APPLICATION OF FUZZY LOGIC APPROACHES TO SAFETY ASSESSMENT IN MARITIME ENGINEERING APPLICATIONS

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ABSTRACT

Safety assessment based on conventional methods such as probability risk assessment (PRA) may not be well suited for dealing with innovative systems having a high level of uncertainty, particularly in the feasibility and concept design stages of a maritime engineering system. By contrast, safety models using fuzzy logic approaches employing fuzzy IF-THEN rules can model the qualitative aspects of human knowledge and reasoning processes without employing precise quantitative analyses. Fuzzy-logic-based approaches may be more appropriately used to carry out risk analysis in the initial design stages of large maritime engineering systems. This provides a tool for working directly with the linguistic terms commonly used in carrying out safety assessment. This paper focuses on the development and representation of linguistic variables to model risk levels subjectively. These variables are then quantified using fuzzy set theory. In this paper, the development of two safety evaluation frameworks using fuzzy logic approaches for maritime engineering safety based decision support in the concept design stage are presented. An example is used to illustrate and compare the proposed approaches. Future risk analysis in maritime engineering applications may take full advantages of fuzzy logic approaches to compliment existing ones.

1. Introduction

Both the report of the Cullen enquiry into the Piper Alpha disaster [Department of Energy, 1990] and the Carver report [House of Lord, 1992] on ship safety have led to a change of policy in maritime safety, replacing prescriptive rules with a more goal-setting regime. Both reports have recommended that safety should be incorporated into the design process from the initial stages. More scientific and rational approaches are required to be developed in order to control major marine accidents. In the UK maritime sector there has been a major change in philosophy in recent years, which has opened up ways for innovative thinking. “The industry guidelines on a framework for risk related decision support” produced by the UK Offshore Operators Association (UKOOA) and the UK Health & Safety Executive (HSE) in 1999 [UKOOA, 1999], provides a sound basis for evaluating the various options in the initial design stages. Based on the guidelines, a qualitative safety model using Mamdani’s fuzzy-logic inference system and an adaptive-fuzzy-logic safety model are proposed for risk analysis in this paper. The proposed frameworks could be useful for a wide range of applications under various

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conditions. They provide a foundation for evaluating various options that need to be considered at the feasibility and concept selection stages of a project, especially with respect to risks associated with major accident hazards such as fire, explosion, impact and loss of stability. They can be incorporated with other existing formal decision making aids such as Multi-Attributes Utility Analysis (MAUA), Analytical Hierarchy Process (AHP) and decision trees if a more detailed or quantitative analysis of the various decision alternatives is desired [UKOOA, 1999]. It should be emphasised that there could be significant uncertainties associated with the information and factors that are used in the decision making process. It is essential to apply common sense engineering judgements to ensure that all significant uncertainties are recognised and addressed.

In recent years, many quantitative safety assessment techniques have been developed and applied by both safety analysts and design engineers. However, there are still some limitations for these techniques to be widely and effectively applied to provide useful solutions to safety based decision making, especially in the concept design stage. This may be due to the following problems:

- Under prevailing circumstances, in particular for systems with a high level of innovation, only limited data is available on system failures for which the statistical accuracy is often poor or only limited operational experience exists. It may be difficult to obtain failure information encompassing the effects of human factors in system reliability with confidence. This is particularly true for maritime engineering systems.
- It is extremely difficult to generate a mathematical model to represent and describe the safety behaviour/discipline of a maritime engineering system. The safety of a system is affected by various factors such as design, manufacturing, installation, commissioning, operations and maintenance.
- A complete quantitative safety assessment involves a significant amount of analytical work and is therefore very costly.
- It is extremely difficult to quantify the effects and consequences of hazards as they involve too many factors with a high level of uncertainty, even in those cases where the physical processes are clearly understood.
- A large number of assumptions, judgements and opinions are involved subjectively in risk quantification process. Therefore, it may require considerable skill for a safety analyst to interpret the results produced.
- Since safety is only one of the important entities involved in the appraisal of the acceptability of an activity, it is often difficult to set up an absolute safety criterion for acceptance standard.

In view of the difficulties as outlined and discussed, it is necessary to develop novel methods in order to ensure that the safety can be properly estimated and integrated into design process at the early stages.

2. Background of Fuzzy Logic Theory

Fuzzy logic systems are knowledge-based or rule-based ones constructed from human knowledge in the form of fuzzy *IF-THEN* rules [Wang L. X., 1997]. An important contribution of fuzzy system theory is that it provides a systematic procedure for transforming a knowledge base into a non-linear mapping. A fuzzy *IF-THEN* rule is an *IF-THEN* statement in which some words are characterised by continuous membership functions. For example, the following is a fuzzy *IF-THEN* rule: *IF the* ***failure rate*** of a hazard is *frequent AND* ***consequence severity*** is *catastrophic AND* ***failure consequence probability*** is *likely, THEN* ***safety estimate*** is *poor.* Linguistic variables *frequent, catastrophic, likely* and *poor* are characterised by the membership functions. A fuzzy system is constructed from a collection of fuzzy *IF-THEN* rules.

The starting point of constructing a fuzzy logic system is to obtain a collection of fuzzy *IF-THEN* rules from human experts or based on the domain knowledge. The next step is to combine these rules into a single system. Different fuzzy systems use different principles for this combination. A fuzzy logic system consists of four components (fuzzy rule base, fuzzy inference engine, fuzzifier and defuzzifier) as shown in Figure 1. The detailed mathematical explanations and illustrations for each component of a fuzzy logic system are out of the scope of this paper (refer to standard fuzzy system texts such as [Klir & Yuan, 1995] for more details).

Consider a fuzzy logic system where $U = U_1 \times U_2 \times \ \ldots \times U_n \subset R^n$ is the input space and $V \subset R$ is the output space. Only the multi-input-single output case is considered here as a multi-output system can always be decomposed into a collection of single-output systems.

A fuzzy knowledge/rule base consists of a set of fuzzy *IF-THEN* rules. Let M be the number of rules in the fuzzy rule base ($l = 1, 2, \ldots, M$ in equation (1)). Specifically, the fuzzy knowledge/rule base comprises the following fuzzy *IF-THEN* rules:

$$Rule^{(l)}: IF\ x_1 \quad is \quad A_1^l \quad and \quad \ldots and \quad x_n \quad is \quad A_n^l, \quad THEN \quad y \quad is \quad B^l \tag{1}$$

where A_i^l ($i = 1, \ldots, n$) and B^l are fuzzy sets in $U_i \subset R$ and $V \subset R$, respectively, and $x = (x_1, x_2, \ldots, x_n)^T \in U$ and $y \in V$ are the input and output (linguistic) variables of the fuzzy system, respectively.

In a fuzzy inference engine, fuzzy logic principles are used to combine the fuzzy *IF-THEN* rules in the fuzzy rule base into a mapping from a fuzzy set *A'* in *U* to a fuzzy set *B'* in *V*. A fuzzy *IF-THEN* rule is interpreted as a fuzzy relation in the input-output product space $U \times V$, and there are a number of implications that specify the fuzzy relation. If the fuzzy rule base consists of a single rule only, the generalised modus ponens (GMP) (equation (2)) specifies the mapping from fuzzy set *A'* in *U* to fuzzy set *B'* in *V*. Given fuzzy set *A'* and fuzzy relation $A \rightarrow B$ in $U \times V$, a fuzzy set *B'* in *V* is inferred as [Wang L. X., 1997]:

$$\mu_{B'}(y) = \sup_{x \in U} \quad t\left[\mu_{A'}(x), \mu_{A \rightarrow B}(x, y)\right] \tag{2}$$

The *sup* represents the compositional rule of inference and is also called the *sup-star composition* [Wang L. X., 1997]. Since any practical fuzzy rule base constitutes more than one rule, the key question here is how to infer with a set of rules. The Mamdani Min Implication operator is used in this study [Mamdani & Assilian, 1975] [Wang L. X., 1997]. Specifically, the fuzzy *IF-THEN* rule *IF* $<FP_1>$ *THEN* $<FP_2>$ is interpreted as a fuzzy relation Q_{MM} in $U \times V$ with the membership function

$$\mu_{Q_{MM}}(x, y) = \min\left[\mu_{FP_1}(x), \mu_{FP_2}(y)\right] \tag{3}$$

Let *A'* be an arbitrary fuzzy set in *U* and be the input to the fuzzy inference engine. Then, by viewing Q_{MM} as a single fuzzy *IF-THEN* rule and using equation (2), the output of the fuzzy inference engine is obtained as:

$$\mu_{B'}(y) = \sup_{x \in U} \quad t\left[\mu_{A'}(x), \mu_{Q_{MM}}(x, y)\right] \tag{4}$$

Minimum Inference Engine is used to perform implication and aggregation processes across the rules [Wang L. X., 1997]:

$$\mu_{B'}(y) = \max_{l=1}^{M}\left[\sup_{x \in U} \min\left(\mu_{A'}(x), \mu_{A_1^l}(x_1), \ldots, \mu_{A_n^l}(x_n), \mu_{B^l}(y)\right)\right] \tag{5}$$

The defuzzifier is defined as a mapping from fuzzy set *B'* in $V \in R$ (which is the output of the fuzzy inference engine) to crisp point $y^{\Delta} \in V$. Conceptually, the task of the defuzzifier is to satisfy a point in *V* that best represents fuzzy set *B'*. This is similar to the mean value of a random variable. However, since *B'* is constructed in some special ways as demonstrated above, we have a number of choices in

determining this representing point. However, only the centre average defuzzifier is applied in this paper. Centre average defuzzifier [Klir & Yuan, 1995] [Wang L. X., 1997] is the most commonly used defuzzifier in fuzzy systems. It is computationally simple and intuitively plausible. The centre average defuzzification method will be described later in this paper.

3. Two Fuzzy-Logic-Based Safety Modelling Approaches

This section delineates two approaches using fuzzy-logic and adaptive-fuzzy-logic for risk analysis in maritime engineering applications.

3.1 Method 1: A Fuzzy-Logic-Based Approach Using Mamdani's Inference System

It is worth noting that many typical safety assessment approaches may have some problems for use in situations where there is a lack of confidence in risk assessment [Wang & Ruxton, 1997]. A fuzzy-logic-based approach may provide a solution as it emulates the reasoning process for synthesising human expert judgements within a specific domain of knowledge, codes and standards based on the guidelines and company policy. A fuzzy-logic-based approach can make use of the experience of experts to construct fuzzy rules. The proposed safety assessment approach for risk analysis consists of three sub-models. Each safety sub-model assesses one particular category or module of risk (i.e. personnel related risk, environment related risk or organisation/business related risk). Then the overall risk level of the system is evaluated by using a mathematical model. The framework of the safety model is shown in Figure 2. Personnel safety sub-model – assesses the personnel related risks that are harmful to a person or group of persons. Environment safety sub-model – assesses the environment related risks. Organisation/business safety sub-model – assesses organisation/business related risk (i.e. financial risk due to potential hazards causing loss of production, damage of the reputation of the organisation or other financial consequences).

Each safety sub-model uses fuzzy logic approach to perform the risk level evaluation. The evaluations obtained from each safety sub-model are fuzzy outputs. Each module of risk analysis yields a linguistic output such as *low risk*, *possible risk*, *substantial risk* or *high risk*. For example, the risk evaluation for personnel related, environment related and organisation/business related categories may be *high risk*, *low risk* and *possible risk*, respectively. After each of the modular risks is evaluated, the overall risk level as a function of the personnel related, environment related and organisation/business related characteristics is determined. The values for overall risk levels are also determined to be one of the four linguistic descriptions. The output is obtained by considering the linguistic estimates for the three categories/modules of risk. The output is a linguistic estimate that represents the risk level associated with the given module/category. The output estimate may be one or a combination of *low risk*, *possible risk*, *substantial risk* and *high risk*.

The numeric risk values obtained from each of the modules and the weights obtained from the AHP analysis are used to derive the final overall risk output, which is a crisp number. This crisp value represents the risk of the specific system or item at specific operation conditions. The following mathematical model is used to quantify the overall risk level:

$$RL = r_P w_p + r_E w_E + r_B w_B \qquad (6)$$

where RL = overall risk level of the system for the given operation conditions,

r_P = the risk associated with the personnel related module/category,

w_P = weighting factor for the personnel related module/category,

r_E = the risk associated with environment related module/category,

w_E = weighting factor for the environment related module/category,

r_B = the risk associated with organisation/business related module/category, and

w_B = weighting factor for the organisation/business related module/category.

Each weighting factor (w_P, w_E or w_B) represents the relative significance of the given risk factor category's contribution to the overall risk level of the system.

An overall flow of information in the formulation of the safety model is depicted in Figure 2. The general approach adopted is similar to that used in fuzzy expert and fuzzy control systems where the knowledge base contains general knowledge pertaining to the problem domain. In the fuzzy logic approach, the knowledge is usually represented by a set of fuzzy rules, which connect antecedents with consequences, premises with conclusions, or conditions with actions. Fuzzy IF-THEN rules are expressions of the form "*IF A, THEN B*", where *A* and *B* are labels of fuzzy sets characterised by appropriate membership functions [Klir & Yuan, 1995]. Through the use of linguistic variables and membership functions, a fuzzy *IF-THEN* rule can easily capture the spirit of a "rule of thumb" used by human. The input membership functions use linguistic variables to describe the parameters used in risk analysis which are developed through statistical data and information analysis, expert experience and engineering judgements, concept mapping and fuzzy modelling. These inputs are then fuzzified to determine the membership degrees in each input class. The development of rules is similar to the development of the input membership functions, which are built upon data and information gathered in knowledge acquisition. The input data (risk parameters) are fed to the fuzzy inference system and are then evaluated using the linguistic rules and fuzzy logic operations. The results are then evaluated in terms of ***risk level*** expressions. Finally, the defuzzification process creates a crisp ranking from the fuzzy conclusion set to express the ***risk levels*** of the subsystem/hazard for prioritising corrective actions and design revisions in safety assessment.

There are five steps in the fuzzy inference process:

- Step 1. Fuzzification of input variables - The first step is to transform the inputs into degrees of match with linguistic values via membership functions.
- Step 2. Application of fuzzy operations (AND or OR) in antecedents. This value will then be applied to the output function. The input to the fuzzy operator may have two or more membership values from fuzzified input variables. The output is a single truth value.
- Step 3. Implication from antecedent to consequent - The single truth value of a rule is determined by AND operator of the rule antecedents. With AND operator, rule evaluation then determines the smallest (minimum) rule antecedent, which is taken to be the truth value of the rule.
- Step 4. Aggregation of consequent across the rules - The output of each rule is combined into a single fuzzy set through the aggregation process.
- Step 5. Defuzzification - Finally, defuzzification process transforms the fuzzy results (i.e. a range of output values from the aggregation process) into a crisp output.

A typical fuzzy safety model includes the development of fuzzy membership functions for representing risk levels, fuzzy rule bases and fuzzy safety expressions. Linguistic variables are employed to develop fuzzy membership functions and they can be used to represent the condition of an attribute at a given interval.

3.1.1 The development of fuzzy linguistic variables and membership functions for representing risk levels

Fuzzy linguistic variables are extensions of numerical variables in a sense that they are able to represent the condition of an attribute at a given interval by taking fuzzy sets as their values [Wang L. X., 1997]. The values obtained in a development of fuzzy linguistic variables are considered as fuzzy measures. These values can then become the criteria for measuring attributes of objects, in this case, risk levels.

The two fundamental parameters used to assess the risk level of a maritime system on a subjective basis are ***failure likelihood*** and ***consequence severity***. Subjective assessment (using linguistic variables instead of ultimate numbers in probabilistic terms) may be more appropriate to conduct analysis on these two parameters as they are always associated with great uncertainty. Thus, these two parameters are represented by natural languages, which can be further described by membership functions. A membership function is a curve that defines how each point in the input space is mapped to a membership value (or degree of membership) between *0* and *1*. The simplest membership functions are formed using straight lines. These straight-line membership functions have the advantage of simplicity. Of these, the simplest are the triangular membership function and trapezoidal membership. The fuzzy membership functions are generated utilising the linguistic categories identified in the knowledge acquisition and they consist of a set of overlapping curves. Six levels of linguistic variables may be used for ***failure likelihood***, five levels for ***consequence severity***, and four levels for ***risk level expressions***. The literature search indicates that four to seven levels of linguistic variables are commonly used to represents risk factors in risk analysis [Bell & Badiru, 1996a] [Bowles & Pelaez, 1995] [Karwowski & Mital, 1986] [Sii, et al., 1999] [Wang, 1997].

Knowledge acquisition, development of fuzzy linguistic variables, development of membership functions and AHP analysis may be required to construct the linguistic risk levels and associated fuzzy membership functions [Klir & Yuan, 1995]. In knowledge acquisition, data collection analysis, expert and engineering judgements, fuzzy modelling and concept mapping are performed sequentially to classify the knowledge. The goal is, based on fuzzy set theory, to establish linguistic variables to develop fuzzy membership functions for representing risks. The arbitrariness and variability associated with combining information from various data and knowledge acquisition channels are the basis for utilising the fuzzy-logic-based approach in the decision making process. Since essentially, what a fuzzy logic system does is to perform this transformation and combination of information from different sources [Wang L. X., 1997].

It is possible to have some flexibility in the definition of membership functions to suit different situations. The application of categorical judgements has been quite positive in several practical situations [Wang, 1997]. It is also usually common and convenient for safety analysts to use categories to articulate safety information.

A linguistic variable may be assigned with a membership function to a set of categories with regard to the particular condition. The typical linguistic variables for ***failure likelihood***, ***consequence severity*** and ***risk levels*** of a particular system may be defined and characterised as follows:

Failure likelihood describes the failure frequencies in a certain period of time, which directly represents the numbers of failures anticipated during the design life span of a particular system or item. Table 1 describes the range of the frequencies of the failure occurrence and defines the fuzzy set of ***failure likelihood***. To estimate ***failure likelihood***, one may choose to use such linguistic variables as "*very low*", "*low*", "*reasonably low*", "*average*", "*frequent*" and "*highly frequent*". Figure 3 shows the fuzzy ***failure likelihood*** set definition.

Consequence severity describes the magnitude of possible consequences, which is ranked according to the severity of the failure effects on three categories of risk separately (i.e. personnel related risk, environment related risk and organisation/business related risk). One may choose to use such linguistic variables as "*negligible*", "*minor*", "*moderate*", "*severe*" and "*catastrophic*" to describe ***consequence severity*** [Wang, 1997]. The fuzzy ***consequence severity*** set definition is shown in Figure 4. Table 2 gives the criteria used to rank the ***consequence severity*** of failure effects.

With reference to the above fuzzy descriptions of ***failure likelihood*** and ***consequence severity***, it may be observed that the linguistic variables are not exclusive, as there are intersections among the defined linguistic variables describing ***failure likelihood*** and ***consequence severity***. Inclusive expressions may make it more convenient for the safety analysts to judge a risk level. Overlapping functions are used to represent various linguistic variables for both attributes because in the analysis of the risks associated with a factor, the risk levels may have "gray" or ill-defined boundaries [Bell & Badiru, 1996a].

3.1.2 A fuzzy rule base

Several sources can be used to derive the fuzzy rules in a rule base. The fuzzy rules may be derived based on statistical studies of the information in previous incident and accident reports or database systems. In-depth literature search may also be helpful. Skilled human analysts often have good, intuitive knowledge of the behaviour of a system and the risks involved in various types of failures without having any quantitative model in mind. Fuzzy rules provide a natural platform for abstracting information based on expert judgements and engineering knowledge since they are expressed in linguistic form rather than numerical variables. Therefore, experts often find fuzzy rules to be a convenient way to express their knowledge of a situation.

In practical applications the fuzziness of the antecedents eliminates the need for a precise match with the inputs. All the rules that have any truth in their premises will fire and contribute to the fuzzy conclusion (risk level expression). Each rule is fired to a degree to which its antecedent matches the input. This imprecise matching provides a basis for interpolation between possible input states.

Rules based on linguistic variables are more natural and expressive than numerical numbers. It is clear that such rules can accommodate quantitative data such as ***failure likelihood*** and qualitative and judgmental data such as ***consequence severity***, and combine them consistently in risk level evaluation.

3.1.3 The development of fuzzy risk level expressions

In safety assessment, it is common to express a ***risk level*** by degrees to which it belongs to such linguistic variables as "*high risk*" (poor safety), "*substantial risk*" (fair safety), "*possible risk*" (average safety) and "*low risk*" (good safety) that are referred to as ***risk level*** expressions. The output set can be defined using fuzzy ***risk level*** expression sets in the same way as the fuzzy inputs. Figure 5 shows the fuzzy ***risk level*** expression sets.

3.2 Method 2: An Adaptive-Fuzzy-Logic Approach

Figure 6 shows a proposed adaptive-fuzzy-logic-based approach in safety modelling with a learning procedure. It is a fuzzy inference system or fuzzy model implemented in the framework of adaptive networks. The fuzzy safety model can be trained using numerical data pairs of a target system supplied by experts via a table-look-up scheme. The adaptive-fuzzy-logic-based approach in risk analysis described in Figure 6 has a learning ability of an artificial neural network. By utilising a one-pass learning operation on training data pairs, the proposed approach can refine fuzzy IF-THEN rules obtained from human experts and numerical data in the fuzzy model to perform risk level estimation of a system. However, if human expertise is not available, membership functions of fuzzy rules can still be designed intuitively based on various knowledge acquisition methods to perform risk analysis.

The general method to generate fuzzy rules from numerical data has been developed and described in detail in [Sii, 2000]. The rules created from both numerical data and human experts can be combined into a common fuzzy rule base. The final system safety assessment is then performed based on this combined rule base.

3.2.1 A generic framework of generating fuzzy rules from numerical data (input-output pairs)

Suppose a set of desired numerical data in the form of input-output pairs is given as:

$$\left(x_1^{(1)}, x_2^{(1)}; y^{(1)}\right), \left(x_1^{(2)}, x_2^{(2)}; y^{(2)}\right), \ldots \quad (7)$$

where x_1 and x_2 are inputs, and y is the output. This simple two input and one output case is chosen here to emphasise and to clarify the basic ideas of this approach. The extensions of this two-inputs-

one-output to general multiple-input-multiple-output cases are straightforward, however, it will not be covered in this paper. The objective here is to generate a set of fuzzy IF-THEN rules from the desired input-output pairs of equation (7), and use these fuzzy IF-THEN rules to determine a fuzzy logic system $f:(x_1, x_2) \rightarrow y$.

Step 1: Divide the input and output spaces into fuzzy regions and assign each region with a fuzzy membership function

Assume that the domain intervals of input variables x_1, x_2, and output variable y are $[x_1^-, x_1^+]$, $[x_2^-, x_2^+]$ and $[y^-, y^+]$, respectively. The domain interval of a variable means that most probably this variable will lie in this interval, but the values of a variable are allowed to lie outside its domain interval. Each domain interval is divided into several fuzzy regions. The lengths of these regions can be equal or unequal. Each region is then assigned with a notation (for example, *F1* stands for *very low* ***failure likelihood***) and its associated membership function. In this paper, the domain interval of x_1 (i.e. ***FL = failure likelihood***) is divided into six regions, the domain region of x_2 (i.e. ***CS = consequence severity***) is divided into five regions, and the domain interval of y (i.e. ***RL = risk level***) is divided into four regions. The shape of each membership function is trapezium with difference sizes (refer to Figures 3, 4, and 5 for details). Other divisions of the domain regions and other shapes of membership functions are also possible to fit a particular situation.

Step 2: Construct fuzzy rules from given numerical data

Determine the degrees of given numerical data in the form of input-output data pairs as shown in equation (7) in different regions. Then assign each of the given input-output pair to the region with maximum degree. Finally, only one fuzzy rule is constructed out of one pair of input-output data.

Step 3: Assign a degree to each rule

Since there are many input-output pairs in numerical data, there is a high probability that there will be some conflicting rules, that is, rules which have the same IF part but have different THEN part. One way to resolve this conflict is to assign a degree to each rule generated from numerical data (input-output pairs) and accept only the rule from a conflict group that has the maximum degree. In this way, not only the conflicting problem is resolved, but also the number of rules is greatly reduced.

The following product strategy is used to assign a degree to each rule: for *Rule # i* "IF x_1 is *A* AND x_2 is *B* THEN *y* is *C*". The degree of this rule, denoted by $D_{(Rule\ \#\ i)}$, is defined as:

$$D_{(Rule\ \#\ i)} = \mu_A(x_1)\ \mu_B(x_2)\ \mu_C(y) \tag{8}$$

where $D_{(Rule\ \#\ i)}$ stands for the degree of the rule, which represents the significance of this particular rule's contribution towards the final evaluation in safety modelling. It is duly dependent on the membership functions of its antecedents (inputs) and consequent (output). $\mu_A(x_1)$, $\mu_B(x_2)$ and $\mu_C(y)$ stand for membership degrees for antecedents *A* and *B* and for consequent *C*, respectively.

In practice, there may often be priori information about numerical data (input-output pairs). For example, if an expert is asked to check a series of data pairs, he may suggest that some of the data pairs are very useful and crucial, but others are unlikely relevant (this may be caused by statistical errors). Therefore, a reliable-level value, $\mu^{(i)}$ between *0* to *1.0* can be assigned to each data pair according to the expert's belief of its contribution to the overall system performance.

Suppose the data pair used to generate *Rule # i* has a reliable-level value of $\mu^{(i)}$. Then the weight of the rule is defined as:

$$W_{(Rule\ \#\ i)} = \mu_A(x_1)\ \mu_B(x_2)\ \mu_C(y)\ \mu^{(i)} \tag{9}$$

In this context, the weight of a rule is defined as the product of the degrees of its components and the reliable-level value $\mu^{(i)}$ assigned to the data pair. This is particularly important in practical applications as real numerical data always have different reliabilities. For more reliable data pairs, higher reliable-

level values are assigned, and lower reliable-level values are given to data pairs with relatively lower reliabilities. In this manner, human experience and judgement about the numerical data (input-output pairs) can be used on a common base. If objectivity is emphasised and human judgement on the numerical data is not desired, the above strategy still works by setting all the reliable-level values of the data pairs equal to unity.

Step 4: Construct a combined fuzzy rule base

A table-look-up representation of a fuzzy rule base is created, which is similar to the one commonly used in control engineering [Altrock, 1995]. In Table 3, the boxes of the base will be filled with fuzzy rules according to the following strategy:

- A combined fuzzy rule base consists of rules generated from numerical data (input-output pairs) and linguistic rules obtained using a table look-up scheme.
- If there is more than one rule in one box of the fuzzy rule base, use the rule that has the maximum degree.
- If a linguistic rule is an AND rule, it fills only one box of the fuzzy rule base.
- If a linguistic rule is an OR rule, it fills all the boxes in the rows or columns corresponding to the regions of the IF part. The THEN part follows if any conditions of the IF part are satisfied.

In this way, both numerical data and linguistic information are codified into a common combined fuzzy rule base framework.

Step 5: Determine a final mapping based on the combined fuzzy rule base

The following defuzzification strategy is used to determine the output y for given input (x_1, x_2). The antecedents of the *ith* fuzzy rule for given inputs (x_1, x_2) are combined by using product operations in order to determine the degree, $\mu_{O^i}^i$, of the output performance corresponding to (x_1, x_2), that is

$$\mu_{O^i}^i = \mu_{I_1^i}(x_1)\mu_{I_2^i}(x_2)D_{(Rule\ \#\ i)}W_{(Rule\ \#\ i)} \tag{10}$$

where O^i denotes the output region of *Rule # i*, I_j^i denotes the input region of *Rule # i* for the *jth* component, $D_{(Rule\ \#\ i)}$ denotes the degree of *Rule # i*, and $W_{(Rule\ \#\ i)}$ denotes the weight of *Rule # i* as assigned. For example, *Rule # 1* in the example in Section 4 gives:

$$\mu_{LR}^1 = \mu_{F1}(FL)\mu_{C1}(CS)\ D_{(Rule\ \#\ 1)}W_{(Rule\ \#\ 1)}$$

where *LR = Low Risk*, *F1 = very low* in *FL* and *C1 = negligible* in *CS*.

It is noted that the degree of the rule is considered twice in the final mapping process, since the weight of the rule is the product of the reliable-level value and the degree of the rule. This double consideration is aimed to lessen the contrast arising between the expert's subjective opinion and the degree of rule (in terms of its significance contribution towards final evaluation in safety modelling).

The centre average defuzzification method is used here to determine the output:

$$y = \frac{\sum_{i=1}^{M} \mu_{O^i}^i \ \bar{y}^i}{\sum_{i=1}^{M} \mu_{O^i}^i} \tag{11}$$

where $\bar{y}^i$ = the centre value of output region O^i and M = the number of fuzzy rules in the combined fuzzy rule base which are involved in the final evaluation.

3.2.2 Development of safety model using adaptive-fuzzy-logic approach

The procedures used in development of a safety model using an adaptive-fuzzy-logic approach are similar to those described in Section 3.1. However, this novel approach can utilise human expertise in the form of fuzzy IF-THEN rules, as well as in conventional knowledge representations in fuzzy modelling to perform risk level estimation. Numerical data in the form of input-output pairs of a system can be employed to create fuzzy rules. Moreover, the flexible structure allows a combination of linguistic data with numerical data into a common fuzzy rule base. The key idea of this approach is to integrate the rules generated from numerical data pairs and linguistic rules obtained from expert judgement and engineering knowledge into a common fuzzy rule base (see Table 4).

4. An Example

An illustrative example of fire due to fuel oil system failure in the engine room of an offshore support vessel is used to demonstrate the two frameworks for risk evaluation. Possible consequences caused by fire in the engine room due to fuel oil system failure include: superficial damage (fire extinguished), minor damage (fire extinguished), significant damage (access to space denied) and severe damage (fire spreading to accommodation, bridge and emergency generator room, loss of lives and/or serious injuries, abandonment of ship, etc.). The fuel oil system failure may also lead to propulsion and machinery failure, which may cause collisions and contacts, grounding or stranding if operating near to offshore platforms in extreme weather conditions.

The fuel oil system failure rate is assumed to be *0.045/year* (i.e. *7.5* on ***failure likelihood*** scale). The ***consequence severity*** values for personnel related risk, environment related risk and organisation or business related risk are *8.5* (somewhere between *severe* and *catastrophic*), *5.5* (somewhere in *moderate*) and *3* (somewhere in *minor*), respectively.

A number between *0* and *10* can be given to represent the ***failure likelihood*** of a system where *0* is *very low* and *10* is *highly frequent*. Another number between *0* and *10* can be given to represent the ***consequence severity*** where *0* is *negligible* and *10* is *catastrophic*. The risk analysis for the fuel system is carried out using the two suggested methods as follows:

4.1 Application of Method 1

The 30 rules in the rule base that are used in this study are listed as follows:

- *Rule # 1*: *IF the* ***failure likelihood*** *is very low AND the* ***consequence severity*** *is negligible, THEN the* ***risk level*** *is low*
- *Rule # 2*: *IF the* ***failure likelihood*** *is very low AND the* ***consequence severity*** *is minor, THEN the* ***risk level*** *is low*
- ***
- ***
- *Rule # 30*: *IF the* ***failure likelihood*** *is highly frequent AND the* ***consequence severity*** *is catastrophic, THEN the* ***risk level*** *is high*

The evaluation of ***risk levels*** for the three categories/modules of risk are performed separately according to the general safety modelling framework. The evaluation of ***risk level*** for the offshore support vessel with the fuel oil system failure rate of *0.045/year* (***failure likelihood*** of *7.5*) and ***consequence severity*** for personnel related risk of *8.5* is performed as follows:

<u>Step 1: Fuzzify Inputs</u>

In this evaluation, 30 rules are considered, however, only four rules are fired contributing to the actual evaluation process. These four rules are:

- Rule # 19: *IF the **failure likelihood** is average AND the **consequence severity** is severe, THEN the **risk level** is substantial*
- Rule # 20: *IF the **failure likelihood** is average AND the **consequence severity** is catastrophic, THEN the **risk level** is substantial*
- Rule # 24: *IF the **failure likelihood** is frequent AND the **consequence severity** is severe, THEN the **risk level** is substantial*
- Rule # 25: *IF the **failure likelihood** is frequent AND the **consequence severity** is catastrophic, THEN the **risk level** is high*

The fuzzification process is described as follows:

- For *Rule # 19*, ***failure likelihood*** at *7.5* corresponds to $\mu_{FL} = 0.7$ for the "*average*" membership function and ***consequence severity*** at *8.5* corresponds to $\mu_{CS} = 0.7$ for the "*severe*" membership function.
- For *Rule # 20*, ***failure likelihood*** at *7.5* corresponds to $\mu_{FL} = 0.7$ for the "*average*" membership function and ***consequence severity*** at *8.5* corresponds to $\mu_{CS} = 0.76$ for the "*catastrophic*" membership function.
- For *Rule # 24*, ***failure likelihood*** at *7.5* corresponds to $\mu_{FL} = 0.66$ for the "*frequent*" membership function and ***consequence severity*** at *8.5* corresponds to $\mu_{CS} = 0.7$ for the "*severe*" membership function.
- For *Rule # 25*, ***failure likelihood*** at *7.5* corresponds to $\mu_{FL} = 0.66$ for the "*frequent*" membership function and ***consequence severity*** at *8.5* corresponds to μ_{CS} = 0.76 for the "*catastrophic*" membership function.

In this manner, each input variable is fuzzified over all the qualifying membership functions required by the rules.

<u>Step 2: Apply Fuzzy Operator</u>

The antecedents of the four rules are evaluated. For example, in applying Rule # 20 the two different pieces of the antecedent (***failure likelihood*** is "*average*" and ***consequence severity*** is "*catastrophic*") yield the fuzzy membership values ($\mu_{FL,20}, \mu_{CS,,20}$) = *(0.7, 0.76)*, respectively. The fuzzy AND operator ($\mu_r = Min(\mu_{FL,r}, \mu_{CS,r})$) simply selects the minimum of the two values, that is *0.7*. The application of the fuzzy operator generates the results as shown in Table 5 for each rule involved in the evaluation process.

<u>Step 3: Apply Implication Method</u>

Implication is implemented for each rule. A consequent is a fuzzy set represented by a membership function which weights appropriately the linguistic characteristics that are attributed to it. In this case the outputs for the four rules are shown in Table 6.

<u>Step 4: Aggregate All Outputs</u>

In this step, the fuzzy sets that represent the outputs of each rule are combined into a single fuzzy set and this only occurs once for each output variable prior to defuzzification. The *max* (maximum) method is used in this study. The aggregation of consequent, that is, ***risk level*** estimate across the rules, is expressed as follows:

$RL=\{Max(\beta_{1,r}, \text{'low'}); Max(\beta_{2,r}, \text{'possible'}); Max(\beta_{3,r}, \text{'substantial'}); Max(\beta_{4,r}, \text{'high'})\}$

For personnel related risk, the ***risk level*** estimate is calculated as follows:

$RL=\{Max(0,0,0,0, \text{'low'}); Max(0,0,0,0, \text{'possible'}); Max(0.7,0.7,0.66,0, \text{'substantial'}); Max(0,0,0,0.66, \text{'high'})\}$

Therefore, *RL = {(0, 'low'); (0,'possible'); (0.7, 'substantial); (0.66, 'high')}*.

The output can be obtained as "*substantial*" with a belief degree of *70%* and "*high*" with *66%*.

Step 5: Defuzzify

Since the aggregation of a fuzzy set encompasses a range of output values, it must be defuzzified in order to obtain a single crisp output value from the set. The centre average calculation method is used here to carry out the defuzzification process. The crisp value y^{Δ} is obtained as follows using the centre average defuzzification method:

$$y^{\Delta} = \frac{8.0 \times 0.7 + 10.0 \times 0.66}{0.70 + 0.66} = 8.97$$

The result of defuzzification produces y^{Δ} = 8.97, which gives the position of ***risk level*** estimation in the axis of the risk level expression in Figure 5. It is obvious that the derived ***risk level*** estimation belongs to *high risk (HS)* with a belief of 99.7% (μ = 0.997) and *substantial risk (SR)* with 0.3% (μ = 0.003).

The environment and business related risks are evaluated in a similar manner. The summary of ***risk levels*** for personnel related, environment related and organisation/business related risks is shown in Table 7.

The AHP analysis can be used to assess the priority weights for different risk categories/modules. In the AHP process, the weights in the additive utility function in the mathematical model can be evaluated. Based on the results obtained in the AHP process, the following statements can be made [Sii, 2000]:

- Personnel related risk is less important than environment related risk ($w_p = 0.31$).
- Environment related risk is more important than organisation or business related risk ($w_E = 0.48$).
- Organisation or business related risk is less important than personnel related risk ($w_B = 0.21$).

Using w_i as the criteria weights, the overall ***risk level*** of the system due to fuel oil system failure can be evaluated using the following mathematical model:

$$RL = r_P w_p + r_E w_E + r_B w_B = 8.97 \times 0.31 + 7.0 \times 0.48 + 5.44 \times 0.21 = 7.28$$

The above *RL* value indicates that the overall risk level is at *7.28* on the ***risk level*** expression scale, that is, within the region of *SR* in Figure 5. This result can be used to carry out safety based decision making as soon as the analysis for other hazards has been carried out in a similar manner.

4.2 Application of Method 2

The defuzzification strategy demonstrated in Section 3.2 is used to determine the output *y* for given input (x_1, x_2) where *y* is *RL*, x_1 is *FL* and x_2 is *CS*. The antecedents of the *i*th fuzzy rule for given inputs (x_1, x_2) are combined using product operations in order to determine the degree, $\mu_{O^i}^{i}$, of the output performance corresponding to (x_1, x_2) by using equation (10).

Personal related: *FL = 7.5* and *CS = 8.5*

In this case there are four rules involved in the final mapping process based on the combined fuzzy rule base. They are *Rules # 19, 20, 24* and *25* shown in Table 4.

Rule # 19: $\mu_{SR}^{19} = \mu_{F4}(FL)\mu_{C4}(CS)D_{(Rule\ \#\ 19)}W_{(Rule\ \#\ 19)} = 0.7 \times 0.7 \times 1.0 \times 1.0 = 0.49$

Rele # 20: $\mu_{SR}^{20} = \mu_{F4}(FL)\mu_{C5}(CS)D_{(Rule\ \#\ 20)}W_{(Rule\ \#\ 20)} = 0.7 \times 0.74 \times 0.72 \times 0.72 = 0.27$

Rule # 24: $\mu_{SR}^{24} = \mu_{F5}(FL)\mu_{C4}(CS)D_{(Rule\ \#\ 24)}W_{(Rule\ \#\ 24)} = 0.66 \times 0.66 \times 1.0 \times 1.0 = 0.44$

Rule # 25: $\mu_{HR}^{25} = \mu_{F5}(FL)\mu_{C5}(CS)D_{(Rule\ \#\ 25)}W_{(Rule\ \#\ 25)} = 0.66 \times 0.72 \times 1.0 \times 1.0 = 0.48$

The centre average defuzzification method described in equation (11) is used here to determine the output:

$$y = \frac{0.49 \times 7.0 + 0.27 \times 7.0 + 0.44 \times 7.0 + 0.48 \times 9.0}{0.49 + 0.27 + 0.44 + 0.48} = 7.57$$

The above value lies in the *SR* region of the risk level expression in Figure 5.

<u>Environment related</u>: $FL = 7.5$ and $CS = 5.5$

In this case *Rules # 18* and *23* are involved in the final mapping process to evaluate the risk level of the system.

Rule # 18: $\mu_{SR}^{18} = \mu_{F4}(FL)\mu_{C3}(CS)D_{(Rule\ \#\ 18)}W_{(Rule\ \#\ 18)} = 0.7 \times 1.0 \times 1.0 \times 1.0 = 0.7$

Rule # 23: $\mu_{SR}^{23} = \mu_{F5}(FL)\mu_{C3}(CS)D_{(Rule\ \#\ 23)}W_{(Rule\ \#\ 23)} = 0.8 \times 1.0 \times 1.0 \times 1.0 = 0.8$

$$y = \frac{0.7 \times 7 + 0.8 \times 7.0}{0.7 + 0.8} = 7.0$$

The above value lies within *SR* region of the risk level expression in Figure 5.

<u>Organisation or business related</u>: $FL = 8.5$ and $CS = 3.0$

In this case, *Rule # 17* and *23* are involved in the final mapping process.

Rule # 17: $\mu_{PR}^{17} = \mu_{F4}(FL)\mu_{C2}(CS)D_{(Rule\ \#\ 17)}W_{(Rule\ \#\ 17)} = 0.7 \times 1.0 \times 0.62 \times 0.62 = 0.27$

Rule # 22: $\mu_{SR}^{22} = \mu_{F5}(FL)\mu_{C2}(CS)D_{(Rule\ \#\ 22)}W_{(Rule\ \#\ 22)} = 0.66 \times 1.0 \times 0.47 \times 0.47 = 0.15$

$$y = \frac{0.27 \times 4.0 + 0.15 \times 7.0}{0.27 + 0.15} = 5.1$$

The above value lies somewhere between *PR* and *SR* regions, with *70%* belief within *PR* region and *30%* belief within *SR* region in Figure 5.

The comparisons made between the results obtained from risk analysis based on this novel approach and the Mamdani-type fuzzy logic approach are shown in Table 8. It can be observed that both the personal related and organisation/business related risk categories obtained using this approach appear to have slightly lower risk levels, compared to the results generated using the safety model incorporating Mamdani-type-fuzzy-logic approach (method 1). This is mainly due to the fact that the Mamdani-type approach does not account for degree and weight of each rule in its rule base as the framework cannot be trained by using real world numerical data in the form of input-output pairs.

The bracketed figures in Table 5 represent results obtained by using adaptive-fuzzy-logic-based approach without training process (i.e. the degree and weight of the rule are objective and they are assigned with 1.0). It can be observed that these results are roughly equivalent to those obtained from the Mamdani-type model (apart from personal related category).

Using w_i as the criteria weights, the overall ***risk level*** of the system due to fuel oil system failure using the adaptive-fuzzy-logic method can be evaluated using the mathematical model as depicted in equation (6):

With training process:

$RL = r_P w_p + r_E w_E + r_B w_B = 7.57 \times 0.31 + 7.0 \times 0.48 + 5.10 \times 0.21 = 6.78$

Without training process:

$RL = r_P w_p + r_E w_E + r_B w_B = 7.50 \times 0.31 + 7.0 \times 0.48 + 5.46 \times 0.21 = 6.83$

It can be seen that both *RLs* above lie in the *SR* region in Figure 5.

5. Conclusions

Most problems found in safety assessment of large maritime or offshore systems with a high level of novelty incorporate built-in or inherent uncertainties; this precludes using conventional approaches that usually require precise quantitative analyses and detailed description of the problem. This paper outlines and explains two concepts for maritime risk analysis using fuzzy-logic-based approaches. Both approaches described in this paper offer a great potential in safety modelling of maritime systems, especially in the initial concept design stages where the related safety information is scanty or with great uncertainty involved. Risk analysis using fuzzy logic and adaptive fuzzy logic approaches can formulate domain human experts' experience and safety engineering knowledge; at the same time information of different properties from various sources can be transformed to become the knowledge base, used in the fuzzy logic inference process. The result of this study has demonstrated that safety modelling based on fuzzy logic approaches provides safety analysts and designers with convenient tools that can be used at various stages of the design process of maritime engineering systems.

The two approaches described in this paper have several advantages over traditional methods. Some of these advantages are described as follows:

- Human expertise – Human expertise in the form of fuzzy IF-THEN rules as well as in conventional knowledge representations can be utilised in fuzzy modelling to solve a practical problem (in this case, it is to estimate risk levels).
- Numerical data pairs – Numerical data pairs (input-output data pairs) of a target system can be used to generate fuzzy rules.
- Flexible – Linguistic data can be combined with numerical data to produce a common fuzzy rule base.
- Fault tolerance - Fuzzy-logic-based system exhibits fault tolerance. The deletion of a rule or an error in a rule does not necessarily deteriorate its performance or destroy the system.
- Training or optimisation – It utilises the numerical data pairs of a target system to optimise or train the fuzzy model. It is possible to allow degree assignment made for each rule generated from the numerical data pair. This also helps to resolve the problem of conflicting rules by choosing the rule with maximum degree.
- Subjectivity – If necessary, expert judgements can be made on the numerical data pairs, in order to assign a relative weight for each rule in the rule base.

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Appendix 1 Operational Meaning of Fuzzy Sets

Fuzzy operators are the class of connecting operators, notably *AND* and *OR*, that combine antecedent fuzzy propositions to produce a composite truth value. The traditional Zadeh fuzzy operators use the min-max rules, but several alternative operator classes exist, such as the classes described by Yager, Schweizer and Sklar, Dubois and Prade, and Dombi. Fuzzy operators determine the nature of the implication and inference process and thus also establish the nature of fuzzy logic for that implementation.

Fuzzy membership is a truth function. For each unique value selected from the domain, the function returns a unique degree of membership in the fuzzy region. This is called a truth function since it reflects the truth of the fuzzy proposition *x is a member of fuzzy set A*.

Degree of membership: In fuzzy set theory, this is the degree to which a variable's value is compatible with the fuzzy set. The degree of membership is a value between 0 (no membership) and 1 (complete membership) and is drawn from the truth function of the fuzzy set. While the values in the domain of a fuzzy set always increase from left to right, the degree of membership follows the shape of fuzzy set's surface. For example, membership values associated with a bell curve rise to a maximum value (usually 1) and then fall back toward the zero membership point. The term truth function is often used interchangeably with degree of membership.

Defuzzification is the process of deriving a scalar, representing a variable's expected value, from a fuzzy set. The defuzzification process isolates a value on the fuzzy set's domain. Domain is the range of real numbers over which a fuzzy set is mapped. A fuzzy set domain can be any set of positive or negative monotonic numbers. Defuzzification is primarily a matter of selecting a point on the fuzzy region's boundary and then dropping a "plumb line" to the domain axis.

Fuzzy rules are statements of knowledge that relate the compatibility of fuzzy premise propositions to the compatibility of one or more consequent fuzzy spaces. In fact, the correspondence or compatibility function between an antecedent fuzzy region and a consequent fuzzy state is determined by the shape of the fuzzy sets, the connector (implication) operators and the preponderance of truth in the region from the current fuzzy state. This means that fuzzy rules operate in a different manner compared to rules in conventional knowledge-based systems (which are concerned with pattern matching and the logical evaluation of discrete expressions).

Table 1 Failure likelihood

Rank	Failure likelihood	Meaning (general interpretation)	Failure rate (generic offshore support vessel interpretation) (1/year)
1	F1: *Very low*	Failure is unlikely but possible during lifetime	$<10^{-7}$
2, 3	F2: *Low*	Likely to happen once during lifetime	$(10^{-7}, 10^{-5}]$
4, 5	F3: *Reasonably low*	Between low and average	$(10^{-5}, 10^{-3}]$
6, 7	F4: *Average*	Occasional failure	$(10^{-3}, 0.1]$
8, 9	F5: *Frequent*	Repeated failure	(0.1, 1]
9.5, 10	F6: *Highly frequent*	Failure is almost inevitable	>1

Table 2: Consequence severity (personnel, environment & organisation/business related risk)

Rank	Consequence severity	Meaning (generic offshore support vessel interpretation)
1	C1: *Negligible*	No injury. No environmental degradation caused. Negligible damage to the system or property.
2, 3	C2: *Minor*	Single or minor injury. Discharge of domestic materials, e.g. food or untreated sewage, or minor spillage of oil or oily mixture. Minor damage, likely business loss in the order of £10, 000 or less.
4, 5, 6	C3: *Moderate*	Multiple injuries. Intermediate spillage of oil, mixture or chemical. Damage requiring shore side support or repair, likely business loss in the order of £100 000.
7, 8	C4: *Severe*	Single fatality or multiple severe injuries. Spillage of large volumes of oil, oily mixture or chemical, e.g. discharge of 10% of the total cargo of a large oil tanker, causing long term damage. Major damage to ship requiring towing or tug assistance or drydocking or lengthy repair, likely business loss in the order of £1 million.
9, 10	C5: *Catastrophic*	Large number of simultaneous deaths. Major spillage of oil, oily mixture or chemicals, e.g. total discharge of a large oil tanker, causing significant long term damage. Total loss of asset, e.g. loss of ship including construction loss or damage about £10 millions or more.

Table 3 Table-look-up illustration of a fuzzy rule base

	C5						
	C4						
	C3						
X_2	***C2***						
	C1						
		F1	***F2***	***F3***	***F4***	***F5***	***F6***
				X_1			

Table 4 A combined rule base (numerical and linguistic)

Rule #	*FL* (***failure likelihood***)	*CS* (***consequence severity***)	*RL* (***risk level***)	Rule's degree	Rule's scale value, $\mu^{(i)}$ [weight]	Rule type
1	F1	*C1*	*LR*	(1.0)	1.0 [1.0]	Linguistic
2	*F1*	*C2*	*LR*	(1.0)	1.0 [1.0]	Linguistic
3	*F1*	*C3*	*PR*	(1.0)	1.0 [1.0]	Linguistic
4	*F1*	*C4*	*PR*	0.61	1.0 [0.61]	Numerical
5	*F1*	*C5*	*PR*	0.65	1.0 [0.65]	Numerical
6	*F2*	*C1*	*LR*	0.62	1.0 [0.62]	Numerical
7	*F2*	*C2*	*PR*	0.45	1.0 [0.45]	Numerical
8	*F2*	*C3*	*PR*	0.41	1.0 [0.41]	Numerical
9	*F2*	*C4*	*PR*	(1.0)	1.0 [1.0]	Linguistic
10	*F2*	*C5*	*SR*	(1.0)	1.0 [1.0]	Linguistic
11	*F3*	*C1*	*PR*	0.36	1.0 [0.36]	Numerical
12	*F3*	*C2*	*PR*	(1.0)	1.0 [1.0]	Linguistic
13	*F3*	*C3*	*PR*	(1.0)	1.0 [1.0]	Linguistic
14	*F3*	*C4*	*SR*	0.58	1.0 [0.58]	Numerical
15	*F3*	*C5*	*SR*	0.58	1.0 [0.58]	Numerical
16	*F4*	*C1*	*PR*	(1.0)	1.0 [1,0]	Linguistic
17	*F4*	*C2*	*PR*	0.62	1.0 [0.62]	Numerical
18	*F4*	*C3*	*SR*	(1.0)	1.0 [1.0]	Linguistic
19	*F4*	*C4*	*SR*	(1.0)	1.0 [1.0]	Linguistic
20	*F4*	*C5*	*SR*	0.72	1.0 [0.72]	Numerical
21	*F5*	*C1*	*PR*	0.80	1.0 [0.80]	Numerical
22	*F5*	*C2*	*SR*	0.47	1.0 [0.47]	Numerical
23	*F5*	*C3*	*SR*	1.0	1.0 [1.0]	Numerical
24	*F5*	*C4*	*SR*	(1.0)	1.0 [1.0]	Linguistic
25	*F5*	*C5*	*HR*	(1.0)	1.0 [1.0]	Linguistic
26	*F6*	*C1*	*SR*	(1.0)	1.0 [1.0]	Linguistic
27	*F6*	*C2*	*SR*	(1.0)	1.0 [1.0]	Linguistic
28	*F6*	*C3*	*SR*	0.72	1.0 [0.72]	Numerical
29	*F6*	*C4*	*HR*	0.90	1.0 [0.90]	Numerical
30	*F6*	*C5*	*HR*	(1.0)	1.0 [1.0]	Linguistic

Table 5 The results for each rule involved in the evaluation process

Rule number	Membership for antecedent 1 (***failure likelihood***), $\mu_{FL,r}$	Membership for antecedents 2 (***consequence severity***), $\mu_{CS,r}$	Fuzzy AND operator (Min), μ_r
Rule #19	0.70	0.70	0.70
Rule # 20	0.70	0.76	0.70
Rule # 24	0.66	0.70	0.66
Rule # 25	0.66	0.76	0.66

Table 6 Output results for the four rules

Rule	Consequence (***risk level*** expressions)	Membership value, μ_r
Rule # 19	*Substantial*	0.70
Rule # 20	*Substantial*	0.70
Rule # 24	*Substantial*	0.66
Rule # 25	*High*	0.66

Table 7 Risk levels for personnel related, environment related and organisation/business related risks

Risk Category	**Risk Level**
Personal related	8.97
Environment related	7
Organisation or business related	5.44

Table 8 Comparisons of results obtained using the two different approaches

Risk category	Adaptive-fuzzy-logic-based approach	Mamdani-type-fuzzy-logic-approach
Personal related	7.57 (7.5)	8.97
Environment related	7.0 (7.0)	7.0
Organisation or business related	5.1 (5.46)	5.5

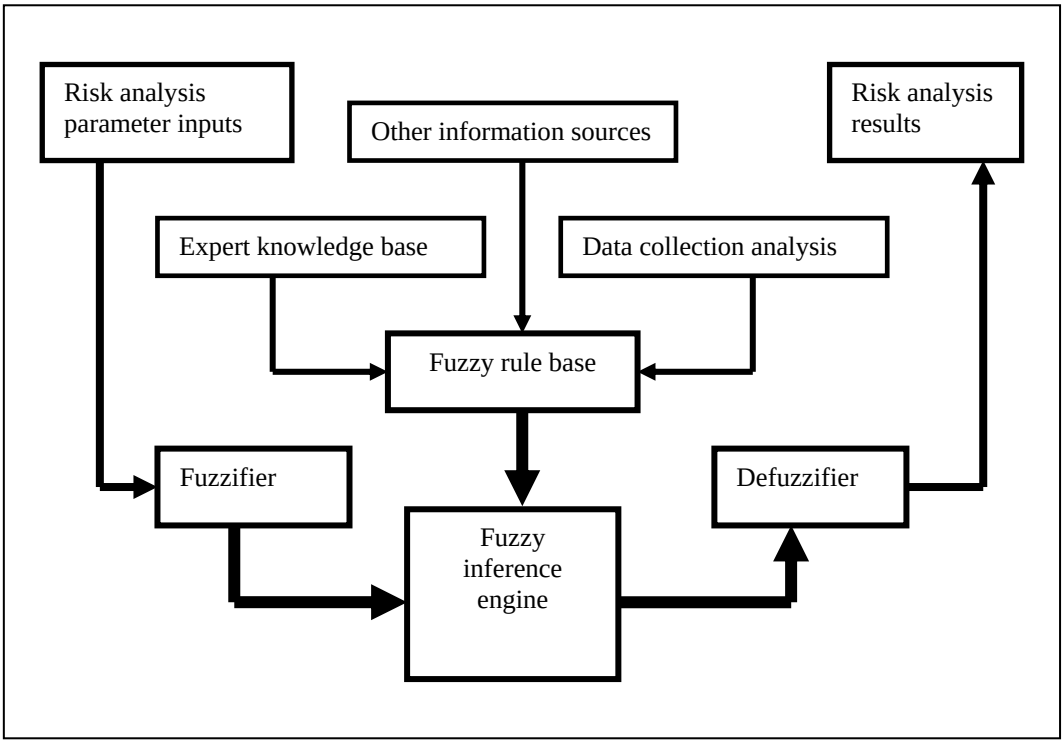

Figure 1 An overview of the safety model for risk analysis

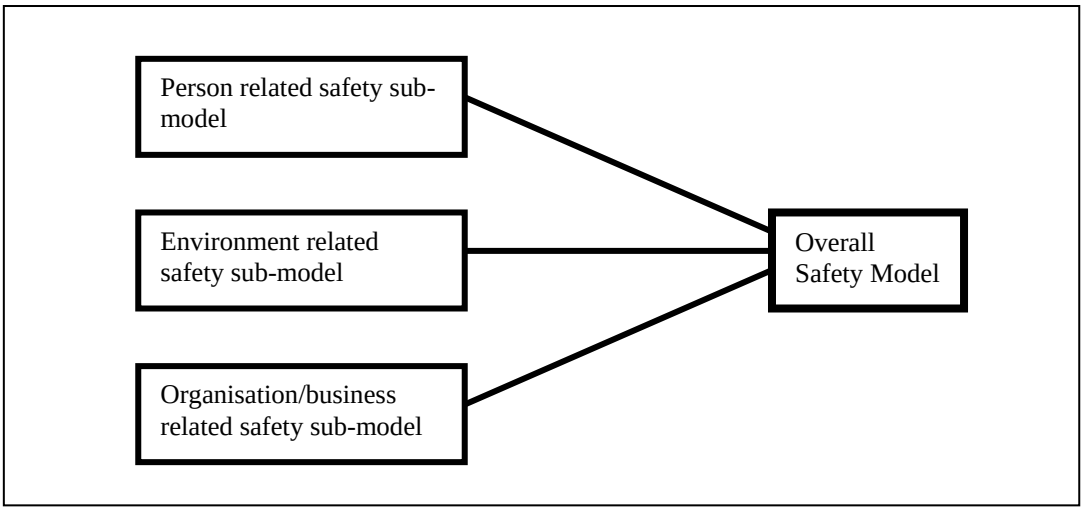

Figure 2 A general safety model framework for risk analysis using fuzzy logic based approaches

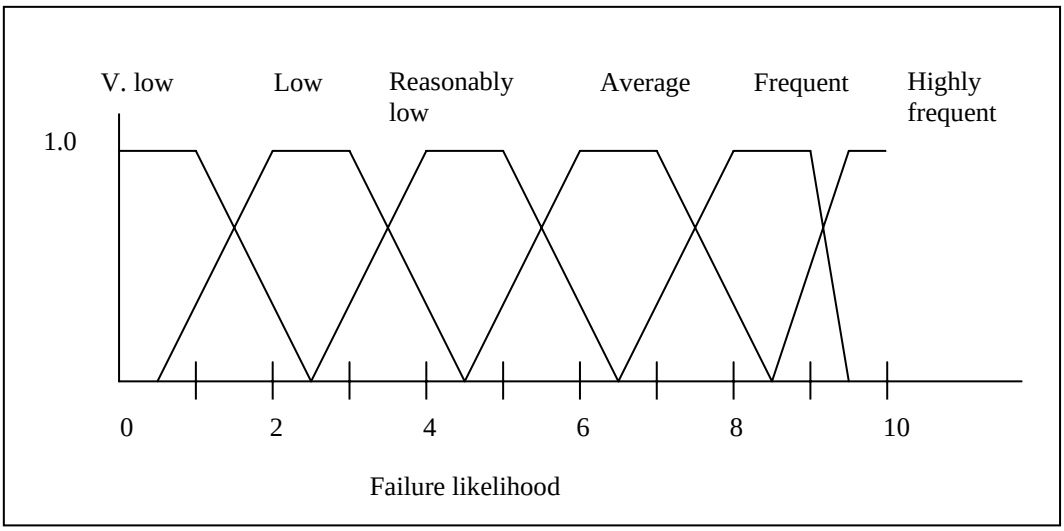

Figure 3 Fuzzy ***failure likelihood*** set definition

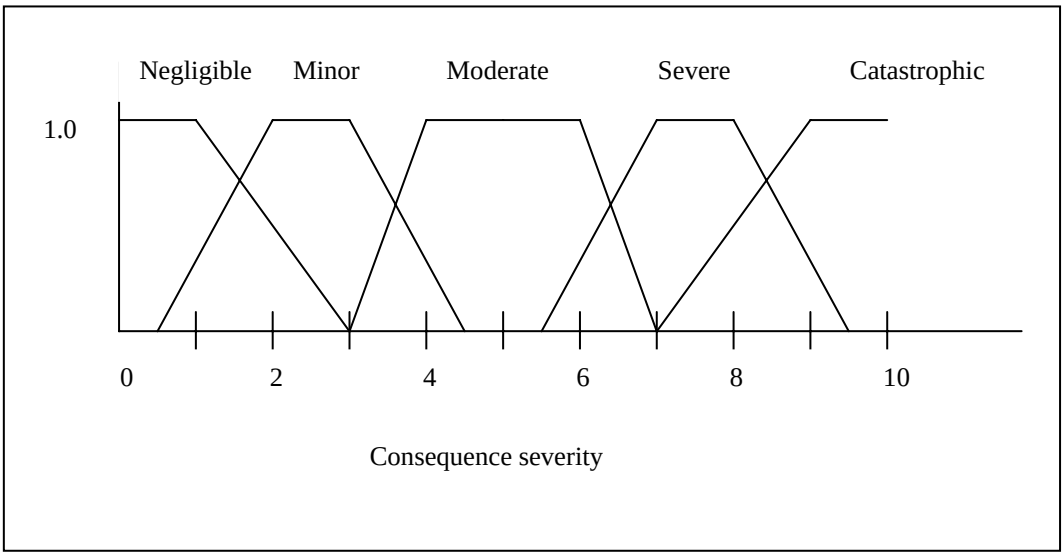

Figure 4 Fuzzy ***consequence severity*** set definition

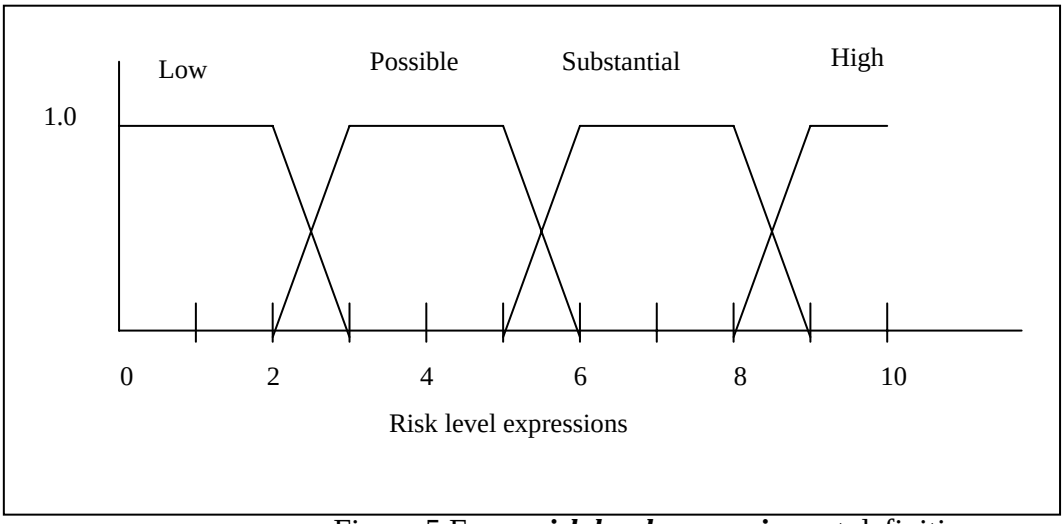

Figure 5 Fuzzy ***risk level expression*** set definition

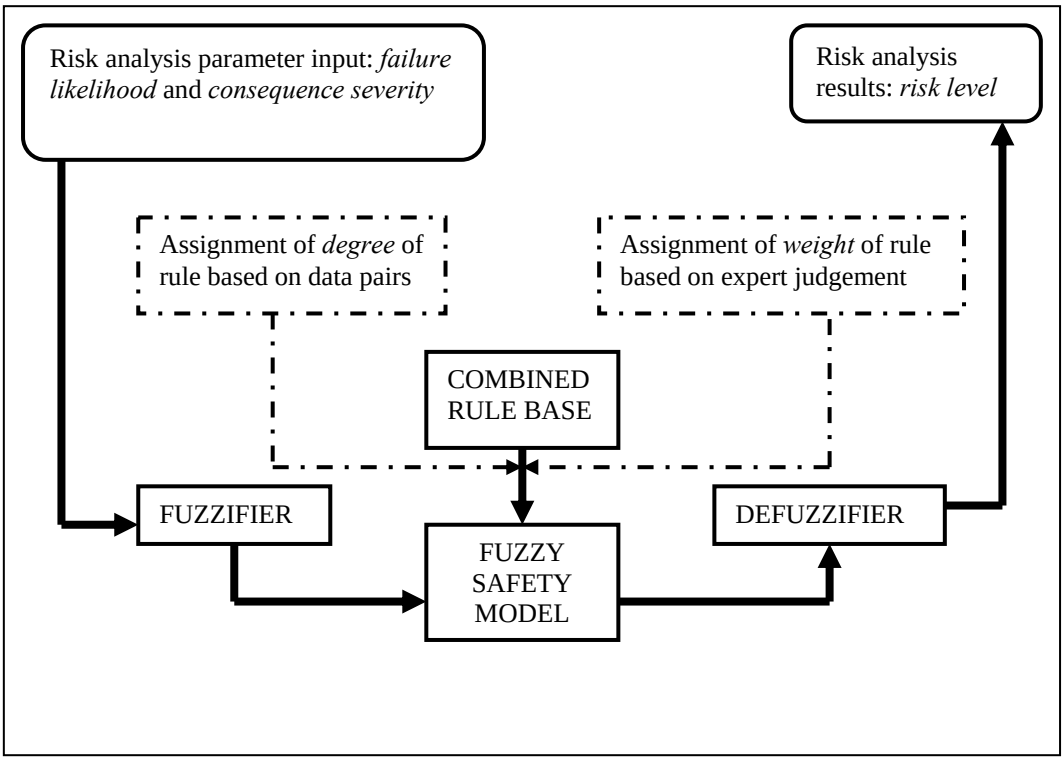

Figure 6: A safety model using an adaptive-fuzzy-logic-based approach