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Information fusion based on reputation and payoff promotes cooperation in spatial public goods game

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ABSTRACT

Reputation information plays an important role in human behavioral interactions. There have been many studies that have considered reputation in spatial public goods game models, but they essentially assumed that reputation will only change the payoff structure of the game (individuals with a good reputation will get more benefits in the future). In fact, individuals with good reputation will have greater influences which can also affect the decision of neighbors around them in the strategy updating process. Grounded on this observation, we proposed an improved strategy learning rule considering both reputation and payoff information. We employ evidence theory to fuse these two aspects of information, based on which individual strategies can be updated. In addition, we construct a weight coefficient to quantify the importance and reliability of reputation. Through numerical simulations, it is unraveled that the reputation effect can greatly attract nearest neighbors to form greater clusters, thus promoting the emergence of cooperation. With the reputation weight increasing gradually, the critical enhancement factor for cooperation to arise is by degrees reduced to the lower boundary, demonstrating that an increasing tendency of strategy adoption relying on reputation is more likely to allow cooperation to thrive. In the region of low reputation weights, reputation only plays a subtle role in inducing cooperation. Within the region of high reputation weights, cooperation is dramatically boosted by reputation and cooperators can swiftly occupy the whole population. Our work may be helpful to further understand the effect of reputation on the emergence of cooperation.

1. Introduction

Cooperative behaviors are prevalent in human society and biological populations, but cooperation means selfish individuals forgo their benefit to help others, which appears not to be favored by natural selection. As such, how cooperation evolves in a competitive world remains a long-standing conundrum across various disciplines [1–3]. Through decades of exploration, the evolutionary game theory has developed into a powerful vehicle for interpreting and resolving the conundrum. Statistical physics [4–9], especially Monte Carlo method [10,11], has also proven to be very relevant and practical for studying human behavior and thus exploring the evolution of cooperation. Nowadays, various game models are employed for representing the cooperation dilemma, such as the Prisoner's Dilemma Game and Snowdrift Game for pairwise

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interaction. Involving group interactions [4], the representative is the Public Goods Game (PGG), which is a more general form of interactions. In a typical public goods game, each player has the temptation to defect for pursuing a higher payoff, but if all players cooperate, the group can maximize their benefits on the whole. As a metaphor for the cooperation dilemma, this situation also probably gives rise to the so-called tragedy of the commons.

Up to now, researchers have proposed diverse cooperation mechanisms [12–19] to explain the cooperation dilemma. Notably, Nowak [20] summed up five main mechanisms for the evolution of cooperation, namely, kin selection, direct reciprocity, indirect reciprocity, network reciprocity, and group selection. In particular for the network reciprocity, Nowak and May [21] carried out a pioneering study on the spatial patterns of individuals, demonstrating that cooperators developing compact clusters can defend against the invasion of defectors. This striking finding has aroused widespread interests of researchers [22,23]. Starting from this, a plethora of research has confirmed that some features, such as heterogeneity [24–28], network structure [29–33], etc., can effectively boost the emergence of cooperation in spatial PGG (SPGG).

In the real world, individuals universally assess the behaviors of others, the information of whom will affect the subsequent decision-making of their interactive object. Reputation mechanism plays a central role in human societies and has aroused widespread interest in the realm of the evolutionary game [34,35]. Early germinating in the indirect reciprocity, reputation was presented by Nowak and Sigmund [36] in the form of image scoring. Furthermore, spreading to the network reciprocity, reputation has also turned out to be an important factor that makes a decisive difference in partnership building and cooperation formation, such as partner switching depending on high reputation [37–39] or inferring reputation to choose partners based on limited cognition [40,41], and reputation threshold-enabled SPGG [42]. As may be a currency valid in many social games, reputation is able to bring us great benefit if traded in a market [43] and even directly change our utility in a game [44]. There are also some studies on behavior experiments showing the importance of reputation in promoting cooperative behavior [45,46]. As for measuring the reputation, some research demonstrated that reputation can be very well quantified as a weighted mean of the fractions of past cooperative acts and the last action performed [47]. In addition, at the stage of strategy transfer, the individual diversity based on reputation is also considered to propose adaptive reputation assortment in SPGG [48]. Recently, from the perspective of gossip and individuals' tolerance, it has been found that cleverly handling the donation information can promote cooperation in PGG [49].

Although previous studies have considered the reputation effect in their models, it is essentially assumed that the reputation of individuals only changes the payoff structure of the game. Such as the assumption that individuals with a good reputation will get more benefits in the future. In fact, reputation may also influence individuals' imitation in the strategy updating process. Since reputation is positively correlated with influence in reality [50], individuals with good reputation also possess greater influence which will affect their surrounding individuals' strategy imitation. Therefore, when individuals choose strategies in uncertain environments, they will learn not only the strategies of individuals with high payoff but also the strategies of individuals with a good reputation. Motivated by this observation, we have improved the traditional payoff-based strategy updating rule by integrating reputation into the learning process. Individuals' strategy updating rule is a mechanism of making decisions by learning and imitating others [23], which can significantly affect the evolutionary equilibrium of the system. Dealing with different factors, many learning mechanisms have been proposed, such as the Fermi rule [6,51], the Markov process-based rule [52,53], the win-stay-lose-shift rule [54,55], the aspiration-driven rule [56], PSO-based rule [57–59] and genetic algorithm based rule [60,61] stemming from intelligent algorithms. Moreover, some research combined different rules to design a sophisticated rule [62,63].

In recent years, evidence theory, as a new framework to describe and deal with uncertainty, has been successfully introduced into evolutionary games for integrating differently sourced information to improve an individual's decision-making [64–70]. In fact, it leverages the way of evidential reasoning to fuse multi-sourced information so as to update the evaluation of uncertainty. Evidence theory is generally considered to be a generalization of Bayesian inference in probability theory. In the real world, people usually assess one thing with uncertainty from many perspectives to acquire different sourced information. How do people integrate all the information to make a decision? Evidence theory provides a scientific method to cope with such a situation. When making decisions in an uncertain environment, people will integrate diversely sourced information comprehensively, and the process of which can be described by reasoning through evidence. As such, this method can also be applied to the process of individual's imitation and learning.

In our research, we regard the probability of imitation according to payoff or reputation as pieces of evidence that can be obtained by individuals. Then we apply evidence theory to fusing the information of the two aspects into a new piece of evidence. According to the new piece of evidence, individuals can make a comprehensive decision. In particular, we build an improved combination rule that considers the importance and reliability of evidence, which can overcome some inherent disadvantages of the traditional Dempster's combination rule. We leverage the evidential reasoning to elaborate an improved strategy updating rule and thus to study the impact of reputation on the emergence of cooperative behavior. To the best of our knowledge, this is the first study that attempts to use evidence theory while considering importance and reliability to merge payoff and reputation information to simulate individual learning in games.

The rest of the paper is organized as follows. In Section 2, we construct our model, concentrating on integrating reputation effect into the learning process by evidential reasoning. Then the corresponding simulation results are demonstrated in Section 3. We further analyze and discuss the role of reputation played in boosting cooperation in detail in this section. At last, the conclusions are summarized in Section 4.

2. Model

2.1. Spatial public goods game

The PGG is conducted on a $L \times L$ square lattice network with periodic boundary conditions. This indicates a kind of typical interactive structure, where every node represents an individual located on the network and has only four neighbors around it. At every phase of evolution, the selected focal individual participates in multi-group PGGs simultaneously, which are centered on himself and his immediate neighbors. In a typical PGG [22], each player in one group can decide to be either a cooperator (C) contributing $\delta = 1$ to the common pool or a defector (D) contributing nothing. After the sum of all contributions multiplied by an investment enhancement factor $r(r > 1)$, the total return is distributed equally among all members of the group. Therefore, the payoff $\pi_{s_i}^g$ of player i with strategy s_i in group $g(g = 1, 2, \ldots, G)$ is given by

$$\pi_{s_i}^g = \begin{cases} \frac{r(N_C^g+1)}{|\Omega_i|+1} - 1, & if\ s_i = C \\ \frac{rN_C^g}{|\Omega_i|+1}, & if\ s_i = D \end{cases}, \tag{1}$$

where Ω_i is the set of neighbors around player i; N_C^g is the number of cooperators in the group game g excluding player i. We assume $r < |\Omega_i| + 1$, which implies that defectors without paying a cost always take a higher payoff than that of cooperators. So that each player has the temptation to defect, but all cooperating is optimal for the group on the whole. This situation yields a social dilemma.

Through multi-group games in one stage, the eventual payoff π_i of player i obtained by accumulating the payoffs from all the G rounds game is

$$\pi_i = \sum_{g=1}^{G} \pi_{s_i}^g \tag{2}$$

2.2. Integrating reputation with payoff in strategy updating

During the process of evolution, we employ the Monte Carlo simulation procedure for random sequential strategy updating [22,23], which provides a more realistic approach than that of updating synchronously with special initial conditions [10]. In particular, each elementary step involves randomly selecting one focal individual i and one of his neighbors j. And then i learns the strategy of j with a certain probability. In the simulation, a complete Monte Carlo step (MCS) is composed of L^*L times of repeated elementary steps, which guarantees that on average every individual in the network has a chance to be selected to update its strategy asynchronously [10].

In general, we employ evidential reasoning to merge two pieces of evidence from payoff and reputation, respectively, so as to construct a sophisticated strategy updating rule. In the following of this section, we elaborate the rule from three parts, namely, payoff-based learning (one piece of evidence from payoff), reputation-based learning (another piece of evidence from reputation), and a special evidential reasoning rule to integrate the two pieces of evidence.

2.2.1. Payoff-based learning

Firstly, we introduce the piece of evidence from payoff-based learning. In the beginning, we adopt random initial conditions to set up the starting configuration. Strategies of all players are initially distributed uniformly at random over the network. Afterwards, based on the difference between their accumulated payoffs from multi-group games, the focal player i imitates the strategy of the neighbor j with the following probability:

$$P_1(s_j \to s_i) = \frac{1}{1 + \exp(-\frac{\pi_j - \pi_i}{\kappa})} \tag{3}$$

where π_i and π_j signify the accumulated payoffs of i and j, respectively; $\kappa(\kappa > 0)$ denotes noise intensity quantifying uncertainty in the strategy imitation. Especially but without loss of generality, κ is set to 0.5 in this study [23,51].

2.2.2. Reputation-based learning

Secondly, we detail another piece of evidence from reputation-based learning. Similar to strategy initialization, every player is randomly assigned a reputation score φ ranging from 0 to $\varphi_{\max}$ at the beginning. The reputation score evolves as time varies. Since reputation in reality is the accumulation of individuals' behavior in a long history [47], we utilize a simple but effective reputation scoring system in which after playing the game, an individual i's reputation score $\varphi_i(t)$ at time step t is reassessed according to its previous reputation score $\varphi_i(t-1)$ at step $t-1$ and his current behavior s_i with his neighbors at step t. Then we obtain the iterative formula for updating reputation between successive time steps as follows.

Table 1
Evidence theory fusing two pieces of evidence from the payoff and reputation.

Decision	BOE	Payoff	Reputation	Combination
$s_j \to s_i$	Y	$m_p(Y)$	$m_r(Y)$	$m_{pr}(Y)$
	N	$m_p(N)$	$m_r(N)$	$m_{pr}(N)$

$$\varphi_i(t) = \varphi_i(t-1) + \Delta_i(t) = \begin{cases} \varphi_i(t-1) + 1, & s_i = C \\ \varphi_i(t-1) - 1, & s_i = D \end{cases} \tag{4}$$

Considering that reputation has bottleneck effects, after a person's reputation reaches an extremely high or low level, doing more good or bad things will have little impact on his reputation, and also for the needs of simulation, we take the boundary conditions of reputation into account. Specifically, when $\varphi_i(t) > \varphi_{\max}$, let $\varphi_i(t)$=$\varphi_{\max}$; when $\varphi_i(t) < 0$, let $\varphi_i(t)$= 0. For a proper upper boundary of reputation score [44], we fix $\varphi_{\max} = 100$.

From the perspective of reputation, the focal player i imitates the strategy of his neighbor j with another independent probability:

$$P_2(s_j \to s_i) = \frac{1}{1 + \exp(-\frac{\varphi_j - \varphi_i}{\kappa} * \frac{\pi_{\max}}{\varphi_{\max}})} \tag{5}$$

where φ_i and φ_j indicate the reputation score of i and j, respectively. Considering that the reputation scoring system is absolutely different from the payoff accumulation rule, so we unify them into the same scale by a normalization coefficient $c = \frac{\pi_{\max}}{\varphi_{\max}}$, where $\pi_{\max} = 4r(1 < r < |\Omega_i| + 1)$ is the maximum accumulated payoff in one round game and $\varphi_{\max}$ is the upper boundary value for reputation score.

2.2.3. Decision-making by evidential reasoning

At last, we utilize a special evidential reasoning rule to integrate the two pieces of evidence from payoff and reputation. At every time step of evolution, it is crucial how a player elaborates the optimal strategy by social learning and flexible decision-making to expand its fitness in competition. Since intelligent individuals are inclined to seek and exploit information from circumstances [64–66], here we assume that the focal player i adjusts its strategy by taking into account both the accumulated payoff and the present reputation. In order to make a comprehensive decision based on the above two probabilities, evidence theory is embedded in our model, which is aimed at integrating information from independent sources to shape a new shred of evidence.

Before developing our sophisticated process of evidential reasoning, we firstly introduce some basic conception about evidence theory. For a concrete problem with uncertainty, the elements may be the set of states in an event or targets to be identified, which is called the framework of discernment $\Theta = \{\theta_1, \theta_2, \ldots, \theta_n\}$ in evidence theory. In the definition, Θ is a finite nonempty set of mutual exclusive and exhaustive hypotheses of the problem and n is the number of the hypotheses. Let 2^Θ (2^Θ=$\{\emptyset, \theta_1, \theta_2, \ldots, \theta_n, \{\theta_1, \theta_2\}, \{\theta_1, \theta_3\}, \ldots, \Theta\}$) denote the power set of the framework of discernment, and any element $A \in 2^\Theta$ represents a proposition of the problem. In evidence theory, information is presented in the form of Basic Probability Assignment (BPA) which is also called mass function m. Based on the framework of discernment Θ, a mass function $m(A)$ is a mapping from 2^Θ to [0, 1] (a probability), i.e., $m: 2^\Theta \to [0, 1]$ which satisfies $\sum_{A \in 2^\Theta} m(A) = 1$ and $m(\emptyset) = 0$. One mass function is just a piece of evidence carrying some information. A can be one focal element of the evidence only if $m(A) > 0$ and here we call it body of evidence (BOE).

Based on the above concepts and definitions, when individual i confronts a choice whether or not to imitate the strategy of its neighbor j, it constructs two different BOE, namely, Y(Yes) and N(No). The BPA can be obtained from reputation and payoff based functions in Eqs. (6) and (7).

$$m_p(Y) = P_1(s_j \to s_i), \quad m_p(N) = 1 - P_1(s_j \to s_i) \tag{6}$$

$$m_r(Y) = P_2(s_j \to s_i), \quad m_r(N) = 1 - P_2(s_j \to s_i) \tag{7}$$

Based on the two separate pieces of evidence as listed in Table 1, we can derive the fused probability m_{pr}. We employ a special evidential reasoning rule which considers importance $\omega\,(0 \le \omega \le 1)$ and reliability $\mu\,(0 \le \mu \le 1)$of evidence, and this approach is well suited in dealing with Multiple Attribute Decision Making (MADM) problems where ambiguity, incompleteness and fuzziness are involved [71–74]. We have the following formula

$$m_{pr} = [m_p \oplus m_r](\theta) = \begin{cases} 0 & \theta = \emptyset \\ \frac{\tilde{m}_{pr}(\theta)}{\sum_{\theta \subseteq \Theta} \tilde{m}_{pr}(\theta)} & \theta \neq \emptyset \end{cases}, \tag{8}$$

where θ is a subset of Θ, which denotes a proposition of the problem, and $\Theta = \{Y, N\}$ is the frame of discernment; here our focal elements (or BOE) are just Y and N, so $\theta \in \{\{Y\}, \{N\}, \{Y, N\}, \emptyset\}$. Then $\tilde{m}_{pr}(\theta)$, namely, the combination of $m_p(\theta)$ and

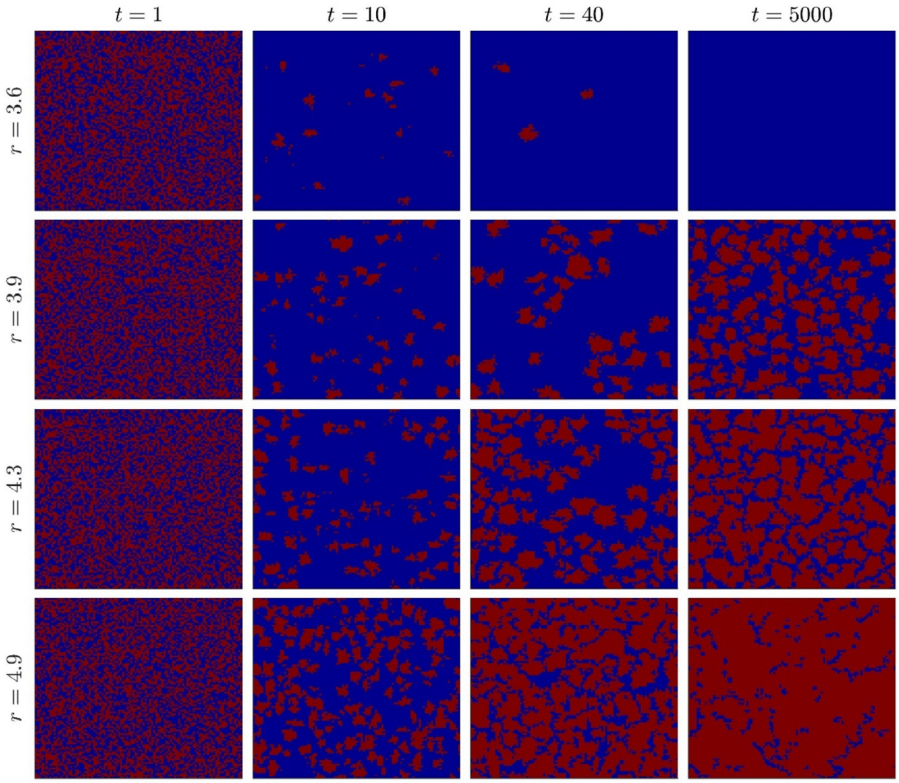

Fig. 1. Spatiotemporal distribution of the two strategies in the SPGG at important time steps $t = 1,\ 10,\ 40$ and 5000 for representative values of enhancement factor $r = 3.6,\ 3.9,\ 4.3$ and 4.9, as obtained with the weak impact of reputation effect $\omega = 0.1$ on the 100×100 square lattice with periodic boundary conditions. The deep red represents cooperators (C), and the deep blue represents defectors (D). The snapshots clearly demonstrate the representative patterns of the competition between cooperators and defectors, which are very close to the classical SPGG. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

$m_r(\theta)$ with importance and reliability can be described as

$$\tilde{m}_{pr}(\theta) = \left[(1-\mu_r)\widehat{m}_p(\theta) + (1-\mu_p)\widehat{m}_r(\theta)\right] + \sum_{A\cap B=\theta} \widehat{m}_p(A)\widehat{m}_r(B)\ , \tag{9}$$

where μ_r and μ_p denote the reliability of reputation and payoff, respectively, and ω_r and ω_p denote the weight of the two factors, respectively, i.e., $\widehat{m}_p(\theta) = \omega_p m_p(\theta),\ \ \widehat{m}_r(\theta) = \omega_r m_r(\theta)$.

In the process of evidence combination, each piece of evidence is profiled by a belief distribution or BPA, based on which we can obtain the joint support $\tilde{m}_{pr}(\theta)$ of a proposition θ from the following two components as in Eq. (9). The first is a bounded sum of individual support for θ, namely, $(1-\mu_r)\widehat{m}_p(\theta) + (1-\mu_p)\widehat{m}_r(\theta)$, and the other is the orthogonal sum of joint support for θ, namely, $\sum_{A\cap B=\theta} \widehat{m}_p(A)\widehat{m}_r(B)$. Notably, the traditional Dempster's combination rule only contains the second part. However, our joint support $\tilde{m}_{pr}(\theta)$ can overcome the Zadeh paradox inherent in Dempster's combination rule.

The importance of a piece of evidence measures the degree of support for a BOE when the evidence points exactly to the BOE. In addition, it is also based on the reliability of the evidence. Reliability is different from importance, but for simplification, we assume that they are equal in this study, i.e., $\omega_p = \mu_p, \omega_r = \mu_r$. To probe into the quantitative effect of reputation, it is also assumed that $\omega_p + \omega_r = 1$. As such, we can tune the fused probability nicely with only one parameter ω_r. Thus, the fused probability is

$$m_{pr}(Y) = \frac{\tilde{m}_{pr}(Y)}{\tilde{m}_{pr}(Y) + \tilde{m}_{pr}(N)}, \tag{10}$$

where the support for imitating the strategy of the chosen neighbor is calculated by $\tilde{m}_{pr}(Y) = [(1-\omega)^2 m_p(Y) + \omega^2 m_r(Y)] + \omega(1-\omega)m_p(Y) * m_r(Y)$ and the support for rejecting imitation is $\tilde{m}_{pr}(N) = [(1-\omega)^2 m_p(N) + \omega^2 m_r(N)] + \omega(1-\omega)m_p(N) * m_r(N)$; ω symbolizes for the importance and reliability of reputation, or a substitute of the symbol ω_r.

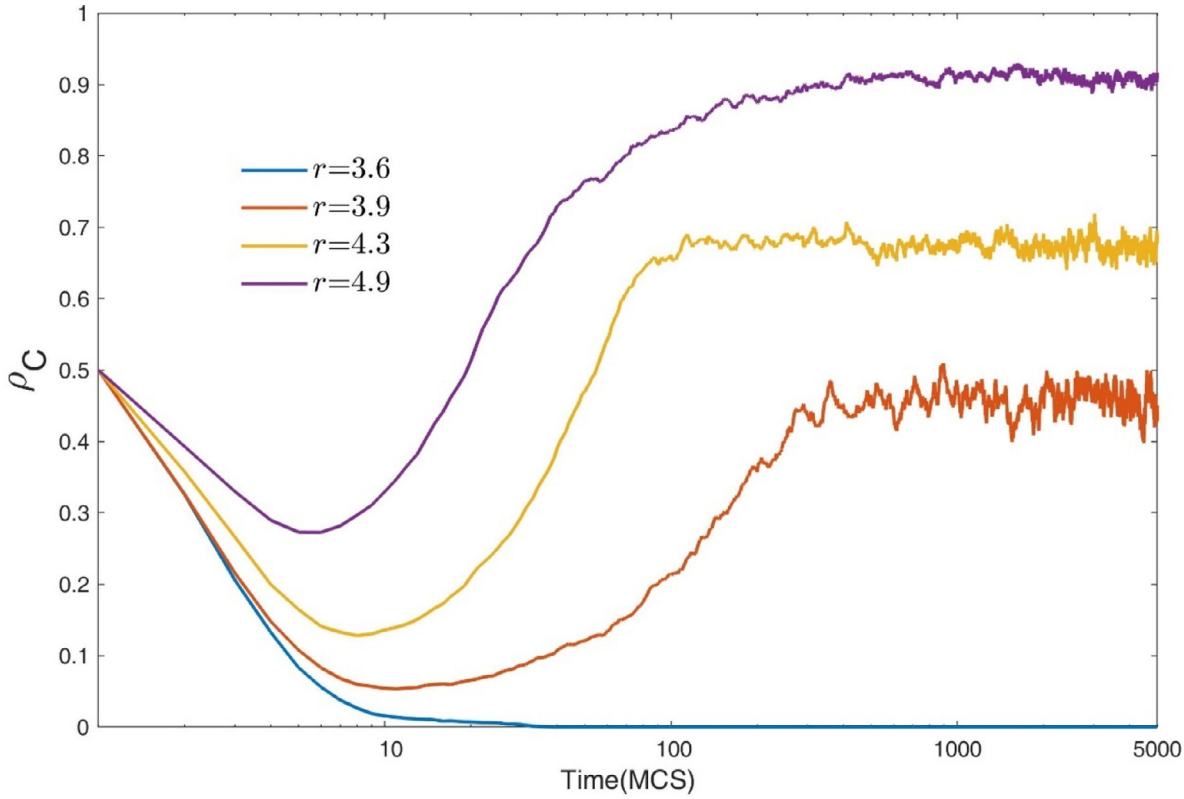

Fig. 2. Temporal evolution of the density of cooperators ρ_C towards its stationary state for a fixed reputation weight $\omega = 0.10$ and different values of the enhancement factor $r = 3.6,\ 3.9,\ 4.3$ and 4.9. Different from the spatiotemporal distribution in Fig. 1, here presented is the visualized fluctuation of ρ_C from the initial state to the final stationary state.

3. Results and discussions

Leveraging efficient computer programs coded by *C* for Monte Carlo simulations, we explore the asymptotic cooperative behavior in the evolutionary process of the population. For the configuration of numerical simulations, there are $N = L \times L$ agents playing the games in the world of cooperators and defectors. In order to make our results stable and valid in the large system-size limit [11], various simulations and verification of validation were conducted on larger size systems such as $L = 200,\ 500$ and 2000. But for clarity, we have just demonstrated the results on the size of 100×100. Moreover, all the following results we report were obtained with an average of 10 independent realizations.

We start by presenting some interesting results in the scenario of specific pairs of parameter combination $(r,\ \omega)$. Since there are merely two parameters that govern our model, we probe into the asymptotic pattern from the view of enhancement factor r and the weight of reputation ω separately.

First, we study various values of enhancement factor at a low weight of reputation $\omega = 0.1$ when the payoff plays a leading role, and our model is compatible with the low-level case (when $\omega = 0$, our model degenerates to the classical case without reputation effect). For the sake of examining the preliminary effect of our modified rule for strategy updating, we select four representative values of enhancement factor $r = 3.6,\ 3.9,\ 4.5$ and 5.3, which cover all typical situations, including low-level cooperation or even cooperators vanishing finally, cooperators and defectors persisting indefinitely, and developing into the prosperous phenomenon of cooperation. As shown in Fig. 1, for each value of the enhancement factor, we display four snapshots at the time steps $t = 1,\ 10,\ 40$ and 5000. And the first one is the strategy distribution for the initial conditions while the last one is for the ultimate stationary state. As time varies, the initial scattered individuals begin to cluster larger and larger for occupying their opponents' territory. For some higher values of the enhancement factor such as $r = 3.9$, 4.5 and 5.3, cooperators coexist with defectors or even defeat defectors in the final asymptotic state ($t = 5000$). However, it is implied that there lies a strict and serious dilemma of cooperation in the PGG. Because while the reputation effect is subtle ($\omega = 0.1$ for $r = 3.6$), the cooperation cannot emerge unless the enhancement factor is large enough. Consequently, in the low-level case where our model is approaching the original version of the SPGG based on the traditional Fermi rule, the population is still confronted with a serious cooperation dilemma to be tackled. In addition, Fig. 2 adds a detailed temporal dimension to Fig. 1, which depicts the fluctuation of the density of cooperators ρ_C in the overall process of evolution from the initial state to the final stationary state. In the region of enhancement factor r that is large enough for cooperation to

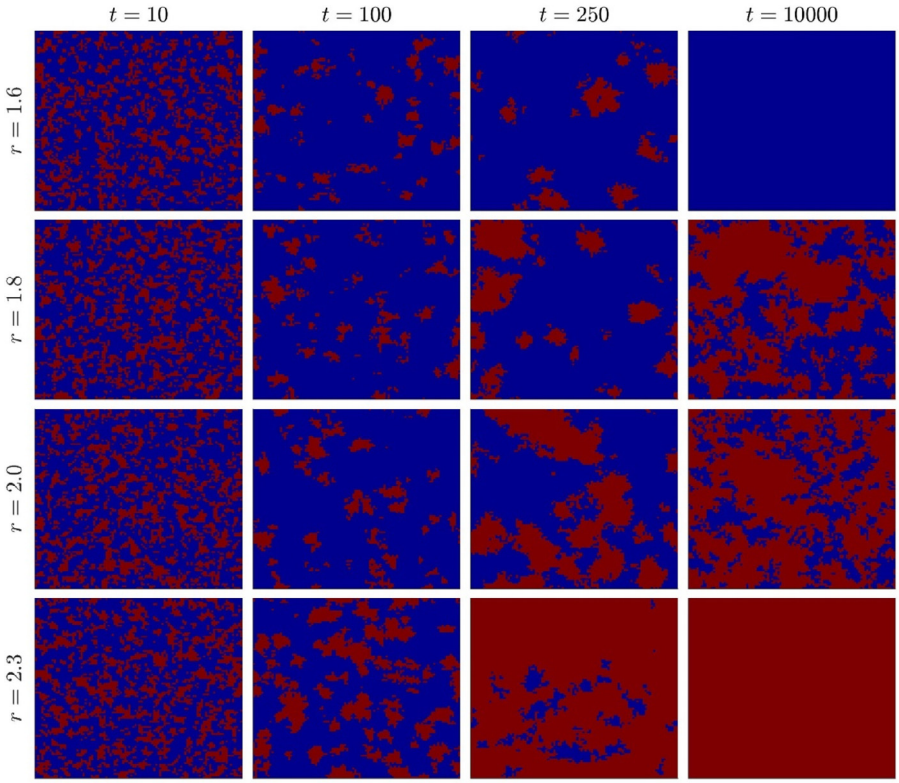

Fig. 3. Spatiotemporal distribution of the two strategies at time steps $t = 10,\ 100,\ 250$ and $10,000$ for different values of enhancement factor $r = 1.6,\ 1.8,\ 2.0$ and 2.3, as obtained with the strong impact of reputation effect $\omega = 0.7$. With the expansion of reputation influence, cooperators can gather swiftly to form large and compact clusters, fighting against defectors at a very small enhancement factor ($r = 1.8$). And at $r = 2.3$ cooperators occupy the whole population.

arise in the stationary state, we notice that the density of cooperators falls sharply to a rather low level firstly and then rises slowly from the bottom until it reaches to the ultimately different levels depending on the enhancement factor. Evidently, the curve for a higher enhancement factor is always superior to that for a lower r. Hence, the classical case is reappeared here and in the positive effect of r on cooperation, our results are consistent with that empirically verified in [75].

In contrast with the case where payoff plays the leading role in the stage of strategy updating and the reputation effect is weak, we increase the weight of reputation ω to a relatively high level 0.7, which greatly boosts the emergence of cooperation. Fig. 3 illustrates that the clusters of cooperators and defectors evolve as time varies. With the expansion of reputation influence, we notice that cooperators can coexist with defectors at a very small enhancement factor ($r = 1.8$) and they gather together to form larger and compact blocks of clusters. Although cooperators are quite disadvantaged at the initial period ($t = 100$), they never vanish and even grow to match defectors in strength at the equilibrium stage ($t = 10,000$). Furthermore, when we increase the enhancement factor r to 2.0, cooperators prevail against defectors. And at last cooperators occupy the whole population at $r = 2.3$. Hence, our results show that good reputation can influence the nearest neighbors or clusters to form a greater group where an individual with a good reputation can expand its fitness, which induces cooperation to thrive in the whole population. Beyond that, by comparing the snapshots vertically, we see that increasing the enhancement factor can also improve the cooperation level in the scenario of strong reputation effect but turns out to be more effective in a limited range. As is also shown in Fig. 4 with the same configuration, the density of cooperators in the population first declines to the bottom around $t = 100$ and then rises sharply to the top level of cooperation except for the scenario of $r = 1.6$, reducing to zero slowly. In addition, we notice that the reputation effect postpones the time point where the density of cooperators is reduced to the lowest level and makes the rising phase of the curve steeper. We also assert that the reputation effect can promote high-level cooperation even in the region of small enhancement factors.

In the context of strong reputation effect, we also examine various weights of reputation $\omega = 0.5,\ 0.6,\ 0.7$ and 0.8 for a lower enhancement factor $r = 2.9$. Fig. 5 visualizes the spatiotemporal distribution of the two strategies. Owing to the low enhancement factor, cooperation fails to arise eventually for $\omega = 0.5$. However, cooperation starts to emerge and even occupy the whole population with the increase of reputation weights by 0.1. We also notice that the size and quantity of cooperator clusters are reduced from $t = 5$ to $t = 100$ for $\omega = 0.5$ and 0.6. In sharp contrast to the trend of $\omega = 0.5$ and 0.6, the clusters of cooperators grow steadily to invade the territory of defectors for $\omega = 0.7$ and 0.8. From the perspective of temporal evolution

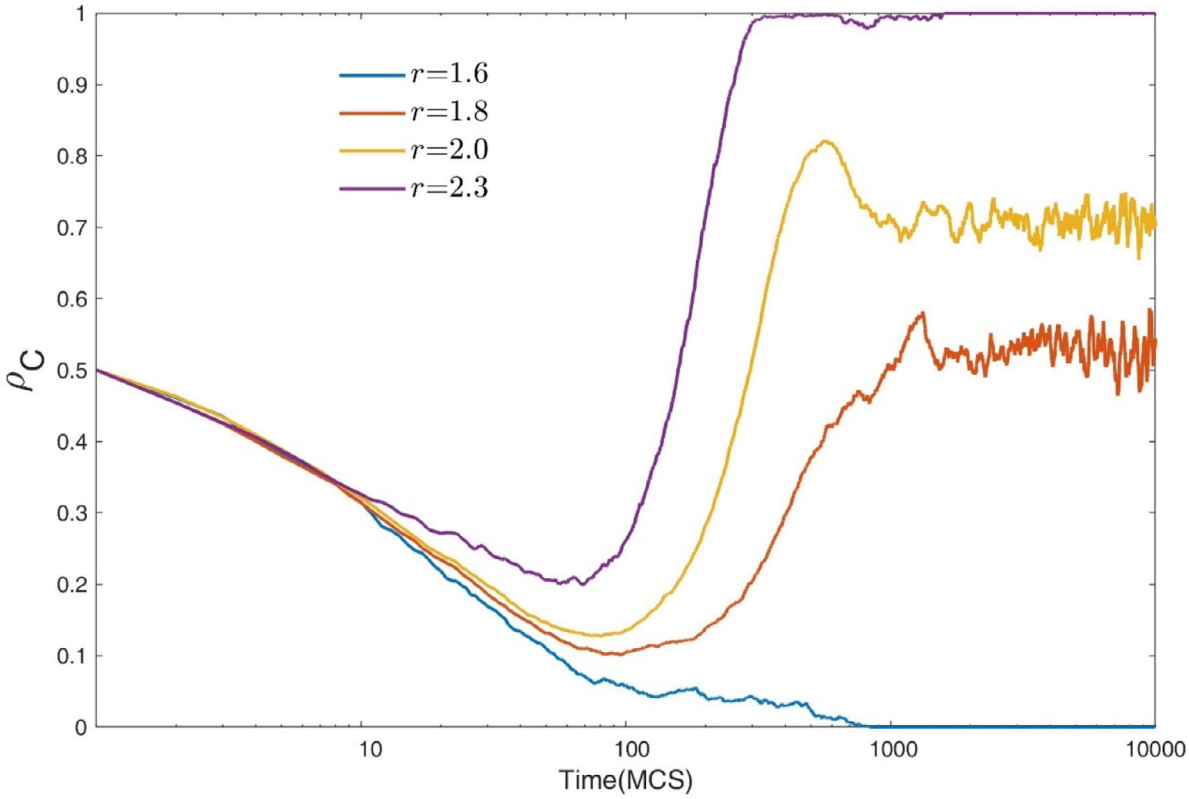

Fig. 4. Temporal evolution of the density of cooperators ρ_C towards its stationary state for a significant reputation weight $\omega = 0.7$ and representative values of the enhancement factor $r = 1.6,\ 1.8,\ 2.0,$ and 2.3. For the three relatively large values of r, the density of cooperators in the population first declines to the bottom at about $t = 100$ and then rises sharply towards the final stationary level of cooperation. Whereas for $r = 1.6$, the density of cooperators is reduced to zero gradually. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

as demonstrated in Fig. 6, there lies remarkable differences between the four curves. The relatively strong reputation effect ($\omega = 0.7$ and 0.8) induces cooperation swiftly and enables the cooperation strategy to spread all over the population. For $\omega = 0.6$, the density of cooperators drops to an extremely low level and takes a very long relaxation time to arrive at the stable stage. So that in the region of high reputation weights, the cooperation level can be enhanced in a nonlinear and dramatic form until cooperators occupy the whole population.

After that, lower weights of reputation are examined further at a proper enhancement factor $r = 3.9$. In the region of low weights of reputation $\omega = 0.1,\ 0.2,\ 0.3$ and 0.4, these results further demonstrate the more details of the subtle effect of reputation from the dimension of temporal evolution. As illustrated in Fig. 7, there are merely small differences between the four curves despite the fact that the reputation weight increases from 0.1 to 0.4. We infer that in the region of low reputation weights, the reputation effect only plays a limited and weak role in enhancing cooperation.

On the whole, we eventually study the asymptotic density of cooperation in the relatively stationary stage for various weights of reputation, represented in Fig. 8. In the extreme case where the piece of evidence from reputation is ignored ($\omega = 0$), the strategy updating rule degenerates into the original version [51] where only payoff information makes sense in strategy updating stage. Cooperation can emerge only if the enhancement factor is high enough ($r > 3.74$). In agreement with the preceding analysis and discussions, for lower weights of reputation ($\omega \in [0, 0.4]$), the reputation effect plays a subtle role, and payoff can give rise to the temptation to defect so that cooperation is quite hard to arise in a large interval of enhancement factor r. However, with the expansion of the reputation weight from 0.4 to 0.8, we discover that the curve moves dramatically towards the direction of decreasing the enhancement factor. This reveals that the reputation effect undermines the cooperation dilemma and even transforms the conundrum into a completely different situation where cooperation emerges efficiently if the reputation works well. In particular, as the reputation weight increases by 0.1, the curve displaces nonlinearly towards the left, especially taking on the trend of enhancing cooperation strikingly in the region of high weights of reputation ($\omega = 0.6$ and 0.7). Moreover, compared with the region of low weights of reputation, the range of enhancement factor r from all defection to all cooperation in the whole population is decreased. Unexpectedly, when the weight of reputation reaches 0.8, the reputation effect leads to an incredibly high cooperation level nearly in the entire domain of the enhancement factor defined in the model.

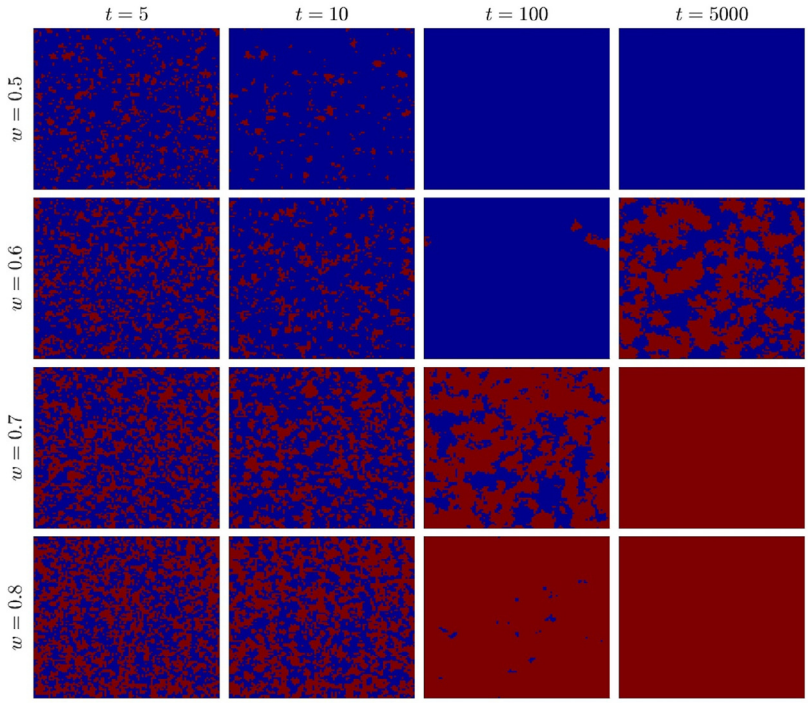

Fig. 5. Spatiotemporal distribution of the two strategies in the SPGG at time steps $t = 5$, 10, 100 and 5000 for different values of reputation weight $\omega = 0.5$, 0.6, 0.7 and 0.8, as obtained with the enhancement factor $r = 2.9$. When $\omega = 0.5$, cooperators disappear eventually owing to the low enhancement factor. However, cooperation starts to emerge at $\omega = 0.6$ and even occupy the whole population when ω is increased to 0.7.

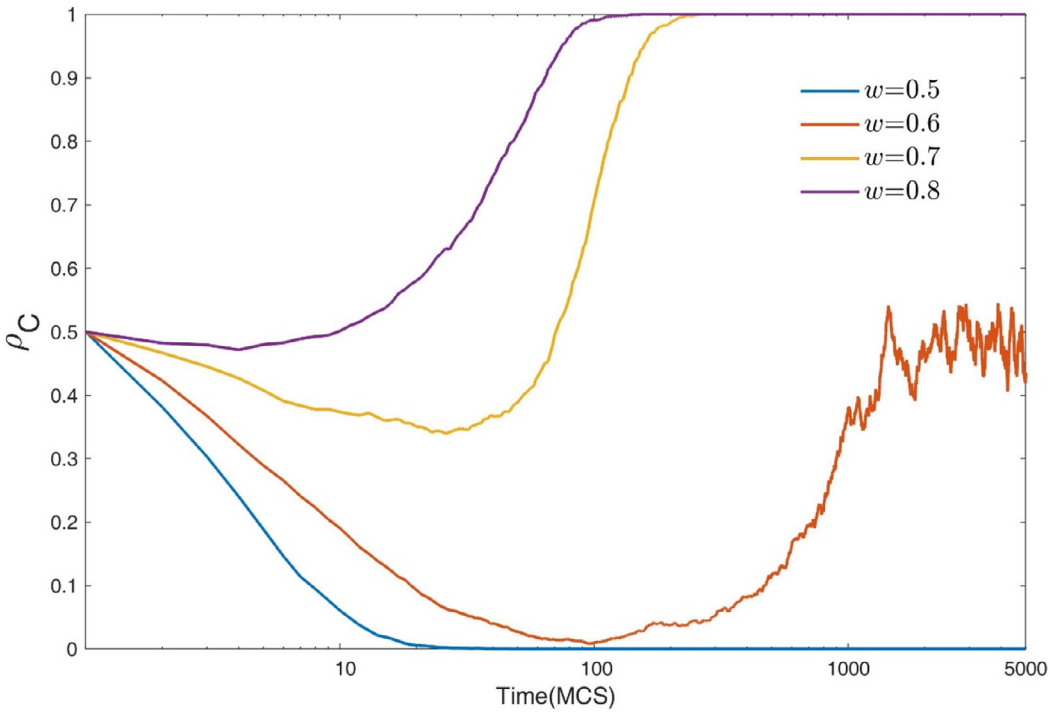

Fig. 6. Temporal evolution of the density of cooperators ρ_C towards its stationary state for a small enhancement factor $r = 2.9$ within the region of high weights of reputation $\omega = 0.5$, 0.6, 0.7 and 0.8. When $\omega = 0.5$, cooperators are quickly eliminated in 50 steps. When $\omega = 0.6$, the density of cooperators firstly drops to an extremely low level and then takes a very long relaxation time to arrive at the stable stage. Whereas when $\omega = 0.7$ and 0.8, the strong reputation effect induces cooperation swiftly and enables the cooperation strategy to spread all over the population.

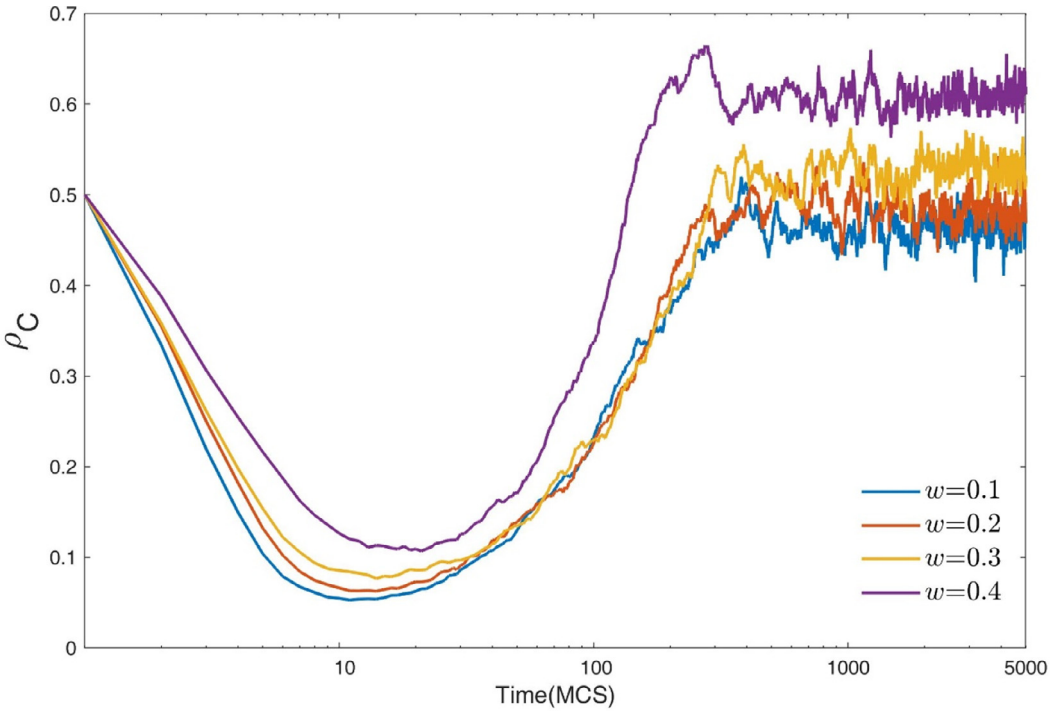

Fig. 7. Temporal evolution of the density of cooperators ρ_C towards its stationary state for a proper enhancement factor $r = 3.9$ in the region of low reputation weights $\omega = 0.1,\ 0.2,\ 0.3$ and 0.4. With the increase of reputation weight from 0.1 to 0.4, there is merely a little improvement in the cooperation level of the population, indicating that in the region of low reputation weights, the reputation effect only plays a limited role in enhancing cooperation.

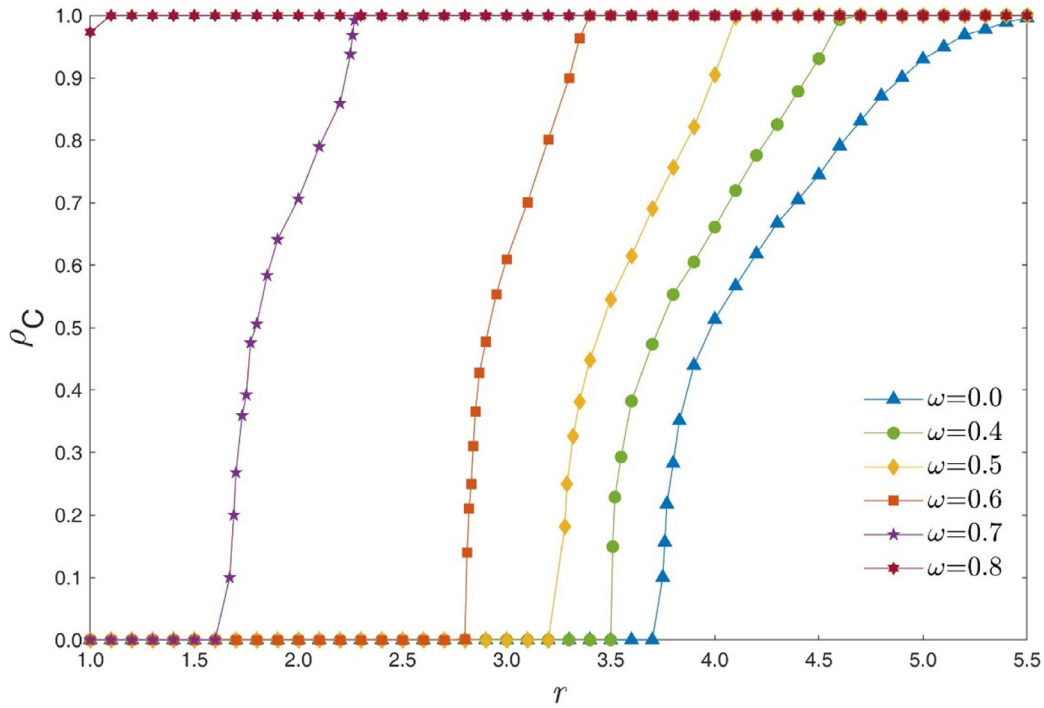

Fig. 8. The asymptotic density of cooperators ρ_C in dependence on the enhancement factor r for typical values of reputation weight $\omega = 0.0,\ 0.4,\ 0.5,\ 0.6,\ 0.7$ and 0.8. For the low weights ($\omega \in [0, 0.4]$), the reputation effect plays a subtle role in promoting cooperation. However, with the expansion of the weight from $\omega = 0.4$ to $\omega = 0.8$, the critical values of r for the emergence of cooperation dramatically drops and the curves show a non-linear movement to the left. Unexpectedly, when the weight reaches 0.8, reputation effect leads to an incredibly high level of cooperation for any values of r in our model. All the simulations have the same initial configuration and conditions as the preceding figures. (For interpretation of the references to colour in this figure legend, the reader is referred to the web version of this article.)

4. Conclusions

Summing up, we have proposed an improved strategy updating rule, which integrates the reputation effect with payoff into the learning process by evidential reasoning. The impact of reputation on the emergence of cooperation is explored by tuning a weight coefficient we introduced. Specifically, we introduce a reputation system to record individuals' cooperative behavior and further assume that an individual's reputation will affect the learning process of its neighbors. Then we describe the two probabilities of imitation based on the payoff and reputation, respectively, as two pieces of evidence that individuals can obtain. Finally, we leverage an improved evidential reasoning rule considering the importance and reliability of evidence to fuse the two aspects of information, based on which individuals can adjust their strategies. We have investigated the critical values of the enhancement factor for the emergence of cooperation for various weights of reputation. Simulations on the square lattice network demonstrate that the reputation effect can greatly influence nearest neighbors to form greater clusters, thus boosting the emergence of cooperation. In particular, with the reputation weight increasing gradually, the critical values for cooperation to arise is by degrees reduced to the lower boundary, indicating that the cooperation dilemma is undermined and an increasing tendency of strategy adoption relying on reputation is more likely to allow cooperation to thrive. In the region of low reputation weights, reputation plays only a subtle role in inducing cooperation. However, within the region of high reputation weights, cooperation is dramatically boosted by reputation and cooperators can swiftly occupy the whole population. These results may be helpful to further understand the facilitative effect of reputation on the emergence of cooperation from the perspective of strategy updating.

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