO) models with ideal and super ideal points. The formulations of DEA-IMRP approach is given in Appendix B. In this example we take DMU_3 (Tai Ping) to illustrate its position in the objective space.

For DMU_3, The data envelopes of DMU_3 for the overall process, stage 1 and stage 2 are generated using the DEA-IMRP approach, as shown in Figs.4~6.

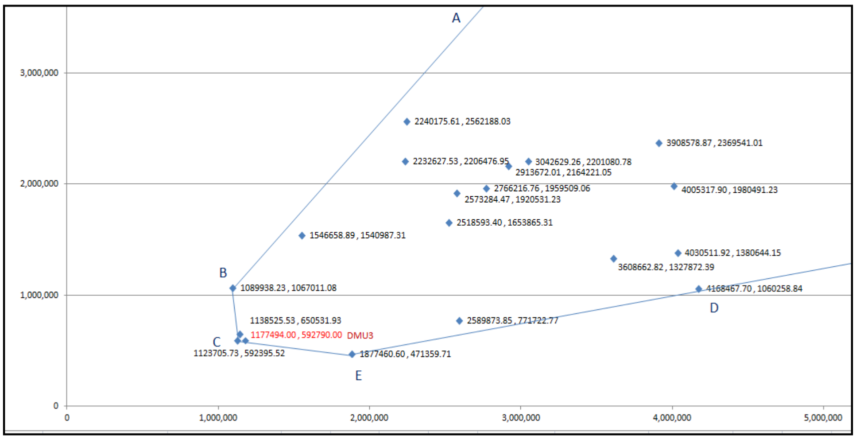

Figure 4. Objective space for DMU_3 of overall process.

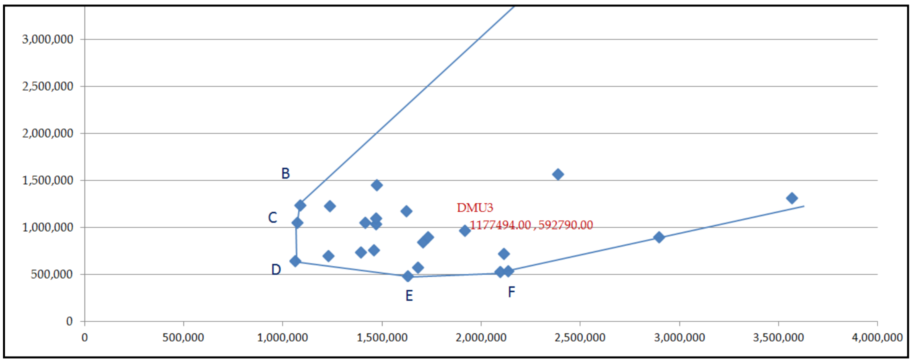

Figure 5. Objective space for DMU_3 of stage 1.

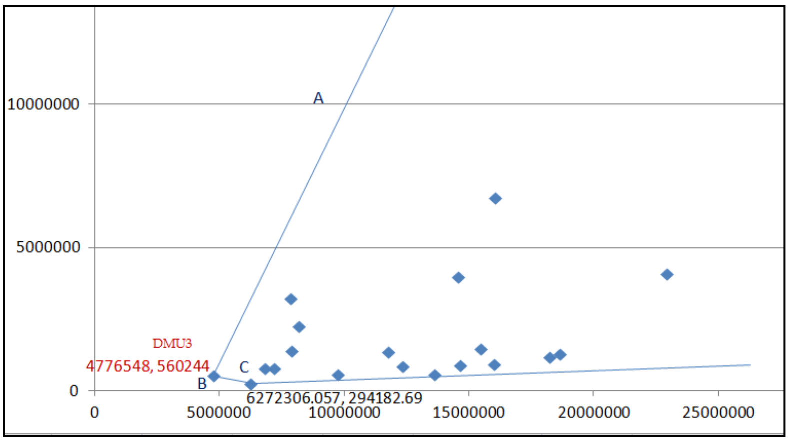

Figure 6. Objective space for DMU_3 of stage 2.

Figs. 4 to 6 show the initial positions of the overall process and sub-stages of DMU_3 in the objective space before performance adjustment. In Figs 4~6, it is clear that DMU_3 need to be improved through resource adjustment.

As shown in Figs. 7-9, after the performance improvement, the positions of DMU_3 are moved onto the efficient frontiers of DMU_3 for the overall process and two sub-stages.

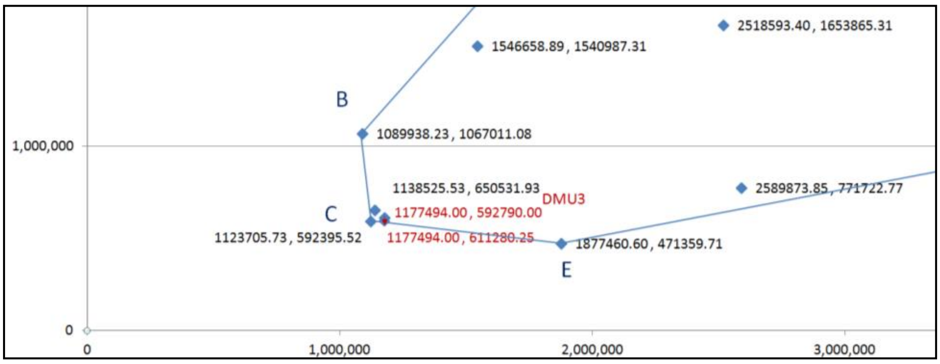

Figure 7. Objective space for DMU_3 of overall process after adjustment.

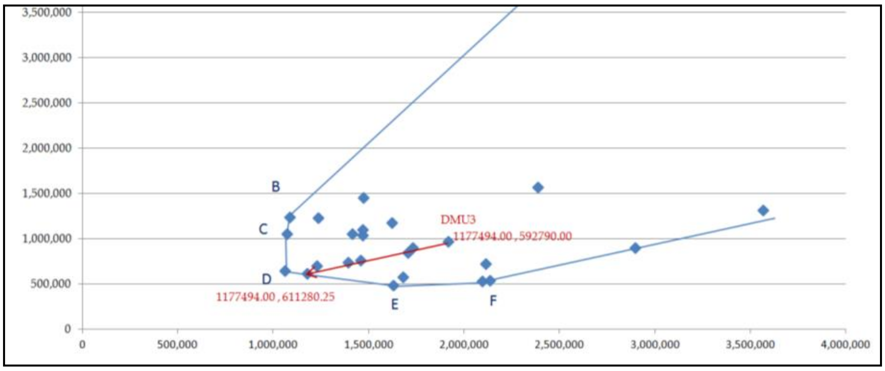

Figure 8. Objective space for DMU_3 of stage 1 after adjustment.

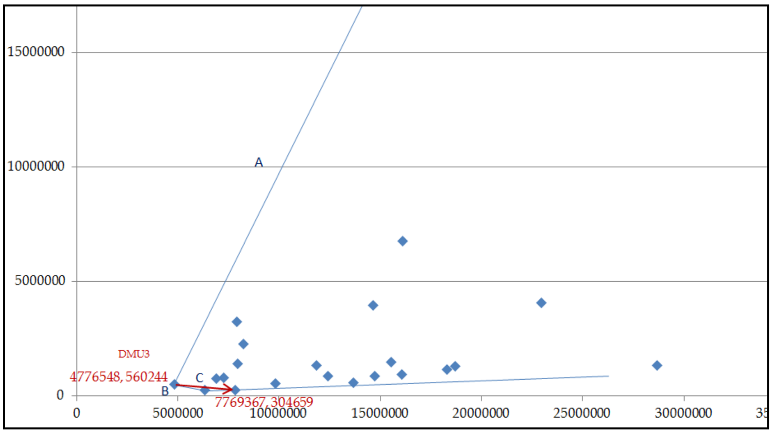

Figure 9. Objective space for DMU_3 of stage 2 after adjustment.

The red arrows in Figs.7 to 9 illustrate the track of the related changes in the overall process and two sub-stages respectively. It should be noted that the efficient frontiers of the overall process and two sub-stages are fixed in the process of performance improvement. Similarly the performance improvement solutions for other DMUs, are analysed, as shown in Appendix C.

After the performance improvement, all DMUs (including overall process and two sub-stages) are projected onto the frontiers respectively in the sense of minimizing the costs of organizational adjustment as defined by Assumption 5. It should be noted that the frontiers of the overall process and two sub-stages remain fixed in the process of performance adjustment.

4.2 A case study

In this subsection a two-stage short process of steel manufacturing is first used as a case to illustrate the applicability of the proposed models (19) and (20) for overall efficiency and its decomposition of two-stage network DEA model. Second, we further explore how to improve the performance for both the overall process and two sub-stages. This example is partially based on the analysis conducted by Zhang *et al.* (2014). The steel manufacturing process normally consists of smelting process and rolling process. In the smelting process the inputs include scrap steel, ferroalloys, the actual smelting time, and electricity. The corresponding output of smelting process is mainly billet. In the second rolling process, the billet produced from the first process should be considered as one of the inputs. The other two inputs for the rolling process are electricity and gas. There is a single output in the rolling process which is the qualified steel. This two-stage network structure can be illustrated as follows:

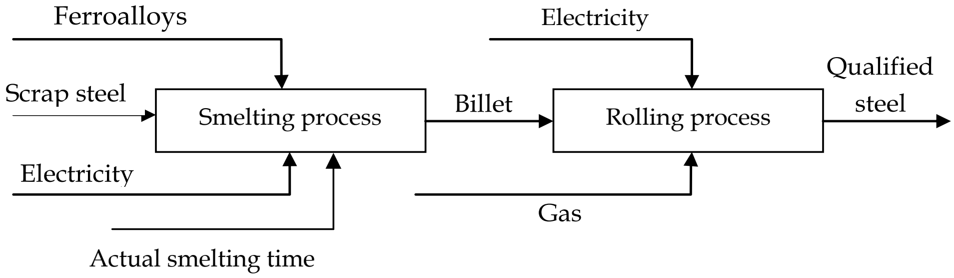

Figure 10. Two-stage network steel manufacturing process.

In this case study the input/output indicators as shown in Table 4 are used.

Table 4. Input and output indicators.

Stages	Inputs		Outputs	
	Variables	Units	Variables	Units
Stage 1: Smelting process	Costs of steel material	10 thousand RMB	Product of billet	Ton
	Costs of ferroalloys	10 thousand RMB		
	Electricity	KWH		
	Actual smelting time	Minutes		
Stage 2: Rolling process	Product of billet	Ton	Product of qualified steel	Ton
	Electricity	KWH		
	Gas	m^3		

A dataset obtained from a steel company in China in 2009 is used for this analysis. There are 15 groups of running data for producing steel product with the same type and specification. The inputs and outputs for the smelting process are as follows:

Table 5. Inputs and outputs of smelting process.

No.	Costs of steel material	Costs of ferroalloys	Electricity	Actual smelting time	Product of billet
1	179.1	16.8	358804	1280	705
2	143.4	13.5	301578	1312	567
3	218.4	19.5	384972	1287	822
4	185.9	18.6	345168	1400	765
5	180.3	17.3	401256	1339	715
6	131.3	12.2	325634	1202	536
7	121.0	12.1	241539	968	450
8	150.2	15.3	225638	1026	598
9	155.3	16.2	296578	1123	576
10	204.2	20.2	423156	1398	758
11	126.2	15.6	262539	1296	562
12	121.8	12.3	225649	1085	519
13	172.6	20.5	425689	1406	760
14	140.4	13.6	250136	1156	628

15	160.4	15.6	312456	1239	758

In Table 5 there are 4 inputs and a single output in the smelting process. The inputs and outputs for the rolling process are shown in Table 6. 3 inputs and a single output are used in the second stage of the rolling process. It should be noted that there is one shared input Electricity for the two sub-stages.

Table 6. Inputs and outputs of rolling process.

No.	Product of billet	Gas	Electricity	Product of qualified steel
1	705	294690	53580	634
2	567	217006	41023	525
3	822	323596	62172	750
4	765	309770	56140	698
5	715	308870	55340	633
6	536	234048	4073	492
7	450	178100	33200	415
8	598	249964	44448	558
9	576	238768	42776	532
10	758	316844	56608	693
11	562	224916	41712	508
12	519	212942	39944	465
13	760	327680	58760	674
14	628	242504	49728	579
15	758	306844	59608	684

Based on the above dataset for the two-stage process of steel manufacturing, the following results are generated for the overall efficiency and the efficiencies for the two stages respectively using the ER decomposition approach. It should be noted that w_1 is equal to w_2 for all 15 DMUs and the reliabilities of two pieces of evidence (r_1 and r_2) are both unity. See Table 7 for details.

Table 7. Assessment results for two-stage short process of steel manufacturing.

No.	Efficiency in model (17) (Stage 1- smelting process)	Efficiency in model (18)(Stage 2 - rolling process)	The ER decomposition and the weights and reliabilities for two stages				
			Efficiency in model (16) (Overall process)	w_{1E}	w_{1N}	w_{2E}	w_{2N}
1	0.892	0.964	0.921	1	1	1	0. 329
2	0.864	1	1	1	1	1	1
3	1	0.983	1	1	1	1	1
4	0.901	0.981	0.955	1	1	1	0. 591
5	0.868	0.949	0.884	1	1	1	0. 191

6	0.904	1	0.927	1	1	1	0. 258
7	0.787	0.993	0.969	1	1	1	0. 889
8	1	1	1	1	1	1	1
9	0.83	0.991	0.935	1	1	1	0. 670
10	0.87	0.980	0.928	1	1	1	0. 501
11	0.942	0.972	0.966	1	1	1	0. 457
12	0.93	0.962	0.922	0. 017	0. 017	1	1
13	0.932	0.950	0.918	0. 020	0. 020	1	1
14	1	0.995	1	1	1	1	1
15	1	0.970	1	1	1	1	1

In this case we take DMU_1 as example to further investigate how to improve its performance in terms of the two stages and the overall process. Models (25a)-(25f) are used to generate performance improvement results for DMU_1, as shown in Table 8. In order to improve the efficiencies of both the overall process and the sub-stages together, DMU_1 needs to

(a) Decrease the Actual smelting time by -165.7265 (unit: minutes), and
(b) Increase the Product of qualified steel by 7.7243 (unit: Ton).

After the adjustment, the weights and reliabilities of two pieces of evidence are shown in Table 8. The efficiencies of DMU_1 for the overall process and two sub-stages are all improved to unity.

Table 8. Resource planning for performance improvement for DMU_1.

DMUs	DMU_1
Costs of steel material	0.000
Costs of ferroalloys	0.000
Electricity (stage 1)	0.000
Actual smelting time	-165.727
Product of billet	0.000
Electricity (stage 2)	0.000
Gas	0.000
Product of qualified steel	7.724
Overall efficiency	1.000
Efficiency of stage 1	1.000
Efficiency of stage 2	1.000
t_1(Equalizer for weights of H1 (Efficient))	0.000
t_2(Equalizer for weights of H2 (Not Efficient))	0.671
w_{1E}(Weight of H1 (Efficient) of stage 1)	1.000
w_{2E}(Weight of H1 (Efficient) of stage 2)	1.000
w_{1N}(Weight of H2(Not Efficient) of stage 1)	1.000
w_{2N}(Weight of H2 (Not Efficient) of stage 2)	0.329
r_{1E}(Weight of H1 (Efficient) of stage 1)	1.000
r_{2E}(Weight of H1 (Efficient) of stage 2)	1.000

r_{1N}(Weight of H2(Not Efficient) of stage 1)	1.000
r_{2N}(Weight of H2 (Not Efficient) of stage 2)	0.329

5. Conclusions and discussions

In this paper we proposed a new approach to decompose an overall DEA model into equivalent two-stage models using a nonlinear technique for efficiency combination, i.e. the ER rule. Different from the existing multiplicative decomposition, additive decomposition and game decomposition, this new ER decomposition approach treats DEA efficiency from each stage through statistical perspective and transforms it into a piece of evidence. In this new approach, the aggregated overall efficiency is equivalent to that generated by traditional DEA model treating each DMU as a "black box". Under this equivalence, we proposed anew multi-stage preemptive nonlinear minimax reference point model to investigate how to improve the performance of the overall process through the performance improvement of DMUs at the two stages by minimizing the costs of organizational adjustment. Such a performance improvement strategy is practical in the sense that the performance improvement of an overall process can only come from the performance improvement of its constituent stages.

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Appendix A. Indicators and original dataset of 24 non-life insurance companies in Taiwan.

Table A-1. Inputs, intermediate products and outputs.

Indicators	Type	Definitions
Operation expenses (X1)	Input	Salaries of the employees and various types of costs incurred in daily operation.
Insurance expenses (X2)	Input	Expenses paid to agencies, brokers, and solicitors; and other expenses associated with marketing the service of insurance.
Direct written premiums (Z1)	Intermediate product	Premiums received from insured clients.
Reinsurance premiums (Z2)	Intermediate product	Premiums received from ceding companies.
Underwriting profit (Y1)	Output	Profit earned from the insurance business
Investment profit (Y2)	Output	Profit earned from the investment portfolio

Table A-2. 24 non-life insurance companies in Taiwan.

No.	Company	Operations expenses (X_1)	Insurance expenses (X_2)	Direct written premiums (Z_1)	Reinsurance premiums (Z_2)	Underwriting profit (Y_1)	Investment profit (Y_2)
1	Taiwan Fire	1,178,744	673,512	7,451,757	856,735	984,143	681,687
2	Chung Kuo	1,381,822	1,352,755	10,020,274	1,812,894	1,228,502	834,754
3	Tai Ping	1,177,494	592,790	4,776,548	560,244	293,613	658,428
4	China Mariners	601,302	594,259	3,174,851	371,863	248,709	177,331
5	Fubon	6,699,063	3,531,614	37,392,862	1,753,794	7,851,229	3,925,272
6	Zurich	2,627,707	668,363	9,747,908	952,326	1,713,598	415,058
7	Taian	1,942,833	1,443,100	10,685,457	643,412	2,239,593	439,039
8	Ming Tai	3,789,001	1,873,530	17,267,266	1,134,600	3,899,530	622,868
9	Central	1,567,746	950,432	11,473,162	546,337	1,043,778	264,098
10	The First	1,303,249	1,298,470	8,210,389	504,528	1,697,941	554,806
11	Kuo Hua	1,962,448	672,414	7,222,378	643,178	1,486,014	18,259
12	Union	2,592,790	650,952	9,434,406	1,118,489	1,574,191	909,295
13	Shingkong	2,609,941	1,368,802	13,921,464	811,343	3,609,236	223,047
14	South China	1,396,002	988,888	7,396,396	465,509	1,401,200	332,283
15	Cathay Century	2,184,944	651,063	10,422,297	749,893	3,355,197	555,482
16	Alianz President	1,211,716	415,071	5,606,013	402,881	854,054	197,947
17	Newa	1,453,797	1,085,019	7,695,461	342,489	3,144,484	371,984
18	AIU	757,515	547,997	3,631,484	995,620	692,731	163,927
19	North America	159,422	182,338	1,141,950	483,291	519,121	46,857

20	Federal	145,442	53,518	316,829	131,920	355,624	26,537
21	Royal &Sunalliance	84,171	26,224	225,888	40,542	51,950	6491
22	Asia	15,993	10,502	52,063	14,574	82,141	4181
23	AXA	54,693	28,408	245,910	49,864	0.1	18,980
24	Mitsui Sumitomo	163,297	235,094	476,419	644,816	142,370	16,976

Appendix B. The formulations of DEA-IMRP approach

First, we restate the equivalent MOO formulations of the overall production process, Stage 1, and Stage 2, respectively. For DMU_0, the MOO formulation of the overall process is discussed as follows.

For overall production process:

$$\min f = \begin{bmatrix} f_1(\lambda) = \sum_{j=1}^{24} \lambda_j x_{1j} \\ f_2(\lambda) = \sum_{j=1}^{24} \lambda_j x_{2j} \end{bmatrix}$$
$$s.t. \begin{cases} \lambda = \left[\lambda_j, j = 1, \dots, 24\right]^T \in \Omega_0 \\ \Omega_0 = \left\{ \lambda \left| \begin{array}{c} \sum_{j=1}^{24} \lambda_j y_{1j} \geq y_{10} \\ \sum_{j=1}^{24} \lambda_j y_{2j} \geq y_{20} \\ \lambda_j \geq 0, j = 1,2, \dots, 24 \end{array} \right. \right\} \end{cases} \tag{B-1}$$

where set Ω_0 is the decision space for the observed DMU_0.

For Stage 1:

$$\min f = \begin{bmatrix} f_1(\lambda) = \sum_{j=1}^{24} \lambda_j x_{1j} \\ f_2(\lambda) = \sum_{j=1}^{24} \lambda_j x_{2j} \end{bmatrix}$$
$$s.t. \begin{cases} \lambda = \left[\lambda_j, j = 1, \dots, 24\right]^T \in \Omega_0 \\ \Omega_0 = \left\{ \lambda \left| \begin{array}{c} \sum_{j=1}^{24} \lambda_j z_{1j} \geq z_{10} \\ \sum_{j=1}^{24} \lambda_j z_{2j} \geq z_{20} \\ \lambda_j \geq 0, j = 1,2, \dots, 24 \end{array} \right. \right\} \end{cases} \tag{B-2}$$

where z_{10} and z_{20} denote the intermediate products: Direct written premiums (Z1) and Reinsurance premiums (Z2) of DMU_0, respectively.

For Stage 2:

$$
\min f = \begin{bmatrix} f_1(\lambda) = \sum_{j=1}^{24} \lambda_j z_{1j} \\ f_2(\lambda) = \sum_{j=1}^{24} \lambda_j z_{2j} \end{bmatrix}
$$
$$
s.t. \begin{cases} \lambda = \left[\lambda_j, j = 1, \dots, 24\right]^T \in \Omega_0 \\ \Omega_0 = \left\{ \lambda \left| \begin{matrix} \sum_{j=1}^{24} \lambda_j y_{1j} \geq y_{10} \\ \sum_{j=1}^{24} \lambda_j y_{2j} \geq y_{20} \\ \lambda_j \geq 0, j = 1,2, \dots, 24 \end{matrix} \right. \right\} \end{cases} \tag{B-3}
$$

The interested readers can refer to Yang *et al.* (2012) and Yang and Xu (2014a) for more details.

Appendix C. The solutions for performance improvement for all DMUs in subsection 4.1.

Table C-1.The solutions for performance improvement for all DMUs.

No.	Company	Operations expenses (X1)	Insurance expenses (X2)	Direct written premiums (Z1)	Reinsurance premiums (Z2)	Underwriting profit (Y1)	Investment profit (Y2)	W_{1E}	W_{2E}	W_{1N}	W_{2N}	t_1	t_2
1	Taiwan Fire	0	0	834919.6	-547208.1	0	11083.8	1	1	0.967	1	0	0.033
2	Chung Kuo	42775.19	-601222.1	-3732772	0	0	0	1	1	0	0	0	1
3	Tai Ping	0	18490.25	2992819	-255585	0	23447.62	1	1	1	0.973	0	0.027
4	China Mariners	0	-427325.3	-1506117	0	0	47151.73	1	1	0.583	1	0	0.417
5	Fubon	0	0	7254475	0	0	0	1	1	1	1	0	0
6	Zurich	0	-271476.5	-3394499	-702002.7	0	6663.095	1	1	0.570	1	0	0.430
7	Taian	0	-682083.4	87225.91	0	2976559	0	0.084	0.084	1	1	0.916	0
8	Ming Tai	0	-794873.5	0	-298753.5	3490654	0	0.082	0.082	1	1	0.919	0
9	Central	0	-702565.5	-9306586	41062.47	0	0	1	1	0.151	1	0	0.849
10	The First	0	-668932.6	0	-175670.3	0	125951.6	1	1	0.580	1	0	0.420
11	Kuo Hua	0	-432328.8	-6032171	0	391803.4	0	1	1	0.119	1	0	0.882
12	Union	0	0	986104.8	-712220.1	0	0	1	1	1	1	0	0
13	Shingkong	0	-546143.1	-1100285	0	2825160	0	0.036	0.036	1	1	0.964	0
14	South China	0	-668211.2	-2262962	-262952.6	0	0	1	1	0.167	1	0	0.833
15	Cathay Century	0	0	0	-327042.6	134192.2	0	1	1	0.949	1	0	0.051
16	Alianz President	0	-222100	-3962703	59345.39	0	0	1	1	0.402	1	0	0.598
17	Newa	-6509.422	-557971.2	0	12694.9	0	0	0	0	1	1	1	0
18	AIU	-433947	0	-2274537	192852.3	0	0	1	1	0.458	1	0	0.542
19	North America	-28381.99	-5214.417	-667150	0	0	0	1	1	0.846	1	0	0.154
20	Federal	0	-3475.352	-31247.94	1325.317	0	0	1	1	0.439	1	0	0.561

21	Royal &Sunalliance	0	-17495.81	-165519.2	-18054.03	0	0	1	1	0.553	1	0	0.447
22	Asia	0	0	0	13560.41	0	0	1	1	1	1	0	0
23	AXA	0	-7349.427	-61476.46	0	0	6443.425	1	1	0.323	1	0	0.677
24	Mitsui Sumitomo	-121309.1	0	-316774.8	-487041.7	0	0	0	0	1	1	1	0