

Dempster-Shafer Theory of Evidence: Potential usage for decision making and risk analysis in construction project management

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Abstract

Decision making and risk analysis are essential tasks along the project life cycle. Researchers have researched different theories and have come up with various tools and methodologies to assist practitioners in assessing risk and making more informative decisions in the case of uncertainty. Many Decision Support Systems (DSSs) are available for practitioners. Unfortunately, in reality such tools are rarely used. Instead, practitioners used to rely upon their personal judgment, past experience, intuition and gut feeling.

Having conducted an extensive literature review, the authors identified the limitations of the extensively used Probability theory (PT), Fuzzy Sets Theory (FST) and the Analytical Hierarchy Process (AHP) for decision making and risk assessment in construction industry. We believe that Dempster-Shafer Theory of Evidence (DST) could provide a valid alternative. Incomplete information and ignorance, which are typical problems in construction industry, could be better handled by using distributed belief assessments. Dempster rule of evidence combination can be used for aggregating risk assessments which may be a novel alternative to the simple averaging procedures adopted by researchers when deploying the FST.

In this first part of the paper there will be a concise illustration of the limitations and the shortcomings of the usage of PT, FST and AHP for handling risk assessment and decision making in construction industry. In the next part there will be a full illustration of the merits and the shortcomings of DST and its potential usage for risk assessment in construction industry. The paper concludes that an evidential reasoning approach based on DST might create a leap in risk assessment and decision making in construction project management. It is the first time that such an approach is ever used for handling construction project risk assessment.

Keywords: decision making, risk assessment, construction project, D-S theory of evidence, Dempster rule of evidence combination.

Introduction

Construction industry has a very risky and complex nature. Risk is associated with every project and each process and decision throughout the project life cycle (BS IEC 62198:2001). Along the project life cycle (PLC) there are many decisions to be made. Unfortunately, the most crucial ones are to be made in the early stages where very limited precise information is available to support the decision maker. Researchers have worked hard to develop models and decision support systems to assist decision makers in making decisions in the case of uncertainty. Several DSS development techniques have been used in construction industry. The choice of an appropriate technique depends mainly upon the difficulty at knowledge acquisition, required data, explanation capacity, difficulty at development and the appropriate domain (Baloi and Price 2003). Probability theory has been extensively deployed in project risk analysis especially project duration risk. Fuzzy Set theory (FST) and

the Analytic Hierarchy Process technique (AHP) are extensively used for risk assessment and decision making in construction industry. These theories and tools have been used by many researchers like: (Cooper et al. 1985); (Kangari and Riggs 1989); Hull (1990); Yeo (1990); (Lai et al. 2008); (Zeng et al. 2007); (Zhang and Zou 2007); (Mahdi and Alreshaid 2005); (Shang et al. 2005); (Al-Harbi 2001); (Leu et al. 2001); (Carr and Tah 2001); (Ziad and Ayyub 1992).

Although there is no best theory of uncertainty (Baloi and Price 2003), the authors argue that risk assessment and decision making in construction industry can be better handled by following an Evidential Reasoning (ER) methodology based on Dempster-Shafer Theory of Evidence.

In this paper, we are illustrating Dempster-Shafer theory of Evidence and the suitability of using it to handle risk assessment and decision making in construction industry. Firstly, there will be a brief summary of the limitations of the widely used PT, FST and AHP for risk assessment and decision making in the case of uncertainty. Secondly, the paper highlights the merits and the limitations of DST and its suitability to handle decision making and risk assessment in construction project management. Finally, a new methodology for risk assessment based on DST is presented. An example is provided to illustrate the possible usage of ER approach for risk assessment.

Literature review; the limitations of PT, FST and AHP

The limitations of PT

Probability theory has been used to model precisely described, repetitive experiments with observable but uncertain outcomes. The two main schools of thought in this field are the frequentist and the Bayesian (Liu et al. 2002). According to Flanagan and Norman (1993), the objectivists, frequentists, believe that probabilities must relate to long frequencies of occurrence. Therefore, only after repeated observations can one determine the relative frequency of events and the associated probabilities. However, in the subjective probability theory, the Bayesian, the probability of an event is the degree of belief or confidence placed in its occurrence by the decision maker on the basis of the evidence available. Unfortunately, objective probabilities are difficult to obtain in the construction industry where many buildings are one-off projects (Flanagan and Norman 1993). As a result, project managers are obliged to fall back on the elicitation of subjective probabilities (Winch 2003). Traditionally the focus has been on quantitative risk analysis. However, dissatisfaction with the inability of this type of approaches to handle subjectivity in risk assessments has led to research into the use of other approaches like the FST (Tah and Carr 2001).

Hence, due to lack of previous data and unique, non-repetitive nature of construction projects, usually probabilistic approach can not be utilized to quantify risks. Instead, individual knowledge, experience, intuitive judgment and rules of thumb should be structured to facilitate risk assessment (Dikmen et al. 2007).

The limitations of FST

According to Dikmen et al. (2007), FST provides a useful way to deal with ill-defined and complex problems in a decision-making environment that incorporates vagueness. However, they claimed that one of the reasons why FST is not widely used in practice may be attributed to its computational complexity. Kangari and Riggs (1989) mentioned another three limitations of the FST. These limitations are:

1. A problem of assigning the membership values of a fuzzy set to represent a linguistic variable. The authors believe that this problem reflects the difficulty in interpreting or understanding the actual

meaning of linguistic variables by different people. It is, by itself, composes another dimension of uncertainty

2. a problem of how to perform arithmetic operations
3. and a problem of associating the final fuzzy set in a series of calculations with a linguistic variable. Kangari and Riggs (1989) mentioned that a generally used technique involves calculating the Euclidean distance between the fuzzy set under question and a set of benchmark fuzzy sets. The fuzzy set under question thus takes on the linguistic characteristic of the closest of the benchmark fuzzy sets.

Tah and Carr (2001) mentioned the limitation of FST when aggregating risk assessments. The existing methods produce an average assessment. The averaging methodology may not be suitable for producing reliable overall assessment. We believe that this limitation in FST is very critical especially when the decision making aim is, for instance, not to assess a risk by itself but to assess the risk level of a project as a whole.

The limitations of AHP

AHP have been widely used by researchers in attempt to tackle the problem of ill-defined and ill-structured problems of decision making and risk assessment. In one of the earliest attempts to use AHP in construction industry, and the first use of it specifically for assessing construction risk, Mustafa and Al-Bahar (1991) found it providing a valuable support for decision making process especially because of the systematic thinking environment it offered. However, they mentioned two concerns regarding the efficient usage of it. These concerns are:

1. Building the hierarchy with (7 ± 2) elements under any node in order to preserve consistency as recommended by the founder of AHP himself. Mustafa and Al-Bahar (1991) illustrated a solution to this case by grouping the elements in clusters and then merging these comparable clusters.
2. The number of judgments required to derive relative priorities. It seems that AHP needs $(n - 1)$ judgments to relate one element to the remaining $(n - 1)$ elements. From these judgments, one can construct all other comparisons by forming ratios. This approach could be used when assessing tangible criteria. However, when one is dealing with intangible criteria it is much difficult. Mustafa and Al-Bahar (1991) argued that in such circumstances, one is no longer certain of the precise correspondence of the strength of a judgment to a numerical value, which represents that judgment, nor is he/she certain of the judgment itself.

According to Sen and Yang (1998), the large number of judgments required often causes inconsistency problem. This will also make conducting sensitivity analysis in AHP is not practical (Belton and Stewart 2002).

These concerns have been mentioned be researchers from AHP practicality point of view. However, there are other concerns from theoretical point of view:

1. According to Belton and Stewart (2002) one of the concerns about the AHP is the Rank Reversal Problem. Rank Reversal refers to the fact that in certain situations, the introduction of a new alternative which does not change the range of outcomes of any criterion may lead to a change in the ranking of the other alternatives (Belton and Stewart 2002). As a result, when new alternatives are added to AHP, the assessments done on the old alternatives have to be discarded. A new assessment procedure has to start from the beginning, taking into account the whole alternatives (Xu and Yang 2001).
2. Belton and Stewart (2002) stated that some concerns have been expressed in literature concerning the appropriateness of the conversion from semantic to numeric scale used to measure the strength of

preference. According to them, the general view, supported by experimental work, seems to be that the extreme points of the scale defined semantically as “absolute preference” is more consistent with a numeric ratio of 1:3 or 1:5 than the 1:9 used in AHP.

3. Sen and Yang (1998) mentioned another theoretical limitation of AHP which is the implicit assumption that elements at any single level in the hierarchy except the bottom one, the alternatives, are preferentially independent. They argued that an evaluation of an attribute in real MCDM problem may most probably depend upon the achievement level of other attributes.

Application of D-S theory of evidence in construction industry

D-S Theory of Evidence

The Dempster-Shafer theory of evidence was established by Shafer (1976) for representing and reasoning with uncertain, imprecise and incomplete information (Smets 1988). Dempster proposed his new system of dealing with uncertainty because of two shortcomings he saw with probability theory:

1. The difficulty of representing ignorance. In probability theory, ignorance is represented by assigning equal prior probabilities to all events which is surrounded with difficulties and limitations. In such representation, there is no distinction between randomness and ignorance.
2. The requirement of subjective belief in an event and its negation to sum to one. Dempster claimed that in many situations evidence that supports one hypothesis should not necessarily decrease the belief in all others (Dempster 1969). In D-S theory, there is no requirement that belief not committed to a given proposition must be committed to its negation. This makes the total allocation of belief can vary to suit the extent of knowledge of the decision maker.

According to Denoeux (1999), D-S theory of evidence distinguishes between uncertainty and ignorance by introducing belief functions that satisfy axioms that are weaker than those of probability functions. Thus, probability functions can be looked at as a subclass of belief functions, and the theory of evidence reduces to probability theory when the probability values are known. In addition, the theory of evidence provides appropriate method, Dempster rule for evidence combination, for computing belief functions for combinations of evidence.

Liu et al. (2002) summarized the advantages of the Dempster-Shafer theory as follows:

1. It has the ability to model information in a flexible way without requiring a probability to be assigned to each element in a set,
2. It provides a convenient and simple mechanism (Dempster's combination rule) for combining two or more pieces of evidence under certain conditions.
3. It can model ignorance explicitly.
4. Rejection of the law of additivity for belief in disjoint propositions.

Liu et al. (2002) also listed the disadvantages of the D-S theory as follows:

1. The theory assumes that pieces of evidence are independent which is not always reasonable to assume independent evidence.
2. The computational complexity of reasoning within the D-S theory could be one of the major points of criticism if the combination rule is not used properly.
3. D-S theory only works on exclusive and exhaustive sets of hypotheses.

As mentioned earlier, there is no best theory to handle uncertainty. D-S theory of evidence is, in comparing with fuzzy set theory and probability theory, richer in terms of semantics. It allows an expression of partial knowledge (Baloi and Price 2003). The authors believe that the merits of DST make researching its potential application in construction industry a valid research project. The authors are proposing using an evidential reasoning approach based on DST for assessing construction risk and making related decisions such as evaluating and ranking construction projects.

Evidential Reasoning for decision making and risk assessment in construction industry

The ER approach is the latest development in MCDA area (Yang 2001). It is based on D-S theory which is powerful evidence combination rule and well suited for handling incomplete uncertainty (Yang and Singh 1994). It uses an evidence-based process to reach a conclusion which differs from traditional MCDA methods (Xu and Yang 2005).

ER uses the concept of ‘degree of belief’ to elicit a decision-maker’s preferences. The degree of belief can be described as the degree of expectation that an alternative will yield an anticipated outcome on a particular criterion (Sonmez et al. 2001). An individual’s degree of belief depends on the knowledge of the subject and the experience. The authors advocate the usage of belief functions for risk assessment due to the fact that it may not always be achievable or practical to expect or force individuals to make certain and precise assessments.

Also, ER approach is different from most conventional MCDM modeling methods in that it employs a belief structure to represent an assessment as a distribution: (Xu and Yang 2001); (Wang and Elhag 2007). It uses an extended decision matrix, in which each attribute of an alternative is described by a distributed assessment using a belief structure. Each belief structure in the belief decision matrix can be transformed into a basic probability assignment (BPA) by combining the relative weight of the criterion and the degrees of belief. Each BPA is viewed as a piece of evidence. The pieces of evidence are subsequently combined into an overall BPA by the Dempster’s rule of evidence combination (Wang and Elhag 2007).

In the extended decision matrix, each attribute of an alternative is described by a distributed assessment (Xu and Yang 2001). Hence, each figure in a decision matrix is actually a distributed assessment, i.e. a set of figures.

Suppose there is a decision making problem with M alternatives, A_i ($i=1, \dots, M$), to choose from and N criteria, C_j ($j=1, \dots, n$), to consider. The next table illustrates the extended decision matrix.

Table :. The extended decision matrix

	$C1$	$C2$	$C3$...	Cn
$A1$	$S(A1(C1))$				
$A2$					
$A3$					
Am					

The assessment of each alternative according to different criteria is to be conducted using K evaluation grades $G = \{G1, G2, G3, \dots, Gk\}$. These grades are to be deployed in each cell of the decision matrix to provide the distributed assessments.

Hence, the assessment of the alternative A1 on a criterion C1 can be represented using the following belief structure:

$$S(A1(C1)) = \{(b11, G1), (b12, G2), (b13, G3), \dots, (b1k, Gk)\}$$

Where $b11, b12, b13, \dots, b1k$ are the degrees of belief that the alternative A1 is assessed to the evaluation grades G1, G2, G3, ..., GK when considering the criterion C1. These evaluation grades could be defined, for instance, as follows: Extremely poor, Poor, Slightly acceptable, Neutral, slightly preferred, Good, Excellent. The degrees of belief are expressed by the decision maker and the value of each of them falls in the range between 0 and 1.

The notion of extended decision matrix can be used successfully for the purpose of construction risk assessment. Having conducted an extensive literature review, we concluded that there is a lack of a comprehensive framework that would enable the different impacts of a risk on specific project objectives, such as, time, cost and quality to be assessed. We are proposing the usage of risk cost as a common scale to measure the risk impact on a construction project objectives. Although many authors recommend the use of 'risk cost' as an assessment scale: (Chan and Au, 2008; Franke, 1987; Paek et al. 1993; Sanchez, 2005; Williams, 1993; and Williams, 1995) and estimating contingency allowances being a well-established practice in the construction industry, pricing risk has never been used systematically to assess construction project risk.

Utilizing 'risk cost' within an ER approach for structuring and enabling the experience and intuition of practitioners to be incorporated will facilitate a realistic and usable assessment method. Risk cost is a common scale and a term widely understood by all parties, making the risk assessment process more functional and acceptable to practitioners bearing in mind the lack of an accepted method for risk assessment in the construction industry (Mulholland and Christian 1999).

We are assessing risk impact as a percentage of project initial cost. Although one may argue against using risk cost as a scale for assessing intangible factors like project quality risk, we believe that it is achievable. A decision maker, who is capable of prioritizing the importance of intangible attributes within the AHP framework, should be able to produce a percentage of project initial cost to reflect the impact of damage on an intangible objective arising from a risk. Moreover, practitioners who are able to size and price residual risks, the unknown unknowns, and provide a contingency allowance for them should be able to size and price the known unknowns: the risks.

Illustrative example

From the previous extended decision matrix, let Ai represents the identified crucial risk factors that may affect a construction project. Also, let Cj represents project objectives on which the risk analyst is to assess risk impacts. Hence, $S(A1(C1))$ will represent the distributed assessment, based on the belief of the risk analyst, of the impact of risk A1 on the objective C1. However, different from a decision making problem where the decision maker is to evaluate the alternatives according to their performance on different attributes, in the case of risk assessment the aim of risk analyst is to assess the different impacts of all risk factors on project objectives.

To put it in context, let Ri represents risk factors, Oj represents project objectives, $S(Rj(Oj))$ represents the distributed assessment of the impact of a risk Ri on a project objective Oj . Having agreed upon that, it is possible now to represent the risk assessment task in the following risk assessment table:

Table 2: The risk assessment table

	$O1$ Project Cost	$O2$ Project Duration	$O3$ Project Quality	$O4$ Others
$R1$	$S(R1(O1))$			
$R2$				
$R3$				
Rm				

In this table risk impact can be assessed on project cost, duration, quality and a fourth objective called “*Others*”. This fourth objectives is considered in order to appreciate any other important objectives the risk analyst may consider when evaluating the risk level of a construction project. This category may include environmental impact, health and safety, company reputation, etc. It is worth noting that the fourth objective may contain more than one objective if they are of the same nature. It is also possible to consider more than four project objectives. This issue depends upon the criticality of required analysis, the complexity of the project under analysis or the preference of the risk analyst.

For assessing risk, the distributed assessment of the impact of the risk $R1$ on the objective $O1$ can be represented as:

$$S(R1(O1)) = \{ (\beta11, G1), (\beta12, G2), (\beta13, G3), (\beta14, G4) \}$$

Evaluation grades

Let the evaluation grades to be expressed as percentages of the project initial cost. For instance $Gi = \{1\%, 2\%, 3\%, 4\%\}$. This means that a risk Ri may have an possible impacts equal to 1%, 2%, 3% or 4% of the project initial cost on the objective Oi . These evaluation grades are used to represent different scenarios where risk may have different impact magnitudes on an objective.

Degrees of belief:

The risk analyst can express his/her degree of belief regarding each grade using a figure between 0 and 1. A number in the range of [0 – 1] will replace each of $\beta11$, $\beta12$, $\beta13$ and $\beta14$.

Hence, in order to assess risk impact on a specific project objective,

1. The risk analyst should be able to have an initial and approximate estimate of the cost of damage that may happen on the objective under analysis if a specific risk happens.
2. He/she has to represent his belief toward the surety of this cost by considering the stated evaluation grades. He/she can express this belief by assigning belief degrees, proportional figures between 0 and 1, to each of the evaluation grades.

The idea, simply, is to distribute the analyst initial and approximate risk cost on a wider and panoramic scale in order to make it as detailed as possible. In other words, the sum of the multiplications between the evaluation grades G_i and the associated degrees of belief β_{ij} should be equal to the initial average risk cost expressed by the risk analyst.

Such assessment methodology can provide the decision maker with panoramic view of risk impact on various project objectives like time, cost, quality, etc. on different levels of the risk bear-down structure of a construction project. In the same time it is very easy to generate an average figure to summaries such detailed assessment.

Risk is usually modelled as a multiplication of *Probability* of occurrence and *Impact*. Hence, the above mentioned impact assessment can be multiplied with a figure, also between 0 and 1, stands for probability of occurrence. For each risk factor, the degrees of belief concerning its potential impact can be multiplied with a probability of impact in order to generate a distributed risk assessment. Also, each risk can be associated with a weight, ω_i , in order to reflect the importance of a risk R_i comparing to the other identified risks. The analyst should provide weights for the identified risks and level these weights so they all sum to 1.

Having assessed the impact, probability of occurrence and weight for each risk, the next step, for the analyst will be aggregating the individual risk assessments in order to get project risk level. Project risk level will stand for the total amount of risk exists in a construction project. According the proposed assessment methodology project risk level will also be presented as a percentage of the project initial cost.

Assessments aggregation

Besides assessing project risk level, aggregating individual risk assessments can be used in order to get different risk levels at different levels of the project risk hierarchy. Moreover, the aggregation can be conducted in order to generate an overall assessment of risk for every project objective. Hence, assessment aggregation can be conducted on two dimensions across the project risk break-down structure rather than the conventional solo direction towards the top of the project risk hierarchy.

In order to conduct such aggregation, Dempster rule of evidence combination can be used. It will provide a novel approach for aggregating different risk assessments. This aggregation method can overcome the limitations of the existing aggregation tools which are mainly averaging tools as illustrated earlier in a previous part of the paper.

Let us assume that we have m risks affecting a project. Each of them is assessed against N grades, and each of them is associated with weight ω_i . From the below figure, the assessment of risk impacts against the defined assessment grade are represented by $\beta_{i,n}$.

The overall assessment β_n on each assessment grade can be generated by using Dempster rule of evidence combination. β_n can be generated by the following non-linear aggregation algorithm:

$$\beta_n = k \left[\prod_{i=1}^m (\omega_i \beta_{i,n} + 1 - \omega_i) - \prod_{i=1}^m (1 - \omega_i) \right]$$

Where K is:

$$k = \left[\sum_{n=1}^N \prod_{i=1}^m (\omega_i \beta_{i,n} + 1 - \omega_i) - N \prod_{i=1}^m (1 - \omega_i) \right]^{-1}$$

In case we have only two risks to be considered i.e., $m=2$, their distributed assessments can be aggregated by the following formulae:

$$\beta_n = \bar{k}(\omega_1\beta_{1,n} + \omega_2\beta_{2,n} - \omega_1\omega_2(\beta_{1,n} + \beta_{2,n} - \beta_{1,n}\beta_{2,n}))$$

$$\bar{k} = \left(\sum_{n=1}^N (\omega_1\beta_{1,n} + \omega_2\beta_{2,n} - \omega_1\omega_2(\beta_{1,n} + \beta_{2,n} - \beta_{1,n}\beta_{2,n})) \right)^{-1}$$

The following figure represents the aggregation process of risk assessments:

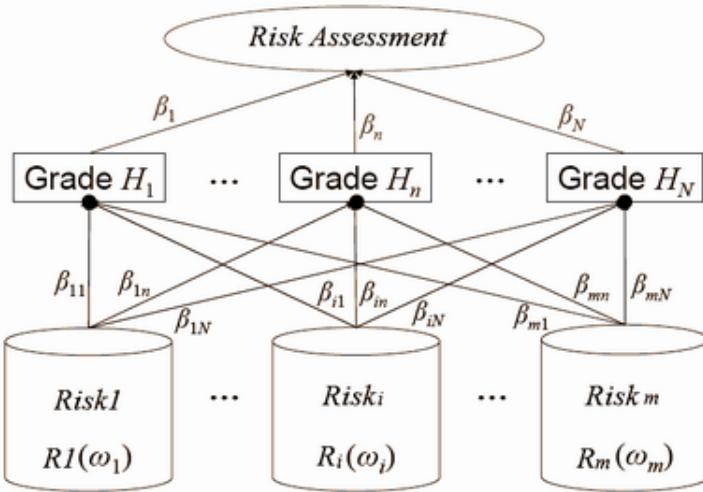


Figure 1: Aggregation of risk assessment

The above illustrated methodology can provide a novel alternative for assessing risk in construction. It can provide detailed assessments of the different impacts of each identified risk on every project objective. The proposed aggregation rule will enable risk analyst to aggregate his/her assessments without averaging them and without loosing their panoramic nature. He/she can conduct an aggregation task to any level of the risk breakdown structure. Moreover, the aggregation can be conducting on every project objectives before aggregating them in order to get a project risk level. This will provide the decision maker with very clear and detailed picture of the task he/she is facing.

The authors argue that such *evidence-based* framework for risk assessment and assessment will have a maximum deployment of the cumulative practical experience of the decision maker which is thought to be essential in decision making. These features will satisfy the practitioners and respond for their aspirations in having practical and usable decision support systems (Taroun 2010).

Conclusion and further research

This paper has illustrated the limitations of the existing methods and theories which are used for risk assessment in construction industry. It has also provided an alternative risk assessment methodology based on an evidential reasoning and Dempster-Shafer Theory of evidence.

To conclude, the existing tools for supporting decision making and risk assessment in construction industry have not fully appreciated the special nature of this industry. Practical experience, personal judgment and intuition are still playing a crucial role in decision making. Besides the technical limitations of any decision making

methodology, its usability and popularity among practitioners should be a matter of concern. Despite the large number of available DSSs for construction practitioners, unfortunately such DSSs are not widely used. The usual practice, however, is to make decisions on the basis of intuition, derived from a mixture of gut feeling, experience, and guesses (Ahmad 1990). Hence, researchers have to facilitate practitioners' experiences by suitable but robust methodologies.

We argue that the proposed ER approach based on D-S theory of evidence could be vital response to the practitioners' aspirations. DST has an advantage over PT and FST being able to represent ignorance or lack of information, which is a typical problem in risk analysis in construction industry. Moreover, it is very appropriate for talking risk assessment because it is very suitable for domains with a hierarchical structure (Liu et al. 2002). The revised Dempster's combination rule of evidence which has been adopted, it is presented in detail in Yang (2001), will be appropriate for aggregating individual risk assessments.

Notably, DST has never been used in the construction project risk assessment domain. We believe that researching the potential applications of DST in risk analysis and decision making in construction industry may be viable and original contribution to knowledge. This paper is part of a wider research project aiming at developing a new model for construction project risk and a new assessment methodology. The proposed model and assessment methodology are to be deployed within a decision support system for project evaluation and risk analysis in construction industry.

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