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Multi-person and multi-attribute design evaluations using evidential reasoning based on subjective safety and cost analyses

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This paper presents an approach for ranking proposed design options based on subjective safety and cost analyses. Hierarchical system safety analysis is carried out using fuzzy sets and evidential reasoning. This involves safety modelling by fuzzy sets at the bottom level of a hierarchy and safety synthesis by evidential reasoning at higher levels. Fuzzy sets are also used to model the cost incurred for each design option. An evidential reasoning approach is then employed to synthesise the estimates of safety and cost, which are made by multiple designers. The developed approach is capable of dealing with problems of multiple designers, multiple attributes and multiple design options to select the best design. Finally, a practical engineering example is presented to demonstrate the proposed multi-person and multi-attribute design selection approach. © 1996 Elsevier Science Limited.

1 INTRODUCTION

The purpose of safety based decision making is to take system safety as a design criterion to produce the best design with both technical and economical constraints being satisfied. If it is possible to carry out safety analysis on a probabilistic basis, Probabilistic Risk Analysis (PRA) methods such as Fault Tree Analysis (FTA) can be used to assess system safety and the information produced can be used to build a multi-objective model for decision making purposes.[1] Multiple Criteria Decision Making (MCDM) techniques can then be employed to process the constructed model to produce efficient design solutions. In many cases, however, it may be difficult or even impossible to precisely determine the parameters of a probability distribution for a given even due to lack of evidence or due to the inability of the safety engineer to make firm assessments.[2] Therefore, one may have to describe a given event in terms of vague and imprecise descriptors such as '*Likely*' and '*Impossible*', terms that are commonly used by safety analysts and engineers. Such judgements are obviously fuzzy and non-probabilistic, and hence non-probabilistic methods such as fuzzy set modelling may be more appropriate to analyze the safety of engineering systems with incomplete information of the kind described above.[2]

After system safety has been assessed using a non-probabilistic safety analysis method, decision making based on such safety analyses can be carried out. The possible applications of this kind of decision making have been widely discussed. A few representative applications are briefly outlined as follows:

1. Risk identification and risk ranking using fuzzy sets and possibility theory.[3] Risks are identified and estimated using traditional approximate reasoning approaches. The synthesised risks are ranked to point out weak links in design and to assist designers to make design decisions.
2. Optimal apportionment of reliability & redundancy in series systems under multiple objectives such as reliability and cost.[4] Fuzzy sets are used to translate imprecise information in objective and constraint functions to formulate fuzzy objective and constraint functions. An optimum solution which gives the highest degree of satisfaction with respect to all the fuzzy objectives and constraints can be identified.

3. Fuzzy apportionment of system reliability.[5] A nonlinear optimisation method based on fuzzy sets is formulated to solve system reliability apportionment problems.

In the above examples, the safety/reliability objective and associated fuzzy constraints are analyzed using traditional fuzzy operations such as intersection and union. This may result in a serious information loss in system synthesis. Furthermore, in the design of large engineering products such as offshore topsides and offshore cranes, an efficient design is usually selected from several design options based on the analysis of multiple objectives such as cost and safety. This is usually carried out by several experienced designers, therefore, synthesis of multiple designers' opinions is required.

In this paper, a methodology involving fuzzy set modelling and evidential reasoning is discussed. This methodology can make full use of the information to obtain safety synthesis without any available information loss. A cost modelling approach using fuzzy sets is also developed to model the cost incurred for each design option. Utility expressions are defined to map the safety associated with each design option and the cost incurred for each design option onto a utility space. Safety and cost objectives described in terms of the utility expressions are then synthesised to obtain the preference description of each design option so as to rank the design options.

2 SAFETY AND COST MODELLING

2.1 Safety modelling

Safety analysis of an engineering system can be conducted using a hierarchical structure as shown in Fig. 1.[2] A failure mode at the bottom level of such a hierarchy can initially be described in terms of failure likelihood, consequence severity and failure consequence probability using linguistic variables. A linguistic variable may be characterised by a membership function to the defined categories with regard to a particular situation.[2] The typical linguistic variables used for describing failure likelihood, consequence severity and failure consequence probability are defined and characterised as shown in Tables 1–3 where seven categories are used.[2] The use of categorical judgements has proven quite successful in many practical situations. Categories provide engineers with measures with which a linguistic variable can be modelled. A linguistic variable may be modelled in terms of membership values with respect to more than one category. For example, in Table 1, 'Highly frequent' is modelled by membership values 0.75 and 1 with respect to categories 6 and 7, respectively. The categories used in describing failure likelihood are the same as those used in describing consequence severity and failure consequence probability. It is obviously possible to have some flexibility

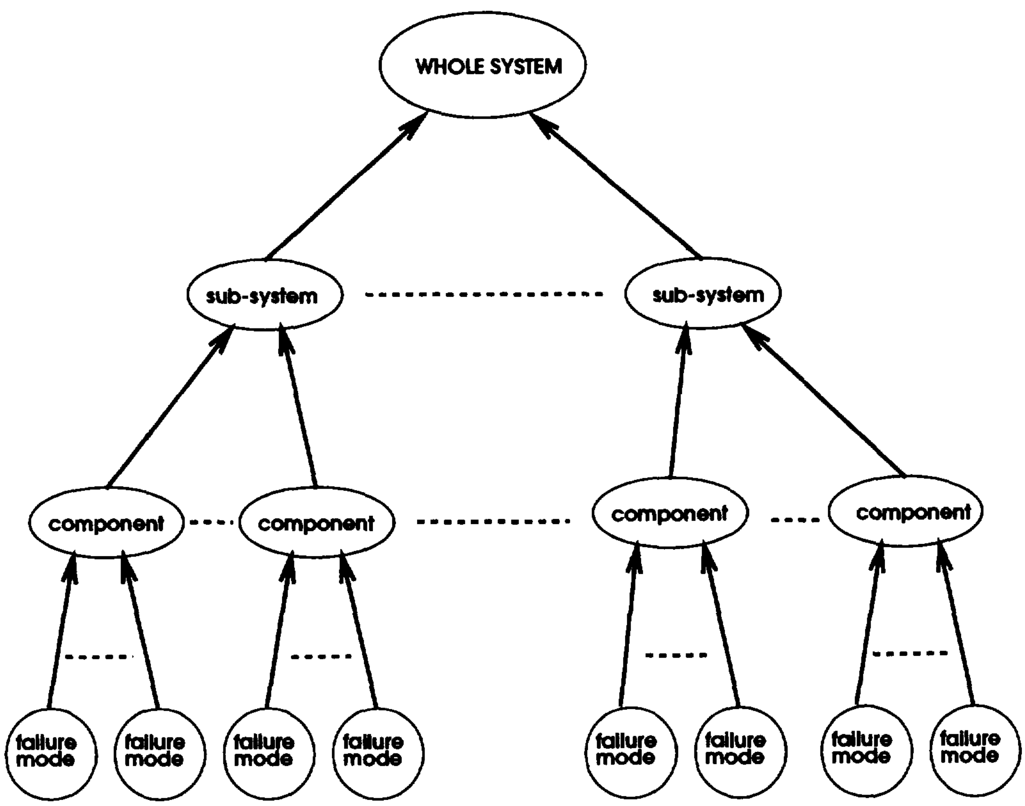

Fig. 1. A safety analysis hierarchy for an engineering system.

Table 1. Failure likelihood

μ_L Linguistic variables:	Categories 1	2	3	4	5	6	7
Highly frequent:	0	0	0	0	0	0.75	1
Frequent:	0	0	0	0	0.75	1	0.25
Reasonably frequent:	0	0	0	0.75	1	0.25	0
Average:	0	0	0.5	1	0.5	0	0
Reasonably low:	0	0.25	1	0.75	0	0	0
Low:	0.25	1	0.75	0	0	0	0
Very low:	1	0.75	0	0	0	0	0

in the definition of membership functions to suit different situations and individuals.[2]

Suppose $U = \{1\ 2 \ldots\ 7\}$ represents the set of seven categories defined for modelling linguistic variables.[2] L^{mk}, C^{mk} and E^{mk} represent the fuzzy sets of the failure likelihood, consequence severity and failure consequence probability associated with failure mode k of component m, respectively. The fuzzy safety description S^{mk} associated with failure mode k of component m can be defined as the product of the fuzzy descriptions of the corresponding failure likelihood, consequence severity and failure consequence probability:[2,6]

$$S^{mk} = C^{mk} o E^{mk} \times L^{mk}$$

where the symbol 'o' represents the composition operation and '$\times$' the Cartesian product operation in fuzzy set theory. The description function of S^{mk} can be described as follows:

$$\mu_{S^{mk}} = \mu_{C^{mk} o E^{mk} \times L^{mk}} = (\mu^1_{S^{mk}}, \mu^2_{S^{mk}}, \cdots, \mu^n_{S^{mk}})$$

where membership degree $\mu^j_{S^{mk}}$ represents the extent to which S^{mk} belongs to the jth category in U ($j = 1, 2, \ldots, 7$). Each element in $\mu_{S^{mk}}$ can be obtained using the maxmin method.[2,6]

Given failure mode k of component m, L^{mk}, C^{mk} and E^{mk} can be characterised by their membership functions with respect to the seven categories. Such membership functions need to be assigned by safety analysts with reference to Tables 1 to 3 regarding the particular situation.

Safety may be expressed more clearly in terms of linguistic variables such as '*Poor*', '*Average*', '*Good*' and '*Excellent*', which are referred to as safety expressions.[2] To evaluate S^{mk} in terms of these linguistic variables, it is necessary to characterise them using membership values with respect to the seven categories already defined. The four safety expressions are defined on the basis of Tables 1 to 3, and shown in Table 4.[2] Again, it is possible to have some flexibility over the values chosen in Table 4.

Using the Best-Fit method described in Ref. 2, the obtained fuzzy safety description S^{mk} of failure mode k of component m can be mapped onto the defined safety expressions. Such an evaluation may be obtained as follows:[2]

$$S(S^{mk}) = \{(\beta^1_{S^{mk}}, \text{"}Poor\text{"}), (\beta^2_{S^{mk}}, \text{"}Average\text{"}), (\beta^3_{S^{mk}}, \text{"}Good\text{"}), (\beta^4_{S^{mk}}, \text{"}Excellent\text{"})\}$$

where each $\beta^j_{S^{mk}}$ ($j = 1, 2, 3, 4$) represents the extent to which S^{mk} is confirmed to the jth safety expression.

After fuzzy descriptions of the failure modes of each component have been evaluated in terms of the safety expressions, it is desirable to synthesise the evaluations to generate an assessment of the safety associated with each component, then the safety associated with each subsystem, if necessary, and finally the safety associated with the system being investigated, as shown by Fig. 1. A novel evidential reasoning approach has been developed on the basis of the Demptster-Shafer theory to deal with such hierarchical evaluation propagation problems without any loss of useful information.[2,8–12] The evidential reasoning approach is capable of combining uncertain

Table 2. Consequence severity

μ_C Linguistic variables:	Categories 1	2	3	4	5	6	7
Catastrophic:	0	0	0	0	0	0.75	1
Critical:	0	0	0	0.75	1	0.25	0
Marginal:	0	0.25	1	0.75	0	0	0
Negligible:	1	0.75	0	0	0	0	0

Table 3. Failure consequence probability

μ_E Linguistic variables:	Categories 1	2	3	4	5	6	7
Definite:	0	0	0	0	0	0.75	1
Highly likely:	0	0	0	0	0.75	1	0.25
Reasonably likely:	0	0	0	0.75	1	0.25	0
Likely:	0	0	0.5	1	0.5	0	0
Reasonably unlikely:	0	0.25	1	0.75	0	0	0
Unlikely:	0.25	1	0.75	0	0	0	0
Highly unlikely:	1	0.75	0	0	0	0	0

evaluations at a single level and implementing hierarchical propagation of such evaluations between different levels. A detailed description of the algorithm of evidential reasoning can be found in Refs 8 and 10. Using the evidential reasoning algorithm, the safety S_i associated with the whole system for design option i can be evaluated as follows:[2]

$$S(S_i) = \{(\beta^1_{S_i}, \text{``}Poor\text{''}), (\beta^2_{S_i}, \text{``}Average\text{''}), (\beta^3_{S_i}, \text{``}Good\text{''}), (\beta^4_{S_i}, \text{``}Excellent\text{''})\}$$

where each $\beta^j_{S_i}$ ($j = 1, 2, 3, 4$) represents the extent to which S_i is confirmed to the jth safety expression.

2.2 Cost modelling

Generally, safety and cost are two conflicting objectives, with higher safety leading to higher costs. This means that if the safety associated with a system is improved, higher costs will usually be incurred. The cost incurred for the safety improvement associated with a design option is usually affected by many factors. Typical factors include:

1. Costs for the provision of redundancies of critical components, provision of protection systems and alarm systems to reduce or eliminate the probabilities of occurrence of undesirable system top events, and use of more reliable components.
2. Cost of labour incurred in redesign of the system.
3. Benefits resulting from the likelihood reduction of occurrence of system failure events and the improvement of system efficiency as a result of the improvement of system safety.

The above factors can have large uncertainties of estimation. In the early stages of a design process, it can be very difficult to assess (1), (2) and (3) in quantitative terms. Therefore, it is not surprising that safety engineers often prefer to estimate costs incurred in safety improvement using linguistic variables. This is particularly true for the preliminary design of large engineering products with a high level of innovation. Under such considerations it may be more appropriate to model the costs incurred in safety improvement associated with a design option using fuzzy sets.

The cost incurred for a design option can be described using linguistic variables such as '*Very low*', '*Low*', '*Moderately low*', '*Average*', '*Moderately high*', '*High*' and '*Very high*', which are referred to as cost expressions. Such linguistic variables can also be described, as shown in Table 5, in terms of membership values with respect to the seven categories already defined.

From Table 5, it can be noted that the cost expressions are not exclusive in the sense that the sum of the membership degrees of the linguistic variables with respect to a category may be greater than 1. Such inclusive expressions may make it more convenient for the safety analyst to judge the costs.

The membership values describing the cost incurred for a design option may be given by designers with reference to Table 5, with each designer having some

Table 4. Safety expressions

μ_{Snk} Linguistic variables:	Categories 1	2	3	4	5	6	7
1. Poor:	0	0	0	0	0	0.75	1
2. Average:	0	0	0	0.5	1	0.25	0
3. Good:	0	0.25	1	0.5	0	0	0
4. Excellent:	1	0.75	0	0	0	0	0

Table 5. Cost expressions

μ_C Linguistic variables:	Categories 1	2	3	4	5	6	7
Very high:	0	0	0	0	0	0.75	1
High:	0	0	0	0	0.75	1	0.25
Moderately high:	0	0	0	0.5	1	0.25	0
Average:	0	0	0.5	1	0.5	0	0
Moderately low:	0	0.25	1	0.5	0	0	0
Low:	0.25	1	0.75	0	0	0	0
Very low:	1	0.75	0	0	0	0	0

flexibility in the formulation of membership values to reflect his own opinion. The cost C_i^l incurred for design option i estimated by designer l can be described in terms of membership values as follows:

$$C_i^l = [\mu_{C_i^l}^1/1, \mu_{C_i^l}^2/2, \mu_{C_i^l}^3/3, \mu_{C_i^l}^4/4, \mu_{C_i^l}^5/5, \mu_{C_i^l}^6/6, \mu_{C_i^l}^7/7]$$

where each $\mu_{C_i^l}^j$ ($j = 1, 2, 3, 4, 5, 6, 7$) represents a degree to which C_i^l belongs to the jth category.

2.3 Safety and cost descriptions in the utility space

The selection of a design proposal is obviously dependent on safety and cost implications in a particular situation. This requires the synthesis of cost and safety analyses for each design option in a rational way. Since safety and cost are both described using fuzzy sets, it is desirable that the evidential reasoning approach be used to carry out such a synthesis in order to avoid loss of useful information. However, as the safety associated with a design option is described in terms of safety expressions and the cost incurred for a design option is described in terms of membership values describing a corresponding linguistic cost variable, it is not convenient to directly implement such a synthesis using the evidential reasoning approach. It is therefore necessary to define a utility space to evaluate safety and cost on the same scale. Four exclusive utility expressions are defined as shown in Table 6; these are '*Slightly preferred*', '*Moderately preferred*', '*Preferred*' and '*Greatly preferred*'. The safety associated with each design option and the cost incurred for each design option are then mapped onto the utility space and expressed in terms of the utility expressions.

From Tables 5 and 6, it can be noted that when the cost incurred for a design option is '*Very high*' the design option is considered to be '*Slightly preferred*' as far as cost is concerned, similarly when the cost is '*Moderately high*' the design option is '*Moderately preferred*', when the cost is '*Moderately low*' the design option is '*Preferred*' and when the cost is '*Very low*' the design option is '*Greatly preferred*'. In addition, when the cost is '*High*' the design option is considered to be something between '*Slightly preferred*' and '*Moderately preferred*', and when the cost is '*Low*' the design option is considered to be something between '*Preferred*' and '*Greatly preferred*'.

From Tables 4 and 6, it can also be noted that when the safety associated with a design option is '*Poor*' the design option is considered to be '*Slightly preferred*' as far as safety is concerned, similarly when the safety is '*Average*' the design option is '*Moderately high*', when the safety is '*Good*' the design option is '*Preferred*' and when the safety is '*Excellent*' the design option is '*Greatly preferred*'.

In the four defined utility expressions in Table 6, '*Slightly preferred*' is described only by the membership values with regard to categories 6 and 7, and '*Greatly preferred*' only by the membership values with regard to categories 1 and 2. The membership functions of '*Preferred*' and '*Moderately preferred*' are not symmetric with respect to categories 3 and 5, respectively, and actually they lay slightly more weight on category 4.

Since the safety expressions and the utility expressions are defined by the same membership functions with respect to the seven categories, safety description S_i can be directly mapped onto the utility

Table 6. Utility expressions

μ Linguistic variables:	Categories 1	2	3	4	5	6	7
1. Slightly preferred:	0	0	0	0	0	0.75	1
2. Moderately preferred:	0	0	0	0.5	1	0.25	0
3. Preferred:	0	0.25	1	0.5	0	0	0
4. Greatly preferred:	1	0.75	0	0	0	0	0

space as follows:

$$U(S_i) = \{(\mu^1_{S_i}, \text{"}\textit{Slightly preferred}\text{"}), (\mu^2_{S_i}, \text{"}\textit{Moderately preferred}\text{"}), (\mu^3_{S_i}, \text{"}\textit{Preferred}\text{"}), (\mu^4{}_{S_i}, \text{"}\textit{Greatly preferred}\text{"})\}$$

where $\mu^j_{S_i} = \beta^j_{S_i}$ $(j = 1, 2, 3, 4)$. Each $\mu^j_{S_i}$ $(j = 1, 2, 3, 4)$ represents a degree of confidence that S_i belongs to the jth utility expression.

Given the membership values of a cost description for a design option with reference to Table 5, the Best-Fit method described in Ref. 2 can also be used to map the fuzzy cost description onto the defined utility expressions. The cost C^l_i incurred in the ith design option estimated by designer l can then be evaluated in terms of the utility expressions as follows:

$$U(C^l_i) = \{(\mu^1_{C^l_i}, \text{"}\textit{Slightly preferred}\text{"}), (\mu^2_{C^l_i}, \text{"}\textit{Moderately preferred}\text{"}), (\mu^3_{C^l_i}, \text{"}\textit{Preferred}\text{"}), (\mu^4_{C^l_i}, \text{"}\textit{Greatly preferred}\text{"})\}$$

where each $\mu^j_{C^l_i}$ $(j = 1, 2, 3, 4)$ represents the extent to which the cost incurred for design option i evaluated by designer l is confirmed to the jth utility expression, and can be obtained as follows.

The Best-Fit method uses the distance between C^l_i and each of the utility expressions to represent the degree to which C^l_i is confirmed to each of the defined utility expressions (i.e., '*Slightly preferred*', '*Moderately preferred*', '*Preferred*' and '*Greatly preferred*'). The distance between the obtained cost description C^l_i and the expression '*Slightly preferred*' is defined as follows:

$$d^l_{i1}(C^l_i, \textit{Slightly preferred}) = \left(\sum_{j=1}^{7} (\mu^j_{C^l_i} - \mu^j_{\textit{Slightly preferred}})^2\right)^{\frac{1}{2}}.$$

Similarly, we can define:

$$d^l_{i2}(C^l_i, \textit{Moderately preferred}) = \left(\sum_{j=1}^{7} (\mu^j_{C^l_i} - \mu^j_{\textit{Moderately preferred}})^2\right)^{\frac{1}{2}}$$

$$d^l_{i3}(C^l_i, \textit{Preferred}) = \left(\sum_{j=1}^{7} (\mu^j_{C^l_i} - \mu^j_{\textit{Preferred}})^2\right)^{\frac{1}{2}}$$

$$d^l_{i4}(C^l_i, \textit{Greatly preferred}) = \left(\sum_{j=1}^{7} (\mu^j_{C^l_i} - \mu^j_{\textit{Greatly preferred}})^2\right)^{\frac{1}{2}}.$$

It should be noted that each d^l_{ij} $(j = 1, 2, 3, 4)$ is an unscaled distance. The closer C^l_i is to the jth expression, the smaller the value of d^l_{ij}. More specifically, d^l_{ij} is equal to zero if C^l_i and the jth utility expression have the same membership function. In such a case, C^l_i should not be evaluated to other expressions at all due to the exclusiveness of the utility expressions. To embody such features, new indices need to be defined based on d^l_{ij} $(j = 1, 2, 3, 4)$.

Suppose d^l_{iJ} $(1 \le J \le 4)$ is the smallest of the obtained distances for C^l_i and let α^l_{i1}, α^l_{i2}, α^l_{i3} and α^l_{i4} represent the reciprocals of the relative distances between the identified fuzzy cost description C^l_i and each of the defined utility expressions with reference to d^l_{iJ}. Then, α^l_{ij} $(j = 1, 2, 3, 4)$ can be defined as follows:

$$\alpha^l_{ij} = \frac{1}{d^l_{ij}/d^l_{iJ}} \quad j = 1,2,3,4,$$

if $d^l_{iJ} = 0 \alpha^l_{iJ}$ is defined to be 1. Normalising α^l_{ij}, $\mu^j_{C^l_i}$ can then be obtained:

$$\mu^j_{C^l_i} = \frac{\alpha^l_{ij}}{\sum_{n=1}^{4} \alpha^l_{in}} \quad j = 1,2,3,4.$$

Each $\mu^j_{C^l_i}$ $(j = 1, 2, 3, 4)$ represents the extent to which C^l_i is confirmed to the jth defined utility expression. It may be noted that if C^l_i is completely confirmed to the jth utility expression then $\mu^j_{C^l_i}$ is equal to 1 and the others are equal to 0. The sum of values of these indices is equal to 1, that is, $\sum_{j=1}^{n} \mu^j_{C^l_i} = 1$. Thus each $\mu^j_{C^l_i}$ could be viewed as a degree of confidence that C^l_i belongs to the jth utility expression.

3 ANALYSIS OF DESIGN OPTIONS

In safety modelling, the safety associated with a failure mode of a component may be judged by multiple designers with reference to Table 1, Table 2 and Table 3. A diagram for synthesising the safety associated with a failure mode of a component is shown in Fig. 2. Suppose there are e designers. Each designer's opinion may have a weight in the design selection process. Given relative weights of designers' opinions, the evidential reasoning approach can be used to synthesise designers' judgements on a failure mode to obtain the evaluation of the safety associated with the failure mode. The safety description of the whole system can finally be synthesised as described in Section 2.1. Then the safety associated with the whole system can be mapped onto the utility expressions. The above procedures can be repeated to obtain the estimates of the safety associated with other design options.

In cost modelling, the cost incurred for each design option can also be judged by e designers, each of whom estimates the cost incurred for each design option in terms of membership values with reference to Table 5. The cost judged by each designer can also be mapped onto the utility expressions as described in the last section. The evidential reasoning approach can be used to synthesise e designers' judgements to obtain the evaluation of the cost incurred for each design option. A diagram of a hierarchy for synthesising the cost incurred for each design option is shown in Fig. 3.

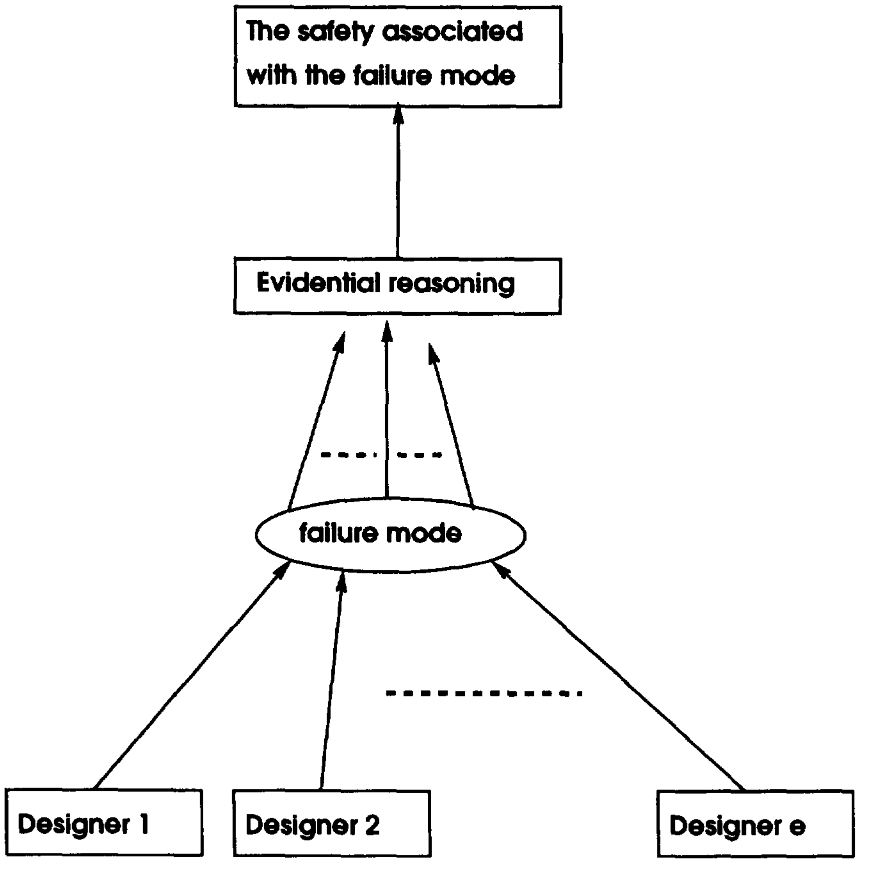

Fig. 2. A diagram for synthesising the safety associated with a failure mode.

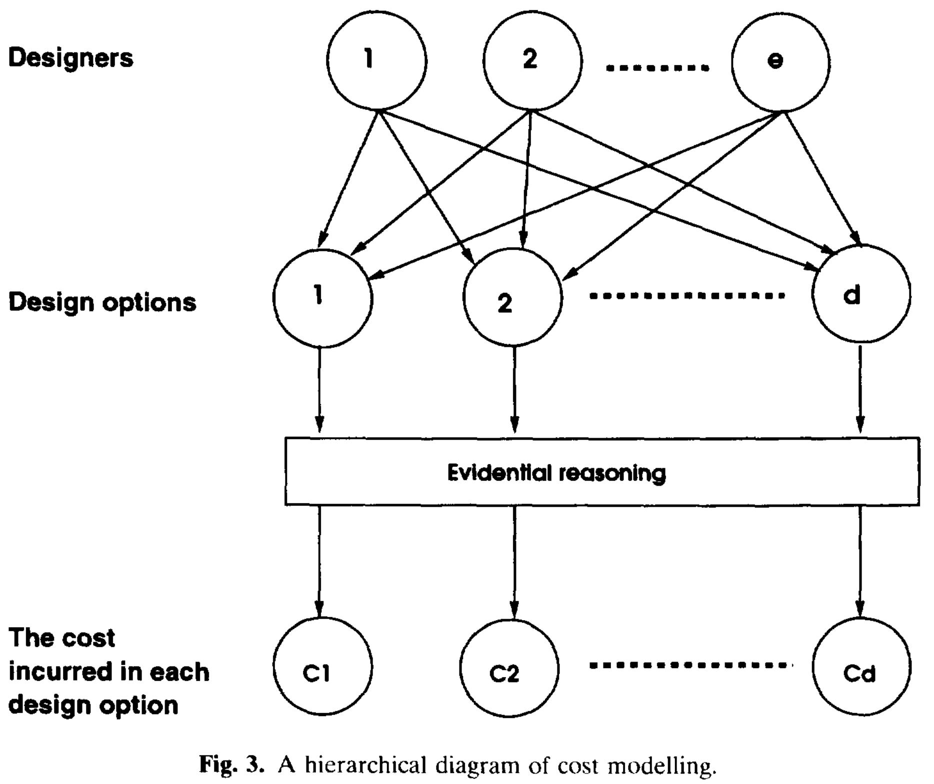

Fig. 3. A hierarchical diagram of cost modelling.

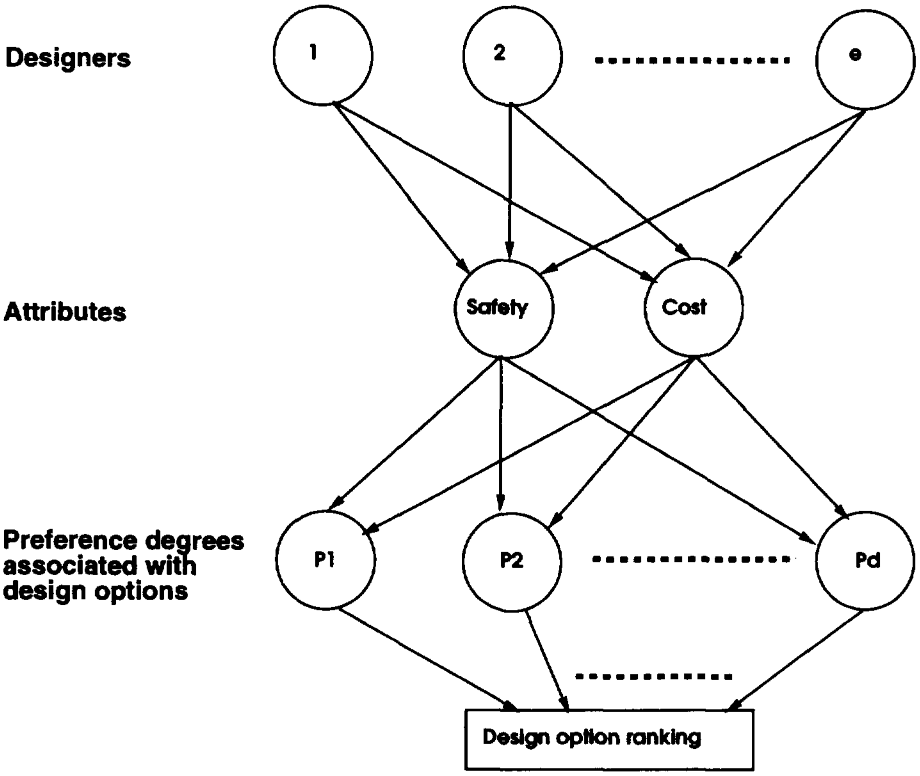

Fig. 4. A hierarchical design selection structure.

The hierarchical design selection framework is shown in Fig. 4. After the safety and cost syntheses for each design option have been carried out on the basis of the information provided by multiple designers, the preference degree associated with each design option can be produced for design selection purposes. Preference degrees associated with design options can be calculated as follows.

Suppose there are d design options in hand. The utility $U(S_i)$ associated with the ith design option and the cost C_i incurred in the ith design option evaluated in terms of the utility expressions can be described as follows.

$$U(S_i) = \{(U^j_{S_i}, U^j), j = 1,2,3,4\} \quad i = 1,2,\cdots,d$$

$$U(C_i) = \{(U^j_{C_i}, U^j), j = 1,2,3,4\} \quad i = 1,2,\cdots,d$$

where U^j ($j = 1, 2, 3, 4$) represent utility expressions '*Slightly preferred*', '*Moderately preferred*', '*Preferred*' and '*Greatly preferred*', respectively, each $\mu^j_{C_i}$ represents the degree to which S_i is confirmed to U^j and $\mu^j_{C_i}$ represents the degree to which $C_i$is confirmed to U^j.

Given the relative importance of cost to safety, denoted by ω, $U(S_i)$ and $U(C_i)$ can be synthesised using the evidential reasoning approach to obtain a preference estimate associated with design option i in terms of the utility expression. Suppose $U(U_i)$ represents such a synthesised preference estimate for design option i. $U(U_i)$ can be described as follows:

$$U(U_i) = \{(\mu^j_i, U^j), j = 1,2,3,4\} \quad i = 1,2,\cdots,d$$

where μ^j_i represents the degree to which the synthesised preference estimate or design option i is confirmed to U^j. $U(U_i)$ therefore provides a descriptive evaluation of design option i.

To calculate preference degrees associated with the design options, it is required to describe the four utility expressions (i.e., '*Slightly preferred*', '*Moderately preferred*', '*Preferred*' and '*Greatly preferred*') using numerical values. The numerical values associated with the defined utility expressions can be calculated by studying the categories and membership values associated with the utility expressions in Table 6. Suppose K'_1, K'_2, K'_3 and K'_4 represent the unscaled numerical values associated with '*Slightly preferred*', '*Moderately preferred*', '*Preferred*' and '*Greatly preferred*', respectively. Then K'_1, K'_2, K'_3 and K'_4 can be calculated as follows:

$$K'_1 = 0.75 \times 6 + 1 \times 7 = 11.5$$

$$K'_2 = 0.5 \times 4 + 1 \times 5 + 0.25 \times 6 = 8.5$$

$$K'_3 = 0.25 \times 2 + 1 \times 3 + 0.5 \times 4 = 5.5$$

$$K'_4 = 1 \times 1 + 0.75 \times 2 = 2.5.$$

The above values give the linear numerical relations between the utility expressions. The normalised vector $[K_1 K_2 K_3 K_4]$ is then obtained as follows where

'*Greatly preferred*' takes the largest value of 1 (i.e., $K_4 = 1$):

$$[K_1\ K_2\ K_3\ K_4] = [0.217\ 0.478\ 0.739\ 1].$$

Note that K_1 is not normalised to 0. This implies that linguistic variable '*Slightly preferred*' still has some preference degree (i.e., 0.217) in the design selection process. Preference degree P_i associated with design option i can be defined as follows:

$$P_i = \sum_{j=1}^{4} \mu_i^j \times K_j + \left(1 - \sum_{j=1}^{4} \mu_i^j\right) \times \frac{1}{4} \times \sum_{j=1}^{4} K_j \quad i = 1,2,\cdots,d$$

where $(1 - \sum_{j=1}^{4} \mu_i^j)$ describes the remaining belief unassigned after commitment of belief to all U^j ($j = 1, 2, 3, 4$) in cost and safety synthesis, and $\frac{1}{4} \times \sum_{j=1}^{4} K_j$ is the average value of the K_js.

Obviously, a larger P_i means that design option i is more desirable. Each P_i ($i = 1, 2, \ldots, d$) represents the extent to which design option i is preferred in comparison with others. The best design with the largest preference degree may be selected on the basis of the magnitudes of P_i ($i = 1, 2, \ldots, d$).

4 AN EXAMPLE OF APPLICATION

An hydraulic hoist transmission system of a marine crane is functionally shown in Fig. 5. This system is used to control the crane motions such as hoisting down loads as required by the operator. It consists of five subsystems, namely the hydraulic oil tank, the auxiliary system, the control system, the protection system and the hydraulic servo transmission system. It is often difficult to assess the safety associated with the system using traditional probabilistic approaches and to estimate the cost incurred for each design option in terms of quantitative cost values. However, it could be comparatively easier to use fuzzy subjective judgements for describing safety and cost objectives in order to select the best design option for this crane hydraulic hoist transmission system.

In this section, the hydraulic hoist transmission system is examined to demonstrate the proposed multi-person and multi-attribute design selection approach. The detailed analysis for design option 1 is presented. The analyses for other design options are carried out in a similar manner.

4.1 Safety modelling and cost modelling

Let us suppose there are four designers and the opinions given by designers 2 and 3 are twice as important as those given by designers 1 and 4. Suppose there are four design options in hand. The safety modelling and cost modelling for the four design options are described as follows.

4.1.1 Option 1

No failure mode eliminated in the design review process.

Let ζ_l represent the relative weight of the lth

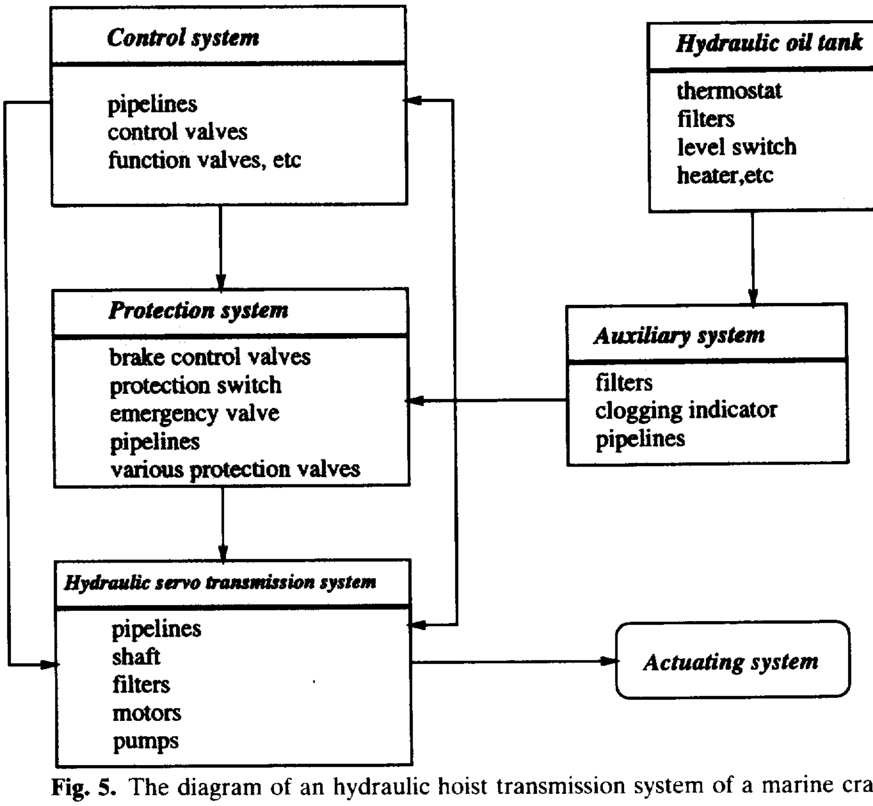

Fig. 5. The diagram of an hydraulic hoist transmission system of a marine crane.

designer's opinion. The matrix ζ can be expressed as follows:

$$\zeta = (1\,2\,2\,1).$$

Suppose λ_l represents a normalised weight of the opinion given by designer l. Given the relative weight of each designer's opinion, λ_l can be obtained as follows:[8,10,11]

$$\lambda_l = \alpha \frac{\zeta_l}{\zeta^l_l}$$

where α is a priority coefficient representing the importance of the role the most important factor plays in evaluation of safety and cost, and ζ^l_l is the largest value of ζ_l ($l = 1, 2, 3, 4$). α can be determined using the following consensus condition:[10,11]

$$\prod_{l=1}^{4} \left(1 - \alpha \frac{\zeta_l}{\zeta^l_l}\right) \le \delta$$

where δ is a sufficiently small non-negative real number representing the degree of approximation in reasoning with subjective judgements provided by the four designers. If all four designers judge that the safety associated with a design option is absolutely '*Good*', for example, the safety of the design option should be '*Good*' with the degree of confidence of over 99.5%. The remaining 0.5% is assigned to δ for conflict resolution in approximate reasoning.[10,11] Note that from the consensus condition a small δ will result in a large α (approaching 1). δ should be sufficiently small in the sense that α is properly assessed with regard to the problem in question.

$$\lambda = [\lambda_1 \lambda_2 \lambda_3 \lambda_4]$$

can thus be obtained as follows with $\delta = 0.0005$:

$$\lambda = [0.48\ 0.96\ 0.96\ 0.48].$$

Each failure mode of each subsystem can be described in terms of failure likelihood, consequence severity and failure consequence probability using membership functions by the four designers. The safety associated with each failure mode of each subsystem can be obtained by processing the corresponding failure likelihood, consequence severity and failure consequence probability descriptions produced by the four designers and then mapped onto the safety expressions using the Best-Fit method.[2]

Suppose each $m^{jl}_{S_1^{mk}}$ ($j, l = 1, 2, 3, 4$) is a basic probability assignment to which the safety evaluation, produced by designer l for failure mode k of subsystem m of design option 1, supports a hypothesis that the safety associated with design option 1 is confirmed to the jth safety expression. Each $m^{jl}_{S_1^{mk}}$ is then given by

$$m^{jl}_{S_1^{mk}} = \lambda_l \times \beta^{jl}_{S_1^{mk}} \quad j, l = 1,2,3,4$$

where each $\beta^{jl}_{S_1^{mk}}$ ($j, l = 1, 2, 3, 4$) represents the extent to which the safety associated with failure mode k of subsystem m estimated by designer l is confirmed to the jth safety expression.

The estimates for the four designers on each failure mode of each subsystem can be synthesised using the evidential reasoning approach to obtain the safety description of each failure mode. Then the safety associated with each subsystem can be obtained by synthesising the safety associated with the corresponding failure modes. Finally, the safety associated with the system can be obtained by synthesising the safety associated with the constituent subsystems using the evidential reasoning approach.

For this design option, the safety associated with the crane hydraulic hoist transmission system is evaluated in Ref. 2 and described as follows:[2]

$$S(S_1) = \{(0.111942, \text{"}\mathit{Poor}\text{"})(0.175782, \text{"}\mathit{Average}\text{"}),, (0.451996, \text{"}\mathit{Good}\text{"}), (0.228256, \text{"}\mathit{Excellent}\text{"})\}.$$

By mapping the safety description of the system onto the utility expressions, $U(S_1)$ is obtained as follows:

$$\begin{aligned} U(S_1) = \{&(0.111942, \text{"}\mathit{Slightly\ preferred}\text{"}),\\ &(0.175782, \text{"}\mathit{Moderately\ preferred}\text{"}),\\ &(0.451996, \text{"}\mathit{Preferred}\text{"}),\\ &(0.228256, \text{"}\mathit{Greatly\ preferred}\text{"})\}. \end{aligned}$$

For this design option, there is no extra cost incurred for elimination of serious failure modes. Therefore, the cost incurred for this design option should be considered to be '*Very low*'. Suppose the four designers judge the cost incurred for this design option as follows:

$$C^1_1 = C^2_1 = C^3_1 = C^4_1 = (1/1, 0.75/2, 0/3, 0/4, 0/5, 0/6, 0/7).$$

By mapping the above cost description onto the utility expressions, the following utility descriptions for cost can be obtained:

$$\begin{aligned} U(C^1_1) &= U(C^2_1) = U(C^3_1) = U(C^4_1)\\ &= \{(0, \text{"}\mathit{Slightly\ preferred}\text{"}),\\ &\quad (0, \text{"}\mathit{Moderately\ preferred}\text{"}),\\ &\quad (0, \text{"}\mathit{Preferred}\text{"}), (1, \text{"}\mathit{Greatly\ preferred}\text{"})\}. \end{aligned}$$

Suppose each $m^{jl}_{C_1}$ ($j, l = 1, 2, 3, 4$) is a basic probability assignment to which the cost evaluation produced by designer l for design option 1 supports a hypothesis that the cost incurred for design option 1 is confirmed to the j utility expression. $m^{jl}_{C_1}$ is then given by:

$$m^{jl}_{C_1} = \lambda_l \times \mu^{jl}_{C_1} \quad j, l = 1,2,3,4$$

where each $\mu^{jl}_{C_1}$ ($j, l = 1, 2, 3, 4$) represents the extent to which the cost C^l_1 estimated by designer l is confirmed to the jth utility expression.

The cost incurred for design option 1 can be obtained by synthesising the estimates made by the four designers using the evidential reasoning approach.

$$U(C_1) = \{(0, \text{“}\textit{Slightly preferred}\text{”}), (0, \text{“}\textit{Moderately preferred}\text{”}), (0, \text{“}\textit{Preferred}\text{”}), (1, \text{“}\textit{Greatly preferred}\text{”})\}.$$

4.1.2 Option 2

Eliminating 'hoist up limit failure' and 'hoist down limit failure' associated with the protection system.

'Hoist up limit failure' and 'hoist down limit failure' associated with the protection system are the sixth failure mode and the seventh failure mode of subsystem 4 described in Ref. 2. If design actions are taken to eliminate these two failure modes, the safety associated with them should be considered to be '*excellent*'. Suppose the four designers judge the safety associated with these two failure modes as follows:

$$S(S_2^{46}(1)) = S(S_2^{46}(2)) = S(S_2^{46}(3)) = S(S_2^{46}(4)) = \{(0, \text{“}\textit{Poor}\text{”}), (0, \text{“}\textit{Average}\text{”}), (0, \text{“}\textit{Good}\text{”}), (1, \text{“}\textit{Excellent}\text{”})\}$$

$$S(S_2^{47}(1)) = S(S_2^{47}(2)) = S(S_2^{47}(3)) = S(S_2^{47}(4)) = \{(0, \text{“}\textit{Poor}\text{”}), (0, \text{“}\textit{Average}\text{”}), (0, \text{“}\textit{Good}\text{”}), (1, \text{“}\textit{Excellent}\text{”})\}$$

where each $S_2^{46}(l)$ ($l = 1, 2, 3, 4$) represents the safety associated with the sixth failure mode of subsystem 4 estimated by designer l, and each $S_2^{47}(l)$ ($l = 1, 2, 3, 4$) represents the safety associated with the seventh failure mode of subsystem 4 estimated by designer l.

The safety associated with these two failure modes for this design option can be obtained by synthesising the four designers' judgements using the evidential reasoning approach.

$$S(S_2^{46}) = \{(0, \text{“}\textit{Poor}\text{”}), (0, \text{“}\textit{Average}\text{”}), (0, \text{“}\textit{Good}\text{”}), (1, \text{“}\textit{Excellent}\text{”})\}$$

$$S(S_2^{47}) = \{(0, \text{“}\textit{Poor}\text{”}), (0, \text{“}\textit{Average}\text{”}), (0, \text{“}\textit{Good}\text{”}), (1, \text{“}\textit{Excellent}\text{”})\}.$$

The above safety descriptions can be used to replace the corresponding descriptions presented in Ref. 2. The evidential reasoning approach is then employed to carry out hierarchical evaluations to obtain the safety synthesis associated with the crane hydraulic hoist transmission system as follows:

$$S(S_2) = \{(0.099336, \text{“}\textit{Poor}\text{”}), (0.151830, \text{“}\textit{Average}\text{”}), (0.372342, \text{“}\textit{Good}\text{”}), (0.343967, \text{“}\textit{Excellent}\text{”})\}.$$

By mapping the safety description of the system onto the utility expressions, $U(S_2)$ can be obtained as follows:

$$U(S_2) = \{(0.099336, \text{“}\textit{Slightly preferred}\text{”}), (0.151830, \text{“}\textit{Moderately preferred}\text{”}), (0.372342, \text{“}\textit{Preferred}\text{”}), (0.343967, \text{“}\textit{Greatly preferred}\text{”})\}.$$

The cost for eliminating the hoist up limit failure and hoist down limit failure associated with the protection system is considered to be '*Moderately low*' and may vary about '*Moderately low*'. Suppose the four designers estimate the cost as follows:

$$C_2^1 = (0/1, 0.2/2, 1/3, 0.5/4, 0/5, 0/6, 0/7).$$

$$C_2^2 = (0/1, 0.1/2, 1/3, 0.6/4, 0.2/5, 0/6, 0/7).$$

$$C_2^3 = (0/1, 0.25/2, 1/3, 0.5/4, 0/5, 0/6, 0/7).$$

$$C_2^4 = (0/1, 0.3/2, 1/3, 0.6/4, 0.1/5, 0/6, 0/7).$$

By mapping the above cost descriptions onto the utility space, the following utility descriptions for cost can be obtained:

$$U(C_2^1) = \{(0.054309, \text{“}\textit{Slightly preferred}\text{”}), (0.066442, \text{“}\textit{Moderately preferred}\text{”}), (0.821848, \text{“}\textit{Preferred}\text{”}), (0.057400, \text{“}\textit{Greatly preferred}\text{”})\}$$

$$U(C_2^2) = \{(0.102638, \text{“}\textit{Slightly preferred}\text{”}), (0.134831, \text{“}\textit{Moderately preferred}\text{”}), (0.657202, \text{“}\textit{Preferred}\text{”}), (0.105330, \text{“}\textit{Greatly preferred}\text{”})\}$$

$$U(C_2^3) = \{(0, \text{“}\textit{Slightlypreferred}\text{”}), (0, \text{“}\textit{Moderatelypreferred}\text{”}), (1, \text{“}\textit{Preferred}\text{”}), (0, \text{“}\textit{Greatlypreferred}\text{”})\}$$

$$U(C_2^4) = \{(0.067060, \text{“}\textit{Slightly preferred}\text{”}), (0.083011, \text{“}\textit{Moderately preferred}\text{”}), (0.777240, \text{“}\textit{Preferred}\text{”}), (0.072689, \text{“}\textit{Greatly preferred}\text{”})\}.$$

The above four judgements can be synthesised to obtain the following evaluation:

$$U(C_2) = \{(0.007316, \text{“}\textit{Slightly preferred}\text{”}), (0.009729, \text{“}\textit{Moderately preferred}\text{”}), (0.967102, \text{“}\textit{Preferred}\text{”}), (0.007588, \text{“}\textit{Greatly preferred}\text{”})\}.$$

4.1.3 Option 3
Eliminating the failure modes involving 'major leak' and 'no output from the package motor' associated with the hydraulic servo transmission system.

'Major leak' and 'no output from the package motor' associated with the hydraulic servo transmission system are the first failure mode and the fourth failure mode of subsystem 5 described in Ref. 2. If design actions are taken to eliminate these two failure modes, the safety associated with them should be considered to be *'excellent'*. Suppose the four designers judge the safety associated with these two failure modes as follows:

$$S(S_3^{51}(1)) = S(S_3^{51}(2)) = S(S_3^{51}(3)) = S(S_3^{51}(4)) = \{(0,\text{"}Poor\text{"}),(0,\text{"}Average\text{"}),(0,\text{"}Good\text{"}),(1,\text{"}Excellent\text{"})\}$$

$$S(S_3^{54}(1)) = S(S_3^{54}(2)) = S(S_3^{54}(3)) = S(S_3^{54}(4)) = \{(0,\text{"}Poor\text{"}),(0,\text{"}Average\text{"}),(0,\text{"}Good\text{"}),(1,\text{"}Excellent\text{"})\}$$

where each $S_3^{51}(l)$ ($l = 1, 2, 3, 4$) represents the safety associated with the first failure mode of subsystem 5 estimated by designer l and each $S_3^{54}(l)$ ($l = 1, 2, 3, 4$) represents the safety associated with the fourth failure mode of subsystem 5 estimated by designer l.

The safety associated with these two failure modes for this design option can be obtained as follows by synthesising the designers' judgements using the evidential reasoning approach.

$$S(S_3^{51}) = \{(0,\text{"}Poor\text{"}),(0,\text{"}Average\text{"}),(0,\text{"}Good\text{"}),(1,\text{"}Excellent\text{"})\}$$

$$S(S_3^{54}) = \{(0,\text{"}Poor\text{"}),(0,\text{"}Average\text{"}),(0,\text{"}Good\text{"}),(1,\text{"}Excellent\text{"})\}.$$

The above safety descriptions can be used to replace the corresponding descriptions presented in Ref. 2. The evidential reasoning approach can then be employed to carry out hierarchical evaluations to obtain the safety synthesis associated with the crane hydraulic hoist transmission system.

$$S(S_3) = \{(0.022057,\text{"}Poor\text{"}),(0.032674,\text{"}Average\text{"}),(0.071220,\text{"}Good\text{"}),(0.844790,\text{"}Excellent\text{"})\}.$$

By mapping the safety description of the system onto the utility expressions, $U(S_3)$ can be obtained as follows:

$$U(S_3) = \{(0.022057,\text{"}Slightly\ preferred\text{"}),(0.032674,\text{"}Moderately\ preferred\text{"}),(0.071220,\text{"}Preferred\text{"}),(0.844790,\text{"}Greatly\ preferred\text{"})\}.$$

The cost for eliminating the failure modes involving 'major leak' and 'no output from the package motor' associated with the hydraulic servo transmission system is considered to be *'Moderately high'* and may vary about *'Moderately high'*. Suppose the four designers estimate the cost as follows:

$$C_3^1 = (0/1,0.2/2,1/3,0.6/4,0.1/5,0/6,0/7).$$

$$C_3^2 = (0/1,0.1/2,1/3,0.6/4,0.2/5,0.1/6,0/7).$$

$$C_3^3 = (0/1,0.3/2,1/3,0.4/4,0/5,0/6,0/7).$$

$$C_3^4 = (0/1,0.3/2,1/3,0.4/4,0/5,0.3/6,0/7).$$

By mapping the above cost descriptions onto the utility expressions, the following utility descriptions for cost can be obtained:

$$U(C_3^1) = \{(0.067604,\text{"}Slightly\ preferred\text{"}),(0.084062,\text{"}Moderately\ preferred\text{"}),(0.777037,\text{"}Preferred\text{"}),(0.071297,\text{"}Greatly\ preferred\text{"})\}$$

$$U(C_3^2) = \{(0.102638,\text{"}Slightly\ preferred\text{"}),(0.134831,\text{"}Moderately\ preferred\text{"}),(0.657202,\text{"}Preferred\text{"}),(0.105330,\text{"}Greatly\ preferred\text{"})\}$$

$$U(C_3^3) = \{(0.067060,\text{"}Slightly\ preferred\text{"}),(0.083011,\text{"}Moderately\ preferred\text{"}),(0.777240,\text{"}Preferred\text{"}),(0.072689,\text{"}Greatly\ preferred\text{"})\}$$

$$U(C_3^4) = \{(0.067060,\text{"}Slightly\ preferred\text{"}),(0.083011,\text{"}Moderately\ preferred\text{"}),(0.777240,\text{"}Preferred\text{"}),(0.072689,\text{"}Greatly\ preferred\text{"})\}.$$

The above four judgements can be synthesised to obtain the following evaluation:

$$U(C_3) = \{(0.017512,\text{"}Slightly\ preferred\text{"}),(0.024162,\text{"}Moderately\ preferred\text{"}),(0.929696,\text{"}Preferred\text{"}),(0.018746,\text{"}Greatly\ preferred\text{"})\}.$$

4.1.4 Option 4

Eliminating the two failure modes associated with the protection system in design option 2 and the two failure modes associated with the hydraulic servo transmission system in design option 3.

The safety associated with these four failure modes has been evaluated in this section and are again represented as follows:

$$S(S_4^{46}) = \{(0,\text{“}Poor\text{”}),(0,\text{“}Average\text{”}),(0,\text{“}Good\text{”}),(1,\text{“}Excellent\text{”})\}$$

$$S(S_4^{47}) = \{(0,\text{“}Poor\text{”}),(0,\text{“}Average\text{”}),(0,\text{“}Good\text{”}),(1,\text{“}Excellent\text{”})\}$$

$$S(S_4^{51}) = \{(0,\text{“}Poor\text{”}),(0,\text{“}Average\text{”}),(0,\text{“}Good\text{”}),(1,\text{“}Excellent\text{”})\}$$

$$S(S_4^{54}) = \{(0,\text{“}Poor\text{”}),(0,\text{“}Average\text{”}),(0,\text{“}Good\text{”}),(1,\text{“}Excellent\text{”})\}.$$

The safety descriptions of the above failure modes can be used to replace the corresponding safety descriptions presented in Ref. 2. The evidential reasoning approach can then be employed to carry out hierarchical evaluations to obtain the safety associated with the hydraulic hoist transmission system.

$$S(S_4) = \{(0.012699,\text{“}Poor\text{”}),(0.018534,\text{“}Average\text{”}),(0.034916,\text{“}Good\text{”}),(0.907020,\text{“}Excellent\text{”})\}.$$

By mapping the above safety description onto the utility space, $U(S_4)$ can be obtained as follows:

$$U(S_4) = \{(0.012699,\text{“}Slightly\ preferred\text{”}),(0.018534,\text{“}Moderately\ preferred\text{”}),(0.034916,\text{“}Preferred\text{”}),(0.907020,\text{“}Greatly\ preferred\text{”})\}.$$

The cost for eliminating the above four failure modes is considered to be ‘*Very high*’ and may vary about ‘*Very high*’. Suppose the four designers estimate the cost as follows:

$$C_4^1 = (0/1,0/2,0/3,0.4/4,1/5,0.3/6,0/7).$$

$$C_4^2 = (0/1,0/2,0/3,0.5/4,1/5,0.2/6,0/7).$$

$$C_4^3 = (0/1,0/2,0/3,0.4/4,1/5,0.2/6,0/7).$$

$$C_4^4 = (0/1,0/2,0/3,0.5/4,1/5,0.25/6,0/7).$$

By mapping the above cost descriptions onto the utility expressions, the following utility descriptions for cost can be obtained:

$$U(C_4^1) = \{(0.059846,\text{“}Slightly\ preferred\text{”}),(0.822751,\text{“}Moderately\ preferred\text{”}),(0.062553,\text{“}Preferred\text{”}),(0.054850,\text{“}Greatly\ preferred\text{”})\}$$

$$U(C_4^2) = \{(0.028571,\text{“}Slightly\ preferred\text{”}),(0.912923,\text{“}Moderately\ preferred\text{”}),(0.031480,\text{“}Preferred\text{”}),(0.027027,\text{“}Greatly\ preferred\text{”})\}$$

$$U(C_4^3) = \{(0.057708,\text{“}Slightly\ preferred\text{”}),(0.826250,\text{“}Moderately\ preferred\text{”}),(0.062819,\text{“}Preferred\text{”}),(0.053223,\text{“}Greatly\ preferred\text{”})\}$$

$$U(C_4^4) = \{(0,\text{“}Slightly\ preferred\text{”}),(1,\text{“}Moderately\ preferred\text{”}),(0,\text{“}Preferred\text{”}),(0,\text{“}Greatly\ preferred\text{”})\}.$$

The above four judgements can be synthesised to obtain the following evaluation:

$$U(C_4) = \{(0.0015137,\text{“}Slightly\ preferred\text{”}),(0.977743,\text{“}Moderately\ preferred\text{”}),(0.005671,\text{“}Preferred\text{”}),(0.004724,\text{“}Greatly\ preferred\text{”})\}.$$

4.2 Design selection

Design selection can be carried out on the basis of the preference degrees associated with the four design options with regard to the particular considerations of cost and safety. The preference degrees associated with the four design options can be obtained by synthesising the safety associated with each design option and the cost incurred for each option using the evidential reasoning approach. If cost and safety are considered to be equally important in the design process (i.e., $\omega = 1$), then preference descriptions and preference degrees associated with the four design options are calculated as follows:

4.2.1 Option 1

$$U(U_1) = \{(0.020365, \text{"}Slightly\ preferred\text{"}), (0.031979, \text{"}Moderately\ preferred\text{"}), (0.082230, \text{"}Preferred\text{"}), (0.845756, \text{"}Greatly\ preferred\text{"})\}$$

$$P_1 = 0.020365 \times 0.217 + 0.031979 \times 0.478 + 0.082230 \times 0.739 + 0.845756 \times 1 + 0.019669 \times 0.6085 = 0.938198.$$

4.2.2 Option 2

$$U(U_2) = \{(0.017574, \text{"}Slightly\ preferred\text{"}), (0.027231, \text{"}Moderately\ preferred\text{"}), (0.881336, \text{"}Preferred\text{"}), (0.057597, \text{"}Greatly\ preferred\text{"})\}$$

$$P_2 = 0.017574 \times 0.217 + 0.027231 \times 0.478 + 0.0881336 \times 0.739 + 0.057597 \times 1 + 0.016263 \times 0.6085 = 0.735630.$$

4.2.3 Option 3

$$U(U_3) = \{(0.014738, \text{"}Slightly\ preferred\text{"}), (0.021939, \text{"}Moderately\ preferred\text{"}), (0.608031, \text{"}Preferred\text{"}), (0.322949, \text{"}Greatly\ preferred\text{"})\}$$

$$P_3 = 0.014738 \times 0.217 + 0.021939 \times 0.478 + 0.608031 \times 0.739 + 0.322949 \times 1 + 0.032342 \times 0.6085 = 0.805649.$$

4.2.4 Option 4

$$U(U_4) = \{(0.005851, \text{"}Slightly\ preferred\text{"}), (0.553364, \text{"}Moderately\ preferred\text{"}), (0.017471, \text{"}Preferred\text{"}), (0.382421, \text{"}Greatly\ preferred\text{"})\}$$

$$P_4 = 0.005851 \times 0.217 + 0.553364 \times 0.478 + 0.017471 \times 0.739 + 0.382421 \times 1 + 0.040904 \times 0.6085 = 0.685990.$$

It can be noted that in this case (i.e., $\omega = 1$) design options 1, 2, 3 and 4 have preference degrees 0.938198, 0.735630, 0.805649 and 0.685990, respectively. Therefore, design option 1 is ranked first, design option 3 second, design option 2 third and design option 4 last. This implies that for $\omega = 1$ the first choice is the one in which no serious failure modes are eliminated, the second choice is the one in which the two failure modes associated with the hydraulic servo transmission system are eliminated, the third choice is the one in which the two failure modes associated with the protection system are eliminated and the last choice is the one in which the four failure modes are eliminated. It can be concluded that when safety and cost are considered to be equally important the existing design is selected without taking any risk reduction measures.

For different ω values, preference degrees associated with the four design options can be calculated in a similar way and are shown in Table 7 and Fig. 6.

From Table 7, the ranking of the four design options with regard to each ω value is made as shown in Table 8. It should be pointed out that the four design options can only be ranked with respect to one ω value. This implies that preference degrees associated with the four design options with respect to different ω values cannot be compared.

When ω is equal to 0.1 and 0.3, respectively, design option 4 has the largest preference degree and is ranked first, design option 3 is ranked second, design option 2 third, and design option 1 last. This is because in these cases safety is considered to be more important than cost in the design process. Therefore, the best design option is the one in which all four serious failure modes are eliminated while the last choice is the one in which no serious failure modes are eliminated. As the importance of cost reduction increases, the ranking of the four design options changes. When ω reaches 0.6, design option 2 becomes the last choice. When ω is equal to 1.5, design option 1 is ranked first, design option 3 second, design option 2 third and design option 4 last. In this case, since cost aspects are considered to be more important than safety aspects in the design process, the best design is the one in which no serious failure modes are eliminated, and the last choice is the one in which the four serious failure modes are eliminated. As the relative importance of cost against safety

Table 7. Preference degrees associated with the four design options

ω	Preference degrees			
	design option 1	design option 2	design option 3	design option 4
0.1	0.698488	0.732714	0.931981	0.956536
0.3	0.720952	0.733107	0.925529	0.948379
0.6	0.776641	0.734066	0.906933	0.921740
1	0.938198	0.735630	0.805649	0.685990
1.5	0.991097	0.734625	0.740475	0.507232
2	0.994650	0.734016	0.733754	0.494754
2.5	0.996093	0.733779	0.731385	0.490583
5	0.997490	0.733329	0.728413	0.485995
8	0.997603	0.733127	0.727599	0.484987
12	0.998070	0.733144	0.727296	0.484326

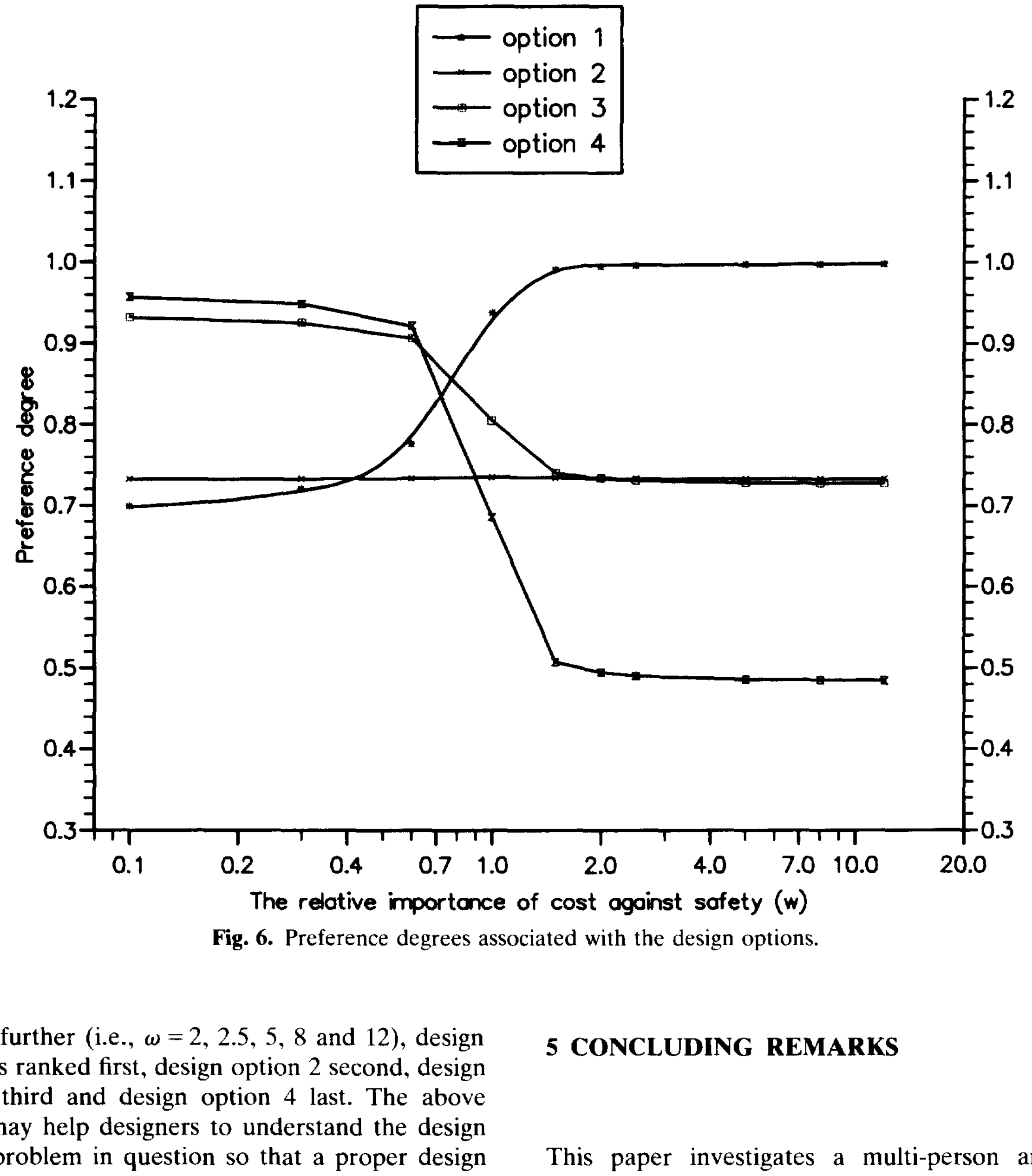

Fig. 6. Preference degrees associated with the design options.

increases further (i.e., $\omega = 2$, 2.5, 5, 8 and 12), design option 1 is ranked first, design option 2 second, design option 3 third and design option 4 last. The above analysis may help designers to understand the design decision problem in question so that a proper design option can be selected with respect to the particular requirements on cost and safety.

Table 8. The ranking of the four design options

ω	Ranking			
	design option 1	design option 2	design option 3	design option 4
0.1	4	3	2	1
0.3	4	3	2	1
0.6	3	4	2	1
1	1	3	2	4
1.5	1	3	2	4
2	1	2	3	4
2.5	1	2	3	4
5	1	2	3	4
8	1	2	3	4
12	1	2	3	4

5 CONCLUDING REMARKS

This paper investigates a multi-person and multi-attribute design selection approach based on subjective safety and cost analyses. This approach can be used as a tool to deal with practical design selection problems particularly in those situations where PRA is not applicable to make safety-based design decisions due to incomplete safety and cost data. This approach provides a rational way of articulating and processing subjective safety and cost information which may be produced by multiple designers with respect to multiple design options.

Although the developed modelling approach only deals with safety and cost objectives in this paper, it can easily be modified to deal with other objectives. It is believed that the developed modelling approach will be potentially useful in safety based decision making as there is often a lack of precise safety data for use in PRA in the initial design of large engineering products.[7]

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