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Withhold-judgment and punishment promote cooperation in indirect reciprocity under incomplete information

JI QUAN[1(a)], XIUKANG YANG[1(b)], XIANJIA WANG[2], JIAN-BO YANG[3], KAIBIAO WU[1] and ZILONG DAI[1]

[1] *School of Management, Wuhan University of Technology, Wuhan 430070, China*
[2] *School of Economics and Management, Wuhan University, Wuhan 430072, China*
[3] *Alliance Manchester Business School, The University of Manchester, Manchester M15 6PB, UK*

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Abstract – Indirect reciprocity is a mechanism that can promote cooperation among populations in which individuals cannot repeatedly interact. Indirect reciprocity evaluates each individual‘s behaviour through social norms, based on which the reputation for each individual can be labelled. In the traditional models, it is usually assumed that all individual reputations are observable and are common knowledge to everyone in the population. In this paper, we relax this assumption and discuss an indirect-reciprocity model under incomplete information in which individuals have private opinions of others. Moreover, based on the observation that some people may have reservations about the current behaviour of an actor, which does not change their previous impressions on him, we generalize this phenomenon as withhold-judgment. We introduce punishment strategy and nine second-order social norms including withhold-judgment and explore how cooperation evolves in both public and private reputation scenarios. We find that social norms that allow for withhold-judgment can maintain high levels of cooperation. Although in the private reputation scenario there is a situation where more and more individuals have divarication over time, causing the reputation system to collapse, social norms that allow for withhold-judgment are still robust even if there are noise, variation, and incomplete information. In addition, we find that the introduction of punishment can promote cooperation, but in some situations, punishment will have a negative impact on social welfare.

Although many models have been proposed to explain the phenomenon of cooperation, such as kin selection [1, 2], network reciprocity [3-5], group selection [6], and so on [7-11], the mechanism for the evolution of cooperation, especially for individuals in a society who cannot interact repeatedly, is still a mystery to human beings [12-14]. Indirect reciprocity that Alexander [15] introduced first made a great effort on this issue. It does not require repeated interactions between the same individuals, which can explain the cooperative behaviour between unfamiliar individuals.

Reputation plays a key role in the indirect reciprocity mechanism. In indirect reciprocity, an individual‘s reputation is evaluated based on social norms [16-24]. Social norms are essentially a mapping from historical information to individual reputation. How to determine appropriate social norms is the focus of academic research. Many kinds of different order social norms are presented in which binary reputation assumption is usually adopted. The first-order social norm is a unary function from an individual‘s behaviour to its reputation. For instance, cooperation can be given a good reputation, whereas defection leads to a bad reputation [25-27]. The second-order social norm is a binary function of the individual‘s behaviour and the reputation of the recipient. Taking account of the recipient's reputation provides more interactive information that allows observers to distinguish between reasonable and unreasonable defection [20, 28]. For instance, it is natural to believe that it is justified for an individual to defect with a bad person, so that the individual’s reputation will not be impaired if he defects against a bad person. The third-order social norm further takes the actor’s reputation into consideration. In this way, higher-order evaluation rules can explain individual behaviour more subtly, but they also require individuals to store and process more information. Based on the third-order social norm and the assumption that all relevant information is public where individuals hold the same opinion on others’ reputation, Ohtsuki and Iwasa [29] proposed the "leading eight" social norm that can maintain the evolutionary stability of cooperation. Pacheco [30] further proposed a simple and

(a) Corresponding Author, E-mail: quanji123@163.com
(b) Joint first author who has equal contributions

robust social norm called "stern judging" under natural selection and variation.

In these traditional indirect reciprocity models, it is generally assumed that the reputation of the focus individual can be directly observed by others. In addition, other individuals can also learn the reputation of the focus individual through communication. Therefore, everyone's information about the reputation of others is the same. However, in the real world, an individual‘s observation may be subject to physiological limitations such as “blurry vision” or “fuzzy mind”. Thus, the information on a person's reputation may be affected by noise [31-37]. Studies have shown that when there is no false information dissemination in society, some social norms can promote the evolution of cooperation, and in the presence of noise interference, appropriate communication is necessary to induce the emergence of cooperative behaviour [38, 39].

Due to the existence of noise and the limitations of communication, it is difficult to reach a consensus on the information about every person's reputation. Once their initial opinions on someone are inconsistent, their views may be further differentiated over time. Hilbe [37] relaxed the public reputation assumption, proving that “leading eight” with private, incomplete and noisy information can not maintain the evolutionary stability of cooperation.

In this paper, we utilize the donation game to explore indirect reciprocity under incomplete information. We made two changes compared to the existing models. First, since punishment is everywhere in reality, and it has received widespread attention in academia [40, 41], thus we introduce a punishment-type strategy in the games. In addition, based on the observation that some people may have reservations about the current behaviour of the actor, which do not change their previous impressions on him, we generalize this phenomenon as withhold-judgment. We discuss nine second-order social norms allowing withhold-judgment. We explore how cooperation evolves in both public and private reputation scenarios when information is noisy and incomplete. We found that in our model when donors cooperate with bad recipients, observers' withhold-judgment will be more conducive to the evolutionary stability of cooperation. However, when donors defect against bad recipients, if observers withhold judgment, it may cause the reputation system to collapse. At the same time, we found that the introduction of punishment will promote cooperation, but in some scenarios, the penalty cost will be higher than the benefits brought by cooperation, which has a negative effect on social welfare and the whole society will even fall into a bigger dilemma.

In our indirect reciprocity model, there is a relatively large and well-mixed population. For each time step, multiple pairs of individuals are randomly sampled from the society to participate in the donation game. For each pair, one individual acts as a donor and the other acts as a recipient randomly. We assume that an individual's reputation is binary, namely, good (G) or bad (B). The donor can cooperate (C), defect (D), or punish (P) against a good or bad recipient. If the donor cooperates with the recipient, the donor will bear a cost c and the recipient will receive benefits b ($b > c > 0$). If the donor defects, both of them have zero payoff. Moreover, we allow the donor to employ the punishment strategy. The cost of punishment is α for the donor and the corresponding fine is β ($\beta > \alpha > 0$) for the recipient. In actual situations, the donor may execute his action incorrectly with probability ε. For instance, a donor is willing to cooperate with a good recipient; however, because of “trembling hand”, he defects. We assume that the recipient participating in the game always knows the donor's action, whereas other individuals can only observe the donor's action with probability $q > 0$. After the interaction, observers update their impressions of the donor independently or non-independently based on their information. The former means that individuals have private opinions on the behaviour of the donor, whereas the latter means that the donor’s reputation is public. In other words, all individuals have the same opinion on it. In the following, we use the private reputation scenario and the public reputation scenario, respectively, to refer to the two situations.

When an individual is an observer, he will observe the interaction between the donor and the recipient, and then update his opinion on the donor based on the social norm n. Social norm n is a mapping from $\{G,B\}\times\{C,D,P\}$ to $\{G,B\}$. It’s based on second-order assessments considering both the reputation of the recipient and the behaviour of the donor. Specifically, if the donor takes action X ($X \in \{C,D,P\}$) towards a recipient with a reputation J ($J \in \{G,B\}$), then the donor will have a new reputation $n(J,X)$ ($n(J,X) \in \{G,B\}$). In this paper, we focus on nine social norms which are listed in Table 1 and use N_i ($i = 1,...,9$), respectively, to refer to them.

		1	2	3	4	5	6	7	8	9
	G	B	B	B	B	B	B	B	B	B
C	G	G	G	G	B	B	B	O	O	O
D	B	O	B	G	O	B	G	O	B	G
P	B	G	G	G	G	G	G	G	G	G

Table 1: Nine social norms. G, B in the yellow head row refers to the reputation of the recipient, namely, good or bad. C, D, and P in the green head column refer to the donor’s action, namely, cooperate, defect or punish. G, B, and O in the blue and red table content refers to the donor’s new reputation after he acts towards a good or bad recipient, respectively. O indicates that observers withhold judgment about the donor's current behaviour and do not change their previous impression on the donor.

One of the common characteristics of the nine social norms we choose is that the standards are the same when they interact with good recipients. In reality, the criteria for evaluating what we should do when interacting with a good person are relatively simple; but the criteria for evaluating what we should do when interacting with a bad person are more difficult. We generally believe that punishing bad people

is pro-social behaviour. However, there is still no consensus on whether we should help or not help bad persons. In this study, we consider social norms allowing withhold-judgment on donors, in which helping or not helping bad recipients does not affect the donor's reputation. Our goal is to test whether social norms allowing withhold-judgment can stabilize cooperation under incomplete information.

The donor with strategy S will take action $S(G)$ toward good recipients and $S(B)$ toward bad recipients. $S(G)$ and $S(B)$ can either be C D or P. That is, the donor might play the same or different actions depending on which tag the recipient player displays [42]. There are $3^2 = 9$ different strategies, namely, $S(G)S(B) \in \{ CC, CD, CP, DC, DD, DP, PC, PD, PP \}$. Although all strategy types are considered, we mainly discuss the competition and evolution between four reasonable strategies, namely, CC, CD, CP and DD in this paper.

The size of the population is N. In each time step, each individual participates in $I = rN$ rounds of interactions, where r denotes the sampling ratio. In each round of interaction, an individual may act as a donor or a recipient with equal probability. After the interaction, the donor‘s reputation will be updated. Let $M_{j,i}^{k}$ denote individual j ’s opinion of individual i after i has participated in the k -th interactions. In the public reputation scenario, $M_{j,i}^{k} = M_{j',i}^{k}$, namely, any two individuals j and j' have the same opinions on individual i. However, in the private reputation scenario, $M_{j,i}^{k}$ and $M_{j',i}^{k}$ can be different because individuals j and j' can have different opinions on individual i due to the different information they have. Let π_i^k denote the payoff of individual i in the k -th interaction, so the average payoff of individual i is $\pi_i = (\sum_{k=1}^{I} \pi_i^k) / (I/2)$. We define the average cooperation rate of the population as $2\sum_{i=1}^{N} I_i^c / (NI)$, where I_i^c denotes the number of times that individual i cooperates in the I rounds interactions of an evolutionary time step. We use $(\sum_{i=1}^{N} \pi_i) / N$ to represent the average payoff of the population or average social welfare.

The number of four strategy types in the population will evolve based on their payoffs. We utilize the strategy to update rules based on the Wright-Fisher process in the simulation. In a new evolutionary time step, each individual breeds several offspring individuals, the number of which is proportional to its fitness. The new offspring individuals adopt the same strategy as their parent with probability $1-\mu$, and randomly select a strategy from the strategy space with probability μ, where μ can be understood as the mutation rate. In particular, we have also considered another form of mutation which is called recombination, i.e., with a small probability μ, the possibility of the first action (taken against a good recipient) being swapped with the second action (taken against a bad recipient) [43]. Then all new individuals form an offspring pool, from which N individuals are randomly selected as the new generation, such that the size of the population remains the same in the evolution.

In the following, we systematically compare the nine social norms and analyze their impacts on the evolution of cooperation under different conditions in the context of public reputation and private reputation, respectively. Initially, each individual's strategy and reputation are randomly assigned in the simulation. Figure 1 shows the evolutionary process of the frequencies of the four strategy types, the corresponding cooperation rate and average payoff of the population, respectively, in 5,000 evolutionary time steps under a specific combination of parameters.

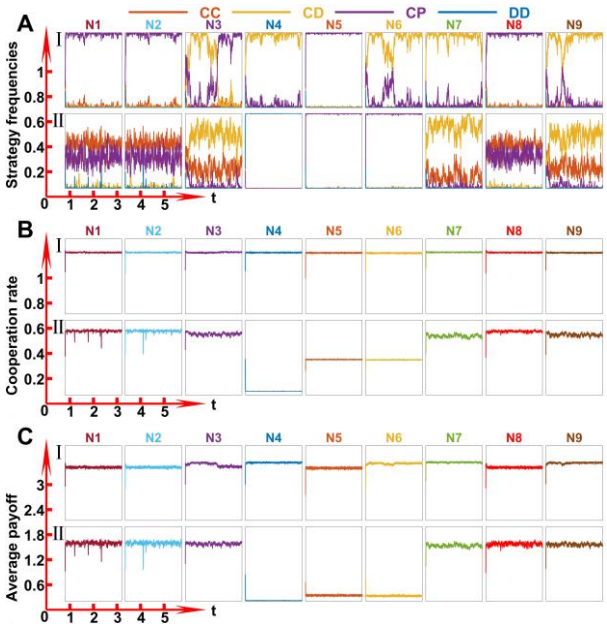

Fig.1: Evolutionary process of the population in nine different social norms when there are four strategy types initially and each accounts for one-fourth. (A) Frequencies of the four strategy types; (B) Cooperation rate; (C) Average payoff. (I) Public reputation scenario; (II) Private reputation scenario. Here $N = 600$, $r = 0.5$, $b = 9$, $c = 3$, $\alpha = 1$, $\beta = 4$, $q = 0.7$, $\varepsilon = 0.1$ and $\mu = 0.005$, which are mostly based on Ref. [18].

The results show that except for the N4 in private reputation scenario, all other eight social norms can maintain the stability of cooperation in both reputation scenarios. Notably, under N1, N2, N5, and N8 social norms, the CP type strategy is always dominant to the CD and DD type strategies in both scenarios, indicating that the introduction of punishment has a positive role in promoting cooperation. Moreover, N1, N2, N3, N7, N8, and N9 social norms can always maintain high social welfare in both reputation scenarios, whereas the N5 social norm has a relatively low social welfare although CP dominates. This also illustrates

that the introduction of the punishment strategy has two sides in some scenarios. On the one hand, it can promote cooperation; on the other hand, it may reduce social welfare. We have noticed that in the private reputation scenario, when the system is dynamically stabilized, as long as there are two strategies coexisting, the frequencies of strategies will fluctuate in the evolution. This is because in our model individuals can only observe the donor's action with a probability. After the interaction, observers updates their impressions of the donor independently based on their information, which means that every individual has a private opinion on the behaviour of the donor. Holding different opinions on the same person will lead to different behaviours of these people, which will result in fluctuations in the frequencies of the strategies.

We have also tested the competition of all nine strategy types. If the other five strategy types are only generated at the mutation stage, or only accounts for a small percentage (lower than 10% totally) in the initial population, our results will not be affected. However, if there are nine strategy types initially and each accounts for one-ninth, our results will change significantly, in which only DD can be stabilized in most social norms. When only two antisocial strategy types, namely, PC and PP are added in the population initially together with the four strategy types, and each account for one-sixth, then CD or CP can be stabilized in the public reputation scenario under most social norms except N2. However, in the private reputation scenario, except for N3 and N9, under all other social norms, only DD can be stabilized. Moreover, our results are robust to the special mutation of recombination and other strategy update rules, such as the Moran process-based rule. In the following, we will further explore the impact of parameters on the cooperation rate and average payoff of the population under these social norms.

Figure 2 shows the effects of observation probability q on the cooperation rate and average payoff of the population. Since there is a variable in each of the figures 2 to 5, in order to test the robustness of our social norms under more severe conditions, we reduce the values of b or β when necessary. As seen in the figure, the increase in q can strengthen the role of social norms, thereby promoting the cooperation rate and also improving the average welfare of the society. Notably, we notice that in the private reputation scene and under the N5 social norm, as the observation probability increases, the cooperation rate always rises, but the average payoff of the society drops first and then rises. This phenomenon can be explained intuitively as follows. When observation probability is low, N5 can hardly play a role in motivating individuals to cooperate, resulting in defection for most individuals in the society, therefore, the average social payoff is nearly zero. When observation probability increases gradually, N5 begins to play a role in promoting cooperation, but the cooperation rate is not high either and defection can induce a large amount of punishment. On average, the penalty cost that the society needs to bear is higher than that of the welfare that cooperation brings to society. Therefore, the average payoff of the population is negative. When observation probability continues to increase close to 1, the role of N5 is fully played, in which situation cooperation rate reaches a very high level, and thus the penalty cost effects are weakened and the overall social welfare is positive. The results in Figure 2 are the average results of ten independent realizations. We have calculated the standard deviations of the results over ten different realizations, which are very small, approximately on the order of 10^{-3}. Thus, we do not show the error bars in Figure 2 and the following figures.

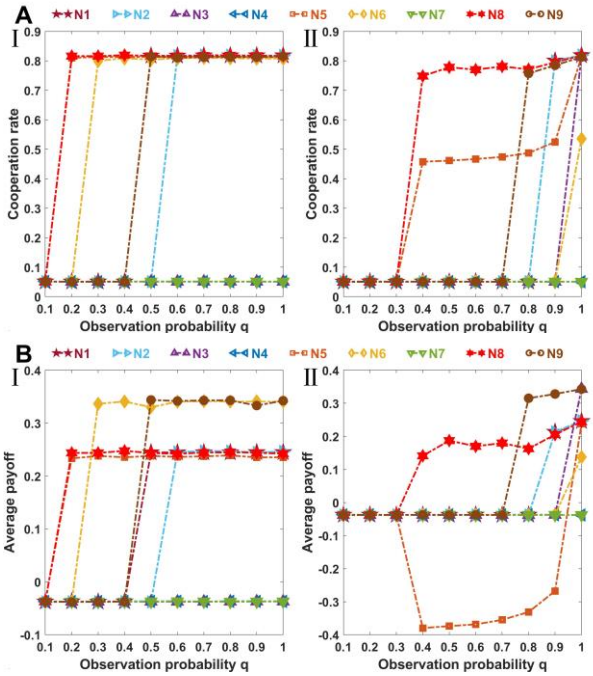

Fig.2: The impact of observation probability q on (A) cooperation rate and (B) average payoff of the population. (I) Public reputation scenario; (II) Private reputation scenario. Here $N = 600$, $r = 0.5$, $b = 4$, $c = 3$, $\alpha = 1$, $\beta = 4$, $\varepsilon = 0.1$ and $\mu = 0.005$.

We have noticed that N8 and N9 can maintain high cooperation rates in both scenarios and the average social payoff is also high. By comparing N8 with N2 (or N5) or comparing N9 with N3 (or N6), we found that it is more conducive to cooperation and high payoff of the society if other individuals withhold their judgments when the donor helps a bad recipient, comparing with the situation that the donor is directly regarded as a bad or good person when he helps a bad recipient. Notably, in the private reputation scene, even if observation probability is low, N8 can still maintain a high cooperation rate and relatively high social welfare. This observation can be explained intuitively as follows. Under noisy and incomplete information situation, if observers evaluate the current behaviour of a donor based on false information, it may further cause a chain reaction, which leads to the collapse of the entire reputation system. However, if observers withhold judgment on the donor's current behaviour, it will hinder the amplification of the chain effect,

thus contributing to the maintenance of the social reputation system. Moreover, under N8, a bad reputation donor can only gain a good reputation by punishing those who have a bad reputation, which is more severe for bad persons. In the situation of reputations being private, the individual's behaviour has a certain probability that cannot be observed. In this case, it is even more important to maintain a good reputation. Because it is costly to be labeled a bad reputation. In fact, CP domination also reveals that good people will be helped, and bad people will be punished. When most people in the population are good, most people will cooperate. Even if the information is not complete, the results will not be affected. Therefore, N8 shows better robustness.

We also found that N1, N4, and N7 perform poorly in the private reputation scenario. Their common feature is to withhold judgment on the defection behaviour of donors. This demonstrates that we can not be indifferent to defection among individuals; otherwise, the society will be in danger. Although in N5 social norm, the introduction of punishment will cause the whole society to bear huge punishment cost, and cause the whole social welfare to fall below zero. However, in the N8 social norm, the CP strategy is always dominant, which illustrates that punishment plays an important role in promoting cooperation.

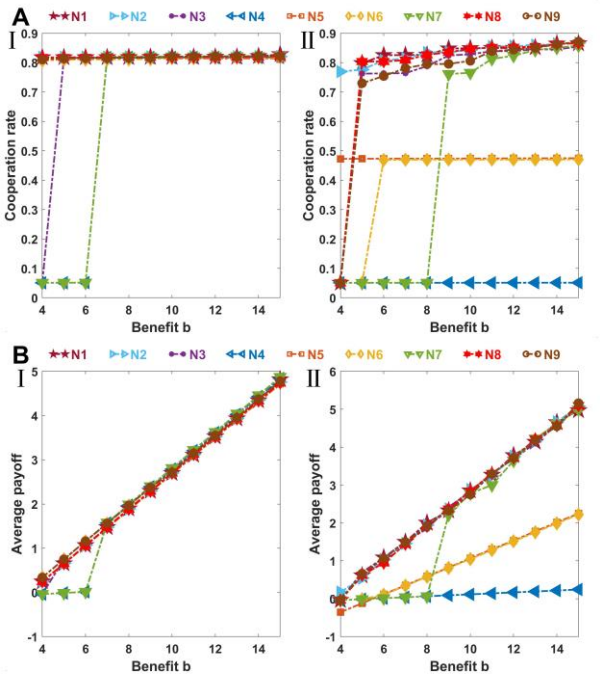

Fig.3: The impact of benefit b on (A) cooperation rate and (B) average payoff of the population. (I) Public reputation scenario; (II) Private reputation scenario. Here $N=600$, $r=0.5$, $c=3$, $\alpha=1$, $\beta=1.5$, $\varepsilon=0.1$, $q=0.7$ and $\mu=0.005$.

Figure 3 shows the impact of benefit b on cooperation rate and average payoff of the population. We have noticed that N1, N2, N3, N8, and N9 can maintain high cooperation rates in both scenarios, whereas N5 and N6 only maintain medium cooperation rates in private reputation scenario, and N4 and N7 are even unable to initiate cooperation in both scenarios when benefit b is small. The common feature of N5 and N6 is that when the donor cooperates with bad recipients, he will have a bad reputation. What social norms N4 and N7 have in common is that when the donor defects against the recipient, other observers' opinions on the donor do not change. All these results further consolidate our previous conclusions that inaction with defection will harm social welfare.

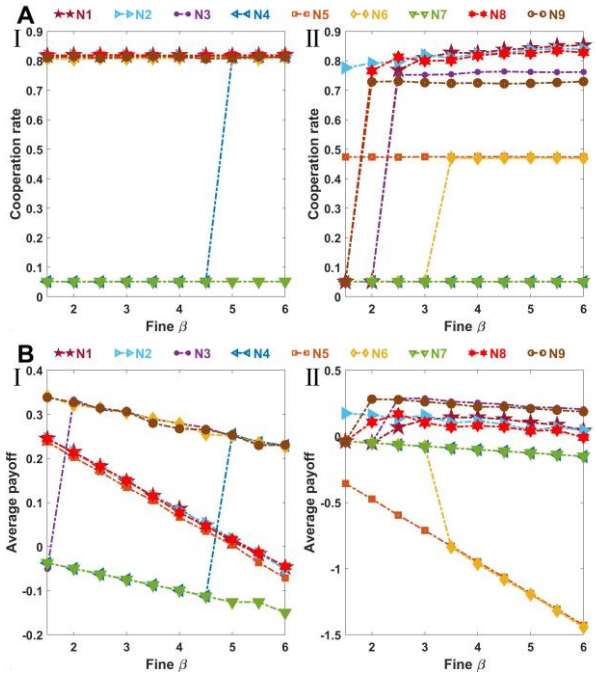

Fig.4: The impact of fine β on (A) cooperation rate and (B) average payoff of the population. (I) Public reputation scenario; (II) Private reputation scenario. Here $N=600$, $r=0.5$, $b=4$, $c=3$, $\alpha=1$, $\varepsilon=0.1$, $q=0.7$ and $\mu=0.005$.

Figure 4 shows the impact of fine β on cooperation rate and average payoff of the population. We have noticed that N4 and N7 are unable to maintain the evolutionary stability of cooperation in both scenarios. Moreover, in the private reputation scenario, except for N4 and N7, all other social norms can maintain the stability of cooperation, although, in N5 and N6 social norms, the cooperation rate only reaches a medium level. On the whole, the increase in the fine favours the evolution of cooperation, but it has a negative impact on the average payoff of society. Notably, N9 can bring relatively high average payoffs of the society in both scenarios. This demonstrates that when the fine of punishment is large, the rules that helping bad recipients not affecting its reputation, and defecting or punishing bad recipients being given a good reputation are conducive to the improvement of social welfare.

Figure 5 shows the impact of error rate ε on the cooperation rate and average payoff of the population. We have noticed that in the public reputation scenario, all social norms

can maintain the evolutionary stability of cooperation. However, in the private reputation scenario, N4 cannot maintain the evolutionary stability of cooperation, and in N5 and N6 social norms, the cooperation rate only reaches a medium level. As the error rate increases, both the cooperation rate and average payoff of the population decline. We can suppose an ideal situation that all individuals are willing to cooperate, but they act incorrectly, such as defection, with probability ε . Defection will trigger punishment, inducing the society to bear more cost, and as ε gradually increases, the frequency of individuals being punished also increases. Therefore, the average payoff of society will decrease in ε .

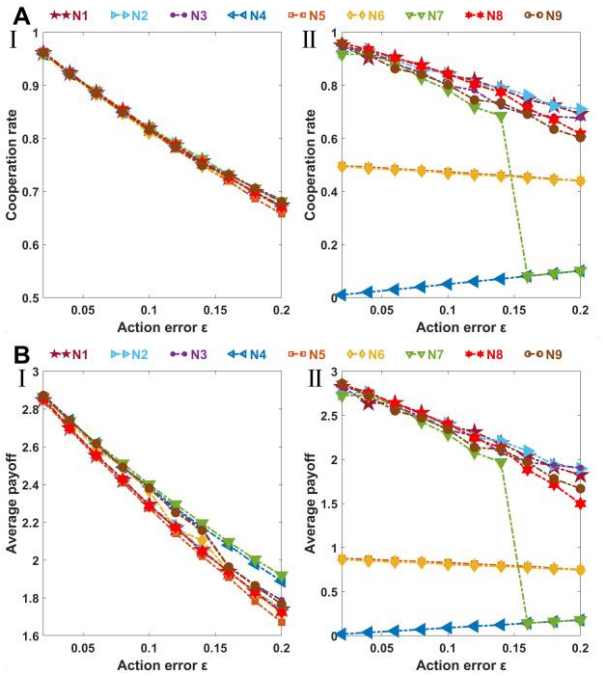

Fig.5: The impact of error rate ε on (A) cooperation rate and (B) average payoff of the population. (I) Public reputation scenario; (II) Private reputation scenario. Here $N = 600$, $r = 0.5$, $b = 9$, $c = 3$, $\alpha = 1$, $\beta = 4$, $q = 0.7$ and $\mu = 0.005$.

In this paper, we have explored cooperation behaviour and the average payoff of the population based on the indirect reciprocity mechanism under incomplete information. Indirect reciprocity investigates how a relatively large group maintains cooperation when individuals take actions and make a judgment based on the information they own. In indirect reciprocity, simple rules such as those based on first-order social norms [25, 26] may not be robust, since individuals may not punish defectors in order to maintain their good image [44]. In contrast, the "leading eight" social norm utilizes more information including the behaviour of donors andthe reputation of both donors and recipients, and can maintain the stability of cooperation [29, 45]. However, their standard model assumes that each interaction between two individuals can be observed by observers who can propagate the results of the interaction to all individuals so that all individuals' opinions will be the same. Social norms such as stern judging have also proven to be successful under public information [30]. However, this rule cannot distinguish between good and bad individuals when information is private [35]. Hilbe [37] relaxed the public information assumption and found that social norms which could maintain cooperation in public reputation scenario failed in private reputation scenario.

A key question is whether simple social norms can be constructed to maintain the evolutionary stability of cooperation in both scenarios. We observe that in the private reputation scenario because previous information held by different individuals is usually different, even if they observe the same interaction, their opinions on the donor may be different. The differences might be magnified over time and eventually lead to the collapse of the reputation system. Therefore, a natural idea is that if we allow other individuals to have reservations about the individual's current behaviour, will it inhibit the spread of the chain effect?

We have discussed nine social norms that allow withhold-judgment and analyzed in detail their impacts on the evolution of cooperation in public and private reputation scenes, respectively. Our experimental results show that under these conditions, most social norms can maintain high cooperation rates. Notably, in the case of private reputation, when the social norm is N8, even if the observation probability is low, the cooperation rate and average income of the society are relatively high, and the CP type strategy is always dominant. From this perspective, we can draw the conclusion that withhold-judgment and punishment can have an important role in the stability of cooperation in indirect reciprocity. We think there are still many other social norms that can be studied in future research, such as those that do not regard "punishing bad people is definitely good", based on which there are many variations. In fact, there are too many possibilities even within the second-order social norms. This issue is extremely complicated but very interesting. We will continue to pay attention to this issue in future research.

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