JMSE **2018**, 3(2), 76–100
doi:10.3724/SP.J.1383.302005
http://www.jmse.org.cn/
http://engine.scichina.com/publisher/CSPM/journal/JMSE

Article

Using Two-layer Minimax Optimization and DEA to Determine Attribute Weights

Yao-Yao Song [1,2], Guo-Liang Yang [1,2,*], Jian-Bo Yang [3], Mohammad Khoveyni [4,5] and Dong-Ling Xu [3]

[1] Institutes of Science and Development, Chinese Academy of Sciences, Beijing 100190, China

[2] University of Chinese Academy of Sciences, Beijing 100049, China; songyaoyao15@mails.ucas.ac.cn

[3] Alliance Manchester Business School, the University of Manchester, Manchester M139SS, UK; Jian-bo.yang@manchester.ac.uk; ling.xu@manchester.ac.uk

[4] Department of Applied Mathematics, Yadegar-e-Imam Khomeini (RAH) Shahre Rey Branch, Islamic Azad University, Tehran 1815163111, Iran; mohammadkhoveyni@gmail.com; m.khoveyni@iausr.ac.ir

[5] Research Center of Modelling and Optimization in Science and Engineering, South Tehran Branch, Islamic Azad University, Tehran 1584743311, Iran

* Correspondence: glyang@casipm.ac.cn

Received: 24 December 2017; Accepted: 28 June 2018; Published: 2 August 2018

Abstract: This study aims to explore a novel method for determining attribute weights, which is a key issue in constructing and analyzing multiple-attribute decision-making (MADM) problems. To this end, a hybrid approach combining the data envelopment analysis (DEA) model without explicit inputs (DEA-WEI) and a two-layer minimax optimization scheme is developed. It is demonstrated that in this approach, the most favorable set of weights is first considered for each alternative or decision-making unit (DMU) and these weight sets are then aggregated to determine the best compromise weights for attributes, with the interests of all DMUs simultaneously considered in a fair manner. This approach is particularly suitable for situations where the preferences of decision-makers (DMs) are either unclear or difficult to acquire. Two case studies are conducted to illustrate the proposed approach and its use for determining weights for attributes in practice. The first case concerns the assessment of research strengths of 24 selected countries using certain measures, and the second concerns the analysis of the performance of 64 selected Chinese universities, where the preferences of DMs are either unknown or ambiguous, but the weights of the attributes should be assigned in a fair and unbiased manner.

Keywords: Data envelopment analysis; Weights; Multiple attribute decision-making; Minimax optimization

1. Introduction

Multiple-attribute decision-making (MADM) prevails in engineering, social, and economic fields. It is used to solve the problem of selecting or ranking in multiple schemes. The attribute set or solution set is an indispensable element of MADM. The solution of MADM should formulate decision rules and reflect the preferences of decision-making units (DMUs) or decision-makers (DMs) so that the result may be scientifically selected. In the process of formulating decision rules, the determination of attribute weights is usually a necessary task and thus has become an important part of MADM problems.

Several methods for determining weights in MADM have been proposed. For example, Eckenrode (1965) suggested six techniques for collecting the judgments of DMs concerning the relative values of attributes, namely, ranking, rating, three versions of paired comparisons, and a method of successive comparisons. Hwang and Yoon (1981) proposed four techniques for weight assignment in MADM, namely, the eigenvector method, the weighted least squares (WLS) method, the entropy method, and the linear programming (LP) technique for multidimensional analysis of preference (LINMAP). Ma et al. (1999) proposed an integrated subjective and objective approach to determine attribute weights. It uses subjective information provided by DMs and objective information based on a decision matrix to form a two-objective programming model for integrating the subjective considerations of DMs and the objective measures of attribute importance. However, Xu (2004) noted that the weights thereby obtained can be quite different from other objective weights, e.g., entropy weights. Shirland et al. (2003) proposed a goal programming model for determining constrained regression estimates of attribute weights using pairwise comparisons by means of triads of attributes. Wang and Lee (2009) proposed an extension of the TOPSIS approach that integrates subjective and objective weights. Their approach is not only based on DMs' judgments but also calculates subjective weights based on Shannon's entropy. Shannon's entropy has also been applied to calculate subjective weights in group decision-making problems. Yang et al. (2017) proposed a three-stage approach for weight assignment in MADM to support the solution of such MADM problems as performance assessment and policy analysis, where (a) the preferences of DMs are either unclear and partial or difficult to acquire and (b) there is a need to consider the best "will" of each DMU in the sense that the DMU's individual way to achieve its best performance should be respected and duly taken into account when its performance is assessed alongside other DMUs.

Yang et al. (2017) summarized the existing main approaches for determining attribute weights and pointed out that, in general, there may be three types of approaches for weight assignment, depending on the information provided and used to identify the weights: **(1) subjective approaches**, e.g., the simple multi-attribute rating technique (SMART) (Edwards, 1977; Edwards and Barron, 1994), the point allocation method (Doyle et al., 1997), the analytic hierarchy process (AHP) (Saaty, 1980 & 1986; Forman and Gass, 1999), rank order distribution weights (Roberts and Goodwin, 2002), and the Delphi method (e.g. Hwang and Yoon, 1981); **(2) objective approaches**, e.g., the entropy method (Hwang and Yoon, 1981; Zeleny, 1982), principal component analysis (Jolliffe 1986), the WLS method (Chu et al., 1979), the projection method (Yue, 2012), the standard deviation integrated approach (Wang and Luo, 2010), and the weight assignment approach based on the quantification of the contrast intensity and the conflicting character of the evaluation criteria (Diakoulaki et al., 1995); **(3) hybrid approaches**, e.g., the integrated subjective and objective approach (Ma et al., 1999), the goal programming model based on pairwise comparisons between attributes (Shirland et al., 2003), the integer linear goal-programming technique based on a training set of choices (Choo and Wedley, 1985), the UTilités Additives (UTA) method (e.g. Jacquet-Lagrèze and Siskos, 1982; Siskos et al., 2005), the extended TOPSIS approach integrating subjective and objective weights (Wang and Lee, 2009), LP for estimating attribute weights by pairwise preference comparisons (Horsky and Rao, 1984), the LP technique for criteria weights (Horowitz and Zappe, 1995), and the minimax disparity approach (Wang and Parkan, 2006; Amin and Emrouznejad, 2006; Amin, 2007; Emrouznejad and Amin, 2010). See also Yang et al. (2017) for a more detailed literature review.

Subjective approaches reflect the subjective judgment or intuition of DMs; however, the analytical results or rankings of alternatives based on weights can be influenced by the lack of experience or knowledge. Objective approaches determine weights by making use of mathematical models or statistical methods, but they often focus on differences in the values of attributes. For instance, in the entropy method, the decision matrix contains a certain amount of information for a set of alternatives or DMUs. The hybrid approach combines the advantages of both the subjective and the objective approaches to determine the weights of attributes in MADM.

The relationship between data envelopment analysis (DEA) without explicit inputs (WEI) and utility theory has been studied by Yang et al. (2014), where the authors investigated the relationship between extended utility functions and DEA-WEI. Specifically, it was argued that the objective function of DEA-WEI resembles the classic utility function, and the link between utility theory in MADM and DEA-WEI was established.

In several real MADM problems, however, for example, the assessment of the academic impact of national research institutes and the assessment of academic capacity of countries, subjective approaches are not applicable because there are no obvious DMs who can provide subjective judgments or intuition on the relative importance of attributes in such problems. Although most of the existing objective approaches can determine weights based on the decision matrix and do not depend on the subjective judgments of DMs, they rely solely on data and do not take into consideration the "will" of DMUs, i.e., a set of the most favorable weights for each DMU. Yang et al. (2017) proposed a new three-stage approach, whereby a set of preliminary weights as well as the most favorable set of weights for each DMU are first considered using a DEA-WEI model, and then these weight sets are aggregated to obtain the best compromise weights for attributes, with the interests of all DMUs taken into account fairly and simultaneously.

However, in the study by Yang et al. (2017), there are two proposed processes that may not match reality, and thus there may pose problems in its application: First, the use of the entropy method to obtain the preliminary weights double counts the role of an attribute and may violate the claimed fairness principle. Secondly, the formation of each DMU's "right to speak" is defined as the gap between the DMU's utility based on preliminary weights and the maximized utility based on an DEA-WEI-like LP model, which is a somehow arbitrary definition and only takes into indirect consideration the DMU's "will" of using its most preferred weights for performance assessment. This may result in deviations from the optimal weight assignment in the sense of genuinely treating each DMU's best "will" fairly. Furthermore, the compromise solution in Yang et al. (2017) in stage 3 is based on an ideal utility vector for all attributes, which is minimized for every DMU, with no special treatment for any DMU to reflect the fairness principle. However, the best approach is to obtain the compromise solution by minimizing the maximum deviation (∞-norm) of the different weight assignment schemes from all DMUs.

Taking into account the above considerations, this study aims to investigate a novel approach for determining the weights of attributes based on a DEA-WEI-like LP model and a two-layer minimax optimization scheme, and to solve the two above-mentioned problems by abandoning the use of the entropy method and the "right to speak", redesigning the model in a sensible and fair manner. In this new approach, a set of the most favorable weights for each DMU is considered, and then these weight vectors are aggregated to obtain the best compromise solution using two-layer minimax optimization as the final attribute weights so that the best interests of all DMUs are taken directly into account fairly and simultaneously.

The remainder of the paper is organized as follows: In Section 2, preliminaries and notations are introduced. Furthermore, DEA, DEA-WEI, and several secondary models in DEA-WEI are briefly reviewed. The new hybrid approach for determining attribute weights is then proposed. In Section 3, Monte Carlo simulation is also conducted to test the characteristics of the proposed approach. Moreover, two case studies are presented to demonstrate the features of the proposed approach, namely the assessment of the research strengths of 24 selected countries/regions and the performance analysis of 64 selected Chinese universities. In Section 4, the paper is concluded.

2. Materials and Methods

2.1. Preliminaries and Notations

In this study, the following notations are used to represent a MADM problem:

$S = \{DMU_j, j = 1,2, \dots, n\}$: a discrete group of *n* possible DMUs or alternatives.

$Y = \{Y_1, Y_2, \dots, Y_s\}$: a set of s attributes. It is assumed without loss of generality that the attributes are additively independent to simplify the discussion.

$W^T = [w_1, w_2, \dots, w_s]^T$: the weight vector of attributes (or weights thereafter), which satisfies $\sum_{r=1}^{s} w_r = 1, w_r \geq 0$. It should be noted that the superscript "T" represents a vector transpose.

$A = [a_{jr}]_{n\times s}$: the decision matrix, where a_{jr} denotes the value of DMU *j* on attribute *r* which is non-negative for $j = 1,2, \dots, n$ and $r = 1,2, \dots, s$.

As in Yang et al. (2017), the decision matrix A is normalized by transforming each of its elements into a corresponding element in the normalized (value) matrix $D = [b_{jr}]_{n\times s}$ using the following linear formula:

$$\begin{cases} \text{Benefit attribute: } b_{jr} = \frac{a_{jr}}{a_r^{max}}, \\ \text{Cost attribute: } b_{jr} = \frac{a_r^{min}}{a_{jr}} \text{ or } 1 - \frac{a_{jr}}{a_j^{max}} \end{cases} \tag{1}$$

When there is no obvious DM to provide value functions for attributes, a piecewise linear value function is defined for each attribute. Let MIN_r and $MEDIAN_r$ be the minimal and median values, respectively, of the attribute r $(r = 1, \dots, s)$. Then, without loss of generality, let

$$\begin{cases} V(MIN_r) = MIN_r \\ V(MEDIAN_r) = (1 + V(MIN_r))/2 \\ V(1) = 1 \end{cases} \tag{2}$$

Figure 1 illustrates a piecewise linear value function $V(b_{jr})$, where b_{jr} denotes the attribute's value after the normalization by formula (1).

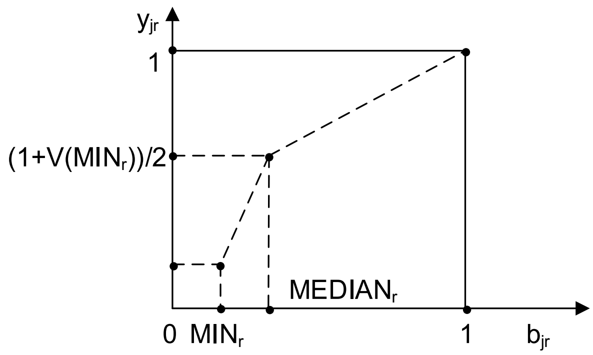

Figure 1. Illustration of a Piecewise Linear Value Function[1]

1 Figure 1 illustrates only, for example, a risk averse value function.

Mathematically, the value function shown in Figure 1 can be given by $y_{jr} = V(b_{jr})$ as follows:

$$y_{jr} = V(b_{jr}) = \begin{cases} \frac{1-MIN_r}{2\times(MEDIAN_r-MIN_r)} b_{jr} + \frac{2\times MEDIAN_r-MIN_r-1}{2\times(MEDIAN_r-MIN_r)} \times MIN_r, \text{if } b_{jr} \in [MIN_r, MEDIAN_r] & (3a) \\ \frac{1-MIN_r}{2\times(1-MEDIAN_r)} b_{jr} + \frac{1-2\times MEDIAN_r+MIN_r}{2\times(1-MEDIAN_r)}, \text{if } b_{jr} \in [MEDIAN_r, 1] & (3b) \end{cases} \quad (3)$$

It can be verified that when $b_{jr} = MEDIAN_r$, the results of formulas (3a) and (3b) are the same. The normalized matrix $DV = [y_{jr}]_{n\times s}$ generated using the value function (3) will be used as the dataset for the analysis. This value function assumes that DMUs or alternatives with median achievements on attributes are given average value. Even though other assumptions may also be made, a more appropriate approach is to obtain as much preference information from DMs as possible, if it is at all possible.

One of the most widely used MADM methods is the simple additive value method, which leads to the overall value of a DMU as follows:

$$\text{Utility}(DMU_j) = \sum_{r=1}^{s} w_r v_j(b_{rj}), j = 1,2, \dots, n. \quad (4)$$

Keeney and Raiffa (1976) showed that the theoretical foundation of this method is utility theory.

2.2. *Combining DEA-WEI and Two-Layer Minimax Optimization*

In Equation (4), a key issue is how to fairly and consistently define the weight w_r, which poses a challenge when there is no obvious decision maker who can provide his/her preferences or when preferences are difficult to obtain clearly. To address this issue, a novel approach is proposed combining DEA-WEI and a two-layer minimax optimization technique to determine weights for multiple attributes.

The first stage in this approach is to use DEA-WEI models to generate a set of attribute weights most favored by each DMU. This set represents the best "will" of each DMU. In the second stage, the most favorable weights of each DMU are aggregated to obtain the best compromise solution as the common attribute weights recommended for all DMUs, where the best "will" of each DMU is respected equally and simultaneously. In the third stage, a two-layer minimax optimization formulation is proposed to obtain the best compromise solution for all DMUs, thus determining the final weights for the attributes.

To test the robustness of the proposed method, Monte Carlo simulation is used to test the ability of the model in the solution process from different perspectives in Section 3.1. To improve the adaptability of the proposed method, in Section 3.2, it is adjusted when there is priori information.

2.2.1. Generating Optimal Weights for Each DMU Based on DEA-WEI Model

DEA was originally developed by Charnes et al. (1978); it is a mathematical programming method for efficiency analysis or performance assessment of DMUs with multiple inputs and multiple outputs. DEA has been widely used in efficiency analysis and performance evaluation of various business and non-profit organizations. The essence of DEA is the principle that each DMU is allowed to generate its most favorable weights for its performance assessment. That is, a DEA model provides the weighting coefficients so that each DMU can assign weights for its best benefit in terms of maximizing its efficiency score. Therefore, each DMU is free to value better whatever it is best at and to ignore the variables with which it does not perform well. In the past several decades, there have been several specific DEA models, the most well-known being the CCR model (Charnes et al., 1978) and the BCC model (Banker et al., 1984). Moreover, the related additive model (Charnes et al., 1985) and the weight-restricted model (Dyson and

Thanassoulis, 1998; Allen et al., 1997) should be mentioned. Systematic reviews on DEA theory and its applications can be found in several studies, e.g., Cooper et al. (2004, 2006), Cook et al. (2009), and Liu et al. (2013).

However, as argued in, e.g., Yang et al. (2014&2017), in several real applications, there are no explicit input data available. It is difficult or sometimes impossible to reformulate data into original inputs and outputs and then apply the classic DEA models to measure the performance of DMUs. In particular, in MADM problems, attributes are normally divided into benefit and cost attributes rather than inputs and outputs in the DEA framework. After the transformation by formula (1), the normalized decision matrix with the highest value of an attribute being preferred is obtained. In this sense, the values of attributes can be regarded as outputs in the context of DEA. Accordingly, Adolphson et al. (1991) first developed DEA models without inputs. Lovell and Pastor (1999) and Caporaletti et al. (1999) systematically studied these DEA models without inputs. Liu et al. (2011) conducted systematic studies on this group of DEA models, which are called DEA-WEI models. Toloo (2013) proposed a new approach for the most efficient unit without explicit inputs. Yang et al. (2014) investigated and linked the DEA-WEI model with quadratic terms to the extended utility function.

In general, DEA-WEI is used for evaluation purposes and does not reflect the technical efficiency of the input-output system of DMUs, which is the normal function of DEA. Yang et al. (2014) investigated the link between the DEA-WEI model and multi-attribute utility theory and noted that the functional form of its objective function resembles the traditional utility function. The DEA-WEI model is different from a classic utility function in that only the functional form of an objective function is determined by the DMs to reflect a subjective emphasis on assessment. The coefficients of the objective function in the DEA-WEI model are determined by a DMU to ensure the most favorable evaluation for this DMU (Yang et al. 2014; Yang et al. 2017).

Yang et al. (2017) showed that the DEA-WEI model can generate an optimal weight vector for each DMU. This weight vector reflects the best "will" of an assessed DMU. In MADM, however, preferential or cognitive constraints may be imposed on the relative importance of each attribute. For instance, as in the AHP method (e.g. Saaty 1980 & 1986), the ratio range of weights of one attribute A1 to another A2 can be set to $\left[\frac{1}{9}, 9\right]$. Therefore, the DEA-WEI model can be more generally reformulated as follows:

$$\max\left\{\theta_0 = \sum_{r=1}^{s} u_r y_{r0} \,\middle|\, \sum_{r=1}^{s} u_r y_{rj} \le 1\,, j = 1,2,\dots,n; u_r \in \Omega\,, r = 1,2,\dots,s\right\}, \tag{5}$$

where the symbol Ω refers to the set of preferential and cognitive constraints on attributes in model (5), and y_{r0} denotes the attribute value of the assessed DMU_0. When there are no explicit DMs, for example, Ω could be defined as $\Omega = \left\{\left(u_i/u_j\right)_{i\neq j} \in \left[\frac{1}{9}, 9\right], i, j = 1,2,\dots,s\right\}$. When there are explicit DMs, Ω may be the set denoting the DMs' prior preference information or value judgments, e.g., $\Omega = \left\{u_i \ge u_j, i \ne j\right\}$. As previously discussed, the essence of DEA or DEA-WEI is that each DMU can provide flexible weights for its inputs/outputs. Therefore, the aim is to obtain the optimal weights for DMU_0 as $(u_1^{0*}, u_2^{0*}, \dots, u_r^{0*})^T$, which is the optimal solution of model (5). See Yang et al. (2017) for details.

2.2.2. Selecting Optimal Weights by Secondary Model

There is the problem that alternative optimal attribute weights commonly exist in the DEA-WEI model (5). A secondary goal can be introduced for obtaining a unique optimal solution for a DMU, such as an aggressive or benevolent goal, which can minimize or maximize the utility of the composite DMUs constructed for other DMUs compared to DMU_0. The aggressive model is given as follows:

$$\min_{u_r} \sum_{r=1}^{s} u_r\left(\sum_{j=1,j\neq 0}^{n} y_{rj}\right) \quad \text{s.t.} \begin{cases} \sum_{r=1}^{s} u_r y_{r0} = \theta_0^* \\ \sum_{r=1}^{s} u_r y_{rj} \leq 1, j = 1,2,\dots,n; j \neq 0 \\ u_r \in \Omega, r = 1,2,\dots,s \end{cases} \tag{6}$$

where θ_0^* is the score of DMU_0 derived from model (5). The subscript $j \neq 0$ indicates that the assessed DMU_0 is excluded from the sequence $j = 1,2,\dots,n$. It can be seen that this model attempts to minimize the utilities of other DMUs and simultaneously maintain the utility of the assessed DMU_0.

The benevolent formulation is achieved by replacing min with max in the objective function of model (6) as follows:

$$\max_{u_r} \sum_{r=1}^{s} u_r\left(\sum_{j=1,j\neq 0}^{n} y_{rj}\right) \text{s.t.} \begin{cases} \sum_{r=1}^{s} u_r y_{r0} = \theta_0^* \\ \sum_{r=1}^{s} u_r y_{rj} \leq 1, j = 1,2,\dots,n; j \neq 0 \\ u_r \in \Omega, r = 1,2,\dots,s \end{cases} \tag{7}$$

where the subscript "0" denotes the assessed DMU_0, and similarly hereinafter.

From the objective function of model (7), the benevolent model intends to maximize the utilities of other DMUs and simultaneously maintain the utility of the assessed DMU_0.

Furthermore, Liang et al. (2008) proposed three other alternative secondary goals, namely, minimizing total deviation from the ideal point (total deviation model), minimizing the maximum deviation (maximum deviation model), and minimizing the mean absolute deviation (absolute deviation model). In fact, the total deviation model can be easily seen to be equivalent to model (7). The maximum deviation model and the absolute deviation model are presented in Appendix A. There are also some other alternative secondary goals, such as those in the methods proposed in Wang and Chin (2010 & 2011) and Sexton et al. (1986).

Using the secondary goal model to obtain a unique optimal solution is not the same as to add weight restrictions because the secondary goal model is used to obtain a unique optimal solution in the existing feasible region instead of reducing the feasible region, as is the case with weight restriction. The secondary goal model is used to ensure the uniqueness of the most favorable weights of the assessed DMU. However, as pointed out by Lin et al. (2016), the secondary models (6), (7), (A1), and (A3) may theoretically exhibit non-uniqueness. However, when the secondary goals mentioned above are used, most cases with multiple optimal solutions can be avoided in real applications. Thus, the secondary LP models have been adopted in this study.

2.2.3. Two-Layer Minimax Optimization for Weight Assignment

Herein the most favorable weight vectors previously generated for individual DMUs are aggregated to obtain the best compromise solution as the common weights of attributes recommended for all DMUs. In the aggregation process, the individual most favorable weight vector $U^{j*} = (u_1^{j*}, u_2^{j*}, \dots, u_s^{j*})^T$, which is obtained from Section 2.2.1 after normalization, i.e., $\sum_{r=1}^{s} u_r^{j*} = 1$, for DMU_j is represented as a reference point. The distance between the best compromise solution and reference point *j* for DMU_j is measured by the ∞-norm, so that the difference between the best compromise weight and the reference weight of each attribute is taken into account fairly in the sense that all DMUs are assumed to be collectively cooperative in terms of minimizing the maximum deviation from any DMU's reference point, or that no DMU is allowed to take advantage over another DMU in this weight aggregation process. That is, this minimax optimization process for weight aggregation operates as an equalizer, so that the difference on any attribute for each DMU is taken into account equally or fairly (Yang, 2000; Yang and Xu, 2014). The best compromise solution is defined as that minimizing the maximum distances for all DMUs, so that the interests of all DMUs are simultaneously taken into consideration without bias. Under this definition, the best compromise solution can be obtained by solving the following two-layer minimax optimization problem:

$$\min\ \tau = \max_{j=1,\dots,n}\left\{t_j = \max_{r=1,2,\dots,s}\left\{\left|w_r - u_r^{j*}\right|\right\}\right\} \quad s.t. \begin{cases} \sum_{r=1}^{s} w_r = 1 \\ w_r \in \psi \\ w_r \geq 0, r = 1,2,\dots,s \end{cases} \tag{8}$$

where $w_r \geq 0$ denotes the compromise weight for the *r*-th attribute, and $\psi = \left\{\frac{w_i}{w_j} \in \left[\frac{1}{9}, 9\right]_{i \neq j}, i, j = 1,2,\dots,s\right\}$.[2] It is worth emphasizing that the two-layer minimax optimization model (10) can be used to fairly generate the final attribute weights for all DMUs. This is due to the fact that the inner-layer of the model functions as an equalizer (Yang, 2000; Yang and Xu, 2014), so that the maximum difference between the best compromise weight and the reference weight for all attributes is minimized for every DMU with no weight difference unfairly given any special treatment for any DMU. Furthermore, the out-layer of the model also functions as an equalizer, so that the maximum distance for all DMUs is minimized, with no distance unfairly given any special treatment for any DMU. This fairness principle is considered to be appropriate for weight assignment in MADM, in particular when there are no obvious DMs.

Figure 2 shows the rationale of the above two-layer minimax optimization approach. It is assumed that there are only two attributes, whose weights are denoted by w_1 and w_2, and there are three DMUs, namely, DMU_1 (Point A), DMU_2 (Point B) and DMU_3 (Point C). Thus, by formula (8), we have $t_1 = \overline{A'\ P}$, where $\overline{A'\ P}$ is the distance between point A' and point P, and similarly $t_2 = \overline{B'\ P}$ and $t_3 = \overline{C'\ P}$. As for every DMU the maximum difference between the best compromise weight and the reference weight for all attributes is minimized and there is no special treatment for any DMU, there are at least two DMUs with the same maximum distance measured by the ∞-norm when an optimal solution is obtained. In this figure, it can be seen that $t_1 = t_3 > t_2$ is the optimal solution, which is a fair principle without bias on any DMU.

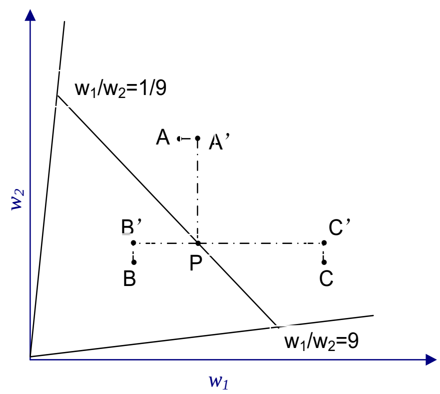

Figure 2. Illustration of Two-Layer Minimax Optimization

Model (8) is a nonlinear non-smooth programming problem and can be equivalently transformed into the following model:

2 Here, the symbol ψ is used to denote the constraints on the attribute's weight, which is different from Ω.

$$\min \tau \quad \text{s.t.} \begin{cases} |w_r - u_r^{1*}| \le \tau, r = 1,2,\dots,s \\ |w_r - u_r^{2*}| \le \tau, r = 1,2,\dots,s \\ \vdots \\ |w_r - u_r^{n*}| \le \tau, r = 1,2,\dots,s \\ \sum_{r=1}^{s} w_r = 1 \\ w_r \in \psi \\ w_r \ge 0, r = 1,2,\dots,s \end{cases} \tag{9}$$

The non-smooth constraint $|w_r - u_r^{j*}| \le \tau\ (j = 1,2,\dots,n)$ can be simply replaced by the equivalent smooth constraints $(w_r - u_r^{j*}) \le \tau$ and $(w_r - u_r^{j*}) \ge -\tau$. Thus, model (9) can be transformed into the following model, which is a standard LP problem and can be easily solved using existing optimization software packages:

$$\min \tau \quad \text{s.t.} \begin{cases} (w_r - u_r^{1*}) \le \tau, r = 1,2,\dots,s \\ (w_r - u_r^{1*}) \ge -\tau, r = 1,2,\dots,s \\ (w_r - u_r^{2*}) \le \tau, r = 1,2,\dots,s \\ (w_r - u_r^{2*}) \ge -\tau, r = 1,2,\dots,s \\ \vdots \\ (w_r - u_r^{n*}) \le \tau, r = 1,2,\dots,s \\ (w_r - u_r^{n*}) \ge -\tau, r = 1,2,\dots,s \\ \sum_{r=1}^{s} w_r = 1 \\ w_r \in \psi, \\ w_r \ge 0, r = 1,2,\dots,s \\ \tau \ge 0, j = 1,2,\dots n \end{cases} \tag{10}$$

Intuitively, it can be deduced from these models that the weight of one attribute, if generated in this weight aggregation process, will be probably larger if all DMUs perform relatively better on this attribute than others. Thus, we have the following hypothesis:

H1: *Given that each attribute is normalized to take values in the interval [0, 1], the weight of an attribute with greater expected value is larger than that of attributes with lower expected values.*

In summary, the rationale behind this new approach is that the weights of attributes are generated by a three-stage process. The first stage is to allow each DMU to propose its most favorable weight vector of attributes to maximize its own utility through the DEA-WEI model. The second stage is to use a properly selected secondary model for obtaining the unique weight vector of attributes for each DMU individually to avoid multiple solutions in the first stage. Which secondary model should be selected depends on whether DMUs are individually cooperative, non-cooperative, or equalitarian-minded when generating their individual most favorable weights. The third stage is to obtain the best compromise solution that minimizes the maximum ∞-norm distances to each unique most favorable weight vector for each DMU found in the second stage.

3. Results

3.1 Numerical Simulations

Monte Carlo simulation is a widely used method for testing the statistical properties of models (e.g., Smith, 1997). Herein, Monte Carlo simulation is used to test the relations between the weights determined by the proposed approach and the characteristics of the existing data (including the expected value and variance) and secondary models.

Without loss of generality, it may be assumed that there are five attributes. Furthermore, Cooper et al. (2000)

proposed a rule of thumb for the number of DMUs required in DEA models, namely, $n \geq \max\{m\times s, 3(m+s)\}$, where n is the number of DMUs, and m and s are the number of inputs and outputs, respectively. Dyson et al. (2001) argued that the number of DMUs should be $n \geq 2(m\times s)$ to achieve a reasonable level of discrimination. To ensure the reliability of the numerical simulations, the minimal number of DMUs was doubled with respect to the number of attributes suggested by Cooper et al. (2000), and it is assumed that there are 30 DMUs.

Experiment 1. Relationship between weights and expectation of attributes' values

For each attribute of these 30 DMUs, data were generated from a half-normal distribution with different combinations of expected value and variance. That is, for these five attributes, data were generated in the range of (0, 1] by a normal distribution, and the corresponding expected values and variances were (0.1, 0.5), (0.3, 0.5), (0.5, 0.5), (0.7, 0.5), and (0.9, 0.5). Thus, the weights for attributes can be obtained using formulas (5) and (10) in a two-stage procedure.

The above numerical experiments were repeated 1000 times, and the average weights for attributes were as follows:

Table 1. Average Attribute Weights

Attributes	Attribute 1	Attribute 2	Attribute 3	Attribute 4	Attribute 5
Expected value	0.1	0.3	0.5	0.7	0.9
Variance	0.5	0.5	0.5	0.5	0.5
Average weights (models 5 and 10)	0.1942	0.1926	0.1989	0.2030	0.2113

From Table 1, it can be seen that the weights of attributes increase as the expected value of these attributes increases. Therefore, there is clear statistical evidence to support hypothesis H1.

Experiment 2. Relationship between weights and variance of attribute values

The variance of attribute values may also affect the weight assignment. To test this, further numerical experiments were conducted, and data were generated from a half-normal distribution with different combinations of variance and fixed expected value. Likewise, for these five attributes, data were generated in the range of (0, 1] by a normal distribution, and the corresponding expected values and variances were (0.5, 0.1), (0.5, 0.3), (0.5, 0.5), (0.5, 0.7), and (0.5, 0.9). Thus, the weights for attributes can be obtained using formulas (6) and (10). The numerical experiments were repeated 1000 times, and the average weights for attributes are given in Table 3.

Table 2. Average Attribute Weights

Attributes	Attribute 1	Attribute 2	Attribute 3	Attribute 4	Attribute 5
Expected value	0.5	0.5	0.5	0.5	0.5
Variance	0.1	0.3	0.5	0.7	0.9
Average weights (models 5 and 10)	0.2110	0.1821	0.2005	0.1999	0.2065
Average weights (models 5,6,10)	0.2091	0.1874	0.1993	0.2031	0.2011
Average weights (models 5,7,10)	0.2787	0.1747	0.1766	0.1883	0.1817
Average weights (models 5, A1,10)	0.2816	0.1746	0.1759	0.1863	0.1816
Average weights (models 5, A3,10)	0.2883	0.1722	0.1736	0.1860	0.1799

As can be seen in Table 2, there is no obvious pattern between the variance of attribute values and the weight assignment.

Experiment 3. Relationship between weights and secondary goals

As was discussed in Section 2.2.2, alternative attribute weights may exist. Thus, the relationship of the weights determined by the proposed approach with four alternative secondary goals and the expected value of the randomized data was further tested.

The previous numerical experiments were repeated 1000 times for model (5) with secondary models (6), (7), (A1), and (A3),[3] and then model (10) was used to obtain the final best compromise weights.

The average weights for attributes are shown in Table 3. It can be seen that the average weights have a strong correlation with the expected values of these attributes for most secondary models (models 6, 7, A1, and A3). Furthermore, the results reveal that the average weights for each attribute are different when different secondary models are used. This demonstrates the importance of examining the situation where performance assessment or policy analysis is conducted, so that a right secondary model can be selected to generate an appropriate unique reference point for each DMU.

Table 3. Average Attribute Weights

Attributes	Attribute 1	Attribute 2	Attribute 3	Attribute 4	Attribute 5
Expected value	0.1	0.3	0.5	0.7	0.9
Variance	0.5	0.5	0.5	0.5	0.5
Average weights (models 5,6,10)	0.2117	0.2023	0.1933	0.1899	0.2029
Average weights (models 5,7,10)	0.1471	0.1757	0.1986	0.2259	0.2527
Average weights (models 5, A1,10)	0.1488	0.1769	0.1984	0.2248	0.2511
Average weights (models 5, A3,10)	0.1505	0.1763	0.1976	0.2247	0.2509

It should be noted that according to Lin et al. (2016), the use of secondary models in this experiment may occasionally not ensure the uniqueness of the attributes weights. However, from a practical viewpoint, this possibility is relatively low. As the numerical experiments were repeated 1000 times, the possible occurrence of multiple optimal weights would have little impact on the average weights. Thus, the robustness and reliability of simulation results can be ensured, and Experiment 3, aiming at determining the influence of choosing different secondary models on attribute weights, is statistically meaningful. Interested readers can refer to Lin et al. (2016) for an iterative method for determining the unique optimal weights.

3.2. Preliminary Weights as Inputs to the Two-Layer Minimax Model

If there is no prior information or DM's preference, it is suggested that the preliminary weights be equally set on attributes ($\omega_r = 1/s, r = 1,2 \ldots, s$), as in Sections 2.2.1–2.2.3 and 3.1, to ensure the fairness of the weighting process because there is no prior information on preference.

However, in real MADM problems, preliminary weights may be provided by DMs or may be generated. Herein, it is demonstrated how the proposed approach can be modified when there is prior information. Let ω_r be the preliminary weight of attribute *r* and $\sum_{r=1}^{s} \omega_r = 1$. Therefore, model (8) can be reformulated as follows:

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3 The details of secondary models (A1) and (A3) are represented in Appendix A.

$$\min\ \tau = \max_{j=1,\dots,n}\left\{t_j = \max_{r=1,2,\dots,s}\left\{\omega_r\left|w_r - u_r^{j*}\right|\right\}\right\}\quad s.t.\begin{cases}\sum_{r=1}^{s} w_r = 1\\ w_r \in \psi\\ w_r \geq 0, r = 1,2,\dots,s\end{cases} \tag{11}$$

Consequently, model (11) can be transformed equivalently into the following model:

$$\min \tau \quad s.t.\begin{cases}\omega_r(w_r - u_r^{1*}) \leq \tau, r = 1,2,\dots,s\\ \omega_r(w_r - u_r^{1*}) \geq -\tau, r = 1,2,\dots,s\\ \omega_r(w_r - u_r^{2*}) \leq \tau, r = 1,2,\dots,s\\ \omega_r(w_r - u_r^{2*}) \geq -\tau, r = 1,2,\dots,s\\ \vdots\\ \omega_r(w_r - u_r^{n*}) \leq \tau, r = 1,2,\dots,s\\ \omega_r(w_r - u_r^{n*}) \geq -\tau, r = 1,2,\dots,s\\ \sum_{r=1}^{s} w_r = 1\\ w_r \in \psi,\\ w_r \geq 0, r = 1,2,\dots,s\\ \tau \geq 0\end{cases} \tag{12}$$

Using model (12), the weight assignment for attributes in MADM problems can be obtained.

3.3. Case Studies

Two cases were investigated to illustrate the proposed approach for determining attribute weights in MADM. The first is the assessment of the research strength of 24 countries/regions to illustrate the comparative advantages of these countries/regions. The second example is the assessment of 64 selected universities under the direct management of the Ministry of Education (MOE) of China. In contrast with the first case, in the second case, there is an explicit DM, which is the MOE, whose prior preference or pre-judgment information could be incorporated for attribute weights.

3.3.1. Assessment of Research Strength of 24 Selected Countries/Regions

Herein, a case study is conducted to apply the proposed weight assignment approach and assess the research strength of 24 countries/regions on nine disciplines[4] related to medical science of the Essential Scientific Indicators (ESI) from Thomson Reuters. See Table 4 for details on these disciplines and their abbreviations.

Table 4. 9 ESI Disciplines and Their Abbreviations

Abbreviations	Disciplines
BB	Biology & Biochemistry
CM	Clinical Medicine
Immu.	Immunology
Mic.	Microbiology
MG	Molecular Biology & Genetics

4 There are 22 disciplines in Essential Scientific Indicators (ESI). Among them, there are nine disciplines related to medical science.

Table 4. *Cont.*

Abbreviations	Disciplines
NB	Neuroscience & Behavior
PT	Pharmacology & Toxicology
PA	Plant & Animal Science
PP	Psychiatry/Psychology

The numbers of SCI papers in these nine disciplines were used to indicate their capacity of publication; thus, the nine indicators refer to the number of SCI papers for each discipline. To achieve a reasonable level of discrimination, according to Dyson et al. (2001), 24 countries/regions whose numbers of SCI papers in each discipline in ESI were among the top 40 were selected. The dataset was from ESI, and the time window was 10 years (from Jan., 2002 to Sept., 2013). The detailed data are shown in Table B1.

In this case study, there is no obvious DM who can provide the weight information on these disciplines, and this type of study is for general policy analysis. In such cases it is difficult or even impossible to have prior information on attribute weights from potential DMs. Thus, it is suitable to use existing data and the proposed approach to generate weights for each attribute (discipline).

First, it was assumed that these nine attributes are all benefit attributes, and the dataset was normalized using formulas (1)–(3) to form a normalized decision matrix and the corresponding matrix after the transformation by value functions.

Secondly, the preliminary weights for the nine attributes were equally distributed, as there is no explicit DM who can provide prior preferential or cognitive information. See Table 5.

Table 5. Preliminary Weights for Nine ESI Disciplines

Discipline	BB	CM	Immu.	Mic.	MG	NB	PT	PA	PP
Preliminary weights	0.1111	0.1111	0.1111	0.1111	0.1111	0.1111	0.1111	0.1111	0.1111

Thirdly, model (5) was used with $\Omega = \left\{\frac{u_i}{u_j} \in \left[\frac{1}{9}, 9\right], i, j = 1,2, \dots, r, i \neq j\right\}$ and secondary models (6), (7), (A1), and (A3) to obtain the unique favorable weights for attributes that reflect the best "will" of each country/region.

Fourthly, model (12) was used to determine the best compromise weights for these attributes as shown in Table 6, in which the weight vectors of model (5) and the four secondary models (6), (7), (A1), and (A3) are listed in detail.

Table 6. Weights for Attributes of Countries/Regions

Disciplines	Weights (models 5,6,12)	Weights (models 5,7,12)	Weights (models 5, A1,12)	Weights (models 5, A3,12)
Biology & Biochemistry	0.1837	0.1837	0.1837	0.1837
Clinical Medicine	0.0204	0.0204	0.0204	0.0204
Immunology	0.0204	0.0204	0.0204	0.0204

Table 6. ***Cont.***

Disciplines	Weights (models 5,6,12)	Weights (models 5,7,12)	Weights (models 5, A1,12)	Weights (models 5, A3,12)
Microbiology	0.1837	0.1837	0.1837	0.1837
Molecular Biology & Genetics	0.0204	0.0204	0.0204	0.0204
Neuroscience & Behavior	0.0204	0.0204	0.0204	0.0204
Pharmacology & Toxicology	0.1837	0.1837	0.1837	0.1837
Plant & Animal Science	0.1837	0.1837	0.1837	0.1837
Psychiatry/Psychology	0.1837	0.1837	0.1837	0.1837

Table 6 shows that the compromise weights for attributes by the four schemes are the same. This fact demonstrates that the results are rather robust. Therefore, using these weights, the utilities of the 24 countries/regions can be calculated as shown in Table 7. USA is ranked first. Japan, Germany, PRC, England, and France are ranked second to sixth. If equal weights are used in the comparison, then USA, Germany, Japan, England, PRC, and France are ranked first to sixth.

Table 7. Utilities of 24 Countries/Regions and Their Ranks

Countries/Regions	Compromise Weights		Equal Weights	
	Utility	Rank	Utility	Rank
USA	1.0000	1	1.0000	1
JAPAN	0.5899	2	0.5820	3
GERMANY	0.5893	3	0.5894	2
PEOPLES R CHINA	0.5845	4	0.5688	5
ENGLAND	0.5779	5	0.5798	4
FRANCE	0.5474	6	0.5476	6
CANADA	0.5469	7	0.5450	7
ITALY	0.5332	8	0.5367	8
SPAIN	0.5283	9	0.5225	9
AUSTRALIA	0.5072	10	0.5110	10
BRAZIL	0.4646	11	0.4499	12
NETHERLANDS	0.4338	12	0.4678	11
INDIA	0.4186	13	0.3553	16
SOUTH KOREA	0.3983	14	0.4007	13
SWITZERLAND	0.3338	15	0.3920	14
SWEDEN	0.3128	16	0.3661	15
BELGIUM	0.2734	17	0.2668	17

Table 7. *Cont.*

Countries/Regions	Compromise Weights		Equal Weights	
	Utility	Rank	Utility	Rank
SCOTLAND	0.1936	18	0.1914	18
POLAND	0.1875	19	0.1688	21
TURKEY	0.1856	20	0.1865	19
TAIWAN	0.1731	21	0.1742	20
DENMARK	0.1412	22	0.1634	22
MEXICO	0.0865	23	0.0601	24
FINLAND	0.0841	24	0.0978	23

3.3.2. Assessment of 64 Chinese Universities

In China, the MOE is the authority that manages Chinese universities, with 64 directly managed universities as its sub-affiliations. The indicators in "Science & Technology (S&T) statistics compilation in 2014", which is published by the MOE of China and Thomson Reuters were used as the data source for analyzing the performance of the S&T activities of the 64 universities.

The attributes used in this case were student per capita (SPC), paper per capita (PPC), patent per capita (PAPC), and technology transfer income per capita (TTIPC). These per capita indicators were obtained from "student", "paper", "patent", and "technology transfer income" after division by "staff". Specifically, "student" denotes the total number of Ph.D. candidates, master students, and undergraduates in a statistical year. "paper" refers to the number of publications in important international and domestic SCI/SSCI journals in a statistical year. "patent" denotes the total number of patent applications and authorized patents in a statistical year. "technology transfer income" refers to the total income from the process of technology transfer in a university in a statistical year. In addition, "staff" refers to the number of employees registered in the statistical year in the universities engaged in teaching, research and development, application of research and development results, and scientific and technological services, as well as those employees supporting these activities.

As the values of attributes differ greatly, the data were normalized and transformed by the value function using formulas (1)–(3) to avoid distortion of the results. The standard decision matrix after normalization and transformation by the value function is shown in Table B2.

Furthermore, it was assumed that there is no prior preference information from DMs, and the preliminary weights were set equally for attributes as follows:

Table 8. Preliminary Weights for Four Attributes

Attributes	SPC (Number/Staff)	PPC (Number/Staff)	PAPC (Number/Staff)	TTIPC (RMB in Thousands/Staff)
Preliminary weights	0.25	0.25	0.25	0.25

The DEA-WEI model (5) with $\Omega = \left\{ \left(u_i/u_j\right)_{i \neq j} \in \left[\frac{1}{9}, 9\right], i, j = 1,2, \dots, s \right\}$ was used to obtain the weight vector for each DMU. Furthermore, different secondary models[5] (models 6, 7, A1, and A3) were used to avoid alternative solutions in model (5). Model (12) was used to obtain the final compromise weights. The results are shown in Table 9.

Table 9. Attribute Weights

Attributes	SPC (Number/Staff)	PPC (Number/Staff)	PAPC (Number/Staff)	TTIPC (RMB in Thousands/Staff)
Compromised weights (models 5,6,12)	0.1045	0.4015	0.3340	0.1600
Compromised weights (models 5,7,12)	0.1793	0.4418	0.1585	0.2204
Compromised weights (models 5, A1,12)	0.1793	0.4418	0.1585	0.2204
Compromised weights (models 5, A3,12)	0.1623	0.4326	0.1985	0.2066

In late 2015, the Chinese central government issued the "Notice of the State Council on Issuing the Overall Plan for Co-ordinately Advancing the Construction of World First-class Universities and First-class Disciplines", where it was announced that China would focus on the construction of a group of world first-class universities. Following this important policy, the importance of scientific publications should be emphasized in the sample universities of this case study, which are representatives of Chinese high-level universities.

Under these circumstances, the DMs in MOE are assumed to have a new preference, namely, to require that the ratio of SPC to PPC and the ratio of PPC to TTIPC should satisfy $w_1 \geq w_2 \geq w_4$. This implies that the staff of the 64 Chinese universities should pay more attention to student training and scientific publications than technology transfer to achieve the goal "world first-class universities".

Hence, the MOE's prior preference information $w_1 \geq w_2 \geq w_4$ was incorporated into model (12) to obtain the final compromise weights, as shown as in the following model:

$$\min \tau \quad \text{s.t.} \begin{cases} \omega_r(w_r - u_r^{1*}) \leq \tau, r = 1,2, \dots, s \\ \omega_r(w_r - u_r^{1*}) \geq -\tau, r = 1,2, \dots, s \\ \omega_r(w_r - u_r^{2*}) \leq \tau, r = 1,2, \dots, s \\ \omega_r(w_r - u_r^{2*}) \geq -\tau, r = 1,2, \dots, s \\ \vdots \\ \omega_r(w_r - u_r^{n*}) \leq \tau, r = 1,2, \dots, s \\ \omega_r(w_r - u_r^{n*}) \geq -\tau, r = 1,2, \dots, s \\ \sum_{r=1}^{s} w_r = 1 \\ w_1 \geq w_2 \geq w_4 \\ w_r \in \psi, w_r \geq 0, r = 1,2, \dots, s \end{cases} \tag{13}$$

5 The properties of different secondary models are shown in Section 2.2.2.

The results are shown in Table 10. Moreover, the approach in Yang et al. (2017) incorporating $w_1 \geq w_2 \geq w_4$ was used to compare the results (see row 6 in Table 10). The comparison clearly demonstrates the robustness of the proposed approach and that the weight of the first indicator (SPC) was double counted and may violate the claimed fairness principle in the approach in Yang et al. (2017).

Table 10. Attribute Weights

Attributes	SPC (Number/Staff)	PPC (Number/Staff)	PAPC (Number/Staff)	TTIPC (RMB in Thousands/Staff)
Compromise weights (models 5,6,13)	0.3351	0.2316	0.2096	0.2237
Compromise weights (models 5,7,13)	0.3900	0.2865	0.0448	0.2787
Compromise weights (models 5, A1,13)	0.3900	0.2865	0.0448	0.2787
Compromise weights (models 5, A3,13)	0.3775	0.2740	0.0824	0.2661
Weights using the approach in Yang *et al.* (2017)	0.4990	0.2356	0.1000	0.1654

In this case, it was further assumed that the universities were cooperative, and the maximum deviation model (A1) was selected as the secondary model to derive a set of multipliers that assigns the maximum possible score to the worst-utility DMU. Therefore, the final results can be obtained using the compromise weights generated from models (5), (A1), and (13), as shown by the blue line in Figure 3. DMU_{31} is one of the most famous universities in China and is ranked first. Although DMU_1 is also one of the most well-known universities in China, the performance of its four attributes was relatively poor because it had an overly large number of employees registered for research related activities in the statistical year. It is interesting to see that DMU_6 is ranked second in this assessment. This is due to the fact that this university performed excellently on at least such attributes as SPC and PPC.

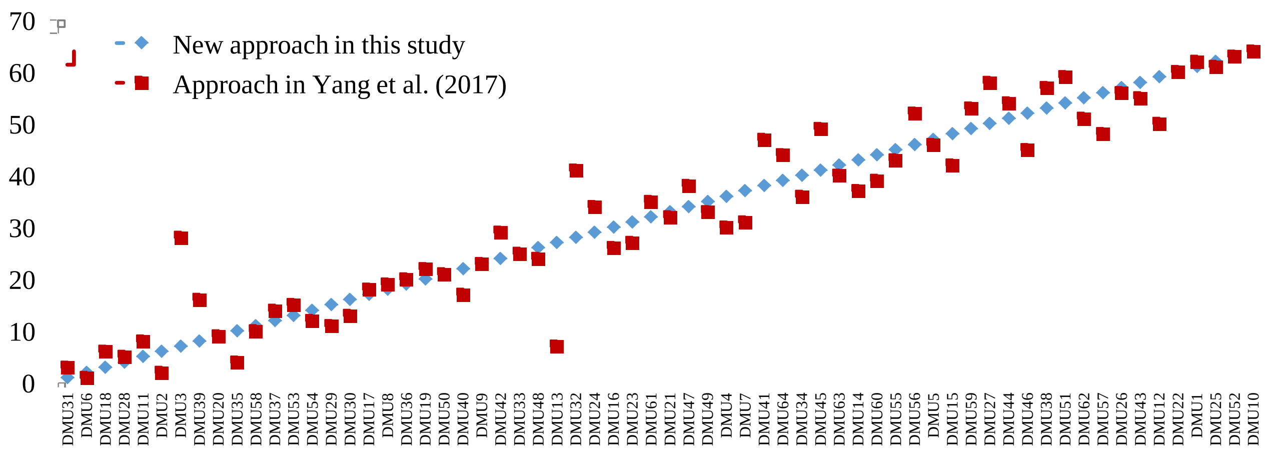

Figure 3. Comparison of Ranks in This Study and the Approach in Yang et al. (2017)

Moreover, the results in this study were compared with the results in Yang et al. (2017) by incorporating the MOE's prior preference information $w_1 \geq w_2 \geq w_4$ into the compromise stage. See model (14) in Yang et al. (2017). Figure 3 shows the comparison of different ranks in this study and the approach in Yang et al. (2017). It can be seen that the double counted weight on SPC causes the rank changes for a number of universities. For example, DMU_3 performs relatively worse on SPC (0.1351), which leads to low ranking if the approach in Yang et al. (2017) is used, which assigns double counted weight on this attribute (0.4990). The ranking changes of other DMUs can be similarly analyzed.

4. Discussion

A hybrid DEA-WEI and two-layer minimax optimization approach was proposed to determine attribute weights for MADM that overcomes the insufficiencies related to double counts and indirect reflection of DMU's "will" in Yang et al. (2017). It comprises multiple main stages. First, each DMU proposes its own most favorable weight vector based on its DEA-WEI model. Second, different secondary models suitable for different cases can be explored to select the optimal attribute weights. Third, a two-layer minimax optimization formulation was proposed to obtain the best compromise solution for all DMUs and determine the final weights for the attributes.

The simulation experiments demonstrated that the weight of an attribute with greater expected value was larger than those of other attributes with lower expected values, and the secondary models had significant impact on the weight assignment. Even though the proposed approach was developed to support weight assignment in situations where the preferences of DMs are either unclear or difficult to acquire, if there are preliminary weights available, the weights can be set as inputs to the new two-layer minimax model so that the variations of alternative values on attributes can be taken into account for more effective alternative ranking.

In addition, two case studies were conducted to illustrate the use of the proposed approach for determining weights. In these case studies, it was demonstrated that the proposed approach provides a fair and flexible method for weight generation, whereby the DM's preferences on weights and different behaviors can be taken into consideration in the process. Furthermore, the analysis demonstrated that different secondary models can lead to different most favorable weight vectors for each DMU and thus different compromise weight vectors, which can lead to different rankings. Consequently, the behavioral implications of these secondary models should be properly understood before a final compromise weight vector is decided. Nevertheless, analyzing the variations of weight and ranking by exploring different behaviors and preferences in MADM should be beneficial.

Acknowledgments: We would like to acknowledge the support from National Natural Science Foundation of China (NSFC No. 71671181) and China Scholarship Council (CSC No. 201304910099). The third author is grateful for the support from the US Air Force Office of Scientific Research under Grant No. FA2386-15-1-5004.

Conflicts of Interest: The authors declare no conflict of interest.

Appendix A

Herein, the following alternative secondary models are presented.

(a) Minimizing maximum deviation (maximum deviation model)

In this approach, the maximal deviation $\propto_j^{'}$ is minimized. The main concept of this approach is to derive a set of multipliers that assigns the maximum possible score to the worst-utility DMU, that is, to improve the worst-utility

DMU as much as possible; the utility of other DMUs may decrease to reduce variations. The secondary goal in this approach can be expressed as follows:

$$\min_{u_r,\alpha_j'} \delta \quad \text{s.t.} \begin{cases} \sum_{r=1}^{s} u_r y_{r0} = \theta_0^* \\ \sum_{r=1}^{s} u_r y_{rj} + \alpha_j' = 1, j = 1,2,\ldots,n \\ \delta - \alpha_j' \geq 0, j = 1,2,\ldots,n \\ \alpha_j' \geq 0, u_r \in \Omega, r = 1,2,\ldots,s; r = 1,2,\ldots,s \end{cases} \tag{A1}$$

This approach may be suitable in the case where DMUs are assumed to be cooperative when they set their individual best weights. An example would be the output evaluation of universities with one single headquarter.

(b) Minimizing the mean absolute deviation (absolute deviation model).

In this approach, the following model is proposed to minimize the mean absolute deviation of DMUs, which attempts to decrease the utility difference among DMUs. Its objective function shows that in this approach, it is attempted to make all DMUs as close as possible to have equal utilities.

$$\min_{u_r,\alpha_j'} \frac{1}{n}\sum_{j=1}^{n}\left|\alpha_j' - \overline{\alpha'}\right| \quad \text{s.t.} \begin{cases} \sum_{r=1}^{s} u_r y_{r0} = \theta_0^* \\ \sum_{r=1}^{s} u_r y_{rj} + \alpha_j' = 1, j = 1,2,\ldots,n \\ \alpha_j' \geq 0, u_r \in \Omega, r = 1,2,\ldots,s; r = 1,2,\ldots,s \end{cases} \tag{A2}$$

where $\overline{\alpha'} = \frac{1}{n}\sum_{j=1}^{n}\alpha_j'$.

By letting $a_j' = \frac{1}{2}\left(\left|\alpha_j' - \overline{\alpha'}\right| + \alpha_j' - \overline{\alpha'}\right)$ and $b_j' = \frac{1}{2}\left(\left|\alpha_j' - \overline{\alpha'}\right| - \left(\alpha_j' - \overline{\alpha'}\right)\right)$, model (A2) can be transformed into the following LP problem:

$$\min_{u_r,\alpha_j'} \frac{1}{n}\sum_{j=1}^{n}\left(a_j' + b_j'\right) \quad \text{s.t.} \begin{cases} \sum_{r=1}^{s} u_r y_{r0} = \theta_0^* \\ \sum_{r=1}^{s} u_r y_{rj} + \alpha_j' = 1, j = 1,2,\ldots,n \\ a_j' - b_j' = \alpha_j' - \frac{1}{n}\sum_{j=1}^{n}\alpha_j', j = 1,2,\ldots,n \\ \alpha_j' \geq 0, a_j' \geq 0, b_j' \geq 0, u_r \in \Omega, r = 1,2,\ldots,s \end{cases} \tag{A3}$$

In the case that an equalitarian principle should be demonstrated, this approach is more suitable than others because it attempts to decrease the utility difference among DMUs.

Appendix B

Table B1. Data on Publications in Nine ESI Disciplines for 24 Selected Countries/Regions

Countries/Regions	BB	CM	Immu.	Mic.	MG	NB	PT	PA	PP
USA	201 787	827 183	57 666	60 652	136 001	136 124	59 649	160 253	139 283
BRAZIL	14 469	55 009	3191	5993	6398	8204	6985	35 977	3025
PEOPLES R CHINA	48 021	116 413	6942	14 230	24 306	15 397	21 188	41 868	5275

Table B1. ***Cont.***

Countries/Regions	BB	CM	Immu.	Mic.	MG	NB	PT	PA	PP
SPAIN	18 869	72 317	4120	8855	10 597	10 838	6250	27 505	9408
INDIA	19 547	39 280	2392	8085	5792	3766	11 850	22 799	970
JAPAN	57 414	172 683	9469	14 423	26 148	24 325	21 121	39 749	5122
GERMANY	46 185	199 385	10 904	16 395	30 450	32 342	14043	40 426	21 723
FRANCE	33 245	128 820	8383	12 709	19 863	17 518	8446	28 660	7613
ITALY	25 252	128 345	6938	6228	14 881	20 082	11373	22 209	7369
CANADA	28 305	114 577	6273	7667	17 431	21 518	7085	34 080	21 349
AUSTRALIA	17 048	85 931	5318	5662	9369	10 205	4836	29 526	15 289
ENGLAND	40 063	194 089	11 471	12 397	27803	27 504	12 608	32 329	30 719
SOUTH KOREA	18 877	61 027	2766	7795	7472	6088	9999	11 213	1972
TURKEY	6523	64 128	875	1926	2132	3989	3014	13 548	2655
NETHERLANDS	12 720	87 678	5369	5397	10 163	12 228	4824	13 159	14 254
BELGIUM	7946	39 865	2401	3529	4664	4698	3258	10 590	4866
MEXICO	4391	10 170	1035	2141	1499	2131	1606	12 152	1286
POLAND	10 154	23 318	1733	1740	3490	3276	3457	14 368	1157
DENMARK	8534	31 047	2209	2343	4076	3732	2221	7888	2087
TAIWAN	8254	39 581	1685	2209	3606	3132	4101	5397	2660
SWITZERLAND	11 049	52 480	4082	3853	8031	8075	3825	10 073	4978
SWEDEN	13 208	52 942	4236	3287	6691	7382	3504	9992	4261
FINLAND	5263	22 945	1313	1697	3160	3383	1680	6110	2676
SCOTLAND	7278	26 342	1706	3255	5205	3664	1728	10 173	3767

Table B2. Data of 64 Chinese Universities after Normalization and Transformation Using Value Function (3)

Universities	Ratio Indicators (Attributes)			
	SPC (Number/Staff)	PPC (Number/Staff)	PAPC (Number/Staff)	TTIPC (RMB in Thousands/Staff)
DMU_1	0.0177	0.1263	0.1037	0.5013
DMU_2	0.7380	1.0000	0.1072	0.0000
DMU_3	0.1351	0.7187	0.5758	1.0000
DMU_4	0.5148	0.5646	0.5443	0.0486
DMU_5	0.3224	0.3627	0.5209	0.2677
DMU_6	0.5346	0.7768	0.6627	0.5957
DMU_7	0.5137	0.5371	0.5542	0.0529

Table B2. ***Cont.***

Universities	Ratio Indicators (Attributes)			
	SPC (Number/Staff)	PPC (Number/Staff)	PAPC (Number/Staff)	TTIPC (RMB in Thousands/Staff)
DMU_8	0.4489	0.6232	0.5367	0.5002
DMU_9	0.5081	0.5045	0.5073	0.4123
DMU_{10}	0.0627	0.1044	0.0128	0.0000
DMU_{11}	0.5110	0.7923	0.2780	0.5119
DMU_{12}	0.5567	0.0358	0.1198	0.0000
DMU_{13}	1.0000	0.2377	0.0000	0.0000
DMU_{14}	0.5200	0.3106	0.6136	0.1467
DMU_{15}	0.5182	0.3576	0.5136	0.0000
DMU_{16}	0.5088	0.5797	0.5259	0.1384
DMU_{17}	0.5105	0.6411	0.1688	0.5133
DMU_{18}	0.5112	0.7080	0.3514	0.6242
DMU_{19}	0.3969	0.6179	0.5998	0.5384
DMU_{20}	0.5084	0.6894	0.5328	0.5096
DMU_{21}	0.5147	0.4658	0.4656	0.1879
DMU_{22}	0.1837	0.3673	0.3081	0.0474
DMU_{23}	0.5580	0.6757	0.2788	0.0000
DMU_{24}	0.4888	0.2585	0.3385	0.5732
DMU_{25}	0.0285	0.5202	0.1678	0.0514
DMU_{26}	0.1563	0.5104	0.4409	0.1065
DMU_{27}	0.0133	0.5202	0.3073	0.5388
DMU_{28}	0.4935	0.6732	0.5272	0.6267
DMU_{29}	0.5149	0.5493	0.7783	0.5173
DMU_{30}	0.5255	0.7201	0.5060	0.3615
DMU_{31}	0.4892	0.9766	0.5281	0.5288
DMU_{32}	0.2062	0.5462	0.5878	0.6865
DMU_{33}	0.5101	0.3118	0.6418	0.5355
DMU_{34}	0.4976	0.3916	0.5884	0.1212
DMU_{35}	0.5119	0.5812	1.0000	0.5095
DMU_{36}	0.4809	0.5433	0.5007	0.5340
DMU_{37}	0.5093	0.6012	0.5048	0.5487
DMU_{38}	0.0210	0.4481	0.5028	0.5100

Table B2. ***Cont.***

Universities	Ratio Indicators (Attributes)			
	SPC (Number/Staff)	PPC (Number/Staff)	PAPC (Number/Staff)	TTIPC (RMB in Thousands/Staff)
DMU_{39}	0.5152	0.2267	0.4946	0.9825
DMU_{40}	0.5182	0.7386	0.5383	0.1894
DMU_{41}	0.2014	0.4911	0.3612	0.5145
DMU_{42}	0.4405	0.5257	0.2583	0.5105
DMU_{43}	0.2680	0.1058	0.5215	0.3097
DMU_{44}	0.1675	0.3072	0.2776	0.5021
DMU_{45}	0.1883	0.4903	0.3250	0.4874
DMU_{46}	0.4625	0.3878	0.2266	0.0000
DMU_{47}	0.5072	0.1452	0.4345	0.5241
DMU_{48}	0.5165	0.5644	0.4918	0.2709
DMU_{49}	0.5548	0.5788	0.1675	0.0460
DMU_{50}	0.5128	0.5595	0.5020	0.3882
DMU_{51}	0.1369	0.3251	0.1256	0.5081
DMU_{52}	0.0364	0.2657	0.0925	0.0126
DMU_{53}	0.4658	0.5918	0.6540	0.5913
DMU_{54}	0.5266	0.5895	0.5748	0.5159
DMU_{55}	0.5091	0.3846	0.3797	0.0627
DMU_{56}	0.1305	0.4131	0.2876	0.5566
DMU_{57}	0.4311	0.2664	0.5041	0.0138
DMU_{58}	0.5290	0.6153	0.5452	0.5228
DMU_{59}	0.0844	0.4668	0.3550	0.5174
DMU_{60}	0.4804	0.4886	0.5310	0.0049
DMU_{61}	0.5095	0.1081	0.6052	0.5412
DMU_{62}	0.2954	0.4019	0.2136	0.1334
DMU_{63}	0.5229	0.4382	0.3024	0.0747
DMU_{64}	0.4131	0.5834	0.1640	0.1484

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