Perturbation Analysis of Evidential Reasoning Rule

Shuai-Wen Tang, Zhi-Jie Zhou, Chang-Hua Hu, Jian-Bo Yang, and You Cao

***Abstract*—Evidential reasoning (ER) rule has been widely used in addressing uncertainty, ignorance, and vagueness information. To explore its performance measure (PM), the perturbation analysis (PA) for the ER rule (ER rule-PA) is conducted, with perturbation taken into consideration. This article aims to analyze the robustness and stability of the ER rule, serving as theoretical basis and technical support for applied research and applications. The combination of two pieces of independent evidence is discussed, and perturbation is added to one piece of evidence. To represent the expected utility of evidence combination under perturbation, perturbation utility is introduced. The novel concept of perturbation coefficient is proposed to characterize the PM of the ER rule (ER rule-PM). The properties of perturbation coefficient are explored to demonstrate the impact of perturbation. The maximum permissible error (MPE) of perturbation coefficient is defined to characterize the acceptability of perturbation. A numerical study is examined to illustrate the implementation process of ER rule-PA. Moreover, a case study of reliability evaluation of aerospace relay is conducted to show the potential applications of ER rule-PA, which makes the proposed method more practical.**

***Index Terms*—Aerospace relay, evidential reasoning (ER) rule, perturbation analysis (PA), perturbation coefficient, perturbation utility.**

I. Introduction

THE EVIDENCE theory was originated from Dempster's [1] work on multivalued mapping in 1967 and formalized by Shafer [2] in 1976. By introducing the concept of belief function, the relationship between proposition and set was established, which is therefore the so-called Dempster–Shafer (D–S) theory or belief function theory [2].

Dempster's rule is simple to understand and satisfies the property of commutativity and associativity, although it has limitations [3]. To address the limitations of the D–S theory like the nondefinition and counter-intuitive problem of Dempster's rule [3], [28], a series of research have been conducted, such as Yager's [4] rule, Dubois and Prade's [5] rule, the PCR5 rule [6], alternative ECRs [7]–[9], etc. These improved methods or new rules have solved some problems such as avoiding the nondefinition and reducing the conflict between two pieces of evidence, but no longer satisfy the properties of Dempster's rule, nor are they a probabilistic or Bayesian reasoning process. As was illustrated in [3], they have led to obvious changes of the specificity of evidence and cannot keep intact the most important property of Dempster's rule that it generalizes Bayes' rule.

To further improve D–S theory, from the perspective of belief function, Li and Deng [29] proposed generalized ordered propositions, where the concept of entropy was introduced to measure uncertainty. Based on Dempster's rule, a new evaluation method for decision making analysis was proposed by Zheng and Deng [30], where basic probability assignment (BPA) distribution was transformed into probability distribution. Besides, in the context of multiple sensors, Xiao [31] proposed a novel data fusion method based on a new belief divergence measure of evidence and the belief entropy.

From the perspective of BPA, to deal with the limitations of evidence-based decision making, Yang and Singh [10] and Yang and Sen [32] first proposed the evidential reasoning algorithm (ERA) in 1994 to fuse weighted evidence in multiple-attribute decision making (MADA). Based on ERA, Yang and Xu [3] proposed the evidential reasoning (ER) rule in 2013. The unique feature of the ER rule is that it takes full account of the weight and the reliability of each piece of evidence, which makes it possible to deal with ambiguity, uncertainty, and incompleteness. Besides, it generalizes the traditional Bayesian inference process and constitutes a generic conjunctive probabilistic reasoning process, which keeps intact the specificity of evidence and well addresses the evidence combination problem with high or complete conflict. It has been proven that both Dempster's rule and original ER algorithm are the special case of the ER rule [3]. With the development of the evidence theory, it has been widely used in many areas like information fusion [12], expert systems [13], [14], fault detection and diagnosis [15], MADA [16], [17], risk analysis [18], [19], image processing [20], regression analysis [21], etc.

However, how to analyze the performance measure (PM) of the ER rule (ER rule-PM) remains to be further studied [3], [22]. In other words, it is necessary to answer the following type of question: "whether the fusion results will keep unchanged or fluctuate within a certain acceptable

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range if the fusion process we have just performed is repeated under identical conditions, but with a given perturbation put into effect?" Generally, perturbation exists in many situations, which is usually transient and intermittent rather than continuous. In engineering practice, as for a specific system, the perturbation may only happen at a certain time and disappear later. Therefore, the system will be intermittently unstable especially when it is not stable enough to resist the perturbation.

Taking the power system as an example, perturbation is often involved in its stability analysis, with the intensity of perturbation distinguished. Small perturbation usually refers to the fluctuation of loads and parameters, such as wind-induced oscillation of overhead lines, while large perturbation is often involved in the sudden change of large-capacity loads, such as the switching of main components, short circuit or open circuit fault of the system. If there are several kinds of perturbation with different intensities, whether the power system has the capability to operate regularly remains to be studied. As was illustrated in [23], perturbation analysis (PA) could be carried out for several PMs of interest. Therefore, PA of the ER rule (ER rule-PA) is of great significance to achieve the above goal.

The aim of this article is to conduct some exploratory research of ER rule-PM through ER rule-PA. The remainder of the article is organized as follows. In Section II, basic framework of ER rule-PA is briefly described. ER rule-PA is performed from the theoretical aspect and some properties are explored in Section III. To facilitate the applications of ER rule-PA, a numerical study is conducted and a generalized method of ER rule-PA is explored. In Section IV, a case study is provided to illustrate the feasibility and effectiveness of ER rule-PA in engineering practice. The conclusion of this article is presented in Section V.

II. Basic Framework of ER Rule-PA

In this section, ER rule will be briefly described [3] and some basic knowledge about PA will be introduced.

A. Brief Description of the ER Rule

In general, a piece of evidence can be profiled by a belief distribution (BD), defined as follows:

$$e_i = \left\{ (\theta, p_{\theta,i})\ \forall\theta \subseteq \Theta, \sum_{\theta\subseteq\Theta} p_{\theta,i} = 1 \right\} \tag{1}$$

where $(\theta, p_{\theta,i})$ is an element of evidence e_i representing that e_i points to the proposition θ to the degree of $p_{\theta,i}$. $(\theta, p_{\theta,i})$ will be referred to as a focal element of e_i if $p_{\theta,i} > 0$. θ can be a single evaluation grade or a subset of evaluation grades. Θ is the frame of discernment including all evaluation grades [33].

There are two important parameters in the ER rule, defined as the weight w_i and the reliability r_i of evidence e_i, with $0 \le w_i \le 1$ and $0 \le r_i \le 1$. The difference between w_i and r_i is mainly reflected in that the weight represents the decision makers' preferences for e_i, while the reliability represents the intrinsic property of the information source where e_i is generated. Moreover, they denote the subjective and objective aspects of evidence, respectively [34].

Remark 1: In engineering practice, the relationship between evidence weight and evidence reliability can be understood as follows. Taking the evaluation system of students in China as an example, the course examination score tends to be more important than the physical examination score, so is the corresponding weight. However, if a student cheats in a course examination such as plagiarizing others' paper, his/her final score is unreliable, which means that the reliability is lower than that of the physical examination score. Although the student's course examination score has a lower reliability, its weight is still higher than that of the physical examination score, Furthermore, the unreliability of the course examination score cannot be changed by teachers or others. It is further indicated that the weight belongs to a subjective concept while the reliability belongs to an objective concept. Thus, to evaluate a student more fairly, both weight and reliability of the examination score should be considered.

The basic probability masses for e_i with weight and reliability considered are then assigned as follows:

$$\tilde{m}_{\theta,i} = \begin{cases} 0 & \theta = \varnothing \\ c_{rw,i} m_{\theta,i} & \theta \subseteq \Theta, \theta \neq \varnothing \\ c_{rw,i}(1 - r_i) & \theta = P(\Theta) \end{cases} \tag{2}$$

where $m_{\theta,i} = w_i p_{\theta,i}$ and $m_{\theta,i}$ denotes the basic probability masses for e_i with weight considered. $c_{rw,i} = 1/(1 + w_i - r_i)$ is a normalization factor such that $\sum_{\theta\subseteq\Theta} \tilde{m}_{\theta,i} + \tilde{m}_{P(\Theta),i} = 1$. $\tilde{m}_{\theta,i}$ denotes the extent to which e_i supports θ with both w_i and r_i taken into consideration. $P(\Theta)$ is the power set, which can also be denoted by 2^{Θ}.

The weighted BD with reliability (WBDR) of e_i can be represented as

$$m_i = \left\{ (\theta, \tilde{m}_{\theta,i})\ \forall\theta \subseteq \Theta;\ (P(\Theta), \tilde{m}_{P(\Theta),i}) \right\}. \tag{3}$$

Suppose that two pieces of evidence e_1 and e_2 are independent as profiled by (1) and their WBDRs are represented by (3). The combined degrees of belief to which e_1 and e_2 jointly support proposition θ, denoted by $p_{\theta,e(2)}$, are obtained by

$$p_{\theta,e(2)} = \begin{cases} 0 & \theta = \varnothing \\ \frac{\hat{m}_{\theta,e(2)}}{\sum_{D\subseteq\Theta} \hat{m}_{D,e(2)}} & \theta \subseteq \Theta, \theta \neq \varnothing \end{cases} \tag{4}$$

$$\hat{m}_{\theta,e(2)} = \left[(1 - r_2)m_{\theta,1} + (1 - r_1)m_{\theta,2}\right] + \sum_{B\cap C=\theta} m_{B,1} m_{C,2}\ \forall\theta \subseteq \Theta \tag{5}$$

where $\hat{m}_{\theta,e(2)}$ denotes the unnormalized combined probability masses to which e_1 and e_2 jointly support proposition θ.

Theorem 1 (Recursive Algorithm of the ER Rule): Suppose L pieces of independent evidence are profiled by (1) and their WBDRs are represented by (3), with $m_{\theta,e(1)} = m_{\theta,1}$ and $m_{P(\Theta),e(1)} = m_{P(\Theta),1}$, the combined degrees of belief to which $e(L)$ jointly support θ are given by

$$p_\theta = p_{\theta,e(L)} = \begin{cases} 0 & \theta = \varnothing \\ \frac{\hat{m}_{\theta,e(L)}}{\sum_{D\subseteq\Theta} \hat{m}_{D,e(L)}} & \theta \neq \varnothing \end{cases} \tag{6}$$

where $\widehat{m}_{\theta,e(L)}$ denotes the unnormalized combined probability masses to which the L pieces of evidence jointly support proposition θ, and is generated by recursively applying the following three equations:

$$m_{\theta,e(i)} = \begin{cases} 0 & \theta = \varnothing \\ \frac{\widehat{m}_{\theta,e(i)}}{\sum_{D\subseteq\Theta}\widehat{m}_{D,e(i)}+\widehat{m}_{P(\Theta),e(i)}} & \theta \neq \varnothing \end{cases} \tag{7}$$

$$\widehat{m}_{\theta,e(i)} = \left[(1-r_i)m_{\theta,e(i-1)} + m_{P(\Theta),e(i-1)}m_{\theta,i}\right] + \sum_{B\cap C=\theta} m_{B,e(i-1)}m_{C,i}\ \forall\theta\subseteq\Theta \tag{8}$$

$$\widehat{m}_{P(\Theta),e(i)} = (1-r_i)m_{P(\Theta),e(i-1)} \tag{9}$$

where $i = 2, 3, \ldots, L$, and $m_{\theta,e(i)}$ denotes the normalized combined probability masses to which the L pieces of evidence jointly support proposition θ.

Proof: See [3]. ■

The impact of w_i and r_i on the combination rule has been well explained in [24].

According to the above discussion, the aggregated assessment for alternatives a_l on criterion G_k can be described as follows:

$$S_k(a_l) = \left\{\left(\theta, p_{\theta,k}(a_l)\right), \theta\subseteq\Theta;\ \left(P(\Theta), p_{P(\Theta),k}(a_l)\right)\right\} \tag{10}$$

where $l = 1, 2, \ldots, M$ and $k = 1, 2, \ldots, K$. $p_{P(\Theta),k}(a_l)$ denotes the combined belief degree that is not assigned to any proposition θ for a_l on G_k. $S_k(a_l)$ is said to be complete if there is $\sum_{\theta\subseteq\Theta} p_{\theta,k}(a_l) = 1$ and incomplete if there is $\sum_{\theta\subseteq\Theta} p_{\theta,k}(a_l) < 1$.

Theorem 2 (Utility Evaluation): Suppose the utility of proposition θ is denoted by $u(\theta)$. Suppose there is no global ignorance, the expected utility of $S_k(a_l)$ is calculated as follows:

$$u(S_k(a_l)) = \sum_{\theta\subseteq\Theta} p_{\theta,k}(a_l)u(\theta). \tag{11}$$

Otherwise, the expected utility of $S_k(a_l)$ would be characterized by the interval $[u_{\min}(S_k(a_l)), u_{\max}(S_k(a_l))]$.

Proof: See [24]. ■

Remark 2: The expected utility $u(S_k(a_l))$ denotes the numerical reasoning output of the ER rule, which is used to measure the intensity of the state of a_l on criterion G_k [10]. In this article, it is identical to the concept of ER rule-PM, which is employed as a quantitative description for the output comparison.

B. Formulation of ER Rule-PA

PA has been developed in discrete event dynamic systems (DEDSs) by Ho *et al.* [25] since the 1970s. The characteristic of PA is that the trajectory of perturbation sample is not obtained by another simulation or experiment, but is constructed with the method of theoretical analysis based on the trajectory of nominal sample. Similar to the definition in [26], PA is an analytical technique that can be used in ER rule-PM by analyzing its sample path. It can literally squeeze out a lot more information than traditionally thought possible from the ER rule. With the method of a paradigm, PA can be better understood. Consider a fusion process as Fig. 1.

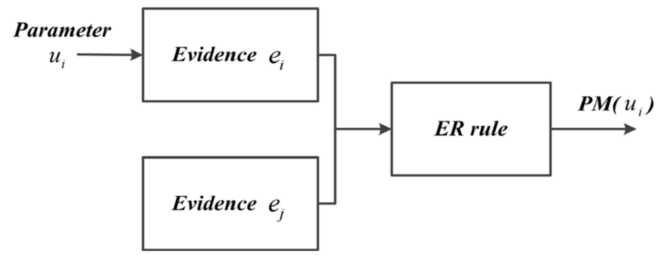


Fig. 1. Example of a fusion process by the ER rule.

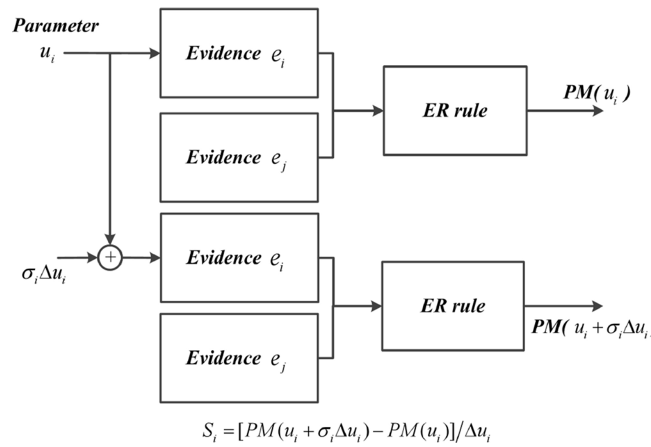


Fig. 2. Calculation of perturbation coefficient of the ER rule.

It can be seen from Fig. 1 that there is an important input called parameter of the ER rule. Meanwhile, there is always an output called PM. To estimate the PM only based on the nominal sample, it is supposed that there is an identical twin to the fusion process in Fig. 1 which means it duplicates exactly the behavior of the original one. The above discussion leads to the following definition.

Definition 1 (Perturbation Coefficient): Suppose the input parameter u_i can be modified by a small amount $\sigma_i\Delta u_i$. Suppose $PM(u_i + \sigma_i\Delta u_i)$ denotes the output of the identical twin fusion process. The perturbation coefficient of the ER rule is calculated as follows:

$$S_i = [PM(u_i + \sigma_i\Delta u_i) - PM(u_i)]/\Delta u_i \tag{12}$$

where σ_i is the intensity of random perturbation. There are two constants ε_1 and ε_2 satisfying $0 < \varepsilon_1 \leq \sigma_i \leq \varepsilon_2$. This is illustrated in Fig. 2.

Here, we provide an idea to control the intensity of random perturbation, that is to say, σ_i should be bounded. Otherwise, the fusion results might be seriously affected. The thresholds ε_1 and ε_2 are preliminarily set for comparison, while their values may be obtained from decision makers based on prior knowledge in engineering practice. Besides, ε_1 and ε_2 belong to the concept of statistics, the actual values should be determined by long-term testing and practice.

Based on Definition 1, if there are as many identical fusion processes as there are parameters of interest, the gradient or

TABLE I
TWO PIECES OF INDEPENDENT EVIDENCE

$p_{\theta,i}$	$\varnothing$	A	B	Θ
e_i	0	$(u_i-U_{l+1})/(U_l-U_{l+1})$	$(U_l-u_i)/(U_l-U_{l+1})$	0
e_j	0	p_j	p_{j+1}	0

coefficient vector can be calculated as follows:

$$\frac{\partial PM}{\partial \Delta u}=\left[\frac{\partial PM}{\partial \Delta u_1},\frac{\partial PM}{\partial \Delta u_2},\dots,\frac{\partial PM}{\partial \Delta u_N}\right]. \tag{13}$$

III. THEORETICAL INFERENCE OF ER RULE-PA

Based on the previous discussions, the input of the ER rule can sometimes be affected by small perturbations, resulting in the low precision and accuracy of fusion results. To further characterize ER rule-PM, ER rule-PA should be conducted from the theoretical aspect.

A. Details of ER Rule-PA

In this section, two pieces of independent evidence e_i and e_j are integrated to illustrate the inference process of ER rule-PA. The basic knowledge is shown as follows.

Definition 2 (Rule-Based Equivalence Transformation Techniques): Suppose there are N distinctive evaluation grades represented by

$$H=\{H_1,H_2,\dots,H_l,\dots H_N\} \tag{14}$$

where H is identical to the frame of discernment Θ. H_l is the lth evaluation grade, and H_l is assumed to be superior to H_{l+1}. Suppose the input of evidence e_i is u_i, which is assessed to grade H_l with belief degree of $\beta_{l,i}(u_i)$, then $\beta_{l,i}(u_i)$ is calculated as follows [27]:

$$\beta_{l,i}(u_i)=\frac{u_i-U(H_{l+1})}{U(H_l)-U(H_{l+1})},\ U(H_{l+1})\le u_i\le U(H_l) \tag{15}$$

$$\beta_{l+1,i}(u_i)=1-\beta_{l,i}(u_i),\ U(H_{l+1})\le u_i\le U(H_l) \tag{16}$$

$$\beta_{k,i}(u_i)=0,\ k=1,2,\dots,N,\ k\ne l,l+1 \tag{17}$$

where $U(H_l)$ denotes a quantitative value that can be judged as a reference value for H_l. Thus, the BD for input parameter u_i can be described as

$$S(u_i)=\{(H_l,\beta_{l,i}),\ l=1,2,\dots,N\},\ i=1,2,\dots,m \tag{18}$$

where m is the total number of evidence.

To simplify the inference process, suppose $\Theta=\{A,B\}$ with A and B mutually exclusive and collectively exhaustive, and the reference values of A and B are U_{l+1} and U_l, respectively. The initial BD of e_j is expressed as follows:

$$e_j=\{(\varnothing,0),\ (A,p_j),\ (B,p_{j+1}),\ (\Theta,1-p_j-p_{j+1})\}. \tag{19}$$

Given an input u_i for e_i, with $U_{l+1}\le u_i\le U_l$, e_i and e_j can be expressed as Table I.

It should be noted that $\beta_{l,i}$ is in essence identical to $p_{\theta,i}$. $\beta_{l,i}$ denotes the belief degrees to which the ith piece of evidence supports the lth evaluation grade, while $p_{\theta,i}$ denotes the belief degrees to which the ith piece of evidence supports proposition θ. Whether $\beta_{l,i}$ or $p_{\theta,i}$ is used depends on the actual frame of discernment.

Suppose w_i and w_j are the weights of evidence e_i and e_j, respectively, and their reliabilities are represented by r_i and r_j, respectively. Based on (6)–(9), the combined degrees of belief $p_{\theta,e(2)}$ and $p_{\Theta,e(2)}$ are calculated as

$$\begin{cases}\widehat{m}_{A,e(2)}=w_i(1-r_j)\frac{u_i-U_{l+1}}{U_l-U_{l+1}}+(1-r_i)w_jp_j+w_iw_j(1-p_{j+1})\frac{u_i-U_{l+1}}{U_l-U_{l+1}}\\ \widehat{m}_{B,e(2)}=w_i(1-r_j)\frac{U_l-u_i}{U_l-U_{l+1}}+(1-r_i)w_jp_{j+1}+w_iw_j(1-p_j)\frac{U_l-u_i}{U_l-U_{l+1}}\\ \widehat{m}_{\Theta,e(2)}=w_j(1-r_i)(1-p_j-p_{j+1})\\ \widehat{m}_{\varnothing,e(2)}=0.\end{cases} \tag{20}$$

Suppose the utility evaluation is set as: $u(A)=1$, $u(B)=0.5$, and $u(\Theta)=0.25$, the expected utility generated using (11) is calculated as (25) shown at the top of the next page.

Suppose u_i is perturbed by a small amount $\sigma_i\Delta u_i$ with $u_i+\sigma_i\Delta u_i\in[U_{l+1},U_l]$, some proper assumptions should be made at first.

1) Δu_i obeys standard Gaussian distribution. This is to simplify the calculation, and to facilitate discussion and simulation.
2) $\sigma_i\Delta u_i$ is independent of r_i and r_j. The reason is that reliability is the inherent property of the information source where evidence is generated, while perturbation is a kind of extrinsic factor and does not affect the generation of evidence.
3) $\sigma_i\Delta u_i$ is independent of w_i and w_j. Even though the importance of a piece of evidence depends on the decision maker who uses the evidence, and perturbation may change his/her mind, it is assumed with the simplest case to simplify the calculation, which can avoid the impact of correlation between the weight and the perturbation.

The above assumptions lead to the following definition.

Definition 3 (Perturbation Utility): Suppose the input of evidence is perturbed, the expected utility u under the perturbation is identical to the perturbation utility u'.

Based on (21)–(24), as shown at the top of the next page, the fusion results are calculated as (26)–(29), shown at the top of the next page. The perturbation utility generated using (25) is calculated as (30), shown at the top of the next page.

It is obvious that there will be $u'=u$ when perturbation is invalid, which means $\sigma_i\Delta u_i=0$. As such, (30) will reduce to (25), which indicates that the essence of ER rule-PA is the ER rule.

In the following discussion, the perturbation coefficient will be used to characterize ER rule-PM. Based on (12), it can be calculated by

$$S(\Delta u_i)=\frac{u'-u}{\Delta u_i}=\frac{\sigma_i[2P(U_l-U_{l+1})-Q]}{4R[R+w_iw_j\sigma_i\Delta u_i(p_j-p_{j+1})]} \tag{31}$$

$$p_{A,e(2)} = \frac{w_i\left(1-r_j\right)(u_i-U_{l+1}) + w_j p_j(1-r_i)(U_l-U_{l+1}) + w_i w_j\left(1-p_{j+1}\right)(u_i-U_{l+1})}{\left[w_i\left(1-r_j\right)+w_j(1-r_i)+w_i w_j\right](U_l-U_{l+1}) + w_i w_j\left[u_i\left(p_j-p_{j+1}\right)+p_{j+1}U_{l+1}-p_j U_l\right]} \tag{21}$$

$$p_{B,e(2)} = \frac{w_i\left(1-r_j\right)(U_l-u_i) + w_j p_{j+1}(1-r_i)(U_l-U_{l+1}) + w_i w_j\left(1-p_j\right)(U_l-u_i)}{\left[w_i\left(1-r_j\right)+w_j(1-r_i)+w_i w_j\right](U_l-U_{l+1}) + w_i w_j\left[u_i\left(p_j-p_{j+1}\right)+p_{j+1}U_{l+1}-p_j U_l\right]} \tag{22}$$

$$p_{\Theta,e(2)} = \frac{w_j(1-r_i)\left(1-p_j-p_{j+1}\right)(U_l-U_{l+1})}{\left[w_i\left(1-r_j\right)+w_j(1-r_i)+w_i w_j\right](U_l-U_{l+1}) + w_i w_j\left[u_i\left(p_j-p_{j+1}\right)+p_{j+1}U_{l+1}-p_j U_l\right]} \tag{23}$$

$$p_{\varnothing,e(2)} = 0 \tag{24}$$

$$u = \frac{2w_i\left(1-r_j\right)(U_l+u_i-2U_{l+1})+2w_j(1-r_i)(U_l-U_{l+1})\left(2p_j+p_{j+1}\right)+2w_i w_j\left[U_l+u_i-2U_{l+1}+2p_{j+1}U_{l+1}+p_j u_i-2p_{j+1}u_i-p_j U_l\right]+w_j(1-r_i)\left[1-p_j-p_{j+1}\right]}{4\left[w_i\left(1-r_j\right)+w_j(1-r_i)+w_i w_j\right](U_l-U_{l+1})+4w_i w_j u_i\left(p_j-p_{j+1}\right)+4w_i w_j\left(p_{j+1}U_{l+1}-p_j U_l\right)} \tag{25}$$

$$p'_{A,e(2)} = \frac{w_i\left(1-r_j\right)(u_i+\sigma_i\Delta u_i-U_{l+1}) + w_j p_j(1-r_i)(U_l-U_{l+1}) + w_i w_j\left(1-p_{j+1}\right)(u_i+\sigma_i\Delta u_i-U_{l+1})}{\left[w_i\left(1-r_j\right)+w_j(1-r_i)+w_i w_j\right](U_l-U_{l+1}) + w_i w_j\left[(u_i+\sigma_i\Delta u)\left(p_j-p_{j+1}\right)+p_{j+1}U_{l+1}-p_j U_l\right]} \tag{26}$$

$$p'_{B,e(2)} = \frac{w_i\left(1-r_j\right)(U_l-u_i-\sigma_i\Delta u_i) + w_j p_{j+1}(1-r_i)(U_l-U_{l+1}) + w_i w_j\left(1-p_j\right)(U_l-u_i-\sigma_i\Delta u_i)}{\left[w_i\left(1-r_j\right)+w_j(1-r_i)+w_i w_j\right](U_l-U_{l+1}) + w_i w_j\left[(u_i+\sigma_i\Delta u)\left(p_j-p_{j+1}\right)+p_{j+1}U_{l+1}-p_j U_l\right]} \tag{27}$$

$$p'_{\Theta,e(2)} = \frac{w_j(1-r_i)\left(1-p_j-p_{j+1}\right)(U_l-U_{l+1})}{\left[w_i\left(1-r_j\right)+w_j(1-r_i)+w_i w_j\right](U_l-U_{l+1}) + w_i w_j\left[(u_i+\sigma_i\Delta u)\left(p_j-p_{j+1}\right)+p_{j+1}U_{l+1}-p_j U_l\right]} \tag{28}$$

$$p'_{\varnothing,e(2)} = 0 \tag{29}$$

$$u' = \frac{2w_i\left(1-r_j\right)(U_l+u_i+\sigma_i\Delta u_i-2U_{l+1})+2w_j(1-r_i)(U_l-U_{l+1})\left(2p_j+p_{j+1}\right)+2w_i w_j\left[U_l+u_i+\sigma_i\Delta u_i-2U_{l+1}+2p_{j+1}(U_{l+1}-u_i-\sigma_i\Delta u_i)+p_j(u_i+\sigma_i\Delta u_i-U_l)\right]+w_j(1-r_i)\left(1-p_j-p_{j+1}\right)}{4\left[w_i\left(1-r_j\right)+w_j(1-r_i)+w_i w_j\right](U_l-U_{l+1})+4w_i w_j(u_i+\sigma_i\Delta u_i)\left(p_j-p_{j+1}\right)+4w_i w_j\left(p_{j+1}U_{l+1}-p_j U_l\right)} \tag{30}$$

---

where P, Q, and R are three constants expressed as follows:

$$\begin{cases} P = w_i\left(1-r_j\right)\left[w_i\left(1-r_j\right)+w_j(1-r_i)\right]+w_i^2 w_j\left(1-r_j\right)\left(2-p_j-p_{j+1}\right) \\ \quad + w_i w_j^2(1-r_i)\left(1+p_j-2p_{j+1}-2p_j^2+p_{j+1}^2+p_j p_{j+1}\right) \\ \quad + w_i^2 w_j^2\left(1-p_j-p_{j+1}+p_j p_{j+1}\right) \\ Q = w_i w_j^2(1-r_i)\left(p_j-p_{j+1}\right)\left(1-p_j-p_{j+1}\right) \\ R = \left[w_i\left(1-r_j\right)+w_j(1-r_i)+w_i w_j\right](U_l-U_{l+1}) \\ \quad + w_i w_j\left[u_i\left(p_j-p_{j+1}\right)+p_{j+1}U_{l+1}-p_j U_l\right]. \end{cases} \tag{32}$$

B. Properties of Perturbation Coefficient

In this section, basic properties of perturbation coefficient will be further studied to analyze the impact of perturbation on the ER rule, and to facilitate the appropriate use of ER rule-PA. The perturbation coefficient $S(\Delta u_i)$ has the following properties.

Theorem 3 (Continuity and Derivability): $S(\Delta u_i)$ is continuous and derivable on its domains.

Proof: See Appendix A. ■

Theorem 4 (Monotonicity): $S(\Delta u_i)$ decreases monotonically when $p_j \geq p_{j+1}$, but increases monotonically in turn.

Proof: See Appendix B. ■

Theorem 5 (Boundedness): There are upper and lower bounds of $S(\Delta u_i)$ as follows:

$$\begin{cases} \min S(\Delta u_i) = \frac{\varepsilon_1\sigma_i\left[2P\left(U_l-U_{l+1}\right)-Q\right]}{4R\left\{\left(U_l-U_{l+1}\right)\left[w_i\left(1-r_j\right)+w_j(1-r_i)+w_i w_j\left(1-\min\left\{p_j,p_{j+1}\right\}\right)\right]\right\}} \\ \max S(\Delta u_i) = \frac{\varepsilon_2\sigma_i\left[2P\left(U_i-U_{i+1}\right)-Q\right]}{4R\left\{\left(U_l-U_{l+1}\right)\left[w_i\left(1-r_j\right)+w_j(1-r_i)+w_i w_j\left(1-\max\left\{p_j,p_{j+1}\right\}\right)\right]\right\}}. \end{cases} \tag{33}$$

Proof: See Appendix C. ■

A special case of (33) is when both two pieces of evidence e_i and e_j are fully reliable and there is no local ignorance, which means $r_i = r_j = 1$ and $p_j + p_{j+1} = 1$. Therefore, (33) will reduce to

$$\begin{cases} \min S(\Delta u_i) = \frac{\varepsilon_1\sigma_i P_0\left(U_l-U_{l+1}\right)}{2w_i w_j R_0\left(U_l-U_{l+1}\right)\left(1-\min\left\{p_j,p_{j+1}\right\}\right)} \\ \max S(\Delta u_i) = \frac{\varepsilon_2\sigma_i P_0\left(U_l-U_{l+1}\right)}{2w_i w_j R_0\left(U_l-U_{l+1}\right)\left(1-\max\left\{p_j,p_{j+1}\right\}\right)} \end{cases} \tag{34}$$

where P_0 and R_0 can be expressed as follows:

$$\begin{cases} P_0 = w_i^2 w_j^2 p_j p_{j+1} \\ R_0 = w_i w_j\left[U_l-U_{l+1}+u_i\left(p_j-p_{j+1}\right)+p_{j+1}U_{l+1}-p_j U_l\right]. \end{cases} \tag{35}$$

To limit the perturbation coefficient within a certain range, and to ensure that perturbation has a smaller impact on the fusion results, the following definition is introduced.

Remark 3: Here, the term "smaller" means that the change of perturbation cannot make the fusion results change greatly. Since the perturbation coefficient $S(\Delta u_i)$ can characterize ER rule-PM, if $S(\Delta u_i)$ keeps stable at a certain value, it can be understood that the influence of perturbation on the fusion results is smaller.

Definition 4 (MPE of Perturbation Coefficient): Suppose that the absolute value of $S(\Delta u_i)$ is no more than ε represents that the perturbation is acceptable, where ε denotes the MPE of perturbation coefficient. We have

$$|S(\Delta u_i)| \leq \varepsilon. \tag{36}$$

TABLE II
TWO PIECES OF INDEPENDENT EVIDENCE

$p_{\theta,i}$	$\varnothing$	A	B	Θ
e_1	0	0.4	0.6	0
e_2	0	0.3	0.7	0

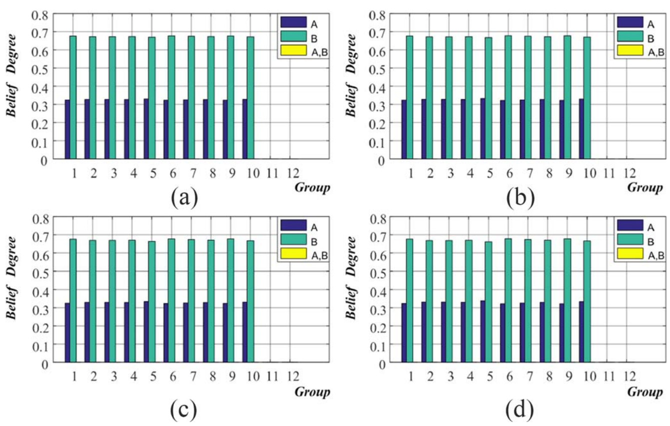


Fig. 3. Final belief degree under perturbation.

In general, ε is a sufficiently small positive number, which is determined by real situations. Definition 4 indicates that in an engineering system, perturbation can be reflected indirectly through MPE, which will better characterize the robustness of the system.

C. Numerical Study

In this section, to demonstrate the implementation process of ER rule-PA, a numerical study is conducted. Suppose $\Theta = \{A, B\}$ with A and B mutually exclusive and collectively exhaustive, and their reference values are 5 and 10, respectively. There are two pieces of independent evidence e_1 and e_2 represented by two BDs shown in Table II, where the input of e_1 is 8. Suppose their weights are given by $w_1 = 0.9$ and $w_2 = 0.6$, respectively, and their reliabilities are given by $r_1 = 0.8$ and $r_2 = 0.5$, respectively.

The BD generated by the ER rule is presented as follows:

$$e = \{(\varnothing, 0), (A, 0.3259), (B, 0.6741), (\Theta, 0)\}. \quad (37)$$

Suppose $u(A) = 1$, $u(B) = 0.5$, and $u(\Theta) = 0.25$, the expected utility is

$$u = \sum_{\theta \subseteq \Theta} p_{\theta,e(2)} u(\theta) = 0.6630. \quad (38)$$

Simulation is conducted by analyzing the changes of perturbation coefficient under different perturbation. A set of random perturbation intensities is assumed as $[\sigma_1, \sigma_2, \sigma_3, \sigma_4] = [0.010, 0.015, 0.020, 0.025]$, and e_1 is the perturbed evidence. The fusion results are shown in Fig. 3.

In Fig. 3, for all $\sigma_i (i = 1, 2, 3, 4)$, ten groups of $\Delta u_j (j = 1, 2, \ldots, 10)$ are generated at random shown in Table III.

The perturbation utility under different perturbation intensities and the expected utility are shown in Fig. 4.

TABLE III
TEN GROUPS OF DIFFERENT PERTURBATION

Δu_j	VALUE	Δu_j	VALUE
Δu_1	0.8884	Δu_6	1.4384
Δu_2	-1.1471	Δu_7	0.3252
Δu_3	-1.0689	Δu_8	-0.7549
Δu_4	-0.8095	Δu_9	1.3703
Δu_5	-2.9443	Δu_{10}	-1.7115

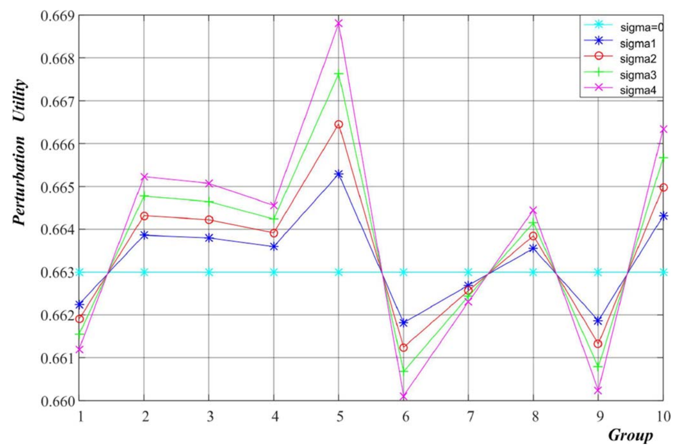


Fig. 4. Comparison of the perturbation utility and the expected utility.

According to Fig. 4, two conclusions can be drawn as follows.

1) Generally, u' fluctuates up and down with the change of Δu_j and σ_i. When Δu_5 changes to Δu_6, the value of u' drops sharply, which is the result of rapid change of perturbation.
2) Specifically, when σ_i is fixed, u' decreases with the increasing of Δu_j, but the inverse increases, which is in accord with (30). Meanwhile, the amplitude of u' is positively correlated with Δu_j. However, no matter how σ_i and Δu_j changes, u' always fluctuates within a certain small range, which means that the small change of perturbation parameters will not cause a great change of the output of the ER rule. This indicates that the robustness and stability of the ER rule are quite strong.

The perturbation coefficient $S(\Delta u_j)(j = 1, 2, \ldots, 10)$ under different perturbation intensities $\sigma_i (i = 1, 2, 3, 4)$ is shown in Table IV.

To characterize the change of perturbation coefficient under different perturbation intensities, $\left|S(\Delta u_j)\right|$ is shown in Fig. 5.

According to Fig. 5, $|S(\Delta u_j)|$ is positively correlated with σ_i. It is obvious that the change of $|S(\Delta u_j)|$ is relatively stable, and the overall fluctuation is small. Moreover, for each perturbation intensity, $|S(\Delta u_j)|$ is maximal under the perturbation Δu_7. This can be explained based on Fig. 4, where all groups of perturbation utility are the closest to the expected utility when $\Delta u_j = 0.3252$.

Based on the above analysis, the complex systems which require high reliability and stability might be severely affected given that $\varepsilon = 0.2\%$. At this moment, certain measures should

TABLE IV
VALUE OF PERTURBATION COEFFICIENTS UNDER DIFFERENT PERTURBATION INTENSITIES

σ_i	$S(\Delta u_j)(\times 10^{-3})$									
σ_1	-0.8450	-0.7512	-0.7481	-0.7338	-0.7771	-0.8244	-0.9375	-0.7296	-0.8260	-0.7650
σ_2	-1.2405	-1.1478	-1.1447	-1.1302	-1.1746	-1.2197	-1.3333	-1.1260	-1.2213	-1.1619
σ_3	-1.6359	-1.5446	-1.5414	-1.5268	-1.5726	-1.6147	-1.7291	-1.5225	-1.6164	-1.5591
σ_4	-2.0311	-1.9416	-1.9384	-1.9235	-1.9713	-2.0094	-2.1248	-1.9192	-2.0112	-1.9566

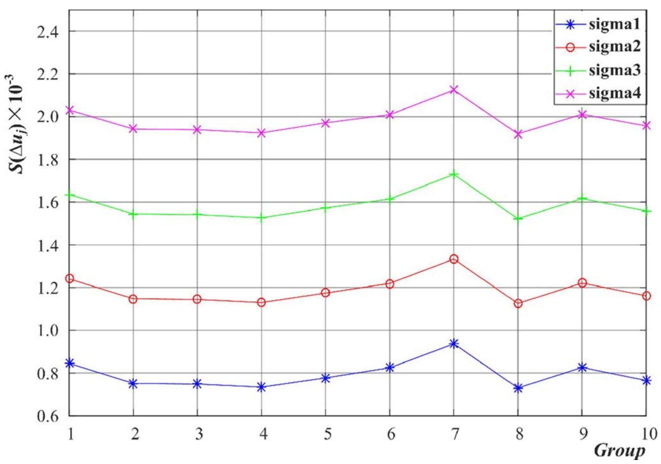


Fig. 5. Comparison of perturbation coefficients under different perturbation intensities.

be taken to eliminate the impact of perturbation to ensure the stable operation of the system.

D. Generalized Method of ER Rule-PA

In the above discussion, the combination of two pieces of independent evidence is conducted from the theoretical aspect. To facilitate the applications of the ER rule, a generalized method of ER rule-PA is explored in this section.

Suppose there are N pieces of independent evidence profiled as follows:

$$e_i = \left\{ (\theta, p_{\theta,i})\ \forall \theta \subseteq \Theta, \sum_{\theta \subseteq \Theta} p_{\theta,i} = 1, i = 1, 2, \dots, N \right\} \quad (39)$$

where $(\theta, p_{\theta,i})$ is an element of evidence e_i, which represents that the evidence points to proposition θ to the degree of belief $p_{\theta,i}$.

To differentiate the perturbed evidence from the normal evidence, suppose there are $M(M \leq N)$ pieces of independent evidence perturbed. Since it has been proven that the fusion results by the ER rule could not be influenced by the order of evidence combination [3], N pieces of evidence are reordered as follows:

$$e = \{e_1, \dots, e_i, \dots, e_M, e_{M+1}, \dots, e_j \dots, e_N\}. \quad (40)$$

The weights and reliabilities of all the evidence are denoted by $w = \{w_1, \dots, w_k, \dots, w_N\}$ and $r = \{r_1, \dots, r_k, \dots, r_N\}$, respectively. The evaluation grades and their reference values are set as Table V.

TABLE V
PARAMETER SETTING OF EVALUATION RESULTS

Evaluation grades	H_1	H_2	$\cdots$	H_l	$\cdots$	H_L
Reference values	U_1	U_2	$\cdots$	U_l	$\cdots$	0

where H_l is superior to H_{l+1} and $\Theta = \{H_1, H_2, \cdots, H_L\}$.

Suppose the first M pieces of evidence are perturbed by a small amount $\sigma_j \Delta u_j$, which is represented as follows:

$$e_j = \left\{ \left(H_l, \frac{u_j + \sigma_j \Delta u_j - U_{l+1}}{U_l - U_{l+1}}\right), \left(H_{l+1}, \frac{U_l - u_j - \sigma_j \Delta u_j}{U_l - U_{l+1}}\right), (H_k, 0) \right\} \quad (41)$$

where $j = 1, 2, \dots, M$, $1 \leq l, k, l+1 \leq L$, $k \neq l, l+1$ and u_j is the initial input of evidence e_j.

Based on the ER rule, the last $(N - M)$ pieces of evidence are integrated and the fusion results are as follows:

$$e(N - M) = \left\{ (H_1, p_1), (H_2, p_2), \dots, (H_L, p_L), \sum_{l=1}^{L} p_l = 1 \right\}. \quad (42)$$

Suppose e_1 is profiled as (43) and that the weight and reliability of $e(N - M)$ are w_S and r_S, respectively

$$e_1 = \left\{ \left(H_1, \frac{u_1 + \sigma_1 \Delta u_1 - U_2}{U_1 - U_2}\right), \left(H_2, \frac{U_1 - u_1 - \sigma_1 \Delta u_1}{U_1 - U_2}\right), (H_l, 0) \right\} \quad (43)$$

where $i = 3, \dots, L$.

Based on (6)–(9), e_1 and $e(N - M)$ can be combined as follows:

$$\begin{cases} \widehat{m}'_{H_1,e(2)} = w_1(1 - r_S)\frac{u_1+\sigma_1\Delta u_1-U_2}{U_1-U_2} + (1 - r_1)w_S p_1 \\ \qquad + w_1 w_S p_1 \frac{u_1+\sigma_1\Delta u_1-U_2}{U_1-U_2} \\ \widehat{m}'_{H_2,e(2)} = w_1(1 - r_S)\frac{U_1-u_1-\sigma_1\Delta u_1}{U_1-U_2} + (1 - r_1)w_S p_2 \\ \qquad + w_1 w_S p_2 \frac{U_1-u_1-\sigma_1\Delta u_1}{U_1-U_2} \\ \dots \\ \widehat{m}'_{H_l,e(2)} = (1 - r_1)w_S p_l\ \forall l = 3, 4, \dots, L \end{cases} \quad (44)$$

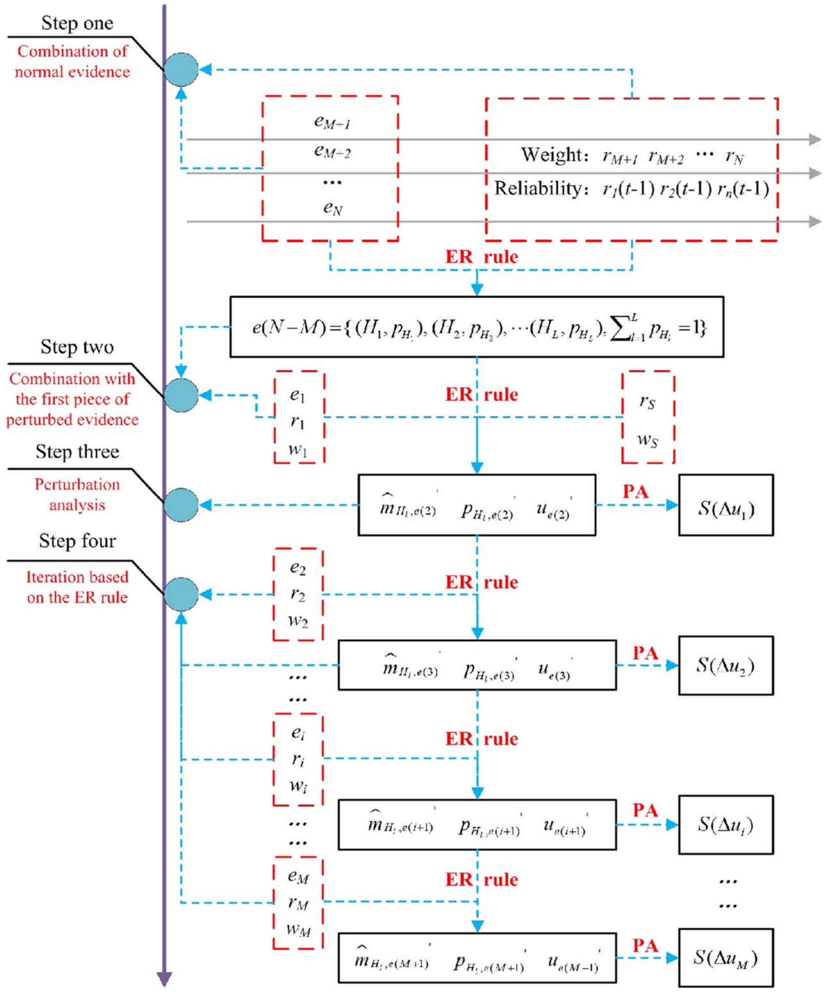


Fig. 6. Overall process of ER rule-PA.

$$\begin{cases} p'_{H_1,e(2)} = \widehat{m}_{H_1,e(2)}/\sum_{l=1}^{L}\widehat{m}_{H_l,e(2)} \\ p'_{H_2,e(2)} = \widehat{m}_{H_2,e(2)}/\sum_{l=1}^{L}\widehat{m}_{H_l,e(2)} \\ \quad\cdots \\ \quad\cdots \end{cases} \tag{45}$$

Suppose $u(H_l) = F_u(l, L), l = 1, 2, \ldots, L$, based on (11), u_2 and $u'_{e(2)}$ are calculated as follows:

$$\begin{aligned} u_2 &= \frac{\sum_{l=1}^{L}\widehat{m}_{H_l,e(2)}F_u(l,L)}{\sum_{l=1}^{L}\widehat{m}_{H_l,e(2)}} \\ &= \frac{\left[(1-r_S)w_1\frac{u_1-U_2}{(U_1-U_2)}+(1-r_1)w_Sp_1+w_1w_Sp_1\frac{(u_1-U_2)}{(U_1-U_2)}\right]}{(1-r_S)w_1+(1-r_1)w_S+w_1w_S\frac{p_1(u_1-U_2)+p_2(U_1-u_1)}{(U_1-U_2)}}F_u(1,L) \\ &\quad+\frac{\left[(1-r_S)w_1\frac{U_1-u_1}{(U_1-U_2)}+(1-r_1)w_Sp_2+w_1w_Sp_2\frac{(U_1-u_1)}{(U_1-U_2)}\right]}{(1-r_S)w_1+(1-r_1)w_S+w_1w_S\frac{p_1(u_1-U_2)+p_2(U_1-u_1)}{(U_1-U_2)}}F_u(2,L) \\ &\quad+\frac{(1-r_S)w_Sp_3F_u(3,L)+\cdots+(1-r_S)w_Sp_LF_u(L,L)}{(1-r_S)w_1+(1-r_1)w_S+w_1w_S\frac{p_1(u_1-U_2)+p_2(U_1-u_1)}{(U_1-U_2)}} \end{aligned} \tag{46}$$

$$\begin{aligned} u'_{e(2)} &= \frac{\sum_{l=1}^{L}\widehat{m}'_{H_l,e(2)}F_u(l,L)}{\sum_{l=1}^{L}\widehat{m}'_{H_l,e(2)}} \\ &= \frac{\left[(1-r_S)w_1\frac{u_1+\sigma_1\Delta u_1-U_2}{(U_1-U_2)}+(1-r_1)w_Sp_1+w_1w_Sp_1\frac{(u_1+\sigma_1\Delta u_1-U_2)}{(U_1-U_2)}\right]}{(1-r_S)w_1+(1-r_1)w_S+w_1w_S\frac{(p_1-p_2)(u_1+\sigma_1\Delta u_1)+p_2U_1-p_1U_2}{(U_1-U_2)}}F_u(1,L) \\ &\quad+\frac{\left[(1-r_S)w_1\frac{U_1-u_1-\sigma_1\Delta u_1}{(U_1-U_2)}+(1-r_1)w_Sp_2+w_1w_Sp_2\frac{(U_1-u_1-\sigma_1\Delta u_1)}{(U_1-U_2)}\right]}{(1-r_S)w_1+(1-r_1)w_S+w_1w_S\frac{(p_1-p_2)(u_1+\sigma_1\Delta u_1)+p_2U_1-p_1U_2}{(U_1-U_2)}}F_u(2,L) \\ &\quad+\frac{(1-r_S)w_Sp_3F_u(3,L)+\cdots+(1-r_S)w_Sp_LF_u(L,L)}{(1-r_S)w_1+(1-r_1)w_S+w_1w_S\frac{(p_1-p_2)(u_1+\sigma_1\Delta u_1)+p_2U_1-p_1U_2}{(U_1-U_2)}} \end{aligned} \tag{47}$$

where $F_u(\bullet)$ is a nonlinear utility function and there is $F_u(1, L) > \cdots > F_u(l, L) > \cdots > F_u(L, L), l \in [2, L-1]$.

Based on (12), the perturbation coefficient $S(\Delta u_1)$ is calculated as follows:

$$S(\Delta u_1) = \frac{u'_{e(2)} - u_2}{\Delta u_1}. \tag{48}$$

It is obvious that there will be $u'_{e(2)} = u_2$ when $\sigma_1 = 0$, which means $S(\Delta u_1) = 0$ and the perturbation is invalid. Thus, ER rule-PA reduces to the ER rule.

According to the recursive algorithm of the ER rule, the remaining $(M-1)$ pieces of evidence are combined in turn, which can be expressed as in Fig. 6. The generalized method of ER rule-PA is concluded as follows.

Step 1: Combine $(N-M)$ pieces of normal evidence based on the ER rule as (1)–(11), with every two pieces of evidence integrated as (20)–(25).

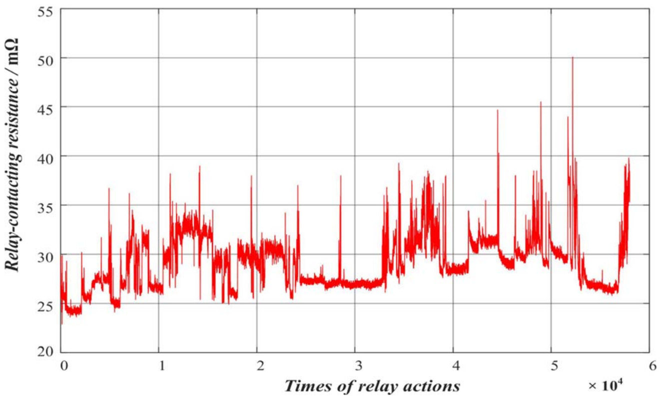


Fig. 7. Testing data curve of relay-contacting resistance.

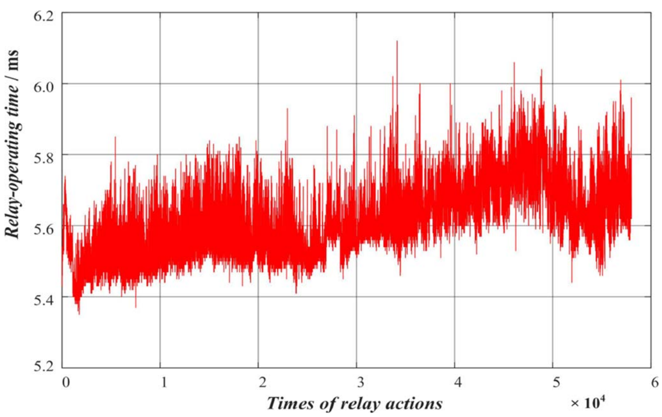


Fig. 8. Testing data curve of relay-operating time.

Step 2: Combine the previously combined assessment result $e(N-M)$ with the first piece of perturbed evidence based on the ER rule as (26)–(30), then $e(N-M+1)$ will be obtained.

Step 3: Conduct PA for the results in step 2 and calculate the perturbation coefficient as (31).

Step 4: Combine $e(N-M+1)$ in step 2 with the remaining $(M-1)$ pieces of evidence by recursively applying the ER rule as step 1. Then, conduct PA as step 3 successively.

Note that the ER rule inherits the basic properties of being associative and commutative, and the fusion results are independent of the order of evidence combination. Hence, no matter how the evidence is combined, the final results will not change. Here, the generalized method of ER rule-PA is developed, in which all the normal evidence is first combined. Then, the first piece of perturbed evidence is combined with the above fusion results $e(N-M+1)$, and PA can be conducted and the perturbation coefficient can be obtained in turn. By recursively combining the rest evidence, PA can be accomplished step by step.

IV. Case Study

To demonstrate the feasibility and effectiveness of ER rule-PA, in this section, a reliability evaluation problem on aerospace relay is examined by introducing perturbation in relay test.

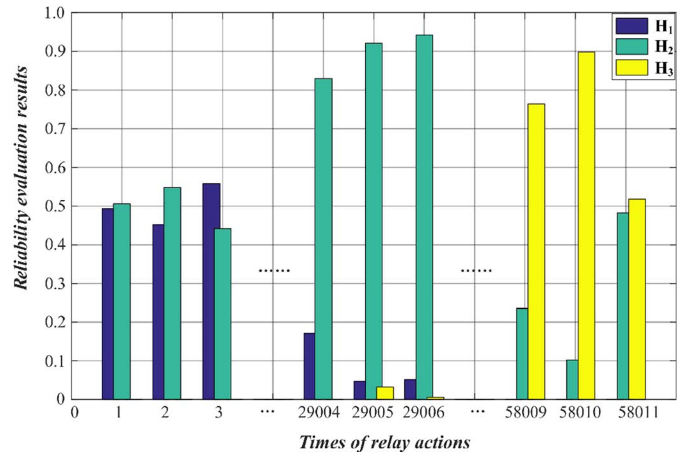


Fig. 9. Reliability evaluation results.

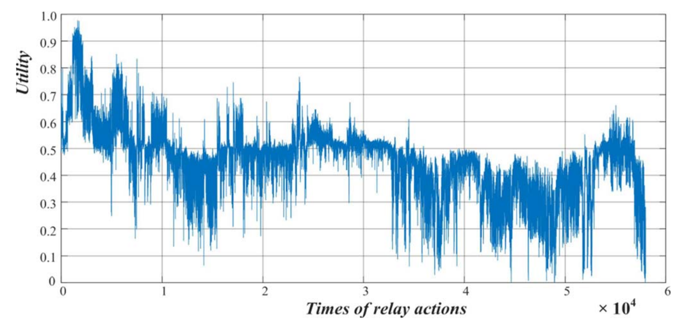


Fig. 10. Expected utility curves without perturbation.

A. Relay Test

Taking the electromagnetic relay tester produced by Beijing Huafeng Company as the experimental platform and JRC-7M aerospace relay as the experimental object, a relay testing platform is built. Since relay-contacting resistance and relay-operating time are two important factors for measuring the reliability of aerospace relay, they have been studied in the experiment. There are 58 011 sets of testing data collected for each factor shown in Figs. 7 and 8.

According to Figs. 7 and 8, the following three conclusions can be drawn.

1) Both values of relay-contacting resistance and relay-operating time witness an obvious tendency of fluctuation, but there is no sign of convergence, which may be affected by some noises.
2) The testing data is basically unchanged between the two adjacent actions. Even though there are obvious fluctuations, they are both within reasonable limits, which indicates that the aerospace relay has certain stability. Here, the term "reasonable limits" means that the testing values of the two factors are normal and do not exceed the fault threshold. Besides, the testing results are consistent with the cognitive results.
3) By calculating the relative change rate of the testing value of the two factors, it is found that the relative change of relay-contacting resistance is obviously larger than that of relay-operating time as the test proceeds. It can be preliminarily speculated that

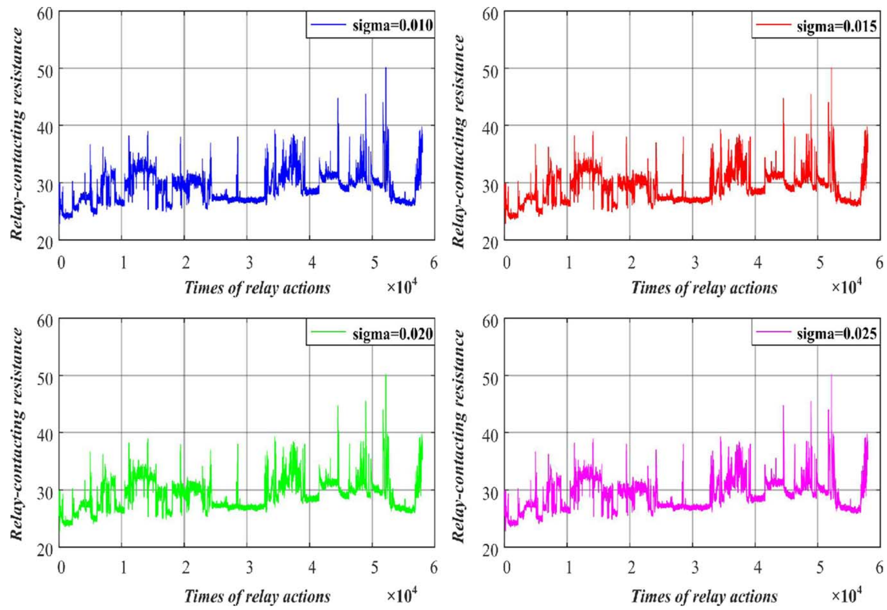


Fig. 11. Testing data curves of relay-contacting resistance under perturbation.

TABLE VI
REFERENCE VALUES FOR EVALUATION GRADES

Factors	H_1	H_2	H_3
Relay-contacting resistance	22.9	28.49	38.2
Relay-operating time	5.35	5.54	5.85

relay-contacting resistance imposes a greater impact on relay performance than that by relay-operating time.

B. Reliability Evaluation for Aerospace Relay Based on the ER Rule

1) Belief Distribution Transformation: According to the characteristics and technical indicators of aerospace relay, three evaluation grades of relay's reliability are defined: "high (H_1)," "medium (H_2)," and "low (H_3)." Therefore, the frame of discernment Θ can be described as $\Theta = \{H_1, H_2, H_3\}$. Then, three reference values which are used as quantitative reference points for the evaluation framework are set, as shown in Table VI.

Based on (15)–(17), all the testing data can be transformed into unified BD form, as is shown in Tables VII and VIII.

2) Reliability Evaluation: According to conclusion 3 in Section IV-A, to characterize the importance differences between the two factors, their weights are set as $w_1 = 0.9$ and $w_2 = 0.6$, respectively.

Considering the high-reliability requirement of aerospace relay and the fine manufacturing process of relay tester, the reliabilities of the two factors should be relatively high, which are set as $r_1 = 0.8$ and $r_2 = 0.85$, respectively. Based on the ER rule, the reliability evaluation results of aerospace relay are shown in Fig. 9.

TABLE VII
INITIAL BD OF RELAY-CONTACTING RESISTANCE

Times of relay actions	Relay-contacting resistance (mΩ)	p_{H_1}	p_{H_2}	p_{H_3}
100	24.1	0.7853	0.2147	0
200	27.1	0.2487	0.7513	0
…	…	…	…	…
29004	27.1	0.2487	0.7513	0
29005	27.3	0.2129	0.7871	0
…	…	…	…	…
58010	38.0	0	0.0206	0.9794
58011	35.8	0	0.2472	0.7528

TABLE VIII
INITIAL BD OF RELAY-OPERATING TIME

Times of relay actions	Relay-operating time (ms)	p_{H_1}	p_{H_2}	p_{H_3}
100	5.55	0	0.9677	0.0323
200	5.53	0.0526	0.9474	0
…	…	…	…	…
29004	5.49	0.2632	0.7368	0
29005	5.59	0	0.8387	0.1613
…	…	…	…	…
58010	5.72	0	0.4194	0.5806
58011	5.62	0	0.7419	0.2581

From Fig. 9, the belief degree of H_3 increases with times of relay actions while that of H_1 decreases, which indicates that the reliability of aerospace relay declines gradually.

Suppose the utility of each evaluation grade is $u(H_1) = 1$, $u(H_2) = 0.5$, and $u(H_3) = 0$, respectively, based on (11), the expected utility curve is presented in Fig. 10.

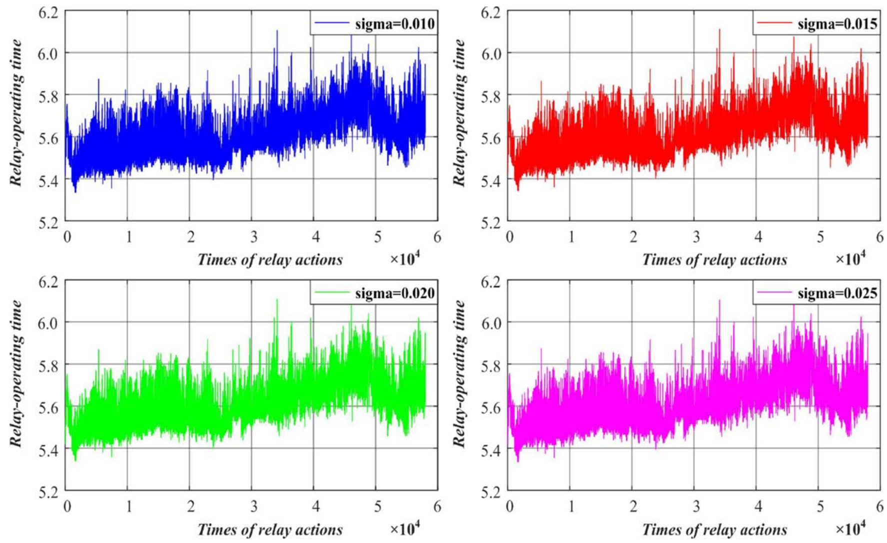


Fig. 12. Testing data curves of relay-operating time under perturbation.

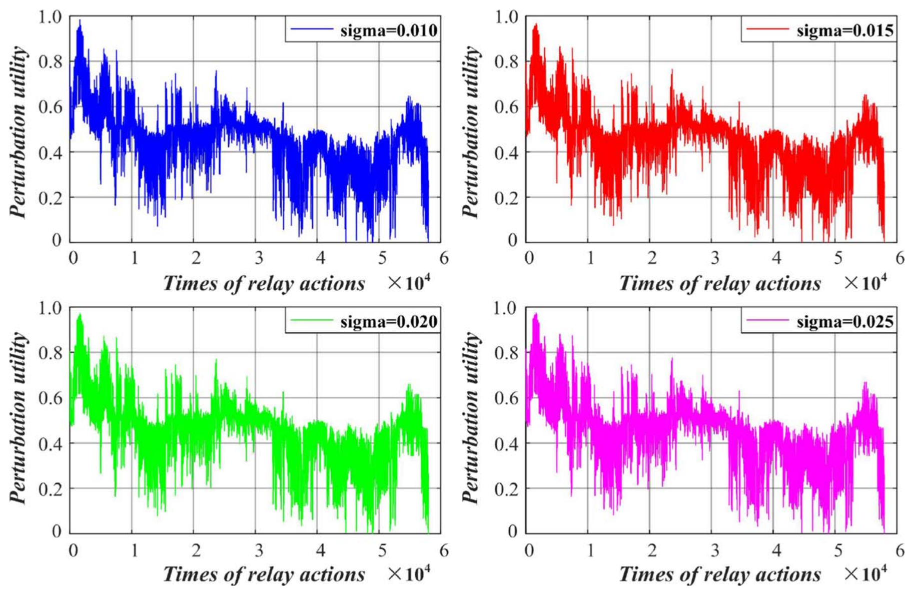


Fig. 13. Perturbation utility curves under different perturbation intensities.

C. Reliability Evaluation for Aerospace Relay Based on ER Rule-PA

1) Generation of Perturbation: To verify the feasibility and effectiveness of ER rule-PA, the sinusoidal perturbation signal is introduced into the experiment, which has the following four characteristics.

1) The amplitude is 1 and the frequency is 100 Hz.
2) The number of sampling points is 58 011.
3) The sampling frequency is 1000 Hz.
4) The perturbation intensity is $[\sigma_1, \sigma_2, \sigma_3, \sigma_4] = [0.010, 0.015, 0.020, 0.025]$.

Through the sampling experiment, 58 011 sets of testing data for relay-contacting resistance and relay-operating time are obtained, as shown in Figs. 11 and 12.

From Figs. 11 and 12, the two factors change slightly after the perturbation signal is introduced. Since there is no significant difference between the above two diagrams, it is necessary to combine evidence. Similarly, the weights of the two factors are given by $w_1 = 0.9$ and $w_2 = 0.6$, with their reliabilities $r_1 = 0.8$ and $r_2 = 0.85$, respectively.

The perturbation utility curves are presented in Fig. 13.

From Figs. 10 and 13, based on (12), the perturbation coefficient $|S(\Delta u_j)|$ is shown in Fig. 14.

According to Fig. 14, the following two conclusions can be drawn.

1) $|S(\Delta u_j)|$ keeps stable at the first stage of relay actions whatever the value of the perturbation intensity σ_i is, which indicates that the robustness of the aerospace relay is strong to some extent.
2) Given that σ_i remains constant, $|S(\Delta u_j)|$ begins to change dramatically as the times of relay actions reach about 46 556, which is consistent with actual situation

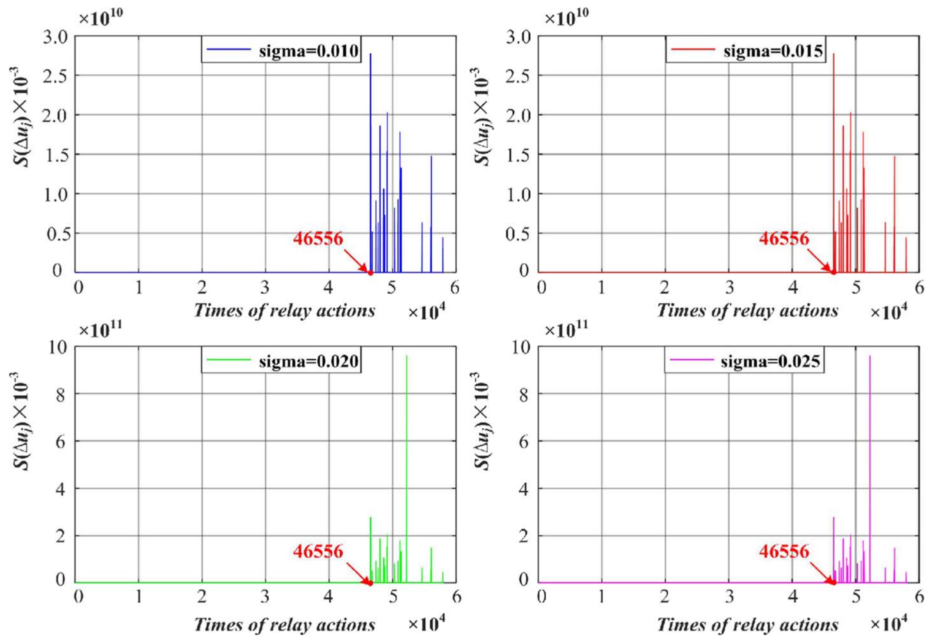


Fig. 14. Comparison of perturbation coefficients under different perturbation intensities.

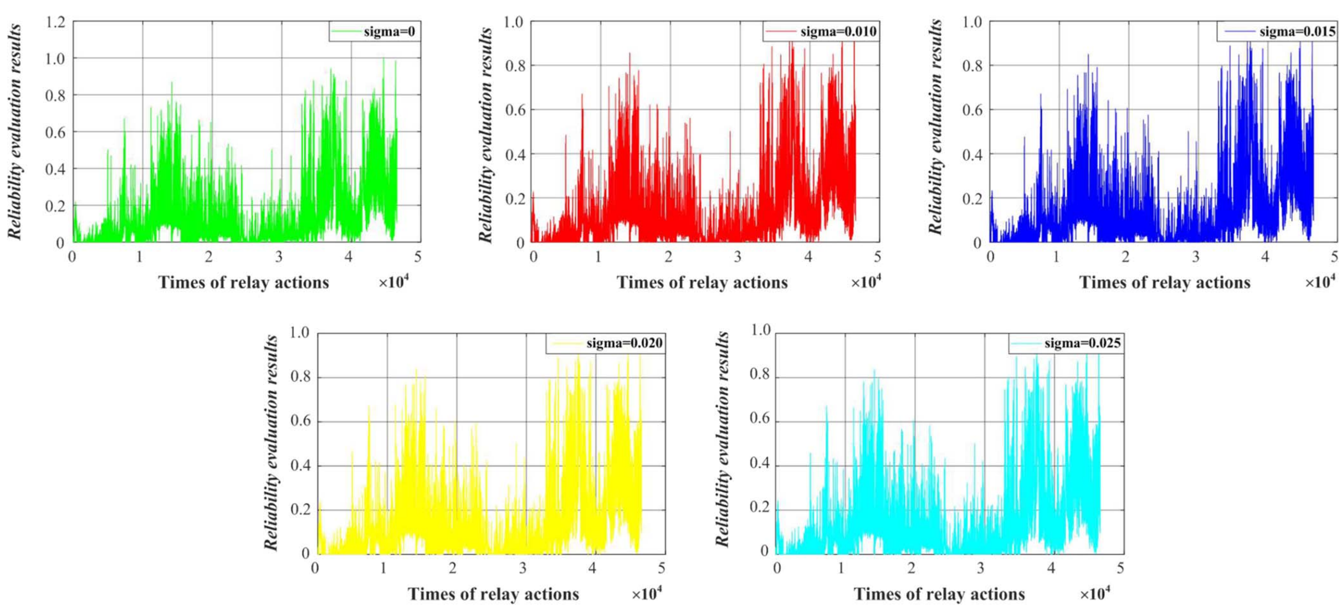


Fig. 15. Comparison among the reliability evaluation results of low grade for relay.

where the performance of aerospace relay decreases as the experiment proceeds. This will lead to its reliability degradation.

Remark 4: Since the requirement of aerospace relay's reliability is very high, another new relay needs to be used to take the place of the old one when the perturbation coefficient varies greatly. Some inspirations can also be obtained in engineering applications, medical areas, security evaluation areas, etc. For instance, if someone is sick, it can be assumed that he/she has been perturbed which results in some small changes in physical function. At this moment, PA can be used to determine whether the patient's condition is serious or not and to find the illness source.

According to the reliability requirement of aerospace relays, the MPE of $S(\Delta u_j)$ is given by $\varepsilon = 0.1\%$. Based on (36), the relay must be replaced when it has acted more than 46 556 times. From the real observation situation, most contacts of relay have been seriously ablated and worn at that time, which will lead to its low reliability and stability.

Remark 5: In this article, we provide an idea to judge the acceptability of perturbation, that is to say, the perturbation has little effect on the system. The threshold ε is preliminarily set for comparison, while how to quickly ascertain the value of ε remains to be explored. Maybe ε can be obtained from expert knowledge or based on statistical method. Once ε is

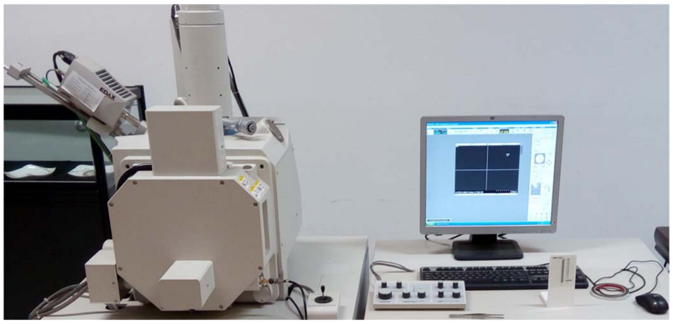
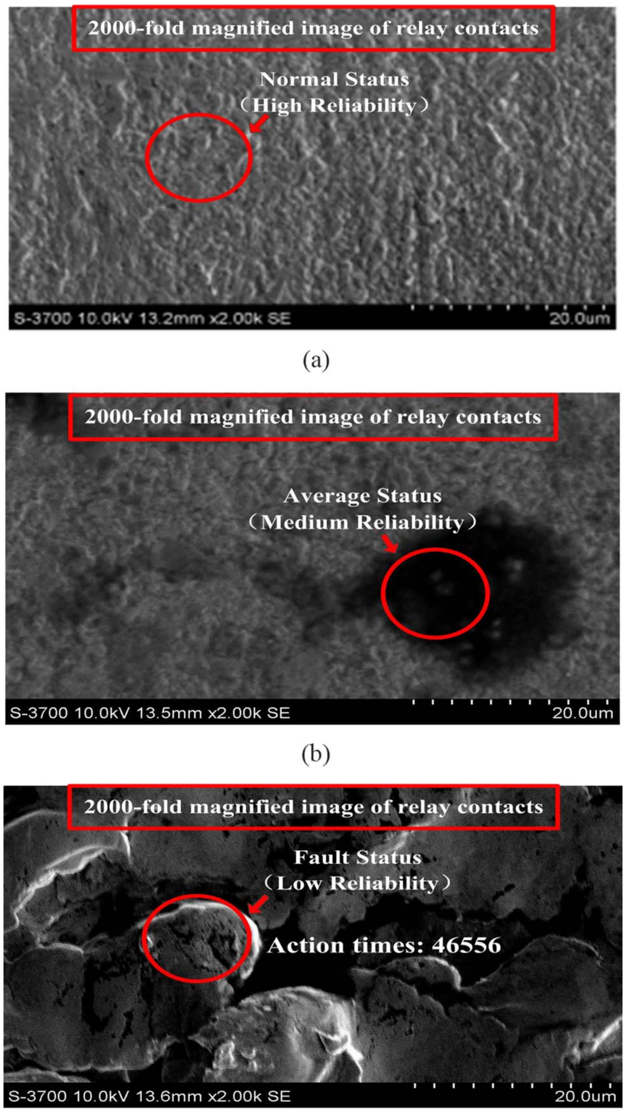


Fig. 16. Diagram of the scanning electron microscope.



(a)

(b)

(c)

Fig. 17. Diagram of relay contacts using the scanning electron microscope. Relay's status with respect to (a) high grade, (b) medium grade, and (c) low grade.

determined, the thresholds ε_1 and ε_2 can be obtained through parameter optimization, which can help to control the intensity of perturbation in engineering practice. However, this article will be done in the future.

To validate the robustness of the ER rule, a comparison among all the reliability evaluation results of low grade for relay with perturbation considered or not is depicted in Fig. 15.

It can be seen from Fig. 15 that the belief degrees to which relay is evaluated to be low are basically consistent given perturbation or not. This indicates that the ER rule is so powerful that the perturbation imposed can hardly affect the fusion results. The main reason why the low grade is concerned is that the reliability of relay decreases gradually, which is bound to decline to failure after a certain number of action times. If the perturbation intensity is big enough, the belief degrees of low grade may rise sharply. However, it does not occur in Fig. 15, which shows that the robustness of the ER rule is considerably strong.

To further verify the effectiveness of the proposed method from the practical application, a comparison with actual image information of relay contacts is presented. The S-3700N multifunctional analytical variable pressure scanning electron microscope, which was produced by Hitachi Company, is used to realize fast and comprehensive scanning observation of relay contacts. The maximum observation area diameter is 203 mm, and the energy spectrum analysis can be carried out when the sample is less than 110 mm in height. The physical picture of the microscope is shown in Fig. 16.

The S-3700N scanning electron microscope is composed of a sample table, an electronic optical system, a display control system, a vacuum system, and a computer. The corresponding parameters are configured as follows.

1) The amplification is between 5 and 300 000 times.
2) The acceleration voltage is between 0.3 and 30 KV.
3) The low vacuum range is between 6 and 270 Pa.
4) The image potential shift is ± 50 μm.
5) The maximum loading sample diameter is 300 mm.

According to the actual observation of relay contacts, the corresponding evaluation grades of relay's reliability ("high," "medium," and "low") are depicted in Fig. 17.

From Fig. 17, when the relay operates 46 556 times, it fails seriously and the corresponding status is in "low reliability." It can be concluded that under the influence of perturbation, the reliability of relay has been decreased to a certain extent, which is exactly consistent with the conclusion of Fig. 14. Therefore, ER rule-PA is proved to be effective.

Based on the above analysis, PA is not only a complement to the properties of the ER rule, but a kind of feasible and effective way to characterize the impact of perturbation.

V. Conclusion

In this article, a new method, referred to as ER rule-PA, is proposed to explore the PM of the ER rule, which provides theoretical and technical support for the evidence theory. The idea of DEDS with its PM and the ER rule are used to be the basis of this article. To reduce the complexity of reasoning, the combination of two pieces of independent evidence is performed from the theoretical aspect. Meanwhile, perturbation is added to one piece of evidence. In addition, the perturbation utility and perturbation coefficient are introduced to study the performance of the ER rule. It is shown that the robustness and stability of the ER rule are quite strong. The inference process of the generalized method of ER rule-PA is deduced for further applications of the ER rule. Finally, the numerical study and the case study are conducted to verify the feasibility and effectiveness of ER rule-PA. It is also shown that ER rule-PA is in essence the ER rule itself when the perturbation is invalid.

There are three main contributions of this article. First, PA is first combined with the ER rule, where perturbation is quantified as a bounded parameter and is added to the input of evidence. Through the comparison of PM of the ER rule, two novel concepts called perturbation utility and perturbation coefficient are introduced, which are employed to measure the influence of perturbation and explore the performance of the ER rule. Second, the MPE of perturbation coefficient is preliminarily defined, which can characterize the acceptability of perturbation. Meanwhile, the value of the MPE can be determined by the real situation based on the expert knowledge, which may provide some references and ideas for engineering applications. Third, a generalized method of ER rule-PA is explored to facilitate the applications of the ER rule, which lays a foundation for the study of properties of the ER rule. Moreover, in engineering practice, it can be used to explore the performance of such complex systems as electric power tower, high bridge, large oil tanks, and rocket structure in that the system may be confronted with the influence of perturbation frequently.

It is worth noting that ER rule-PA can be applied not only to engineering practice for dealing with such cases as fault diagnosis and performance analysis of systems, but to other areas such as health care in disease diagnosis and medical decision making. In this article, the inference process of combining independent evidence for PA is explored in detail, and all the evidence is assumed to share the same belief structure. Furthermore, the weight and reliability of each piece of evidence are given subjectively. When the belief structure or the weight/reliability of criteria is different, it will be challenging but interesting to deal with such situations, which requires further research.

Appendix A
Proof of Theorem 3

For all $\Delta u_i \in [U_{l+1} - u_i, U_l - u_i]$ and an arbitrary small amount Δu_0 in (31), there is

$$\begin{aligned} &S(\Delta u_i + \Delta u_0) - S(\Delta u_i) \\ &\quad = \frac{\sigma_i\big[2P(U_l - U_{l+1}) - Q\big]}{4R\big[R + w_i w_j \sigma_i(\Delta u_i + \Delta u_0)\big(p_j - p_{j+1}\big)\big]} \\ &\qquad - \frac{\sigma_i\big[2P(U_l - U_{l+1}) - Q\big]}{4R\big[R + w_i w_j \sigma_i \Delta u_j\big(p_j - p_{j+1}\big)\big]} \\ &\quad = \frac{w_i w_j \sigma_i^2 \Delta u_0 R\big(p_{j+1} - p_j\big)\big[2P(U_l - U_{l+1}) - Q\big]}{4R'\big[R' + w_i w_j \sigma_i \Delta u_0 R\big(p_j - p_{j+1}\big)\big]} \end{aligned} \tag{49}$$

where $R' = R[R + w_i w_j \sigma_i \Delta u_i (p_j - p_{j+1})]$.

It is obvious that $\lim_{\Delta u_0 \to 0}[S(\Delta u_i + \Delta u_0) - S(\Delta u_i)] = 0$. Thus, $S(\Delta u_i)$ is continuous on $[U_{l+1} - u_i, U_l - u_i]$.

Based on (49), for all $\Delta u_i \in [U_{l+1} - u_i, U_l - u_i]$ and an arbitrary small amount Δu_0, we have

$$\begin{aligned} &\lim_{\Delta u_0 \to 0} \frac{S(\Delta u_i + \Delta u_0) - S(\Delta u_i)}{\Delta u_0} \\ &\quad = \lim_{\Delta u_0 \to 0} \frac{w_i w_j \sigma_i^2 R\big(p_{j+1} - p_j\big)\big[2P(U_l - U_{l+1}) - Q\big]}{4R'\big[R' + w_i w_j \sigma_i \Delta u_0 R\big(p_j - p_{j+1}\big)\big]} \\ &\quad = w_i w_j \sigma_i^2 R\big(p_{j+1} - p_j\big)\big[2P(U_l - U_{l+1}) - Q\big]/4R'^2. \end{aligned} \tag{50}$$

The result of (50) exists, which means $S(\Delta u_i)$ is derivable on its domains.

Appendix B
Proof of Theorem 4

Based on Theorem 3 and (31), we have

$$K = \frac{dS(\Delta u_i)}{d\Delta u_i} = \frac{w_i w_j \sigma_i^2\big(p_{j+1} - p_j\big)\big[2P(U_l - U_{l+1}) - Q\big]}{4R\big[R + w_i w_j \sigma_i \Delta u_i\big(p_j - p_{j+1}\big)\big]^2}. \tag{51}$$

There are two cases to be considered.

1) If $p_j \geq p_{j+1}$, there will be $K \leq 0$, which means $S(\Delta u_i)$ decreases monotonically on the interval $[U_{l+1} - u_i, U_l - u_i]$.
2) If $p_j < p_{j+1}$, there will be $K > 0$, which means $S(\Delta u_i)$ increases monotonically on the interval $[U_{l+1} - u_i, U_l - u_i]$.

In conclusion, $S(\Delta u_i)$ decreases monotonically when $p_j \geq p_{j+1}$, but the inverse increases.

Appendix C
Proof of Theorem 5

As Δu_i satisfies $u_i + \sigma_i \Delta u_i \in [U_{l+1}, U_l]$, there are two cases to be considered.

1) If $p_j \geq p_{j+1}$, the maximum and minimum values of the denominator in (31) are given by

$$\begin{aligned} \max\ & 4R\big[R + w_i w_j \sigma_i \Delta u_i\big(p_j - p_{j+1}\big)\big] \\ &= 4R\big[R + w_i w_j (U_l - u_i)\big(p_j - p_{j+1}\big)\big] \\ &= 4R\big\{\big[w_i\big(1 - r_j\big) + w_j(1 - r_i) + w_i w_j\big](U_l - U_{l+1}) \\ &\qquad + w_i w_j\big[u_i\big(p_j - p_{j+1}\big) + p_{j+1}U_{l+1} - p_j U_l\big] \\ &\qquad + w_i w_j (U_l - u_i)\big(p_j - p_{j+1}\big)\big\} \\ &= 4R\big\{(U_l - U_{l+1})\big[w_i\big(1 - r_j\big) + w_j(1 - r_i) \\ &\qquad + w_i w_j\big(1 - p_{j+1}\big)\big]\big\} \end{aligned} \tag{52}$$

$$\begin{aligned} \min\ & 4R\big[R + w_i w_j \sigma_i \Delta u_i\big(p_j - p_{j+1}\big)\big] \\ &= 4R\big[R + w_i w_j (U_{l+1} - u_i)\big(p_j - p_{j+1}\big)\big] \\ &= 4R\big\{\big[w_i\big(1 - r_j\big) + w_j(1 - r_i) + w_i w_j\big](U_l - U_{l+1}) \\ &\qquad + w_i w_j\big[u_i\big(p_j - p_{j+1}\big) + p_{j+1}U_{l+1} - p_j U_l\big] \\ &\qquad + w_i w_j (U_{l+1} - u_i)\big(p_j - p_{j+1}\big)\big\} \\ &= 4R\big\{(U_l - U_{l+1})\big[w_i\big(1 - r_j\big) + w_j(1 - r_i) \\ &\qquad + w_i w_j\big(1 - p_j\big)\big]\big\}. \end{aligned} \tag{53}$$

As $\varepsilon_1 \leq \sigma_i \leq \varepsilon_2$, the upper and lower bounds of the perturbation coefficient are given by

$$\begin{cases} \min S(\Delta u_i) = \dfrac{\varepsilon_1 \sigma_i [2P(U_l - U_{l+1}) - Q]}{4R\{(U_l - U_{l+1})[w_i(1-r_j) + w_j(1-r_i) + w_i w_j (1-p_{j+1})]\}} \\ \max S(\Delta u_i) = \dfrac{\varepsilon_2 \sigma_i [2P(U_l - U_{l+1}) - Q]}{4R\{(U_l - U_{l+1})[w_i(1-r_j) + w_j(1-r_i) + w_i w_j (1-p_j)]\}}. \end{cases} \tag{54}$$

2) If $p_j < p_{j+1}$, similar to the above analysis, the upper and lower bounds of the perturbation coefficient are given by

$$\begin{cases} \min S(\Delta u_i) = \dfrac{\varepsilon_1 \sigma_i [2P(U_l - U_{l+1}) - Q]}{4R\{(U_l - U_{l+1})[w_i(1-r_j) + w_j(1-r_i) + w_i w_j (1-p_j)]\}} \\ \max S(\Delta u_i) = \dfrac{\varepsilon_2 \sigma_i [2P(U_l - U_{l+1}) - Q]}{4R\{(U_l - U_{l+1})[w_i(1-r_j) + w_j(1-r_i) + w_i w_j (1-p_{j+1})]\}}. \end{cases} \tag{55}$$

Based on the above discussion, it can be concluded that there are two bounds of $S(\Delta u_i)$ as (33).

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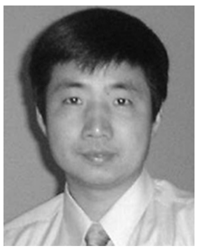

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