Multiple Attribute Design Evaluation of Large Engineering Products Using The Evidential Reasoning Approach

Summary: This paper reports the application of an evidential reasoning approach to design selection of retro-fit options for complex engineering products. The particular selection problem investigated in this paper is initially modelled by means of techno-economic analysis and may be viewed as a multiple attribute decision making (MADM) problem with a hierarchical structure of attributes which may be measured for each design option using numerical values or subjective judgements with uncertainty. In this paper the evaluation model is discussed at first. Techniques for articulating original evaluation data are also explored. The evidential reasoning approach is then summarised which has been developed on the basis of decision theory and the Dempster-Shafer theory in artificial intelligence. A real world design selection problem involving the retro-fitting of a typical short sea ferry is then examined in detail using the approach.

1. Introduction
Evaluations of different design options for a complex engineering product often need to take into account many performance attributes so that the economic and technical aspects of the product can be comprehensively assessed. Such an attribute often represents a general performance index and may be described by and evaluated through its associated contributing attributes. This leads to a certain type of evaluation framework such as a hierarchical structure or even a general network structure. In a relatively simple hierarchical structure, an attribute may be broken down into lower level attributes and attributes at the bottom level are directly measurable by means of either numerical values or subjective judgements with uncertainty.

An evaluation analysis model for design selection of retro-fit options with a hierarchy of attributes was developed by Evans [3], in which a four-level structure of attributes was established, taking account of both commercial and technical measures of performance. The initial evaluations of all the attributes at each retro-fit option are articulated using normalised

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numerical (percentage) values or certain scaling standards. The simple weighting method was then adopted to obtain the overall disutility value of each option. The options are ranked on the basis of the magnitude of their disutility values.

Some of the attributes involved in the model, however, are inherently subjective and as such it may be more natural to evaluate them initially using subjective judgements with uncertainty. Furthermore, the implementation of the simple weighting method requires such assumptions as linearity of utilities, independence of preferences and direct compensation among attributes in the hierarchy [4, 13]. As such assumptions may not be acceptable, it is advisable to be cautious when using this technique to deal with MADM problems.

This paper is devoted to applying the evidential reasoning approach to treat the above design selection problem. This approach has been developed to deal with MADM problems with both quantitative and qualitative attributes where each qualitative attribute can have its own hierarchy of relevant attributes which could be assessed using subjective judgements with complete or incomplete uncertainty [11, 12, 13, 14]. One of the advantages of this approach is that it can deal with incomplete uncertainty in a more natural yet rational way. Since certainty could be viewed as a special case of uncertainty, the application of this approach to the selection of the retro-fit options for this case study would then be based on the transformation of the original data into equivalent subjective statements with complete uncertainty. If the precise numerical values are not available, it is then more natural to articulate subjective judgements with uncertainty as original evaluation data.

The technique for the data transformation is first discussed and the main calculation procedure of the evidential reasoning approach is presented. The application of this approach to the selection of retro-fit options for a typical short sea ferry is demonstrated in the last section using the same data as given by Evans [3]. A hypothetical example based on the above evaluation model is also constructed and investigated. In this example, some of the subjective attributes are assessed by deliberately adopting incomplete uncertain judgements which may be expected from design engineers. The purpose of examining the hypothetical example is to demonstrate the potential of the evidential reasoning approach to deal with complex design decision problems in a more realistic way.

2. Evaluation Model for Ship Retro-Fit Options

2.1 Hierarchical Attribute Structure

The technical problem deals with the examination of three retro-fit options for a short sea roll-on roll-off ferry to enhance its damage stability characteristics (roll-on roll-off is simply called ro-ro which means that vehicles drive on/off ferries). The selection of a retro-fit option demands a clear definition of the necessary attributes and their associated contributing attributes which could influence the ship's operation. These attributes include both the
commercial and the technical aspects of the option. Such attributes are defined and arranged in a hierarchical structure as shown in Figure 1 [3]. These attributes and their relationships are used to convert the commercial aspects of the option into a comparative format.

In Figure 1, the selection of a retro-fit option is based on the assessment of the option over two general attributes (*Ship Operation* and *Installation*) which are broken down into lower level attributes within a hierarchy. These lower level attributes define the general attributes and are easier to evaluate at each option. The attributes at the bottom level (Level 4) of the hierarchy are directly measurable and is referred to as basic attributes. These basic attributes are defined and described by Evans [3], and they may be divided into two types, quantitative ones and qualitative ones, although Evans tried to use numbers to measure all the attributes.

Evans measured most quantitative basic attributes by means of extra cost incurred due to the required modifications for each retro-fit option, such as all the basic attributes associated with the upper level attributes *Installation* and *Running Cost*, though some of those basic attributes could be more naturally assessed in a qualitative manner. Other quantitative basic attributes were measured by using numbers due to their numerical nature. For instance, *Operating Time Factor* was expressed as a percentage of the total increase in time spent in port operating the device; *Loss of Lane Length* as a percentage of a total loss of cargo space; and *Changes to Ventilation* as an increase in ventilation rate or air changes per hour.

Qualitative attributes are those which may not be readily assessed by using exact numbers in the first instance due to their subjective nature, although one often tries to associate numbers with his judgements for simplicity or because of the ignorance of other methods which may be more rational and easier to apply. In his report, Evans expressed the qualitative attributes in various ways. For instance, *Human Error Factor* was expressed as a percentage of the possibility of human error; *Loss of Stowage Flexibility* as an expected loss of stowage flexibility; *Collision Resistance* as a linear scale of 0 to 1, and *Pilot Access* as a change in the accessibility for the pilot to board/leave the ship underway.

The classification of quantitative and qualitative attributes is summarised as in Table 1.

<table>
<thead>
<tr>
<th>Table 1</th>
<th>Classification of Attributes</th>
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<tbody>
<tr>
<td><strong>Quantitative Attributes</strong></td>
<td><strong>Qualitative Attributes</strong></td>
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<tr>
<td>Operating Time Factor</td>
<td>Device Cost</td>
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<tr>
<td>Loss of Lane Length</td>
<td>Fitting Cost</td>
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<tr>
<td>Changes to Ventilation</td>
<td>Out of Service Cost</td>
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<tr>
<td>Increased Berthing Time</td>
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<tr>
<td>Increased Power</td>
<td>Auxiliary Modification</td>
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<tr>
<td>Maintenance</td>
<td>Stabiliser Installation</td>
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<tr>
<td>Modify Linkspan/Quay</td>
<td>Ventilation Modifications</td>
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</tbody>
</table>
In the attribute hierarchy as shown in Figure 1, not all attributes at a single level play the same role in the evaluation of an attribute associated at a level immediately above. For instance, *Ship Operation* is evaluated through *Port Activity* and *Ship Inherent* where *Port Activity* may be relatively more important in the evaluation. The relative importance of the attributes at a single level with regard to an associated attribute at a level immediately above therefore needs to be taken into consideration. In fact, different weights were used to represent relative importance [3], as shown by the numbers in the brackets in Figure 1. It may be noted that these weights are normalised so that the sum of the weights of the attributes at a single level with respect to an associated upper level attribute is one. For instance, the three attributes *Integrity*, *Daily Operation Requirements* and *Running Cost* are associated with the *Ship Inherent* and the sum of their weights is one.

### 2.2 Articulation of Evaluation Data

The influence of each retro-fit option for a ship can initially be assessed on each basic attribute. Such an assessment may be articulated and represented using a subjective statement with uncertainty or using a numerical value, depending upon whether the attribute is inherently qualitative (subjective) or quantitative (mathematically measurable).

With regard to an attribute such as *Collision Resistance*, for example, it may be stated that the modification of a particular retro-fit option for a ship is "Minor" or "Moderate". In the statement, "Minor" and "Moderate" may be referred to as evaluation grades describing the degrees of modification. Such a grade represents a distinct modification state of an attribute for a particular retro-fit option. It is also possible that the degree of modification is something between "Minor" and "Moderate". For instance, it could be judged that the modification of the retro-fit option in terms of *Collision Resistance* is "Minor" to an extent of $\beta_n$ and "Moderate" to an extent of $\beta_{n+1}$ with $\beta_n$, $\beta_{n+1} \geq 0$ and $\beta_n + \beta_{n+1} \leq 1$. Here $\beta_n$ and $\beta_{n+1}$ represent uncertainty, indicating the degrees of confidence (belief) in the evaluation. $\beta_n$ may simply be referred to as a confidence degree. In this paper, $\beta_n + \beta_{n+1} < 1$ or $= 1$ is referred to as incomplete or complete uncertainty, respectively.

Other grades such as "None", "Major" and "Fundamental" may also be used to describe the degrees of modification. Generally, a set of such grades may be employed. Suppose $H_n$ stands for a general grade. A set of evaluation grades may then be defined by

$$H = \{ H_1, \ldots, H_n, \ldots, H_N \}$$

Large modifications for a ship lead to increase of cost and technical complexity. It may therefore be less favourable. The grade "Minor" may thus, in a certain sense, be preferred to
the grade "Major". Suppose $H_{n+1}$ is preferred to $H_n$. A particular set of evaluation grades may then be defined by

$$H = \{ H_1, H_2, H_3, H_4, H_5 \} = \{ \text{Fundamental, Major, Moderate, Minor, None} \}$$  \hspace{1cm} (2)$$

Such grades may be quantified using a scale. Suppose $p(H_n)$ represents the scale of $H_n$. If $p(H_n)$ is set to be a real number in the closed interval $[0, 1]$ which may be referred to as the preference degree space [12, 13], the set of evaluation grades is then quantified by

$$p\{H\} = [ p(H_1) \cdots p(H_n) \cdots p(H_N) ]^T$$  \hspace{1cm} (3)$$

where $p(H_n)$ ($n=1, \ldots, N$) satisfy the following basic conditions

$$p(H_1) = 1, \quad p(H_N) = 0, \quad \text{and} \quad p(H_{n+1}) < p(H_n) \quad n=1, \ldots, N-1$$  \hspace{1cm} (4)$$

and the consistency conditions [12, 13].

If the state of an option on a basic attribute is precisely evaluated to $H_n$, then the state may be quantified by $p(H_n)$. If the state is evaluated to $H_n$ and $H_{n+1}$ to the degrees of $(1-\beta^n)$ and $\beta^n$, respectively, then the state could be quantified by such an expected value as $\beta^n p(H_n) + (1-\beta^n) p(H_{n+1})$ [12, 13] which is referred to as the rejection degree of the option on the given attribute. In this way, a subjective judgement could be quantified on each attribute separately. More complex evaluations can be taken into account where the subsets of $H$ such as $\{H_n, H_{n+1}\}$ could be treated as a single grade as well.

Some attributes are numerically measurable using mathematical means. Such a measure can be transformed through normalisation or more generally utility conversion, so that the value of such an attribute at an option will lie within the closed interval $[0, 1]$. In fact, percentage values are often used for such normalisation. Furthermore, such a normalised value could be transformed to be equivalent to a subjective judgement with complete uncertainty. Suppose $y_{ij}$ is the value of option $i$ on attribute $j$ with $0 \leq y_{ij} \leq 1$. If $p(H_{n+1}) \leq y_{ij} \leq p(H_n)$, then it may be equivalent to state that option $i$ on attribute $j$ is evaluated to $H_n$ and $H_{n+1}$ with the confidence degrees of $(1-\beta)$ and $\beta$, respectively, where $\beta$ is given by

$$\beta = \frac{p(H_n) - y_{ij}}{p(H_n) - p(H_{n+1})}$$  \hspace{1cm} (5)$$
By equivalence it is meant that the value obtained by quantifying the statement is the same as the numerical value $y_{ij}$. Such a transformation is useful when there are both quantitative and qualitative basic attributes in a design selection problem. The transformation could be justified on the ground that certainty may be viewed as a special case of uncertainty.

All basic attributes in Figure 1, whether quantitative or qualitative, are measured using percentage values in the original report so that the simple weighting approach could be adopted for the selection of retro-fit options [3]. In this approach, the overall weights of attributes at a single level are first calculated by assuming an overall additive utility function. In other words, the utility of an attribute at a single level is the weighted sum of the utilities of the attributes associated at a level immediately below.

In Figure 1, for example, the overall weight of Operating Time Factor could be calculated as $0.65 \times 0.6 \times 0.85 \times 0.15 = 0.049725$. Thus, the hierarchical attribute structure can be simplified by a decision matrix, where the two main attributes Ship Operation and Installation are replaced by the 24 basic attributes on which each retro-fit option is evaluated. A MADM method, such as the simple weighting method, may then be used to deal with the selection problem based upon the decision matrix [3].

This simple weighting approach suffers from the demerits such as linearity of utilities, independence of preferences and direct compensation among attributes in the hierarchy which may not be acceptable to decision makers (design engineers). This may lead to irrational evaluations and consequently discourage the use of the approach for hierarchical information synthesis. Furthermore, the approach requires the precise numerical evaluations of inherently qualitative attributes. To deal with these problems, the next section introduces a new hierarchical evaluation approach based on decision theory and the Dempster-Shafer theory. This approach can handle subjective judgements with complete or incomplete uncertainty in a rational way and it does not suffer from the demerits as mentioned above.

3. **Hierarchical Analysis through Evidential Reasoning**

3.1 **Design Evaluation via Evidence Combination**

As indicated in Figure 1, the modification state of an attribute is determined by attributes associated at a level immediately below. If an attribute is only associated with one lower level attribute whose modification state is judged to be absolutely "Minor", then the modification state of the upper level attribute must be "Minor". Generally, an attribute may be associated with several lower level attributes. If the modification states of the lower level attributes are all evaluated to be absolutely "Minor", then the modification state of the upper level attribute associated should certainly be "Minor".

However, such identical evaluations are rarely available in practice. Problems may then arise as to how different evaluations of lower level attributes may be synthesised in a rational
way so as to attain a proper evaluation for the modification of an associated upper level attribute. The problem may be generalised to one of addressing how each retro-fit option could be ultimately assessed and measured based upon the initial evaluations of the option on the basic attributes. The evidential reasoning approach provides an alternative way to deal with such uncertain synthesis problems by means of evidence combination for multiple attributes. This approach can model the narrowing of the hypothesis set with the accumulation of evidence. In other words, the chance that a hypothesis is true will be improved if more pieces of evidence support the hypothesis.

In Figure 1, for example, whether the modification of an attribute at a level immediately above the bottom level would be "Fundamental", "Major", "Moderate", "Minor" or "None" is regarded as a hypothesis. A given judgement about the modification of a basic attribute for an option is viewed as a single piece of evidence. If the modification of the basic attribute is, to certain extent, judged to be "Minor", then the modification of the associated upper level attribute would, to some degree, be "Minor" as well. The evidential reasoning approach provides a systematic way of synthesising such uncertain modification evaluations of basic attributes to produce an evaluation for an associated upper level attribute.

To apply the approach, the mutual exclusiveness and exhaustiveness of all hypotheses have to be satisfied. It is therefore necessary that all the evaluation grades be defined as distinct grades. In other words, if one of the grades is absolutely confirmed, all the other grades must not be confirmed at all; if more than one grade is confirmed simultaneously, the total degree of confidence must be one or smaller than one. In addition to this requirement, the grades must cover all possible grades the designer may need to use to judge degrees of modification.

3.2 Basic Evidential Reasoning Model and Algorithm

An evaluation analysis model is used as a framework in which to conduct multiple attribute analysis and reasoning with uncertain decision knowledge. A basic evaluation analysis model may be constructed as shown in Figure 2, in which only a single level of basic attributes are involved. This model is however a basic element in a general framework for hierarchical attribute analysis.

In Figure 2, $e'_k$ denotes a basic attribute such as Operating Time Factor, which can be directly evaluated for a given option. The set of basic attributes for evaluation of an upper level attribute, denoted by $y_k$, is defined by

$$E_k = \{ e'_k \cdots e'_k \cdots e'_k \}$$

(6)
In Figure 1, if \( y_k \) denotes Cargo Handling, for example, then there will be five basic attributes associated with it, i.e. \( L_k = 5 \). Thus, \( e'_k = \text{Operating Time Factor} \), \( e''_k = \text{Human Error Factor} \), \( e'''_k = \text{Loss of Lane Length} \), \( e''''_k = \text{Loss of Stowage Flexibility} \), and \( e''''''_k = \text{Change to Ventilation} \).

Define \( m^n_{ki} \) as a basic probability assignment to which a basic attribute \( e'_k \) supports a hypothesis that the modification state of an attribute \( y_k \) at an option \( a \) is confirmed to a grade \( H_n \). Suppose \( m \) is a probability function. Then, the basic probability assignment \( m^n_{ki} \) could be denoted using the function \( m \) as follows

\[
m^n_{ki} = m(H_n / e'_k(a))
\] (7)

The above equation is used to show the relationship between the basic probability assignment \( m^n_{ki} \) and other parameters concerned, including evaluation grade \( (H_n) \), basic attribute \( (e'_k) \), and design option \( (a) \). The basic probability assignment \( m^n_{ki} \) can be calculated from the given confidence degree and the normalised relative weight for \( e'_k \).

Similarly, define \( m^n_k \) as an overall probability assignment to which the state of an attribute \( y_k \) at an option \( a \) is confirmed to a grade \( H_n \) by the whole attribute set \( E_k \) associated with \( y_k \). \( m^n_k \) could then be denoted as follows

\[
m^n_k = m(H_n / E_k(a))
\] (8)

The overall probability assignment \( m^n_k \) is obtained by combining all basic probability assignments \( m^n_{ki} \) \( (j=1, \ldots, L_k) \) for all the basic attributes associated with \( y_k \). This combination process is carried out using the evidential reasoning algorithm as discussed later.

Suppose \( \beta^n_k(a_r) \) denotes a confidence degree associated with the state of a basic attribute \( e'_k \) at an option \( a_r \) being evaluated to \( H_n \). Then, an uncertain subjective judgement for evaluation of the state of \( e'_k(a_r) \) is expressed by the following expectation

\[
S(e'_k(a_r)) = \{(\beta^n_k(a_r), H_n) \mid n = 1, \ldots, N; \text{ and } \sum_{n=1}^{N} \beta^n_k(a_r) \leq 1\}
\] (9)

which indicates that the state of \( e'_k \) at a design option \( a_r \) is evaluated to a grade \( H_n \) with a confidence degree of \( \beta^n_k(a_r) \) for \( n=1, \ldots, N \). In the above formula, we assume that the state of a basic attribute \( e'_k \) at \( a_r \) may be evaluated to any individual evaluation grade defined in \( H \). More general uncertain subjective judgements can be handled as well [13]. \( S(e'_k(a_r)) \) is then quantified using its rejection degree, defined as the following expected scale

\[
p_{r,ki} = p(e'_k(a_r)) = \sum_{n=1}^{N} \beta^n_k(a_r) p(H_n)
\] (10)
Thus, the scales $p(H_n) \ (n=1, \ldots, N)$ must be defined so that in addition to the basic conditions defined in the previous section the following consistency condition can be satisfied as well, that is, for two designs $a_r$ and $a_h$

$$S(e^j_r(a_r)) \text{ is preferred to } S(e^j_h(a_h)) \text{ if and only if } p_{r,kj} < p_{h,kj} \quad (11)$$

Suppose $\zeta^j_k$ expresses the relative weight of the attribute $e^j_k$ in evaluation of $y_k$, and $\zeta^j_k$ can be articulated as a uniform weight as follows

$$\zeta_k = [\zeta^1_k \cdots \zeta^l_k \cdots \zeta^L_k]^T, \quad \sum_{j=1}^{L_k} \zeta^j_k = 1, \quad 0 \leq \zeta^j_k \leq 1 \quad (12)$$

$\zeta^j_k$ can be readily obtained using any well-known weight assignment method, such as the eigenvector method [4, 6].

Let $e^j_k$ be the most important attribute in the attribute set $E_k$, called the key attribute, i.e.

$$\zeta^j_k = \max_j \{\zeta^1_k \cdots \zeta^l_k \cdots \zeta^L_k\} \quad (13)$$

Transform the weight $\zeta^j_k$ so that the highest weight becomes one, that is

$$\overline{\zeta}^j_k = \frac{\zeta^j_k}{\zeta^l_k} \quad j = 1, \ldots, L_k \quad (14)$$

Suppose for the key attribute the following relation is true

$$m^j_{k\beta}(a) = \alpha_k \beta^j_{k\beta}(a), \quad 0 < \alpha_k \leq 1 \quad (15)$$

which means that if the modification state of the most important evidence $e^j_k$ associated with the attribute $y_k$ is absolutely confirmed to a grade $H_n$ at an option $a$ (i.e. $\beta^j_{k\beta}(a) = 1$), then the state of $y_k$ at $a$ will be confirmed to the same grade to a degree of $\alpha_k$.

Then, define a normalised weight for the attribute $e^j_k$ as $\lambda^j_k$. $\lambda^j_k$ is obtained by

$$\lambda^j_k = \alpha_k \overline{\zeta}^j_k \quad j = 1, \ldots, L_k \quad (16)$$

which represents the normalised relative importance of the attribute $e^j_k$ in evaluation of $y_k$ where $0 \leq \lambda^j_k \leq 1$. $\lambda^j_k$ will be used to calculate the basic probability assignments.
In the above equations, $\alpha_k$ can be interpreted as a priority coefficient representing the degree of significance for the role of the most importance attribute in the evaluation of the attribute $y_k$. $\alpha_k$ is assigned by satisfying the following relation

$$\prod_{j=1}^{L_k} \left(1 - \alpha_k \frac{\zeta_j}{\zeta_k}\right) \leq \delta, \quad \delta \geq 0$$

(17)

where $\delta$ is a small positive real number. It may be taken so that $1.0 \times 10^{-6} \leq \delta \leq 1.0 \times 10^{-2}$. Such an assignment means that if $H_j$ is absolutely confirmed by all the attributes $E_k$ then the upper level attribute associated will be confirmed to $H_j$ to an extent of $(1-\delta) \times 100$ percent. More details about the assignment of $\delta$ can be found in reference [13].

The basic probability assignment $m_{ij}^n$ is then calculated by

$$m_{ij}^n(a) = \lambda^i_k \beta^a_j(a)$$

(18)

The above equation means that the fact that the state of a basic attribute $e_k^j$ is absolutely evaluated to an evaluation grade $H_n$ only supports to the extent of $\lambda^i_k$ the hypothesis that the state of the associated attribute $y_k$ is confirmed to the same grade $H_n$. It is obvious that the following inequality is true

$$\sum_{n=1}^{N} m_{ij}^n(a) \leq 1$$

(19)

which means that the sum of the basic probability assignments from a basic attribute $e_k^j$ to all individual evaluation grades $H_n$ ($n=1, \ldots, N$) does not exceed one.

Suppose $m_{ij}^u$ denotes the basic probability assignment to the whole set of evaluation grades represented by $H$, which is the remaining belief unassigned after commitment of belief to all individual evaluation grades $H_n$ ($n=1, \ldots, N$), that is,

$$m_{ij}^u(a) = 1 - \sum_{n=1}^{N} m_{ij}^n(a)$$

(20)

Thus, $m_{ij}^u$ reflects the degree of incomplete uncertainty. Such uncertainty is not assigned to individual grades but to the whole set. The evidential reasoning approach works in a way that the degree of incomplete uncertainty will be reduced with the accumulation of evidence.

A basic probability assignment matrix, denoted by $M(y_k / E_k)$, is defined to summarise the basic probability assignments for evaluation of $y_k$ at an option $a$ through the whole set of evidence $E_k$.
The overall probability assignment \( m^\psi_k \) is calculated by combining the above basic probability assignments using the evidential reasoning algorithm. More generally, let \( \psi \) be a subset of evaluation grades, or \( \psi \subseteq H \), and \( m^\psi_k \) an overall probability assignment to which the state of \( y_k \) at \( a_r \) is confirmed to \( \psi \) by the attribute set \( E_k \), or

\[
m^\psi_k(a_r) = m(\psi / E_k(a_r))
\]  

(22)

If \( m^\psi_k \) for all \( \psi \subseteq H \) are generated from \( M(y_k / E_k) \), then the state of \( y_k \) at \( a_r \) is expressed by the following expectation

\[
S(y_k(a_r)) = \{m(\psi / E_k(a_r)), \ \psi\}, \text{ for all } \psi \subseteq H \}
\]  

(23)

The rejection degree of \( y_k(a_r) \) i.e. \( p(y_k(a_r)) \), is used to quantify \( S(y_k(a_r)) \), and is defined as the following expected scale

\[
p_{r,k} = p(y_k(a_r)) = \sum_{\psi \subseteq H} m(\psi / E_k(a_r))p(\psi)
\]  

(24)

where \( p(\psi) \) is the scale of \( \psi \) and is defined as the average of \( p(H_n) \) for all \( H_n \subseteq \psi \). A qualitative attribute quantified by a rejection degree possesses the basic property of its marginal utilities being monotonic. In other words, for two design options, \( a_r \) and \( a_h \), \( S(y_k(a_r)) \) is preferred to \( S(y_k(a_h)) \) if and only if \( p_{r,k} < p_{h,k} \). Such quantification can thus form a rational basis for further decision analysis.

To introduce the evidential reasoning algorithm, define a subset \( e_{l(i)} \) of \( E_k \) and a combined probability assignment \( m^\psi_{l(i)} \) as follows

\[
e_{l(i)} = \{e^i_k \cdots e^i_k\}, \quad 1 \leq i \leq L_k; \quad m^\psi_{l(i)} = m(\psi / e_{l(i)})
\]  

(25)

where \( m(\psi / e_{l(i)}) \) is a combined probability assignment to \( \psi \) confirmed by \( e_{l(i)} \). Then, the evidential reasoning algorithm can be stated as follows

\[
\{H_j\}: \quad m^j_{l(i+1)} = K_{l(i+1)}(m^j_{l(i)}m^j_{k,j+1} + m^j_{l(i)}m^H_{k,j+1} + m^H_{l(i)}m^j_{k,i+1}), \quad j = 1, \ldots, N
\]
\[ \{H\}: \quad m^H_{i,(i+1)} = K_{i,(i+1)} m^H_{i,(i)} m^H_{k,(j+1)} \]

\[ K_{i,(i+1)} = \left[ 1 - \sum_{r=1}^{N} \sum_{j \neq r}^{N} m^r_{i,(i)} m^r_{k,(j+1)} \right]^{-1} \]

\( i = 1, \cdots, L_k - 1 \) \( (26) \)

### 3.3 Hierarchical Evidence Propagation

It can be proven from the algorithm that \( m^\psi_{i,(i)} \) is the overall probability assignment \( (\psi \subseteq H) \) confirmed by \( E_k \) and \( m^\psi_{i,(i)} = 0 \) for any \( \psi \subseteq H \) other than \( \psi = H_j \) and \( H \), or

\[
m^j_k = m(H_j / E_k) = m^j_{i,(i)} , \quad j = 1, \cdots, N, \quad \text{and} \quad m^H_k = m(H / E_k) = m^H_{i,(i)} \quad (27)
\]

\[
m(\psi / E_k) = m^\psi_{i,(i)} = 0 \quad \text{for any} \quad \psi \subseteq H \quad \text{but} \quad \psi \neq H_j \quad (j=1, \cdots, N) \quad \text{and} \quad H \quad (28)
\]

Consequently, the state of the \( k \)th attribute can be evaluated in terms of the modification expressions defined in \( H \) as follows

\[
S(y_k) = \{(m^j_k, H_j), \quad \text{for} \quad j = 1, \cdots, N\} \quad (29)
\]

that is, the \( k \)th attribute is evaluated to \( H_j \) with a degree of confidence of \( m^j_k, j=1, \ldots, N \). Such an evaluation is generated by synthesising the given judgements of the relevant basic attributes.

The rejection degree of the \( r \)th design option on the \( k \)th attribute \( y_k(a_r) \), i.e. \( p(y_k(a_r)) \), is used to quantify \( S(y_k(a_r)) \) and is obtained as the following expected scale

\[
p_{r,k} = p(y_k(a_r)) = \sum_{n=1}^{N} m^n_{i,(i)} p(H_n) + m^H_{i,(i)} p(H) \quad (30)
\]

where \( p(H) = \sum_{n=1}^{N} p(H_n) / N \), being the average value of \( p(H_n) \).

In Figure 1, if the \( k \)th attribute at level 3 is viewed as \( y_k \), then the modification state of the attribute can be evaluated and quantified in the same way. Suppose \( m^j_{i,k} \) is the obtained overall probability assignment that the state of the \( k \)th attribute at level 3 is confirmed to \( H_j \). Similarly, the evaluation of each attribute at this level (level 3) immediately above the bottom
level could be obtained. A further problem is then to produce an evaluation of an attribute at an upper level (level 2). Suppose there are \( L_{2l} \) attributes at level 3 associated with the \( l \)th attribute \( y_{2l} \) at level 2. The set of the attributes at level 3 associated with the attribute \( y_{2l} \) at level 2 is defined by

\[
F_{2l} = \{c_1^{2l}, \ldots, c_{L_{2l}}^{2l}\}
\]  

(31)

At this stage, the evaluations of the attributes at level 3 have already been generated. So, that the modification state of the \( k \)th attribute at level 3 is confirmed to \( H_j \) to an extent of \( m_j^{l} \) \( (j=1, \ldots, N) \) could be viewed as a piece of evidence while the modification state of the \( l \)th attribute \( y_{2l} \) at level 2 may be assumed to be evaluated to any of \( H_j \) \( (j=1, \ldots, N) \). \( m_j^{l} \) is a degree of confidence that the state of the \( l \)th attribute at level 2 is confirmed to \( H_j \). The problem then becomes how to obtain \( m_j^{l} \) from \( m_j^{l_k} \) \( (j=1, \ldots, N; k=1, \ldots, L_{2l}) \). This problem can be readily solved in the same way as in the last subsection.

The modification state of the \( l \)th attribute at level 2 can then be evaluated and quantified by

\[
S(y_{2l}) = \{(m_j^{l}, H_j), \ j=1,\ldots,N\}
\]  

(32)

\[
p_r(y_{2l}) = p(y_{2l}(a_r)) = \sum_{n=1}^{N} m_{l_{2l}(l_{2l})}^{n} p(H_n) + m_{l_{2l}(l_{2l})}^{n} p(H)
\]  

(33)

In a similar way, the evaluations can be propagated from level 2 to level 1. Suppose \( m_j^{l} \) is the obtained overall probability assignment that the state of the \( l \)th attribute at level 1 is evaluated to \( H_j \). Let \( m_j^{l} \) be a degree of confidence to which the state of the \( r \)th option is confirmed to \( H_j \). Then, \( m_r^{l} \) can be obtained from \( m_j^{l} \) using the evidential reasoning algorithm. The modification state of the \( r \)th option can thus be evaluated and quantified by

\[
S(a_r) = \{(m_r^{n}, H_n), \ n=1,\ldots,N\}
\]  

(34)

\[
p_r = p(a_r) = \sum_{n=1}^{N} m_r^{n} p(H_n) + m_r^{n} p(H)
\]  

(35)

The options can finally be compared with each other based on the magnitude of the rejection degrees obtained. In other words, the option \( a_r \) is not preferred to the option \( a_h \) if and only if \( p(a_r) \geq p(a_h) \). This is because as a whole the modifications associated with the retro-fit option \( a_r \) is judged to be larger than those for \( a_h \) so that the retro-fit option \( a_r \) leads to a greater increase of cost and technical complexity.
4. An Application

4.1 Original Problem

The evidential reasoning approach has been applied to the retro-fitting of a typical short sea ferry for compliance with the requirements of SOLAS 90 (Safety of Life at Sea Regulations). The options considered to meet the increased stability requirements are the provision of sponsons (additional structures to increase the buoyancy of a ferry near the waterline), movable transverse bulkheads (internal walls used for dividing up the ferry into smaller compartments to improve safety) and the provision of buoyant wing compartments. Ro-ro ferries, in general, have large open decks and provision of additional stability in the intact condition allows them to have enhanced survivability characteristics if the water should get onto the open decks.

In this analysis, each option is considered to be compared with the original ship which has a disutility value of zero. Hence, a total disutility value which is near to zero corresponds to a high measure of merit since the departure from the initial disutility value is relatively small. The problem is then how to evaluate and rank these three retro-fit options by taking into account the attributes as shown by Figure 1.

To attain compliance with SOLAS 90, the option of retro-fitting with sponsons would need to be fitted with a pair of 1.2 meters wide sponsons over 44 percent of the ship's length. The percentage evaluations of the option with regard to the original ship in terms of the basic attributes are as shown in column 3 of Table 2 (Option S1). The detailed description of these evaluations is given by Evans [3]. For instance,

i> Increased Berthing Time: The addition of sponsons below the waterline would increase the ship's resistance to motion and as such hinder the ship's manoeuvrability while berthing. The envisaged increase in berthing time was estimated as 5 percent longer than before the retro-fitting. The performance index is given as 5.

ii> Increased Power: The addition of sponsons below the waterline will seriously alter the ship's underwater hull form and its resistance to motion. The extent to which the beam can be increased for a given length of ship is limited to ensure that the ship does not require excessive horsepower in relation to its displacement and speed. The fuel cost was estimated to increase by 5 percent resulting in an additional operating cost of £51000 per annum.

iii> Maintenance: The use of sponsons will increase maintenance costs through the need to inspect these void spaces for corrosion and structural integrity. The estimated cost of increased maintenance is 7 percent of the ship's total cost.
To attain compliance with SOLAS 90, the option of retro-fitting with movable transverse bulkheads would need to be fitted with 2 sets of movable transverse bulkhead doors. The percentage evaluations for this option are given in column 4 of Table 2. For instance,

i> Human Error Factor: With 16 operations per day the possibility of human error in closing and securing the doors is estimated at 7 percent.

ii> Collision Resistance: The retro-fitting of movable partial height, transverse bulkhead doors does not alter the collision resistance of the ship and as such the performance index is set to 100.

iii> Additional Crew: With the need to operate these doors 16 times a day, and the need for quick turnaround, an extra crew member is required on the vehicle deck to assist in the operation of these doors.

iv> Device Cost: The cost of two sets of bulkhead doors is estimated as £270000, or 0.3% of the total ship cost.

Table 2: Original Data for Evaluation of Attributes at Each Option

<table>
<thead>
<tr>
<th>Level 4</th>
<th>Criteria</th>
<th>Option S1</th>
<th>Option S2</th>
<th>Option S3</th>
</tr>
</thead>
<tbody>
<tr>
<td>Operating Time Factor</td>
<td>% of extra time</td>
<td>0</td>
<td>5</td>
<td>22.55</td>
</tr>
<tr>
<td>Human Error Factor</td>
<td>% increase</td>
<td>0</td>
<td>7</td>
<td>7</td>
</tr>
<tr>
<td>Loss of Lane Length</td>
<td>% loss</td>
<td>0</td>
<td>4</td>
<td>13</td>
</tr>
<tr>
<td>Loss of Stowage Flexibility</td>
<td>% loss</td>
<td>0</td>
<td>4</td>
<td>30</td>
</tr>
<tr>
<td>Change to Ventilation</td>
<td>% alteration</td>
<td>0</td>
<td>0</td>
<td>10</td>
</tr>
<tr>
<td>Berthing Operations</td>
<td>% increase in time</td>
<td>5</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Collision Resistance</td>
<td>% lost potential</td>
<td>86</td>
<td>100</td>
<td>0</td>
</tr>
<tr>
<td>Change to Intact Condition</td>
<td>% decrease in the natural roll period</td>
<td>25</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Pilot Access</td>
<td>% change</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Passenger Access</td>
<td>% change</td>
<td>20</td>
<td>0</td>
<td>15</td>
</tr>
<tr>
<td>Vehicle Access</td>
<td>% change</td>
<td>0</td>
<td>0</td>
<td>24</td>
</tr>
<tr>
<td>Additional Crew</td>
<td>yes/no (1/0)</td>
<td>0</td>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>Increased Power</td>
<td>% increase</td>
<td>5</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Maintenance</td>
<td>% increase</td>
<td>7</td>
<td>1</td>
<td>2</td>
</tr>
<tr>
<td>Modified Linkspan</td>
<td>% cost of ship</td>
<td>0.1</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Modified Passenger Access</td>
<td>% cost of ship</td>
<td>0.01</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Device Cost</td>
<td>% cost of ship</td>
<td>2.54</td>
<td>0.3</td>
<td>1.4</td>
</tr>
<tr>
<td>Fitting Cost</td>
<td>% cost of ship</td>
<td>0.65</td>
<td>0.07</td>
<td>0</td>
</tr>
<tr>
<td>Out of Service Cost</td>
<td>% cost of ship</td>
<td>0.42</td>
<td>0</td>
<td>0.3</td>
</tr>
<tr>
<td>Modify Access Equipment</td>
<td>% cost of ship</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Modify LSA Equipment</td>
<td>% cost of ship</td>
<td>0.2</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>
The provision of buoyant wing compartments on both sides of the ship would also ensure compliance with the SOLAS 90 requirements. The percentage evaluations of this option are given by column 5 of Table 2. For example,

i> Operating Time Factor: The operation of doors within the buoyant compartments would restrict cargo flow due to their arrangement. The number of doors to be operated would escalate with the number of voyages per day. It is envisaged that the operation of the doors would take approximately 9 minutes. Assuming a loading/discharging time of 40 minutes the operation time factor increases by 22.5 percent.

ii> Loss of Stowage Flexibility: The construction of the entrance into the buoyant wing compartment will provide considerable difficulty for vehicles other than cars. The loss of stowage flexibility is therefore regarded as quite considerable at 30 percent.

iii> Change to Ventilation: The venting of the vehicle deck will be restricted in the wing compartments due to its construction. The degree of change is assumed as 10 percent.

iv> Out of Service Cost: The additional time, outside the normal refit, to complete this work is estimated to be 14 days which translates to an expected cost of £243600.

### 4.2 Problem Transformation

The selection problem of the retro-fit options as represented by Figure 1 and Table 2 could be dealt with using the simple weighting approach briefly described in section 2.2. This approach, however, suffers from the demerits such as linearity of utilities, independence of preferences and direct compensation among attributes. This section is therefore devoted to applying the evidential reasoning approach to analyse the problem.

To do so, the original data presented in Table 2 needs to be transformed into equivalent statements using the evaluation grades as defined in section 2.2. The set of evaluation grades is therefore defined and quantified by

\[
H = \{ H_1, H_2, H_3, H_4, H_5 \} = \{ \text{Fundamental, Major, Moderate, Minor, None} \}
\]

\[
p\{H\} = ( p(H_1), p(H_2), p(H_3), p(H_4), p(H_5) ) = ( 1, 0.7, 0.5, 0.3, 0 )
\]

Then, each percentage value in Table 2 could be transformed to be equivalent to a statement. For instance, the Berthing Operations for the option of retro-fitting with sponsons has a performance index of 0.05 (5 percent). As \(0 < 0.05 < 0.3\), it could be equivalent to state

<table>
<thead>
<tr>
<th>Auxiliary Modification</th>
<th>% cost of ship</th>
<th>0.09</th>
<th>0</th>
<th>0</th>
</tr>
</thead>
<tbody>
<tr>
<td>Stabiliser Installation</td>
<td>% cost of ship</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Ventilation Cost</td>
<td>% cost of ship</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>
that in terms of Berthing Operations the modification of this option from the original ship is "Minor" to an extent of 0.1667 and "None" to 0.8333. This is because the numerical value obtained by quantifying the statement using the technique as presented in section 3.2 is exactly 0.05. In this way, all the percentage values listed in Table 2 could be transformed into equivalent statements. By equivalence it is meant that a percentage value and the quantification of its transformed statement is the same. Table 3 is obtained by transforming the data of Table 2.

In Table 3, the grades "Fundamental", "Major", "Moderate", "Minor" and "None" are abbreviated by "FU", "MA", "MO", "MI" and "NO", respectively. The real numbers in the brackets following the abbreviations denote the degrees of confirmation. The statement examined above is thus represented by "MI(0.1667), NO(0.8333)", as shown by column 2 and row 7 of Table 3. It may be noted that the state of each basic attribute at an option may be confirmed to one or more of the defined grades simultaneously and the sum of the degrees of such confirmation is one. Thus all the statements in Table 3 have complete uncertainty.

<table>
<thead>
<tr>
<th>Table 3</th>
<th>Equivalent Statements for Evaluation of Attributes at Each Option</th>
</tr>
</thead>
<tbody>
<tr>
<td>Level 4</td>
<td>Option S1</td>
</tr>
<tr>
<td>Operating Time Factor</td>
<td>NO(1)</td>
</tr>
<tr>
<td>Human Error Factor</td>
<td>NO(1)</td>
</tr>
<tr>
<td>Loss of Lane Length</td>
<td>NO(1)</td>
</tr>
<tr>
<td>Loss of Stowage Flexibility</td>
<td>NO(1)</td>
</tr>
<tr>
<td>Change to Ventilation</td>
<td>NO(1)</td>
</tr>
<tr>
<td>Berthing Operations</td>
<td>MI(0.1667), NO(0.8333)</td>
</tr>
<tr>
<td>Collision Resistance</td>
<td>FU(0.5333), MA(0.4667)</td>
</tr>
<tr>
<td>Change to Intact Condition</td>
<td>MI(0.8333), NO(0.1667)</td>
</tr>
<tr>
<td>Pilot Access</td>
<td>NO(1)</td>
</tr>
<tr>
<td>Passenger Access</td>
<td>NO(1)</td>
</tr>
<tr>
<td>Vehicle Access</td>
<td>NO(1)</td>
</tr>
<tr>
<td>Additional Crew</td>
<td>NO(1)</td>
</tr>
<tr>
<td>Increased Power</td>
<td>MI(0.1667), NO(0.8333)</td>
</tr>
<tr>
<td>Maintenance</td>
<td>MI(0.2333), NO(0.7667)</td>
</tr>
<tr>
<td>Modified Linkspan</td>
<td>MI(0.0033), NO(0.9967)</td>
</tr>
<tr>
<td>Modified Passenger Access</td>
<td>MI(0.0003), NO(0.9997)</td>
</tr>
<tr>
<td>Device Cost</td>
<td>MI(0.0847), NO(0.9153)</td>
</tr>
<tr>
<td>Fitting Cost</td>
<td>MI(0.0217), NO(0.9783)</td>
</tr>
<tr>
<td>Out of Service Cost</td>
<td>MI(0.0140), NO(0.9860)</td>
</tr>
<tr>
<td>Modify Access Equipment</td>
<td>NO(1)</td>
</tr>
<tr>
<td>Modify LSA Equipment</td>
<td>MI(0.0067), NO(0.9933)</td>
</tr>
</tbody>
</table>
4.3 Hierarchical Analysis Using the Evidential Reasoning Approach

The evidential reasoning analysis can then be applied to generate the evaluation of each option by hierarchically synthesising these obtained degrees of confirmation on the basis of the attribute hierarchy and the relative weights of the attributes as shown in Figure 1. It is assumed that the modification state of an attribute at a single level is almost certainly "Minor" if the modification states of all the attributes associated at a level immediately below are regarded to be absolutely "Minor". It is assumed that "almost certainly" is modelled by assigning $\delta = 0.01$. Such an assumption is necessary for conflict resolution using the evidential reasoning approach although alternative values may also be assigned to $\delta$.

A priority coefficient $\alpha$ and normalised relative weights $\lambda$ can then be calculated. For instance, the attribute Cargo Handling at level 3 is associated with the attributes Operating Time Factor, Human Error Factor, Loss of Lane Length, Loss of Stowage Flexibility and Change to Ventilation at level 4 as shown in Figure 1. Since option 1 is not modified in terms of these five lower level attributes, it should be judged that option 1 is not modified either in terms of Cargo Handling to an extent of over 99 percent. $\alpha$ associated with Cargo Handling can then be readily obtained by solving the following inequality

$$
\left(1 - \alpha \frac{0.15}{0.55}\right)\left(1 - \alpha \frac{0.10}{0.55}\right)\left(1 - \alpha \frac{0.55}{0.55}\right)\left(1 - \alpha \frac{0.15}{0.55}\right)\left(1 - \alpha \frac{0.05}{0.55}\right) \leq \delta = 0.01
$$

(38)

An approximate procedure is suggested as follows to obtain $\alpha$. Firstly, assign an initial value to $\alpha$ by assuming that all the attributes are of equal importance, that is

$$
\alpha^0 = 1 - \delta^{1/5} = 1 - 0.01^{1/5} = 0.6019
$$

(39)

Then, assign $\alpha^t$ so that $\alpha^t$ increases from $\alpha^{t-1}$ by for example 0.001 ($t = 1, 2, ...$) until the inequality is satisfied. This simple procedure is of general use as it is easy to implement for any number of attributes and for any relative weights. Note that the initially assigned $\alpha^0$ is the lower bound of $\alpha$ satisfying the inequality. $\alpha$ for Cargo Handling is then assigned to 0.9759.

Let $y_k$ stand for Cargo Handling and $E_k$ for the five associated basic attributes. The basic probability assignment matrix for evaluation of Cargo Handling through the five basic attributes can then be constructed. These basic probability assignments can be combined using the evidential reasoning algorithm as presented in the previous section. The generated evaluation for Cargo Handling is as shown by column 2 and row 2 of Table 4, that is

"NO(0.9903), H(0.0097)". This evaluation states that there is almost no difference at all
between option 1 and the original ship in terms of *Cargo Handling*. The real number 0.0097 in $H(0.0097)$ stands for the remaining uncertainty unassigned to the defined single evaluation grades. Such propagation of the evaluations from level 4 to level 3 is rational in the sense that $\delta$ has been assigned to 0.01.

In a similar way, all the attributes at level 3 can be evaluated through the attributes at level 4. The results are shown by Table 4. The attributes at level 2 can in turn be evaluated through the attributes at level 3, as shown by Table 5. Table 6 shows the evaluations of *Ship Operation* and *Installation* at each option, which are generated through the attributes at level 2. Finally, the evaluations of these three options are obtained in terms of the degrees of modification, as shown by row 2 of Table 7.

### Table 4
**Obtained Judgements for Evaluation of Attributes at Each Option**

<table>
<thead>
<tr>
<th>Level 3</th>
<th>Option S1</th>
<th>Option S2</th>
<th>Option S3</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cargo Handling</td>
<td>NO(0.9903), H(0.0097)</td>
<td>MI(0.0757), NO(0.9126)</td>
<td>MI(0.5112), NO(0.4737)</td>
</tr>
<tr>
<td>Berthing Operation</td>
<td>MI(0.1667), NO(0.8333)</td>
<td>H(0.0117)</td>
<td>H(0.0151)</td>
</tr>
<tr>
<td>Integrity</td>
<td>FU(0.0065), MA(0.0057) MI(0.8091), NO(0.1619) H(0.0168)</td>
<td>FU(0.0122), NO(0.9710) H(0.0168)</td>
<td>NO(0.9902), H(0.0098)</td>
</tr>
<tr>
<td>Daily Operation Requirement</td>
<td>MI(0.0456), NO(0.9348) H(0.0196)</td>
<td>NO(0.9901), H(0.0099) MI(0.3224), NO(0.6470) H(0.0306)</td>
<td></td>
</tr>
<tr>
<td>Running Cost</td>
<td>MI(0.1160), NO(0.8727) H(0.0113)</td>
<td>FU(0.0021), NO(0.9861), H(0.0116)</td>
<td>MI(0.0004), NO(0.9896)</td>
</tr>
<tr>
<td>Transfer Facilities</td>
<td>MI(0.0019), NO(0.9882) H(0.0099)</td>
<td>NO(0.9902), H(0.0098) MI(0.9902), H(0.0098)</td>
<td></td>
</tr>
<tr>
<td>Procurement</td>
<td>MI(0.0407), NO(0.9490) H(0.0103)</td>
<td>MI(0.0045), NO(0.9857) H(0.0098)</td>
<td>MI(0.0215), NO(0.9685) H(0.0100)</td>
</tr>
<tr>
<td>Secondary Modification</td>
<td>MI(0.0001), NO(0.9900) H(0.0099)</td>
<td>NO(0.9901), H(0.0099)</td>
<td>NO(0.9901), H(0.0099)</td>
</tr>
</tbody>
</table>

### Table 5
**Obtained Judgements for Evaluation of Attributes at Each Option**

<table>
<thead>
<tr>
<th>Level 2</th>
<th>Option S1</th>
<th>Option S2</th>
<th>Option S3</th>
</tr>
</thead>
<tbody>
<tr>
<td>Port Activities</td>
<td>MI(0.0007), NO(0.9810) H(0.0183)</td>
<td>MI(0.0626), NO(0.9177) H(0.0197)</td>
<td>MI(0.4573), NO(0.5184) H(0.0244)</td>
</tr>
<tr>
<td>Ship Inherent</td>
<td>FU(0.0001), MA(0.0001) MI(0.1404), NO(0.8370) H(0.0224)</td>
<td>FU(0.0013), MI(0.0001) NO(0.9826), H(0.0160)</td>
<td>MI(0.0011), NO(0.9830) H(0.0159)</td>
</tr>
<tr>
<td>Port</td>
<td>MI(0.0019), NO(0.9883)</td>
<td>NO(0.9902), H(0.0098) MI(0.9902), H(0.0098)</td>
<td></td>
</tr>
</tbody>
</table>
Table 6  Obtained Judgements for Evaluation of Attributes at Each Option

<table>
<thead>
<tr>
<th>Level 1</th>
<th>Option S1</th>
<th>Option S2</th>
<th>Option S3</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ship Operation</td>
<td>MI(0.0049), NO(0.9767), H(0.0184)</td>
<td>MI(0.0230), NO(0.9592), H(0.0178)</td>
<td>MI(0.2253), NO(0.7486), H(0.0261)</td>
</tr>
<tr>
<td>Installation</td>
<td>MI(0.0006), NO(0.9873), H(0.0121)</td>
<td>NO(0.9879), H(0.0121)</td>
<td>MI(0.0002), NO(0.9877), H(0.0121)</td>
</tr>
</tbody>
</table>

Table 7  Ranking of The Three Options

<table>
<thead>
<tr>
<th>Selection</th>
<th>Option S1</th>
<th>Option S2</th>
<th>Option S3</th>
</tr>
</thead>
<tbody>
<tr>
<td>Evaluations</td>
<td>MI(0.0023), NO(0.9789), H(0.0188)</td>
<td>MI(0.0109), NO(0.9705), H(0.0186)</td>
<td>MI(0.1195), NO(0.8553), H(0.0252)</td>
</tr>
<tr>
<td>Disutilities</td>
<td>0.0101</td>
<td>0.0126</td>
<td>0.0485</td>
</tr>
<tr>
<td>Ranking</td>
<td>1</td>
<td>2</td>
<td>3</td>
</tr>
</tbody>
</table>

4.4  Result Analysis

The disutilities and the ranking of the three retro-fit options are shown by the last two rows of Table 7. The most favourable option is the sponson, followed closely by the movable bulkheads and then by the wing compartments, The results are in harmony with the analysis conducted by Evans [3].

The sponsons prove beneficial in the analysis for short sea ro-ros as the loss of Collision Resistance and the decrease in the natural roll period associated with Change to Intact Condition do not play an important role in the evaluation of the whole ship. This is also because the increase in power and hence increased fuel costs have a small overall effect on the total economics. This option does not affect the ro-ro concept, disrupting the through flow of vehicles, which for the short sea ro-ro has a critical effect on the operating economics.

The retro-fitting with buoyant wing compartments is the least preferred option in this analysis due to their disruptive effects on Loss of Stowage Flexibility (the through flow of traffic) and on Cargo Handling.

4.5  Design Evaluation with Incomplete Uncertainty

The transformed data as shown in Table 3 include only complete uncertainty for both quantitative and qualitative attributes. It is quite likely that subjective judgements with incomplete uncertainty could be provided for evaluation of qualitative attributes. This may be incurred because of the complexity and novelty of a design problem or because a team is
involved for design evaluation with each member having different views in the evaluation. To
demonstrate the potential of the evidential reasoning approach for dealing with complex
problems with incomplete uncertainty, the above example is modified so that incomplete
uncertainty is introduced in evaluation of the qualitative attributes as classified in Table 1.

The evaluations of the three retro-fit options on the seven qualitative attributes are
modified as shown in Table 8 and the other evaluations on all the quantitative attributes are
the same as given in Table 3. In Table 8, one can find that incomplete uncertainty is assumed.
For instance, the state of Options S2 on Human Error Factor is assigned to Moderate (MO),
Minor (MI) and None (NO) to the degrees of 0.1, 0.2 and 0.6, respectively, and the sum of
these degrees is 0.9 less than one. Such incomplete uncertain judgements may result from the
inability of design engineers to provide precise judgements.

<table>
<thead>
<tr>
<th>Level 4</th>
<th>Option S1</th>
<th>Option S2</th>
<th>Option S3</th>
</tr>
</thead>
<tbody>
<tr>
<td>Human Error Factor</td>
<td>NO(1)</td>
<td>MO(0.1000), MI(0.2000),</td>
<td>MO(0.0500), MI(0.3000),</td>
</tr>
<tr>
<td></td>
<td></td>
<td>NO(0.6000)</td>
<td>NO(0.5000)</td>
</tr>
<tr>
<td>Loss of Stowage Flexibility</td>
<td>NO(1)</td>
<td>MI(0.2000), NO(0.7000)</td>
<td>MO(0.1000), MI(0.8000),</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>NO(0.0500)</td>
</tr>
<tr>
<td>Collision Resistance</td>
<td>FU(0.4000), MA(0.4000),</td>
<td>FU(0.8000), MA(0.15000),</td>
<td>NO(1)</td>
</tr>
<tr>
<td></td>
<td>MO(0.1000),</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>MI(0.8000), NO(0.1500)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Change to Intact Condition</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Pilot Access</td>
<td>NO(1)</td>
<td>NO(1)</td>
<td>NO(1)</td>
</tr>
<tr>
<td>Passenger Access</td>
<td>MI(0.6000), NO(0.3000)</td>
<td>NO(1)</td>
<td>MO(0.1000), MI(0.4000),</td>
</tr>
<tr>
<td>Vehicle Access</td>
<td>NO(1)</td>
<td>NO(1)</td>
<td>NO(0.4500)</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>NO(0.1500), MI(0.7000),</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>NO(0.1000)</td>
</tr>
</tbody>
</table>

The results of combining the given judgements and transformed statements from the
bottom level to the level immediate above (i.e. level 3) are shown in Table 9. From the table,
one can clearly see the effectiveness of the evidential reasoning approach both in reducing the
degrees of uncertainty unassigned to individual evaluation grades and in narrowing the
hypothesis set with the accumulation of evidence.

The evaluation of Option S3 on Cargo Handling, for example, are associated with five
basic attributes including Operating Time Factor, Human Error Factor, Loss of Lane Length,
Loss of Stowage Flexibility, and Change to Ventilation. Two of the five basic attributes are
qualitative ones, i.e. Human Error Factor and Loss of Stowage Flexibility. The initial
evaluations of Option S3 on the two attributes are shown by the second block row of the last
column in Table 8. Both evaluations involve incomplete uncertainty as the total assigned
degree of confidence in individual evaluation grades is each 85 percent only. The remaining confidence degree of 15 percent is assigned to none of the five individual evaluation grades but to the whole evaluation set. The combined evaluation of Option S3 on Cargo Handling is shown in the second block row of the last column in Table 9. The uncertainty assigned the whole set \((H)\) is now reduced to 1.57 percent. This reduction results from the accumulation of the evaluations for the five lower level attributes.

At Option S3, the states of the five basic attributes associated with Cargo Handling are evaluated to the three grades Moderate, Minor and None to various degrees. While all the five attributes are evaluated to the last two grades to large degrees, only the two qualitative attributes Human Error Factor and Loss of Stowage Flexibility are evaluated to Moderate to the degrees of 5 percent and 10 percent, respectively (see Table 8). Since the evaluations to Moderate from the two qualitative attributes are much less significant and are not supported by the other attributes, the evaluation for Cargo Handling is thus narrowed to the two grades Minor and None to the large degrees of nearly 51 percent and 48 percent, respectively.

The final evaluations for the three retro-fit options as shown in Table 10 are similar to those as obtained in Table 7 and both rankings are the same. This is because the modifications provided as in Table 8 are not significant. Such analysis is however useful to investigate the robustness of the evaluations obtained.

<table>
<thead>
<tr>
<th>Level 3</th>
<th>Option S1</th>
<th>Option S2</th>
<th>Option S3</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cargo Handling</td>
<td>NO(0.9903), H(0.0097)</td>
<td>MO(0.0003), MI(0.0832),</td>
<td></td>
</tr>
<tr>
<td>Berthing Operation</td>
<td>MI(0.1667), NO(0.8333)</td>
<td>NO(0.9039), H(0.0127)</td>
<td></td>
</tr>
<tr>
<td>Integrity</td>
<td>FU(0.0173), MA(0.0173),</td>
<td>FU(0.0094), MA(0.0018),</td>
<td></td>
</tr>
<tr>
<td></td>
<td>MO(0.0043), MI(0.7559),</td>
<td>NO(0.9720), H(0.0168)</td>
<td></td>
</tr>
<tr>
<td></td>
<td>NO(0.1417), H(0.0636)</td>
<td>NO(0.9902), H(0.0098)</td>
<td></td>
</tr>
<tr>
<td>Daily Operation Requirements</td>
<td>MI(0.0374), NO(0.9385)</td>
<td>NO(0.9901), H(0.0099)</td>
<td></td>
</tr>
<tr>
<td></td>
<td>H(0.0241)</td>
<td>MO(0.0446), MI(0.2970),</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>NO(0.6112), H(0.0471)</td>
<td></td>
</tr>
<tr>
<td>Running Cost</td>
<td>MI(0.1160), NO(0.8727),</td>
<td>FU(0.0021), MI(0.0002),</td>
<td></td>
</tr>
<tr>
<td></td>
<td>H(0.0113)</td>
<td>NO(0.9861), H(0.0116)</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>MI(0.0004), NO(0.9896)</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>H(0.0100)</td>
<td></td>
</tr>
<tr>
<td>Transfer Facilities</td>
<td>MI(0.0019), NO(0.9882),</td>
<td>NO(0.9902), H(0.0098)</td>
<td></td>
</tr>
<tr>
<td></td>
<td>H(0.0099)</td>
<td>NO(0.9902), H(0.0098)</td>
<td></td>
</tr>
<tr>
<td>Procurement</td>
<td>MI(0.0407), NO(0.9490),</td>
<td>MI(0.0045), NO(0.9857),</td>
<td></td>
</tr>
<tr>
<td></td>
<td>H(0.0103)</td>
<td>H(0.0098)</td>
<td></td>
</tr>
<tr>
<td>Secondary Modification</td>
<td>MI(0.0001), NO(0.9900)</td>
<td>NO(0.9901), H(0.0099)</td>
<td></td>
</tr>
<tr>
<td></td>
<td>H(0.0099)</td>
<td>NO(0.9901), H(0.0099)</td>
<td></td>
</tr>
</tbody>
</table>
Table 10  Ranking of The Three Options

<table>
<thead>
<tr>
<th>Selection</th>
<th>Option S1</th>
<th>Option S2</th>
<th>Option S3</th>
</tr>
</thead>
<tbody>
<tr>
<td>Evaluations</td>
<td>MI(0.0023), NO(0.9790), H(0.0188)</td>
<td>MI(0.0121), NO(0.9690), H(0.0189)</td>
<td>MO(0.0002), MI(0.1183), NO(0.8562), H(0.0253)</td>
</tr>
<tr>
<td>Disutilities</td>
<td>0.0101</td>
<td>0.0131</td>
<td>0.0482</td>
</tr>
<tr>
<td>Ranking</td>
<td>1</td>
<td>2</td>
<td>3</td>
</tr>
</tbody>
</table>

5. Concluding Remarks

This paper demonstrates how the evidential reasoning approach could be used as an alternative tool to deal with real world complex decision analysis problems in engineering design. Such problems often involve both quantitative attributes and inherently subjective (qualitative) attributes. The evaluations of subjective attributes are usually associated with uncertainty. Although in traditional evaluation methods, such as the simple weighting method, subjective attributes could be assessed using some scaling values such as percentage values, the new evidential reasoning approach provides a more natural yet rational approach for articulating and processing information for evaluation of subjective attributes with complete or incomplete uncertainty. Furthermore, when dealing with multiple attribute decision making (MADM) problems, this new approach does not suffer from the demerits such as linearity of utilities, independence of preferences and direct compensation among attributes.

As has been indicated by Evans [3], more detailed analysis for selection of retro-fit options would need to be carried out on a ship by ship basis using further levels of attributes. Indeed, different operating scenarios may need to be taken into account for evaluation of an attribute at an option. However, the evidential reasoning approach sets no limit on the number of attribute levels of a problem.

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REFERENCES


