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Use of Advances in Technology for Maritime Risk Assessment

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The maritime industry is moving toward a "goal-setting" risk-based regime. This opens the way to safety engineers to explore and exploit flexible and advanced risk modeling and decision-making approaches in the design and operation processes. In this article, following a brief review of the current status of maritime risk assessment, a design/operation selection framework and a design/operation optimization framework are outlined. A general discussion of control engineering techniques and their application to risk modeling and decision making is given. Four novel risk modeling and decision-making approaches are then outlined with illustrative examples to demonstrate their use. Such approaches may be used as alternatives to facilitate risk modeling and decision making in situations where conventional techniques cannot be appropriately applied. Finally, recommendations on further exploitation of advances in general engineering and technology are suggested with respect to risk modeling and decision making.

KEY WORDS: Control theory; decision making; risk assessment; risk modeling

1. MARITIME RISK ASSESSMENT

Reliability analysis and safety analysis are two different processes, although there is a considerable overlap (and often confusion) between them. They both refer to the studies of process and equipment failures or operability. Reliability analysis of an item studies its characteristics expressed by the probability that it will perform a required function under stated conditions for a stated period of time. If such an analysis is extended to include the study of the consequences of the failures of the item in terms of possible damage to property, injury/death of people, and/or the degradation of the environment, the study is referred to as safety analysis (risk assessment).

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In the maritime industry, over recent years, quite a few serious accidents including the capsize of the *Herald of Free Enterprise* and the *Exxon Valdez* tragedy have shocked the public and attracted great attention to safety. Studies on how similar accidents may be prevented have been actively carried out at both the national and international levels. After Lord Carver's report on the investigation of the capsize of the *Herald of Free Enterprise* was published in 1992, the U.K. Maritime & Coastguard Agency (MCA) quickly responded and in 1993 proposed to the International Maritime Organization (IMO) that formal safety assessment should be applied to ships to ensure a strategic control of safety and pollution prevention (MSA, 1993, 1996; Sekimizu, 1997). The guidelines for the application of formal safety assessment have been recently approved for rule/regulation making purposes by the IMO. At the moment, one of the major concerns on the practical application of formal ship safety assessment is associated with the simplification of the approach and the study of trial test cases for producing more detailed guidelines to facilitate its

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application while human and organizational elements that significantly influence quality, safety, etc., also need to be addressed in detail accordingly.

In the U.K. offshore industry, a safety case approach was introduced in 1993 following the public inquiry into the *Piper Alpha* accident of July 6, 1988. The safety case regulations were amended in 1996 to include verification of safety-critical elements. The Offshore Installations and Wells (Design and Construction, etc.) Regulations 1996 (DCR'96) were introduced to deal with various stages of the life cycle of the installation (HSE, 1996). The main feature of the new offshore safety regulations in the United Kingdom is the absence of a prescriptive regime, defining specific duties of the operator as regard to what are adequate means. The regulations set forth a high-level safety objective while leaving the selection of particular arrangements to deal with hazards in the hands of the operator. This is in recognition of the fact that hazards related to an installation are specific to its function and site conditions.

Recently, the industrial guidelines on a framework for risk-related decision support have been produced by the U.K. Offshore Operators Association (UKOOA) (UKOOA, 1999). In general, the framework could be usefully applied to a wide range of situations. In particular, it provides a sound basis for evaluating the various options that need to be considered at the feasibility and concept selection stages of a project. It can also be combined with other formal decision-making aids such as Analytical Hierarchy Process (AHP) (UKOOA, 1999).

As far as port safety is concerned, the guidelines indicating a general framework on port safety in the United Kingdom came from "Safety in Docks—Port Regulations and Guidance" (Health & Safety Commission, 1988). The current status of port safety shows that there is a close relation between the MCA and the port authorities in order to ensure adequate levels of safety and pollution prevention in U.K. ports. It is again a case of leaving the operators to decide on the ways to deal with possible hazards instead of setting the path that they should follow in each case. What is needed is an application of formal safety assessment methods for handling situations arising in any kind of terminal with just minor modifications in the factors influencing them. This means that the methods applied in a chemical refinery dock when the vessel is undertaking loading or unloading of cargo can be equally applied, with the domain knowledge, to a container or a Ro/Ro terminal. The "Port Marine Safety Code" recently produced by the DETR (Department of the Environment, Transport and the Regions, U.K.) introduces a national standard for every aspect of port marine safety in the United Kingdom (DETR, 2000). It has also stimulated research in the areas of port safety assessment.

Many leading maritime organizations have started the move away from prescription toward a risk-based regime to assist in maintaining capability throughout the life cycle of maritime products. Such a change will create new perspectives on risk modeling and safety-based decision making. It is believed that a change from "tell me what to do" to "shown me how to do it" and to "involve me in it" will also take place in the maritime industry. This can certainly encourage safety engineers to develop and apply more flexible risk modeling and decision-making approaches from the advances in general engineering and technology.

2. MAJOR PROBLEMS IN MARITIME RISK ASSESSMENT

A ship, an offshore installation, or a port system is usually an expensive, large, and complex engineering structure, made up of many subsystems that must be carefully integrated to form a complete working system. Each product may be a unique commission ordered by a customer for a specific purpose and location.

When designing a large maritime product, at the initial design stages, there are usually several design options produced for selection. Selecting the most effective design option is usually time consuming (Moan & Berge, 1997). The decisions made at the early design stages may have a more significant impact on system performance than those at any other stage in its life cycle. It should be noted that when such options are produced at the top level, only nonnumerical data, which could be subjective, may be available. The information available for making risk-based decisions on which option to select at this design stage may be incomplete or the level of uncertainty associated with the failure data may be unreasonably high. As a design proceeds to a more detailed stage, the selection of design options at lower levels is required and again the similar process for selecting a particular design option may be required where such problems still exist. As design further proceeds, decision making may move from design selection to design optimization where quantitative risk-assessment-based decisions are made. It should be noted that the decision-making process at all levels may involve the treatment of uncertain or incomplete information.

Maritime product design is a broad-based activity. The design process combines creativity, empiricism, theory, and practice while the range of influencing factors and diversity of applications may require the latest technology to be utilized. There are many difficulties in the general design process of maritime engineering products in the context of risk assessment due to their made-to-order nature. The typical ones include:

1. The nonexistence or inadequacy of historical data for many novel designs.
2. The impracticability of full-scale experimentation with many new design aspects due to a high level of cost, though computer simulation might be potentially available or possible.
3. Difficulty of replacing or modifying them (a new build or a new design) once on location and in operation.

The operation of a ship, an offshore installation, or a port is also associated with a high level of uncertainty because it usually operates in a very changeable environment while human error and organizational malfunctions play an important role in many possible accidents. To facilitate safety-based design/operation decision making, it is required to model risks in various situations with confidence. This certainly needs flexible risk modeling and decision-making techniques to be developed and applied.

3. RISK ASSESSMENT AND DECISION MAKING

The design process of a maritime engineering product may be highly simplified as a sequence, including, for instance, feasibility design stage, concept design stage, preengineering stage, detail engineering stage, construction stage, commissioning, and start up stage (Vinnem & Hope, 1986). Both the feasibility and concept design stages usually form the initial design stages of a maritime engineering product. Selecting the most effective design option is usually time consuming; late decisions may eventually jeopardize the balance of the whole project (Moan & Berge, 1997; Patel, 1993).

The purpose of the study in the feasibility design stage is to evaluate whether or not further development of a design project is technically feasible and commercially favorable. The safety evaluation at the feasibility design stage usually plays a relatively subordinate role regarding whether to develop the project or not. The results should therefore be given as a ranking of the alternatives rather than as estimates of absolute levels of risk. As a result, in the feasibility stage, risk analysis is carried out to compare/rank alternative solutions. In addition, it will also identify areas of uncertainty where detailed studies may need to be carried out later. Risk analysis during the concept design stage of a maritime product aims at providing safety-related input in the process of developing an acceptable design.

A general design selection framework is shown in Fig. 1. At the initial design stages, incomplete data and high level of uncertainty may not allow traditional methods to be effectively and efficiently applied to model safety and other design objectives for making design/operation decisions or selecting the most desirable options. Therefore, flexible algorithms for modeling safety and other design objectives in such situations are needed.

Once the best design option is chosen, the design is further developed. More information becomes available for more detailed safety analysis. Decision making may need to be carried out at the next level. At this stage, it may be the case that only part of the information is complete for safety modeling while the remainder is still incomplete. This may also be true for modeling other objectives, such cost, that are considered in the decision-making process. As the design proceeds, it moves to a stage where sufficient data allows design optimization to be carried out. A framework for design/operation optimization is shown in Fig. 2.

The test case studies also need to be carried out in order to provide guidelines for design/operation decision making.

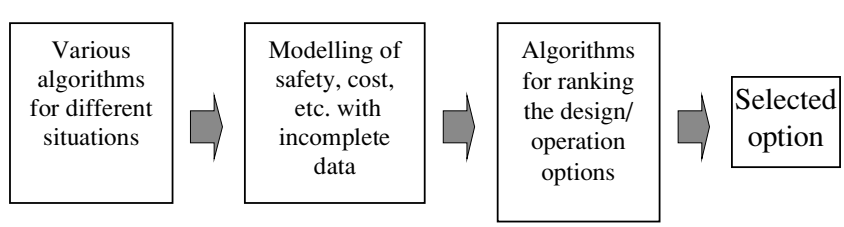

Fig. 1. A general framework for design/operation selection.

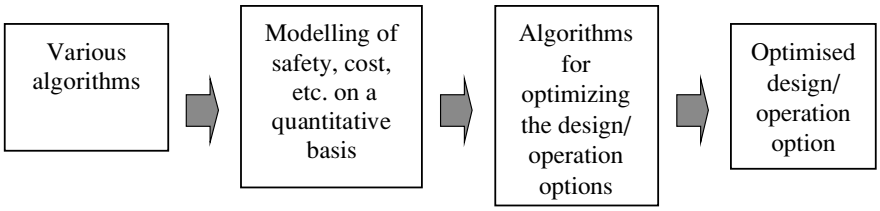

Fig. 2. A general framework for design/operation optimisation.

4. CONTROL ENGINEERING TECHNIQUES AND THEIR APPLICATION TO RISK MODELLING AND DECISION MAKING

Traditionally, risk assessment is usually carried out either using a top-down approach or a bottom-up approach, depending on the availability of failure data, the level of the analysis required, the degree of complexity of the interrelationships of the design, and the level of innovation in the design (Wang *et al.*, 1995). A top-down safety assessment of a system starts with the identification of the top events that can be obtained from previous accident and incident reports of similar systems. Once the top events required to be studied further are determined, the causes leading to them can be identified deductively in increasing detail until the causes are identified at the required level of resolution. It should be pointed out that the success of a top-down safety assessment is highly dependent on the reliability of failure data in previous incident and accident reports, where the proper investigation and appropriate data recording of incidents and accidents are vital. In a bottom-up safety assessment process, a system can be divided into subsystems that can be further broken down to the component level in order to identify all possible hazards. In such a process, theoretically the identification of all possible hazards is achievable although in practice it is always possible to have omissions. All combinations of possible failure events at both the component and the subsystem levels may be studied to identify the possible serious failure events. Finally, risk evaluation and design review can be conducted. Compared to the top-down safety assessment, the bottom-up analysis provides safety engineers with a high level of confidence that the possible hazards and serious failure system events are identified with less/no omissions. Consequence analysis can also be carried out to study the possible effects caused by the occurrence of each identified top event.

In general, risk assessment consists of the following phases, as demonstrated in the methodology proposed by the UK MCA (MSA, 1993, 1996):

1. Identification of hazards.
2. Assessment of the risks associated with those hazards.
3. Identification of ways of managing the risks identified.
4. Cost-benefit assessment of the options.
5. Making decisions on which options to select.

The above can be applied to different phases of the design and operation processes of maritime engineering systems. Many techniques widely used in control engineering may be adopted to deal with such problems in risk modeling and decision making. Some typical examples include:

1. Approximate reasoning approach for dealing with problems associated with a high level of uncertainty (Wang *et al.*, 1995; Wang, 1997a). This includes subjective safety-based decision-making methods (Wang *et al.*, 1996; Wang & Kieran, 2000), evidential reasoning techniques, fuzzy set modeling methods, and the Dempster-Shafer method for risk modeling and decision making (Sii *et al.*, 2000b, 2001b).
2. Safety-based design/operation optimization approach (Wang *et al.*, 1996; Wang & Kieran, 2000).
3. Application of artificial neural network approach for risk estimation (Sii *et al.*, 2000a; Sii, 2001; Sii *et al.*, 2001a).
4. Application of the approaches developed in operational research (for example, delay time concept) for maintenance optimization (Pillay, 2001; Pillay *et al.*, 2001).

It should be mentioned that the above is only a partial list that has been investigated by the authors, and there are more techniques in general engineering and technology that may be applied to facilitate risk modeling and decision making in maritime design and operations.

In the above examples or techniques, (1) and (3) are developed to model situations under a high level of uncertainty. Decision making under uncertainty

has been studied by many researchers and different approaches such as the empirical Bayesian method and fuzzy set method have been exploited. This is also true in the maritime context although only limited practical applications of such approaches can be found. Examples of techniques (2) and (4) can be used to improve both design aspects and operational strategies based on quantitative analysis. This article is aimed at demonstrating how some advances in general engineering and technology can be exploited for risk modeling and decision making in maritime design and operations. The above approaches possess potential as valuable aids and effective alternatives in maritime risk-assessment-based design and operations. It is believed that practical applications of these approaches will result from utilization by organizations who deal with safety-based selection problems having a high level of uncertainty and insufficient safety data or with safety-based optimization problems. In such cases, the use of the above methodologies could have a highly beneficial effect. In general, the following areas in risk assessment can significantly benefit from advances in general engineering and technology:

1. Hazard identification.
2. Risk estimation.
3. Risk-based design/operation strategy selection.
4. Risk-based design/operation optimization.
5. Risk-based operations/maintenance.

5. FOUR RISK MODELING AND DECISION-MAKING APPROACHES

In order to demonstrate the application of the advances in general engineering and technology, the following four novel risk modeling and decision-making approaches are described:

1. An approximate reasoning approach.
2. A risk estimation framework using artificial neural network (ANN) techniques.
3. An approach for optimizing maintenance strategies using the delay time concept.
4. An optimization approach for studying both design and operational aspects using multiple objective decision-making approaches.

Possible applications of the above four methods are briefly outlined hereunder, in the context of design and/or operation selection and optimization.

5.1. An Approximate Reasoning Approach

The last two decades have seen many studies in the areas of nonprobabilistic theories and their applications (Apostolakis *et al.*, 1993). Coolen and Newby (1994) studied Bayesian modeling with imprecise prior probabilities. An extension of the standard Bayesian approach based on the theory of imprecise probabilities and intervals of measures was developed to reflect expert opinions using prior distributions. The opinions of several experts can be combined using the approach developed. Karwowski and Mital (1988) investigated modeling of risk using approximate reasoning and fuzzy sets. Linguistic variables were used to assess the risk of an event and hazardous events were modeled using fuzzy set theory (Keller & Kara-Zaitri, 1989). Fuzzy set theory was used for uncertainty analysis (Chun & Ahn, 1992). The potential applicability of fuzzy set theory to uncertainty analysis of accident progression event trees with imprecise and uncertain branch probabilities and/or with a number of phenomenological uncertainty issues was examined as a possible alternative procedure to that used in probabilistic risk assessment.

In general, uncertainties in maritime risk assessment are highly relevant to impression associated with the complexity of a system as well as vagueness of human judgments. Our literature search indicates that to deal with uncertainty problems in the maritime context, an approximate reasoning approach is expected to have the following characteristics:

1. Risks are modeled in a hierarchical process where the loss of data in synthesis is acceptably low.
2. Experts/safety analysts can judge an event using a number of measurable parameters on a subjective basis and their judgments can be synthesized on a rational basis.
3. Several objectives including safety should be modeled consistently in order to combine them to enable the designers to make the most suitable decisions at the appropriate stage in a new design/operation process.

Many developed methods have been investigated in terms of their applicability in the context of maritime design and operations and it seems that no method available has the above characteristics. This article gives a novel method based on fuzzy set theory and the evidential reasoning approach. This method,

capable of addressing the above characteristics, is described as follows.

In many circumstances, it may be difficult for conventional techniques such as quantitative risk analysis to be applied due to the high level of uncertainty in failure data or the qualitative nature of failure data. Safety analysts may often have to use subjective descriptors to describe the safety associated with an event or an element. To assess the fuzzy safety associated with an event or an element, it is required to synthesize the associated occurrence likelihood, consequence severity, and failure consequence probability (the probability that consequences happen given the occurrence of the event) (Karwowski & Mital, 1986; Wang *et al.*, 1995). These three parameters can be judged by safety analysts using fuzzy sets and the judgments produced can then be synthesized. To estimate the failure likelihood, for example, one may often use such variables as "*highly frequent,*" "*frequent*," "*reasonably frequent*," "*average*," "*reasonably low*," "*low*," and "*very low*;" to estimate the consequence severity, one may often use such variables as "*catastrophic*," "*critical*," "*marginal*," and "*negligible*;" and to estimate the failure consequence probability, one may often use such variables as "*definite*," "*highly likely*," "*reasonably likely*," "*likely*," "*reasonably unlikely*," "*unlikely*," and "*highly unlikely*." It should be pointed out that the consequence severity, the failure likelihood, or the failure consequence probability of an event or an element may not belong to only one of the linguistic descriptions used to describe the respective extent. For example, if the consequence severity of an event or an element is between "*catastrophic*" and "*critical*" or even between "*catastrophic*" and "*negligible*," it may be described in terms of several linguistic variables and the membership degrees belonging to them.

Linguistic variables used in describing failure likelihood, consequence severity, and failure consequence probability can be characterized by their membership functions to a set of categories that describe the degrees of failure likelihood, severity class, and failure consequence probability and that are usually graduated from low to high. It is often recommended that the number of categories be restricted to no more than seven to remain within the practical bounds of human discrimination (Karwowski & Mital, 1986). For instance, if $U = \{1, 2, 3, \ldots, 7\}$ represents a set of categories, the linguistic variable "*catastrophic*" may be modeled by:

$$\text{“}catastrophic\text{”} = \{1/0, \ldots, 5/0, 6/0.75, 7/1.0\}$$

where the integers in the numerators of each term within the brackets represent the categories and the real numbers in the denominators stand for the membership degrees.

The membership values for the components in U belonging to the linguistic variable "*catastrophic*" can thus be denoted as follows:

$$\mu_{catastrophic} = (0, \ldots 0, 0.75, 1.0).$$

If L, C, and E represent the fuzzy sets of the failure likelihood, consequence severity, and failure consequence probability of a failure mode, the fuzzy safety score S can be defined using the following fuzzy set manipulation:

$$S = C^{\circ} E \times L$$

$$\mu_S = \mu_{C^{\circ} E \times L} = \left(\mu_S^1, \ldots, \mu_S^j, \ldots\right)$$

where "$\circ$" represents the composition operation and "$\times$" the Cartesian product operation in fuzzy set theory.

The Cartesian product of E and L is defined by

$$\mu_{E \times L} = \left(\mu_{E \times L}^{ij}\right)_{7 \times 7}$$

where $\mu_{E \times L}^{ij} = \min(\mu_E^i, \mu_L^j)$, $i, j = 1, 2, \ldots, 7$; $\mu_S^j = \max(\min(\mu_C^1, \mu_{E \times L}^{1j}), \ldots, \min(\mu_C^7, \mu_{E \times L}^{7j}))$, $j = 1, 2, \ldots, 7$; μ_S is the description function of safety score S in terms of membership degrees representing the extent to which S belongs to the elements in U.

To express the subjective safety more explicitly, linguistic variables such as "*poor,*" "*average,*" "*fair,*" and "*good*" can be used. For instance, it may be quite clear to state that the safety of a failure mode is to a large extent "*good*." Such linguistic variables ("*poor,*" "*average,*" "*fair,*" and "*good*") are referred to as *safety expressions*. The safety expressions may also be characterized by membership degrees to each element in U. The fuzzy safety description of an event can then be mapped back onto the defined *safety expressions*. The safety can then be obtained as follows:

$$S(S_1) = \{(\beta_1', poor'), (\beta_2', fair'), (\beta_3', average'), (\beta_4', good')\}$$

where β_n ($m = 1, 2, 3$ or 4) represents the extent to which the safety of the event belongs to the mth safety expression.

A safety model is usually a hierarchical structure with multiple layers where:

1. Judgments on an event at the bottom level of the hierarchy made by multiple experts need to be synthesized.
2. Safety synthesis needs to be carried out at the next level.
3. Safety synthesis is progressed up to the top level where the safety estimation of the system can be obtained.

The hierarchical structure of a safety model can be formulated by studying the system under investigation. The system is composed its constituent subsystems, which can be further broken down to the component level. Each component is associated with certain failure modes. The subsystems, components, and failure modes may carry different weights when synthesizing the safety of the system in such a hierarchy. The weight of an element in a synthesis level may be judged on a subjective basis in terms of its contribution to the safety of the associated element in the upper level. An operation process can also be represented in such a hierarchical structure. For example, the safety associated with the initial shooting operation of fish nets of a fishing vessel is determined by the operations of the associated derricks, lazy deckies, nets, and winches. The safety associated with the operation of each derrick, lazy decky, net, or winch may be further determined by human error, external events, etc. (Pillay & Wang, 2003).

The technique that is used to carry out the above synthesis is the evidential reasoning approach, which is based on the principle that if more pieces of evidence (each may carry different weight) support a hypothesis then it is more likely that it is true (Yang & Sen, 1994; Yang & Singh, 1994; Yang, 2001; Yang & Xu, 2002a). The evidential reasoning approach has the advantage of synthesizing safety estimates without loss of any data and also that uncertainties in safety estimates are handled in a rational manner.

Suppose M_n^m ($m = 1, 2, 3$ or 4; $n = 1, \ldots,$ or N) is a degree to which $S(S_n)$ (safety judged by expert n) supports the hypothesis that the safety evaluation associated with a failure event is H_m ($H_1 =$ *"poor;"* $H_2 =$ *"fair;"* $H_3 =$ *"average;"* or $H_4 =$ *"good"*). Then, M_n^m can be obtained as follows:

$$M_n^m = \lambda_n \times \beta_n$$

where λ_n is the normalized relative weight of expert n in the safety estimation process.

Suppose M_n^H ($n = 1, \ldots,$ or N) is the remaining belief unassigned after commitment of belief to all H_m ($m = 1, 2, 3$ and 4) for $S(S_n)$. M_n^H can be obtained as follows:

$$M_n^H = 1 - \sum_{m=1}^{4} M_n^m.$$

Suppose MM_n^m ($m = 1, 2, 3$ or 4; $n = 1, \ldots,$ or N) represents the degree to which the safety associated with the event belongs to H_m as a result of the synthesis of the judgments produced by safety analysts $1, \ldots,$ and n. Suppose MM_n^H represents the remaining belief unassigned after commitment of belief to all H_m ($m = 1, 2, 3$ and 4) as a result of the synthesis of the judgments produced by safety analysts $1, \ldots,$ and n. The algorithm for synthesizing the analysts' judgments to obtain the safety evaluation associated with the event can be stated as follows (Wang, 1994; Yang *et al.*, 1994):

Initial conditions: $MM_1^m = M_1^m \quad MM_1^H = M_1^H$

$$\{H_m\} \quad MM_{n+1}^m = K_{n+1}\left(MM_n^m M_{n+1}^m + MM_n^m M_{n+1}^H + MM_n^H M_{n+1}^m\right)$$

$$m = 1, 2, 3, 4$$

$$\{H\} \quad MM_{n+1}^H = K_{n+1} MM_n^H M_{n+1}^H$$

$$K_{n+1} = \left[1 - \sum_{T=1}^{4} \sum_{R=1 R \neq T}^{4} MM_n^T M_{n+1}^R\right]^{-1}$$

$$n = 1, \ldots, N-1$$

$N - 1$ iterations of the above algorithm are required to obtain the degree (i.e., MM_n^m) to which the safety evaluation associated with the event belongs to H_m ($m = 1, 2, 3$ and 4). The safety evaluation associated with the failure event can then be presented in the following form:

$$S(S_{the\ event}) = \{(\beta^1, \text{"}poor\text{"}), (\beta^2, \text{"}fair\text{"}), (\beta^3, \text{"}average\text{"}), (\beta^4, \text{"}good\text{"})\}$$

where β^m ($m = 1, 2, 3$ and 4) is equal to MM_N^m.

It is worth mentioning that the order in which safety estimates are combined does not make any difference in terms of the final synthesis using the above algorithm. It is also worth mentioning that the sum of β^m ($m = 1, 2, 3$ and 4) may not be equal to 1 at the end of synthesis using the above algorithm. This is because there may be still some unassigned belief to $S(S_{the\ event})$ due to the incompleteness of the safety estimates that are combined. The evidential reasoning algorithm has the advantage that in theory the total

unassigned belief decreases as more safety estimates are synthesized.

In a hierarchical structure with multiple layers, the above synthesis can be used to obtain the safety estimate for an event at the bottom level. Then the evidential reasoning algorithm can be used again to obtain safety synthesis at the next level in the hierarchy. Such a synthesis can be eventually progressed up to the top level where the safety associated with the system can be obtained as follows:

$$S(S) = \{(\beta 1, \text{“}poor\text{”}), (\beta 2, \text{“}fair\text{”}), (\beta 3, \text{“}average\text{”}), (\beta 4, \text{“}good\text{”})\}$$

where βm ($m = 1, 2, 3$ or 4) represents the extent to which the safety of the system belongs to the mth safety expression.

Cost can also be modeled in a similar manner. Given the relative importance of cost against safety, the safety and cost estimates can be synthesized, using the evidential reasoning approach, to obtain the preference estimate $U(i)$ associated with design/operation option i as follows:

$$U(i) = \{(\mu_{Ui}^1, \text{“}slightly\ preferred\text{”}), (\mu_{Ui}^2, \text{“}moderately\ preferred\text{”}), (\mu_{Ui}^3, \text{“}preferred\text{”}), (\mu_{Ui}^4, \text{“}greatly\ preferred\text{”})\}$$

where each μ_{Ui}^m ($m = 1, 2, 3, 4$) represents an extent to which the utility associated with design/operation option i belongs to the mth *utility expression* (“*slightly preferred*,” “*moderately preferred*,” “*preferred*,” or “*greatly preferred*”).

Preference degree P_i associated with design/operation option i is obtained by (Wang *et al.*, 1996):

$$P_i = \sum_{j=1}^{4} \mu_{Ui}^j \times K_j + \left(1 - \sum_{j=1}^{4} \mu_{Ui}^j\right) \times 1/4 \times \sum_{j=1}^{4} K_j$$

where K_1, K_2, K_3, K_4 are the utility degrees associated with the four *utility expressions,* respectively; $(1 - \sum_{j=1}^{4} \mu_{Ui}^j)$ describes the remaining belief unassigned after commitment of belief in the synthesis of cost and safety descriptions; and $1/4 \times \sum_{j=1}^{4} K_j$ is the average value of the K_js. It is worth mentioning that K_1, K_2, K_3, K_4 may not be fixed for different applications and they may be determined by appropriate expert judgments.

Obviously, the larger P_i is the more desirable design/operation option i. The best design/operation option with the largest preference degree can be selected on the basis of the magnitudes of P_i ($i = 1, 2, \ldots D$) if there are several design options available in the design process.

It is noted that only two design/operational objectives are considered in the description of this method. If more design objectives, such as reliability, are dealt with, this method can be easily extended to carry multiple objective decision making. This method may be more appropriate for use in situations where a design of a maritime engineering product is at the initial stages or there is a lack of adequate data for use in quantitative risk assessment.

The above approximate reasoning approach incorporating an evidential reasoning technique is related to but different from traditional methods or frameworks for multiple criteria decision analysis such as expected utility theory. The major differences may be twofold. First, the approximate reasoning approach operates directly on degrees of belief for criteria aggregation rather than utility. This allows uncertainties such as ignorance and vagueness as well as various types of information, which could be of both a quantitative and qualitative nature, to be aggregated consistently without losing their original features as no averaging is imposed during the aggregation process (Yang, 2001; Yang & Xu, 2002a). However, many uncertainties, including ignorance and vagueness, cannot be properly handled using traditional expected utility theory. Second, although the approximate reasoning approach uses utility concepts and measures to indicate average performances at the end of criteria aggregation and provides a generic nonlinear process, it does not require an assumption about the functional structure for criteria aggregation except for assuming that criteria provide independent evidence for assessing alternative designs and also assuming a piecewise linear marginal utility function for a quantitative criterion and equal-distance utility distributions for the assessment grades of a qualitative criterion (Yang & Sen, 1996, 1997; Yang, 2001; Yang & Xu 2002b). Nevertheless, if a multiple criteria decision analysis problem does not have such uncertainties as ignorance and vagueness and a clear structure for its criteria aggregation could be assumed rationally, well-known additive or multiplicative utility function methods could be used.

5.1.1. An Example for Demonstrating the Approximate Reasoning Approach

A real hydraulic hoisting transmission system of an offshore crane is used to demonstrate the above approach (Wang, 1994; Wang *et al.*, 1995). For demonstration purposes, the detailed modeling process will not be described here. Four safety analysts/engineers with sufficient engineering and risk assessment knowledge were used to give their judgments. The opinions given by safety analysts 2 and 3 are twice as important as those given by designers 1 and 4 (the relative importance of safety analysts may be determined by comparing their experience and their confidence level in modeling and also considering the role each plays in the decision-making process). There are four design options, each of which corresponds to certain levels of safety and cost.

Using the approach described above, the preference degrees associated with the four options can be obtained as follows (Wang *et al.*, 1996):

Option 1: $P_1 = 0.938$
Option 2: $P_2 = 0.736$
Option 3: $P_3 = 0.806$
Option 4: $P_4 = 0.686$

The ranking of the four design options is as follows:

Ranking	Options	Preference degrees
1	Option 1	$P_1 = 0.938$
2	Option 3	$P_3 = 0.806$
3	Option 2	$P_2 = 0.736$
4	Option 4	$P_4 = 0.686$

The ranking of the design options varies with the relative importance of cost against safety. Fig. 3 shows the preference degrees associated with the four design options at different values of relative importance of cost against safety (Wang, 1994; Wang *et al.*, 1995). For example, when the cost factor is considered to be five times as important as the safety factor, then the ranking of the four design options is (1) option 1, (2) option 2, (3) option 3, and (4) option 4. Given the relative importance of safety against cost, the ranking of design options can be found in Fig. 3.

From the above example, it can be seen that the experience and qualifications of safety analysts would to a certain extent determine the results of an application of the above approach. However, it is believed that in most cases the application of such a subjective safety-based decision-making process should have a consistency in terms of the influences of the experience and qualifications of safety analysts. This is partially true when in the process of synthesizing the subjective estimates at the bottom level, produced by safety analysts, an evidential reasoning approach, developed based on the Dempster-Shafer theory, is employed (Yang, 2001). Therefore, if more safety analysts are employed who have some experience and qualifications in safety and cost modeling, then reasonably consistent results of an application are expected. In addition, the use of three parameters (failure likelihood, failure consequence severity, and failure consequence probability) may make it easy for a designer/safety engineer to model an event at the bottom level.

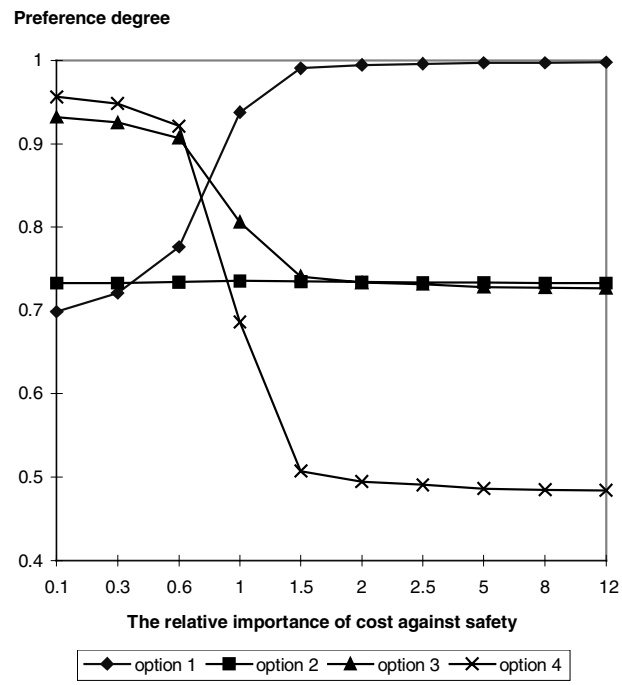

Fig. 3. Ranking of the four design options.

A number of novel safety-based decision-support frameworks for modeling situations with a high degree of uncertainty in safety data have also been investigated recently by the authors. These include:

1. A safety model for risk analysis of offshore engineering products using approximate reasoning and evidential reasoning approaches (Sii, *et al.*, 2001(c), 2001(d), 2001(e), 2001(f); Sii & Wang, 2002).
2. A design-decision-support framework for evaluation of design options/proposals using a fuzzy-logic-based composite structure methodology (Sii, *et al.*, 2001(f), 2001(g)).

3. A design-decision-support framework for evaluation of design options using a composite structure methodology based on the approximate reasoning approach and evidential reasoning method (Sii *et al.*, 2001f).

The first framework is designed for risk analysis of an engineering system having a hierarchical structure involved in safety assessment. In (1) above, safety-rule-based modeling can be employed where the safety estimate of an event is obtained by comparing the memberships of the three parameters (failure rate, consequence severity, and failure consequence probability) with the safety rules in the rule base. The other two frameworks are used for design-decision support, using a fuzzy-logic-based method and using a composite structure grounded in the approximate reasoning approach and evidential reasoning method, respectively. They are suggested for safety-based design evaluation of large engineering products especially at the initial stages. They are multiple attribute decision-making (MADM) frameworks that provide a juxtaposition of cost, safety, and technical performance and other objectives of a system during evaluation to assist decision makers in selecting the winning design/procurement proposal that best satisfies the requirement in hand. It has also been shown that the formal decision-making techniques such as Analytical Hierarchy Process (AHP) and the Delphi method can be incorporated with the proposed frameworks in carrying out safety-based design-support evaluation (Sii *et al.*, 2001f).

5.2. A Risk Estimation Framework Using ANN Techniques

There has been little reported in the application of ANNs in risk assessment. However, it has been noted that the advantages of ANNs can contribute to risk modeling, especially in situations where conventional methods could not be used with confidence to describe the relationship between the input and output variables or there is an inconsistency in input-output relationships (Sii, 2001). An inconsistency in input-output relationship here refers to situations where conventional mathematical models failed to be applied to delineate the input-output relationship due to lack of precise knowledge, or information/data with a high level of fuzziness or ambiguity, or differences (if not contradictory) of opinions about that relationship among the risk analysts. Under such circumstances ANNs may be more appropriate for eliciting the true input-output relationship. Different types of neural networks, such as the Multi-Layer Perceptron (MLP), the radial basis function networks (RBF), and B-spline (Haykin, 1999) networks, etc. can be used to model a system for risk assessment. In the development of an ANN model, success depends upon a clear understanding of the actual problem, as the selection of network inputs, number of hidden layers and number of neurons in each layer, the nonlinear transfer function, and the training algorithm should be based on the features of the problem to be modeled. General guidance on how the features of the problem should influence such choices regarding the model is briefly described as follows (Sii, 2001):

1. There could be multiple inputs and multiple outputs.
2. Experience indicates that one hidden layer would be enough to deal with the majority of risk modeling problems. Using more than one hidden layer will increase the computational load but may achieve faster learning or better generalization.
3. If one hidden layer is used, then the number of neurons in the hidden layer is approximately equal to the number of inputs $\times$ $\log_2$ number of patterns used for training (Roskilly & Mesbahi, 1996).
4. A sigmoid transfer function is usually used while other types of transfer functions may also be applicable.
5. A fast back propagation training algorithm can be used, which is available in the Neural Network Toolbox in MATLAB.

As ANNs learn by example, defining and preparing the training data set is also important. The training data must sample every possibility of the problem under all possible working conditions. The data sets, including the input training set and the desired output, should be as orthogonal as possible, that is, the variables contained in the data sets should be independent with no correlation. Once the problem description and data for the training sets are produced, the rest of the development of the ANN will simply fall into place. ANN testing is performed with a set of test data that is different from the training data used.

The risk estimation framework incorporating ANNs comprises the following steps, as shown in Fig. 4 (Sii, 2001):

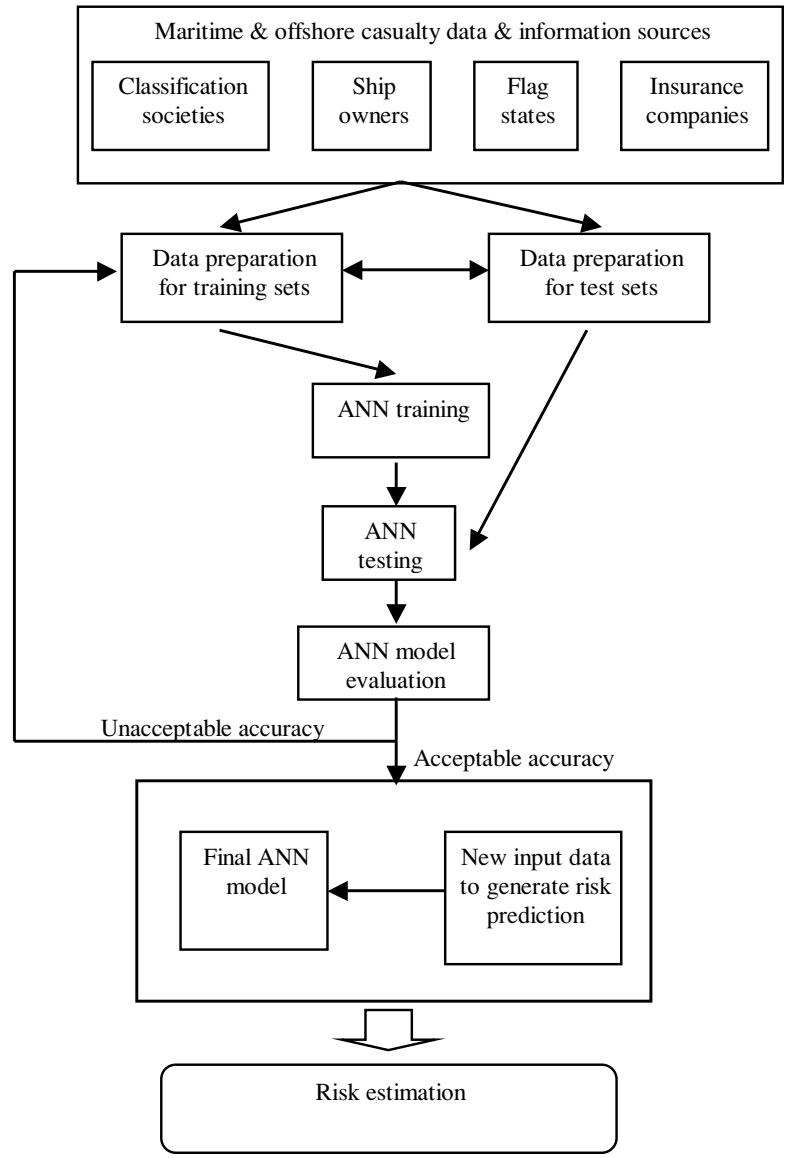

Fig. 4. The risk estimation framework incorporating ANN.

Step 1: Collect Data. Collect data sets, number series, or system information that have a relationship to or influence on a system failure from relevant sources such as classification societies, ship owners, flag states, insurance companies, and experts.

Step 2: Prepare Data. Define and prepare training input sets and decide how to handle the gathered information for presentation to the ANNs. Determine the range of data and set minimum and maximum values to these levels.

Step 3: Extract Test Data Set. In order to be able to test the trained network, it is common to set aside some of the data for testing (cross-validation). Usually, the total data set is divided into two, one for training and the other for testing.

Step 4: Train the Network. Select suitable network architecture by setting the number of network input equal to the number of input variables and the number of network output equal to that of problem output. Select the number of hidden layers and the number of neurons in each hidden layer.

Step 5: Test the ANN. Apply the test data sets to the trained ANN model to test its performance.

Step 6: Evaluate the ANN Model. If the risk estimation generated by the model lies within acceptable accuracy, then proceed to the next step. Otherwise repeat Steps 2 to 6 all over again until the risk estimation produced falls within the acceptable accuracy. For various applications, accuracy-level requirements would be different and are judged subjectively by the user.

Step 7: Use the ANN Model to Carry Out Risk Prediction. Feed new casualty data to the ANN model to perform risk estimation.

Step 8: The risk estimation or prediction generated by the ANN model can be applied to safety-based design and operation-support system as a source of expert input.

5.2.1. *An Example for Demonstrating the Risk Estimation Framework Using ANN Techniques*

A hypothetical generic vessel is used to demonstrate the risk estimation framework using ANN techniques. The failure of the vessel is determined by the vessel's design features and the ship owner's management quality. The vessel's design features include fire-fighting capability, navigation equipment level, and redundancy of machinery. The ship owner's management quality includes the quality of ship owner management and the quality of operation (Sii, 2001).

An MLP network is chosen for modeling the vessel and its structure is shown in Fig. 5. It can be seen that the neural network model has 5 inputs and 1 output. After performing several experiments on the optimal number of hidden neurons to be used for ANN training and learning, 20 hidden neurons are selected for the first hidden layer (In Fig. 5, 4 nodes, and two small circles with dotted lines that are not explicitly shown, are used to represent the actual 20 nodes for the hidden layer.) The 5 nodes (neurons) in the input layer correspond to the quality of ship owner's management, quality of operation, fire-fighting capacity, navigation equipment level, and machinery redundancy, respectively. The 1 node (neuron) in the output layer corresponds to the possibility of vessel failure.

The data used for ANN model training and learning is shown in Table I and is within the following guidelines (Sii, 2001):

– IF either one or more of the factors from ship owner management quality or vessel's design features is classified as "Very Low," THEN the possibility of vessel failure is predicted to be "Very High."

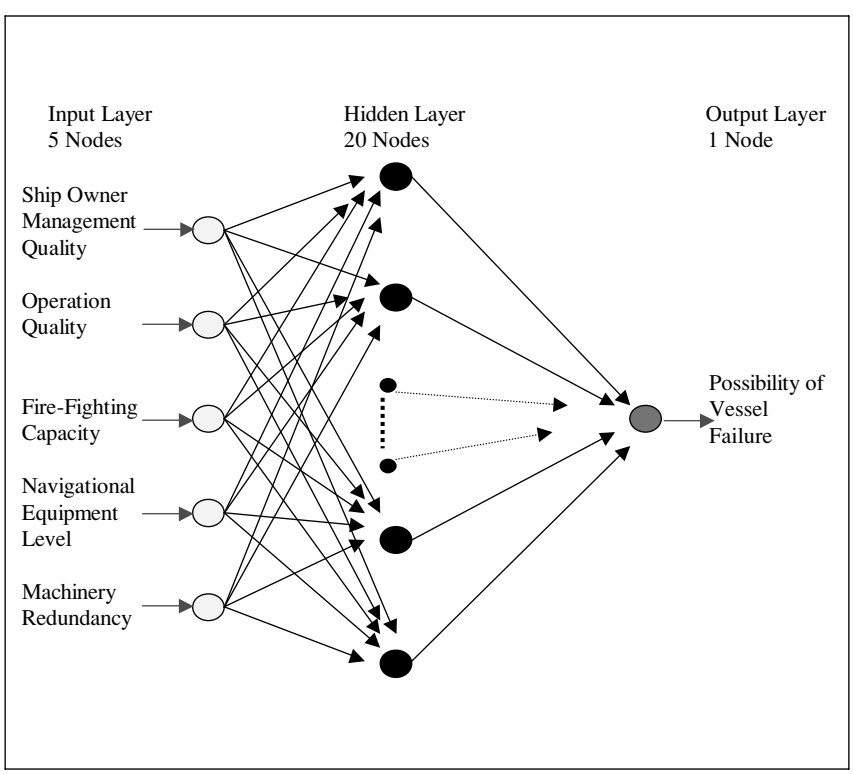

Fig. 5. The neural network for predicting vessel failure.

Table I. Input Data

Ship Owner's Management Quality	Operation Quality	Fire-Fighting Capability	Navigation Equipment Level	Machinery Redundancy	Possibility of Vessel Failure
Very low	Very high	Very high	Very high	Very high	Very high
Very high	Very low	Very high	Very high	Very high	Very high
Very high	Very high	Very low	Very high	Very high	Very high
Very high	Very high	Very high	Very low	Very high	Very high
Very high	Very high	Very high	Very high	Very low	Very high
Very high	Very high	Very high	Very high	Very high	Very low
High	High	High	High	High	Low
Average	Average	Average	Average	Average	Average
Very low	Very low	Very low	Very low	Very low	Very high
Very low	Low	Average	High	Very high	Very high
Low	Very low	Very high	Average	High	Very high
Average	High	Very low	Very high	Low	Very high
Very high	Low	High	Very low	Average	Very high
High	Very high	Low	Low	Very low	Very high
High	Very high	Low	Average	Low	High
Low	Average	Very high	Low	High	High
Average	Low	Low	Very high	Very high	High
Very high	High	Average	High	Low	High
Low	Average	High	Low	Average	High
Low	High	Low	Very high	Low	High
High	Low	Very high	Low	Low	High
Low	Low	High	Low	Low	Very high
Low	High	Low	Low	Low	Very high
Low	Low	Low	Low	Very high	Very high
Low	Low	Low	Low	Low	Very high

- IF either one or more of the factors from ship owner management or vessel's design features is classified as "Low," THEN the possibility of vessel failure is predicted to be "High."
- The rest of the predicted possibilities of vessel failure will be computed according to the average scale values of the five factors listed below:

Level	Scale Values
Very high	0.9, 1.0
High	0.7, 0.8
Average	0.4, 0.5, 0.6
Low	0.2, 0.3
Very low	0.0, 0.1

The trained ANN model was applied to predict 10 different test cases. The predicted outputs are shown in Table II. The predicted results were found to be good as they follow the predefined hypothetical criteria closely. For example, the possibility of vessel failure is predicted to be high IF ship owner management quality is low, operation quality is very high, fire-fighting capability is very high, navigation equipment level is average, and machinery redundancy is very high.

In the above, data generated using relatively simple rules were used to test the proposed risk estimation framework. In many cases, data directly obtained from insurance companies, classification societies, and ship operators can be directly used without incorporating any specific rules similar to the above. For example, a set of data on the relationship between ship age, ship tonnage, and ship failure probability may be obtained from a classification society. An ANN model can be trained using such data. After the training, this ANN model can be used to predict the failure probability for a bulk carrier if the age and tonnage are provided. Several more test case studies have been carried out to test the proposed risk estimation framework using ANN techniques and results have been satisfactory. The detailed information can be found in Sii (2001).

It has been demonstrated that ANNs have the following characteristics:

- The capability of learning a set of nonlinear patterns.

Predicted Results by ANN Model

Ship Owner's Management Quality	Operation Quality	Fire-Fighting Capability	Navigation Equipment Level	Machinery Redundancy	Possibility of Vessel Failure
Low	Very high	Very high	Average	Very high	High
Very high	High	Average	Low	Very low	Very high
Low to very low	High	Average	Low	Very high	Very high
High	Very low	High	Average	High to very High	Very high
High	High	Very low	High	Average	Very high
High	Average	High	High	Very high	Low
Average	High	Average	Average to high	Average	Average
Low to very low	Very high	Very high	Average	High	High to very high
Low	Low	Low	Low	Average	Very high
High	Average to low	Very high	Very high	Very high	High to very high

- Te ability to generalize and interpolate accurately within the range of the training data.
- A risk-predicting or forecasting model needs consistent, sufficient independent variables (features) used to train and test the ANNs.
- Ease of application, especially using the existing software packages (for example, the Neural Network Toolbox in MATLAB).
- ANNs are powerful tools and a complement to statistical techniques when the data are incomplete or "noisy," or when many hypotheses are to be pursued and high computational rates are required. With their unique features, they can lead to a powerful decision-making, predicting, and forecasting tool in certain situations.

5.3. An Approach for Optimizing Maintenance Strategies Using the Delay-Time Concept

Maintenance costs form a significant part of the overall operating costs in maritime operations. Maintenance also affects reliability and can thus have environmental and safety consequences. The International Management Code for the Safe Operation of Ships and for Pollution Prevention (ISM Code) addresses management aspects. The importance of maintenance is demonstrated by the fact that it is the only shipboard activity to have one whole element assigned to it (i.e., ISM Code element 10) (IMO, 1997).

ISM Code element 10, focusing on maintenance of ship and equipment, *inter alia*, states that: "The Company should establish procedures in its SMS (Safety Management System) to identify equipment and technical systems the sudden operational failure of which may result in hazardous situations. The SMS should provide for specific measures aimed at promoting the reliability of such equipment or systems." This is consistent with what reliability centered maintenance (RCM) delivers. RCM focuses the maintenance resources only on those items that affect system reliability, thereby making the maintenance program cost effective in the long run.

The recent preventive maintenance (PM) developments have a more generic framework with an aim of maximizing the profitability of a working system, which is also demonstrated by another maintenance management approach, "Total Productive Maintenance" (TPM) developed by Nakajima of the Japan Institute of Plant Maintenance (JIPM) (Nakajima, 1997). In such a philosophy of maximizing profitability, different elements, such as down time, cost, and safety criticality, may need to be studied together.

In PM, maintenance activities are performed before equipment failure. PM involves the repair, replacement, and maintenance of equipment in order to avoid unexpected failure during use.

In the maritime industry, there are some specific problems with regard to maintenance that need to be considered when developing a maintenance model. These problems include:

- The high degree of isolation from repair and spares facilities.
- The high cost of transport unit (i.e., the ship).
- The high cost of a maritime system out of service.
- Varying costs, availability, and quality of labor and spares throughout the world.

- Maritime personnel are operators as well as maintainers.
- The frequency with which personnel join and leave ships/offshore installations, creating a need for continuity of ship maintenance plans.
- Severe safety and insurance conditions, necessitating rigorous survey requirements.

The time between the first identification of abnormalities (initial point) and the actual failure time (failure point) will vary depending on the deterioration rate of the component. This time period is called the delay time or opportunity window to carry out maintenance or an inspection (Christer & Walker, 1984a; Christer *et al.*, 1995). The time to failure of equipment is a function of its maintenance concept, and to capture this interaction the conventional time to first failure of reliability theory requires enrichment. This may be achieved using the delay-time concept. Considerable work has been carried out on the modeling of this concept to production plants (Christer & Walker, 1984a; Christer *et al.*, 1995; Christer *et al.*, 1998). Other works include the application to gearbox failure on buses (Leung & Kit-leung, 1996) and preventive maintenance modeling for a vehicle fleet (Christer & Walker, 1984b).

The flowchart in Fig. 6 illustrates a proposed approach for optimizing inspection strategies. The proposed approach is an integration of two models: the downtime estimation model and cost estimation model. These two models require failure data and a probability distribution function of the delay time. Each model developed will produce an optimal inspection period with respect to downtime and cost, respectively. A best compromise is then achieved by plotting $D(T)$ (downtime) against $C(T)$ (expected cost).

After studying operating practice and the existing maintenance and failure data, the system can be modeled using the following assumptions:

- Inspections take place at regular time intervals of T hours and each requires a constant time.
- Downtime owing to inspection $= d$.
- Average downtime for breakdown repair $= d_b$.
- Arrival rate of defects per unit time $= k$.

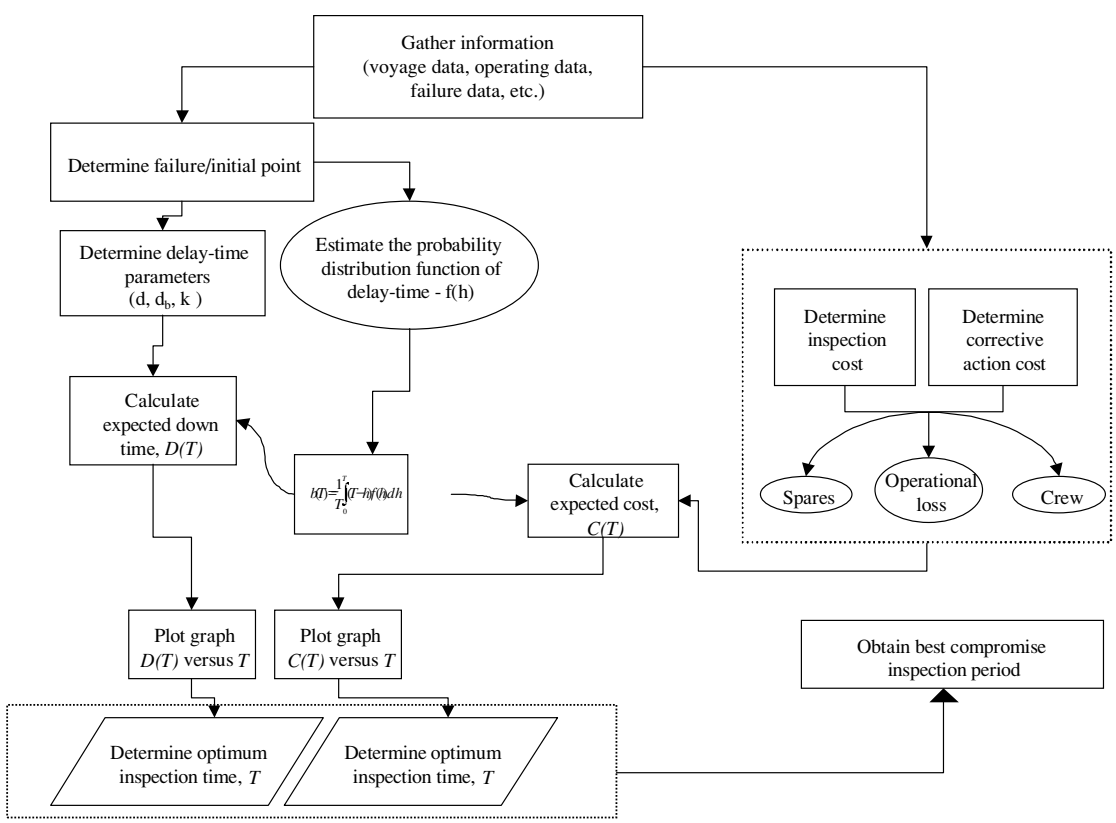

Fig. 6. A proposed approach flowchart.

- Failures are repaired immediately with downtime $d_b \ll T$.
- Inspections are perfect in that any defect present will be identified.
- Defects identified will be repaired within the inspection period.
- The time of origin of faults is uniformly distributed over the time between inspections.
- The delay time is independent of its time of origin.

It could be argued that some of the above assumptions may not be practical. For example, inspections/repairs could never be carried out perfectly. However, such assumptions, which have been widely used by many safety/reliability researchers, are made mainly for demonstrating the proposed method with ease. Even without the above assumptions the proposed approach for optimizing maintenance strategies using the delay-time concept is applicable.

A fault arising within a period $(0, T)$ has a delay time h (if a fault arises at $T - h$ within a period $(0, T)$), the occurrence probability of this event being $f(h)\Delta h$ where $f(h)$ is the probability density function of h and Δh is the infinitesimal increase of h. This fault will be repaired as a breakdown repair if the fault arises in the period $(0, T - h)$; otherwise an inspection repair is carried out. Summing up all possible values of h, the probability of a fault arising as a breakdown $b(T)$ can be expressed as:

$$b(T) = \frac{1}{T}\int_0^T (T-h)\, f(h)\, dh.$$

$b(T)$ is independent of k but dependent on h. A delay time can only be estimated or identified when the defect has occurred and led to a breakdown failure. Hence if $b(T)$ is the probability of a defect arising as a breakdown failure, and a breakdown failure can exist when a defect has arisen, then it is fair to say that $b(T)$ is a conditional probability (keeping in mind that this expression excludes sudden failure, i.e., no opportunity window).

Consequently, the expected downtime per unit time function $D(T)$ is given below:

$$D(T) = \left\{ \frac{d + kTb(T)\, d_b}{T + d} \right\}.$$

The product of kT will give the expected number of defects within the time horizon considered. This is normally based on some historic data gathered for the equipment or system.

Substituting $b(T)$ gives:

$$D(T) = \left\{ \frac{d + kT\left[\frac{1}{T}\int_0^T (T-h)\, f(h)\, dh\right] d_b}{T + d} \right\}.$$

Delay-time distributions can be estimated using subjective or objective methods. Several models have been developed for these two approaches (Baker & Wang, 1992; Baker & Wang, 1993; Wang, 1997b). The objective models generally require a large amount of data complemented with survey questionnaires, which should reflect the operations of the analyzed system over a considerable period of time. A truncated standard normal distribution with $\mu = 0$ and $\sigma^2 = 1$ is used for simplification purposes. When $f(h)$ follows a standard normal distribution truncated at 0, then,

$$f(h) = \frac{2}{\sqrt{2\pi}} e^{-h^2/2}.$$

The distribution is normalized in such a way that it integrates to one. If sufficient data for delay-time modeling is available, then variances for a truncated standard normal distribution or even other types of distributions can be specified using a trial and error approach to produce the best results. However, determination of such variances is out of the scope of this article.

$D(T)$ is obtained as follows:

$$D(T) = \left\{ \frac{d + kT\left[\frac{1}{T}\int_0^T (T-h)\left(\frac{2}{\sqrt{2\pi}} e^{-h^2/2}\right) dh\right] d_b}{T + d} \right\}.$$

The expected cost model estimates the expected cost per unit time of maintaining the equipment on an inspection regime of period T. There are three cost elements that need to be considered in this modeling phase. These three elements are:

- Cost of a breakdown.
- Cost of an inspection repair.
- Cost of an inspection.

The expected cost per unit time of maintaining the equipment on an inspection system of period T is obtained as follows (Pillay, 2001):

$$C(T) = \frac{[kT\{\mathrm{Cost_B}\, b(T) + \mathrm{Cost_{IR}}[1 - b(T)]\} + \mathrm{Cost_i}]}{(T + d)},$$

where $\mathrm{Cost_B}$ = breakdown repair cost, $\mathrm{Cost_{IR}}$ = inspection repair cost, and $\mathrm{Cost_i}$ = inspection cost.

The detailed modeling of $\mathrm{Cost_B}$, $\mathrm{Cost_{IR}}$, and $\mathrm{Cost_i}$ can be found in Pillay (2001).

Substituting $b(T)$, the following is obtained:

$$C(T) = \frac{\left[kT\left\{\text{Cost}_\text{B}\left[\frac{1}{T}\int_0^T (T-h)\left(\frac{2}{\sqrt{2\pi}}e^{-h^2/2}\right)dh\right] + \text{Cost}_\text{IR}\left\langle 1 - \left[\frac{1}{T}\int_0^T (T-h)\left(\frac{2}{\sqrt{2\pi}}e^{-h^2/2}\right)dh\right]\right\rangle\right\} + \text{Cost}_\text{i}\right]}{(T+d)}.$$

Both $D(T)$ and $C(T)$ are a function of T. If $D(T)$ and $C(T)$ are plotted against T, then the best inspection strategy can be determined by studying the particular requirements on $D(T)$ and $C(T)$.

It is worth mentioning that in some cases it could be relatively difficult to quantify costs with confidence. For example, for a fishing vessel, there are several elements that need to be considered when representing downtime as loss value in terms of dollars, such as wages, loss of catch (seasonal variation), reduction in catch due to shorter stay at sea (brought on by consumption of perishables), etc. Therefore, decisions on the inspection strategy may not be made solely based on $C(T)$. In such cases, $D(T)$-based decisions on the inspection strategy may be more reasonable. If uncertainties in both $D(T)$ and $C(T)$ are considered, then decisions on the inspection strategy can be made based on a combination of $D(T)$ and $C(T)$.

In the decision-making process, expected safety criticality defining the level of risk associated with a system can also be modeled using the delay-time concept (Pillay, 2001). The expected safety criticality can be used together with $D(T)$ and $C(T)$ to determine the inspection strategy. Within the acceptable level of risk, the inspection strategy can be determined by minimizing $D(T)$ or $C(T)$ or by considering $D(T)$ and $C(T)$ in an integrated manner.

5.3.1. An Example for Demonstrating the Approach for Optimizing Maintenance Strategies Using the Delay-Time Concept

The application of the delay-time concept to determine the optimum inspection interval is demonstrated using a real hydraulic winch operating system on a fishing vessel. This vessel is a 1266 GRT (gross tonnage), deep-sea trawler with an L.O.A. (length overall) of 60 meters (Pillay, 2001). The winches are used to deploy the nets and haul the catch onto the ship. The supporting winches, that is, the gilson winch and tipping winches, are not considered in this example. The main pumps provide the hydraulic power to the port and starboard winches as well as the net drum motor. The 1010 pumps are used to control the tension and balance the loads on the main winches.

The following information was gathered for this particular system, which included a combination of logged records and reports complemented by expert judgments (where no data was available):

Inspection downtime (d)
$= 15$ minutes $= 0.010417$ days.
Downtime for breakdown repair (d_b)
$= 4.5$ days.
Total operating hours of winch (for 25 voyages)
$= 1344$ hours $= 56$ days.
Arrival rate of defects (k)
$= 0.535$ per day (30 failures for 25 voyages).

The actual process of carrying out an inspection itself would take about 45 minutes for this particular system, for which no spares are carried in the fishing vessel due to the nature of the fishing industry. Most of the inspections can be carried out when the hydraulic system is not operating. An inspection includes visual inspection and off-load and function testing. Hence, the downtime caused by inspection would be much lower than 45 minutes. From experience, only 15 minutes is required to carry out an on-load pressure test for such a system. Therefore, the inspection downtime, d, is set to be 15 minutes or 0.010417 days.

The downtime for breakdown repair takes into account any logistic delays that may occur while waiting for spares to be sent from shore suppliers. Most fishing vessels carry a minimum amount of spares on board. Hence, should a breakdown occur at sea on the hydraulic system, the ship might be operationally crippled for a period of time. From experience, this period could be a few hours or days, depending on the position of the vessel at the time of breakdown.

Substituting the values obtained for the hydraulic system gives the following equation (Pillay, 2001):

$$D(T) = \left\{\frac{0.01041 + (0.535T)\left[\frac{1}{T}\int_0^T (T-h)\left(\frac{2}{\sqrt{2\pi}}e^{-h^2/2}\right)dh\right]4.5}{T + 0.01041}\right\}.$$

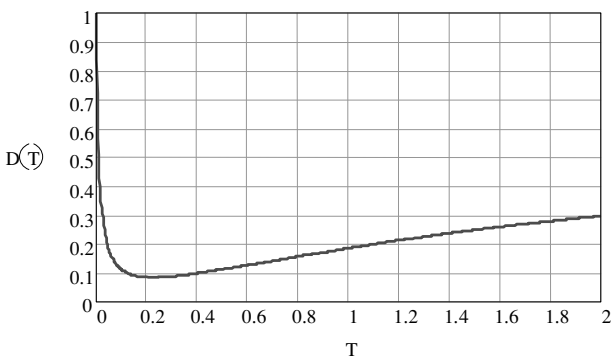

Fig. 7. Optimal inspection period based on minimum $C(T)$.

If $D(T)$ is plotted against T, the result can be shown in Fig. 7. It can be noted that that the inspection period is estimated to be 5.18 operating hours (0.216 days) when the downtime is minimized. In a similar way, the expected cost model can also be formulated (Pillay, 2001). The $C(T)$ curve is shown in Fig. 8 (Pillay, 2001). It can be noted that the inspection time is 7.24 operating hours (0.302 days) when the cost is minimized. When both $D(T)$ and $C(T)$ are used to determine the best inspection interval, $D(T)$ can be plotted again $C(T)$ as shown in Fig. 9. It can be noted that no inspection interval gives minimum $D(T)$ and $C(T)$ simultaneously. The best compromise interval should be between points 1 and 2. Of course, if necessary, more objectives, such as safety criticality, can be added to make operational decisions with an aim of achieving an optimal profitability.

The use of a delay-time model within a preventive maintenance system would be useful to determine inspection strategies. It may be worth mentioning that in the above, the optimal inspection interval is produced based on the synthesis of both the cost and downtime. Inspections carried out during the operation phase of machinery will reveal any failures that have already been initiated at an earlier time. Early detection of the "abnormal" condition could reduce the occurrence likelihood of breakdown failures, which may take long time to repair, and lead to necessary measures to mitigate possible consequences. Inspection repairs, which usually take a relatively short time, may be conducted before failures propagate to become more serious. The approach described is applicable to situations where repair time is either larger or smaller than the inspection interval in principle since the criteria considered are not only the downtime.

The inspection regime can be integrated into the existing maintenance procedures in order to help make operational decisions. The effectiveness of the proposed approach can be improved if sufficient data is available in order to generate a true probability distribution function for the delay time.

5.4. An Optimization Approach for Studying Both Design and Operational Aspects

As described previously, in the maritime industry, there is a tendency that designers and operators are given more flexibility to use a variety of methods to deal with their design as far as the goal can be achieved. Various formal decision-making techniques can be used to make design/operation optimization. In risk-based optimization, one or more objectives are usually involved. Typical objectives would include safety and cost. There are usually several design parameters that determine the design/operation objectives. Typical examples are where risk reduction

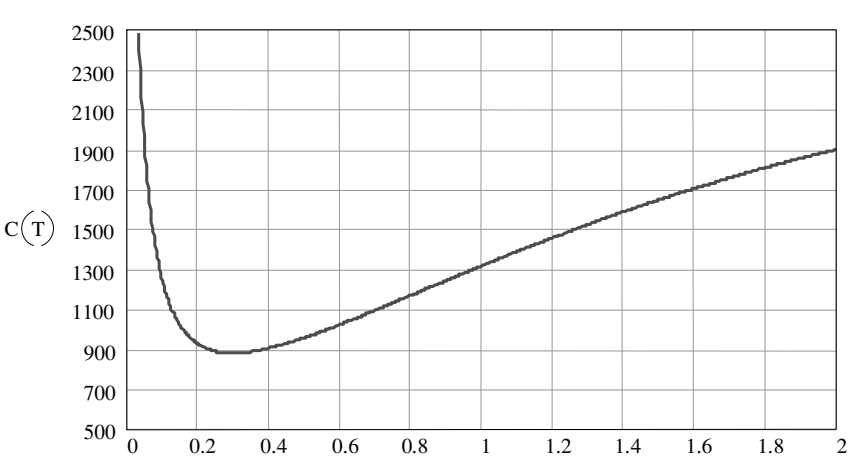

Fig. 8. Optimal inspection period based on $C(T)$.

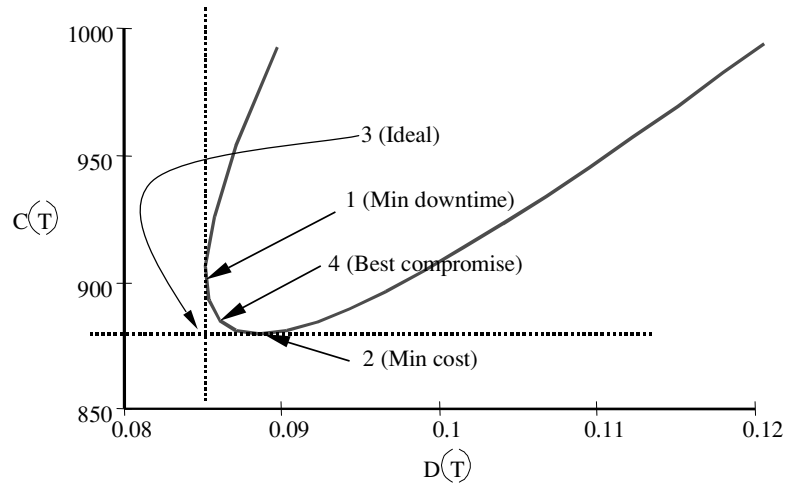

Fig. 9. Expected cost $C(T)$ against expected downtime $D(T)$.

actions are required and how maintenance policy is optimized. Techno-economic modeling is usually carried out in the design decision-making process.

Techno-economic modeling has been extensively discussed (Goss, 1989; Rasmussen, 1990; Carpenter & Fleming, 1991), but not many practical applications are reported. This is particularly true for maritime safety assessment. This could be largely because of the uncertain value placed on human life and difficulties of qualifying risks (House of Lords, 1992). However, it has been noted that if the uncertainty regarding the risks of a large marine or offshore product is not unacceptably high, a techno-economic analysis may be beneficially conducted to process the safety information to make design decisions.

It is generally impossible to have a design that could maximize safety (i.e., minimize risks) and minimize the safety-related life cycle cost simultaneously. A compromise is therefore required. The decision as to which objective is to be stressed is dependent on the particular situation in hand. The appropriate level of safety then becomes dependent on the relative importance of the two criteria. If the nondominated design options for such a situation have to be obtained, it becomes feasible to use a formal Multiple Criteria Decision-Making (MCDM) tool to arrive at efficient or optimal decisions.

The risk function can be expressed in terms of the occurrence probabilities of the system top events with each system and top event being weighted on the basis of the severity of its possible consequences (Wang, 1994). The safety-related life cycle cost of a system may be modeled by taking into account the top-event-caused consequences, repair cost, maintenance cost, design review cost, etc. A techno-economic model combines the safety (risk) function with the economic function. Such a model is given by:

min:	Risk
min:	Cost
subject to:	Constraints

The above model implies that the design actions and operation strategies should be implemented efficiently to minimize the risks of the system and the safety-related life cycle cost (Wang *et al.*, 1996). In the above model, risk and cost may be two conflicting objectives. It is possible to use MADM techniques to process the model to obtain the best design aspects and maintenance policies (i.e., where the minimal cost is obtained at a given risk level or the minimal risk is obtained given a level of cost).

In the safety-based decision-making process, it may be required to take into account more design/operation aspects with a general goal of maximizing the profitability of the system. It should be noted that if the goal for optimization is profitability (of course, this should be achieved under the condition that safety should be maintained to a certain level that satisfies the corresponding safety regulations), then more design/operation functions such as maintainability, serviceability, durability, etc. may also need to be taken into account. Such functions may be added to the above model in order to obtain an overall picture of the system performance in terms of different design/operational objectives. All such functions may also be combined in order to achieve an optimal profitability subject to a set of constraints. One of the constraints is the required level of safety, which could either be defined numerically or descriptively.

For example, for the U.K. offshore industry, the occurrence likelihood of events causing a loss of integrity of the safety refuge should be less than 10^{-3} per platform year and the associated risks should be reduced to ALARP (As Low As Reasonably Practicable) (Spouse, 1997).

The above multiple-criteria decision-making optimization approach is different from traditional methods such as expected utility theory. In this approach, risk and cost are modeled in terms of failure likelihood, consequence severity, design actions, and operation strategies rather than in term of utilities. The condition for applying such an approach is that both risk and cost should be modeled on a quantitative basis. The designer can have an in-depth picture of how risk and cost can influence each other in a quantitative way. Of course, if the situation does not allow the quantitative mathematical model for such an optimization analysis to be conducted, then other methods, such as a subjective approach or an expected utility decision-making approach, may be a better alternative.

5.4.1. An Example for Demonstrating the Optimization Approach

Three top events, T_1, T_2, and T_3, are identified for a hydraulic hoisting transmission system of a real offshore crane (Wang *et al.*, 1996). Ten cut sets are associated with T_1, 43 cut sets with T_2, and 14 cut sets with T_3. The parameters used for formulating the techno-economic model can be found in detail in Wang (1994).

Suppose c significant cut sets associated with T_1, T_2, and T_3 are considered for reduction in the design review process. Let $X = MTBM$ (mean time between maintenance), y_j = the probability reduction of occurrence of cut set j as a result of a design review action ($j = 1, 2, \ldots, c$), and $Y = [y_1, y_2, \ldots, y_c]^T$. A techno-economic model is formulated as follows:

$$\begin{aligned}
\text{min: Cost} &= \text{Cost}_T^*(X) + \text{Cost}_M^*(X) \\
&\quad + \text{Cost}_R^* + \text{Cost}_D^*(X, Y) \\
\text{min: Risk} &= \left[\sum_{i=1}^{n} K_i \times P_i(X) \right. \\
&\quad \left. + \sum_{j=1}^{c} K_{Dj} \times (P_{Dj}(X) - y_j) \right] \\
\text{subject to: } X_{\max} &\geq X \geq X_{\min} 0 \leq y_i \leq P_D j(X) \\
&\qquad (j = 1, 2, \ldots, c)
\end{aligned}$$

where COST_T^* represents top-event-caused cost after the modification; COST_M^*-maintenance cost after the modification; COST_R^*-repair cost after the modification; COST_D^* = design review cost after the modification; K_i = the weighting factor representing the severity of possible consequences caused by the occurrence of top event T_i; $X_{\max}$ = the largest $MTBM$ value used in the safety analysis; $X_{\min}$ = the smallest $MTBM$ value used in the safety analysis; $K_{Dj} = K_i$ if the jth cut set is associated with top event T_I; $P_i(X)$ represents the probability of occurrence of top event T_i; $P_{Dj}(X)$ represents the original probability of occurrence of the jth cut set before a design review action is taken; n = number of top events; and c = number of cut sets considered in risk reduction.

The formulated model can be processed to obtain the optimization results shown in Fig. 10 using formal multiple objective decision-making techniques (Wang *et al.*, 1996). All designs in the frontier in Fig. 10 are efficient ones. Every design in the frontier corresponds to a design/operation vector. For example, at point 4, cost and risk are equal to £193k and 0.12, respectively; the mean time between maintenance is equal to 6,269 hours and five cut sets need to be eliminated. From Fig. 10, it can be seen that cost is significantly reduced with a slight increase of risk from point 1 to point 5 in the frontier, and that risk is significantly reduced with a slight increase of cost from point 2 to point 6. These two sections should be avoided in the design if risk reduction is not of paramount importance relative to cost or cost reduction is not of paramount importance relative to risk. A practical efficient design can be at some point in the section between 5 and 6 in Fig. 10, depending on the particular requirements on cost and safety to be considered by the decision maker.

6. A BRIEF DISCUSSION OF THE ABOVE FOUR METHODS

As shown in Fig. 1, in the beginning of the initial stages of safety assessment, design selection needs to be carried out in situations where a high level of uncertainty is associated with the estimates of safety, cost, reliability, etc. due to incomplete data. An approximate reasoning approach may be more appropriately used for selecting the best alternative where expert judgments may need to be synthesized. The approximate reasoning approach can also be used for hazard identification, risk estimation, cost-benefit assessment, and decision making for design/operation

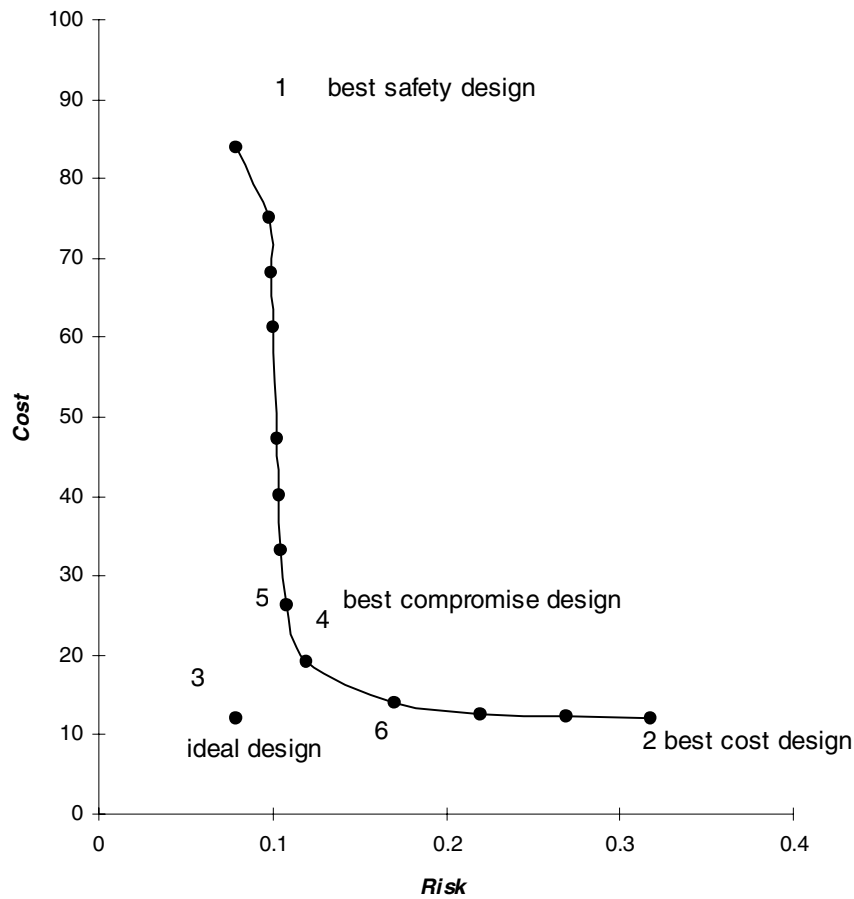

Fig. 10. The optimization results.

selection in Fig. 1. Another alternative method is the risk estimation framework using artificial neural network (ANN) techniques. Such a framework can be best applied in situations where there is a large volume of data that is "messy," there are complex relationships among the data, and a number of people are involved in tackling a problem with their own areas of expertise when the model works as a coordinator.

As shown in Fig. 2, after the best design option has been chosen, the safety assessment proceeds further and it will reach a stage where risk-based design/operation optimization is preferably carried out. At this stage, safety may be assessed using appropriate risk assessment techniques in terms of occurrence likelihood of major hazards and magnitudes of consequences. The optimization approach for studying both design and operational aspects using multiple-objective decision-making approaches described can be employed to achieve an optimal safety within both the technical and economic constraints. As far as the operational aspects are concerned, the approach for optimizing maintenance strategies using the delay-time concept described can be employed to improve the inspection process.

It has been recognized that a framework with a holistic nature is desirable for risk assessment of large engineering systems where appropriate risk modeling and decision-making tools can be selected for use at different stages of the design process and operations. This is particularly true for maritime risk assessment. All the four methods described in this article can be included in such a holistic framework. It is also worth bearing in mind the fact that these techniques are still in their infancy and are not yet universally used in industries, although both safety engineers and decision makers can benefit from the potential of these approaches for risk modeling and decision making in certain situations.

All such risk modeling and decision-making tools can be integrated in the sense that they form a general structure to facilitate risk-based design and operations of large and complex marine engineering systems.

7. CONCLUSION

The safety culture in many industries, including the maritime sector, has been changing over the last several years. This may be demonstrated by the implementation of the ISM Code and the introduction of formal safety assessment for rule-making purposes in the marine industry, enforcement of the goal setting safety case regulations in the U.K. offshore industry, and the introduction of the port marine safety code in the U.K. port sector. In general, many industries are moving toward a "goal setting" risk-based regime. This gives more flexibility to safety engineers to employ the latest risk modeling techniques and decision-making/optimization tools. It may be very beneficial that many advances that have been developed and are being developed in general engineering and technology are further explored, exploited, and applied in order to facilitate risk modeling and decision making. In fact, it is widely accepted that any developed safety analysis approach should preferably be introduced into a commercially stable environment in order that the applications have the chance to become established and prove feasible, otherwise it is more likely that their full potential will not be realized. Therefore, emphasis should be directed to apply them in the maritime environment.

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