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Uncertainty and Preference Modelling for Multiple Criteria Vehicle Evaluation

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Abstract

A general framework for vehicle assessment is proposed based on both mass survey information and the evidential reasoning (ER) approach. Several methods for uncertainty and preference modeling are developed within the framework, including the measurement of uncertainty caused by missing information, the estimation of missing information in original surveys, the use of nonlinear functions for data mapping, and the use of nonlinear functions as utility function to combine distributed assessments into a single index. The results of the investigation show that various measures can be used to represent the different preferences of decision makers towards the same feedback from respondents. Based on the ER approach, credible and informative analysis can be conducted through the complete understanding of the assessment problem in question and the full exploration of available information.

Keywords: Multiple criteria analysis; Vehicle evaluation; Uncertainty modeling; Evidence theory

1. Introduction

Uncertainty is one of the main concerns in most decision making processes and has been researched intensively[1-6]. Although the types and sources of uncertainty for various problems may be different, they share some common features[7]. As an empirical study, this paper is aimed to investigate pragmatic ways for handling uncertainty in assessment of vehicle attributes.

Evaluating vehicle attributes is an important activity of a process which can help both OEMs (i.e., original equipment manufacturers) and consumers understand the quality and attractiveness of various vehicles. To facilitate vehicle attribute assessment, different types of surveys and tests may be conducted for gathering first-hand data. These surveys and tests might be conducted internally by an OEM, independent specialist consultancy companies, consumer groups, or government departments.

One example is the survey carried out by J.D. Power and Associates, Automotive Performance Execution And Layout Study (APEAL). It is a mail survey that asks the consumers what they like and don't like about the various vehicle attributes listed on the questionnaire. It measures customer ratings on retail purchasers and lessees from randomly drawn samples on over 100 vehicle attributes including ride and handling, engine and transmission, comfort and convenience and so on[8].

In APEAL, vehicle attributes are rated using a ten point scale on which 1 might represent 'unacceptable' while 10 might represent 'outstanding'. To make the data as accessible as possible, experts at J. D. Power convert thousands of questions and answers into scores in a few categories, which are then aggregated to give vehicle attributes an overall score. This pivotal figure is then expressed as a percentage and an overall rating[9].

Another example is the rating listed in the Consumer Reports published by Consumers Union[10-12]. Consumer Reports rates the performance of the vehicles tested by the experts of Consumers Union at its specialized auto-test facility and compiles regularly updated charts showing which models perform best and worst overall. Assessments of vehicles on many designated attributes are based on various scales among which an ordinal 5-point scale may take the following form:

- 1 = "Poor"
- 2 = "Fair"
- 3 = "Good",
- 4 = "Very Good"
- 5 = "Excellent"

The examples show that to distinguish or rank vehicle attributes, more than one rating or survey on vehicles may be obtained (especially for big OEMs who might even have their own surveys on vehicles). It is obvious that each survey has its own style and specific attribute settings. The statements used in one survey may somewhat differ from those used in other surveys, although they may belong to the same or similar attributes.

It is often the case that an analyst needs to use multiple surveys in assessing vehicle attributes. Hence, it is important to combine assessments from diverse surveys in order to produce comprehensive and consistent assessments. This requires transformation or mapping of survey data from various formats to a common format. While linear functions are most common and might perform well for certain transformations, a question arises as to whether nonlinear transformation functions might perform better in other cases. This forms one of the research questions investigated in this paper. It is likely that a subject (consumer or expert assessor) may not provide assessments for a vehicle on certain attributes. It is also possible that in a hierarchy of detailed vehicle attributes, there may be partial or no assessments on certain attributes. Such missing information can cause problems in follow-up analysis and need to be handled properly in order to provide reliable and non-distorted overall assessments on vehicles. This paper is devoted to investigating these issues for objective and consistent vehicle attribute assessment based on multiple surveys.

In this paper, a general framework for vehicle attribute assessment is firstly constructed on the basis of the Evidential Reasoning (ER) approach. The procedure for constructing and using this general framework in assessment of vehicle attributes is discussed in Section 2. In Section 3, various methods for estimating missing information in surveys are investigated. Sections 4 and 5 are devoted to studying the impacts of nonlinear mapping functions and utility functions on overall assessments. The application of these methods for dealing with uncertainty is demonstrated using a case study in Section 6, and the paper is concluded in Section 7.

2. The General Assessment Framework and Related Issues

The information propagation and aggregation in assessing vehicle attributes can be conducted using the Evidential Reasoning (ER) approach[13]. Based on different survey results or original ratings, the comprehensive assessments or overall ratings of vehicle attributes can be generated using the IDS Multi-criteria Assessor software[14], which has been developed on the basis of the ER approach[15]. The ER approach and the IDS software provide a methodological basis and a tool for this research.

As stated in Section 1, multiple surveys, different forms of questionnaires and various rating scales may be used for assessing vehicle attributes. It is usually desirable to take into account as much evidence as available and appropriate for vehicle assessment, including imprecise and incomplete information that may exist in surveys. Next, we propose and discuss a generic procedure for assessing and ranking vehicle attributes under the above-mentioned scenario.

2.1. Proposed generic procedure

The proposed generic procedure includes six main steps as follows.

(1) Collect and validate data from all surveys for all related vehicles.
(2) Construct a detailed vehicle attribute hierarchy.
(3) Construct an attribute assessment hierarchy based on evidence aggregation logic and algorithms.
(4) Transform survey data to assessments measured on a common scale using mapping functions.
(5) Use the IDS software to aggregate all transformed assessments for a vehicle attribute.
(6) Rank vehicle attributes based on overall assessments.

These steps are normally followed in the above order, though there may be interactions between these steps in the process of vehicle assessment. For example, the order between step (3) and step (4) does not have to be followed strictly. Next, we discuss a general hierarchy structure for assessing different attributes of a vehicle.

2.2. General hierarchy structure for attribute assessment

Fig. 1 illustrates a general hierarchy for assessing vehicle attributes. A vehicle attribute might have information from multiple surveys and might have more than one related statement within a survey. Different criteria might be used to relate survey data to a vehicle attribute and hence assess that particular vehicle attribute. For example, a criterion could be that the mean rating for a survey statement of an OEM's vehicle should be greater than the mean rating for that survey statement of a competitor OEM's vehicle. The hierarchy incorporates sub-criteria within a criterion and the hierarchy could be expanded both horizontally and vertically. The ER approach, discussed in Sub-section 2.4, is capable of handling complex hierarchies. The survey data is usually collected at the leaf node of a branch and then the ER approach is used to aggregate the data bottom-up for each vehicle attribute.

As mentioned above, different surveys use different scales for obtaining ratings of the statements in the questionnaire. For aggregating survey information related to vehicle attributes, survey data have to be transformed to a common scale. In the next sub-section, we discuss a representative common scale that can be used for assessing vehicle attributes.

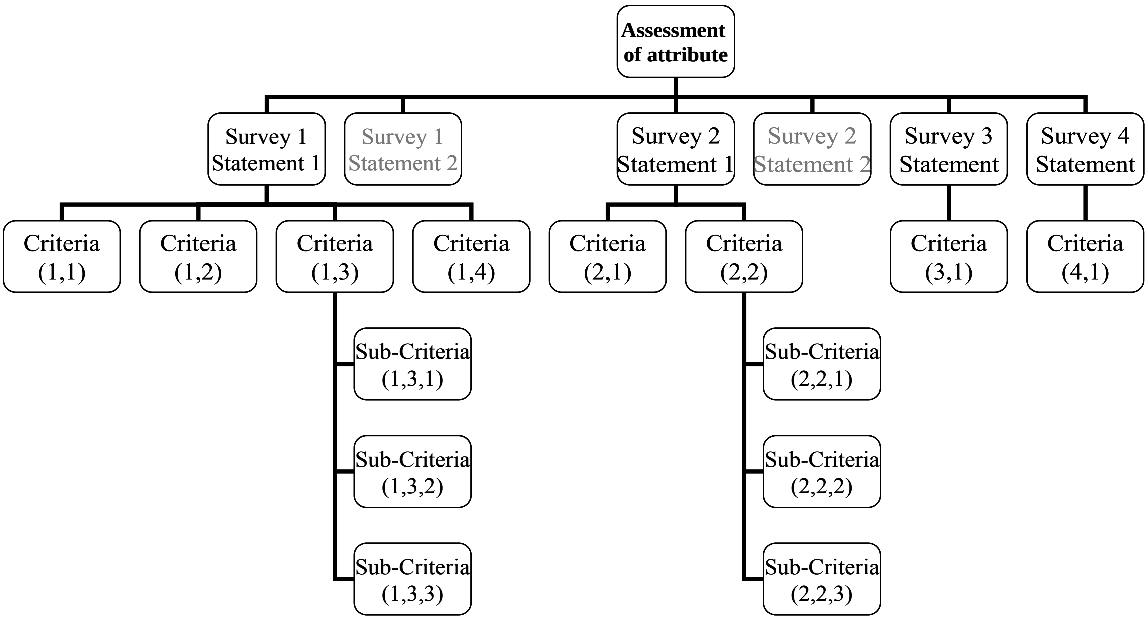

Fig. 1. A general hierarchical structure for attribute assessment

2.3. The common scale

Assessments on an attribute in various surveys may be measured on different scales, which may be judgmental, cardinal, or ordinal. Also, a scale may have various grades. As such, a common scale is employed in the general assessment framework, so that all assessments generated from various surveys can be measured on the same scale and direct comparisons among the assessments can be made. For example, a viable common scale with an ordinal five grades from 1 to 5 is listed below. To take into account missing information, one more grade (unassigned priority) can be added to the common scale.

- 5 = "Top priority"
- 4 = "High priority"
- 3 = "Average priority"
- 2 = "Low priority"
- 1 = "No priority"
- UN = "Unassigned priority"

We use the above common scale for illustrating our results in the coming sections.

2.4. The Evidential Reasoning algorithm and its logic

The ER algorithm is developed on the basis of a multi-attribute evaluation framework[16] and the evidence combination rule of the Dempster-Shafer (D–S) theory[17]. It can be used to aggregate criteria of a multilevel structure. It is employed in the general assessment framework for propagating and aggregating information. The rationale and logic of the ER approach has been discussed in Refs. 15 and 18-20.

The main function of the ER algorithm in the general assessment framework is to calculate the combined degrees of belief of a group of criteria for generating an assessment on their parent criterion using a group of recursive formulas. Such calculations are conducted from the lowest level to the top level in an assessment hierarchy as shown in Fig. 1, and eventually the overall assessment can be obtained in terms of a distributed assessment. To assist in ranking a series of vehicle attributes, the concept of expected utility may be used as an auxiliary measure.

A general-purpose multiple criteria decision analysis tool, the Intelligent Decision System (IDS) software[14] can be used for this purpose, which is based on the Evidential Reasoning (ER) approach. IDS provides Windows-based graphical interfaces to support the building of decision models where all vehicle attributes can be assessed on a general hierarchy using a belief structure. The rank order of all vehicle attributes can be generated on the basis of utility scores. With the help of the IDS software, the general assessment framework can be employed efficiently and easily.

Among the advantages of using this framework for assessing vehicle attributes, the explicit representation and propagation of uncertainty in the information aggregation process is of particular interest. By allowing imprecise and incomplete information to be modeled for assessment of vehicle attributes, the framework can be widely applicable and can produce transparent and reliable results in a consistent and transparent way. Imprecise and incomplete information in the assessment of vehicle attributes may be incurred from missing information in an original survey, from data transformation and representation, and so on. It is crucial to measure and express all types of uncertainty properly before the process of assessing vehicle attributes is started.

As such, three related issues will be studied in more detail in the following sections. First, the measurement of uncertainty caused due to missing information in original surveys is studied. The estimation of missing information is also discussed for certain circumstances. Secondly, the possibility and properties of using nonlinear functions for data transformation are investigated. Thirdly, the properties of using nonlinear functions as utility function in aggregating distributed assessments into a single index are studied for the cross comparison and rank ordering of attributes.

The skewness of evaluation rating may also be a source for uncertainty in both data survey and transformation stages. This type of uncertainty can be dealt with by using nonlinear functions as mapping function or as utility function in some circumstances as discussed in Sections 4 and 5.

3. Missing Information

3.1 Measurement of uncertainty

Missing information often exists in a survey. There are basically two types of missing information in a survey. One is failed observation, which means that there is no valid information from a respondent. This may be because the respondent has no idea or simply refuses to give his or her opinion about all survey statements. The other is failed data or missing information in a valid observation, which means that a respondent does give valid responses to at least one, but not all, questions in a questionnaire. This might be because the respondent only has part of the knowledge required to answer the full questionnaire or the questionnaire is time consuming to complete. In the former situation, the failed observation either can be taken as no information and be counted in the sample or can simply be deleted and not counted in the sample. In the latter situation, the observation with failed data or missing information should be taken into account as it is normal for a respondent to have only partial knowledge about a specific vehicle but still have interests to participate in the survey and provide what he or she knows. Note that we assume that the respondent does not deliberately (or by any other means) give incorrect response when he or she has the knowledge about the vehicle.

For a single vehicle attribute, the total number of valid observations is the sample size, and the number of observations which have no valid response is the amount of missing information. The amount of missing information is divided by the sample size to give the level of missing information or unassigned belief degree which is defined as the measure of uncertainty in question. Based on this definition, uncertainty caused due to missing information can be determined by Eq. (1) as follows.

$$UN = \frac{T_s - M_s}{T_s} \tag{1}$$

where UN — unassigned belief degree or the measure of uncertainty

T_s — sample size for evaluation of a vehicle attribute

M_s — number of respondents who provide valid response for evaluation of a vehicle attribute.

Sometimes criteria used for relating survey data to a vehicle attribute might need information from two or more vehicles. For example, relative position of two vehicles in a particular survey could be used for assessing the importance of an attribute. In such cases, Eq. (2) can be used for taking into account the impact of missing information for both vehicles. Note that Eq. (2) can be easily extended for taking into account data from more than two vehicles.

$$UN = \varphi_{s1} \cdot \frac{T_{s1} - M_{s1}}{T_{s1}} + \varphi_{s2} \cdot \frac{T_{s2} - M_{s2}}{T_{s2}} \tag{2}$$

where UN — unassigned belief degree or the measure of uncertainty for comparison of two vehicles

T_{s1} — sample size for the first vehicle

T_{s2} — sample size for the second vehicle

M_{s1} — number of respondents who provide valid response for the first vehicle

M_{s2} — number of respondents who provide valid response for the second vehicle

$\varphi_{s1}, \varphi_{s2}$ — the respective weights for the two vehicles in determining uncertainty, and $\varphi_{s1} + \varphi_{s2} = 1$.

One can set $\varphi_{s1} = \varphi_{s2} = \frac{1}{2}$ if the impact of individual vehicle's sample size is not to be accounted for or can set $\varphi_{s1} = \frac{T_{s1}}{T_{s1} + T_{s2}}$ and $\varphi_{s2} = \frac{T_{s2}}{T_{s1} + T_{s2}}$ if the impact of individual sample sizes is to be accounted for. Other methods of setting the weights φ_{s1} and φ_{s2} might be used according to individual circumstances and the preferences of the analyst.

For example, if the sample size and the number of valid responses in a sample for an attribute of the first vehicle are 200 and 192 respectively, the level of missing information or the uncertainty in the evaluation on that attribute is 0.04 by Eq. (1). If we set $\varphi_{s1} = \varphi_{s2} = \frac{1}{2}$ and the sample size and the number of valid responses for the same attribute of the second vehicle are 240 and 228, the level of missing information or the uncertainty in the evaluation comparison of the two vehicles on the attribute is 0.045 by Eq. (2).

A special case in Eq. (2) that needs to be noted is that T_{s1} or T_{s2} or both are equal to 1. If T_{s1} is equal to 1 and M_{s1} is equal to 0 or T_{s2} is equal to 1 and M_{s2} is equal to 0, the uncertainty degree calculated using Eq. (2) may not be equal to 1 though it should be so. As in this case there must be no valid response for one of the two vehicles, the value of uncertainty obtained using Eq. (2) does not make sense if $UN \neq 1$. In fact, if M_{s1} or M_{s2} is less than a certain threshold value, the corresponding UN should be artificially assigned to 1, which means that the comparison between the two vehicles is improper if there is a lack of valid information.

3.2 Accommodating uncertainty in questionnaire

In a large scale survey, it is normal to have some subjects who might not have complete knowledge about the survey or can not provide full confidence in some assessments. To accommodate such a scenario, we propose below a new form of questionnaire that would give more freedom and flexibility to the respondents and could result in the collection of original and better quality data.

In the proposed questionnaire, the uncertainty in the subjects' response is explicitly captured by allowing the subject to provide a distributed assessment as shown in Table 1. An overall assessment on the attribute can then be calculated by Eq. (3) and Table 2.

$$BI_n = \sum_{j=1}^{m} \zeta_j \cdot BICB_{n,j} \tag{3}$$

where BI_n — the mean belief degree on grade n $(n = 1, \cdots, N)$ of an attribute

m — total number of respondents,

$BICB_{n,j}$ — belief degree on grade n for an attribute given by respondent j,

ζ_j — the weight of the j^{th} respondent in evaluating an attribute, and $\sum_{j=1}^{m} \zeta_j = 1$.

Typically, if all respondents are given the same importance, $\zeta_j = \frac{1}{m}$. Table 1 takes a traditional survey format as a special case, where a respondent ticks only one box (with 100% degree of belief), and allows more flexible yet realistic answers to survey questions. The above-suggested format of questionnaire is only one example out of many possibilities. It can be designed to be more user-friendly but without losing its essential characteristics.

3.3. Estimation of missing information

In this subsection, we discuss some cases in which missing information on an attribute can be estimated as a function of related information on other attributes. We propose some functions for estimating the missing information in those cases.

Table 1 Proposed model to account for uncertainty in assessment

Survey Statement						
Assessment grade	Worst (1)	Poor (2)	Average (3)	Good (4)	Excellent (5)	Unsure (6)
Degree of belief						

***Remark*:** please tick the box under a grade or give percentage values in the boxes under the grades that fit the statement.

Table 2 Aggregation of assessments on a survey statement's rating

Grade	Description	Belief degree (%)	Remark
1	Worst	BI_1	
2	Poor	BI_2	
3	Average	BI_3	
4	Good	BI_4	
5	Excellent	BI_5	
6	Uncertainty	BI_6	
Total belief degree			$0 \leq$ and ≤ 100 %

In a survey, there are often many attributes to be assessed which may be grouped in a hierarchy. There might also be cases where a set of attributes from various surveys are related in one way or other. In vehicle evaluation, for example, independent survey providers may disclose their survey results and OEMs may have their own evaluations, although statements for a similar attribute may be different in such various surveys. If such relationships can be explicitly expressed in a hierarchy, the following approach can be used for estimating missing information from surveys.

Suppose an attribute A can be expressed fully by a set of sub-attributes a_i, $i = 1, 2, \ldots, k$, and $k \geq 2$. Then, the attribute A is dependent on its sub-attributes which are assumed to be mutually independent for assessment. Suppose assessments on attributes provided in survey(s) include missing information. It is also assumed that the same scale is used in the assessments. If different evaluation scales are used, assessments given on various scales can be transformed to a common scale using the mapping functions discussed in Section 4.

Depending on the characters of missing information there could be six different cases to consider. The first case is one where all assessments are given. This is an ideal case and does not require any estimation of missing information. The second case is one where neither a parent attribute nor its sub-attributes are assessed. In this case, there might be no need or it might be inappropriate to estimate missing information simply because of the complete lack of information. The other four cases are discussed in detail below.

Case 1 — Assessment on the high level (or parent) attribute is unknown, but all assessments on its sub-attributes are known.

Eq. (4) can be used to estimate the unknown assessment of attribute A.

$$At_j = \sum_{i=1}^{k} \omega_i \cdot av_{i,j} \qquad j = 1, 2, \ldots\ldots, N \tag{4}$$

where At_j — estimated assessment value on the j^{th} grade of the attribute A

ω_i — weight of the i^{th} sub-attribute in the assessment on its parent attribute, and $\sum_{i=1}^{k} \omega_i = 1$

$av_{i,j}$ — assessment value on the j^{th} grade of the i^{th} sub-attribute a_i

k — number of sub-attributes related to the same parent attribute A

N — number of grades on the assessment scale

In this case, the estimated assessment on A is of full confidence as long as the assessments on a_i, $i = 1, 2, \ldots, k$ are of full confidence. More precisely, the estimated assessment on A determined by Eq. (4) has the same confidence degree as the sub-attributes.

Case 2 — Assessment on the parent attribute is unknown, and some assessments on its sub-attributes are known.

If there are s out of k sub-attributes with unknown assessments, there will be obviously s degrees of freedom in estimating the unknown assessments. In other words, s conditions are needed to estimate the unknown assessments. If s conditions indeed exist and can be identified, the estimation of the unknown assessments can be uniquely made with full confidence. Otherwise, necessary conditions have to be subjectively established and the confidence of the estimation may vary according to the reliability of the subjective conditions established.

For example, if the weights of the related sub-attributes can be generated by a fuzzy AHP procedure[21], the weighted average of known assessments can be used as the estimation of the unknown assessments. Among a variety of ways for establishing the necessary conditions, the following Eq. (5) is a simple and convenient one.

$$av_{k_1,j} = av_{k_2,j} = \cdots = av_{k_s,j} = \frac{1}{|I_2|} \sum_{i_2 \in I_2} av_{i_2,j}$$

$$k_1, k_2, \ldots\ldots, k_s \in I_1 \tag{5}$$

where I_1 — subscript set for the sub-attributes with unknown assessments

I_2 — subscript set for the sub-attributes with known assessments

$|I_2|$ — number of sub-attributes in I_2

$av_{i_2,j}$ — known assessment on the j^{th} grade of sub-attribute i_2

With s given conditions, the assessment on the unknown attribute A can be made using Eq. (6).

$$At_j = \sum_{i_2 \in I_2} \omega_{i_2} \cdot av_{i_2,j} + \sum_{i_1 \in I_1} \omega_{i_1} \cdot av_{i_1,j}$$

$$j = 1, 2, \ldots\ldots, N \tag{6}$$

It is hard to determine the confidence of the estimation given by Eq. (6), if $s \neq 0$. However for each estimate, under certain conditions, it is possible to determine an interval within which the true value might exist. As the value of each grade $av_{i,j}$ is defined in the closed interval [0, 1], Eq. (7) gives the maximum estimate of At_j by setting all $av_{i_1,j} = av_{i_1 j}^{\max} = 1$ for $i_1 \in I_1$, and Eq. (8) gives the minimum estimate of At_j by setting all $av_{i_1,j} = 0$ for $i_1 \in I_1$, respectively.

$$At_j^{\max} = \sum_{i_2 \in I_2} \omega_{i_2} \cdot av_{i_2,j} + \sum_{i_1 \in I_1} \omega_{i_1} \cdot av_{i_1,j}^{\max}$$

$$j = 1, 2, \ldots\ldots, N \tag{7}$$

$$At_j^{\min} = \sum_{i_2 \in I_2} \omega_{i_2} \cdot av_{i_2,j} + \sum_{i_1 \in I_1} \omega_{i_1} \cdot av_{i_1,j}^{\min}$$

$$j = 1, 2, \ldots\ldots, N \tag{8}$$

where $At_j^{\max}$ — maximum estimate of At_j

$At_j^{\min}$ — minimum estimate of At_j

At the extreme points of $av_{i_1 j}^{\max} = 1$ and $av_{i_1,j}^{\min} = 0$, we have,

$$At_j^{\max} = \sum_{i_2 \in I_2} \omega_{i_2} \cdot av_{i_2,j} + \sum_{i_1 \in I_1} \omega_{i_1}$$

$$j = 1, 2, \ldots\ldots, N$$

$$At_j^{\min} = \sum_{i_2 \in I_2} \omega_{i_2} \cdot av_{i_2,j}$$

$$j = 1, 2, \ldots\ldots, N$$

Therefore, we have the estimation interval [$At_j^{\min}$, $At_j^{\max}$] for At_j with confidence. Alternatively, we have the following relationship:

$$0 \leq At_j^{\min} \leq At_j \leq At_j^{\max} \leq 1$$

In a similar way and based on Eqs. (7) and (8), the estimation interval for each unknown sub-attribute can be deduced to give Eqs. (9) and (10) respectively.

$$av_{k_h,j}^{\max} = \min\left[1,\ \frac{1}{\omega_{k_h}}\left(At_j^{\max} - \sum_{i_2 \in I_2} \omega_{i_2} \cdot av_{i_2,j}\right)\right]$$

$$j = 1, 2, \ldots\ldots, N \tag{9}$$

$$av_{k_h,j}^{\min} = \max\left[0,\ \frac{1}{\omega_{k_h}}\left(At_j^{\min} - \sum_{i_2 \in I_2} \omega_{i_2} \cdot av_{i_2,j} - \sum_{\substack{i_1 \in I_1 \\ i_1 \neq k_h}} \omega_{i_1}\right)\right]$$

$$j = 1, 2, \ldots\ldots, N \tag{10}$$

$$k_h \in I_1 \text{ or } 1 \le h \le s$$

where $av_{k_h,j}^{\max}$ — maximum estimate of $av_{k_h,j}$

$av_{k_h,j}^{\min}$ — minimum estimate of $av_{k_h,j}$

For given s and $s \geq 1$, we can select any unknown sub-attribute and determine its estimation interval using Eqs. (7) and (8) or (9) and (10), depending on whether or not the selected sub-attribute has sub-attributes of its own or not. Then, based on the same rationale and the estimation interval of the first estimated sub-attribute, the estimation interval for the second unknown sub-attribute can be determined using Eqs. (7) and (8) or (9) and (10) again. This process can be repeated until the estimation intervals for the $s + 1$ (i.e., s sub-attributes and one parent attribute) unknown attributes are obtained.

For example, suppose a parent attribute A has six sub-attributes a_i, $i = 1, 2, \ldots, 6$ and the sub-attributes from a_1 to a_4 have complete assessment information as shown in Table 3, while the remaining two sub-attributes have unknown assessments. The estimated values and the estimation intervals of assessments for the last two sub–attributes and the parent attribute are calculated using Eqs. (5) to (10) and are listed in Table 3. As the estimation intervals for all the grades of the two unknown sub-attributes are outside the rational interval [0, 1] of the grade definition, the applicable estimation intervals need to be adjusted into the closed interval [0, 1].

It is interesting to note that in the tests we conducted, the applicable estimation intervals, after adjustment, for all the sub-attributes are equal to the complete closed interval [0, 1]. This is in fact the nature of Eqs. (9) and (10). Therefore, Eqs. (9) and (10) or to guess the estimation interval of an unknown sub-attribute is actually meaningless in this case. However, the estimation interval of the unknown parent attribute is relatively small and stable. In many cases, the estimation interval for the parent attribute is of importance and is used for determining the final assessment interval of a detailed vehicle attribute or the vehicle itself under uncertainty.

Case 3 — Assessment on the parent attribute is known, and some assessments on its sub-attributes are unknown.

If there are s out of k sub-attributes with unknown assessments, there will obviously be (s-1) degrees of freedom in estimating unknown assessments. If (s - 1) conditions are met, the unknown assessments can be estimated with confidence. Otherwise, necessary conditions have to be subjectively established and the confidence of the estimation obtained under these conditions may vary according to the reliability of the subjective conditions established.

Table 3 The maximum and minimum values of missing assessments

Attribute	Grade 1	Grade 2	Grade 3	Grade 4	Grade 5	ω	Remark
At (value)	0.0175	0.1475	0.33	0.4125	0.0925		Predicted
At (interval)	[0.01, 0.31]	[0.11, 0.41]	[0.24, 0.54]	[0.27, 0.57]	[0.07, 0.37]		
a_1	0.1	0.2	0.6	0.1	0	0.1	known
a_2	0	0	0	0.8	0.2	0.2	known
a_3	0	0.3	0.6	0	0.1	0.3	known
a_4	0	0	0	1	0	0.1	known
a_5 (value)	0.025	0.125	0.3	0.475	0.075	0.05	Predicted
a_5 (interval)	[0, 1]	[0, 1]	[0, 1]	[0, 1]	[0, 1]		
a_6 (value)	0.025	0.125	0.3	0.475	0.075	0.25	Predicted
a_6 (interval)	[0, 1]	[0, 1]	[0, 1]	[0, 1]	[0, 1]		

For example, if we assume that each grade value of the unknown sub-attributes are equally correlated to the grade values of the parent attribute and the known sub-attributes, and

$$av_{k_1,j} = av_{k_2,j} = \cdots = av_{k_s,j}$$

$$k_1, k_2, \ldots\ldots, k_s \in I_1 \tag{11}$$

Eq. (12) can be used to estimate the grade values of the unknown sub-attributes.

$$av_{k_h,j} = \left(At_j - \sum_{i_2 \in I_2} \omega_{i_2} \cdot av_{i_2,j} \right) \Big/ \sum_{i_1 \in I_1} \omega_{i_1}$$

$$k_h \in I_1 \tag{12}$$

Eq. (6) is used to pair with the (s-1) given conditions expressed in Eq. (11) to form s equations. The estimation intervals of these s unknown sub-attributes can be determined using Eqs. (9) and (10) in which both $At_j^{\min}$ and $At_j^{\max}$ should be substituted by At_j. The parent attribute with six sub-attributes as discussed in Case 2 is used to demonstrate the estimation of the two unknown sub-attributes. At this time, suppose the assessment on the parent attribute is known as shown in Table 4. As both $At_j^{\min}$ and $At_j^{\max}$ are substituted by At_j in Eqs. (9) and (10), the estimation intervals for a_5 and a_6 are reduced to a smaller range after adjustment or deleting void area produced by Eqs. (9) and (10).

Table 4 The estimation values of missing assessments for two sub-attributes

Attribute	Grade 1	Grade 2	Grade 3	Grade 4	Grade 5	ω	Remark
At	0.0175	0.1475	0.33	0.4125	0.0925		known
a_1	0.1	0.2	0.6	0.1	0	0.1	known
a_2	0	0	0	0.8	0.2	0.2	known
a_3	0	0.3	0.6	0	0.1	0.3	known
a_4	0	0	0	1	0	0.1	known
a_5 (value)	0.025	0.125	0.3	0.475	0.075	0.05	Predicted
a_5 (interval)	[0, 0.15]	[0, 0.75]	[0, 1]	[0, 1]	[0, 0.45]		
a_6 (value)	0.025	0.125	0.3	0.475	0.075	0.25	Predicted
a_6 (interval)	[0, 0.03]	[0, 0.15]	[0.16, 0.36]	[0.37, 0.57]	[0, 0.09]		

Case 4 — Assessment on the parent attribute is known, but all the assessments on its sub-attributes are unknown.

Generally speaking, for any given sub-attribute h, its unknown assessments can be determined by Eq. (13), assuming that all other sub-attributes are guessed in advance.

$$av_{h,j} = \frac{1}{\omega_h}\left(At_j - \sum_{\substack{i=1 \\ i \neq h}}^{k} \omega_i \cdot av_{i,j} \right)$$

$$h = 1, 2, \ldots, k; j = 1, 2, \ldots\ldots, N \tag{13}$$

Similar to Case 3, if we assume that each grade value of the unknown sub-attributes is equally correlated to the grade values of the parent attribute and Eq. (11) is used as the necessary ($k-1$) conditions, Eq.(14) can be used to estimate the grade values of the unknown sub-attributes..

$$av_{h,j} = At_j$$

$$h = 1, 2, \ldots, k; j = 1, 2, \ldots\ldots, N \tag{14}$$

To obtain the maximum estimation value of $av_{h,j}$, let all other $av_{i,j} = 0$, $i = 1, 2, \ldots, k$ and $i \neq h$ in Eq. (13). Then, we have

$$av_{h,j}^{\max} = \min[1, \frac{At_j}{\omega_h}]$$

$$h = 1, 2, \ldots, k;\ j = 1, 2, \ldots, N \tag{15}$$

Similarly, to obtain the minimum estimation value of $av_{h,j}$, let all other $av_{i,j} = 1$, $i = 1, 2, \ldots, k$ and $i \neq h$. Eq.(13) gives

$$av_{h,j}^{\min} = \max[0, \frac{1}{\omega_h}(At_j - \sum_{\substack{i=1 \\ i \neq h}}^{k} \omega_i)]$$

$$h = 1, 2, \ldots, k; j = 1, 2, \ldots, N \tag{16}$$

To demonstrate Eqs. (14) – (16), the example employed in Case 2 and Case 3 is again used for Case 4 with only assessment for the parent attribute being given as shown in Table 5. It is interesting to find that the applicable estimation intervals after adjustment are likely to fall into a closed interval that is smaller than [0, 1].

Table 5 The estimation values of missing assessments for all sub-attributes

Attribute	Grade 1	Grade 2	Grade 3	Grade 4	Grade 5	ω	Remark
At	0.0175	0.1475	0.33	0.4125	0.0925		known
a_1(value)	0.018	0.148	0.330	0.413	0.093	0.1	Predicted
a_1 (interval)	[0, 0.175]	[0, 1]	[0, 1]	[0, 1]	[0, 0.925]		
a_2(value)	0.018	0.148	0.330	0.413	0.093	0.2	Predicted
a_2 (interval)	[0, 0.088]	[0, 0.738]	[0, 1]	[0, 1]	[0, 0.463]		
a_3(value)	0.018	0.148	0.330	0.413	0.093	0.3	Predicted
a_3 (interval)	[0, 0.058]	[0, 0.492]	[0, 1]	[0, 1]	[0, 0.308]		
a_4(value)	0.018	0.148	0.330	0.413	0.093	0.1	Predicted
a_4 (interval)	[0, 0.175]	[0, 1]	[0, 1]	[0, 1]	[0, 0.925]		
a_5 (value)	0.018	0.148	0.330	0.413	0.093	0.05	Predicted
a_5 (interval)	[0, 0.35]	[0, 1]	[0, 1]	[0, 1]	[0, 1]		
a_6 (value)	0.018	0.148	0.330	0.413	0.093	0.25	Predicted
a_6 (interval)	[0, 0.07]	[0, 0.59]	[0, 1]	[0, 1]	[0, 0.37]		

Except for Case 1, there might be multiple solutions for all the other three cases. Additional conditions or constraints are therefore needed for finding a specific solution in each of these three cases. In Eqs. (5) to (16), we assumed that all unknown assessments are equal. This is the simplest yet viable assumption for estimating missing information and it is most likely to generate the mean values for missing assessments. Without doubt, if more dedicated conditions can be established under specific circumstances, better estimates for unknown assessments can be generated with confidence.

In the next section, we investigate and discuss the affects of non-linear mapping functions for transforming survey data on to the common scale.

4. Mapping Function

As discussed in previous section, in many circumstances, there is a need for transforming assessments from an original survey to a desired scale which may differ from the evaluation scale used in the original survey. Linear mapping functions such as the one shown in Fig. 2 are the simplest ones. However, there are also various non-linear functions that could be used for data transformation. To demonstrate some properties of non-linear functions for data transformation, a quadratic function $f(x) = ax^2 + bx + c$ is taken as an example in this section.

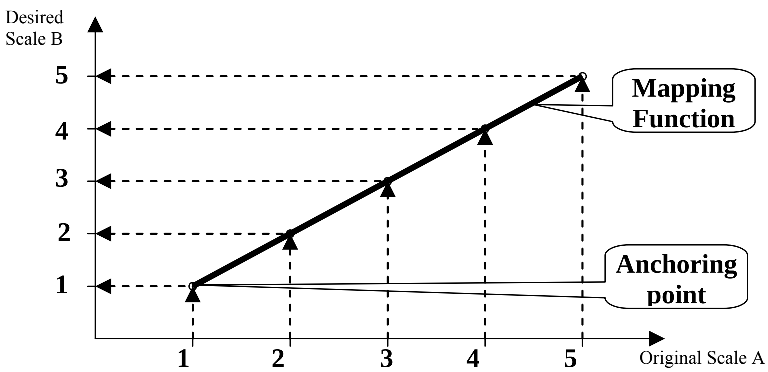

Fig. 2. Linear mapping from one scale to another

In a quadratic function, there could be at most three unknown parameters *a*, *b* and *c* and hence three conditions are needed to determine a quadratic function. For simplicity, we assume that the anchoring point and the top rank point of a mapping function are the same in original scale A and desired scale B. In other words, the lowest grade in the original scale A for a survey is mapped to the lowest grade in the desired scale B and the highest grade in the original scale A is mapped to the highest grade in the desired scale B, as shown in Fig. 2. Furthermore, we assume that the middle point of assessment on the original scale is mapped into t on the desired scale. Based on these assumptions, the following relationship can be established.

$$f(x,t) = a(t)x^2 + b(t)x + c(t) \tag{17}$$

where x — independent variable which stands for the assessment grade on the original scale

t — value of $f(x_m, t)$ at the middle point x_m of the original scale, which is a parameter used to control the shape of the parabola

$f(x,t)$ — assessment on the desired scale corresponding to x, given the parameter t

Assuming that the original scale A and the desired scale B have five assessment grades as shown in Fig. 2, the values for the parameters in Eq. (17) would then be:

$$a(t) = \frac{3-t}{4},\ b(t) = \frac{3t-7}{2},\ c(t) = \frac{15-5t}{4}.$$

Fig. 3 shows a set of parabolas following Eq. (17) with $x_m = 3$ as the middle point in the original scale. Note that, if $t = 3$, then $a = 0$ and $c = 0$, and $f(x)$ becomes a straight line or a linear function as shown in Fig. 3. If $t < 3$, the grades between the lowest and highest grades on the desired scale are devaluated in comparison with the corresponding grades on the original scale. If $t > 3$, the grades between the lowest and highest grades on the desired scale are appreciated in comparison with the corresponding grades on an original scale.

The extents of devaluation or appreciation are quite different for different grades between the lowest and highest grades. These properties can be used to reduce or enlarge the range of assessments. For example, suppose that the original evaluations for six attributes are as shown in Table 6. A number in a column under a grade is the belief degree of the grade assessed for the corresponding attribute. The mean grade in the last column for an attribute is the average of grade numbers (i.e., 1 for the lowest grade and 5 for the highest grade) multiplied by corresponding belief degrees.

Take $t = 2$ for example. $a(t) = 0.25$, $b(t) = -0.5$ and $c(t) = 1.25$, and the mapping function becomes a convex curve. The five grades 1, 2, 3, 4 and 5 on the original scale are mapped to 1, 1.25, 2, 3.25 and 5 respectively on the desired scale. To find the belief degrees of an assessment on each grade on the desired scale, piecewise linear approach can be employed.

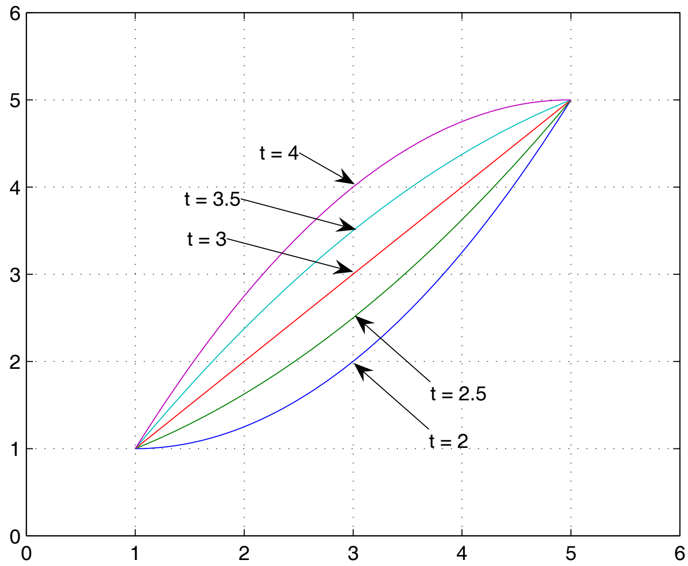

Fig. 3. Parabolas of the quadratic function with different value for parameter t

If a grade on the original scale coincides with a grade on the desired scale after mapping, then the belief degree of the grade for an attribute on the original scale is directly transformed to the grade on the desired scale. Otherwise, if a grade on the original scale moves to a place in between two adjacent grades, say Grade h and Grade h+1, on the desired scale after mapping, then the belief degree of the grade for an attribute on the original scale is transformed to the grades on the desired scale in the following way.

Suppose that Bo represent the belief degree of a grade for an attribute on the original scale before mapping. The grade on the original scale is mapped to a place Gd between Grade h and Grade h+1 on the desired scale. Then,

$$Bd_h = Bo \times \left(1 - \frac{Gd - Gd_h}{Gd_{h+1} - Gd_h}\right)$$

$$Bd_{h+1} = Bo \times \frac{Gd - Gd_h}{Gd_{h+1} - Gd_h}$$

where: Bd_h, Bd_{h+1} — belief degrees of Grade h and h+1 for an attribute on the desired scale respectively after mapping

Gd_h, Gd_{h+1} — scale values of Grade h and h+1 on the desired scale respectively

Gd — scale values between Grade h and Grade h+1 on the desired scale after mapping, which is mapped from the grade on the original scale

After executing above calculations for all belief degrees of assessments on an attribute, the belief degrees of assessments on each grade on the desired scale for the same attribute should be added up to give the total that is the data in each cell in Table 7.

As expected in the example of data mapping from Table 6 to Table 7, after transformation, all the ratings are devaluated or shifted towards lower grades. The distribution range of the mean grades over the six attributes is also changed from [3.6, 4] on the original scale to [2.85, 3.4625] on the desired scale after mapping. It means that the distribution of the average ratings has been dispersed by the mapping function.

However, it is noticeable that the rank order of the six attributes based on the mean grades has changed after mapping in this example. Therefore, only when a nonlinear mapping function truly reflects a desired evaluation transformation, this approach can be used.

Table 6 Assessments on the original scale

Attribute	Grade 1	Grade 2	Grade 3	Grade 4	Grade 5	Mean
a_1	0	0	0.6	0.2	0.2	3.6
a_2	0	0.2	0.2	0.3	0.3	3.7
a_3	0	0	0.2	0.8	0	3.8
a_4	0	0.1	0.1	0.7	0.1	3.8
a_5	0	0.15	0.15	0.3	0.4	3.95
a_6	0	0	0	1	0	4

Table 7 Assessments on the desired scale using quadratic mapping function with t =2

Attribute	Grade 1	Grade 2	Grade 3	Grade 4	Grade 5	Mean
a_1	0	0.6	0.15	0.05	0.2	2.85
a_2	0.15	0.25	0.225	0.075	0.3	3.125
a_3	0	0.2	0.6	0.2	0	3
a_4	0.075	0.125	0.525	0.175	0.1	3.1
a_5	0.1125	0.1875	0.225	0.075	0.4	3.4625
a_6	0	0	0.75	0.25	0	3.25

To use a nonlinear mapping function without altering the rank order of a group of alternatives, the nonlinear mapping function should be applied to the mean grades instead of every individual grades. For example, if the same quadratic function with $t = 2$ is used to map the mean grades of the six attributes listed in Table 6, the rank order of their mean grades after mapping will be the same as the ranking before mapping. The mean grades before and after mapping are listed in the two columns under Example 1 in Table 8, which shows that the distribution interval for the mean grades is changed from [3.6, 4] to [2.69, 3.25]. To further demonstrate this property of non-linear function, a ten attribute example, named Example 2, is also given in Table 8. By comparison of the two columns under Example 2, the distribution interval is changed from [3.03, 4] to [2.03, 3.25]. However, the relative rankings of the attributes are not changed in both examples.

Table 8 Mean grade transformation using nonlinear mapping function

Alternative	Example 1		Example 2	
	Mean grade before mapping	Mean grade after mapping	Mean grade before mapping	Mean grade after mapping
a_1	3.6	2.69	3.03	2.0302
a_2	3.7	2.8225	3.08	2.0816
a_3	3.8	2.96	3.19	2.1990
a_4	3.8	2.96	3.26	2.2769
a_5	3.95	3.1756	3.3	2.3225
a_6	4	3.25	3.36	2.3924
a_7			3.4	2.44
a_8			3.5	2.5625
a_9			3.59	2.6770
a_{10}			4	3.25
Interval	[3.6, 4]	[2.69, 3.25]	[3.03, 4]	[2.03, 3.25]

These examples show that a nonlinear function used to transform evaluation data plays an important part in changing the distribution range. This property can be used to change the distribution pattern of assessments to a desirable one. Apart from the quadratic function discussed in this section, other types of nonlinear functions can also be used as mapping function. Some properties of the cubic function will be discussed in the next section. It is important to choose an appropriate nonlinear function as the mapping function. Such a choice is domain specific and requires expert knowledge.

5. Utility Function

The concept of expected utility has been introduced in the ER approach for rank ordering when the distributed overall assessments are not sufficient to show differences in ranking. As an auxiliary measure, utility is applied to the general vehicle assessment framework to help intuitively rank the detailed attributes or vehicles. Suppose $u(H_j)$ is the utility of the grade H_j with

$$u(H_{j+1}) > u(H_j) \quad \text{if } H_{j+1} \text{ is preferred to } H_j \tag{18}$$

If all assessments are complete and precise, the expected utility of an alternative A can be calculated by

$$u(A) = \sum_{j=1}^{N} \beta_j u(H_j) \tag{19}$$

where H_j — j^{th} grade on an evaluation scale, $j = 1, 2, \ldots, N$

β_j — belief degree evaluated on grade H_j

$u(A)$ — expected utility of the alternative A.

$u(H_j)$ may be estimated using probability assignment methods[22-23] or by constructing regression models using partial rankings or pairwise comparisons[20]. In most cases, a linear function of $u(H_j)$ may be preferred because of its simplicity, although a nonlinear function may also be used to calculate utility in certain circumstances. It should be noted, however, that a utility function is used to capture the decision maker's preferences and as such it should be constructed using preference information provided by the decision maker. In this section, a cubic function is used to demonstrate the features of nonlinear functions in dealing with uncertainty and preference in assessment propagation and aggregation. A general cubic function can be expressed as follows.

$$f(x) = ax^3 + bx^2 + cx + d \tag{20}$$

As there are four unknown parameters a, b, c and d in Eq. (20), four conditions need to be given to determine the cubic function. Among various ways of giving the four conditions, for illustration purpose, fixing the middle point and the two ends of the definition interval for the utility function is a straightaway selection. The fourth condition could be determination of the slope at any one of the known points so that the shape of the cubic function inside the definition interval can be controlled explicitly.

In a general way, let (x_l, y_l), (x_h, y_h) and (x_m, y_m) be the starting, ending and middle points, respectively, in the definition interval of the utility function. In this example the slope of the utility function at the middle point is supposed to be given. Let $y = f(x)$, and $\frac{dy}{dx} = s$ at $x = x_m$. The four unknown parameters a, b, c and d can be determined by Eqs. (21) to (24).

$$a = \frac{A(y_m - y_h) - C(x_m^2 - x_h^2) - sA(x_m - x_h) + 2Cx_m(x_m - x_h)}{A(x_m^3 - x_h^3) - B(x_m^2 - x_h^2) - 3Ax_m^2(x_m - x_h) + 2Bx_m(x_m - x_h)} \tag{21}$$

$$b = \frac{C - Ba}{A} \tag{22}$$

$$c = s - 3x_m^2 a - 2x_m b \tag{23}$$

$$d = y_h - ax_h^3 - x_h^2 b - x_h c \tag{24}$$

where $A = 2x_m(x_l - x_m) - (x_l^2 - x_m^2)$

$B = 3x_m^2(x_l - x_m) - (x_l^3 - x_m^3)$

$C = s(x_l - x_m) - (y_l - y_m)$

For example, suppose the values for the three known points be given by

$x_l = 1, y_l = 0;$

$x_h = 5, y_h = 1;$

$x_m = 3, y_m = 0.5.$

The distinct shapes of the cubic function in the definition interval [1, 5] are shown in Fig. 4 with different slope values at the middle point. In Fig. 4, line $y1$ corresponds to $s_1 = 0$, $y2$ to $s_2 = 1/5$, $y3$ to $s_3 = 1/4$, $y4$ to $s_4 = 1/3$ and $y5$ to $s_5 = 1/2$. Fig. 5 describes the slope change along the transverse axis x of the cubic functions in the definition interval.

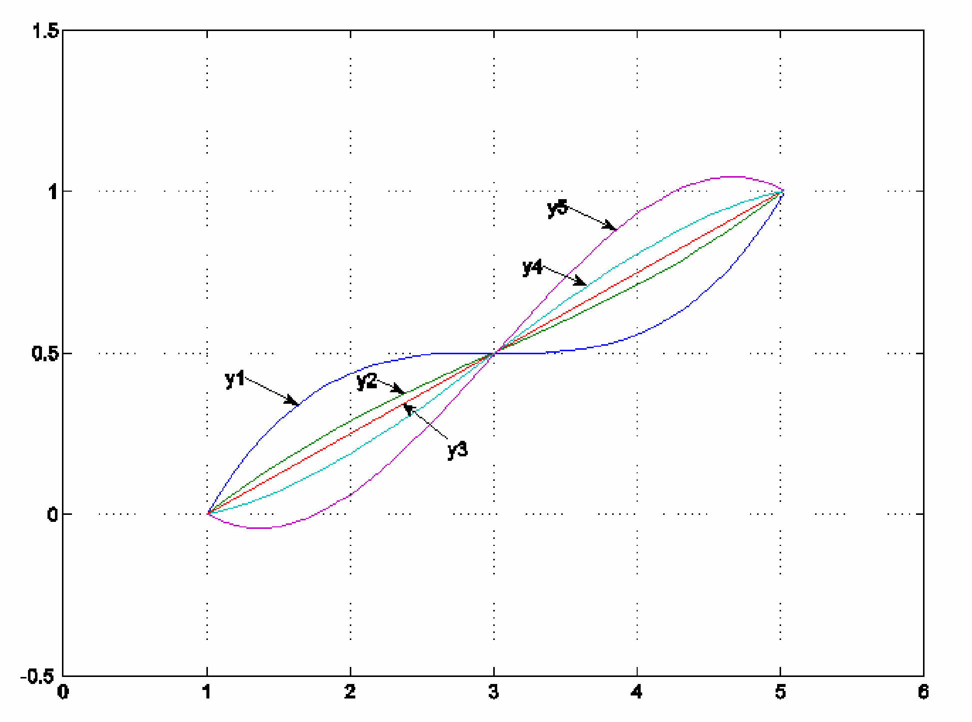

Fig. 4. Cubic utility functions with varying slopes at the middle point

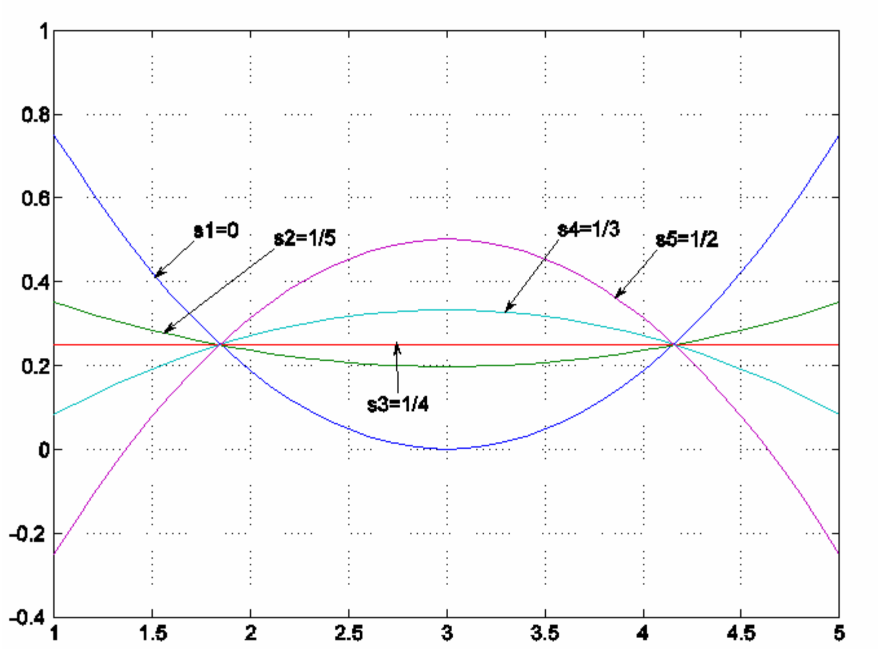

Fig. 5. Slope changes for various cubic utility functions along the assessment scale x

In general, utility function is non-decreasing, which means the slope at any point is non-negative, i.e., $\frac{dy}{dx} = 3ax^2 + 2bx + c \geq 0$ in the whole definition area $[x_l, x_h]$. This gives the minimum slope $s_{min} = 0$ and the maximum slope $s_{max} = 3/8$ at the middle point x_m in the above example. Also, care should be taken while using some of the curves shown in Fig. 4, as slope at some points of those curves is negative.

The six attributes listed in Table 6 are again taken as example for demonstrating the effect of using a nonlinear function as utility function. The expected utilities calculated with different slopes at the middle point are listed in Table 9. Except for $s = 1/4$ which means that the cubic function degenerates into a linear function, the rank order of the six attributes' utilities has changed compared with the original rank order generated by the mean grade, because of the use of the nonlinear utility functions. This is a noticeable feature of using a nonlinear function as utility function. It follows that if a nonlinear function truly reflects the preferences of the decision maker on the various grades of an evaluation scale it can be used as utility function. Otherwise, a linear utility function may be the best choice in the ER approach.

Table 9 Expected utility calculated using different cubic functions

Alternative	Mean	$s = 0$	$s = 1/5$	$s = 1/4$	$s = 1/3$	$s = 1/2$
a_1	3.6	0.6125	0.6425	0.65	0.6625	0.6875
a_2	3.7	0.65625	0.67125	0.675	0.68125	0.69375
a_3	3.8	0.55	0.67	0.7	0.75	0.85
a_4	3.8	0.5875	0.6775	0.7	0.7375	0.8125
a_5	3.95	0.709375	0.731875	0.7375	0.746875	0.765625
a_6	4	0.5625	0.7125	0.75	0.8125	0.9375

6. Application Examples

To investigate how to handle uncertainty using the methods discussed in previous sections, a case study using the general assessment framework discussed in Section 2 is conducted and reported in this Section. Due to confidentiality reasons, we have masked the original data used in our case study and only discuss the important aspects of the results. In this study, more than a hundred detailed vehicle attributes are rank-ordered using data from four different surveys. A hierarchy for assessing a detailed vehicle attribute more or less follows the hierarchy shown in Fig. 1. The IDS software is applied to assess and rank the vehicle attributes in the case study.

6.1 Estimate a missing assessment

To demonstrate our method for estimating missing assessments, we considered four vehicle attributes A7D01, A7D02, A7D03, and A7D04. These four vehicle attributes are the sub-attributes of the parent attribute A7D. Of these five attributes, A7D01, A7D02, A7D03 are completely assessed in the four surveys and their aggregated assessments are shown in Table 10 (see rows 5-7 of Table 10). Attribute A7D04 has some missing assessments and so does the attribute A7D. So, the problem at hand fits exactly to the description of Case 2 in Sub-section 3.3. Since the parent attribute assessments are unknown and some of the sub-attributes assessments are also unknown, Eqs. (5)–(10) are used for obtaining the estimates of value and interval (i.e., minimum and maximum values) of the unknown assessments. In estimating the missing assessments of the parent attribute A7D, all the sub-attributes are assumed to have equal importance as shown in Table 10. Rows 2-4 of Table 10 show the estimated value and interval for the parent attribute A7D and rows 8-10 show the estimated value and interval for the sub-attribute A7D04.

Table 10 Estimate value and estimation interval for the missing assessment

Attribute	Grade 1	Grade 2	Grade 3	Grade 4	Grade 5	ω	Remark
A7Dmin	0.1206	0.0509	0.2374	0.0509	0.2902		Predicted
A7Dexp	0.1608	0.0678	0.3166	0.0678	0.3869		
A7Dmax	0.3706	0.3009	0.4874	0.3009	0.5402		
A7D01	0.201	0.0663	0.3095	0.0663	0.3568	0.25	Known
A7D02	0.2060	0.0724	0.3377	0.0724	0.3116	0.25	Known
A7D03	0.0754	0.0648	0.3025	0.0648	0.4925	0.25	Known
A7D04min	0	0	0	0	0	0.25	Predicted
A7D04exp	0.1608	0.0678	0.3166	0.0678	0.3869		
A7D04max	1	1	1	1	1		

6.2 Using a quadratic mapping function

To illustrate the effect of using a quadratic mapping function, Criteria (1, 1) (which is a basic criteria) in Fig. 1 is taken as an example. Suppose that all the assessments on Criteria (1, 1) are to be transformed to a new scale using Eq. (17) before applying the ER approach for aggregation. As a test, the middle point t at x_m= 3 is set to be 2, 2.5, 3, 3.5 and 4 respectively and for each value of t the assessments on Criteria (1, 1) can be transformed to the new scale in an individual distribution. Based on these assessments on the new scale the new rank orders of all attributes can be obtained using the IDS software as described in Sub-section 2.4. The table in Appendix A lists the top 50 ranked attributes for different values of t.

From the table in Appendix A, we can see that the average utility and the ranking of the top eighteen attributes are unchanged by using a quadratic mapping function for Criteria (1, 1). The reason is that for most of these attributes the belief degrees of assessments from Survey 1 (see Fig. 1) completely belong to the highest grade. Hence any changes in Criteria (1, 1) transformation would not affect these attributes. The ranking of the attributes in rows 19-20 (i.e., A5C and A3A) is not affected by the transformation, but the average utility of these attributes is affected. From row 21 onwards both the ranking and average utility of the attributes is affected because of the quadratic transformation function. This study shows clearly the impact of using a quadratic mapping function on the ranking of vehicle attributes.

6.3 Using a cubic curve as utility function

To illustrate the effect of using a cubic utility function, the five cubic curves from $y1$ to $y5$ expressed in Eq. (20) and shown in Fig. 4 are used to calculate the utility values for the five grades of the desired scale. The utility values of the five grades for different cubic functions are listed in Table 11. It is obvious that at s = 1/4 the cubic curve degenerates into a straight line. Note that at s = 1/2 the cubic curve, i.e. $y5$ in Fig. 4, has negative slope near the two ends of the definition interval [0, 1].

Table 11 The utility values at every grades for given s values

$u(H_j)$	Grade 1	Grade 2	Grade 3	Grade 4	Grade 5
s = 0	0	0.4375	0.5	0.5625	1
s = 1/5	0	0.2875	0.5	0.7125	1
s = 1/4	0	0.2500	0.5	0.7500	1
s = 1/3	0	0.1875	0.5	0.8125	1
s = 1/2	0	0.0625	0.5	0.9375	1

The table in Appendix B ranks the top 50 attributes using different cubic functions for the overall utility. Again, many of the attributes (seventeen to be specific) at the top are unaffected by the cubic utility function. The reason is that for most of these attributes the belief is associated completely to Grade 5 and so the overall utility is always one irrespective of the utility function. For the attribute in row 19 (A6I), the ranking is not affected even though the average utility is changed. From row 20 onwards, the average utility and the ranking of the attributes is affected by the cubic utility function. For the attribute "A7B02" (shown in bold in Appendix B), the average utility is reduced (and the rank order lowered) when $s < ¼$ and the average utility is increased (and the rank order rose) when $s > ¼$. On the other hand for the attribute "A3B04" (shown in bold italic in Appendix B), the observed behavior is totally opposite. This study clearly shows that using a cubic utility function affects the rank ordering of the attributes in more than one way.

7. Concluding Remarks

A general framework for assessing vehicle attributes using survey information was investigated. To make better use of the framework, four issues related to decision making under uncertainty are studied in the context of the ER algorithm. Applicable approaches were investigated and devised for dealing with uncertainty that may result from both original surveys and assessment aggregation. The new findings are as follows.

(1) The format of a conventional questionnaire, which only allows simple assessment on a single grade for a question, can be improved by giving respondent more choices for answering the questionnaires in a flexible yet realistic manner in order to take advantage of the features of the ER approach with the belief degree structure. Uncertainty present in a survey can be counted by means of the amount of missing evaluations divided by the total number of valid responses.

(2) For partly missing information in an assessment hierarchy, an estimate or estimation interval may be generated for each piece of the missing information. These estimates and estimation intervals can be directly used in the original ER and novel interval-based ER method[24-26] to make the final evaluation more informative and realistic.

(3) Nonlinear mapping function can be used for assessment transformation. Different functions and even different parts of a function can have significant effects in assessment transformation. However, it should be used with care as it may change the rank order of a

group of alternatives based on the mean grades of the assessments. Only if a nonlinear function truly represents the transformation nature then it can be used as a mapping function. The choice of a transformation function is domain specific and requires expert knowledge.

(4) Similarly, a nonlinear function can be used as utility function for assessment aggregation. However, it may also change the rank order of a group of alternatives based on mean grades of the assessments. Only if a nonlinear function truly represents the decision maker's preferences then it can be used as a utility function. Based on the results of this research, it is highly recommended that a linear function be used as utility function if there is no strong evidence to support the use of any type of nonlinear functions.

Uncertainty in decision making process is complicated. In this paper, we investigated the basic approaches and their features in dealing with uncertainty possibly present in the process of vehicle evaluation. It is expected that the research findings would help make better use of survey information and deal with uncertainty in an objective and a consistent way. These can be taken as the basis for further study in this area.

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Appendix A Rank order contrast while using a quadratic mapping function for Criteria (1,1)

Mid-point	t = 2		t = 2.5		t = 3		t = 3.5		t = 4	
Rank order	Average utility	Attribute	Average utility	Attribute	Average utility	Attribute	Average utility	Attribute	Average utility	Attribute
1	1	A3B09	1	A3B09	1	A1A12	1	A3B09	1	A3B09
2	1	A6C	1	A6C	1	A3A07	1	A6C	1	A6C
3	1	A6D03	1	A6D03	1	A3B09	1	A6D03	1	A6D03
4	1	A6K06	1	A6K06	1	A5C04	1	A6K06	1	A6K06
5	1	A5C04	1	A5C04	1	A5F02	1	A5C04	1	A5C04
6	1	A6C08	1	A6C08	1	A6	1	A6C08	1	A6C08
7	1	A6D04	1	A6D04	1	A6C	1	A6D04	1	A6D04
8	1	A7B06	1	A7B06	1	A6C08	1	A7B06	1	A7B06
9	1	A1A12	1	A1A12	1	A6D	1	A1A12	1	A1A12
10	1	A5F02	1	A5F02	1	A6D01	1	A5F02	1	A5F02
11	1	A6D	1	A6D	1	A6D02	1	A6D	1	A6D
12	1	A3A07	1	A3A07	1	A6D03	1	A3A07	1	A3A07
13	1	A6	1	A6	1	A6D04	1	A6	1	A6
14	1	A6D01	1	A6D01	1	A6D09	1	A6D01	1	A6D01
15	1	A6D09	1	A6D09	1	A6K06	1	A6D09	1	A6D09
16	1	A6D02	1	A6D02	1	A7B06	1	A6D02	1	A6D02
17	0.9872	A6C13	0.9872	A6C13	0.9872	A6C13	0.9872	A6C13	0.9872	A6C13
18	0.9813	A6I	0.9813	A6I	0.9813	A6I	0.9813	A6I	0.9813	A6I
19	0.9744	A5C	0.9758	A5C	0.9772	A5C	0.979	A5C	0.9807	A5C
20	0.9392	A3A	0.9421	A3A	0.9451	A3A	0.9492	A3A	0.953	A3A
21	0.9286	A7B02	0.9286	A7B02	0.9288	A3A02	0.9381	A3A02	0.9468	A3A02
22	0.9147	A3A02	0.9217	A3A02	0.9286	A7B02	0.9319	A6E04	0.9389	A5A07
23	0.9137	A6E04	0.9195	A6E04	0.9253	A6E04	0.9286	A7B02	0.9384	A3A04
24	0.9131	A1A06	0.9171	A1A06	0.9212	A1A06	0.9285	A5A07	0.9383	A6E04
25	0.9102	A6H	0.9134	A6H	0.9173	A5A07	0.9278	A3A04	0.9355	A6E02
26	0.9044	A3B01	0.9107	A3B01	0.917	A3B01	0.9272	A1A06	0.9339	A3B01
27	0.9007	A5A07	0.909	A5A07	0.9165	A6H	0.9258	A3B01	0.9328	A1A06
28	0.9	A1A03	0.9079	A3A04	0.9163	A3A04	0.9253	A6E02	0.9299	A3B04
29	0.8995	A3A04	0.906	A6E02	0.9142	A6E02	0.9212	A6H	0.9289	A5C05
30	0.8979	A6E02	0.9044	A3B04	0.9113	A3B04	0.9209	A3B04	0.9286	A7B02
31	0.8975	A3B04	0.9	A1A03	0.9087	A5C05	0.919	A5C05	0.9256	A6H
32	0.8914	A5C05	0.9	A5C05	0.9009	A1B	0.9112	A3B06	0.9224	A3B06
33	0.8914	A1B	0.8962	A1B	0.9	A1A03	0.9068	A1B	0.9124	A1B
34	0.8822	A3B06	0.8907	A3B06	0.8992	A3B06	0.9	A1A03	0.9077	A3A01
35	0.8781	A6C12	0.8796	A6C12	0.8811	A6C12	0.8936	A3A01	0.9	A1A03
36	0.875	A6C05	0.875	A6C05	0.8787	A3A01	0.8841	A6C12	0.8979	A5A05
37	0.8547	A3A01	0.8667	A3A01	0.875	A6C05	0.8833	A5A05	0.887	A6C12
38	0.8436	A5A05	0.8558	A5A05	0.868	A5A05	0.875	A6C05	0.8834	A3B08
39	0.8381	A2A01	0.8444	A2A01	0.8508	A2A01	0.8648	A3B08	0.875	A6C05
40	0.8248	A1A02	0.8293	A3B08	0.8452	A3B08	0.8595	A2A01	0.8679	A2A01
41	0.8152	A3B	0.8273	A1A02	0.8297	A1A02	0.8395	A1A07	0.852	A1A07
42	0.8138	A5C02	0.8213	A3B	0.8274	A3B	0.8369	A3B	0.8456	A3B
43	0.8134	A3B08	0.8199	A5C02	0.8265	A1A07	0.8351	A5C02	0.8438	A5C02
44	0.8101	A6A05	0.8178	A1A07	0.826	A5C02	0.8343	A1A02	0.8409	A6A05
45	0.8088	A1A07	0.816	A6A05	0.8218	A6A05	0.8316	A6A05	0.8387	A1A02
46	0.8024	A5A01	0.8045	A5A01	0.8065	A5A01	0.8139	AX000	0.8302	A3A03
47	0.7986	A1A	0.7986	A1A	0.8011	AX000	0.8138	A3A03	0.8266	AX000
48	0.7868	A4A05	0.7911	AX000	0.7986	A1A	0.8106	A5A01	0.8185	A3A06
49	0.7838	A2B01	0.7868	A4A05	0.7964	A3A03	0.8034	A3A06	0.8146	A5A01
50	0.781	AX000	0.7846	A3A03	0.7892	A1A05	0.7997	A1A05	0.8098	A1A05

Appendix B Rank order contrast while using a cubic utility function

M.P.-slope	s= 0		s = 1/5		s = 1/4		s = 1/3		s = 1/2	
Rank order	Average utility	Attribute	Average utility	Attribute	Average utility	Attribute	Average utility	Attribute	Average utility	Attribute
1	1	A3B09	1	A3B09	1	A3B09	1	A3B09	1	A3B09
2	1	A6C	1	A6C	1	A6C	1	A6C	1	A6C
3	1	A6D03	1	A6D03	1	A6D03	1	A6D03	1	A6D03
4	1	A6K06	1	A6K06	1	A6K06	1	A6K06	1	A6K06
5	1	A5C04	1	A5C04	1	A5C04	1	A5C04	1	A5C04
6	1	A6C08	1	A6C08	1	A6C08	1	A6C08	1	A6C08
7	1	A6D04	1	A6D04	1	A6D04	1	A6D04	1	A6D04
8	1	A7B06	1	A7B06	1	A7B06	1	A7B06	1	A7B06
9	1	A1A12	1	A1A12	1	A1A12	1	A1A12	1	A1A12
10	1	A5F02	1	A5F02	1	A5F02	1	A5F02	1	A5F02
11	1	A6D	1	A6D	1	A6D	1	A6D	1	A6D
12	1	A3A07	1	A3A07	1	A3A07	1	A3A07	1	A3A07
13	1	A6	1	A6	1	A6	1	A6	1	A6
14	1	A6D01	1	A6D01	1	A6D01	1	A6D01	1	A6D01
15	1	A6D09	1	A6D09	1	A6D09	1	A6D09	1	A6D09
16	1	A6D02	1	A6D02	1	A6D02	1	A6D02	1	A6D02
17	0.9872	A6C13	0.9872	A6C13	0.9872	A6C13	0.9872	A6C13	0.9872	A6C13
18	0.9774	A6I	0.9805	A6I	0.9813	A6I	0.9826	A6I	0.9852	A6I
19	0.9743	A5C	0.9766	A5C	0.9772	A5C	0.9782	A5C	**0.9821**	**A7B02**
20	0.9391	A3A	0.9439	A3A	0.9451	A3A	0.9471	A3A	0.9801	A5C
21	0.9193	A3A02	0.9269	A3A02	0.9288	A3A02	**0.9464**	**A7B02**	0.9688	A6C05
22	0.9165	A6E04	0.9235	A6E04	**0.9286**	**A7B02**	0.9319	A3A02	0.9511	A3A
23	0.9111	A6H	0.9189	A1A06	0.9253	A6E04	0.9282	A6E04	0.9383	A3A02
24	0.9098	A1A06	**0.9179**	**A7B02**	0.9212	A1A06	0.925	A1A06	0.9375	A6H12
25	0.9089	A5A07	0.9156	A5A07	0.9173	A5A07	0.9201	A5A07	0.9341	A6E04
26	0.9083	A3B01	0.9154	A6H	0.917	A3B01	0.92	A3B01	0.9327	A1A06
27	0.9057	A3A04	0.9153	A3B01	0.9165	A6H	0.9198	A3A04	0.9268	A3A04
28	0.9047	A6E02	0.9142	A3A04	0.9163	A3A04	0.9183	A6H	0.9258	A3B01
29	***0.9026***	***A3B04***	0.9123	A6E02	0.9142	A6E02	0.9174	A6E02	0.9257	A5A07
30	0.9	A1A03	***0.9096***	***A3B04***	***0.9113***	***A3B04***	***0.9142***	***A3B04***	0.9237	A6E02
31	0.8976	A1B	0.9064	A5C05	0.9087	A5C05	0.9125	A5C05	0.9219	A6H
32	0.8972	A5C05	0.9003	A1B	0.9009	A1B	0.9063	A6C05	0.9202	A5C05
33	0.89	A3B06	0.9	A1A03	0.9	A1A03	0.9023	A3B06	***0.9201***	***A3B04***
34	0.8778	A6C12	0.8974	A3B06	0.8992	A3B06	0.902	A1B	0.9085	A3B06
35	**0.875**	**A7B02**	0.8805	A6C12	0.8811	A6C12	0.9	A1A03	0.9043	A1B
36	0.8644	A3A01	0.8759	A3A01	0.8787	A3A01	0.8835	A3A01	0.9	A1A03
37	0.855	A5A05	0.8654	A5A05	0.875	A6C05	0.8822	A6C12	0.893	A3A01
38	0.8459	A1A02	0.8563	A6C05	0.868	A5A05	0.8723	A5A05	0.8845	A6C12
39	0.8447	A5C02	0.8488	A2A01	0.8508	A2A01	0.8541	A2A01	0.8812	A1A07
40	0.8409	A2A01	0.8431	A3B08	0.8452	A3B08	0.8487	A3B08	0.881	A5A05
41	0.8346	A3B08	0.833	A1A02	0.8297	A1A02	0.8447	A1A07	0.8607	A2A01
42	0.8164	A3B	0.8297	A5C02	0.8274	A3B	0.8311	A3B	0.859	AX000
43	0.8138	A6A05	0.8252	A3B	0.8265	A1A07	0.8245	A6A05	0.8558	A3B08
44	0.8051	A3A06	0.8202	A6A05	0.826	A5C02	0.8244	A1A02	0.8384	A3B
45	0.7876	A1A	0.8156	A1A07	0.8218	A6A05	0.8204	AX000	0.8306	A5A01
46	0.7861	A4A05	0.8017	A5A01	0.8065	A5A01	0.8198	A5C02	0.8298	A6A05
47	0.7837	A2B01	0.7964	A1A	0.8011	AX000	0.8146	A5A01	0.8136	A1A02
48	0.7832	A3A03	0.7937	A3A03	0.7986	A1A	0.8125	A6H12	0.8096	A3A03
49	0.7824	A5A01	0.7909	A3A06	0.7964	A3A03	0.8022	A1A	0.8095	A1A
50	0.7813	A6C05	0.7895	AX000	0.7892	A1A05	0.8008	A3A03	0.8073	A5C02