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Social exclusion with dynamic cost on the evolution of cooperation in spatial public goods games

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ABSTRACT

Social exclusion, as a special form of punishment, can effectively promote cooperation in spatial public goods game (SPGG). Previous research usually assumes that the unit cost of exclusion and the probability of exclusion success are statically fixed. As such, the total cost of an excluder can only be described as a linear function of the number of defectors. However, the scale return effect generally results in nonlinear characteristics of costs. In this study, we have thus relaxed the linear assumption and introduced a state-dependent kind of social exclusion with dynamic costs. Specifically, we describe the unit cost of exclusion as a function of the state of the group in which individuals are located. Due to the scale effect of punishment, an increase in the number of defectors in the group will result in a decrease in the unit cost of exclusion. We explore the impacts of this exclusion-type strategy in the SPGG with nonlinear exclusion cost on the evolution of cooperation. We further investigate parameters such as the cost parameter, the probability of exclusion success and the dilemma strength of the PGG on the evolution of cooperation. Simulations are performed on a square lattice with the traditional neighborhood structure and the Moore neighborhood structure, respectively. The results show that social exclusion with state-dependent costs can promote cooperation within a wide range of parameters in both cases. Especially, when the probability of exclusion success is high, a very strong dilemma strength can lead the system to form huge cooperative clusters to beat defectors. We further confirm that the Moore neighborhood case is more conducive to the evolution of cooperation in our social exclusion mechanism with dynamic cost compared with the traditional neighborhood case. These results allow us to better understand the role of the dynamic cost of punishment in the emergence of prosocial behavior.

1. Introduction

Cooperation, which is prevalent in society, plays an important role in the development of human civilization [1–3]. However, theoretically explaining the emergence of cooperation under the framework of Darwin's theory of evolution is difficult [4,5]. The term social dilemma is commonly used in the field of economics to describe the situation of cooperation [6–9]. Social dilemma refers to all situations when individual interests conflict with collective interests. Public goods game (PGG)

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with multi-person interaction is a typical theoretical model that is used to describe the situation of social dilemmas [10]. From the perspective of economics, people are egoistic, and they tend to exploit others to obtain the maximum benefits. Everyone's pursuit of maximizing his or her own interests will also lead to the "tragedy of the commons [11]", and only cooperation can achieve a win-win situation.

Based on the simple game model, numerous studies have used evolutionary games to explain the phenomenon of cooperation [12–19] and even aimed at finding rules to promote more large-scale cooperation [20–27]. These studies have suggested some effective mechanisms, including indirect [28–31], direct [32,33], and network reciprocity [34–37]. Network reciprocity explains the emergence of cooperation from the structure of interactions between individuals, and with the development of network science, this framework has aroused widespread interest in academia. In a structured population, PGG is also known as spatial PGG (SPGG). The traditional SPGG model features two kinds of strategies, namely, cooperation and defection. Since punishment behavior can be seen everywhere in society [38], it has also aroused widespread concern in recent years. Thus, besides cooperation and defection, punishment has also been introduced as another type of strategy into the SPGG model [39–41]. The consensus has been reached that punishment—paying a cost itself while causing the punished individuals to pay a higher cost—can effectively promote cooperation. Till now, different types of punishments have been proposed, including peer [42–46], public pool [47–50], self-organized [51], conditional [52], probabilistic sharing [53], tolerance-based punishments [54–56], and so on [57].

Although punishments can effectively promote cooperation, this mechanism can lead to a decrease in the average payoff of the group owing to the bilateral cost loss of punishers and individuals who are punished [58,59]. A special punishment called social exclusion has been proposed recently [60–71]. In contrast to traditional punishments, exclusion can threaten defectors by expelling them out of the group with a certain probability. Defectors who are driven out cannot get anything, and only the remaining survivals can share cooperative benefits. Concerning which of the two strategies of expulsion and punishment is more evolutionary advantage, Liu et al. [63] proposed that exclusion can dominate punishment when they coexist. However, Sui et al. [66] showed that rationality can change the ranking of peer punishment and exclusion. They argued that the peer punishment strategy will dominate under weak rational conditions, whereas the exclusion strategy will dominate under strong rationality. In addition, different kinds of exclusion in the group have also attracted attention. For instance, Li et al. [62] proposed the concept of random sequential exclusion and found that sequential exclusion has greater advantages than synchronous exclusion for the evolution of cooperation when three types of strategies exist. Quan et al. [67] extended their model to the four types of strategies situation and verified that asynchronous exclusion is also better than synchronous exclusion in both well-mixed and structured populations.

However, extant research on exclusion assumes that the unit cost of exclusion or the probability of exclusion success is statically fixed. Thus, the total cost of exclusion can only be described as a linear function of the number of defectors. The principle of economics dictates that the production of goods has a scale effect, which makes costs generally exhibit nonlinear characteristics. Within a certain range, the increase in production quantity results in a decline in the unit cost. Based on this theory, the unit exclusion cost will also depend on the number of defectors in the group. Thus, the unit exclusion cost in each group varies dynamically as the system status changes in the evolutionary process. Based on this observation, we introduce a kind of social exclusion with state-dependent cost that allows the unit cost of exclusion to be a function of the state of the group. Under the scale effect assumption, an increase in the number of defectors results in a decrease in the unit cost of exclusion in the corresponding group. We explore the impact of the introduction of this exclusion-type strategy whose cost depends on the state of the system on the evolution of cooperation. Parameters that can adjust the probability of exclusion success, exclusion cost, and the dilemma strength of the game have been introduced in our model. Simulations are performed on a square lattice network with the traditional neighborhood structure and the Moore neighborhood structure, respectively. The results show that cooperation can emerge under strong dilemma strength and moderate probability of exclusion success in both cases. In addition, the Moore neighborhood case is more conducive to the evolution of cooperation in our social exclusion mechanism with dynamic cost compared with the traditional neighborhood case.

The rest of the article is arranged as follows. In Section 2, we introduce the SPGG model with the exclusion type strategy and dynamic cost. Then we show the main simulation results and give sufficient discussions in Section 3. In Section 4, we summarize the study.

2. Model

In the SPGG model, there are N individuals with each locating on a node of the network and playing the PGG with its direct neighbors. We only study homogeneous network structure in this paper that each individual is surrounded by the same number of neighbors, which is denoted as $G-1$. Thus every individual belongs to G different groups and each group has G players. Thus, each individual will participate in G rounds of PGGs at the same time. One round is centered on himself and the other $G-1$ rounds are centered on each of his neighbors. In the traditional PGG model, there are two types of strategies, namely, cooperation and defection. Cooperators contribute one unit to a common pool, and defectors contribute nothing. Then all contributions are multiplied by a synergy factor $r > 1$, and are equally allocated among all group members. If the team is fully cooperative, the group will yield the maximal profits $r-1$ for each individual. However, each selfish individual has a motive to take a free ride of others, and the defection of everyone will result in the whole group getting nothing.

In this study, we introduce another strategy type called pool exclusion (E). This strategy will not only contribute one unit but also share the total exclusion cost to expel all defectors in the group. Let δ denote the unit exclusion cost for expelling each defector. Assume that under the pool exclusion, each defector can be successfully expelled from the group with probability β. Defectors who have been expelled cannot share the benefit of public goods. Different from previous studies, we assume that the unit exclusion cost δ depends on the status of the group, which is related to the number of defectors in the group. Because of the scale effects, the greater the number of defectors, the smaller the unit cost of exclusion. Specifically, the unit cost of exclusion can be described as a decreasing function of the number of defectors as

$$\delta = \varphi + \frac{G - n_D}{G} \times \theta, \tag{1}$$

where n_D denotes the number of defectors in the group, ϕ and θ are adjustment factors that denote the fixed and variable part cost, respectively. Thus, the total cost that each excluder is responsible for in the group is $\frac{\delta n_D}{n_E}$. The payoff of each type of strategies C, D and E in one PGG can be calculated as

$$\Pi_C = \frac{r(n_C + n_E + 1)}{G - k} - 1, \tag{2}$$

$$\Pi_D = \begin{cases} 0 & \text{expelled with probability } \beta \\ \dfrac{r(n_C + n_E)}{G - k} & \text{otherwise} \end{cases}, \tag{3}$$

and

$$\Pi_E = \frac{r(n_C + n_E + 1)}{G - k} - 1 - \frac{\delta n_D}{n_E}, \tag{4}$$

respectively, where n_C, n_D and n_E denote the number of cooperators, defectors and pool excluders in the group, respectively, excluding itself, and k denotes the number of defectors who are expelled from the group. Evidently, k obeys the binomial distribution with the parameter β. The total payoffs of each individual is the sum of the proceeds of the G rounds of PGGs in which they participate.

Individuals will update their strategies based on their payoffs in the interactions. We use the imitation rule based on the Fermi function in this study. The imitation process consists of some basic Monte Carlo steps (MCSs). During each MCS, one individual (denoted as i) is selected randomly from the population to update his strategy, and then one of i's neighbors (denoted as j) is selected randomly from all of his neighbors. By comparing the cumulative payoffs of the two, the probability that individual i adopts the strategy of individual j can be described as

$$P_{i \to j} = \frac{1}{1 + \exp\left(-\frac{\Pi_j - \Pi_i}{\kappa}\right)}, \tag{5}$$

where Π_i and Π_j are the cumulative payoffs of individuals i and j in the G-round games in which they participate, and $\kappa > 0$ is the noise intensity. $\kappa \to 0$ indicates the deterministic imitation rule, that is to say, the strategy with higher payoffs will be imitated with probability one. Whereas $\kappa \to +\propto$ represents the completely random imitation, and individuals will imitate their selected neighbors with probability 0.5 regardless of their cumulative payoffs. Each MCS contains N independent processes to ensure that each individual has one chance on average of being selected to update his strategy.

3. Results and discussion

Simulations are performed on a 100×100 square lattice with periodic conditions and thus $N = 10^4$. We focus on the average level of cooperation in the population ρ_{C+E}, which is defined as

$$\rho_{C+E} = \frac{1}{N}(N_C + \mathrm{N}_E), \tag{6}$$

where N_C and N_E denote the number of cooperators and excluders in the population, respectively. Consistent with previous research [17,21,46,47,51,52,72] and without loss of generality, we also set $\kappa/G = 0.1$ in this study. The simulation begins with a random initialization that each type of strategy is distributed on each node of the network with the same probability. We have explored two situations in this study. In the first case, each individual has four direct neighbors ($G = 5$) and in the second case, each individual has eight direct neighbors or called Moore neighbors ($G = 9$). The proportions of cooperators and excluders can be stabilized by 10^4 MCSs for most parameters in both situations, and for some phase transition points, we extend the MCSs to 5×10^4. The results we report are an average of 10 independent realizations.

In order to investigate the impact of exclusion with state-dependent cost on the evolution of cooperation, we fix ϕ= 0.1 to maintain the minimum unit exclusion cost and $\theta = 0.8$ to adjust the degree of impacts of the number of defectors in the group on the unit cost of exclusion. We focus on the combined effects of different exclusion probabilities β and synergy coefficients r on the level of cooperation under dynamic unit exclusion costs. In order to compare the results of the

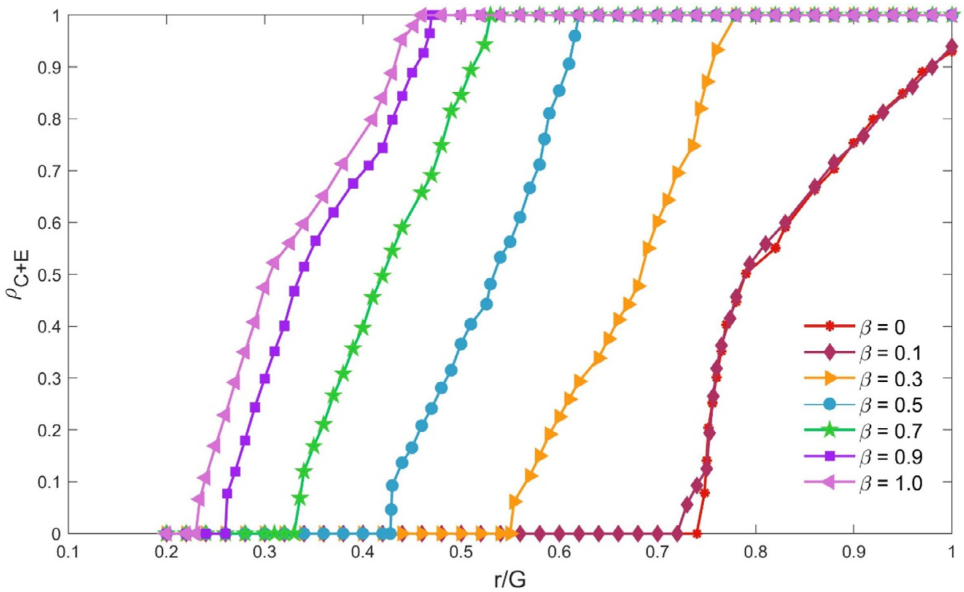

Fig. 1. When $G = 5$, comparison curves of the average level of cooperation in the population with r/G under different values of β, for fixed $\kappa/G = 0.1$, $\phi = 0.1$ and $\theta = 0.8$. When $\beta = 0$ and 0.1, cooperation emerges at $r/G = 0.75$ and 0.73, respectively, and the system cannot reach the state of full cooperation when $r/G \le 1$. When $\beta = 0.3$, cooperation emerges at $r/G = 0.55$, and defection disappears at $r/G = 0.78$. When β increases to 0.5, 0.7, 0.9, and 1, respectively, cooperation emerges at $r/G = 0.43$, 0.34, 0.26 and 0.23 correspondingly, while defection disappears at $r/G = 0.62$, 0.53, 0.47 and 0.45, correspondingly. Obviously, the increase in the probability of exclusion success greatly lowers threshold values of $r/G$for the emergence of cooperation and the disappearance of defection.

two situations with different neighbor numbers, we introduce a universal scaling parameter r/G to quantify the dilemma strength in different size games [73–76]. Obviously, the higher the values of r/G, the weaker the dilemma of the games.

We first give the results when $G = 5$. Fig. 1 shows some curves for comparison when β is fixed at 0.1, 0.3, 0.7 and 0.9, respectively. Specifically, two extreme values of $\beta = 0$ and 1 are also added. Each curve represents the relationship of the average level of cooperation and r/G in the interval [0.2, 1] when the evolutionary system reaches a stable state. The effect of the exclusion probability and dilemma strength on cooperation can be observed in Fig. 1. Evidently, a higher exclusion probability can promote the emergence of cooperation with a smaller r/G. Specifically, when $\beta = 0$, defection always dominates at low values of r/G, and cooperation cannot appear until $r/G = 0.75$. Not surprisingly, this value is the same as the threshold that was previously studied with only two strategies of cooperation and defection [72]. We observe that the curve of the cooperation rate when $\beta = 0.1$ is almost the same as $\beta = 0$ over the entire interval. By observing the evolution of the system, we find the reason for this result is that excluders are eliminated at the beginning because of the low probability of exclusion success and the additional exclusion costs. Thus, only cooperators and defectors are left to play games, which coincides with the previous condition. When β increases to 0.3, cooperation can emerge at $r/G = 0.55$. The threshold has been greatly reduced compared to the situation of $\beta = 0.1$. Moreover, full cooperation can occur when $r/G = 0.78$. The comparisons of the two curves show that improving the exclusion probability can effectively accelerate the emergence of cooperation and the disappearance of defection. With the gradual increase of β, the critical value of r/G for the emergence of cooperation is getting smaller and smaller. Intuitively, the increased probabilities of exclusion success make the living conditions of defectors harsh. Even at a very low level of r/G, the excluders can guarantee their own profits by successfully expelling the defectors. Especially, when $\beta = 1$, full cooperation can occur at $r/G = 0.23$.

Notably, the three types of strategies, namely, cooperation, defection, and exclusion display a rock-scissors-paper cycle. Defectors exploit cooperators, cooperators crush excluders, and excluders sanction defectors. In order to observe the evolution of the three mutually influential strategies, we select some representative values of r/G and β to show the evolutionary processes of the strategy frequencies and their spatial distributions. Fig. 2 shows the evolutionary process of the three types of strategies with time for four different values of r/G when $\beta = 0.5$. Intuitively, under medium values of β and low values of r/G, excluders cannot acquire enough gains after paying the cooperation and exclusion costs. Thus, it can be seen from panel (a) corresponding to a low value of r/G that both cooperation and exclusion are quickly dominated by defection. As r/G increases to 0.44 as in panel (b), cooperation can emerge and excluders and defectors coexist. However, the system can only be stabilized at a relatively low level of cooperation. When r/G increases to 0.54 as seen in panel (c), the frequency of exclusion first declines and then rises until finally being stabilized at a relatively high level. This can be intuitively explained as follows. When r/G increases to a relatively high level, although defectors predominate at the beginning, as the number of defectors and cooperators increases and decreases, respectively, the unit exclusion cost for excluders decreases due to the

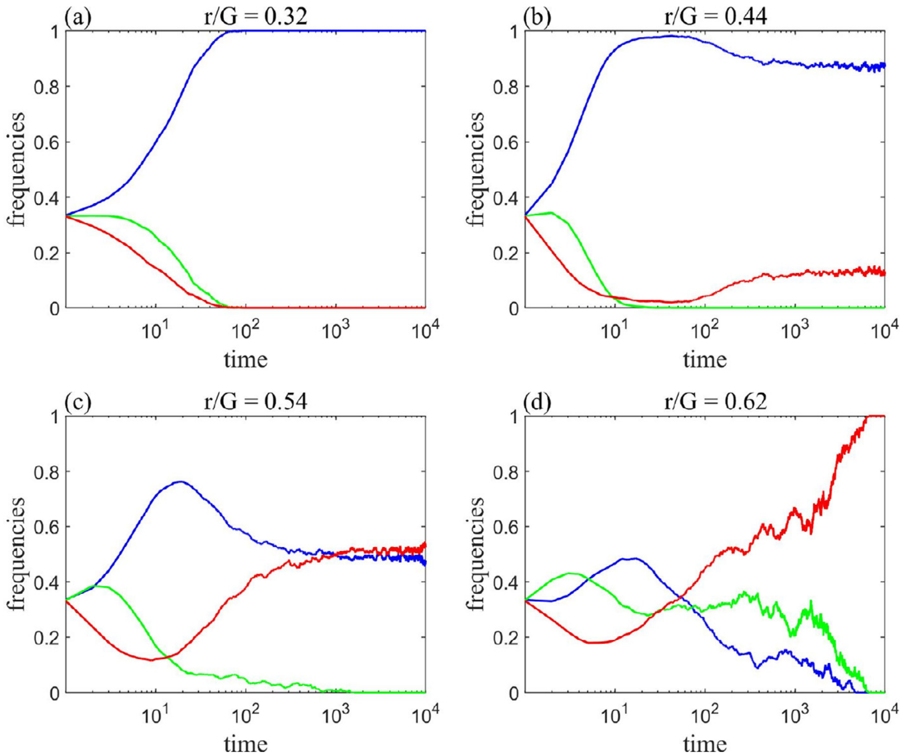

Fig. 2. Evolutionary processes of the frequencies of three types of strategies *C*(green), *D*(blue), and *E*(red) with time when β = 0.5. (a) r/G = 0.32; (b) r/G = 0.44; (c) r/G = 0.54; (d) r/G = 0.62. The fixed parameters are $\kappa/G = 0.5, \phi = 0.1, \theta = 0.8$ and $G = 5$. In panel (a), cooperation and exclusion gradually disappear in the evolutionary process and defection dominates at 90 MCSs. In panel (b) and panel (c), the proportion of cooperators gradually decreases, and finally drops to zero at around 50 MCSs and 1600 MCSs, respectively. The proportion of excluders first drops and then rises, finally stabilized and coexisted with defectors, varying oppositely with the proportion of defectors. In panel (d), the proportion of defectors first rises and then drops, and finally disappears at around 5000 MCSs. The system reaches a state of full cooperation. (For interpretation of the references to colour in this figure legend, the reader is referred to the web version of this article.)

scale effect. After cooperators are eliminated, defectors can no longer be sheltered by cooperators. At this time, the proportion of excluders is gradually increasing. Interestingly, the reduction of the defectors will, in turn, lead to an increase in the unit cost of exclusion, which leads to a decrease in the proportion of excluders. Eventually, under the dual effects of exclusion cost and the scale return, the proportion of excluders and defectors reached a certain balance. When r/G increases to 0.62 as in panel (d), cooperators and excluders have a competitive advantage, and defectors are eliminated in this situation. In the end, the system terminates into a state of full cooperation.

Fig. 3 shows the spatiotemporal distributions of the three types of strategies in the SPGG at different MCSs when $\beta = 0.5$. The four sets of subgraphs are the simulation results corresponding to r/G =0.32, 0.44, 0.54 and 0.62, respectively. When r/G is small, moderate values of probability β cannot protect cooperators and excluders from surviving, and the system terminates into the state of full defection. As seen in panels (a) of Fig. 3, which depicts the process of the gradual disappearance of cooperation and exclusion when $r/G = 0.32$. Specifically, before 50 MCSs, excluders and cooperators can form small clusters to fight against defectors, but they failed finally and are eventually eliminated in 10^4 MCSs. Panels (b) demonstrates that as r/Gincreases to 0.44, excluders can gradually form huge clusters to coexist with defectors. The increase in the number of defectors also leads to a reduction in the unit cost of exclusion, making it is possible for excluders to unite to resist the aggression of defectors in this strong level of dilemma situation. When r/G reaches 0.54, as seen in panel (c), the clusters of cooperation continue to grow until reaching an extremely huge one. When the system is stable, excluders and defectors coexist, and the system reaches a higher level of cooperation. Panel (d) shows that when $r/G = 0.62$, which corresponds to a weaker dilemma situation, the system terminates into a state of full cooperation.

Fig. 4 shows the evolutionary processes of the three types of strategies with time under four different values of β when $r/G = 0.5$. Evidently, the increase in the probability of exclusion success reduces the survival rate of defectors, which can significantly promote the evolution of cooperation. Especially, as seen in the figure, when $\beta = 0.5$, excluders can coexist with defectors when the evolutionary system reaches a stable state. When $\beta = 0.9$, the system can reach a full cooperation state with the coexistence of excluders and cooperators.

Fig. 5 shows the spatiotemporal distribution of the three types of strategies in the SPGG at different MCSs when $r/G =$ 0.5. The four sets of subgraphs are the simulation results corresponding to β =0.3, 0.5, 0.7 and 0.9, respectively. In panel (a),

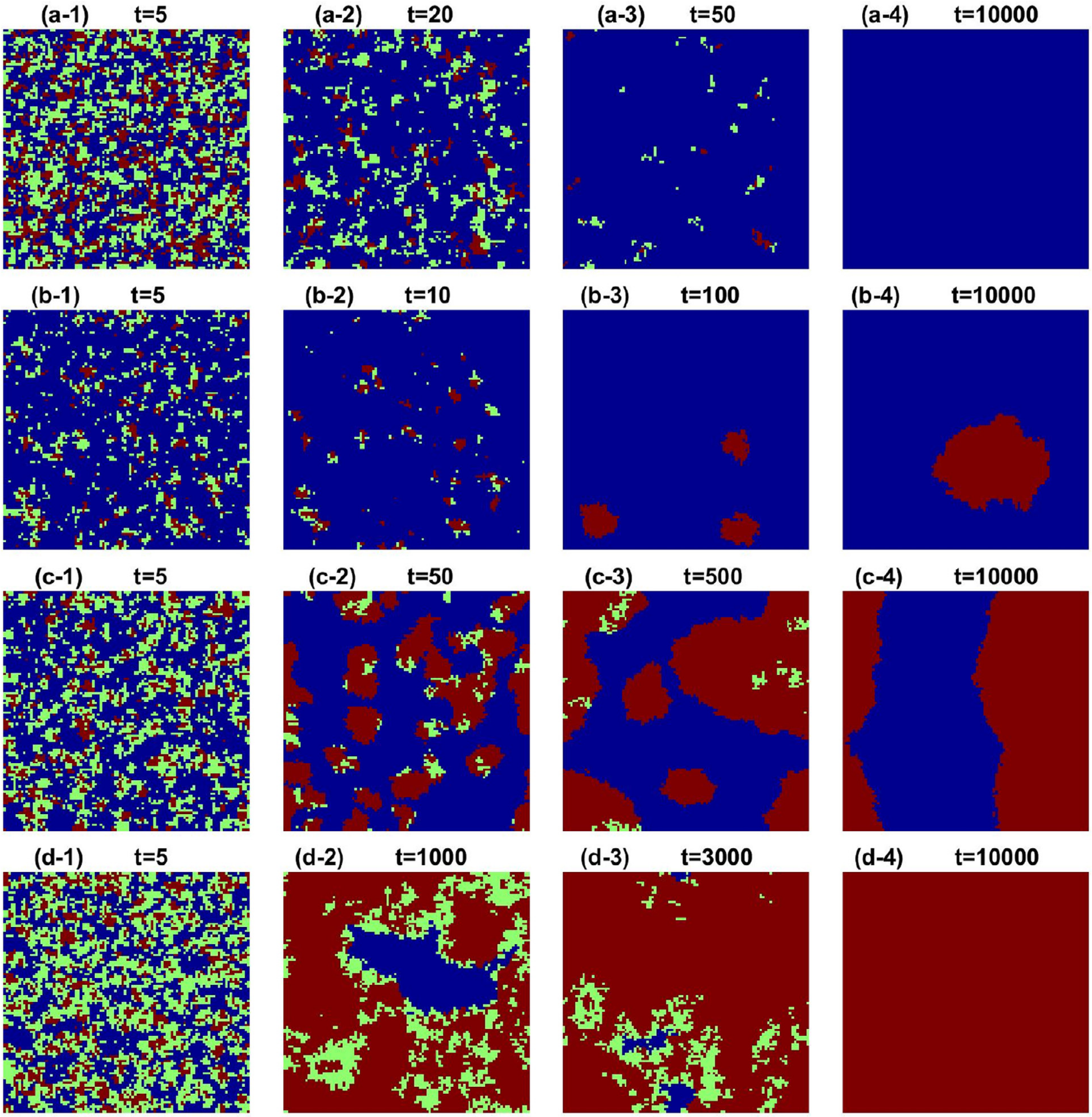

Fig. 3. Spatiotemporal distribution of the three types of strategies in the SPGG at different MCS when $\beta = 0.5$. (a) $r/G = 0.32$; (b) $r/G = 0.44$; (c) $r/G = 0.54$; (d) $r/G = 0.62$. The fixed parameters are $\kappa/G = 0.1$, $\phi = 0.1$, $\theta = 0.8$ and $G = 5$. Among them, the blue, green and red represent defectors, cooperators, and excluders, respectively. Panel (a) shows that when r/G is small, even though excluders and defectors can form small clusters, they could not resist the invasion of defection. In panel (b) and panel (c), as r/G increases, excluders can form small clusters to survive at the beginning, and then clusters continue to gather to form larger clusters, and finally, clusters of excluders can coexist with defectors. In panel (d), as r/G increases further, excluders can beat defectors by forming huge cooperative clusters, and the system terminates into the state of full exclusion. (For interpretation of the references to colour in this figure legend, the reader is referred to the web version of this article.)

excluders and cooperators can form small clusters to fight against defectors at the beginning, but they are not dominant due to the low values of β . The system terminates into the state of full defection. In panel (b), as β increases to 0.5, excluders can form small clusters, and then small clusters converge to form larger clusters gradually. Finally, excluders survived by forming a huge cluster that is surrounded by defectors, leading the system to a medium level of cooperation. In panel (c), as β increases to 0.7, cooperators and excluders can gather together to form huge clusters to fight with defectors, leading the system to a high level of cooperation. Interestingly, we observe that when the system reaches stability, cooperators separate

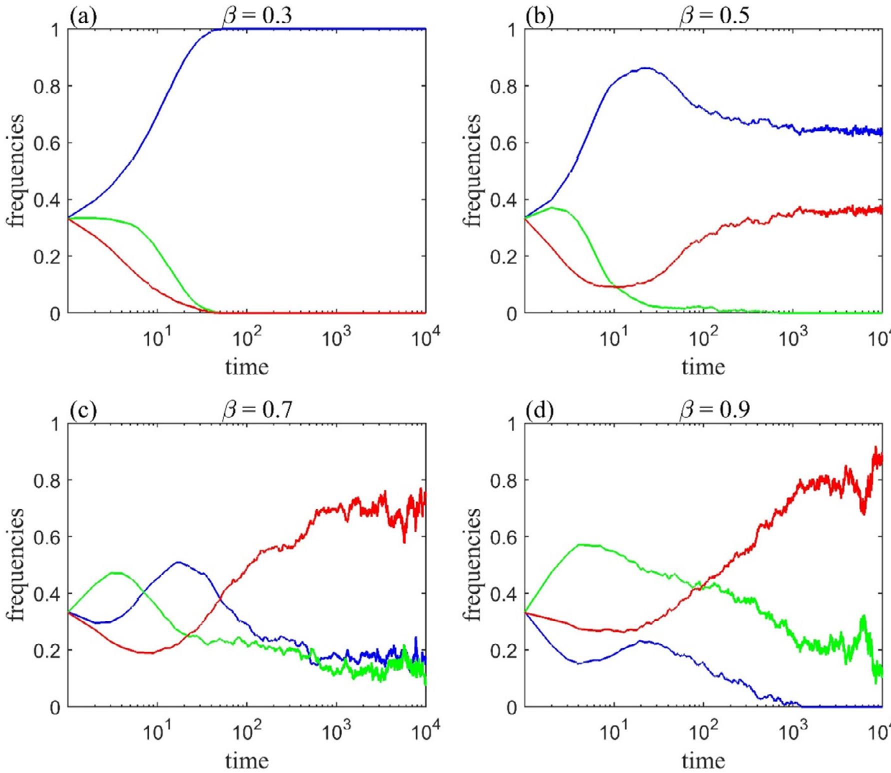

Fig. 4. Evolutionary processes of the fractions of three strategies *C*(green), *D*(blue), and *E*(red) with time when $r/G = 0.5$. (a) $\beta = 0.3$; (b) $\beta = 0.5$; (c) $\beta = 0.7$; (d) $\beta = 0.9$. The fixed parameters are $\kappa/G = 0.1$, $\phi = 0.1$, $\theta = 0.8$ and $G = 5$. In panel (a), defectors dominate at around 60 MCSs, with both cooperators and excluders disappeared in the evolutionary process. In panel (b), cooperators disappear at around 500 MCSs. The proportions of excluders and defectors fluctuate until reaching a balance. The system terminates into the coexistence of excluders and defectors at 10,000 MCSs with a medium level of cooperation. In panel (c), the proportion of excluders first drops and then rises. Excluders coexist with cooperators and defectors finally, leading the system to a high level of cooperation. When β increases further, as seen in panel (d), defectors disappear and the system terminates into a full cooperation state with the coexistence of excluders and cooperators. (For interpretation of the references to colour in this figure legend, the reader is referred to the web version of this article.)

defectors from excluders, which provides a space for the defectors to survive. In panel (d), when β reaches 0.9, cooperators and excluders beat defectors by forming huge cooperative clusters in the early period of the evolution, which results in the disappearance of defectors. Cooperators and excluders can coexist randomly because of the same benefits when there are no defectors.

In order to observe the results under more parameter combinations, we relax the fixation of θ to explore the effects of dilemma strength, cost, and exclusion probability on the evolution of cooperation. In our model, since the unit exclusion cost depends on the state of the system, therefore, the change in θ makes the unit exclusion cost more dynamic. Fig. 6 gives the color projection of the average level of cooperation on the two-dimensional (2D) planes, namely, r/G vs β plane and r/G vs θ plane, or called 2D heat maps [77–80]. In the upper three figures, from left to right, three values of $\theta = 0.6$, 0.8 and 1.0 are considered, respectively. In each figure, the entire r/G vs β plane is divided into three parts, corresponding to the three system states, namely, full cooperation, full defection, and the coexistence of cooperation and defection. Evidently, as the cost parameter θ increases, the area where cooperation emerges gradually shrinks. In the lower three figures, from left to right, three values of $\beta = 0.3$, 0.6 and 0.9 are considered, respectively. As the exclusion probability parameter β increases, the area where cooperation emerges gradually expands.

We have also explored the Moore neighborhood situation in which there are eight direct neighbors for each individual, thus $G = 9$. Figs. 7 and 8 demonstrate the corresponding results. In order to compare with the previous results of $G = 5$, parameters are fixed at the same values in the two cases. Fig. 7 shows the average level of cooperation in the population with r/G under seven different values of β, which corresponds to Fig. 1. Comparing these two figures, we find that under the same parameters, in the case of $G = 9$, cooperation can emerge at a smaller r/G threshold. Moreover, the threshold of r/G for the disappearance of defection is also greatly reduced when $G = 9$. Therefore, we can make a preliminary judgment that under our exclusion mechanism with dynamic cost, the population structure based on the Moore neighborhood is more conducive to the evolution of cooperation.

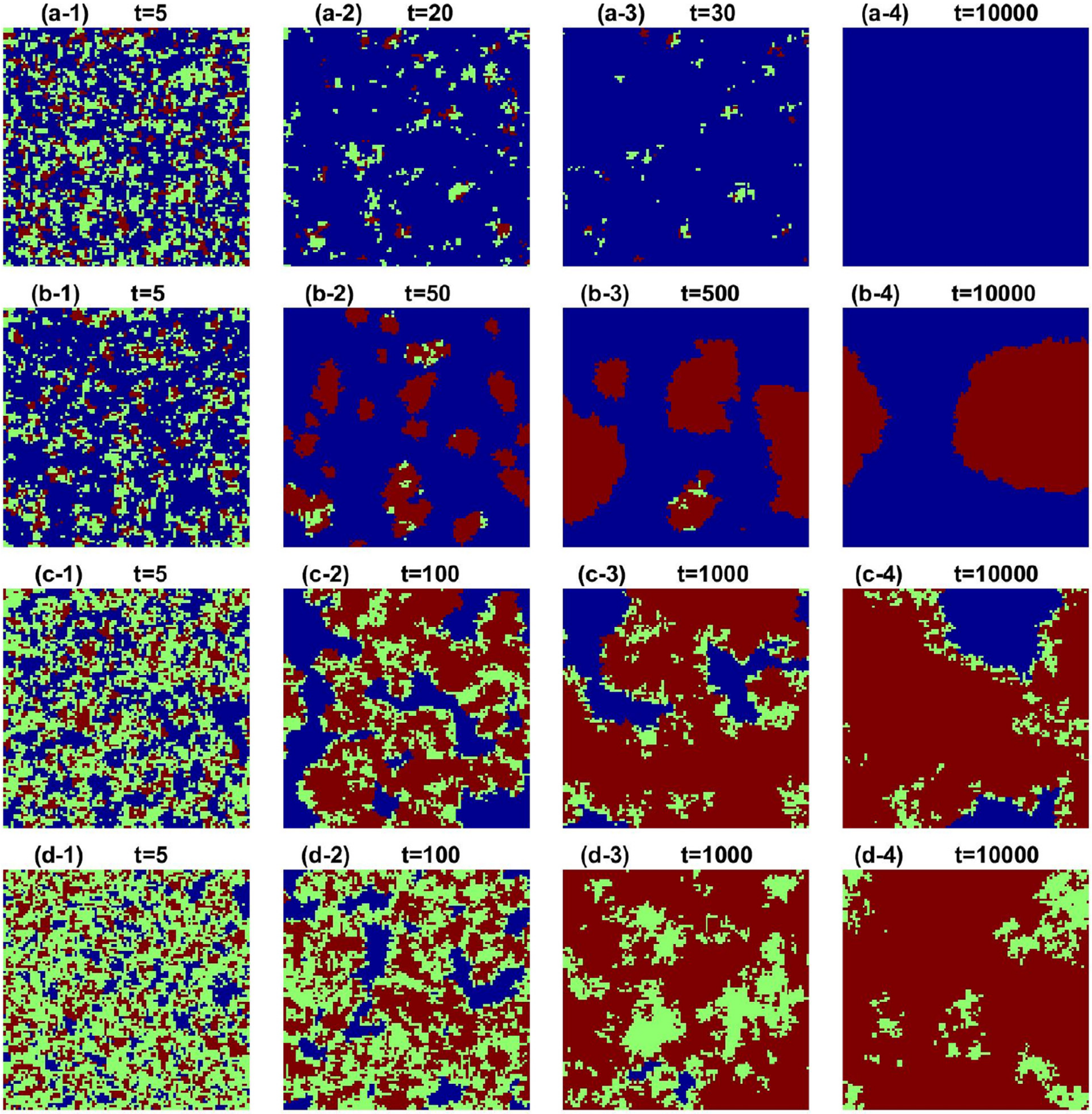

Fig. 5. Spatiotemporal distribution of the three types of strategies in the SPGG at different MCS when $r/G = 0.5$. (a) $\beta = 0.3$; (b) $\beta = 0.5$; (c) $\beta = 0.7$; (d) $\beta = 0.9$. The fixed parameters are $\kappa/G = 0.1$, $\phi = 0.1$,$\theta = 0.8$ and $G = 5$. Among them, the blue, green and red represent defectors, cooperators, and excluders, respectively. In panel (a), cooperators and excluders cannot survive due to low values of β. The system terminates into the state of full defection. In panel (b), as β increases, cooperators disappear at nearly 500 MCSs, but excluders can form small clusters gradually; small clusters converge to form larger clusters to compete with defectors. The system terminates into the state of the coexistence of defectors and excluders. In panel (c), cooperators and excluders can gather together to form huge clusters to fight with defectors. The system terminates into the coexistence of the three strategies at 10,000 MCSs. In panel (d), cooperators and excluders beat defectors by forming huge cooperative clusters in the early period of the evolution, which results in the disappearance of defectors. (For interpretation of the references to colour in this figure legend, the reader is referred to the web version of this article.)

In order to verify whether this conclusion is also true under other parameter combinations, we have drawn the 2D heat maps of the Moore neighborhood case in Fig. 8, corresponding to the traditional neighborhood case as in Fig. 6. By comparing the corresponding parameter areas of the three states in the two figures, we find that the threshold values for both the emergence of cooperation and the disappearance of defection are reduced in the Moore neighborhood case, which further confirms that the Moore neighborhood is more conducive to the evolution of cooperation in our social exclusion mechanism with dynamic cost.

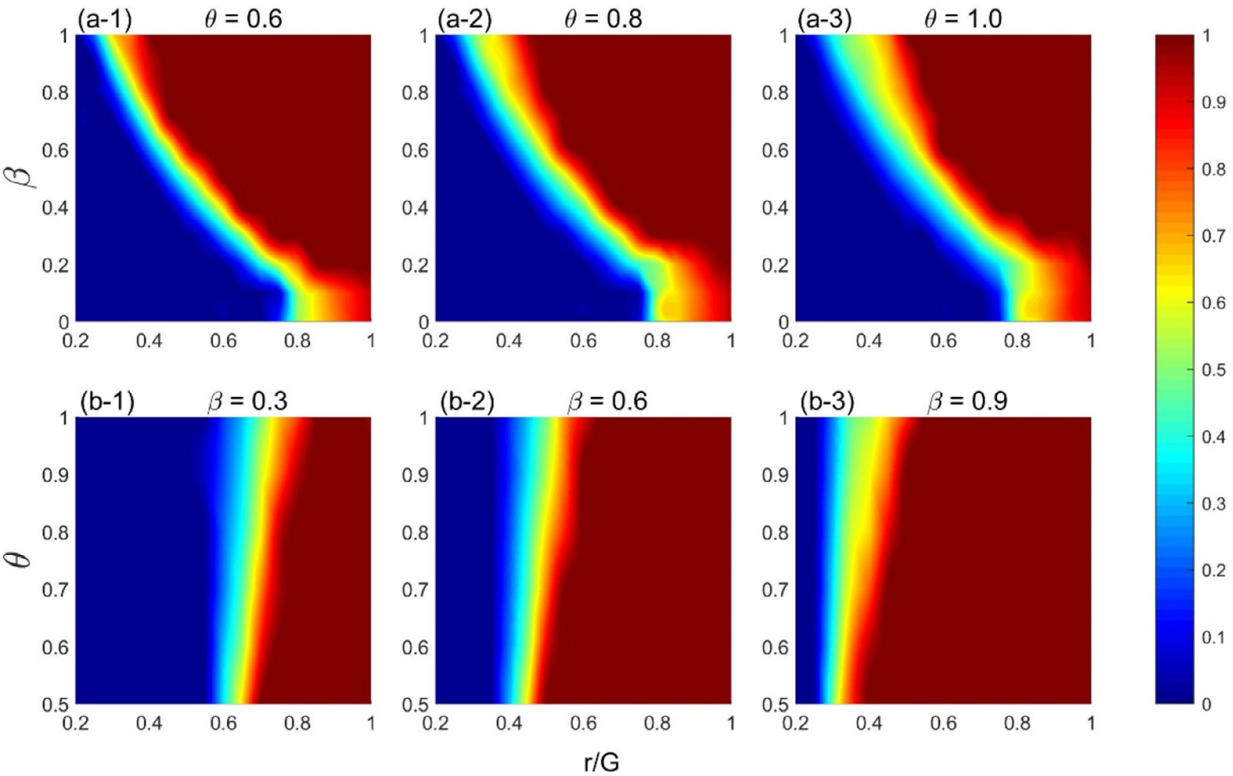

Fig. 6. When $G = 5$, the 2D heat maps of the average level of cooperation. (a) r/G vs β plane; (b) r/G vs θ plane. The color bar on the right relates different colors to different levels of cooperation. From dark blue to deep red, the level of cooperation is gradually increasing from zero to one. In panel (a), from left to right, three values of $\theta = 0.6$, 0.8 and 1.0 are considered, respectively. As the cost parameter θ increases, the area where cooperation emerges gradually shrinks. In panel (b), from left to right, three values of $\beta = 0.3$, 0.6 and 0.9 are considered, respectively. As the exclusion probability parameter β increases, the area where cooperation emerges gradually expands. (For interpretation of the references to colour in this figure legend, the reader is referred to the web version of this article.)

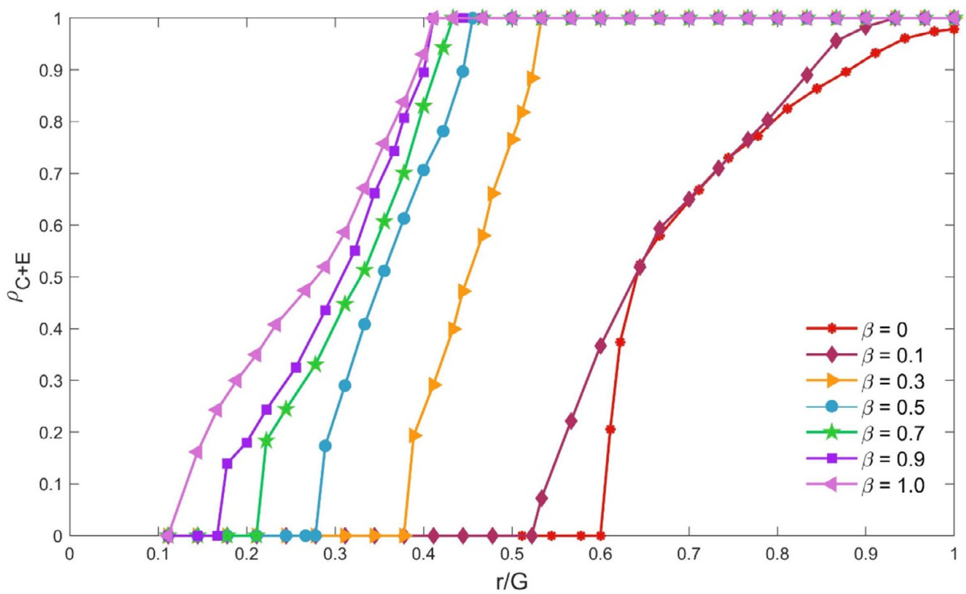

Fig. 7. In the Moore-neighborhood case ($G = 9$), comparison curves of the average level of cooperation in the population with r/G under different values of β, for fixed $\kappa/G = 0.1$, $\phi = 0.1$ and $\theta = 0.8$. When $\beta = 0$, cooperation emerges at $r/G = 0.61$, and the system cannot reach the state of full cooperation when $r/G \leq 1$. When $\beta = 0.1$, cooperation emerges at $r/G = 0.53$, and defection disappears at $r/G = 0.93$. When β increases to 0.3, 0.5, 0.7, 0.9 and 1.0, respectively, cooperation emerges at $r/G = 0.39$, 0.29, 0.22, 0.18, and 0.14, correspondingly, while defection disappears at $r/G = 0.53$, 0.46, 0.43, 0.41, and 0.41, correspondingly. Obviously, the increase in the probability of exclusion success greatly lowers threshold values of r/G for the emergence of cooperation and the disappearance of defection.

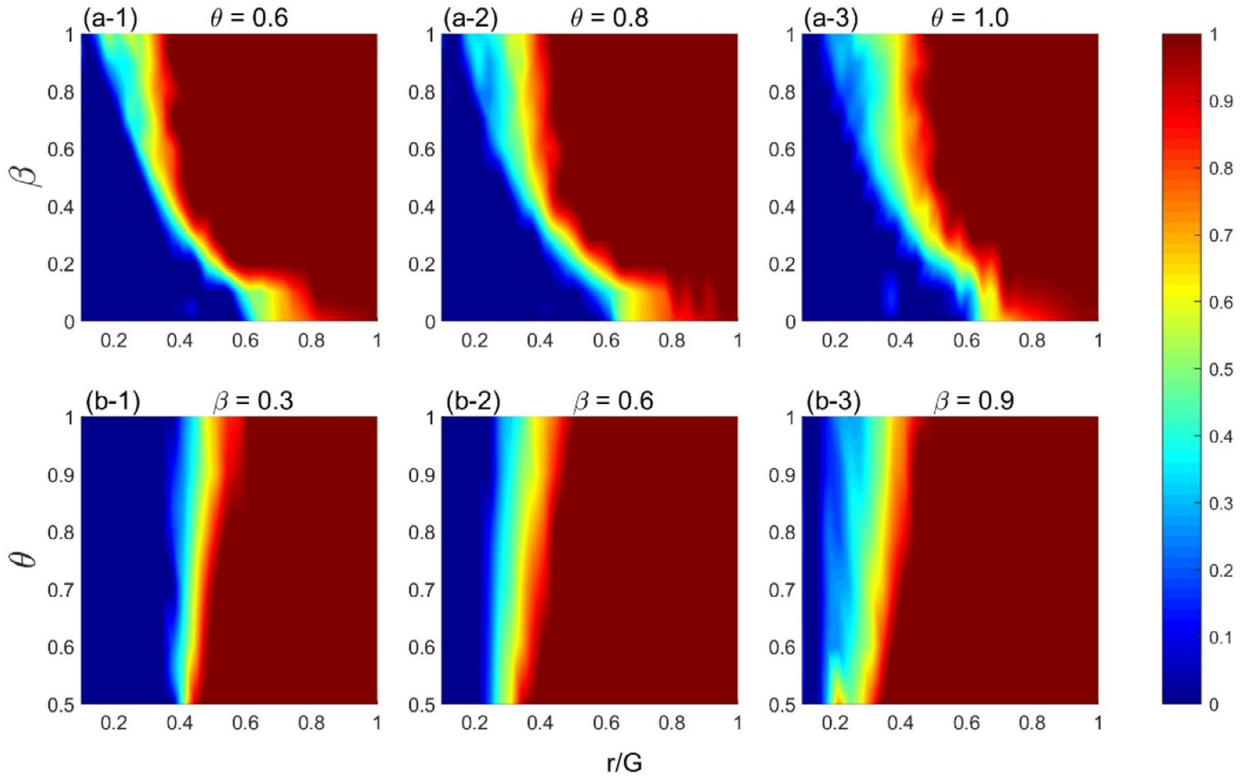

Fig. 8. In the Moore-neighborhood case ($G = 9$), the 2D heat maps of the average level of cooperation. (a) r/G vs β plane; (b) r/G vs θ plane. All parameters are fixed at the same values as the case of $G = 5$ for comparison. By comparing the parameter areas for the emergence of cooperation in Fig. 6 and Fig. 8, we further confirm that population structure based on the Moore neighborhood is more conducive to the evolution of cooperation in our social exclusion mechanism with dynamic cost.

4. Conclusion

In this paper, an evolutionary SPGG model with three types of strategies, namely, cooperation, defection and exclusion on a square lattice structured population has been elaborated. Moreover, two types of neighborhood structures, namely, the traditional neighborhood and the Moore neighborhood are explored, respectively. Exclusion is a prosocial strategy that voluntarily pays a cost to expel defectors in the group. With this strategy, defectors will be expelled from the group with a certain probability. Defectors who are driven out will get nothing and only the surviving ones can share the cooperative benefits. As a special type of punishment, exclusion has been proven to promote cooperation under some conditions. However, current research on exclusion usually assumes that the unit cost of exclusion is fixed, and the individual's total exclusion cost is linearly related to the number of defectors. However, costs are often characterized by nonlinearities due to economies of scale. Based on this motivation, we introduced a kind of exclusion with dynamic costs in the SPGG model. In our model, the unit cost of exclusion is a function of the state of the group, and the more defectors, the smaller the unit cost of exclusion. We establish a probabilistic exclusion model and explored the impact of the introduction of the exclusion-type strategy with state-dependent cost on the evolution of cooperation. We investigate parameters such as the probability of exclusion success, exclusion cost, and the dilemma strength of the game on the cooperation level of the population. Through simulation experiments, we find that this type of exclusion could promote cooperation within a wide range of parameters. Dynamic cost resulting from the scale effect is more conducive to the emergence of prosocial behavior. Especially, we also confirm that the Moore neighborhood case is more conducive to the evolution of cooperation in our social exclusion mechanism with dynamic cost compared with the traditional neighborhood case.

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