Chaos, Solitons and Fractals 131 (2020) 109517

Contents lists available at ScienceDirect

Chaos, Solitons and Fractals

Nonlinear Science, and Nonequilibrium and Complex Phenomena

journal homepage: www.elsevier.com/locate/chaos

Frontiers

Reputation evaluation with tolerance and reputation-dependent imitation on cooperation in spatial public goods game

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ARTICLE INFO

Article history:
Received 2 July 2019
Revised 10 October 2019
Accepted 8 November 2019
Available online 14 November 2019

Keywords:
Spatial public goods game
Reputation
Tolerance
Evolution of cooperation

ABSTRACT

Reputation exists widely in human society and plays a significant role in people's behavioral norms. People with a good reputation are more likely to be noticed in society, so their behaviors are more likely to be imitated and spread. However, traditional imitation models in evolutionary games based on the Fermi rule ignore this reality. Based on this observation, this paper improves the existing spatial public goods game models in two aspects. First, we introduce a reputation evaluation mechanism considering tolerance, allowing individuals to defect in a harsh environment in which the number of detected neighbors in the group is large whereas not affecting their reputation. Moreover, different from the traditional random selection of neighbors as imitation objects in the process of individual strategy evolution, this paper assumes that individuals with a good reputation have a higher probability of being selected, whereas whether to imitate this individual depends on the Fermi function based on the payoff difference. Through simulation experiments, we found that reputation-dependent imitation can improve the cooperation level of the population. We also found that small values of tolerance in reputation evaluation provide a better environment for the survival of cooperators. This mechanism allows us to better understand the pervasive phenomenon of cooperation in the social system.

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1. Introduction

Cooperation is ubiquitous in a wide range of biological and social systems, which conflicts with the theory of natural selection. So far, cooperation dilemma has been a hot topic in various fields and has attracted the focus of many researchers [1–4]. Evolutionary game theory has been proved to be one of the most effective frameworks in solving this problem [5–7]. The public goods game (PGG) provides a theoretical model for studying the collective cooperative behavior of multi-agent interaction [8,9]. In a typical public goods game, there are N persons in the group, and each person's strategy can only be cooperation or defection. The cooperators contribute a fixed share to the public pool, while the defectors do not contribute. Then, all the contributions are added and multiplied by an enhancement factor r. The total revenue is allocated equally to all individuals regardless of their strategies. Obviously, this creates a free opportunity for defectors to obtain the same benefits as cooperators, which is the reason for the social dilemma.

In order to explain this dilemma, many mechanisms have been proposed. Nowak summarized five main mechanisms for the evolution of cooperation, namely, kin selection, direct reciprocity, indirect reciprocity, network reciprocity and group selection [10]. With the rapid development of social network theory, network reciprocity has received extensive attention in recent years. It has been proved that some spatial structure can effectively improve the level of cooperation in the PGG [11–13]. Following this direction, exploring more realistic and subtle microscopic mechanisms to explain the evolution of cooperation in the spatial PGG (SPGG) has attracted extensive attention from scholars. It has been found that social diversity [14–17], reward and punishment [18–26], social exclusion [27–30], heterogeneity of payoffs [31–36], coevolution of strategies and interactions [37–39], asymmetric information sharing updating [40], voluntary participation [41–43] and some strategy update rules [44,45] can promote the evolution of cooperation. In addition to theoretical models, literature based on behavioral experiments investigating the evolution of cooperation has also emerged recently [46–48].

Unlike other biological systems, the real human society relies on the credit system, and reputation is very important for a person to survive in the society, such as business transactions, credit

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https://doi.org/10.1016/j.chaos.2019.109517
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loans, and academic research. Therefore, it is natural to introduce individual reputation and study the effect of reputation information on the evolution of cooperation. In fact, since Nowak [49] first linked the choice of strategies with the score of an individual's image in the games, proving that individual image had a great influence on the evolution of cooperation, reputation has been widely studied in evolutionary games [50–58]. At present, different models have been proposed and the reputation is considered from different perspectives. For instance, based on the observation that individuals tend to choose to cooperate with individuals with higher reputation, some literature assumed that reputation affects individuals' choice of partners [59–63] or link weight between partners [64]. Others assumed that reputation affects the strategy update rule of individuals [65–67]. There are also studies assuming that reputation affects individual payoffs [68,69] or investments [70,71]. In addition, it is found that the heterogeneity [72,73] and diversity [74] of reputation also have certain impacts on the evolution of cooperation.

Notably, in a complex social environment, reputation is closely related to one's influence. People with good reputations are more likely to be noticed, and their behaviors are more likely to be imitated and spread. Moreover, when assessing the behavior of others, it is reasonable to consider the environment in which they are located since we always show some degree of tolerance. However, traditional SPGG models based on the Fermi-rule imitation ignore this reality, and also no exceptions are allowed in the reputation evaluation rule. In view of this situation, this paper improves the existing model from two aspects. First, we introduce a reputation evaluation mechanism that considers tolerance, allowing individuals to defect in a hostile environment in which there are too many defected neighbors in their group whereas not affecting their reputation. Then, different from the traditional random selection of neighbors as imitation objects in the strategy updating, this paper assumes that players with good reputation have a higher probability of being selected, whereas whether to imitate this player depends on the payoff difference based on the Fermi function. We simulated this model on a square lattice and focus on the self-organizing cooperative behavior between selfish individuals. Through simulation experiments, we have found that reputation-dependent imitation can promote the evolution of cooperation. We have also observed that a smaller tolerance threshold creates a better environment for cooperation.

The remainder of the article is arranged as follows. The evolutionary SPGG model considering reputation is introduced in detail in Section 2, including reputation evaluation with tolerance and reputation-dependent imitation. Then, the specific process and details of the simulation, as well as the results and discussion of the experiments are present in Section 3. Finally, we summarize the full context in Section 4.

2. Model

2.1. Spatial public goods game

In the SPGG, each individual is confined on a node of the network and interacts with its direct neighbors. Let Ω_i denote the set of direct neighbors of individual i. At each stage, individual $i$has to participate in $|\Omega_i|+1$ groups of PGGs simultaneously, in which one group is centered on himself and the other $|\Omega_i|$ groups are centered on each of his $|\Omega_i|$ neighbors, respectively. In the multi-group games, individual i must decide whether to contribute a unit to the public pool. In each group game, all contributions in the public pool of the group are multiplied by an enhancement factor $r(r>1)$, and the total amount is allocated equally among each individual of this group. Let $s_i$denote the strategy of player i. Consistent with other literature, individuals who contribute are called cooperators, whereas those who do not contribute are called defectors. That is, player i can choose either cooperation ($s_i = C$) or defection ($s_i = D$). In the gth ($1 \le g \le |\Omega_i|+1$) group game (denote the set of this group as G_g), the payoffs of player i as a cooperator π_C^g and as a defector π_D^g are

$$\pi_C^g = \frac{r\left(N_C^g + 1\right)}{|G_g|} - 1 \tag{1}$$

and,

$$\pi_D^g = \frac{rN_C^g}{|G_g|} \tag{2}$$

respectively, where N_C^g is the number of cooperators excluding player i in his g-th group game. Then the total payoff π_i of individual i participating the $|\Omega_i|+1$ groups of games is

$$\pi_i = \sum_{g=1}^{|\Omega_i|+1} \pi_{s_i}^g \tag{3}$$

2.2. Reputation evaluation with tolerance

We introduce reputation in the model and all players are assigned a dynamic reputation value based on their behavior in the past. In the evolutionary process, if a player contributes to the public pool, his reputation value will increase by one unit. In addition, we consider tolerance, allowing individuals to defect in a harsh environment in which the number of detected neighbors in the group is large whereas not affecting their reputation. Specifically, we set a tolerance threshold T. If a player chooses to defect and the number of cooperators N_c in the group is less than the tolerance threshold T, his reputation value will remain unchanged. If the player chooses to defect and the number of cooperators N_c in the group is larger than or equal to the tolerance threshold T, his reputation value will be reduced by one unit. The reputation of player i at time t can be summarized as

$$R_i(t) = \begin{cases} R_i(t-1) + 1 if s_i(t) = C \\ R_i(t-1) if s_i(t) = D and N_c < T \\ R_i(t-1) - 1 if s_i(t) = D and N_c \ge T \end{cases}. \tag{4}$$

Let the player's reputation be in the interval [0, 20]. We set the following boundary conditions. If a player's reputation exceeds 20, his reputation will remain at 20. Similarly, if the player's reputation is below 0, his reputation will remain at 0.

2.3. Reputation-dependent imitation

In the traditional imitation process based on the Fermi rule, at each stage, an individual is randomly selected (denote as i) and given the opportunity to update his strategy. Then, one of its four neighbors is randomly selected (denote as j), and then individual i imitates the strategy of individual j with a probability based on the payoff difference of the Fermi function.

Different from the traditional process, reputation information is considered in this study. Specifically, after player i is randomly selected to update his strategy, player i selects one of his four neighbors j according to the probability that is proportional to the neighbors' reputation R_j. This rule can ensure that individuals with a good reputation have a higher probability of being noticed and imitated. By comparing the payoff difference of player i and player j, whether player i imitates the strategy of player j depends on the probability defined by the Fermi function

$$P\left(s_i \leftarrow s_j\right) = \frac{1}{1 + \exp\left(\left(\pi_i - \pi_j\right)/\kappa\right)}, \tag{5}$$

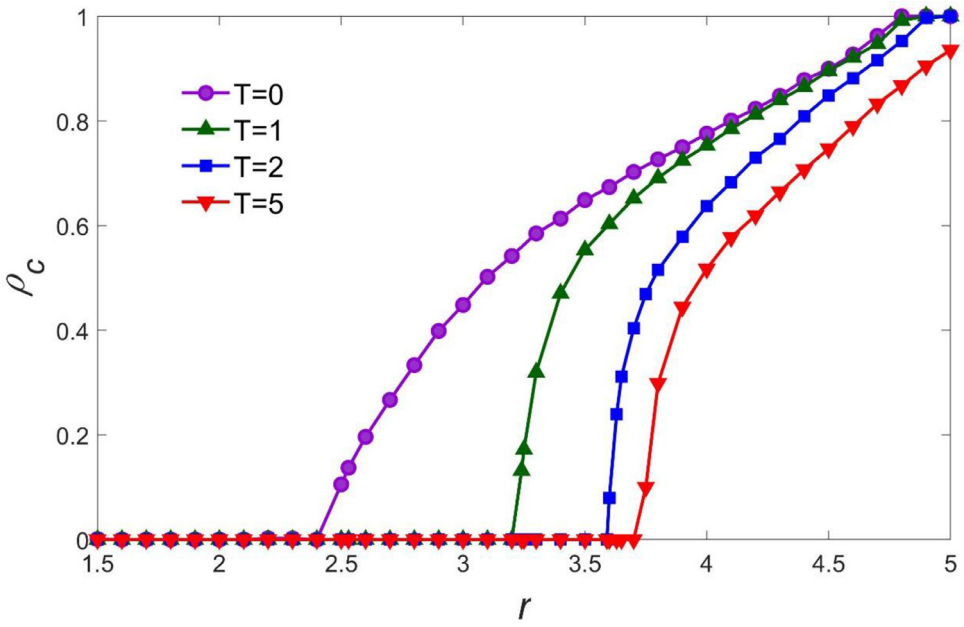

Fig 1. Frequency of cooperators ρ_C as a function of enhancement factor r for different values of tolerance T. Each data is obtained by averaging the proportion of cooperators in the last 1000 iterations after the system reaching evolutionary stable. The curves show the critical values of $r$for the phase transition to occur for different T. For instance, the phase transition points for the emergence of cooperation corresponding to T=0, 1, 2, 5 are r= 2.45, 3.25, 3.6 and 3.75, respectively. Notably, reputation-dependent imitation can make cooperative behavior easier to spread, thus accelerating the gathering of cooperators, which is more conducive to the evolution of cooperation. Moreover, small values of tolerance in the reputation evaluation provide a better environment for the survival of cooperators.

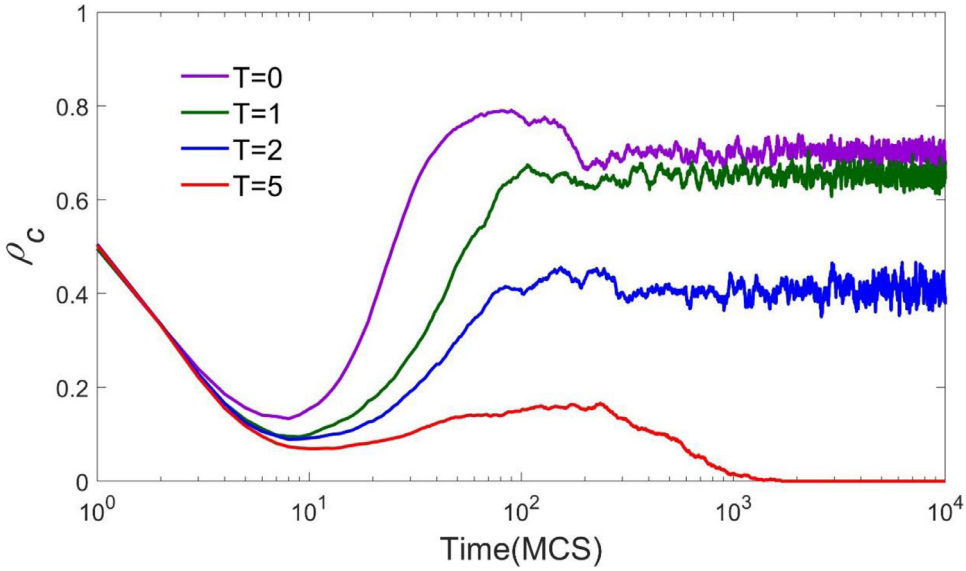

Fig 2. Frequency of cooperators ρ_C varies with time for different tolerance thresholds T and fixed enhancement factor$r = 3.7$. In all situations, the system can reach evolutionarily stable in 10^4 iterations. For $T = 0$, 1 and 2, the frequency of cooperators firstly falls and then rises gradually towards a dynamic stable and non-zero level, whereas $T = 5$, cooperation frequency finally drops to zero. Note that the frequency of cooperators in a smaller tolerance threshold environment is superior to that of a larger one, which implies that tolerance inhibits the evolution of cooperation.

where π_i and π_j denote the accumulated payoff of player i and j, respectively. κ ($\kappa > 0$) is a noise parameter that describes uncertainty in the strategy updating. When $\kappa \to 0$, player i will definitely imitate player j when player j has a higher payoff. $\kappa \to \infty$ means that whether player i imitates player j is completely random. Based on experience and other literature, we set $\kappa = 0.5$ in this study.

3. Results and discussion

We carried out Monte Carlo simulations on a $N = 100 \times 100$ square lattice with periodic boundary conditions. We utilized the asynchronous strategy update rule. For most parameter values, the system can be stabilized after 10^4 iterations. At some critical point,

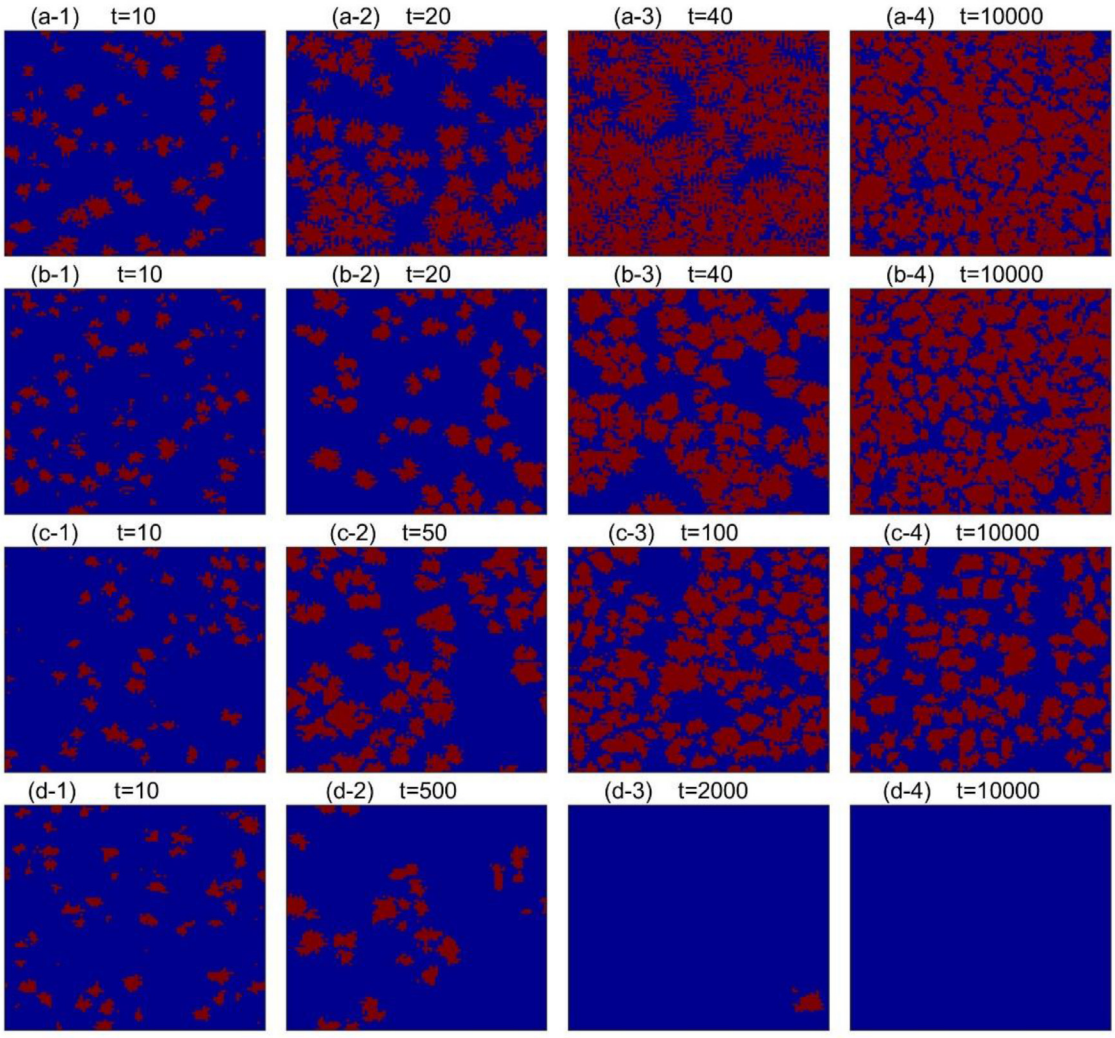

Fig 3. Snapshots of the spatial distributions of strategies at four representative time steps for different tolerance thresholds (a) $T = 0$; (b) $T = 1$; (c) $T = 2$and (d) $T = 5$ and fixed enhancement factor $r = 3.7$. From top to bottom, each row represents a situation corresponding to a different tolerance threshold T; from left to right, each column represents a different time step t. Red pixels represent cooperators and blue pixels for defectors. Notably, defectors pervade the population under a relaxed environment of reputation evaluation ($T = 5$). However, for a rigorous environment ($T = 0$, 1 and 2), after 10 time steps, the surviving cooperators gradually gather and form stable clusters to defense defectors. Therefore, the rigorous reputation evaluation environment can induce the formation and development of clusters of cooperators.

we iterated 5×10^4 times. In each iteration, we performed 10,000 updates to ensure that each node has an opportunity to adjust its strategy on average. All the results are obtained by averaging 10 independent experiments. Initially, strategies C and D are randomly distributed on each node with equal probability, and the reputation value of each player is also evenly distributed in the interval [0, 20]. We focus on the average cooperation level of the population which is defined as

$$\rho_C = \frac{1}{N}\sum_{i=1}^{N} s_i. \tag{6}$$

We study the effects of reputation-dependent imitation and tolerance level Ton the evolution of cooperation. Fig. 1 shows the average level of cooperation as a function of enhancement factor r for four different values of T. When $T = 0$, players are in the most rigorous evaluation environment. That is, as long as players choose to defect, their reputation will be strictly reduced by one unit. $T = 5$ refers to the most relaxed evaluation environment. At this time, the reputation is increased by one unit for a cooperative act, whereas remaining unchanged when defecting. From Fig. 1, we observe that for each $T(T = 0, 1, 2$ and $5)$, there is a corresponding critical value of r, above which phase transition occurs and cooperation can emerge. When $T = 5$, the current model degenerates into the traditional SPGG without reputation, and our simulation results are completely consistent with those in the previous study [75]. Specifically, when $r < 3.75$, all players choose to defect, and the proportion of cooperators is 0. When $r = 3.75$, phase transition occurs and the number of cooperators in the population gradually increases. When T decreases, the critical value of r for the phase transition also decreases, indicating that the higher the tolerance, the more unfavorable the evolution of cooperation. Especially, when Tdrops from 2 to 0, the critical value of r for phase transition also drops from 3.25 to 2.45. The effect of r on the cooperation level of the population is also affected by the value of T. On the whole, reputation-dependent imitation can enhance the evolution of cooperation. Moreover, small values of tolerance in the reputation evaluation provide a better environment for the survival of cooperators.

The evolutionary processes of cooperation with time under four diverse tolerance thresholds are compared in Fig. 2 for fixed en-

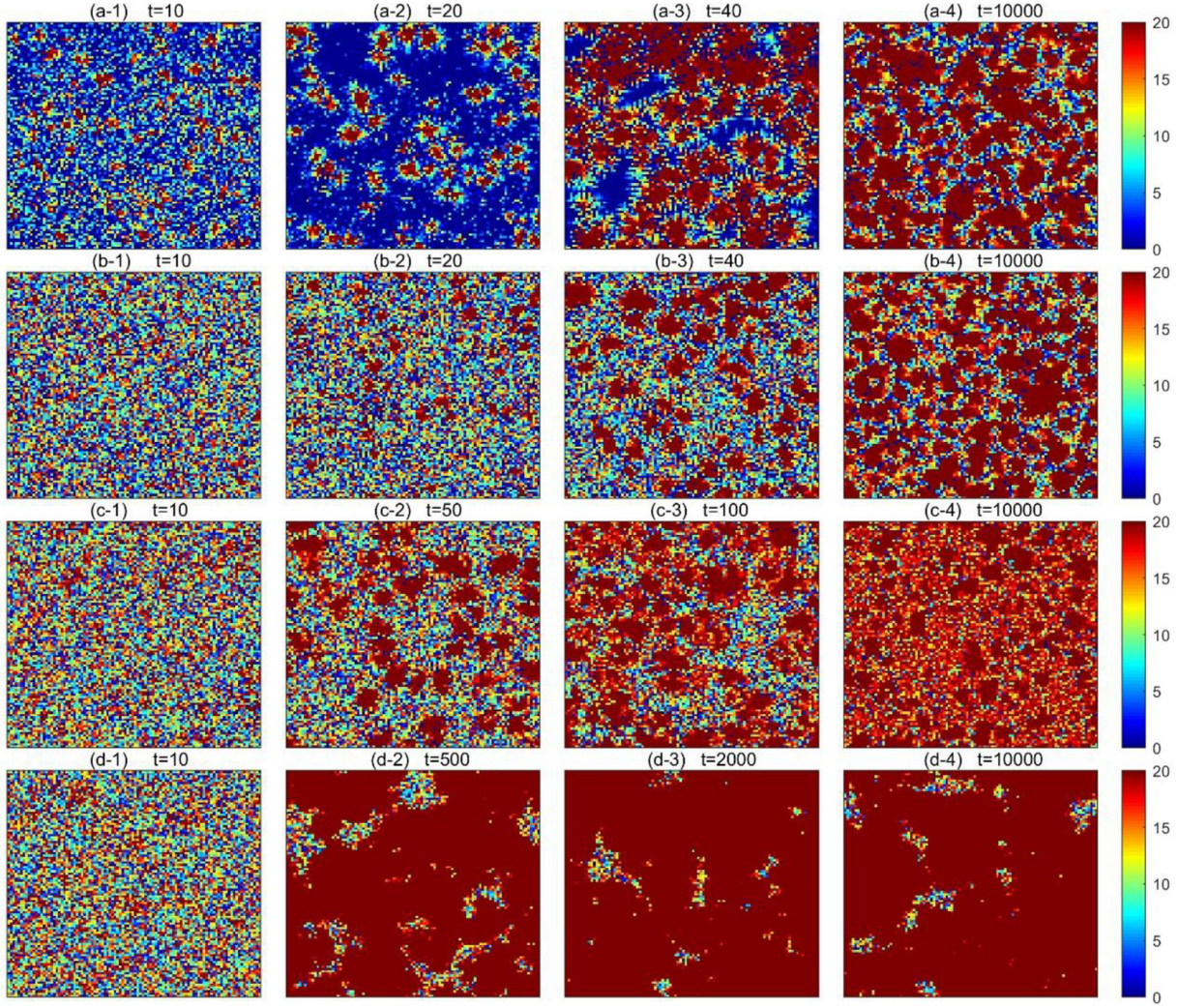

Fig 4. Snapshots of the spatial distributions of individuals' reputation at four representative time steps for different tolerance thresholds (a) $T = 0$; (b) $T = 1$; (c) $T = 2$and (d) $T = 5$ and fixed enhancement factor $r = 3.7$. From top to bottom, each row represents a situation corresponding to a different tolerance threshold T; from left to right, each column represents a different time step t. The color bar relates the reputation values corresponding to different colors. It should be noted that the reputation distribution snapshots are not exactly consistent with those of the strategy distribution snapshots in Fig. 3, especially in the situations corresponding to large T. Because the reputation evaluation criteria are different under different T, in particular, when $T = 5$, defection does not affect an individual's reputation. Therefore, the reputation values under different T cannot be compared. In fact, it can be seen from the figure that when $T = 5$, although the system reaches the full defection state after the stability, high-reputation individuals are almost still occupying the whole population.

hancement factor $r = 3.7$. This value is especially representative because cooperation cannot emerge when reputation is not introduced as in the traditional SPGG. However, in our model, if reputation information can affect the imitation process of individuals' strategy updating, cooperation can survival at varying degrees for different values of T. As can be seen from the figure, when in strict evaluation environments ($T = 0$, 1 and 2), the frequency of cooperators firstly falls to the bottom and then rises gradually towards the stationary level. After about $t = 100$, ρ_C fluctuates violently, which means that there is fierce competition between cooperators and defectors within the population. However, in the most relaxed evaluation environment ($T = 5$), cooperation frequency finally drops to zero. That is probably because initially the cooperators are randomly distributed in the population, and the defectors can obtain a higher payoff from the population. Since reputation evaluation is not strict, other players will imitate the behavior of the defectors so that it is difficult for the cooperators to survive. However, when $T = 0$, 1 and 2, the impact of reputation on the formation of cooperation clusters emerges, thus the cooperation level of the population gradually rises to a dynamic stable and non-zero level. Comparing the four curves, we can see that the risk of cooperators induced to become defectors is constantly changing, and the introduction of the tolerance threshold keeps the frequency of cooperators at a moderate level. In addition, the frequency of cooperators with a smaller tolerance threshold is superior to that with a larger tolerance threshold. Dovetailed with Fig. 1, we consolidate that smaller tolerance thresholds or a strict reputation evaluation system can promote the cooperation level of the population.

In order to further understand the emergence of cooperation more from the spatial distribution of strategies and observe the dynamic formation of cooperator clusters, the spatial pattern of defectors and cooperators is studied. Fig. 3 displays in more detail the evolutionary process of the spatiotemporal distribution of the two strategies at several different time steps. We observe that under small values of tolerance ($T = 0$, 1 and 2), there are only sporadic clusters of cooperation in the population at initial times when t is small ($t \leq 10$), but reputation still played a significant role in promoting the formation of cooperative clusters. This is pos-

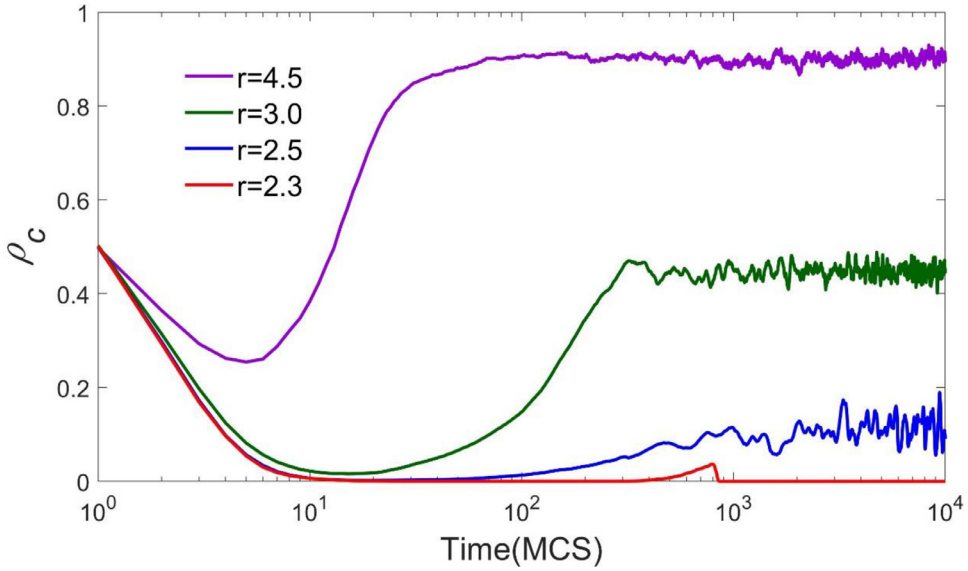

Fig 5. Evolutionary processes of the frequency of cooperators ρ_C with time for different values of enhancement factor r in the most rigorous environment $T = 0$. Four representative values of $r = 4.5$, 3.0, 2.5 and 2.3 are considered, respectively. In this particular rigorous situation of $T = 0$, cooperation can emerge even at a relatively small enhancement factor $r = 2.5$. With the increase of the enhancement factor, the benefit of cooperation expands and players who cooperate can get higher potential payoffs, which is conducive to the evolution of cooperation. Especially, when $r = 4.5$, the cooperation level of the population gradually rises and reaches an extremely high level. Although a tiny tolerance threshold will create a rigorous environment, it is conducive to the long-term survival of cooperators.

sibly because initially scattered cooperators are difficult to form dense clusters and cannot identify defectors by their randomly distributed reputations;therefore, it is difficult for cooperators to resist the invasion of defectors, resulting in defectors occupying almost the entire population. Then with time goes by, the role of reputation becomes more prominent, which creates an appropriate environment for the survival of cooperators. As more and more cooperators survive, the remaining cooperators gradually gather and form stable clusters, and the frequency of cooperators is significantly increased. At $t = 10000$, cooperators control almost the whole population. For $T = 5$, scattered cooperators gradually form clusters, but this state only lasts for a period of time, after 500 time steps, defectors occupy the dominant territory. The results indicate that the most relaxed environment is not suitable to help cooperators or resist the invasion of defectors, and finally, the cooperators in the population disappear completely. Instead, a rigorous environment, namely a small tolerance threshold T, can induce the formation and development of clusters of cooperators.

In order to explore the role of reputation on the evolution of cooperation, we draw the snapshots of the distributions of individuals' reputation corresponding to Fig. 3. The reputation values of individuals are evenly distributed between [0, 20] initially in all situations. As seen in Fig. 4, when individuals are in the most strict evaluation environment ($T = 0$), then with time goes by, high-reputation individuals gradually increase and gather to form clusters. At $t = 10000$, the population reaches a state in which high-reputation cooperators are surrounded by low-reputation defectors, which is consistent with the spatial distribution of strategies in Fig. 3. The arrival of this state is probably because high-reputation cooperators have greater influence to affect low-reputation individuals around them to imitate their strategies. When the evaluation environment is most relaxed ($T = 5$), as shown in Figs. 2–4, after about 500 time steps, the cooperation level of the population can reach the highest level (Fig. 2), and high-reputation individuals expand to almost the entire population (Fig. 4). Interestingly, at $t = 10000$, the cooperation level of the population drops to zero (Fig. 3); however, high-reputation individuals are still occupying the whole population (Fig. 4). This is because cooperation makes the reputation of individuals rise rapidly, whereas defection has no effect on the reputation of individuals when $T = 5$. In this situation, individuals can accumulate a high reputation rapidly and the clusters of high-reputation individuals can easily spread to the whole population. Therefore, even if the number of cooperators in the group gradually decreases to zero ($t = 10000$), large clusters of high-reputation individuals are still occupying almost the whole population.

We have also probed into the impact of enhancement factor on the evolution of cooperation in the most rigorous environment ($T = 0$). Fig. 5 demonstrates the evolutionary process of frequency of cooperators in the population with time. When the enhancement factor r is relatively small ($r = 2.3$), the temptation to defect is large so that the frequency of cooperators continues to decrease, eventually leading to the disappearance of all cooperative individuals in the population. In addition, as the enhancement factor increases, players can get higher potential payoffs, and there is a critical value of the enhancement factor, above which the reputation mechanism begins to function, and players tend to cooperate. For instance, the value of $r = 2.5$makes cooperators become active, thus making the cooperation level of the whole population gradually rise and maintain at a non-zero level. From Fig. 5, we can clearly discern that cooperation is most easily stabilized in the evolution when the enhancement factor r is set to 4.5. The number of cooperators in the population gradually increases after 10 steps. Then the cooperation frequency rises rapidly to a high level after dozens of time steps and finally stabilizes in about 90 steps. The results show that the rigorous environment for reputation evaluation is conducive to the construction of a cooperative atmosphere. Although a tiny tolerance threshold creates a rigorous environment, it is conducive to the long-term survival of the cooperators.

In addition, we visualize the spatial distribution of strategies in the context of a rigorous evaluation environment ($T = 0$) in Fig. 6, from which we can observe clearly the spatial pattern for clusters of defectors and cooperators. The results show that when the en-

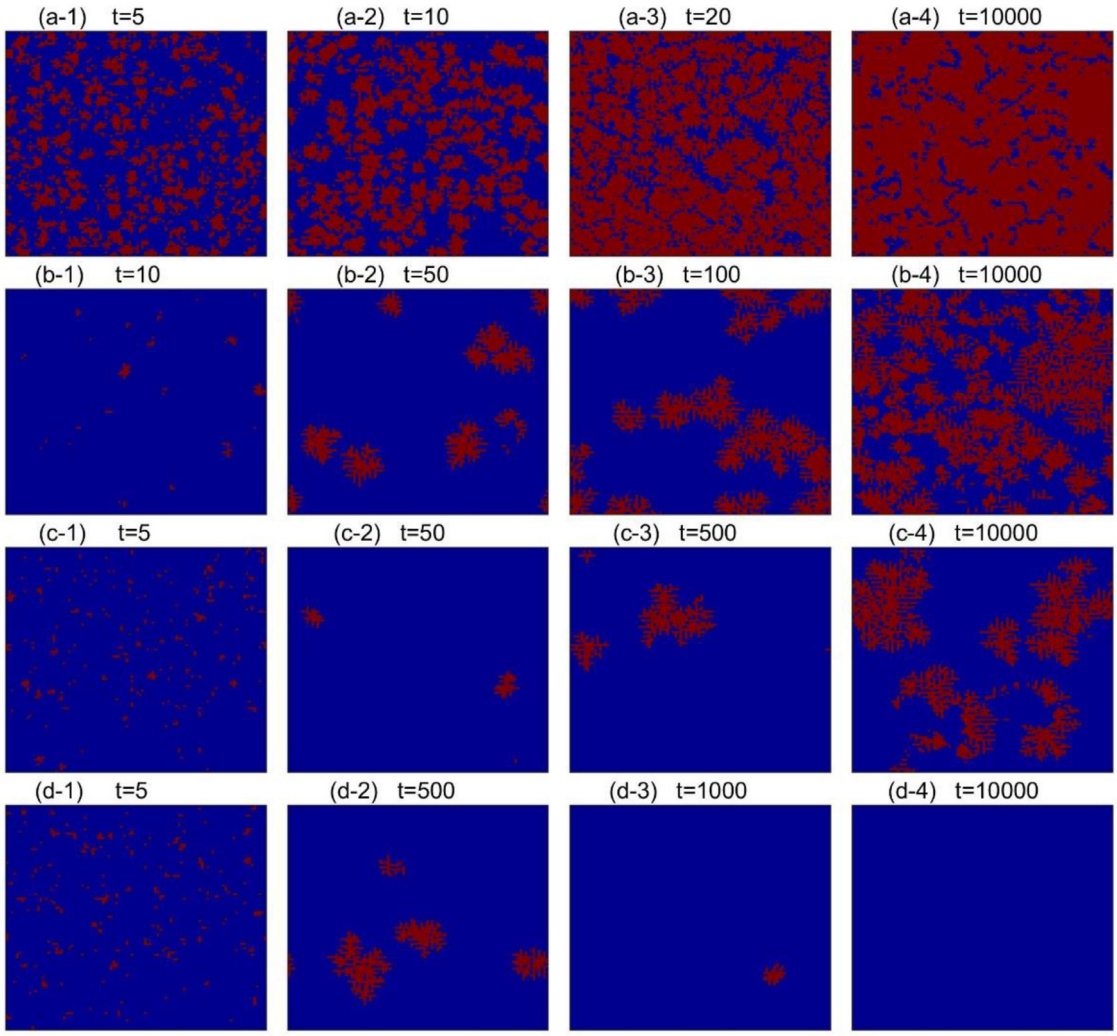

Fig 6. Snapshots of the spatial distributions of strategies at four representative time steps for different values of enhancement factor (a) $r = 4.5$; (b) $r = 3.0$; (c) $r = 2.5$ and (d) $r = 2.3$ in the most rigorous environment $T = 0$. From top to bottom, each row represents a situation corresponding to a different enhancement factor r; from left to right, each column represents a different time step t. Cooperation is coded by deep red color while defection is coded by deep blue color. In the harshest reputation evaluation environment, reputation plays a remarkable role in constraining the behavior of defectors so that more players choose to cooperate when r is not too small. Therefore, with the expansion of the enhancement factor ($r = 3.0$and 4.5), we can see that cooperators gradually gather to form compact clusters, and the cooperation level of the population gradually improves.

hancement factor r is small ($r = 2.3$), the clusters of cooperators gradually decrease, and defectors are more frequently imitated by their neighbors. Thus, at the final stable status, defectors expand to the whole population. However, in the harshest reputation evaluation environment, reputation plays a significant role, thus, with the expansion of the enhancement factor ($r = 3.0$and 4.5), cooperators gradually gather to form compact clusters, and the cooperation level of the population gradually improves. Finally, cooperators occupy the dominant territory of the population, and the cooperation strategy wins. It should be noted that although cooperation has not yet reached the full level, it is still at a very high level, which shows the superiority of the reputation-dependent imitation mechanism. These results demonstrate that a rigorous reputation evaluation environment helps reduce the risk of cooperators exploited by defectors, making players with high reputation easily become a model for other players to follow and increasing the frequency of cooperators among the population.

At last, we plot the snapshots of the spatial distributions of individuals' reputation corresponding to Fig. 6. In our model, reputation is evenly distributed initially in all simulations. As seen in Fig. 7, when $r = 2.3$, low-reputation individuals gradually increase in the evolution and dominate the whole population finally. This is consistence with the situation that the cooperation level of the population drops to zero at $t = 10000$ in Fig. 6. As the enhancement factor r increases to a relatively large value, we can clearly see that high-reputation individuals gradually increase and gather to form clusters in the evolutionary process. Especially, when $r = 4.5$, high-reputation individuals almost occupy the whole population finally, which is also consistent with the corresponding situation in Fig. 6. This can be well explained from the perspective of the reputation dependent learning mechanism. In our model, when individuals update their strategies, high-reputation individuals who are more inclined to cooperate usually have stronger influence, and since r is relatively large, their strategies are more easily imitated by low-reputation defectors around them. Then these low-reputation defectors gradually accumulate reputation and become high-reputation cooperators. Thus, high-reputation coopera-

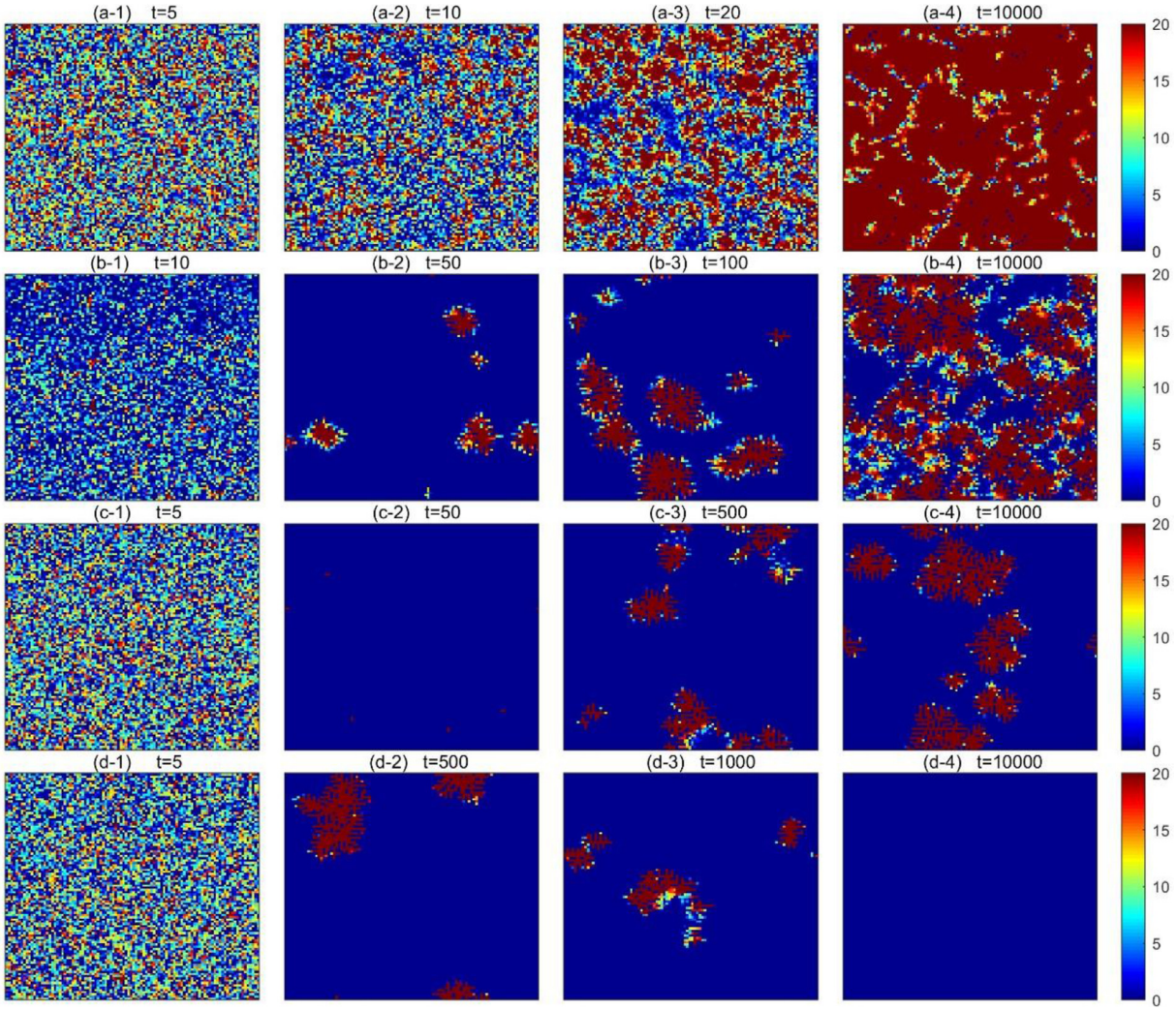

Fig 7. Snapshots of the spatial distributions of individuals' reputation at different time steps for different values of enhancement factor (a) $r = 4.5$; (b) $r = 3.0$; (c) $r = 2.5$ and (d) $r = 2.3$ in the most rigorous environment $T = 0$. From top to bottom, each row represents a situation corresponding to a different r; from left to right, each column represents a different time step t. The color bar relates the reputation values corresponding to different colors. Here, the criteria for reputation evaluation in different rows are the same. So the reputation can be compared. Notably, the evolution of individuals' reputation is highly consistent with the evolution of individuals' strategies as seen in Fig. 6.

tors can gradually form close clusters and expand, which eventually makes the level of cooperation increased significantly.

4. Conclusions

In this paper, we improved the existing SPGG models in two aspects. First, we introduced a reputation evaluation mechanism considering tolerance. Second, we modified the traditional strategy update rule based on the Fermi rule. We assume that individuals with a good reputation have a higher probability of being selected by their neighbors, whereas whether to imitate this individual depends on the Fermi function based on the payoff difference. We investigated the influence of tolerance threshold and reputation-dependent imitation on the evolution of cooperation. A large number of simulation experiments indicated that integrating reputation into imitation plays an important role in the emergence of cooperation, especially when the reputation evaluation is rigorous, or the tolerance threshold is tiny. In conclusion, reputation-dependent imitation and rigorous reputation evaluation environments are conducive to the long-term survival of the cooperators. This mechanism allows us to better understand the pervasive phenomenon of cooperation in the social system.

Declaration of Competing Interest

We wish to confirm that there are no known conflicts of interest associated with this publication and there has been no significant financial support for this work that could have influenced its outcome.

Acknowledgements

This research was supported by the National Natural Science Foundation of China (No. 71871173, 71871171, 71501149) and the Fundamental Research Funds for the Central Universities of China (WUT: 2019VI029). The fifth author is grateful for the support by NSFC-Zhejiang Joint Fund for the Integration of Industrialization and Informatization (U1709215).

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