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Consumer preference prediction by using a hybrid evidential reasoning and belief rule-based methodology

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ABSTRACT

Consumer preference prediction is a key factor to the success of new product development. This paper presents a hybrid evidential reasoning (ER) and belief rule-based (BRB) methodology for consumer preference prediction and a novel application to orange juices. The orange juices are distinguished by their values of sensory attributes, which are grouped for simplicity into different categories such as appearance, aroma, texture, flavour, and aftertaste. The ER approach is used to aggregate consumer preferences for category attributes into an overall preference, and the BRB methodology is used to model the casual relationships between category attributes and their sensory attributes. The casual relationships between the overall preference and the sensory attributes of orange juices are trained and tested using real data and memorized for prediction or new product design. A case study involving 16 orange juices is conducted using the proposed hybrid ER and BRB methodology to demonstrate its novel applications. The results show that the hybrid ER and BRB methodology can fit and predict consumer preferences with high accuracy.

1. Introduction

Consumer preferences for orange juices are closely related to their sensory attributes, but cannot usually be characterized by sensory attributes linearly. External preference mapping (EPM) (Arditti, 1997; Faber, Mojet, & Poelman, 2003; Geel, Kinear, & de Kock, 2005; Guinard, Uotani, & Schlich, 2001; Heyd & Danzart, 1998; Martínez, Cruz, Hough, & Vega, 2002; van Kleef, van Trijp, & Luning, 2006) is a methodology that has been widely used for sensory analysis. The EPM models consumer preferences as a polynomial function of the first two principal components (PCs) that are extracted from the sensory data of orange juices, but it suffers from the drawback that not all consumer preferences can be well fitted and accounted for (Faber et al., 2003). Artificial neural networks (ANNs) (Boccorh & Paterson, 2002; Bomio, 1998; Krishnamurthy, Srivastava, Paton, Bell, & Levy, 2007; Tan, Gao, & Gerrard, 1999; Zhang & Chen, 1997) have also been used for modelling the relationships between consumer preferences and sensory attributes. They model consumer preferences as a complicated nonlinear function of sensory attributes, which is defined by a multilayer network with sensory attributes as inputs, consumer preferences as output, and one or more hidden layers. However, the relationships between consumer preferences and sensory attributes modelled by ANNs are a black box, which makes the relationships difficult to interpret and understand.

Recently, Yang, Wang, Xu, Chin, and Chatton (submitted for publication) developed a belief rule-based (BRB) methodology for quality assessment, target setting and consumer preference prediction in retro design of food and drink products. The BRB methodology does not need to specify any functional forms. It characterizes the causal relationships between consumer preferences and product attributes using belief rule bases (BRBs). Each BRB is a collection of belief rules which are the generalization of traditional IF–THEN rules and can be built using expert knowledge and real data for food and drink products. To avoid building a large BRB, the BRB methodology requires product attributes to be properly grouped. For each group (or category) attribute, a sub-belief rule base is built to represent its relationships with its product attributes. The casual relationships between consumer preferences and group attributes are also characterized by a sub-belief rule base. In other words, the BRB system for food or drink product is composed of some sub-BRBs. It is found, however, that when the number of group attributes is over three or four, the sub-BRB for characterizing the casual relationships between consumer preferences and group attributes is still too big to be built and contains too many parameters such as rule weighs and belief degrees to be estimated. To avoid this difficulty, we propose in this paper a hybrid evidential

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reasoning and belief rule-based methodology for consumer preference prediction for sensory products. The evidential reasoning (ER) approach is used to aggregate consumer preferences for group attributes and the BRB methodology is used to model the casual relationships between consumer preferences for group attributes and their sensory attributes.

The paper is organized as follows. Section 2 introduces the hybrid ER and BRB methodology for consumer preference prediction. Section 3 conducts a case study using the real data collected from a company that designs and manufactures sensory products to illustrate the applications of the proposed hybrid ER and BRB methodology in predicting consumer preferences for orange juices. The paper concludes in Section 4.

2. The hybrid ER and BRB methodology

Orange juices are usually distinguished by their values of a large number of sensory attributes. It is impractical if not impossible to use a large number of sensory attributes to build a BRB directly. To simplify the structure of the model to be developed, it is required that the sensory attributes of orange juices be properly grouped by their characteristics into different categories such as appearance, aroma, texture, flavour, aftertaste and so on. Let A_j $(j = 1,\ldots,K)$ be K group (category) attributes, each containing m_j sensory attributes as shown in Fig. 1. For such a model structure, there are K BRBs to be developed, each for one group attribute, and consumer preferences for group attributes will be aggregated by using the ER approach.

2.1. Preference mapping for group attributes by BRBs

A BRB is a collection of belief rules which are the generalization of traditional IF–THEN rules. In traditional IF–THEN rules, the consequence of each rule is certain such as *High*, *Medium* or *Low*; while the consequences of belief rules can be certain or uncertain, characterized by belief structures such as {(*High*, 100%)}, which represents a certainly (100%) *High* consequence, and {(*High*, 30%), (*Medium*, 60%), (*Low*, 10%)}, which represents that the consequence could be *High* to the extent of 30%, *Medium* to the extent of 60% and *Low* to the extent of 10%, where 30%, 60% and 10% are belief degrees. If the belief degrees for the consequence of a belief rule are summed to one, then the belief rule is said to be complete; otherwise it is said to be incomplete and the remaining belief degree which is not assigned to any possible consequence is referred to as ignorance. For example, IF taste is *Good* and flavor is *Average* THEN quality is {(*Good*, 35%), (*Average*, 45%), (*Poor*, 15%)}, which is

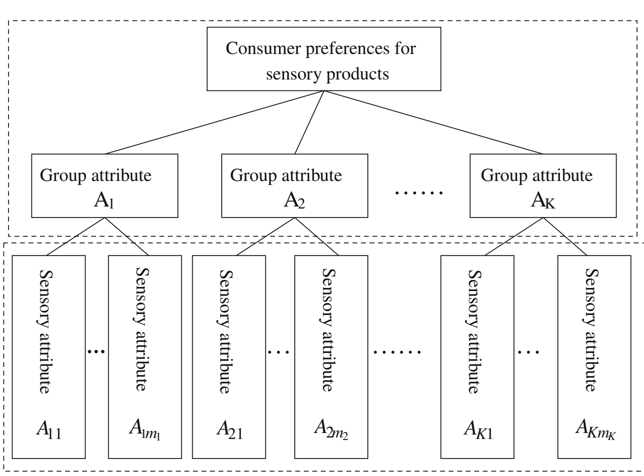

Fig. 1. The hybrid ER and BRB structure for consumer preference prediction.

an incomplete belief rule and the missing belief degree 5% (=100% − 35% − 45% − 15%) is called ignorance.

Each belief rule plays a different role in a BRB and is thus assigned a rule weight. Likewise, each antecedent attribute in the "IF" part of a belief rule also plays a different role and is thus given an attribute weight. Mathematically, a belief rule can be defined as (Yang, Liu, Wang, Sii, & Wang, 2006):

IF $A_1^k \wedge A_2^k \wedge \cdots \wedge A_T^k$ THEN $\{(C_1,\beta_{k1}), (C_2,\beta_{k2}),\ldots,(C_N,\beta_{kN})\}$ with a rule weight $0 \leqslant \theta_k \leqslant 1$ and attribute weights $\delta_1,\ldots,\delta_T \geqslant 0$ satisfying $\sum_{j=1}^{T}\delta_j = 1$, where $\{(C_1,\beta_{k1}), (C_2,\beta_{k2}),\ldots,(C_N,\beta_{kN})\}$ is a belief structure satisfying $\beta_{kl} \geqslant 0$ $(l = 1,\ldots,N)$ and $\sum_{l=1}^{N}\beta_{kl} \leqslant 1$.

The above belief rule can be understood as IF (A_1 is A_1^k, A_2 is A_2^k,…, and A_T is A_T^k) THEN consequence is $\{(C_1,\beta_{k1}), (C_2,\beta_{k2}),\ldots,(C_N,\beta_{kN})\}$, where $A_1,\ldots,A_T$ are the antecedent attributes of the belief rule, $C_1,\ldots,C_N$ are assessment grades for its consequence, $A_j^k \in \{C_{j1},\ldots,C_{jT_j}\}$ is a possible assessment grade for antecedent attribute A_j, $j \in \{1,\ldots,T\}$, and β_{kl} is the belief degree to which C_l is believed to be the consequence, $l \in \{1,\ldots,N\}$. If $\sum_{l=1}^{N}\beta_{kl} = 1$, the belief rule is said to be complete; otherwise it is incomplete.

To build a belief rule or BRB, discrete assessment grades such as *High*, *Medium* and *Low* need to be defined for each antecedent attribute and consequence. More specifically, for the hierarchical structure in Fig. 1, discrete assessment grades have to be defined for each group and sensory attribute. The number of grades for each group and sensory attribute can be either the same or different, depending upon the need of real applications. Once the discrete assessment grades are defined, the BRB for each group attribute can be built using expert knowledge initially and then trained by using the BRB learning algorithm (Xu et al., 2007; Yang, Liu, Xu, Wang, & Wang, 2007).

Table 1 shows the generic forms of the BRBs for group attributes, where $C_1,\ldots,C_T$ are assessment grades for group attributes, $A_{h1},\ldots,A_{hm_h}$ are sensory attributes related to group attribute A_h, $A_{hj}^k \in \{C_{hj1},\ldots,C_{hjT_{hj}}\}$ $(k = 1,\ldots,M_h)$ are assessment grades for A_{hj} $(j = 1,\ldots,m_h)$, M_h $(h = 1,\ldots,K)$ are the numbers of belief rules, which are equal to the numbers of all possible combinations of different assessment grades for antecedent attributes.

In the case that a group attribute is involved with many sensory attributes, it is impractical to build a BRB using relevant sensory attributes directly. To avoid building a large BRB, it is required that the BRBs for those group attributes with too many sensory attributes be built using the first two or three principal components (PCs). This is somewhat similar to what is used in the EPM.

Suppose A_h is a group attribute related to many sensory attributes $A_{h1},\ldots,A_{hm_h}$, whose values for orange juice P_i $(i = 1,\ldots,n)$ are denoted by $\mathbf{x}_{ih} = (x_{ih1},\ldots,x_{ihm_h})$. Let $R_h = (r_{jk})_{m_h\times m_h}$ be their correlation matrix, whose elements are computed by

$$r_{jk} = \frac{\sum_{i=1}^{n}(x_{ihj}-\bar{x}_{hj})(x_{ihk}-\bar{x}_{hk})}{\sqrt{\sum_{i=1}^{n}(x_{ihj}-\bar{x}_{hj})^2\cdot\sum_{i=1}^{n}(x_{ihk}-\bar{x}_{hk})^2}},\quad j,k = 1,\ldots,m_h, \tag{1}$$

where $\bar{x}_{hj} = \frac{1}{n}\sum_{i=1}^{n}x_{ihj}$ from $j = 1$ to m_h are the average values of x_{hj} over the n orange juice products. Denote by λ_j $(j = 1,\ldots,m_h)$ the eigenvalues of the characteristic equation $|R_h - \lambda I| = 0$ and $\mathbf{u}_j = (u_{j1},\ldots,u_{jm_h})^T$ the corresponding eigenvectors derived from $R_h\mathbf{u}_j = \lambda_j\,\mathbf{u}_j$, where λ_j $(j = 1,\ldots,m_h)$ are sorted in descending order, i.e. $\lambda_1 \geqslant \lambda_2 \geqslant \cdots \geqslant \lambda_{m_h}$, and $\mathbf{u}_j$ satisfy the condition $\mathbf{u}_j^T\mathbf{u}_j = 1$. Then, the m_h PCs are obtained as

$$F_{hj} = u_{j1}\left(\frac{x_{h1}-\bar{x}_{h1}}{\sqrt{\mathrm{var}(x_{h1})}}\right) + \cdots + u_{jm_h}\left(\frac{x_{hm_h}-\bar{x}_{hm_h}}{\sqrt{\mathrm{var}(x_{hm_h})}}\right),\quad j = 1,\ldots,m_h, \tag{2}$$

where F_{hj} $(j = 1,\ldots,m_h)$ are the jth PC for group attribute A_h and $\sqrt{\mathrm{var}(x_{hj})}$ the standard deviation of x_{hj} over the n orange juice products. That is

Table 1
BRBs for group attributes A_h (h = 1,...,K).

Rule	Rule weight	Antecedent attributes (weight)				Consequence (A_h)			
		$A_{h1}(\delta_{h1})$	$A_{h2}(\delta_{h2})$	...	$A_{hm_h}(\delta_{hm_h})$	C_1	C_2	...	C_T
1	θ_{h1}	A_{h1}^1	A_{h2}^1	...	$A_{hm_h}^1$	$\beta_{11}^{(h)}$	$\beta_{12}^{(h)}$	...	$\beta_{1T}^{(h)}$
2	θ_{h2}	A_{h1}^2	A_{h2}^2	...	$A_{hm_h}^2$	$\beta_{21}^{(h)}$	$\beta_{22}^{(h)}$	...	$\beta_{2T}^{(h)}$
⋮	⋮	⋮	⋮	...	⋮	⋮	⋮	...	⋮
M_h	θ_{hM_h}	$A_{h1}^{M_h}$	$A_{h2}^{M_h}$	...	$A_{hm_h}^{M_h}$	$\beta_{M_h1}^{(h)}$	$\beta_{M_h2}^{(h)}$	...	$\beta_{M_hT}^{(h)}$

$$\sigma_{hj} = \sqrt{\text{var}(x_{hj})} = \sqrt{\frac{1}{n}\sum_{i=1}^{n}(x_{ihj} - \bar{x}_{hj})^2}, \quad j = 1, \ldots, m_h. \tag{3}$$

The PCs explain as much of the variation as possible in the original sensory data. In particular, the first PC explains the maximal amount of variance of the original sensory data, the second PC explains the maximal remaining variance in the data, and so on. The percentage of total variation of the original sensory data explained by each PC is defined as $100 \times \lambda_j / \sum_{i=1}^{m_h} \lambda_i$. Accordingly, the accumulative contribution ratio (ACR) of the first L_h ($L_h \leqslant m_h$) PCs is computed by

$$ACR = \left(\sum_{i=1}^{L_h} \lambda_i \middle/ \sum_{j=1}^{m_h} \lambda_j\right) \times 100\%. \tag{4}$$

It is required that $ACR \geqslant 75\%$ in determining how many PCs should be used for building a BRB for a group attribute. Once the PCs for building a BRB are determined, the original sensory data will be transformed into corresponding PC scores by using the following equation:

$$F_{hj}(i) = u_{j1}\left(\frac{x_{ih1} - \bar{x}_{h1}}{\sqrt{\text{var}(x_{h1})}}\right) + \cdots + u_{jm_h}\left(\frac{x_{ihm_h} - \bar{x}_{hm_h}}{\sqrt{\text{var}(x_{hm_h})}}\right),$$
$$i = 1, \ldots, n; j = 1, \ldots, L_h, \tag{5}$$

where L_h is the number of PCs used for building a BRB. Note that when PCs are used for building a BRB, assessment grades should be defined for each PC rather than for each relevant sensory attribute.

2.2. Data transformation and rule activation

Let $C_{hj1}, \ldots, C_{hjT_{hj}}$ be a set of assessment grades defined for sensory attribute A_{hj} or principal component F_{hj} and x_{hj} be a numerical value of A_{hj} or PC score of F_{hj}, which lies between two adjacent assessment grade values, say $u(C_{hjl})$ and $u(C_{hjl+1})$, $1 \leqslant l \leqslant T_{hj} - 1$. Then, the numerical value (or PC score) x_{hj} can be transformed into the belief structure $\{(C_{hjl}, \alpha_{hjl}), (C_{hjl+1}, \alpha_{hjl+1})\}$ by using the following piecewise linear function (Yang, 2001):

$$\alpha_{hjl} = \frac{u(C_{hjl+1}) - x_{hj}}{u(C_{hjl+1}) - u(C_{hjl})} \quad \text{and} \quad \alpha_{hjl+1} = \frac{x_{hj} - u(C_{hjl})}{u(C_{hjl+1}) - u(C_{hjl})}, \tag{6}$$

where $u(C_{hjl+1})$ and $u(C_{hjl})$ are grade values of C_{hjl+1} and C_{hjl}, respectively. The belief structure can be rewritten in full as $\{(C_{hj1}, 0), \ldots, (C_{hjl-1}, 0), (C_{hjl}, \alpha_{hjl}), (C_{hjl+1}, \alpha_{hjl+1}), (C_{hjl+2}, 0), \ldots, (C_{hjT_{hj}}, 0)\}$, which means the numerical value x_{hj} is only assessed to grades C_{hjl} and C_{hjl+1}. All the sensory data and PC scores that have been chosen for building BRBs can be transformed into belief structures in terms of their numerical values and assessment grades in this way.

Consider a set of sensory data or PC scores $\mathbf{x}_h = (x_{h1}, \ldots, x_{hm_h})$, each of which has been transformed into the belief structure $\left\{(C_{hjl}, \alpha_{hjl}) | l = 1, \ldots, T_{hj}, \sum_{l=1}^{T_{hj}} \alpha_{hjl} = 1\right\}$ (j = 1,...,m_h) by Eq. (6). This set of sensory data or PC scores will match to some extent the antecedent attribute values of each belief rule in Table 1 and possibly activate the belief rules. Take the following belief rule for example.

IF $A_{h1}^k \wedge A_{h2}^k \wedge \cdots \wedge A_{hm_h}^k$ THEN $\{(C_1, \beta_{k1}^{(h)}), (C_2, \beta_{k2}^{(h)}), \ldots, (C_T, \beta_{kT}^{(h)})\}$.

From the belief structures $\left\{(C_{hjl}, \alpha_{hjl}) | l = 1, \ldots, T_{hj}, \sum_{l=1}^{T_{hj}} \alpha_{hjl} = 1\right\}$ (j = 1,...,m_h) transformed from $\mathbf{x}_h = (x_{h1}, \ldots, x_{hm_h})$, one can easily find the belief degree $\alpha_{h1}^{(k)} \in \{\alpha_{h11}, \ldots, \alpha_{h1T_{h1}}\}$ to which x_{h1} is assessed to grade $A_{h1}^k \in \{C_{h11}, \ldots, C_{h1T_{h1}}\}$, the belief degree $\alpha_{h2}^{(k)} \in \{\alpha_{h21}, \ldots, \alpha_{h2T_{h2}}\}$ to which x_{h2} is assessed to grade $A_{h2}^k \in \{C_{h21}, \ldots, C_{h2T_{h2}}\}$, and all the other belief degrees including $\alpha_{hm_h}^{(k)} \in \{\alpha_{hm_h1}, \ldots, \alpha_{hm_hT_{hm_h}}\}$ to which x_{hm_h} is assessed to grade $A_{hm_h}^k \in \{C_{hm_h1}, \ldots, C_{hm_hT_{hm_h}}\}$. Apparently, $\alpha_{hj}^{(k)}$(j = 1,...,m_h) represent the extent to which each antecedent attribute of the above belief rule is matched by the input data $\mathbf{x}_h = (x_{h1}, \ldots, x_{hm_h})$. Since each antecedent attribute plays a different role in the belief rule, the overall degree γ_{hk} to which the above belief rule is matched can be defined as (Yang et al., 2006)

$$\gamma_{hk} = \prod_{j=1}^{m_h} \left(\alpha_{hj}^{(k)}\right)^{\bar{\delta}_{hj}}, \quad k = 1, \ldots, M_h, \tag{7}$$

where $\bar{\delta}_{hj} = \delta_{hj} / \max_{l=1,\ldots,m_h} \{\delta_{hl}\}$ so that $0 < \bar{\delta}_{hj} \leqslant 1$ and $\delta_{hj} > 0$ is the relative importance weight of A_{hj} which is either a sensory attribute or a PC. The overall degree γ_{hk} is referred to as matching degree.

It is easy to prove that $0 \leqslant \gamma_{hk} \leqslant 1$, $\gamma_{hk} = 1$ if $\alpha_{hj}^{(k)} = 1$for all j = 1,...,m_h, and $\gamma_{hk} = 0$ if there exists some $\alpha_{hj}^{(k)} = 0$ for $j \in \{1, \ldots, m_h\}$. The contribution of an antecedent attribute towards γ_{hk} is positively related to its relative importance weight, which means that an important antecedent attribute plays a greater role in the determination of γ_{hk}.

Once a belief rule is matched with a nonzero matching degree, it will be activated. As a result, an activation weight is defined to show to what degree the belief rule is activated. According to Yang et al. (2006), the activation weight for an activated belief rule can be defined as

$$w_{hk} = \frac{\theta_{hk}\gamma_{hk}}{\sum_{l=1}^{M_h} \theta_{hl}\gamma_{hl}} = \frac{\theta_{hk}\left[\prod_{j=1}^{m_h}\left(\alpha_{hj}^{(k)}\right)^{\bar{\delta}_{hj}}\right]}{\sum_{l=1}^{M_h}\theta_{hl}\left[\prod_{j=1}^{m_h}\left(\alpha_{hj}^{(l)}\right)^{\bar{\delta}_{hj}}\right]}, \quad k = 1, \ldots, M_h, \tag{8}$$

where θ_{hk} is the rule weight of the kth belief rule. The bigger the matching degree γ_{hk}, the bigger the activation weight w_{hk}. If a belief rule is not matched (i.e. $\gamma_{hk} = 0$) or has a zero weight ($\theta_{hk} = 0$), then it will not be activated (i.e. $w_{hk} = 0$). After the examination of the belief rules in Table 1 one by one, all the belief rules that have been assigned a nonzero activation weight will be activated simultaneously.

2.3. Rule inference and preference aggregation by the ER approach

Once a belief rule is activated, it will produce a consequence characterized by a belief structure. Such a belief structure can be viewed as a piece of evidence and needs to be combined with the other consequences (evidence) produced by other activated belief rules to provide a conclusion. The ER approach (Yang, 2001) developed for multiple attribute decision analysis provides a useful analytical algorithm for combining multiple pieces of evidence and is thus adopted by the BRB methodology.

The ER approach treats activation weights as the relative importance of each consequence and aggregates the consequences produced by all the activated belief rules in Table 1 using the equations below (Wang, Yang, & Xu, 2006):

$$\alpha_{hj} = \frac{\mu_h * \left[\prod_{k=1}^{M_h}(w_{hk}\beta_{kj}^{(h)} + 1 - w_{hk}) - \prod_{k=1}^{M_h}(1 - w_{hk})\right]}{1 - \mu_h * \left[\prod_{k=1}^{M_h}(1 - w_{hk})\right]}, \quad j = 1, \ldots, T, \tag{9}$$

$$\mu_h = \left[\sum_{l=1}^{T}\prod_{k=1}^{M_h}(w_{hk}\beta_{kl}^{(h)} + 1 - w_{hk}) - (T_h - 1)\prod_{k=1}^{M_h}(1 - w_{hk})\right]^{-1}, \tag{10}$$

where w_{hk} is determined by Eq. (8) and α_{hj}, which satisfies $\alpha_{hj} \geqslant 0$ and $\sum_{j=1}^{T}\alpha_{hj} = 1$, is the belief degree to which the conclusion is assessed to grade $C_j \in \{C_1, \ldots, C_T\}$. The conclusion provided by the ER approach is a belief structure: $\{(C_1, \alpha_{h1}), \ldots, (C_T, \alpha_{hT})\}$, which serves as the assessment of group attribute A_h in Fig. 1.

The assessment results of group attributes are then aggregated using the ER approach to generate an overall assessment $\{(C_1,\alpha_1),\ldots,(C_T,\alpha_T)\}$,where $\alpha_1,\ldots,\alpha_T$ are the belief degrees determined by the following equations:

$$\alpha_j = \frac{\mu * \left[\prod_{h=1}^{K}(w_h\alpha_{hj} + 1 - w_h) - \prod_{h=1}^{K}(1 - w_h)\right]}{1 - \mu * \left[\prod_{h=1}^{K}(1 - w_h)\right]}, \quad j = 1, \ldots, T, \tag{11}$$

$$\mu = \left[\sum_{j=1}^{T}\prod_{h=1}^{K}(w_h\alpha_{hj} + 1 - w_h) - (T - 1)\prod_{h=1}^{K}(1 - w_h)\right]^{-1}, \tag{12}$$

where w_h ($h = 1,\ldots,K$) are the relative importance weights of the K group attributes satisfying $\sum_{h=1}^{K} w_h = 1$ with $w_h \geqslant 0$ for $h = 1,\ldots,K$.

2.4. Preference fitting and parameter learning

Let $O = f(\mathbf{x})$ be the final output of the hybrid ER and BRB model in Fig. 1, which is a belief structure characterized by the overall assessment $\{(C_1,\alpha_1),\ldots,(C_T,\alpha_T)\}$ determined by Eqs. (11) and (12). This belief structure needs transforming into a numerical value to match consumer preferences for orange juices that are often characterized by a mean score or rating. Denote by $u(C_j)$ ($j = 1,\ldots,T$) the scores or ratings for assessment grades $C_1,\ldots,C_T$, through which the overall assessment $O = f(\mathbf{x}) = \{(C_1,\alpha_1),\ldots,(C_T,\alpha_T)\}$ can be converted into an expected numerical value by the following equation:

$$O = f(\mathbf{x}) = \sum_{j=1}^{T}\alpha_j u(C_j). \tag{13}$$

The converted numerical values serve as consumer preference predictions for orange juices by the hybrid ER and BRB model and provide a good basis for fitting consumer preferences and learning model parameters.

Suppose we have collected n sets of sensory attribute data $\mathbf{x}_i$ and consumer preferences y_i that are characterized by mean scores or ratings for n orange juice products $P_i(i = 1,\ldots,n)$. By Eq. (13),n prediction values $O_i(i = 1,\ldots,n)$ could be generated from the hybrid ER and BRB model once the BRBs in Table 1 and the relative importance weights for group attributes in Eqs. (11) and (12) are given or known. These prediction values may usually be different from consumer preferences for orange juices. Let ε_i be their deviations defined by

$$\varepsilon_i = y_i - O_i, \quad i = 1,\ldots,n. \tag{14}$$

It is most desirable that these deviations be kept as small as possible. Based upon this point of view, we construct the following optimization model for parameter estimation:

Table 2
Sensory profiles of 16 orange juices and consumer preferences for them.

Attribute	Orange juices															
	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
Appearance																
Colour	84.4	26	68.4	28	45.4	45.1	30.1	58.5	28	32	68.3	83.2	79.4	21.8	80	83.2
Bitty	22.1	19.9	74.6	58.3	44.4	28.1	1.5	16.3	36.7	38.6	5.8	7.7	54.2	3.5	51.1	11.9
Frothy	46.3	19.8	15.6	7.3	9.5	21.9	2.2	11.9	43.9	52	14.9	2.8	1.8	27.8	37.6	20.5
Aroma																
Citrus	34.8	34.7	26.1	28.5	33.7	23.3	32.1	16.4	27.9	25.1	29.2	17.6	24.1	31.1	22.7	17.6
Marmalade	21.6	17.4	12.3	8	13.9	19.8	13.9	21.5	9.7	10.5	11.1	14.8	12.9	12.1	11.5	7.3
Orange	47	44.3	54.3	45.2	52.6	48	45.8	51.1	44.6	42.5	42.6	39.2	42.1	32.5	43.8	31.4
Other fruit	4.1	1.8	0.1	1.4	1.4	0.7	2.6	0.1	1.3	0.9	2.4	2.6	1.8	0.2	0.3	2
Off	0.8	0.3	1.2	2	0.9	1.7	3.4	2.9	3.1	1.2	4.1	3	1.9	2.6	3.1	1.8
Texture																
Bitty	34.5	40.6	67.7	65.6	33.3	52.7	0.3	34.8	45.6	59.4	1.6	2.9	65	2.6	75.4	6.7
Body	53.3	39.1	51.6	53.4	44.6	53.7	31	51.3	46.2	52.3	44.8	50.7	45	44.3	52.7	41.4
Flavour																
Sweet	54.4	36.2	31.4	55.7	49.9	59.2	36.1	50.9	36.9	61.9	50.7	56.9	55.5	63.7	36.9	45.6
Sour	39.3	54.1	60.8	37.8	47	37.1	55.6	46.1	52.5	41.7	45.2	36.2	42.8	36.4	56	55.1
Citrus	55.5	57.1	56.1	59.9	58.2	56.8	48.8	51.1	52.3	62.3	52.3	54.2	54	50.8	45.8	38.7
Orange	18	9.8	2.9	12.8	12.3	10.4	17.8	16	8.8	10.4	18.4	25.3	13.2	26.7	11.9	31.1
Artificial orange	30.3	25.8	44.3	23.9	32.3	26.9	31.3	28.3	41.8	23.9	25.3	21.9	29.2	19.2	38.5	32.3
Other fruit	2.5	0.9	0.2	0.7	0.6	0.2	2.4	1.8	1.7	0.8	1	1	1.9	0.9	0.5	0.3
Bitter	37.1	46.6	41.3	32.9	38.9	29.2	42.3	30.5	44.1	30.1	43	34.8	43.9	25.3	45.9	42.7
Off	1.7	1.3	0.4	2.3	3.3	2.4	9.2	5.2	1.7	2.6	4.6	5.3	9.9	5.8	11	10.6
Aftertaste (afterfeel)																
Mouthcoating	36.4	30	28.8	44.5	40.9	44.1	46.1	44.7	47.4	36.7	47.6	49	45.7	48.4	45.1	54.9
Astringent	52.1	60.4	67.3	60.7	62.5	62.2	65.7	69.1	77	59.7	65.2	62.2	65.2	60.8	69.8	72.1
Irritant	36	33.1	41.7	41.5	41.9	45.6	48.9	53.4	56.6	36.7	44.4	47.2	52.5	46.7	51.3	59
Consumer preference	5.97	5.8	5.82	5.4	6.18	5.62	6.12	6.02	6.01	6.03	5.4	5.75	5.66	6.35	*5.37*	*5.57*

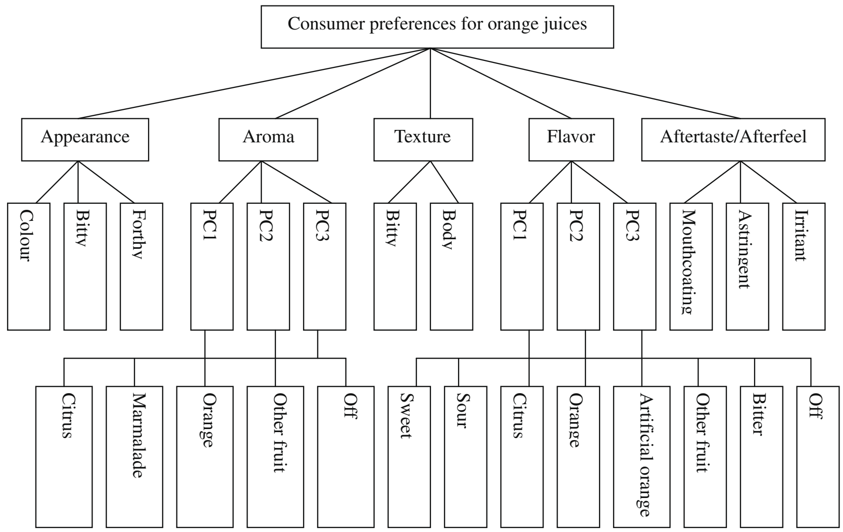

Fig. 2. Hierarchical structure for consumer preference prediction of orange juices.

$$\text{Minimize} \quad J = \sum_{i=1}^{n} \varepsilon_i^2 = \sum_{i=1}^{n} (y_i - O_i)^2 \tag{15}$$

$$\text{Subject to} \quad \sum_{j=1}^{T} \beta_{kj}^{(h)} = 1, \quad h = 1, \ldots, K; \quad k = 1, \ldots, M_h, \tag{16}$$

$$\sum_{j=1}^{m_h} \delta_{hj} = 1, \quad h = 1, \ldots, K, \tag{17}$$

$$\sum_{h=1}^{K} w_h = 1, \tag{18}$$

$$0 \leqslant \theta_{hk} \leqslant 1, \quad h = 1, \ldots, K; \quad k = 1, \ldots, M_h, \tag{19}$$

$$\beta_{kj}^{(h)} \geqslant 0, \quad h = 1, \ldots, K;$$

$$k = 1, \ldots, M_h; \quad j = 1, \ldots, T, \tag{20}$$

$$\delta_{hj} \geqslant 0, \quad h = 1, \ldots, K; \quad j = 1, \ldots, m_h, \tag{21}$$

$$w_h \geqslant 0, \quad h = 1, \ldots, K, \tag{22}$$

where the constraints (16) and (20) are on the belief degrees in the consequence of each belief rule in Table 1, (18) and (22) on the relative importance weights of the K group attributes, (17) and (21) on the relative importance weights of the sensory attributes or PCs related to the same group attribute, and (19) is the constraint on the rule weights of the BRBs in Table 1. When PCs are used for building a BRB, it is required that the relative importance weight of the first PC be no less than that of the second PC, which is in turn no less than the relative importance weight of the third PC. So, additional constrains need to be added when PCs are employed to build the BRBs in Table 1.

Table 3
Principal components for aroma.

Item	Principal components		
	PC1	PC2	PC3
Eigenvalue	1.823	1.261	0.887
% variance	36.456	25.218	17.742
Cumulative %	36.456	61.673	79.415
Eigenvector			
Citrus	0.455	0.457	−0.362
Marmalade	0.506	−0.250	0.605
Orange	0.502	−0.473	0.031
Other fruit	0.210	0.711	0.477
Off	−0.490	−0.015	0.525

Table 4
Principal components for flavor.

Item	Principal components		
	PC1	PC2	PC3
Eigenvalue	3.652	2.301	1.018
% variance	45.656	28.763	12.727
Cumulative %	45.656	74.418	87.145
Eigenvector			
Sweet	−0.497	0.092	−0.003
Sour	0.499	−0.015	−0.107
Citrus	−0.243	−0.548	0.141
Orange	−0.171	0.576	−0.054
Artificial orange	0.449	−0.158	−0.016
Other fruit	0.001	0.114	0.968
Bitter	0.445	0.041	0.162
Off	0.126	0.566	−0.018

Table 5
Assessment grades for sensory attributes, principal components and consumer preferences.

Attribute	Grades		
	Low	Medium	High
Appearance			
Colour	21.80	53.10	84.40
Bitty	1.50	38.05	74.60
Frothy	1.80	26.90	52.00
Aroma			
PC1	−3.798	0	3.789
PC2	−2.992	0	3.773
PC3	−2.997	0	3.954
Texture			
Bitty	0.3	37.85	75.4
Body	31	42.35	53.7
Flavour			
PC1	−3.877	0	4.485
PC2	−3.37	0	4.418
PC3	−2.262	0	2.619
Aftertaste (afterfeel)			
Mouthcoating	28.8	41.85	54.9
Astringent	52.1	64.55	77
Irritant	33.1	46.05	59
Consumer preference	5	6	7

By solving the above optimization model, all the parameters in Table 1 and the relative importance weights for group attributes can be learned, based on which consumer preferences for the n orange juices can be best fitted and preferences for new orange juices can be predicted. This will be illustrated in the next section through a case study.

3. Application to a case study

Orange juices are most widely consumed beverages in the world. Table 2 shows the sensory profiles of 16 orange juices and consumer preferences for them provided by a well-known company that designs and manufactures sensory products. The 16 or-

Table 6
Belief rule base for appearance.

Rule	Rule weight	Attribute			Appearance		
		Colour	Bitty	Frothy	Low	Medium	High
1	0.845	Low	Low	Low	0.001	0.051	**0.948**
2	0.032	Low	Low	Medium	0.158	0.024	0.818
3	0.825	Low	Low	High	0.996	0.002	0.002
4	0.486	Low	Medium	Low	0.995	0.000	0.005
5	0.554	Low	Medium	Medium	0.008	0.021	**0.971**
6	1.000	Low	Medium	High	0.265	0.022	0.713
7	0.719	Low	High	Low	0.992	0.001	0.008
8	0.928	Low	High	Medium	0.928	0.012	0.060
9	0.863	Low	High	High	0.027	0.010	**0.963**
10	0.005	Medium	Low	Low	0.988	0.004	0.008
11	0.992	Medium	Low	Medium	0.980	0.013	0.007
12	0.169	Medium	Low	High	0.701	0.294	0.005
13	1.000	Medium	Medium	Low	0.003	0.046	**0.951**
14	0.039	Medium	Medium	Medium	0.994	0.003	0.003
15	0.038	Medium	Medium	High	0.598	0.009	0.393
16	0.069	Medium	High	Low	0.810	0.187	0.003
17	1.000	Medium	High	Medium	0.106	0.851	0.043
18	0.167	Medium	High	High	0.045	0.021	**0.934**
19	0.906	High	Low	Low	0.647	0.007	0.346
20	0.865	High	Low	Medium	1.000	0.000	0.000
21	1.000	High	Low	High	0.500	0.067	0.433
22	0.192	High	Medium	Low	0.961	0.035	0.004
23	1.000	High	Medium	Medium	0.961	0.031	0.009
24	0.348	High	Medium	High	0.884	0.000	0.116
25	1.000	High	High	Low	0.256	0.742	0.002
26	0.959	High	High	Medium	0.682	0.282	0.036
27	0.801	High	High	High	0.115	0.001	**0.884**

Table 7
Belief rule base for aroma.

Rule	Rule weight	Attribute			Aroma		
		PC1	PC2	PC3	Low	Medium	High
1	1.000	Low	Low	Low	0.001	0.046	**0.953**
2	0.041	Low	Low	Medium	0.185	0.024	0.791
3	1.000	Low	Low	High	0.996	0.002	0.002
4	1.000	Low	Medium	Low	0.995	0.000	0.005
5	1.000	Low	Medium	Medium	0.127	0.135	0.738
6	1.000	Low	Medium	High	0.040	0.016	**0.944**
7	1.000	Low	High	Low	0.992	0.002	0.007
8	1.000	Low	High	Medium	0.931	0.012	0.057
9	1.000	Low	High	High	0.020	0.010	**0.970**
10	0.958	Medium	Low	Low	0.988	0.004	0.008
11	0.180	Medium	Low	Medium	0.978	0.015	0.008
12	0.172	Medium	Low	High	0.602	0.297	0.101
13	1.000	Medium	Medium	Low	0.209	0.256	0.535
14	0.036	Medium	Medium	Medium	0.997	0.000	0.003
15	0.034	Medium	Medium	High	0.493	0.009	0.498
16	0.908	Medium	High	Low	0.837	0.160	0.003
17	1.000	Medium	High	Medium	0.442	0.520	0.038
18	0.194	Medium	High	High	0.043	0.019	**0.937**
19	1.000	High	Low	Low	0.945	0.013	0.042
20	0.807	High	Low	Medium	1.000	0.000	0.000
21	1.000	High	Low	High	0.803	0.053	0.144
22	0.243	High	Medium	Low	0.961	0.035	0.004
23	1.000	High	Medium	Medium	0.627	0.036	0.337
24	0.295	High	Medium	High	0.976	0.000	0.024
25	1.000	High	High	Low	0.506	0.492	0.002
26	1.000	High	High	Medium	0.166	0.390	0.443
27	1.000	High	High	High	0.085	0.001	**0.914**

ange juices are divided into two sample sets for training and testing, respectively. The first 14 orange juices, which are competitive products, are treated as training samples and the last two from the company are treated as testing samples.

Since the orange juices are distinguished by five group attributes that are further composed of 21 sensory attributes, as shown in Fig. 2, it is not realistic to build a huge big BRB for the five group attributes to link them with consumer preferences. For facilitating the model to be developed, we build a BRB for each group attribute, respectively, and then aggregate the five group attributes with the ER algorithm. The two group attributes, aroma and flavor, are also involved with too many sensory attributes and we thus use PCs to build a BRB for each of them, respectively. The first three PCs for aroma and flavor are shown in Tables 3 and 4, which account for 79.4% and 87.1% of the total variance of the sensory data for aroma and flavor, respectively.

Table 5 shows the assessment grades, *High*, *Medium* and *Low*, defined for the sensory attributes and PCs that are used to build BRBs, where the maximum values of the sensory attributes and PCs are defined as *High* and the minimum values as *Low*. The maximum and minimum values of the PCs are computed by solving the following pair of linear programming (LP) models for $h = 2$ and 4, respectively:

$$\text{Max/Min} \quad F_{hj} = u_{j1}\left(\frac{x_{h1} - \bar{x}_{h1}}{\sqrt{\text{var}(x_{h1})}}\right) + \cdots + u_{jm_h}\left(\frac{x_{hm_h} - \bar{x}_{hm_h}}{\sqrt{\text{var}(x_{hm_h})}}\right) \quad j = 1, 2, 3 \tag{23}$$

$$\text{Subject to} \quad x_{hj}^{\min} \leqslant x_{hj} \leqslant x_{hj}^{\max}, \quad j = 1, \ldots, m_h.$$

The five group attributes and the consequence of each belief rule are also assessed by *High*, *Medium* and *Low*. Since consumer preferences are between mean scores 5 and 7, scores 7, 6 and 5 are thus defined as *High*, *Medium* and *Low*.

Based upon the above definitions for assessment grades, 117 belief rules are to be developed in total, nine of which are for texture and 27 are for appearance, aroma, flavor, and aftertaste, respectively. By solving the optimization model expressed by (15)–(22) using the training data set, we get the results of the 117 belief rules and their rule weights, which are provided in Tables 6–10, and the relative importance weights for the five group attributes: appearance, aroma, texture, flavor and aftertaste as (0.3814, 0.0090, 0.0048, 0.3188, 0.2860), which shows that consumer preferences for orange juices are not very much related to aroma and texture. In other words, consumers do not care too much about aroma and texture of orange juices.

Tables 6–10 reveal some useful patterns that consumers prefer for orange juices. For example, consumers prefer (*Low*, *Low*, *Low*) and (–, *High*, *High*) among the combinations of three antecedent attributes and (*Low*, *Low*) as well as (*High*, *High*) among the combinations of two antecedent attributes, where the symbol '–' represents that the first antecedent attribute could be assessed to any grade of *High*, *Medium* and *Low*. In another word, when the last two antecedent attributes are both assessed to *High*, the first antecedent attribute has little impact on consumer preferences. Besides the above patterns, consumers also prefer (*Low*, *Medium, Medium*) *and* (*Medium*, *Medium, Low*) for appearance and (*Low*, *Medium*, *High*) for aroma, flavor and aftertaste. It is obvious that the BRBs

Table 8
Belief rule base for texture.

Rule	Rule weight	Attribute		Texture		
		Bitty	Body	Low	Medium	High
1	1.000	Low	Low	0.001	0.046	**0.953**
2	1.000	Low	Medium	0.995	0.000	0.005
3	1.000	Low	High	0.923	0.006	0.071
4	0.953	Medium	Low	0.988	0.004	0.008
5	1.000	Medium	Medium	0.205	0.253	0.542
6	1.000	Medium	High	0.835	0.162	0.003
7	1.000	High	Low	0.988	0.005	0.007
8	0.242	High	Medium	0.961	0.035	0.004
9	1.000	High	High	0.090	0.001	**0.909**

Table 9
Belief rule base for flavor.

Rule	Rule weight	Attribute			Flavor		
		PC1	PC2	PC3	Low	Medium	High
1	0.023	Low	Low	Low	0.001	0.061	**0.938**
2	0.030	Low	Low	Medium	0.220	0.025	0.756
3	0.973	Low	Low	High	0.996	0.002	0.002
4	1.000	Low	Medium	Low	0.995	0.000	0.005
5	0.802	Low	Medium	Medium	0.278	0.345	0.377
6	1.000	Low	Medium	High	0.023	0.014	**0.963**
7	1.000	Low	High	Low	0.069	0.004	**0.927**
8	0.734	Low	High	Medium	0.842	0.013	0.145
9	1.000	Low	High	High	0.020	0.010	**0.970**
10	0.449	Medium	Low	Low	0.988	0.004	0.008
11	0.175	Medium	Low	Medium	0.978	0.015	0.008
12	0.144	Medium	Low	High	0.690	0.302	0.008
13	0.819	Medium	Medium	Low	0.214	0.292	0.493
14	0.023	Medium	Medium	Medium	0.959	0.038	0.003
15	0.037	Medium	Medium	High	0.195	0.009	0.796
16	0.935	Medium	High	Low	0.847	0.149	0.004
17	0.931	Medium	High	Medium	0.702	0.262	0.036
18	0.203	Medium	High	High	0.042	0.019	**0.939**
19	1.000	High	Low	Low	0.075	0.144	0.782
20	0.195	High	Low	Medium	1.000	0.000	0.000
21	1.000	High	Low	High	0.821	0.051	0.127
22	0.322	High	Medium	Low	0.962	0.034	0.004
23	1.000	High	Medium	Medium	0.866	0.030	0.104
24	0.356	High	Medium	High	0.993	0.000	0.007
25	1.000	High	High	Low	0.693	0.305	0.002
26	1.000	High	High	Medium	0.945	0.016	0.040
27	0.759	High	High	High	0.090	0.001	**0.909**

Table 10
Belief rule base for aftertaste.

Rule	Rule weight	Attribute			Aftertaste		
		Mouthcoating	Astringent	Irritant	Low	Medium	High
1	0.914	Low	Low	Low	0.001	0.048	**0.952**
2	0.040	Low	Low	Medium	0.184	0.024	0.792
3	1.000	Low	Low	High	0.996	0.002	0.002
4	1.000	Low	Medium	Low	0.995	0.000	0.005
5	1.000	Low	Medium	Medium	0.037	0.077	0.886
6	1.000	Low	Medium	High	0.040	0.016	**0.944**
7	0.957	Low	High	Low	0.834	0.024	0.142
8	0.894	Low	High	Medium	0.923	0.012	0.064
9	1.000	Low	High	High	0.020	0.010	**0.970**
10	1.000	Medium	Low	Low	0.988	0.004	0.008
11	0.088	Medium	Low	Medium	0.978	0.015	0.008
12	0.156	Medium	Low	High	0.667	0.313	0.021
13	1.000	Medium	Medium	Low	0.579	0.300	0.121
14	0.027	Medium	Medium	Medium	0.993	0.004	0.003
15	0.035	Medium	Medium	High	0.408	0.009	0.582
16	0.949	Medium	High	Low	0.840	0.157	0.003
17	1.000	Medium	High	Medium	0.640	0.324	0.035
18	0.234	Medium	High	High	0.043	0.019	**0.937**
19	0.969	High	Low	Low	0.977	0.009	0.014
20	0.068	High	Low	Medium	1.000	0.000	0.000
21	0.966	High	Low	High	0.787	0.054	0.159
22	0.320	High	Medium	Low	0.961	0.035	0.004
23	0.798	High	Medium	Medium	0.351	0.034	0.614
24	0.209	High	Medium	High	0.761	0.000	0.239
25	1.000	High	High	Low	0.523	0.475	0.002
26	1.000	High	High	Medium	0.965	0.010	0.025
27	1.000	High	High	High	0.088	0.001	**0.911**

Table 11
Fitting performance of the 16 orange juices by the hybrid ER and BRB methodology.

Orange juice	Overall assessment			Consumer preference	
	Low	Medium	High	Actual mean score	Fitted or predicted mean score
1	0.496	0.038	0.466	5.97	5.969
2	0.584	0.028	0.388	5.80	5.804
3	0.449	0.286	0.265	5.82	5.816
4	0.759	0.078	0.163	5.40	5.405
5	0.341	0.133	0.526	6.18	6.184
6	0.664	0.052	0.284	5.62	5.620
7	0.412	0.059	0.529	6.12	6.117
8	0.453	0.075	0.472	6.02	6.018
9	0.487	0.020	0.493	6.01	6.006
10	0.442	0.085	0.473	6.03	6.031
11	0.755	0.091	0.154	5.40	5.399
12	0.589	0.067	0.344	5.75	5.755
13	0.508	0.327	0.165	5.66	5.656
14	0.302	0.049	0.648	6.35	6.346
15	*0.772*	*0.081*	*0.147*	*5.37*	*5.374*
16	*0.729*	*0.044*	*0.226*	*5.50*	*5.497*

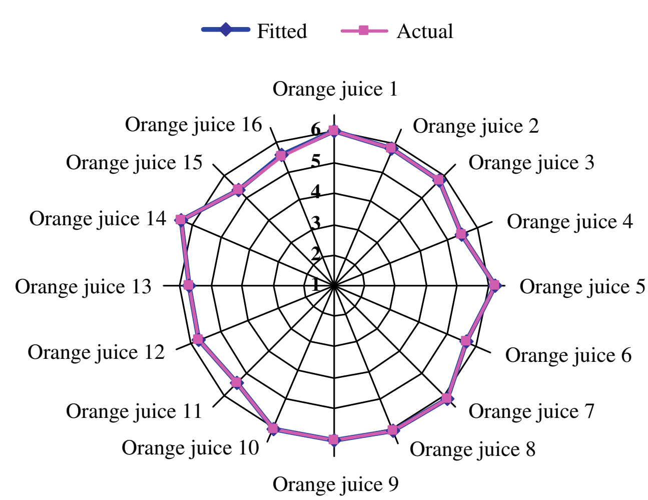

Fig. 3. Fitting performance of the 16 orange juices by the hybrid ER and BRB methodology.

in Tables 6–10 provide a very good understanding of consumer preferences for orange juices and much useful information for new orange juice product design.

Table 11 and Fig. 3 show the fitting performance of the hybrid ER and BRB methodology for the 16 orange juices. It is very clear that the hybrid ER and BRB methodology achieves desirable performance for both training and testing samples. In order to compare with other methodologies, we have modeled and tested the two data sets using artificial neural networks (ANNs), multiple regression analysis (MRA) and principal components (PCs)-based regression analysis (PCRA), respectively. The results are presented in Table 12, from which it is seen that MRA achieves perfect performance for training data set, but performs poorly for testing data set. This poor performance can be improved by using the PCRA, which extracts three PCs from the data set of 21 sensory attributes as explanatory (independent) variables. Better performance, however, is achieved by the artificial neural network with the 21 sensory attributes as inputs, 11 hidden neurons and consumer preference as output. Compared with the hybrid ER and BRB methodology, the neural network is a black box, whose relationships between the output and inputs are not easy to interpret, whereas the BRBs are transparent and the belief rules are interpretable and revisable.

The testing results in Tables 11 and 12 show the superiority and validity of the hybrid ER and BRB methodology. The validated hybrid ER and BRB model can then be used for consumer preference prediction for new orange juices. Table 13 shows four design alternatives for new orange juices and consumer preference predictions for them by the validated hybrid ER and BRB model. It is evident that the four design alternatives can all achieve maximum cus-

Table 12
Fitting performance of the 16 orange juices by artificial neural networks (ANNs), multiple regression analysis (MRA) and principal components (PCs)-based regression analysis.

Orange juice	Consumer preference	Fitted or predicted consumer preference					
		ANNs	Accuracy (%)	MRA	Accuracy (%)	PCRA	Accuracy (%)
1	5.97	5.994	99.59	5.970	100	5.955	99.76
2	5.80	5.797	99.88	5.800	100	5.951	97.46
3	5.82	5.808	99.86	5.820	100	5.790	99.55
4	5.40	5.402	99.95	5.400	100	5.818	92.36
5	6.18	6.179	99.92	6.180	100	5.878	95.04
6	5.62	5.618	99.97	5.620	100	5.805	96.71
7	6.12	6.100	99.73	6.120	100	5.977	97.72
8	6.02	6.031	99.79	6.020	100	5.814	96.61
9	6.01	6.010	99.94	6.010	100	5.831	97.09
10	6.03	6.003	99.54	6.030	100	5.821	96.53
11	5.40	5.405	99.88	5.400	100	5.904	90.64
12	5.75	5.718	99.36	5.750	100	5.867	98.05
13	5.66	5.659	99.95	5.660	100	5.841	96.74
14	6.35	6.349	99.95	6.350	100	5.874	92.56
15	*5.37*	*5.384*	*99.82*	*7.109*	*67.62*	*5.767*	*92.68*
16	*5.50*	*5.407*	*98.36*	*7.079*	*71.29*	*5.836*	*93.83*

Table 13
Consumer preference prediction for new orange juice design alternatives.

Attribute	Design alternatives for new orange juices			
	1	2	3	4
Appearance				
Colour	21.80	54.50	21.80	52.89
Bitty	1.50	26.41	1.50	27.29
Frothy	6.85	1.80	2.26	1.80
Aroma				
Citrus	33.76	20.40	18.23	16.40
Marmalade	17.36	15.32	11.80	11.42
Orange	41.37	35.04	41.92	38.26
Other fruit	3.34	1.05	1.66	2.96
Off	0.92	0.65	0.90	3.79
Texture				
Bitty	26.92	14.40	54.97	32.52
Body	40.42	31.00	48.48	52.48
Flavour				
Sweet	63.63	62.30	63.66	63.69
Sour	36.20	36.20	36.20	36.20
Citrus	62.30	62.30	62.30	62.30
Orange	19.16	15.68	20.43	21.74
Artificial orange	19.21	19.20	19.20	19.20
Other fruit	2.50	2.50	2.50	2.50
Bitter	46.60	46.60	46.60	46.60
Off	4.23	6.01	3.60	2.95
Aftertaste (afterfeel)				
Mouthcoating	30.40	32.87	34.74	39.05
Astringent	75.24	77.00	77.00	77.00
Irritant	59.00	59.00	59.00	59.00
Consumer preference	6.95	6.95	6.95	6.95

tomer satisfaction. The new orange juices designed by the four alternatives are therefore predicted to be more popular than any of the 16 orange juices.

4. Conclusions

Consumer preference prediction is an important issue to retro design of sensory products. In this paper we have developed a hybrid ER and BRB methodology for consumer preference prediction for orange juices that are distinguished by a large number of sensory attributes. Since the sensory attributes have to be grouped into many categories such as appearance, aroma, texture, flavour and aftertaste for ease to interpret, the ER approach has therefore been suggested instead of BRB to aggregate category attributes to avoid building a huge big BRB for them, whereas the BRB methodology has been employed to model the casual relationships between category attributes and their sensory attributes. An optimization model has been constructed to learn the overall relationships between consumer preferences and sensory attributes and model parameters such as belief degrees, rule weights and the weights for sensory and category attributes, etc. The proposed hybrid ER and BRB methodology and its mathematical model have been examined and validated using the real data for 16 orange juices. It has been shown that the hybrid ER and BRB methodology outperforms ANNs, MRA and PCRA and can fit and predict consumer preferences for orange juices with high accuracy. It therefore provides a novel and systematic way for consumer preference study and a useful tool for consumer preference prediction for sensory products.

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