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Dynamic scale return coefficient with environmental feedback promotes cooperation in spatial public goods game

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Abstract. In the traditional spatial public goods game (SPGG), the total contributions of each group are linearly amplified by the same enhancement factor, which does not coincide with the real situation. Although some literature has considered the non-linearity and heterogeneity of scale returns between groups, the differences are completely generated by stochastic mechanisms. In addition, the coefficient will not change once assigned in previous studies. In this paper, we consider an environmental feedback mechanism and introduce a dynamic scale return coefficient into the payoff function. We observe that people who have a good record of cooperation working together can produce greater synergies and can have a larger return coefficient. We use the concept of reputation to describe individuals' historical cooperation records. Therefore, we assume that the scale return coefficient is governed by the difference between the average reputation of the group and the whole population, which will also evolve over time. In addition, we set a reputation amplitude to control the extent to which reputation differences between groups affect payoffs. By simulation experiments, we reveal that dynamic scale return coefficient based

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on reputation difference between groups helps to promote the evolution of cooperation. Moreover, the magnitude of the reputation amplitude directly affects the level of cooperation and the emergence of cooperative clusters. Our model provides a new way to introduce the environmental feedback mechanism to payoffs function and facilitate the in-depth study of the impact of reputation on cooperation.

Keywords: agent-based models, evolutionary game theory, nonlinear dynamics

Contents

1. Introduction

From politics, economy to culture, cooperation exists in every aspect of social life. However, this phenomenon contradicts Darwin's theory of natural selection [1, 2], which holds that selfish individuals do not cooperate for personal benefit. Therefore, research on the widespread emergence of cooperation has become a hot topic for experts and scholars in many fields [3–5]. Evolutionary game theory [6–8] is considered to be one of the important theoretical frameworks for discussing this issue. Among this direction, the public goods game (PGG) becomes a classic model of studying group interaction [9–14]. Based on the PGG, various cooperation mechanisms have been put forward to illustrate the cooperative behavior between selfish individuals, such as volunteering [15–18], punishment [19–23], reward [24–26], exclusion [27–30], conditional strategies [31] and social diversity [32–35]. Among these mechanisms, many are discussed under structured populations [18, 20, 21, 23, 31–35]. Public goods game played in a structured population is also known as spatial public goods game (SPGG).

However, most of the work on the SPGG model is based on the assumption of homogeneous payoff structure, in which every cooperator contributes the same amount of investment, and the total contributions of each group are linearly amplified by the same enhancement factor, which is not consistent with the actual situation. Till now, there have been some literature attempts to explore the impact of heterogeneous payoff

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structure on cooperation. Some focus on the investment heterogeneity, such as dynamic allocation of investment over time [36], selective investment in which each individual pays different investments to different groups [37–42], conditional investment [43, 44] and age-related investment [45]. The others consider heterogeneous enhancement factors in the games [46–49]. In addition, heterogeneity of profit allocation has also been introduced [50–52].

Although various types of heterogeneity are considered in the model, previous work all assumed that the public resource in the games remains constant over time. Hilbe *et al* [53] introduced the idea that the public resource is changeable and depends on the strategic choices of individuals. They proposed a dynamic model that strategic choices in one round affect game payoffs in subsequent rounds by using the theory of stochastic games. Su *et al* [54] theoretically extended this framework by modeling the evolutionary dynamics with stochastic game transitions. Both studies have verified that this type of environmental feedbacks helps to overcome social dilemmas. We borrowed their thoughts of environmental feedback in the games, in which individuals' cooperative or defective strategies change the environment they face, thereby increasing or decreasing the return coefficient of the game, and then the return coefficient will affect all individuals' payoffs. Different from them, we assume that individuals' historical behavior, not just their current strategies, will affect the subsequent public resources.

We used the concept of reputation to record individuals' historical cooperation information. It is worth mentioning that the definition of reputation here is essentially different from that in indirect reciprocity. In indirect reciprocity models, individuals do not repeat their interactions and the essence of reputation is to influence other possible individuals' strategic choices, that is, people with a good reputation are more likely to get help from others [55]. However, in our research, individuals need to repeat their interactions. Reputation does not represent the evaluation of other individuals on him, but merely a variable that records one's cooperative behavior. In fact, in recent years, with reputation has aroused widespread concern in human society, reputation has been widely employed to explain cooperative behavior between selfish individuals. There are currently two main directions for the introduction of reputation. One is to use social norms to update an individual's reputation, such as in the indirect reciprocal model [56–58]. The other is to use the number of historical cooperative action to update one's reputation, and there are many studies that used this method to define reputation [59–64]. For instance, based on this definition, Wang *et al* [59] introduced reputation into the strategy selection, in which reputable individuals have a greater chance of being selected and being imitated by their neighbors. Chen *et al* [60] applied a reputation threshold to control the difference in Fermi probability of individuals' strategy imitation, and proved that this threshold can promote the evolution of cooperation. Some other scholars have also employed reputation mechanisms into individuals' strategy update rules for research [61, 62]. Besides, Wang *et al* [63] introduced reputation tolerance to control the number of participants in the game and further overmaster group formation through member threshold. And Yang *et al* [64] introduced a reputation threshold to dominate the formation of groups. Ding *et al* [43] proposed a mechanism, in which individuals decide to invest based on their tolerance to their neighbors' reputation. Furthermore, some scholars explored the impact of reputation

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on the evolution of cooperation by constructing new strategies or combining them with other mechanisms [65–67].

The scale-return effect in multi-person cooperation has been noticed [29, 68]. It is easy to understand environmental feedback in games through the concept of scale return. In reality, if members' cooperative records in a team are generally high, it indicates that these members are more trustworthy and more likely to trust their companions. Therefore, they are more willing to form a strong alliance relationship, so that they can generate greater synergy in the production of public goods, thus achieving a higher scale return. Conversely, when members' reputations in a team are generally low, the cohesiveness of the team tends to be low, which results in a smaller scale return when creating public resources. With this in mind, we introduce a reputation-based scale return coefficient to describe the effect of environmental feedback on individuals' payoffs. Here, the scale return coefficient is governed by the difference between the average reputation of the group and the whole population. Moreover, this coefficient will scale up or down the total contribution within a group in the form of a power exponent, thereby affecting the payoffs of all individuals in the entire group. Based on this assumption, if one group's average reputation is higher than the population's average reputation, then all individuals in this group will acquire more payoffs. Whereas if the average reputation of a group is lower than the population's average reputation, the overall payoffs of individuals in this group will decline. In addition, a reputation amplitude is also introduced to adjust the extent to which reputational differences affect the scale return parameter.

The rest of this paper is arranged as follows. We introduce the SPGG model considering the dynamic scale return coefficient with environmental feedback in the second section. Simulation results of the model and relevant discussions have been presented in the third section. Finally, the fourth section summarizes the main conclusions of this paper.

2. Model

2.1. Spatial public goods game with scale return coefficient

In the SPGG, there are N individuals in the population with each individual being located on a node of the network. Let Ω_i denote the set of nodes that directly connected to individual i. Thus individual i has $|\Omega_i|$ directly connected neighbors. In each round, individual i participates in $|\Omega_i| + 1$ groups of games at the same time, where one group is centered on himself and the rest are centered on his $|\Omega_i|$ neighbors. Moreover, each individual adopts the same strategy (cooperation or defection) in all group games. If individual i cooperates, his contribution is one ($s_i = 1$), otherwise as a defector his contribution is zero ($s_i = 0$). In the initial round, each individual is randomly assigned a cooperative or defective strategy with equal probability.

After each round of the game, the contributions of all individuals in the group are aggregated. Considering the synergy and discounting effects of cooperation in social dilemmas [69], here, we introduce a scale return coefficient θ as a power exponent of the total contribution to reflect these effects of cooperation [29]. Subsequently, the

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nonlinearly scaled contribution is multiplied by an enhancement factor r and then evenly distributed among all individuals of the group regardless of their strategies. Therefore, the payoff of individual i in group l (denote the set of individuals in group l as G_l) is

$$p_i^l = \frac{1}{|G_l|} r \cdot \left(\sum_{x \in G_l} s_x \right)^{\theta} - s_i, \tag{1}$$

where s_i represents the contribution or the strategy of individual i; $\sum_{x \in G_l} s_x$ indicates the total contribution of all individuals in group l; $|G_l|$ is the number of individuals in group l; and $r > 0$ is the enhancement factor.

Then, the total payoff of individual i participating in the $|\Omega_i| + 1$ rounds of games is

$$p_i = \sum_{l \in G_i} p_i^l. \tag{2}$$

2.2. Dynamic scale return coefficient with environmental feedback

We introduce a variable θ as the scale return coefficient in the previous section. Here, we assume that this coefficient is not randomly generated but is dominated by an environmental feedback mechanism. Our logic is that individuals' cooperative behavior will increase the subsequent scale return coefficient of the game in their group through environmental feedback. We use the concept of reputation to record individuals' cooperative behavior. Thus, we set the scale return coefficient as a function of the difference between the average reputation of the group and the whole population to reveal the impact of reputation difference between groups on the payoff structure. Therefore, the scale return coefficient θ in group l can be defined as

$$\theta^l = 1 + \alpha \left(\sum_{x \in G_l} \bar{R}_x - \sum_{y \in G} \bar{R}_y \right) / R_{\max}, \tag{3}$$

where $\sum_{x \in G_l} \bar{R}_x$ is the average reputation of individuals within group l; $\sum_{y \in G} \bar{R}_y$ is the average reputation of the whole population; $R_{\max}$ is the maximal difference in reputation of individuals in the population; and $\alpha \geqslant 0$ is an adjustable reputation amplitude that regulates the impact of reputation difference on the payoff. Obviously, when the average reputation of a group is greater than that of the whole population, all individuals of the group are allocated more payoff. Whereas when the average reputation of a group is less than that of the whole population, all individuals of the group get relatively lower payoff. Initially, each individual is randomly endowed with an integer reputation value in the interval $[0, R_{\max}]$. Without loss of generality, we set $R_{\max} = 100$.

2.3. Evolutionary rules of strategy and reputation

We use the asynchronous strategy update rule in the study. The update process consists of many basic Monte Carlo steps (MCSs). During each MCS, an individual x and one of his neighbors y are randomly selected in succession. Then individual x adopts the strategy of y with a probability defined by the Fermi function

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$$W(s_y \to s_x) = \frac{1}{1 + \exp\left[(p_x - p_y)/\kappa\right]} \tag{4}$$

where p_x and p_y is the payoff of individual x and y, respectively. κ stands for noise intensity, which implies uncertainty in the strategy imitation [70, 71]. When $\kappa \to 0$, as long as the payoff of individual y is greater than that of x, individual x will adopt the strategy of y in certainty. When $\kappa \to \infty$, individual x adopts the strategy of y with probability 1/2 regardless of their payoffs. Without loss of generality, we set $\kappa = 0.5$ in this study [11].

After individual x updates their strategy, their reputation will also be updated. We employ the simplest reputation update rule [60] in this study, in which an individual's reputation value increases by one if their strategy is cooperative, otherwise their reputation reduces by one. According to this rule, the reputation value of individual x at round t is

$$R_i\,(t) = R_i(t-1) - (-1)^{s_i(t)}, \tag{5}$$

where $R_i(t-1)$ represents the reputation value of individual i at round $t-1$, and $s_i(t)$ is the strategy of individual i at round t. In the process of reputation updating, we assume that the reputation values of all individuals are within interval $[0, 100]$. So when an individual's reputation value accumulates to the upper limit 100, their reputation value will not continue to rise. Similarly, when an individual's reputation value falls to the lower limit 0, their reputation value will not continue to decline. Each MCS contains N independent processes, so that each individual has one opportunity on average to update their payoff and reputation in one step.

3. Results and discussion

The simulations are performed on a square lattice network of $L \times L$ with periodic boundary conditions. We focus on the average cooperation rate of the population ρ_c, which is defined as the proportion of cooperators in the population, namely,

$$\rho_c = \frac{1}{N}\sum_{i}^{N} s_i. \tag{6}$$

All the following results we report are performed at $L = 100$. We have also verified that for larger size networks, such as $L = 200$ and 500, the results are also robust. When $L = 100$, the frequency of each strategy in the system can be stabilized after 20 000 MCSs. In the following results, each value of ρ_c is the average result of the last 1000 steps of the total 20 000 steps. Moreover, every data point is the average result of ten independent runs. In order to increase the robustness of the results, for all phase transition points, we also increased the number of independent runs to 100 as in [32] and found that the results did not change.

First, in order to investigate the effects of dynamic scale return coefficients that result from inter-group differences in reputation on the evolution of cooperation, figure 1 shows the comparison curves of cooperation density ρ_c with enhancement factor r for

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four different values of α. As the figure illustrates, reputation amplitude α has a positive impact on the evolution of cooperation. The increase in α reduces critical values of r above which cooperation can exist. Specifically, when $\alpha = 0$, individuals' reputation has no effects on their payoffs, in which environmental feedback mechanism does not work and the model degenerates into the basic version. In this situation, we can notice that cooperation emerges when $r > 3.7$, which is consistent with previous research [72]. On the contrary, when $\alpha = 1$, the impact of reputation difference between groups is distinct, which means that the difference of payoffs between a high reputation group and a lower one will be magnified. In this case, cooperation emerges when $r > 2$, which indicates that cooperators can exist at a very small enhancement factor r due to the effect of reputation feedback. In summary, dynamic and heterogeneous scale return coefficient governed by reputation difference between groups drives the evolution of cooperation.

In order to compare the dynamic process and stabilization time of the system under different enhancement factors, figure 2 displays the evolutionary processes of cooperation density for four representative values of r and fixed $\alpha = 1$. When $r \geqslant 2.7$, as shown by the red, yellow and purple curves in figure 2, the proportion of cooperators declines first and then increases gradually, and finally stabilizes at a high level. When $r = 2$, as shown by the blue curve in figure 2, even if reputation amplitude α is large, which means the impact of reputation difference on payoffs is great, cooperation cannot be maintained because of small values of enhancement factor. Thus, defectors occupy the whole population and cooperative strategy completely disappears finally. In short, the impact of reputation differences among groups on the evolution of cooperation is restricted by the enhancement factor r.

In order to observe the evolution of strategies on the spatial network, figure 3 illustrates the snapshots of individuals' strategy distribution over time for three representative values of enhancement factor r and fixed $\alpha = 1$. Comparing the results of $r = 2.7$, 3.5 and 4.6, as shown in (a)–(c) of figure 3, we notice that for the same time step, as r increases, the proportion of red parts expands, which implies that the level of cooperation is rising. This result further consolidates the conclusions in figure 2. Moreover, when $t = 10$, as shown in (a-1)–(c-1) of figure 3, red regions are only scattered in the blue regions, which represents that cooperators in the population are few. As time step t increases from 10 to 1000, the red blocks gradually expand and connect together, which reveals that cooperators in the population gradually form compact clusters, constantly eroding the area of defectors. We believe that this rapid evolutionary process is related to the environmental feedback mechanism. As cooperators can obtain a higher reputation, the average reputation of a group with more cooperators is greater than the average reputation of the population. Thus, cooperative groups can obtain more benefits and form stable clusters to survive, finally making the cooperative strategy continue to expand in the cluster. Figure 4 further depicts the snapshots of individuals' reputation distribution over time. Comparing figure 3 with figure 4, we can easily find that the distribution of individuals' reputation and strategy are highly consistent in the evolutionary process.

Next, we further study the evolution process and stabilization time of cooperation under different values of reputation amplitude. Figure 5 shows the evolutionary processes of the cooperation density for four different values of α and fixed $r = 3.2$. When

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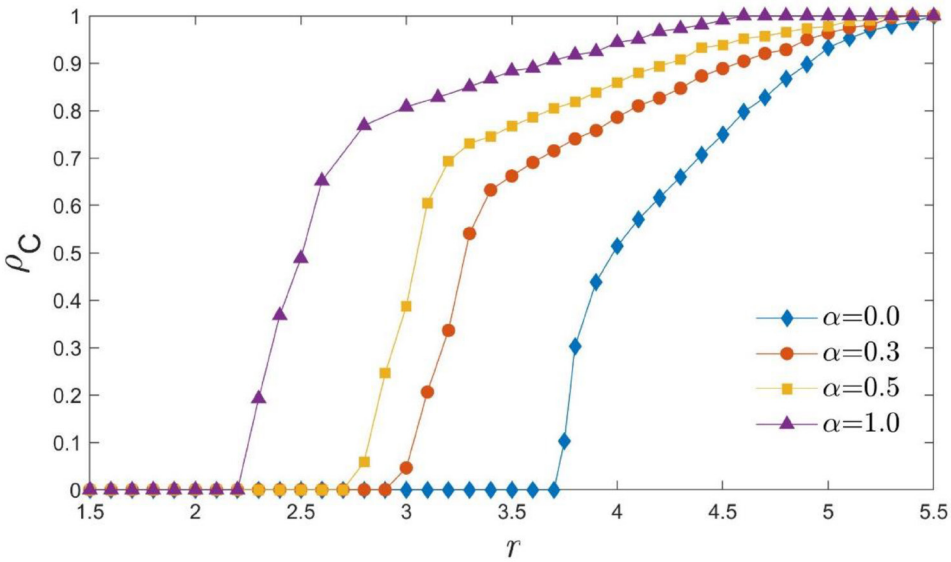

Figure 1. Comparison curves of cooperation density ρ_c with enhancement factor r when the reputation amplitude α is 0, 0.3, 0.5 and 1, respectively, for fixed $\kappa = 0.5$. Compared with $\alpha = 0$, when $\alpha > 0$, much smaller values of enhancement factor can promote the emergence of cooperation, which indicates that the introduction of a dynamic scale return coefficient with environmental feedback is conducive to cooperation. Furthermore, as reputation amplitude α expands, cooperation can occur at a smaller enhancement factor and can be maintained over a wider range of parameters.

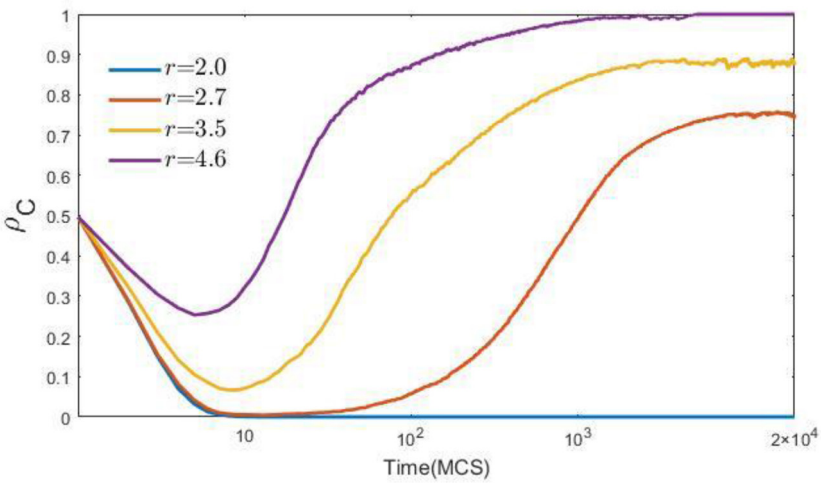

Figure 2. Evolutionary processes of the cooperation density when the enhancement factor r is 2, 2.7, 3.5 and 4.6, respectively, for fixed $\alpha = 1$ and $\kappa = 0.5$. When reputation amplitude α is large, as enhancement factor r increases, the proportion of cooperators in the population ascends, and a higher level of cooperation can be achieved after the system reaching a stable state. However, if the enhancement factor is too small, even if the reputation amplitude is large, it cannot promote the emergence of cooperation.

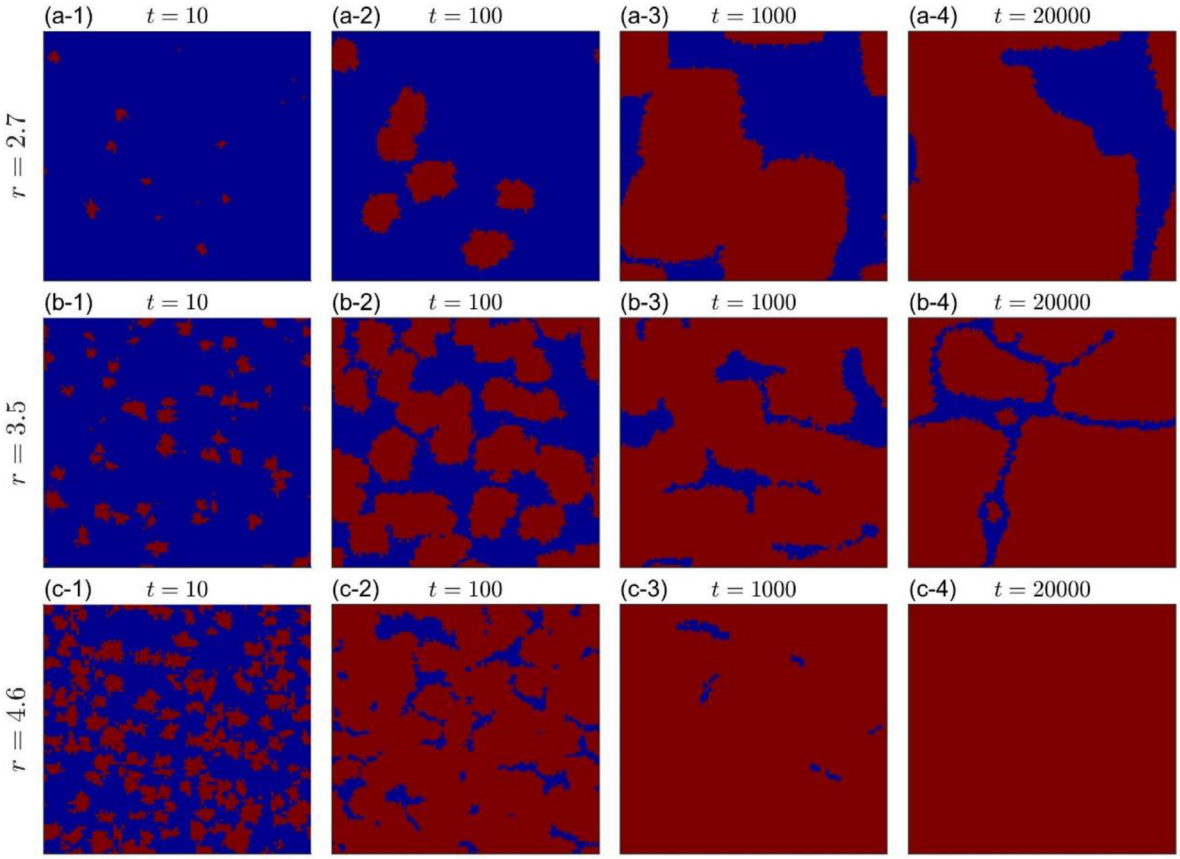

Figure 3. Snapshots of individuals' strategy distribution over time when (a) $r = 2.7$, (b) $r = 3.5$ and (c) $r = 4.6$, respectively, for fixed $\alpha = 1$ and $\kappa = 0.5$. Red parts represent cooperation and blue parts represent defection. From left to right, each column is the results corresponding to $t = 10, 100, 1000$ and 20 000, respectively. For the same reputation amplitude α, as enhancement factor r increases, the red regions are expanding, in which situation, cooperators are more likely to form tightly connected clusters to resist the invasion of defectors and occupy the territory of defectors.

$\alpha = 0$, the environmental feedback mechanism based on the reputation difference does not work. In this case, as shown by the blue curve, the cooperation level drops to zero finally, which reveals that cooperators disappear and defectors occupy the entire population. The other three curves in figure 5 correspond to situations with reputation feedback. It is easy to observe that under the influence of reputation feedback, the cooperation level of the population shows a trend of decline first and then rise, and finally stabilizes at a high level. By comparing the three curves of red, yellow and blue, we can discover that with the increase in the reputation amplitude α, the level of cooperation in the whole population shows a consistent improvement. This is because the expansion of the reputation amplitude increases the heterogeneity of scale return coefficient between groups, and the difference in the payoff of different groups will be magnified. Therefore, cooperators are more likely to huddle together in order to obtain high levels of reputation and payoff, and finally the level of cooperation achieves an increase within the entire population.

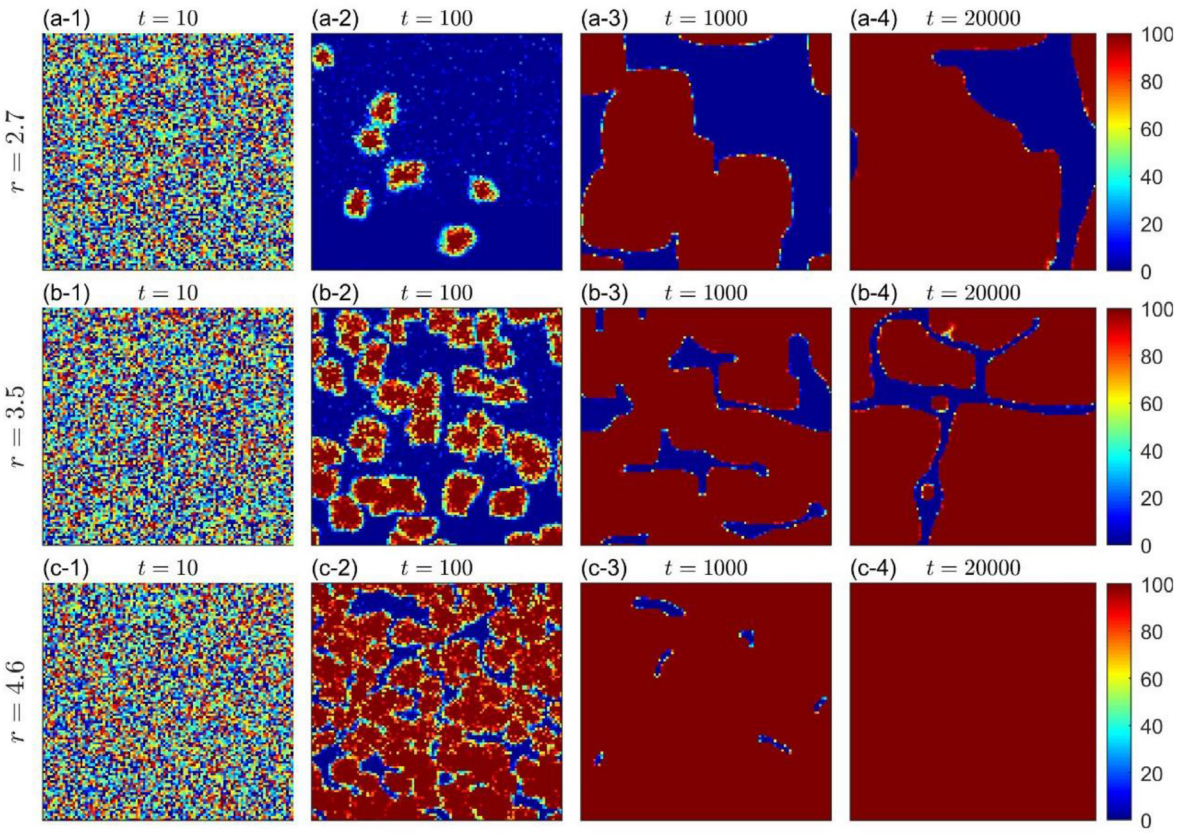

Figure 4. Snapshots of individuals' reputation distribution over time when (a) $r = 2.7$, (b) $r = 3.5$ and (c) $r = 4.6$, respectively, for fixed $\alpha = 1$ and $\kappa = 0.5$. Different colors represent different values of reputation which are defined by the color bar. From left to right, each column is the results corresponding to $t = 10, 100, 1000$ and 20 000, respectively. As shown in the figure, the evolution of individual's reputation is highly consistent with the evolution of individual's strategy in figure 3.

Corresponding to the results in figures 5 and 6 further depicts snapshots of strategy distribution over time for three different values of α and fixed $r = 3.2$. At the initial stage, individuals are randomly assigned strategies and thus cooperators and defectors in the population each account for half. When $t = 5$, red blocks are scattered in the blue ocean, which means that the number of cooperators in the population is sharply reduced compared with that at $t = 0$. This is because the advantage of cooperation is not obvious due to the random distribution of reputation at the beginning, and defection is more competitive. When time step t increases from 5 to 1000, cooperators gradually form larger and larger clusters to resist the invasion of defectors under the feedback of reputation. The system reaches a steady state before $t = 20\ 000$ in all situations finally. However, the situations are slightly different for different values of amplitude. When reputation amplitude is small, as shown in (a-1) and (b-1) of figure 6, the large cluster collapses into a plurality of small clusters; whereas when $\alpha = 1$, as shown in (c-1) of figure 6, cooperative clusters can continue to expand and to be firmly connected. In short, a larger reputation amplitude is more likely to promote the formation and growth of cooperative clusters. This conclusion indicates that when heterogeneity

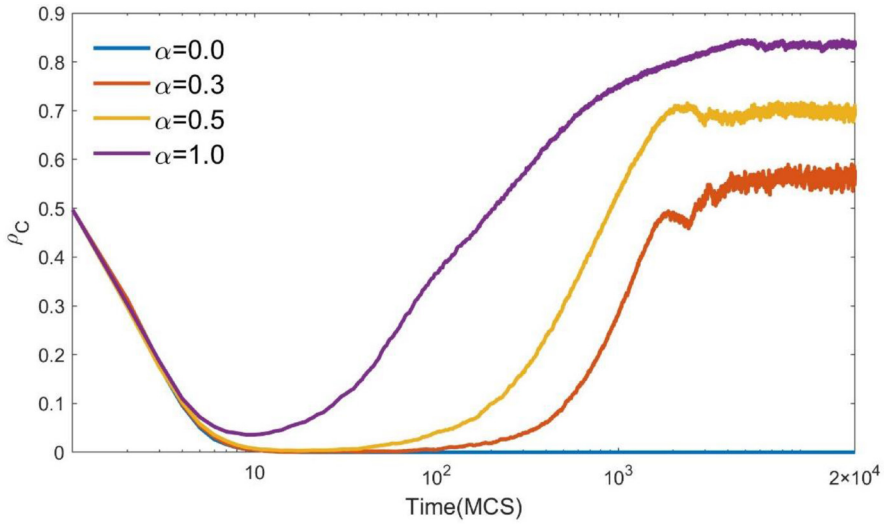

Figure 5. Evolutionary processes of the cooperation density when the reputation amplitude α is 0, 0.3, 0.5 and 1, respectively, for fixed $r = 3.2$ and $\kappa = 0.5$. Obviously, when $\alpha = 0$, the proportion of cooperators is reduced to zero finally and defectors occupy the entire population, whereas when $\alpha > 0$, cooperation can continue to be maintained. Moreover, the larger the values of reputation amplitude α, the higher the proportion of cooperators when the system reaches a stable state.

of scale return coefficient between groups is enhanced, it is more conducive to the evolution of cooperation, which is consistent with the previous study [68]. However, different from the previous research which based on the assumption that heterogeneity is controlled by a random distribution, we assume that the scale return coefficient depends on the average reputation difference between groups. In other words, heterogeneity is endogenous in this paper, so it is more realistic. Figure 7 further depicts the snapshots of individuals' reputation distribution over time corresponding to figure 6, which once again verifies the consistency between the distribution of individual's reputation and the distribution of individual's strategy in the evolutionary process.

In order to verify the universality of this feedback mechanism on promoting the evolution of cooperation, we further perform some simulation experiments in other kinds of structured populations. Considering many realistic population structures exhibit small-world characteristics, we adopt the WS small-world network [73] in this exploration. This structure can be generated by random reconnection of edges on the basis of nearest-neighbor-coupled networks. Let the number of individuals in the network $N = 10\ 000$. In the initial nearest-neighbor-coupled network, the degree of each node is four, with each node being connected to its left and right two nodes. In the generation process of the WS small-world network, a parameter p is introduced to represent the probability of edge reconnection. When $p = 0$, the network structure does not change, corresponding to the original nearest-neighbor-coupled network; when $p = 1$, all edges are re-connected randomly, corresponding to the random network; when the value of p is small, the WS small-world network can be generated. We selected three representative values of p in our simulation experiments, namely, $p = 0.05$, 0.1 and 0.2. Since the network is randomly generated, for each set of parameters, ten independent networks have been randomly generated, and for each network, ten independent runs have been

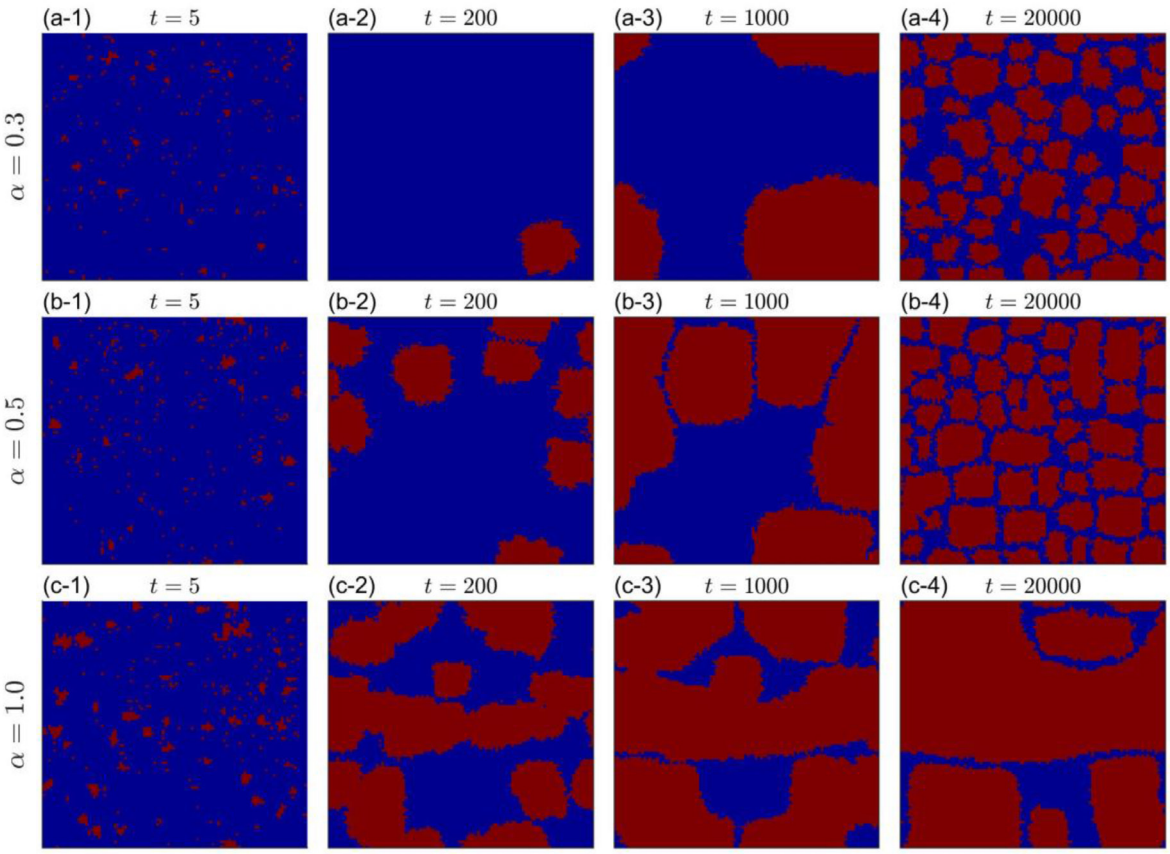

Figure 6. Snapshots of individuals' strategy distribution over time when (a) $\alpha = 0.3$, (b) $\alpha = 0.5$ and (c) $\alpha = 1$, respectively, for fixed $r = 3.2$ and $\kappa = 0.5$. Red parts represent cooperation and blue parts represent defection. From left to right, each column shows the results corresponding to $t = 5, 200, 1000$ and 20 000, respectively. When the enhancement factor is fixed, the greater the value of reputation amplitude α, the easier it is to form large and firm cooperative clusters to resist the invasion of defection when the system reaches a stable state.

performed. The final result for each combination of parameters we present is the average of the 100 independent results.

Figure 8 shows the relationship between the average cooperation density in the population and enhancement factor r on the WS small-world networks with different values of p and α for fixed $\kappa = 0.5$. We consider two values of $\alpha = 1$ and $\alpha = 0$, which correspond to the situations with dynamic scale return coefficient and without dynamic scale return coefficient, respectively. Obviously, no matter what the probability of edge reconnection is, the cooperation rate of $\alpha = 1$ is significantly higher than that of $\alpha = 0$, which indicates that the introduction of the dynamic scale return coefficient promotes the emergence of cooperation. Moreover, as can be seen from the figure, in the dynamic scale return coefficient situation, when r is less than 3.7, small values in p can slightly promote cooperation, but when r is greater than 3.7, large values in p can promote the population to reach full cooperation more quickly. From the perspective of threshold

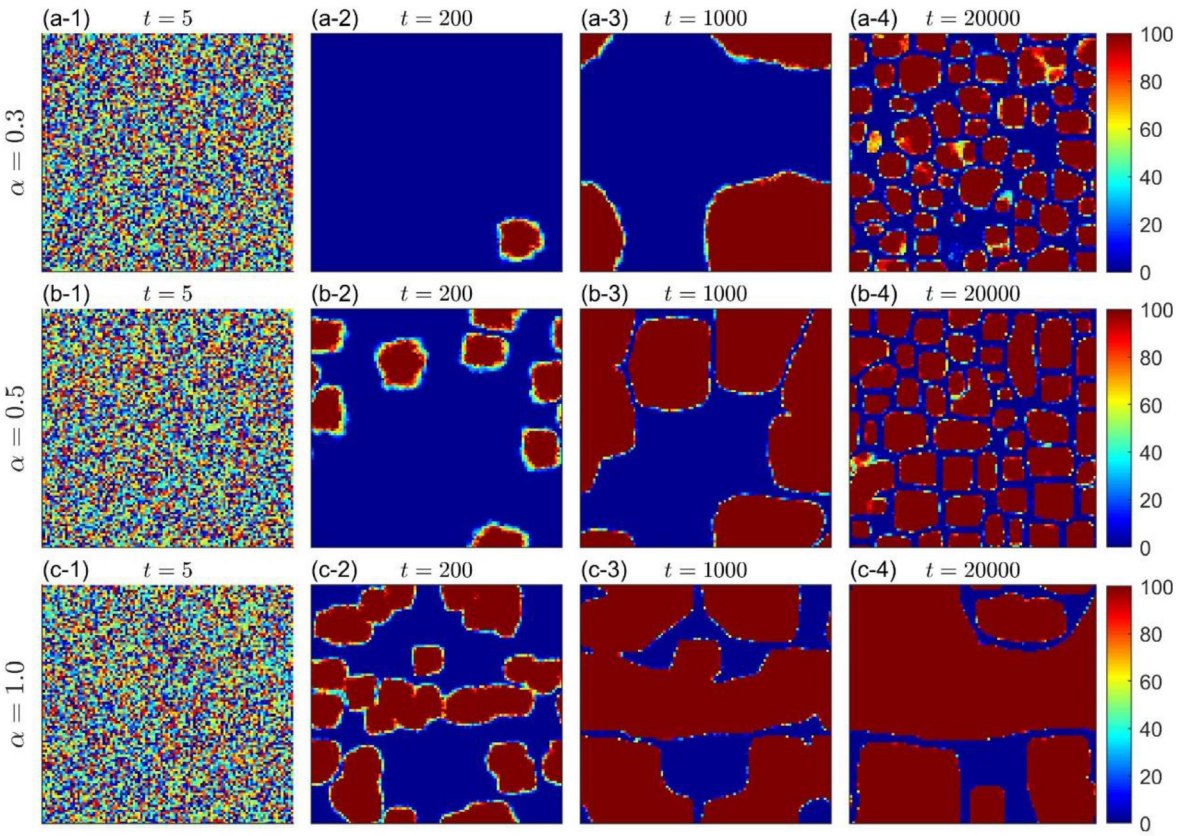

Figure 7. Snapshots of individuals' reputation distribution over time when (a) $\alpha = 0.3$, (b) $\alpha = 0.5$ and (c) $\alpha = 1$, respectively, for fixed $r = 3.2$ and $\kappa = 0.5$. Different colors represent different values of reputation which are defined by the color bar. From left to right, each column shows the results corresponding to $t = 5, 200, 1000$ and 20 000, respectively. Obviously, the evolution of individual's reputation is consistent with the evolution of individual's strategy in figure 6.

values for the emergence of cooperation, compared with the lattice, the WS small-world networks cannot promote the emergence of cooperation to a larger extent under our feedback mechanism. Since the average degree of the two kinds of networks is the same, we suspect that this result is related to long-range edges on the WS small-world network. The long-range edges greatly reduce the average path length of the network and also reduces the heterogeneity of the scale return coefficient between groups. Therefore, when r is small, it cannot promote cooperation more effectively.

In order to observe the stable time of the evolutionary system, figure 9 presents the evolutionary processes of the cooperation density over time for different values of r when p is fixed at 0.05, 0.1 and 0.2, respectively. We select four representative values of r near the phase transition point. By comparing the three subfigures, it is evident that small values of p greatly prolongs the time required for the system to reach stability. Notably, when $p = 0.05$, for the values of r ranging from 3.6 to 3.8, the proportion of cooperation has slowly increased over a long period of time until it is stable.

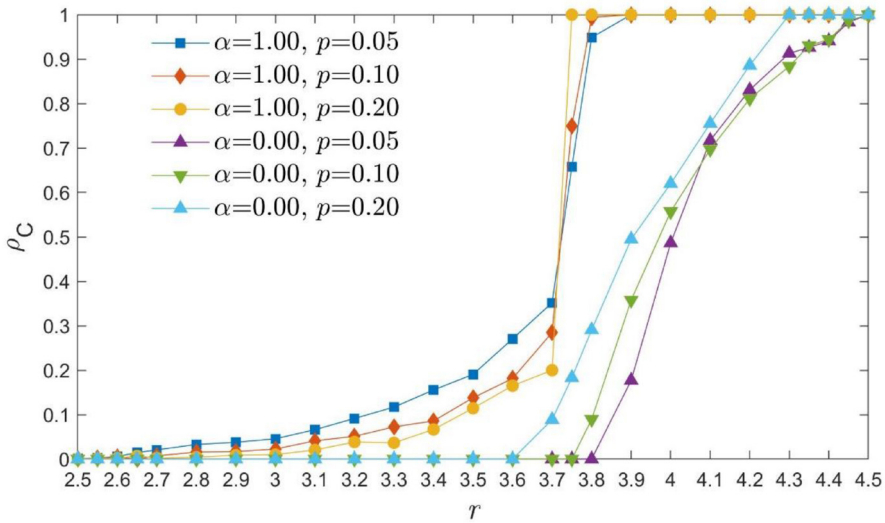

Figure 8. Comparison curves of cooperation density ρ_c with enhancement factor r when the reputation amplitude α is 1 and 0, and the probability of edge reconnection p is 0.05, 0.1 and 0.2, respectively, on the WS small-world networks, for fixed $\kappa = 0.5$. $\alpha = 1$ corresponds to the situation with dynamic scale return coefficient and $\alpha = 0$ corresponds to the situation without dynamic scale return coefficient. Obviously, no matter what the probability of edge reconnection is, the cooperation rate of $\alpha = 1$ is significantly higher than that of $\alpha = 0$, which indicates that the introduction of the dynamic scale return coefficient promotes the emergence of cooperation.

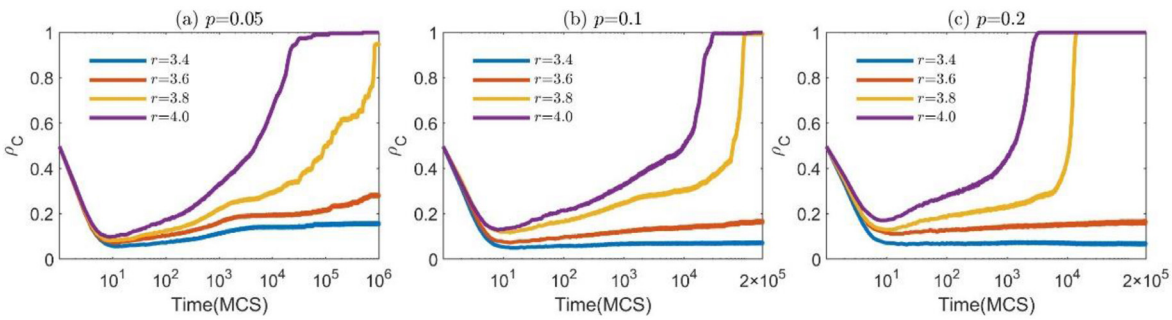

Figure 9. Evolutionary processes of the cooperation density at four different values of r on the WS small-world networks for fixed $\alpha = 1$ and $\kappa = 0.5$. (a) $p = 0.05$; (b) $p = 0.1$; (c) $p = 0.2$. When p is small, the time required for the system to reach stability is significantly increased. Specifically, when $p = 0.05$, the system achieves stability in 10^6 steps, whereas $p = 0.1$ or 0.2, the system can reach evolutionary stable within 2×10^5 steps.

4. Conclusion

In summary, we have explored the impact of dynamic scale return coefficients on the emergence of cooperation by considering a reputation-based environmental feedback mechanism. The magnitude of the scale return coefficient in a group is manipulated by the difference between the average reputation of the group and that of the whole

population. In addition, we set an amplitude parameter to adjust the impact of reputation differences among groups on individuals' payoff function. Simulation results present that this positive feedback mechanism between individuals' historical behavior and subsequent payoffs favors the evolution of cooperation. Specifically, the increase in reputation amplitude lowers the critical values of enhancement factors above which cooperation can emerge, and promotes the level of cooperation for certain range of enhancement factors. In addition, we also observed that large values of amplitude factor make the impact of reputation differences between groups more prominent, thus promoting the formation of greater cooperative clusters and achieving higher levels of cooperation. These results verify that reputation-based environmental feedback has an important impact on the evolution of cooperation, and help us to further understand the impact of reputation on individuals' cooperative behavior. However, we only consider the most simple network structures, namely, lattice and WS small-world networks in this work. On the one hand, our model can be extended to more complex structured populations. On the other hand, the impact of other types of environmental feedback on the evolution of cooperation remains to be further considered. We believe that repeated games with environmental feedback will become a hot topic in future research.

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