Multiple Attribute Design Evaluation of Large Engineering Products Using The Evidential Reasoning Approach[1]

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Summary: *This paper reports the application of an evidential reasoning approach to design selection of retro-fit options for complex engineering products. The particular selection problem investigated in this paper is initially modelled by means of techno-economic analysis and may be viewed as a multiple attribute decision making (MADM) problem with a hierarchical structure of attributes which may be measured for each design option using numerical values or subjective judgements with uncertainty. In this paper the evaluation model is discussed at first. Techniques for articulating original evaluation data are also explored. The evidential reasoning approach is then summarised which has been developed on the basis of decision theory and the Dempster-Shafer theory in artificial intelligence. A real world design selection problem involving the retro-fitting of a typical short sea ferry is then examined in detail using the approach.*

1. Introduction

Evaluations of different design options for a complex engineering product often need to take into account many performance attributes so that the economic and technical aspects of the product can be comprehensively assessed. Such an attribute often represents a general performance index and may be described by and evaluated through its associated contributing attributes. This leads to a certain type of evaluation framework such as a hierarchical structure or even a general network structure. In a relatively simple hierarchical structure, an attribute may be broken down into lower level attributes and attributes at the bottom level are directly measurable by means of either numerical values or subjective judgements with uncertainty.

An evaluation analysis model for design selection of retro-fit options with a hierarchy of attributes was developed by Evans [3], in which a four-level structure of attributes was established, taking account of both commercial and technical measures of performance. The initial evaluations of all the attributes at each retro-fit option are articulated using normalised

[1] Journal of Engineering Design, Vol.8, No.3, pp.211-230, 1997

numerical (percentage) values or certain scaling standards. The simple weighting method was then adopted to obtain the overall disutility value of each option. The options are ranked on the basis of the magnitude of their disutility values.

Some of the attributes involved in the model, however, are inherently subjective and as such it may be more natural to evaluate them initially using subjective judgements with uncertainty. Furthermore, the implementation of the simple weighting method requires such assumptions as linearity of utilities, independence of preferences and direct compensation among attributes in the hierarchy [4, 13]. As such assumptions may not be acceptable, it is advisable to be cautious when using this technique to deal with MADM problems.

This paper is devoted to applying the evidential reasoning approach to treat the above design selection problem. This approach has been developed to deal with MADM problems with both quantitative and qualitative attributes where each qualitative attribute can have its own hierarchy of relevant attributes which could be assessed using subjective judgements with complete or incomplete uncertainty [11, 12, 13, 14]. One of the advantages of this approach is that it can deal with incomplete uncertainty in a more natural yet rational way. Since certainty could be viewed as a special case of uncertainty, the application of this approach to the selection of the retro-fit options for this case study would then be based on the transformation of the original data into equivalent subjective statements with complete uncertainty. If the precise numerical values are not available, it is then more natural to articulate subjective judgements with uncertainty as original evaluation data.

The technique for the data transformation is first discussed and the main calculation procedure of the evidential reasoning approach is presented. The application of this approach to the selection of retro-fit options for a typical short sea ferry is demonstrated in the last section using the same data as given by Evans [3]. A hypothetical example based on the above evaluation model is also constructed and investigated. In this example, some of the subjective attributes are assessed by deliberately adopting incomplete uncertain judgements which may be expected from design engineers. The purpose of examining the hypothetical example is to demonstrate the potential of the evidential reasoning approach to deal with complex design decision problems in a more realistic way.

2. Evaluation Model for Ship Retro-Fit Options

2.1 Hierarchical Attribute Structure

The technical problem deals with the examination of three retro-fit options for a short sea roll-on roll-off ferry to enhance its damage stability characteristics (roll-on roll-off is simply called ro-ro which means that vehicles drive on/off ferries). The selection of a retro-fit option demands a clear definition of the necessary attributes and their associated contributing attributes which could influence the ship's operation. These attributes include both the commercial and the technical aspects of the option. Such attributes are defined and arranged in a hierarchical structure as shown in Figure 1 [3]. These attributes and their relationships are used to convert the commercial aspects of the option into a comparative format.

In Figure 1, the selection of a retro-fit option is based on the assessment of the option over two general attributes (*Ship Operation* and *Installation*) which are broken down into lower level attributes within a hierarchy. These lower level attributes define the general attributes and are easier to evaluate at each option. The attributes at the bottom level (Level 4) of the hierarchy are directly measurable and is referred to as basic attributes. These basic attributes are defined and described by Evans [3], and they may be divided into two types, quantitative ones and qualitative ones, although Evans tried to use numbers to measure all the attributes.

Evans measured most quantitative basic attributes by means of extra cost incurred due to the required modifications for each retro-fit option, such as all the basic attributes associated with the upper level attributes *Installation* and *Running Cost*, though some of those basic attributes could be more naturally assessed in a qualitative manner. Other quantitative basic attributes were measured by using numbers due to their numerical nature. For instance, *Operating Time Factor* was expressed as a percentage of the total increase in time spent in port operating the device; *Loss of Lane Length* as a percentage of a total loss of cargo space; and *Changes to Ventilation* as an increase in ventilation rate or air changes per hour.

Qualitative attributes are those which may not be readily assessed by using exact numbers in the first instance due to their subjective nature, although one often tries to associate numbers with his judgements for simplicity or because of the ignorance of other methods which may be more rational and easier to apply. In his report, Evans expressed the qualitative attributes in various ways. For instance, *Human Error Factor* was expressed as a percentage of the possibility of human error; *Loss of Stowage Flexibility* as an expected loss of stowage flexibility; *Collision Resistance* as a linear scale of 0 to 1, and *Pilot Access* as a change in the accessibility for the pilot to board/leave the ship underway.

The classification of quantitative and qualitative attributes is summarised as in Table 1.

Table 1 Classification of Attributes

Quantitative Attributes		Qualitative Attributes
Operating Time Factor	*Device Cost*	*Human Error Factor*
Loss of Lane Length	*Fitting Cost*	*Loss of Stowage Flexibility*
Changes to Ventilation	*Out of Service Cost*	*Collision Resistance*
Increased Berthing Time	*Modify Access Equipment*	*Change in The Intact Condition*
Additional Crew	*Modify LSA Equipment*	*Pilot Access*
Increased Power	*Auxiliary Modification*	*Passenger Access*
Maintenance	*Stabiliser Installation*	*Vehicle Access*
Modify Linkspan/Quay	*Ventilation Modifications*	

In the attribute hierarchy as shown in Figure 1, not all attributes at a single level play the same role in the evaluation of an attribute associated at a level immediately above. For instance, *Ship Operation* is evaluated through *Port Activity* and *Ship Inherent* where *Port Activity* may be relatively more important in the evaluation. The relative importance of the attributes at a single level with regard to an associated attribute at a level immediately above therefore needs to be taken into consideration. In fact, different weights were used to represent relative importance [3], as shown by the numbers in the brackets in Figure 1. It may be noted that these weights are normalised so that the sum of the weights of the attributes at a single level with respect to an associated upper level attribute is one. For instance, the three attributes *Integrity*, *Daily Operation Requirements* and *Running Cost* are associated with the *Ship Inherent* and the sum of their weights is one.

2.2 Articulation of Evaluation Data

The influence of each retro-fit option for a ship can initially be assessed on each basic attribute. Such an assessment may be articulated and represented using a subjective statement with uncertainty or using a numerical value, depending upon whether the attribute is inherently qualitative (subjective) or quantitative (mathematically measurable).

With regard to an attribute such as *Collision Resistance*, for example, it may be stated that the modification of a particular retro-fit option for a ship is "*Minor*" or "*Moderate*". In the statement, "*Minor*" and "*Moderate*" may be referred to as evaluation grades describing the degrees of modification. Such a grade represents a distinct modification state of an attribute for a particular retro-fit option. It is also possible that the degree of modification is something between "*Minor*" and "*Moderate*". For instance, it could be judged that the modification of the retro-fit option in terms of *Collision Resistance* is "*Minor*" to an extent of β^n and "*Moderate*" to an extent of β^{n+1} with β^n, $\beta^{n+1} \geq 0$ and $\beta^n + \beta^{n+1} \leq 1$. Here β^n and β^{n+1} represent uncertainty, indicating the degrees of confidence (belief) in the evaluation. β^n may simply be referred to as a confidence degree. In this paper, $\beta^n + \beta^{n+1} < 1$ or $= 1$ is referred to as incomplete or complete uncertainty, respectively.

Other grades such as "*None*", "*Major*" and "*Fundamental*" may also be used to describe the degrees of modification. Generally, a set of such grades may be employed. Suppose H_n stands for a general grade. A set of evaluation grades may then be defined by

$$H = \{ H_1 \cdots H_n \cdots H_N \} \tag{1}$$

Large modifications for a ship lead to increase of cost and technical complexity. It may therefore be less favourable. The grade "*Minor*" may thus, in a certain sense, be preferred to the grade "*Major*". Suppose H_{n+1} is preferred to H_n. A particular set of evaluation grades may then be defined by

$$\begin{aligned} H &= \{\ H_1 \quad H_2 \quad H_3 \quad H_4 \quad H_5\ \} \\ &= \{\ Fundamental \quad Major \quad Moderate \quad Minor \quad None\ \} \end{aligned} \tag{2}$$

Such grades may be quantified using a scale. Suppose $p(H_n)$ represents the scale of H_n. If $p(H_n)$ is set to be a real number in the closed interval [0 1] which may be referred to as the preference degree space [12, 13], the set of evaluation grades is then quantified by

$$p\{H\} = [\ p(H_1)\ \cdots\ p(H_n)\ \cdots\ p(H_N)\ ]^T \tag{3}$$

where $p(H_n)$ ($n=1, \ldots, N$) satisfy the following basic conditions

$$p(H_1) = 1,\ p(H_N) = 0,\ \ and\ \ p(H_{n+1}) < p(H_n) \qquad n=1\ \cdots\ N\text{-}1 \tag{4}$$

and the consistency conditions [12, 13].

If the state of an option on a basic attribute is precisely evaluated to H_n, then the state may be quantified by $p(H_n)$. If the state is evaluated to H_n and H_{n+1} to the degrees of $(1-\beta^n)$ and β^n, respectively, then the state could be quantified by such an expected value as $\beta^n\ p(H_n) + (1-\beta^n)\ p(H_{n+1})$ [12, 13] which is referred to as the rejection degree of the option on the given attribute. In this way, a subjective judgement could be quantified on each attribute separately. More complex evaluations can be taken into account where the subsets of H such as $\{H_n\ H_{n+1}\}$ could be treated as a single grade as well.

Some attributes are numerically measurable using mathematical means. Such a measure can be transformed through normalisation or more generally utility conversion, so that the value of such an attribute at an option will lie within the closed interval [0 1]. In fact, percentage values are often used for such normalisation. Furthermore, such a normalised value could be transformed to be equivalent to a subjective judgement with complete uncertainty. Suppose y_{ij} is the value of option i on attribute j with $0 \le y_{ij} \le 1$. If $p(H_{n+1}) \le y_{ij} \le p(H_n)$, then it may be equivalent to state that option i on attribute j is evaluated to H_n and H_{n+1} with the confidence degrees of $(1-\beta)$ and β, respectively, where β is given by

$$\beta = \frac{p(H_n) - y_{ij}}{p(H_n) - p(H_{n+1})} \tag{5}$$

By equivalence it is meant that the value obtained by quantifying the statement is the same as the numerical value y_{ij}. Such a transformation is useful when there are both quantitative and qualitative basic attributes in a design selection problem. The transformation could be justified on the ground that certainty may be viewed as a special case of uncertainty.

All basic attributes in Figure 1, whether quantitative or qualitative, are measured using percentage values in the original report so that the simple weighting approach could be adopted for the selection of retro-fit options [3]. In this approach, the overall weights of attributes at a single level are first calculated by assuming an overall additive utility function. In other words, the utility of an attribute at a single level is the weighted sum of the utilities of the attributes associated at a level immediately below.

In Figure 1, for example, the overall weight of *Operating Time Factor* could be calculated as $0.65 \times 0.6 \times 0.85 \times 0.15 = 0.049725$. Thus, the hierarchical attribute structure can be simplified by a decision matrix, where the two main attributes *Ship Operation* and *Installation* are replaced by the 24 basic attributes on which each retro-fit option is evaluated. A MADM method, such as the simple weighting method, may then be used to deal with the selection problem based upon the decision matrix [3].

This simple weighting approach suffers from the demerits such as linearity of utilities, independence of preferences and direct compensation among attributes in the hierarchy which may not be acceptable to decision makers (design engineers). This may lead to irrational evaluations and consequently discourage the use of the approach for hierarchical information synthesis. Furthermore, the approach requires the precise numerical evaluations of inherently qualitative attributes. To deal with these problems, the next section introduces a new hierarchical evaluation approach based on decision theory and the Dempster-Shafer theory. This approach can handle subjective judgements with complete or incomplete uncertainty in a rational way and it does not suffer from the demerits as mentioned above.

3. Hierarchical Analysis through Evidential Reasoning

3.1 Design Evaluation via Evidence Combination

As indicated in Figure 1, the modification state of an attribute is determined by attributes associated at a level immediately below. If an attribute is only associated with one lower level attribute whose modification state is judged to be absolutely "*Minor*", then the modification state of the upper level attribute must be "*Minor*". Generally, an attribute may be associated with several lower level attributes. If the modification states of the lower level attributes are all evaluated to be absolutely "*Minor*", then the modification state of the upper level attribute associated should certainly be "*Minor*".

However, such identical evaluations are rarely available in practice. Problems may then arise as to how different evaluations of lower level attributes may be synthesised in a rational way so as to attain a proper evaluation for the modification of an associated upper level attribute. The problem may be generalised to one of addressing how each retro-fit option could be ultimately assessed and measured based upon the initial evaluations of the option on the basic attributes. The evidential reasoning approach provides an alternative way to deal with such uncertain synthesis problems by means of evidence combination for multiple attributes. This approach can model the narrowing of the hypothesis set with the accumulation of evidence. In other words, the chance that a hypothesis is true will be improved if more pieces of evidence support the hypothesis.

In Figure 1, for example, whether the modification of an attribute at a level immediately above the bottom level would be "*Fundamental*", "*Major*", "*Moderate*", "*Minor*" or "*None*" is regarded as a hypothesis. A given judgement about the modification of a basic attribute for an option is viewed as a single piece of evidence. If the modification of the basic attribute is, to certain extent, judged to be "*Minor*", then the modification of the associated upper level attribute would, to some degree, be "*Minor*" as well. The evidential reasoning approach provides a systematic way of synthesising such uncertain modification evaluations of basic attributes to produce an evaluation for an associated upper level attribute.

To apply the approach, the mutual exclusiveness and exhaustiveness of all hypotheses have to be satisfied. It is therefore necessary that all the evaluation grades be defined as distinct grades. In other words, if one of the grades is absolutely confirmed, all the other grades must not be confirmed at all; if more than one grade is confirmed simultaneously, the total degree of confidence must be one or smaller than one. In addition to this requirement, the grades must cover all possible grades the designer may need to use to judge degrees of modification.

3.2 Basic Evidential Reasoning Model and Algorithm

An evaluation analysis model is used as a framework in which to conduct multiple attribute analysis and reasoning with uncertain decision knowledge. A basic evaluation analysis model may be constructed as shown in Figure 2, in which only a single level of basic attributes are involved. This model is however a basic element in a general framework for hierarchical attribute analysis.

In Figure 2, e_k^j denotes a basic attribute such as *Operating Time Factor*, which can be directly evaluated for a given option. The set of basic attributes for evaluation of an upper level attribute, denoted by y_k, is defined by

$$E_k = \{\ e_k^1 \ \cdots \ e_k^j \ \cdots \ e_k^{L_k}\ \} \tag{6}$$

In Figure 1, if y_k denotes *Cargo Handling*, for example, then there will be five basic attributes associated with it, i.g. L_k=5. Thus, e_k^1=*Operating Time Factor*, e_k^2=*Human Error Factor*, e_k^3=*Loss of Lane Length*, e_k^4=*Loss of Stowage Flexibility*, and e_k^5=*Change to Ventilation*.

Define m_{kj}^n as a basic probability assignment to which a basic attribute e_k^j supports a hypothesis that the modification state of an attribute y_k at an option a is confirmed to a grade H_n. Suppose m is a probability function. Then, the basic probability assignment m_{kj}^n could be denoted using the function m as follows

$$m_{kj}^n = m(H_n / e_k^j(a)) \tag{7}$$

The above equation is used to show the relationship between the basic probability assignment m_{kj}^n and other parameters concerned, including evaluation grade (H_n), basic attribute (e_k^j), and design option (a). The basic probability assignment m_{kj}^n can be calculated from the given confidence degree and the normalised relative weight for e_k^j.

Similarly, define m_k^n as an overall probability assignment to which the state of an attribute y_k at an option a is confirmed to a grade H_n by the whole attribute set E_k associated with y_k. m_k^n could then be denoted as follows

$$m_k^n = m(H_n / E_k(a)) \tag{8}$$

The overall probability assignment m_k^n is obtained by combining all basic probability assignments m_{kj}^n ($j=1, ..., L_k$) for all the basic attributes associated with y_k. This combination process is carried out using the evidential reasoning algorithm as discussed later.

Suppose $\beta_{kj}^n(a_r)$ denotes a confidence degree associated with the state of a basic attribute e_k^j at an option a_r being evaluated to H_n. Then, an uncertain subjective judgement for evaluation of the state of $e_k^j(a_r)$ is expressed by the following expectation

$$S(e_k^j(a_r)) = \{(\beta_{kj}^n(a_r),\ \ H_n),\ \ n = 1, \cdots, N;\ \text{and} \sum_{n=1}^{N} \beta_{kj}^n(a_r) \ \leq\ 1\} \tag{9}$$

which indicates that the state of e_k^j at a design option a_r is evaluated to a grade H_n with a confidence degree of $\beta_{kj}^n(a_r)$ for $n=1, ..., N$. In the above formula, we assume that the state of a basic attribute e_k^j at a_r may be evaluated to any individual evaluation grade defined in H. More general uncertain subjective judgements can be handled as well [13]. $S(e_k^j(a_r))$ is then quantified using its rejection degree, defined as the following expected scale

$$p_{r,kj} = p(e_k^j(a_r)) = \sum_{n=1}^{N} \beta_{kj}^n(a_r) p(H_n) \tag{10}$$

Thus, the scales $p(H_n)$ ($n=1, ... , N$) must be defined so that in addition to the basic conditions defined in the previous section the following consistency condition can be satisfied as well, that is, for two designs a_r and a_h

$$S(e_k^j(a_r)) \text{ is preferred to } S(e_k^j(a_h)) \text{ if and only if } p_{r,kj} < p_{h,kj} \tag{11}$$

Suppose ζ_k^j expresses the relative weight of the attribute e_k^j in evaluation of y_k, and ζ_k^j can be articulated as a uniform weight as follows

$$\zeta_k = [\zeta_k^1 \cdots \zeta_k^j \cdots \zeta_k^{L_k}]^T, \quad \sum_{j=1}^{L_k} \zeta_k^j = 1, \quad 0 \le \zeta_k^j \le 1 \tag{12}$$

ζ_k^j can be readily obtained using any well-known weight assignment method, such as the eigenvector method [4, 6].

Let e_k^I be the most important attribute in the attribute set E_k, called the key attribute, i.g.

$$\zeta_k^I = \max_j \{\zeta_k^1 \cdots \zeta_k^j \cdots \zeta_k^{L_k}\} \tag{13}$$

Transform the weight ζ_k^j so that the highest weight becomes one, that is

$$\bar{\zeta}_k^j = \zeta_k^j / \zeta_k^I \qquad j = 1, \cdots, L_k \tag{14}$$

Suppose for the key attribute the following relation is true

$$m_{kI}^n(a) = \alpha_k \beta_{kI}^n(a), \quad 0 < \alpha_k \le 1 \tag{15}$$

which means that if the modification state of the most important evidence e_k^I associated with the attribute y_k is absolutely confirmed to a grade H_n at an option a (i.g. $\beta_{kI}^n(a) = 1$), then the state of y_k at a will be confirmed to the same grade to a degree of α_k.

Then, define a normalised weight for the attribute e_k^j as λ_k^j. λ_k^j is obtained by

$$\lambda_k^j = \alpha_k \bar{\zeta}_k^j \qquad j = 1, \cdots, L_k$$

(16)

which represents the normalised relative importance of the attribute e_k^j in evaluation of y_k where $0 \le \lambda_k^j \le 1$. λ_k^j will be used to calculate the basic probability assignments.

In the above equations, α_k can be interpreted as a priority coefficient representing the degree of significance for the role of the most importance attribute in the evaluation of the attribute y_k. α_k is assigned by satisfying the following relation

$$\prod_{j=1}^{L_k}(1-\alpha_k \frac{\zeta_k^j}{\zeta_k^I}) \leq \delta, \quad \delta \geq 0 \tag{17}$$

where δ is a small positive real number. It may be taken so that $1.0 \times 10^{-6} \leq \delta \leq 1.0 \times 10^{-2}$. Such an assignment means that if H_j is absolutely confirmed by all the attributes E_k then the upper level attribute associated will be confirmed to H_j to an extent of $(1-\delta)\times 100$ percent. More details about the assignment of δ can be found in reference [13].

The basic probability assignment m_{kj}^n is then calculated by

$$m_{kj}^n(a) = \lambda_k^j \beta_{kj}^n(a) \tag{18}$$

The above equation means that the fact that the state of a basic attribute e_k^j is absolutely evaluated to an evaluation grade H_n only supports to the extent of λ_k^j the hypothesis that the state of the associated attribute y_k is confirmed to the same grade H_n. It is obvious that the following inequality is true

$$\sum_{n=1}^{N} m_{kj}^n(a) \leq 1 \tag{19}$$

which means that the sum of the basic probability assignments from a basic attribute e_k^j to all individual evaluation grades H_n ($n=1, \ldots, N$) does not exceed one.

Suppose m_{kj}^H denotes the basic probability assignment to the whole set of evaluation grades represented by H, which is the remaining belief unassigned after commitment of belief to all individual evaluation grades H_n ($n=1, \ldots, N$), that is,

$$m_{kj}^H(a) = 1 - \sum_{n=1}^{N} m_{kj}^n(a) \tag{20}$$

Thus, m_{kj}^H reflects the degree of incomplete uncertainty. Such uncertainty is not assigned to individual grades but to the whole set. The evidential reasoning approach works in a way that the degree of incomplete uncertainty will be reduced with the accumulation of evidence.

A basic probability assignment matrix, denoted by $M(y_k / E_k)$, is defined to summarise the basic probability assignments for evaluation of y_k at an option a through the whole set of evidence E_k

$$M(y_k / E_k) = \begin{bmatrix} m_{k1}^1 & \cdots & m_{k1}^n & \cdots & m_{k1}^N & m_{k1}^H \\ \vdots & \cdots & \vdots & \cdots & \vdots & \vdots \\ m_{kj}^1 & \cdots & m_{kj}^n & \cdots & m_{kj}^N & m_{kj}^H \\ \vdots & \cdots & \vdots & \cdots & \vdots & \vdots \\ m_{kL_k}^1 & \cdots & m_{kL_k}^n & \cdots & m_{kL_k}^N & m_{kL_k}^H \end{bmatrix} \begin{matrix} \{e_k^1(a)\} \\ \vdots \\ \{e_k^j(a)\} \\ \vdots \\ \{e_k^{L_k}(a)\} \end{matrix} \qquad (21)$$

The overall probability assignment m_k^n is calculated by combining the above basic probability assignments using the evidential reasoning algorithm. More generally, let ψ be a subset of evaluation grades, or $\psi \subseteq H$, and m_k^ψ an overall probability assignment to which the state of y_k at a_r is confirmed to ψ by the attribute set E_k, or

$$m_k^\psi(a_r) = m(\psi / E_k(a_r)) \qquad (22)$$

If m_k^ψ for all $\psi \subseteq H$ are generated from $M(y_k / E_k)$, then the state of y_k at a_r is expressed by the following expectation

$$S(y_k(a_r)) = \{(m(\psi / E_k(a_r), \quad \psi), \quad \text{for all } \psi \subseteq H\} \qquad (23)$$

The rejection degree of $y_k(a_r)$ i.e. $p(y_k(a_r))$, is used to quantify $S(y_k(a_r))$, and is defined as the following expected scale

$$p_{r,k} = p(y_k(a_r)) = \sum_{\psi \subseteq H} m(\psi / E_k(a_r) p(\psi) \qquad (24)$$

where $p(\psi)$ is the scale of ψ and is defined as the average of $p(H_n)$ for all $H_n \subseteq \psi$. A qualitative attribute quantified by a rejection degree possesses the basic property of its marginal utilities being monotonic. In other words, for two design options, a_r and a_h, $S(y_k(a_r))$ is preferred to $S(y_k(a_h))$ if and only if $p_{rk} < p_{hk}$. Such quantification can thus form a rational basis for further decision analysis.

To introduce the evidential reasoning algorithm, define a subset $e_{I_k(i)}$ of E_k and a combined probability assignment $m_{I_k(i)}^\psi$ as follows

$$e_{I_k(i)} = \{e_k^1 \cdots e_k^i\}, \quad 1 \le i \le L_k; \quad m_{I_k(i)}^\psi = m(\psi / e_{I_k(i)}) \qquad (25)$$

where $m(\psi / e_{I_k(i)})$ is a combined probability assignment to ψ confirmed by $e_{I_k(i)}$. Then, the evidential reasoning algorithm can be stated as follows

$$\{H_j\}: \; m_{I_k(i+1)}^j = K_{I_k(i+1)}(m_{I_k(i)}^j m_{k,i+1}^j + m_{I_k(i)}^j m_{k,i+1}^H + m_{I_k(i)}^H m_{k,i+1}^j), \; j = 1, \cdots, N$$

$$\{H\}: \quad m^H_{I_k(i+1)} = K_{I_k(i+1)} m^H_{I_k(i)} m^H_{k,i+1}$$

$$K_{I_k(i+1)} = \left[1 - \sum_{\tau=1}^{N} \sum_{\substack{j=1 \\ j\neq\tau}}^{N} m^{\tau}_{I_k(i)} m^{j}_{k,i+1}\right]^{-1}$$

$$i = 1, \cdots, L_k - 1 \tag{26}$$

3.3 Hierarchical Evidence Propagation

It can be proven from the algorithm that $m^{\psi}_{I_k(L_k)}$ is the overall probability assignment ($\psi \subseteq H$) confirmed by E_k and $m^{\psi}_{I_k(L_k)}$=0 for any $\psi \subseteq H$ other than $\psi = H_j$ and H, or

$$m^j_k = m(H_j / E_k) = m^j_{I_k(L_k)}, \quad j = 1, \cdots, N, \quad \text{and} \quad m^H_k = m(H / E_k) = m^H_{I_k(L_k)} \tag{27}$$

$$m(\psi / E_k) = m^{\psi}_{I_k(L_k)} = 0 \text{ for any } \psi \subseteq H \text{ but } \psi \neq H_j \ \ (j = 1, \cdots, N) \text{ and } H \tag{28}$$

Consequently, the state of the kth attribute can be evaluated in terms of the modification expressions defined in H as follows

$$S(y_k) = \{(m^j_k, \ H_j), \ \text{for } j = 1, \cdots, \mathrm{N}\} \tag{29}$$

that is, the kth attribute is evaluated to H_j with a degree of confidence of m^j_k, $j=1, ..., N$. Such an evaluation is generated by synthesising the given judgements of the relevant basic attributes.

The rejection degree of the rth design option on the kth attribute $y_k(a_r)$, i.e. $p(y_k(a_r))$, is used to quantify $S(y_k(a_r))$ and is obtained as the following expected scale

$$p_{r,k} = p(y_k(a_r)) = \sum_{n=1}^{N} m^n_{I_k(L_k)} p(H_n) + m^H_{I_k(L_k)} p(H) \tag{30}$$

where $p(H) = \sum_{n=1}^{N} p(H_n) / N$, being the average value of $p(H_n)$.

In Figure 1, if the kth attribute at level 3 is viewed as y_k, then the modification state of the attribute can be evaluated and quantified in the same way. Suppose m^j_{3k} is the obtained overall probability assignment that the state of the kth attribute at level 3 is confirmed to H_j. Similarly, the evaluation of each attribute at this level (level 3) immediately above the bottom level could be obtained. A further problem is then to produce an evaluation of an attribute at an upper level (level 2). Suppose there are L_{2l} attributes at level 3 associated with the lth attribute y_{2l} at level 2. The set of the attributes at level 3 associated with the attribute y_{2l} at level 2 is defined by

$$F_{2l} = \{c_{2l}^{1} \cdots c_{2l}^{k} \cdots c_{2l}^{L_{2l}}\} \tag{31}$$

At this stage, the evaluations of the attributes at level 3 have already been generated. So, that the modification state of the kth attribute at level 3 is confirmed to H_j to an extent of m_{3k}^{j} ($j=1, ..., N$) could be viewed as a piece of evidence while the modification state of the lth attribute y_{2l} at level 2 may be assumed to be evaluated to any of H_j ($j=1, ..., N$). m_{2l}^{j} is a degree of confidence that the state of the lth attribute at level 2 is confirmed to H_j. The problem then becomes how to obtain m_{2l}^{j} from m_{3k}^{j} ($j=1, ..., N; k=1, ... , L_{2l}$). This problem can be readily solved in the same way as in the last subsection.

The modification state of the lth attribute at level 2 can then be evaluated and quantified by

$$S(y_{2l}) = \{(m_{2l}^{j},\ \ H_j),\ \ j = 1, \cdots, N\} \tag{32}$$

$$p_{r(2l)} = p(y_{2l}(a_r)) = \sum_{n=1}^{N} m_{I_{2l}(L_{2l})}^{n} p(H_n) + m_{I_{2l}(L_{2l})}^{H} p(H) \tag{33}$$

In a similar way, the evaluations can be propagated from level 2 to level 1. Suppose m_{1l}^{j} is the obtained overall probability assignment that the state of the lth attribute at level 1 is evaluated to H_j. Let m_{r}^{j} be a degree of confidence to which the state of the rth option is confirmed to H_j. Then, m_{r}^{j} can be obtained from m_{1l}^{j} using the evidential reasoning algorithm. The modification state of the rth option can thus be evaluated and quantified by

$$S(a_r) = \{(m_{r}^{n},\ \ H_n),\ \ n = 1, \cdots, N\} \tag{34}$$

$$p_r = p(a_r) = \sum_{n=1}^{N} m_{r}^{n} p(H_n) + m_{r}^{H} p(H) \tag{35}$$

The options can finally be compared with each other based on the magnitude of the rejection degrees obtained. In other words, the option a_r is not preferred to the option a_h if and only if $p(a_r) \geq p(a_h)$. This is because as a whole the modifications associated with the retro-fit option a_r is judged to be larger than those for a_h so that the retro-fit option a_r leads to a greater increase of cost and technical complexity.

4. An Application

4.1 Original Problem

The evidential reasoning approach has been applied to the retro-fitting of a typical short sea ferry for compliance with the requirements of SOLAS 90 (Safety of Life at Sea Regulations). The options considered to meet the increased stability requirements are the provision of sponsons (additional structures to increase the buoyancy of a ferry near the waterline), movable transverse bulkheads (internal walls used for dividing up the ferry into smaller compartments to improve safety) and the provision of buoyant wing compartments. Ro-ro ferries, in general, have large open decks and provision of additional stability in the intact condition allows them to have enhanced survivability characteristics if the water should get onto the open decks.

In this analysis, each option is considered to be compared with the original ship which has a disutility value of zero. Hence, a total disutility value which is near to zero corresponds to a high measure of merit since the departure from the initial disutility value is relatively small. The problem is then how to evaluate and rank these three retro-fit options by taking into account the attributes as shown by Figure 1.

To attain compliance with SOLAS 90, the option of retro-fitting with sponsons would need to be fitted with a pair of 1.2 meters wide sponsons over 44 percent of the ship's length. The percentage evaluations of the option with regard to the original ship in terms of the basic attributes are as shown in column 3 of Table 2 (Option S1). The detailed description of these evaluations is given by Evans [3]. For instance,

i> *Increased Berthing Time*: The addition of sponsons below the waterline would increase the ship's resistance to motion and as such hinder the ship's manoeuvrability while berthing. The envisaged increase in berthing time was estimated as 5 percent longer than before the retro-fitting. The performance index is given as 5.

ii> *Increased Power*: The addition of sponsons below the waterline will seriously alter the ship's under water hull form and its resistance to motion. The extent to which the beam can be increased for a given length of ship is limited to ensure that the ship does not require excessive horsepower in relation to its displacement and speed. The fuel cost was estimated to increase by 5 percent resulting in an additional operating cost of £51000 per annum.

iii> *Maintenance*: The use of sponsons will increase maintenance costs through the need to inspect these void spaces for corrosion and structural integrity. The estimated cost of increased maintenance is 7 percent of the ship's total cost.

To attain compliance with SOLAS 90, the option of retro-fitting with movable transverse bulkheads would need to be fitted with 2 sets of movable transverse bulkhead doors. The percentage evaluations for this option are given in column 4 of Table 2. For instance,

i> *Human Error Factor*: With 16 operations per day the possibility of human error in closing and securing the doors is estimated at 7 percent.

ii> *Collision Resistance*: The retro-fitting of movable partial height, transverse bulkhead doors does not alter the collision resistance of the ship and as such the performance index is set to 100.

iii> *Additional Crew*: With the need to operate these doors 16 times a day, and the need for quick turnaround, an extra crew member is required on the vehicle deck to assist in the operation of these doors.

iv> *Device Cost*: The cost of two sets of bulkhead doors is estimated as £270000, or 0.3% of the total ship cost.

Table 2 Original Data for Evaluation of Attributes at Each Option

Level 4	Criteria	Option S1	Option S2	Option S3
Operating Time Factor	% of extra time	0	5	22.55
Human Error Factor	% increase	0	7	7
Loss of Lane Length	% loss	0	4	13
Loss of Stowage Flexibility	% loss	0	4	30
Change to Ventilation	% alteration	0	0	10
Berthing Operations	% increase in time	5	0	0
Collision Resistance	% lost potential	86	100	0
Change to Intact Condition	% decrease in the natural roll period	25	0	0
Pilot Access	% change	0	0	0
Passenger Access	% change	20	0	15
Vehicle Access	% change	0	0	24
Additional Crew	yes/no (1/0)	0	1	0
Increased Power	% increase	5	0	0
Maintenance	% increase	7	1	2
Modified Linkspan	% cost of ship	0.1	0	0
Modified Passenger Access	% cost of ship	0.01	0	0
Device Cost	% cost of ship	2.54	0.3	1.4
Fitting Cost	% cost of ship	0.65	0.07	0
Out of Service Cost	% cost of ship	0.42	0	0.3
Modify Access Equipment	% cost of ship	0	0	0
Modify LSA Equipment	% cost of ship	0.2	0	0

Auxiliary Modification	% cost of ship	0.09	0	0
Stabiliser Installation	% cost of ship	0	0	0
Ventilation Cost	% cost of ship	0	0	0

The provision of buoyant wing compartments on both sides of the ship would also ensure compliance with the SOLAS 90 requirements. The percentage evaluations of this option are given by column 5 of Table 2. For example,

i> *Operating Time Factor*: The operation of doors within the buoyant compartments would restrict cargo flow due to their arrangement. The number of doors to be operated would escalate with the number of voyages per day. It is envisaged that the operation of the doors would take approximately 9 minutes. Assuming a loading/discharging time of 40 minutes the operation time factor increases by 22.5 percent.

ii> *Loss of Stowage Flexibility*: The construction of the entrance into the buoyant wing compartment will provide considerable difficulty for vehicles other than cars. The loss of stowage flexibility is therefore regarded as quite considerable at 30 percent.

iii> *Change to Ventilation*: The venting of the vehicle deck will be restricted in the wing compartments due to its construction. The degree of change is assumed as 10 percent.

iv> *Out of Service Cost*: The additional time, outside the normal refit, to complete this work is estimated to be 14 days which translates to an expected cost of £243600.

4.2 Problem Transformation

The selection problem of the retro-fit options as represented by Figure 1 and Table 2 could be dealt with using the simple weighting approach briefly described in section 2.2. This approach, however, suffers from the demerits such as linearity of utilities, independence of preferences and direct compensation among attributes. This section is therefore devoted to applying the evidential reasoning approach to analyse the problem.

To do so, the original data presented in Table 2 needs to be transformed into equivalent statements using the evaluation grades as defined in section 2.2. The set of evaluation grades is therefore defined and quantified by

$$\begin{aligned} H &= \{\, H_1 \quad H_2 \quad H_3 \quad H_4 \quad H_5 \,\} \\ &= \{\, \textit{Fundamental} \quad \textit{Major} \quad \textit{Moderate} \quad \textit{Minor} \quad \textit{None} \,\} \end{aligned} \tag{36}$$

$$\begin{aligned} p\{H\} &= (\, p(H_1) \quad p(H_2) \quad p(H_3) \quad p(H_4) \quad p(H_5)) \\ &= (\, 1 \quad 0.7 \quad 0.5 \quad 0.3 \quad 0\,) \end{aligned} \tag{37}$$

Then, each percentage value in Table 2 could be transformed to be equivalent to a statement. For instance, the *Berthing Operations* for the option of retro-fitting with sponsons has a performance index of 0.05 (5 percent). As $0 < 0.05 < 0.3$, it could be equivalent to state that in terms of *Berthing Operations* the modification of this option from the original ship is "*Minor*" to an extent of 0.1667 and "*None*" to 0.8333. This is because the numerical value obtained by quantifying the statement using the technique as presented in section 3.2 is exactly 0.05. In this way, all the percentage values listed in Table 2 could be transformed into equivalent statements. By equivalence it is meant that a percentage value and the quantification of its transformed statement is the same. Table 3 is obtained by transforming the data of Table 2.

In Table 3, the grades "*Fundamental*", "*Major*", "*Moderate*", "*Minor*" and "*None*" are abbreviated by "*FU*", "*MA*", "*MO*", "*MI*" and "*NO*", respectively. The real numbers in the brackets following the abbreviations denote the degrees of confirmation. The statement examined above is thus represented by "*MI*(0.1667), *NO*(0.8333)", as shown by column 2 and row 7 of Table 3. It may be noted that the state of each basic attribute at an option may be confirmed to one or more of the defined grades simultaneously and the sum of the degrees of such confirmation is one. Thus all the statements in Table 3 have complete uncertainty.

Table 3 Equivalent Statements for Evaluation of Attributes at Each Option

Level 4	Option S1	Option S2	Option S3
Operating Time Factor	NO(1)	MI(0.1667), NO(0.8333)	MI(0.7517), NO(0.2483)
Human Error Factor	NO(1)	MI(0.2333), NO(0.7667)	MI(0.2333), NO(0.7667)
Loss of Lane Length	NO(1)	MI(0.1333), NO(0.8667)	MI(0.4333), NO(0.5667)
Loss of Stowage Flexibility	NO(1)	MI(0.1333), NO(0.8667)	MI(1)
Change to Ventilation	NO(1)	NO(1)	MI(0.3333), NO(0.6667)
Berthing Operations	MI(0.1667), NO(0.8333)	NO(1)	NO(1)
Collision Resistance	FU(0.5333), MA(0.4667)	FU(1)	NO(1)
Change to Intact Condition	MI(0.8333), NO(0.1667)	NO(1)	NO(1)
Pilot Access	NO(1)	NO(1)	NO(1)
Passenger Access	MI(0.6667), NO(0.3333)	NO(1)	MI(0.5000), NO(0.5000)
Vehicle Access	NO(1)	NO(1)	MI(0.8000), NO(0.2000)
Additional Crew	NO(1)	FU(1)	NO(1)
Increased Power	MI(0.1667), NO(0.8333)	NO(1)	NO(1)
Maintenance	MI(0.2333), NO(0.7667)	MI(0.0333), NO(0.9667)	MI(0.0667), NO(0.9333)
Modified Linkspan	MI(0.0033), NO(0.9967)	NO(1)	NO(1)
Modified Passenger Access	MI(0.0003), NO(0.9997)	NO(1)	NO(1)
Device Cost	MI(0.0847), NO(0.9153)	MI(0.0100), NO(0.9900)	MI(0.0467), NO(0.9533)
Fitting Cost	MI(0.0217), NO(0.9783)	MI(0.0023), NO(0.9977)	NO(1)
Out of Service Cost	MI(0.0140), NO(0.9860)	NO(1)	MI(0.0100), NO(0.9900)
Modify Access Equipment	NO(1)	NO(1)	NO(1)
Modify LSA Equipment	MI(0.0067), NO(0.9933)	NO(1)	NO(1)

Auxiliary Modification	MI(0.0030), NO(0.9970)	NO(1)	NO(1)
Stabiliser Installation	NO(1)	NO(1)	NO(1)
Ventilation Cost	NO(1)	NO(1)	NO(1)

4.3 Hierarchical Analysis Using the Evidential Reasoning Approach

The evidential reasoning analysis can then be applied to generate the evaluation of each option by hierarchically synthesising these obtained degrees of confirmation on the basis of the attribute hierarchy and the relative weights of the attributes as shown in Figure 1. It is assumed that the modification state of an attribute at a single level is almost certainly "*Minor*" if the modification states of all the attributes associated at a level immediately below are regarded to be absolutely "*Minor*". It is assumed that "almost certainly" is modelled by assigning $\delta = 0.01$. Such an assumption is necessary for conflict resolution using the evidential reasoning approach although alternative values may also be assigned to δ.

A priority coefficient α and normalised relative weights λ can then be calculated. For instance, the attribute *Cargo Handling* at level 3 is associated with the attributes *Operating Time Factor*, *Human Error Factor*, *Loss of Lane Length*, *Loss of Stowage Flexibility* and *Change to Ventilation* at level 4 as shown in Figure 1. Since option 1 is not modified in terms of these five lower level attributes, it should be judged that option 1 is not modified either in terms of *Cargo Handling* to an extent of over 99 percent. α associated with *Cargo Handling* can then be readily obtained by solving the following inequality

$$\left(1-\alpha\frac{0.15}{0.55}\right)\left(1-\alpha\frac{0.10}{0.55}\right)\left(1-\alpha\frac{0.55}{0.55}\right)\left(1-\alpha\frac{0.15}{0.55}\right)\left(1-\alpha\frac{0.05}{0.55}\right) \le \delta = 0.01 \tag{38}$$

An approximate procedure is suggested as follows to obtain α. Firstly, assign an initial value to α by assuming that all the attributes are of equal importance, that is

$$\alpha^0 = 1-\delta^{1/5} = 1-0.01^{1/5} = 0.6019 \tag{39}$$

Then, assign α^t so that α^t increases from α^{t-1} by for example 0.001 (*t = 1, 2, ...*) until the inequality is satisfied. This simple procedure is of general use as it is easy to implement for any number of attributes and for any relative weights. Note that the initially assigned α^0 is the lower bound of α satisfying the inequality. α for *Cargo Handling* is then assigned to 0.9759.

Let y_k stand for *Cargo Handling* and E_k for the five associated basic attributes. The basic probability assignment matrix for evaluation of *Cargo Handling* through the five basic attributes can then be constructed. These basic probability assignments can be combined using the evidential reasoning algorithm as presented in the previous section. The generated evaluation for *Cargo Handling* is as shown by column 2 and row 2 of Table 4, that is "*NO(0.9903), H(0.0097)*". This evaluation states that there is almost no difference at all between option 1 and the original ship in terms of *Cargo Handling*. The real number 0.0097 in *H(0.0097)* stands for the remaining uncertainty unassigned to the defined single evaluation grades. Such propagation of the evaluations from level 4 to level 3 is rational in the sense that δ has been assigned to 0.01.

In a similar way, all the attributes at level 3 can be evaluated through the attributes at level 4. The results are shown by Table 4. The attributes at level 2 can in turn be evaluated through the attributes at level 3, as shown by Table 5. Table 6 shows the evaluations of *Ship Operation* and *Installation* at each option, which are generated through the attributes at level 2. Finally, the evaluations of these three options are obtained in terms of the degrees of modification, as shown by row 2 of Table 7.

Table 4 Obtained Judgements for Evaluation of Attributes at Each Option

Level 3	Option S1	Option S2	Option S3
Cargo Handling	NO(0.9903), H(0.0097)	MI(0.0757), NO(0.9126) H(0.0117)	MI(0.5112), NO(0.4737) H(0.0151)
Berthing Operation	MI(0.1667), NO(0.8333)	NO(1)	NO(1)
Integrity	FU(0.0065), MA(0.0057) MI(0.8091), NO(0.1619) H(0.0168)	FU(0.0122), NO(0.9710) H(0.0168)	NO(0.9902), H(0.0098)
Daily Operation Requirements	MI(0.0456), NO(0.9348) H(0.0196)	NO(0.9901), H(0.0099)	MI(0.3224), NO(0.6470) H(0.0306)
Running Cost	MI(0.1160), NO(0.8727) H(0.0113)	FU(0.0021), MI(0.0002) NO(0.9861), H(0.0116)	MI(0.0004), NO(0.9896) H(0.0100)
Transfer Facilities	MI(0.0019), NO(0.9882) H(0.0099)	NO(0.9902), H(0.0098)	NO(0.9902), H(0.0098)
Procurement	MI(0.0407), NO(0.9490) H(0.0103)	MI(0.0045), NO(0.9857) H(0.0098)	MI(0.0215), NO(0.9685) H(0.0100)
Secondary Modification	MI(0.0001), NO(0.9900) H(0.0099)	NO(0.9901), H(0.0099)	NO(0.9901), H(0.0099)

Table 5 Obtained Judgements for Evaluation of Attributes at Each Option

Level 2	Option S1	Option S2	Option S3
Port Activities	MI(0.0007), NO(0.9810) H(0.0183)	MI(0.0626), NO(0.9177) H(0.0197)	MI(0.4573), NO(0.5184) H(0.0244)
Ship Inherent	FU(0.0001), MA(0.0001) MI(0.1404), NO(0.8370) H(0.0224)	FU(0.0013), MI(0.0001) NO(0.9826), H(0.0160)	MI(0.0011), NO(0.9830) H(0.0159)
Port	MI(0.0019), NO(0.9883)	NO(0.9902), H(0.0098)	NO(0.9902), H(0.0098)

Ship	H(0.0098) MI(0.0041), NO(0.9836) H(0.0123)	MI(0.0004), NO(0.9877) H(0.0119)	MI(0.0021), NO(0.9858) H(0.0121)

Table 6 Obtained Judgements for Evaluation of Attributes at Each Option

Level 1	Option S1	Option S2	Option S3
Ship Operation	MI(0.0049), NO(0.9767) H(0.0184)	MI(0.0230), NO(0.9592) H(0.0178)	MI(0.2253), NO(0.7486) H(0.0261)
Installation	MI(0.0006), NO(0.9873) H(0.0121)	NO(0.9879), H(0.0121)	MI(0.0002), NO(0.9877) H(0.0121)

Table 7 Ranking of The Three Options

Selection	Option S1	Option S2	Option S3
Evaluations	MI(0.0023), NO(0.9789) H(0.0188)	MI(0.0109), NO(0.9705) H(0.0186)	MI(0.1195), NO(0.8553) H(0.0252)
Disutilities	0.0101	0.0126	0.0485
Ranking	1	2	3

4.4 Result Analysis

The disutilities and the ranking of the three retro-fit options are shown by the last two rows of Table 7. The most favourable option is the sponson, followed closely by the movable bulkheads and then by the wing compartments, The results are in harmony with the analysis conducted by Evans [3].

The sponsons prove beneficial in the analysis for short sea ro-ros as the loss of *Collision Resistance* and the decrease in the natural roll period associated with *Change to Intact Condition* do not play an important role in the evaluation of the whole ship. This is also because the increase in power and hence increased fuel costs have a small overall effect on the total economics. This option does not affect the ro-ro concept, disrupting the through flow of vehicles, which for the short sea ro-ro has a critical effect on the operating economics.

The retro-fitting with buoyant wing compartments is the least preferred option in this analysis due to their disruptive effects on *Loss of Stowage Flexibility* (the through flow of traffic) and on *Cargo Handling*.

4.5 Design Evaluation with Incomplete Uncertainty

The transformed data as shown in Table 3 include only complete uncertainty for both quantitative and qualitative attributes. It is quite likely that subjective judgements with incomplete uncertainty could be provided for evaluation of qualitative attributes. This may be incurred because of the complexity and novelty of a design problem or because a team is involved for design evaluation with each member having different views in the evaluation. To demonstrate the potential of the evidential reasoning approach for dealing with complex problems with incomplete uncertainty, the above example is modified so that incomplete uncertainty is introduced in evaluation of the qualitative attributes as classified in Table 1.

The evaluations of the three retro-fit options on the seven qualitative attributes are modified as shown in Table 8 and the other evaluations on all the quantitative attributes are the same as given in Table 3. In Table 8, one can find that incomplete uncertainty is assumed. For instance, the state of *Options S2* on *Human Error Factor* is assigned to *Moderate* (*MO*), *Minor* (*MI*) and *None* (*NO*) to the degrees of 0.1, 0.2 and 0.6, respectively, and the sum of these degrees is 0.9 less than one. Such incomplete uncertain judgements may result from the inability of design engineers to provide precise judgements.

Table 8 Modified Judgements for Evaluation of Qualitative Attributes at Each Option

Level 4	Option S1	Option S2	Option S3
Human Error Factor	NO(1)	MO(0.1000), MI(0.2000), NO(0.6000)	MO(0.0500), MI(0.3000), NO(0.5000)
Loss of Stowage Flexibility	NO(1)	MI(0.2000), NO(0.7000)	MO(0.1000), MI(0.8000), NO(0.0500),
Collision Resistance	FU(0.4000), MA(0.4000), MO(0.1000),	FU(0.8000), MA(0.15000),	NO(1)
Change to Intact Condition	MI(0.8000), NO(0.1500)	NO(1)	NO(1)
Pilot Access	NO(1)	NO(1)	NO(1)
Passenger Access	MI(0.6000), NO(0.3000)	NO(1)	MO(0.1000), MI(0.4000), NO(0.4500)
Vehicle Access	NO(1)	NO(1)	MO(0.1500), MI(0.7000), NO(0.1000)

The results of combining the given judgements and transformed statements from the bottom level to the level immediate above (i.g. level 3) are shown in Table 9. From the table, one can clearly see the effectiveness of the evidential reasoning approach both in reducing the degrees of uncertainty unassigned to individual evaluation grades and in narrowing the hypothesis set with the accumulation of evidence.

The evaluation of *Option S3* on *Cargo Handling*, for example, are associated with five basic attributes including *Operating Time Factor*, *Human Error Factor*, *Loss of Lane Length, Loss of Stowage Flexibility*, and *Change to Ventilation*. Two of the five basic attributes are qualitative ones, i.g. *Human Error Factor* and *Loss of Stowage Flexibility*. The initial evaluations of *Option S3* on the two attributes are shown by the second block row of the last column in Table 8. Both evaluations involve incomplete uncertainty as the total assigned degree of confidence in individual evaluation grades is each 85 percent only. The remaining confidence degree of 15 percent is assigned to none of the five individual evaluation grades but to the whole evaluation set. The combined evaluation of *Option S3* on *Cargo Handling* is shown in the second block row of the last column in Table 9. The uncertainty assigned the whole set (*H*) is now reduced to 1.57 percent. This reduction results from the accumulation of the evaluations for the five lower level attributes.

At *Option S3*, the states of the five basic attributes associated with *Cargo Handling* are evaluated to the three grades *Moderate, Minor* and *None* to various degrees. While all the five attributes are evaluated to the last two grades to large degrees, only the two qualitative attributes *Human Error Factor* and *Loss of Stowage Flexibility* are evaluated to *Moderate* to the degrees of 5 percent and 10 percent, respectively (see Table 8). Since the evaluations to *Moderate* from the two qualitative attributes are much less significant and are not supported by the other attributes, the evaluation for *Cargo Handling* is thus narrowed to the two grades *Minor* and *None* to the large degrees of nearly 51 percent and 48 percent, respectively.

The final evaluations for the three retro-fit options as shown in Table 10 are similar to those as obtained in Table 7 and both rankings are the same. This is because the modifications provided as in Table 8 are not significant. Such analysis is however useful to investigate the robustness of the evaluations obtained.

Table 9 Obtained Judgements for Evaluation of Attributes at Each Option

Level 3	Option S1	Option S2	Option S3
Cargo Handling	NO(0.9903), H(0.0097)	MO(0.0003), MI(0.0832), NO(0.9039), H(0.0127)	MO(0.0007), MI(0.5077), NO(0.4758) H(0.0157)
Berthing Operation	MI(0.1667), NO(0.8333)	NO(1)	NO(1)
Integrity	FU(0.0173), MA(0.0173), MO(0.0043), MI(0.7559), NO(0.1417), H(0.0636)	FU(0.0094), MA(0.0018), NO(0.9720), H(0.0168)	NO(0.9902), H(0.0098)
Daily Operation Requirements	MI(0.0374), NO(0.9385) H(0.0241)	NO(0.9901), H(0.0099)	MO(0.0446), MI(0.2970), NO(0.6112), H(0.0471)
Running Cost	MI(0.1160), NO(0.8727) H(0.0113)	FU(0.0021), MI(0.0002) NO(0.9861), H(0.0116)	MI(0.0004), NO(0.9896) H(0.0100)
Transfer Facilities	MI(0.0019), NO(0.9882) H(0.0099)	NO(0.9902), H(0.0098)	NO(0.9902), H(0.0098)
Procurement	MI(0.0407), NO(0.9490) H(0.0103)	MI(0.0045), NO(0.9857) H(0.0098)	MI(0.0215), NO(0.9685) H(0.0100)
Secondary Modification	MI(0.0001), NO(0.9900) H(0.0099)	NO(0.9901), H(0.0099)	NO(0.9901), H(0.0099)

Table 10 Ranking of The Three Options

Selection	Option S1	Option S2	Option S3
Evaluations	MI(0.0023), NO(0.9790) H(0.0188)	MI(0.0121), NO(0.9690) H(0.0189)	MO(0.0002), MI(0.1183), NO(0.8562), H(0.0253)
Disutilities	0.0101	0.0131	0.0482
Ranking	1	2	3

5. Concluding Remarks

This paper demonstrates how the evidential reasoning approach could be used as an alternative tool to deal with real world complex decision analysis problems in engineering design. Such problems often involve both quantitative attributes and inherently subjective (qualitative) attributes. The evaluations of subjective attributes are usually associated with uncertainty. Although in traditional evaluation methods, such as the simple weighting method, subjective attributes could be assessed using some scaling values such as percentage values, the new evidential reasoning approach provides a more natural yet rational approach for articulating and processing information for evaluation of subjective attributes with complete or incomplete uncertainty. Furthermore, when dealing with multiple attribute decision making (MADM) problems, this new approach does not suffer from the demerits such as linearity of utilities, independence of preferences and direct compensation among attributes.

As has been indicated by Evans [3], more detailed analysis for selection of retro-fit options would need to be carried out on a ship by ship basis using further levels of attributes. Indeed, different operating scenarios may need to be taken into account for evaluation of an attribute at an option. However, the evidential reasoning approach sets no limit on the number of attribute levels of a problem.

ACKNOWLEDGEMENT

The authors are indebted to Mr J.P. Evans of Transmarine Ltd, Wallsend Research Station, Wallsend, UK for his permission to allow the authors to use the hierarchical evaluation model on the original data which he developed. This work is partly supported by the UK Engineering and Physical Science Research Council under Grant no.GR/F 95306.

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