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Reputation-based conditional compassion promotes cooperation in spatial public goods games

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Abstract. Compassion, as a typical altruistic social preference, can explain the phenomenon of cooperation to a certain extent. However, compassion is conditional in reality, and it can only be triggered under certain conditions. Based on this observation, we introduce a kind of reputation-based compassion mechanism in spatial public goods games. We set a proportion of individuals in the population as being compassionate, and only individuals with relatively low payoffs and not too low reputations may receive sympathy from other individuals who are compassionate in the group. We find that compared with randomly selecting individuals who are compassionate, individuals in the top rank of payoff or reputation being compassionate are more likely to promote cooperation; in this case, even if one percent of individuals in the population are compassionate, the level of cooperation can be significantly improved. The higher the payoff redistribution ratio, the higher the cooperative rate in the population. By classifying individuals and comparing their payoffs before and after compassion, we find that most of the payoff redistribution occurs at the boundary of the cooperator clusters. The reduction of payoff gaps affects the strategy imitation, that is, the possibility of

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low-payoff individuals imitating the strategies of high-payoff individuals can be increased. These results expand our understanding of conditional compassion in promoting cooperation.

Keywords: agent-based models, evolutionary game theory

Contents

1. Introduction

Cooperation is ubiquitous in social and biological systems, which is incompatible with Darwin's view of natural selection. How cooperative behavior is generated and maintained in a competitive world is the core issue of natural and social sciences [1–3]. To account for that, evolutionary game theory, as a powerful and effective mathematical framework, is introduced to solve the cooperation dilemma [4, 5]. The public goods game (PGG) provides a theoretical model for studying the cooperative behavior of multi-agent interaction and is a typical paradigm to explain the emergence of cooperation among selfish individuals [6, 7]. In a typical PGG, each individual's strategy can only be cooperation or defection. The cooperators contribute a fixed share to the public pool, while the defectors do not. Total income is allocated equally among all individuals, regardless of their strategy. Defectors can get the same benefits as cooperators without paying the cost. This phenomenon is called free-riding behavior, which is the main dilemma of cooperation in the PGG.

Statistical physics provides an effective method of studying collective behavior through individual interactions [8–11]. It emphasizes the importance of the interactive network structure and introduces the spatial PGG (SPGG) as a null model in studying the evolution of cooperation. In particular, Perc *et al* [9] made a detailed review on the research progress of statistical physics in our understanding of human cooperation, as well as the importance of the Monte Carlo simulations and the theory of collective behavior of interacting particles near phase transition points for understanding counterintuitive evolutionary outcomes. It has been proved that some spatial structures can

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effectively promote cooperation, and cooperators can resist the invasion of defectors by forming clusters [12–14]. Besides the interactive structure between individuals, some other incentive mechanisms have also been proposed to enrich the research on cooperation evolution, such as reward [15–18], punishment [19–24], and exclusion [25–29]. The above works discussed the contribution of incentive strategies to the construction of a stable and secure cooperative society in different situations. In addition, Chen *et al* [17] found that in different population structures, the combined-strategy mechanism is always more effective than the single-strategy mechanism in promoting cooperation, such as the combination of punishment and reward, or the combination of exclusion and punishment [25].

Different from other species, human beings have unique social morality, emotion, and rationality. They can consider the impact of changes in the interests of others on their utility and social fitness. Sometimes, to pursue some non-material benefits, they are willing to sacrifice their interests. Social preference can be used to describe their behavioral choices. The typical social preferences include reciprocity [30–34], fairness [35–40], and altruism [41–46]. Many classic game experiments reflect the altruism of individuals, such as the Ultimatum Games experiment [47, 48], Dictator Games experiment [49–51], Gift Exchange Game experiment [52–54], and Trust Game experiment [55–57].

Compassion is a typical altruistic social preference that means people would sacrifice their interests to help vulnerable groups. In recent years, studies have demonstrated the positive effect of compassion on group cooperation efficiency and personal life satisfaction from the perspective of psychology and sociology [48, 56–59]. However, in daily life, when someone asks for our help, instead of helping him unconditionally, we often judge whether he is worthy of help based on our situation. It has been proved through social experiments [60, 61] that people have different views on the pursuit of income gap reduction because of their different evaluations of the characteristics of the same society they being in. For instance, Fehr [38] designed an experiment that considers an environment where higher earnings in a real-effort task are typically associated with higher effort and varies how fair and transparent this relationship is. The results show that greater inequality does not lead to one's antisocial behavior. On the contrary, it depends on whether the increase in inequality can be attributed to the efforts made, that is, the fairness of the income-generating process. Only when morally problematic activities lead to greater inequality will low-income individuals engage in more 'money burning' activities aiming at reducing inequality. From a personal point of view, the causal effect of sympathy in altruistic behavior is related to the individual's state and identity [37–39, 62, 63]. DeSteno *et al* [63] argued that perceptions of increased similarity mediated the enhanced levels of compassion and a few individuals in a good situation would feel the same burden and pressure as others. Szolnoki and Perc [64] regarded tolerance as sympathy for enduring a difficult environment, and revealed the fascinating subtleties of temporal and spatial dynamics arising from the competition of subsystem solutions in a structured population.

The behavioral choices of individuals in multi-agent interaction are influenced by their own beliefs, which are their opinions on other people's current or historical rounds

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of information. The reputation mechanism is seen as an effective way to provide information about the potential of both parties to establish a successful relationship [65, 66]. Nowak and Sigmund [67] firstly introduced the prototype of the reputation known as the image score. Subsequently, several works confirmed the efficiency of reputation in promoting cooperation [68–74]. For instance, Fu *et al* [74] emphasized the importance of reputation when individuals adjust their partnership. These previous works provide objective and rational evidence for individuals to determine the degree of cooperation with others.

Inspired by the above research, we introduce a kind of reputation-based compassion mechanism in SPGG. We set a part of individuals in the population as being compassionate, and only individuals with relatively low payoffs and not too low reputations may receive sympathy from other individuals who are compassionate in the group. Monte Carlo simulations are used to simulate the interaction between multiple agents and the update process of individual information. We try to figure out if setting individuals with a high payoff or high reputation to be compassionate can better promote cooperation, compared with randomly selecting sympathizers from the population. If the proportion of sympathizers with a high payoff or high reputation is very small, can the level of cooperation be significantly improved in the population and how small can the proportion be? Moreover, based on the strategy of their neighbors, individuals are divided into four types: central cooperator (C), boundary cooperator (C_{b}), central defector (D), and boundary defector (D_{b}). By comparing their fitness before and after compassion, we try to observe the trend and direction of payoff redistribution among individuals, so that we can better understand the effect of payoff redistribution on strategy imitation.

The rest of this paper is arranged as follows. In the second section, we introduce the SPGG model considering the payoff redistribution caused by sympathy. In the third section, the simulation results and related discussions are presented. Finally, the fourth part summarizes the main conclusions and prospects for further research.

2. Model

2.1. SPGG considering conditional compassion based on reputation

In the SPGG, there are N individuals. Each individual is confined on a node of the square lattice network with periodic boundaries and interacts with its four direct neighbors. In each round of the interaction, an individual participates in five groups of games simultaneously. One group is centered on himself and the other four are centered on each of his neighbors. Here, let S_i denote the strategy of individual i. As the traditional PGG defined, individuals who contribute one unit to the public pool are called cooperators ($S_i = 1$), while those who contribute nothing are called defectors ($S_i = 0$). Each one's strategy remains the same in the five group interactions.

After one round of the game, the contributions of each group member are summed. The sum is multiplied by an enhancement factor $r(r > 0)$ and then evenly distributed to each member regardless of its strategy. Thus, the payoff of individual i who is the

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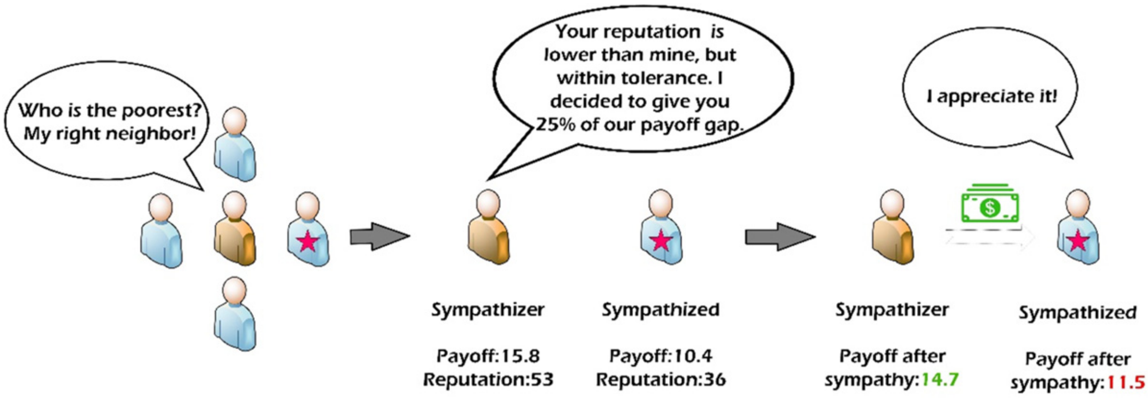

Figure 1. The main flow of reputation-based conditional compassion on the square lattice network. The individual in the yellow shirt is compassionate, and the poorest neighbor selected is marked with a red star. Although the reputation of the sympathized one is lower than that of the sympathizer, it is within his tolerance. Thus, the sympathizer is still willing to give 25% of their payoff gap to the sympathized one being selected.

center one of group G_l (we denote the node-set as G_l) is calculated as

$$P_i^l = \frac{1}{5} r \left(\sum_{x \in G_l} S_x \right) - S_i, \tag{1}$$

where $\sum_{x \in G_l} S_x$ is the sum of all members' contributions in group G_l.

Then, the total payoff of individual i participating in the five rounds of games is

$$P_i = \sum_{l \in G_i} P_i^l. \tag{2}$$

When the calculation of payoff is finished, we introduce a reputation-based conditional compassion mechanism which leads to the redistribution of payoff. Figure 1 shows the main idea of this process. Essentially, the preference of compassion is defined as that some individuals are willing to distribute part of their payoff to the poorest neighbor after judging his reputation.

First, individual i choose the poorest neighbor j in G_l:

$$j = \arg \min_{x \in G_l} P_x. \tag{3}$$

Then, the trigger of compassion needs to be examined. Let $\theta (0 \leqslant \theta \leqslant 1)$ denote the proportion of a certain high-level characteristic of the population so that individual i whose payoff or reputation is at the top θ of the population is believed to have sympathy. Let *Tol* denote the maximum tolerance of the sympathizer to the reputation of the individual being sympathized. We use the constraint condition of the proportion of

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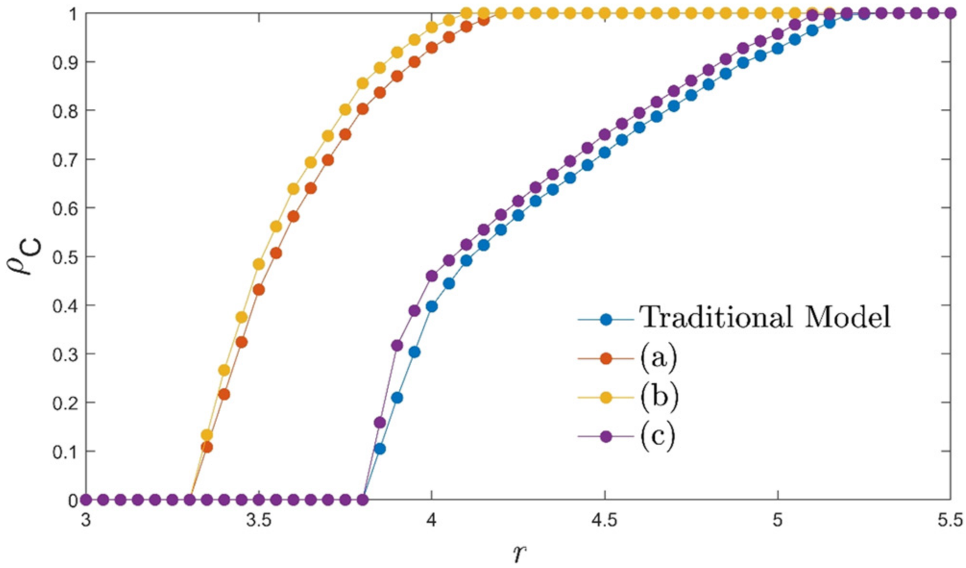

Figure 2. Comparison curves of cooperation density ρ_C with enhancement factor r when selecting individuals in different ways of endowing them compassion. The parameters are fixed as $\theta = 10\%$, and $p = 0.5$. Individuals at the top rank of (a) payoff, (b) reputation, and (c) randomly selected are compassionate, respectively. In the traditional model and (c) model, cooperation emerges at $r = 3.87$. When individuals in the top rank of (a) payoff or (b) reputation are compassionate, cooperation emerges at $r = 3.4$ and achieves full cooperation and maintains stability at $r = 4.1$. This shows that compassion can promote cooperation, and the effect is more obvious when individuals in the top rank of payoff or reputation are compassionate.

payoff redistribution $\tilde{p}$ as

$$\tilde{p} = \begin{cases} p(0 < p \leqslant 0.5) & \text{if } \underset{\max}{Rank}(C_i) \leqslant \theta N \text{ and } R_i - R_j \leqslant Tol \\ 0 & \text{otherwise} \end{cases}, \tag{4}$$

where C_i is the characteristic (here is payoff P_i or reputation R_i) of individual i, while $\underset{\max}{Rank}(C_i)$ represents the order of C_i in the population from large to small. Individuals in the top rank of payoff or reputation are compassionate. Only the reputation difference of the sympathizer and the sympathized one are within the tolerance Tol, the compassion condition can be triggered, and at this time, $\tilde{p} = p \in (0, 0.5]$. If the trigger condition is not met, $\tilde{p} = 0$.

Finally, sympathizer i will give proportion $\tilde{p}$ of his payoff difference to individual j. That is

$$\begin{aligned} F_i &= P_i - \tilde{p} \cdot (P_i - P_j) \\ F_j &= P_j + \tilde{p} \cdot (P_i - P_j), \end{aligned} \tag{5}$$

where F_i and F_j represent the fitness of sympathizer i and the one being sympathized j.

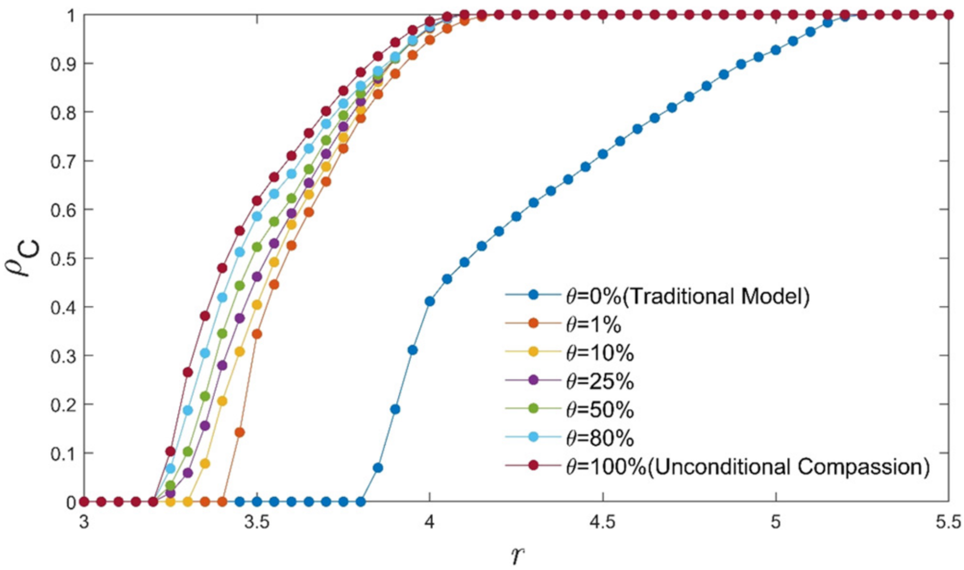

Figure 3. Comparison curves of cooperation density ρ_C with enhancement factor r when we set θ proportion of individuals in the population being compassionate. The proportion of payoff distribution is fixed at $p = 0.5$. Except for the two extreme cases, cooperation emerges at $r = 3.3$ to $r = 3.4$. Analyzing the results in the figure, we can conclude that when individuals with high payoff have compassion, even if the proportion of them is only 1%, the level of cooperation can be significantly improved.

2.2. Evolutionary rules of strategy and reputation

We adopt the strategy of synchronous updating in this study, that is, everyone's strategy is updated at the same time based on the fitness of the previous step. The update process consists of many basic Monte Carlo steps (MCSs). In each MCS, an individual x and one of his neighbor y is chosen randomly in succession. The probability of individual x imitating the strategy of y is defined by the Fermi function

$$W(S_x \to S_y) = \frac{1}{1 + \exp[(F_x - F_y)/\kappa]}, \tag{6}$$

where F_x and F_y is the fitness of individual x and y after redistribution, respectively. κ represents noise intensity which means indeterminacy during the process of imitation. When $\kappa \to 0$, individual x adopts the strategy of y so long as F_x is larger than F_y. On the contrary, when $\kappa \to \infty$, there is a half chance that individual x takes the strategy of y without considering their fitness. Consistent with previous research, we set $\kappa = 0.5$ in this study.

Individuals' reputation also evolves with their strategy. We use the simplest rule, namely, cooperation plus one and defection minus one to update their reputation. Thus, the reputation value of individual i $(0 \leqslant R_i \leqslant 100)$ at time t is

$$R_i(t) = R_i(t-1) - (-1)^{S_i(t)}, \tag{7}$$

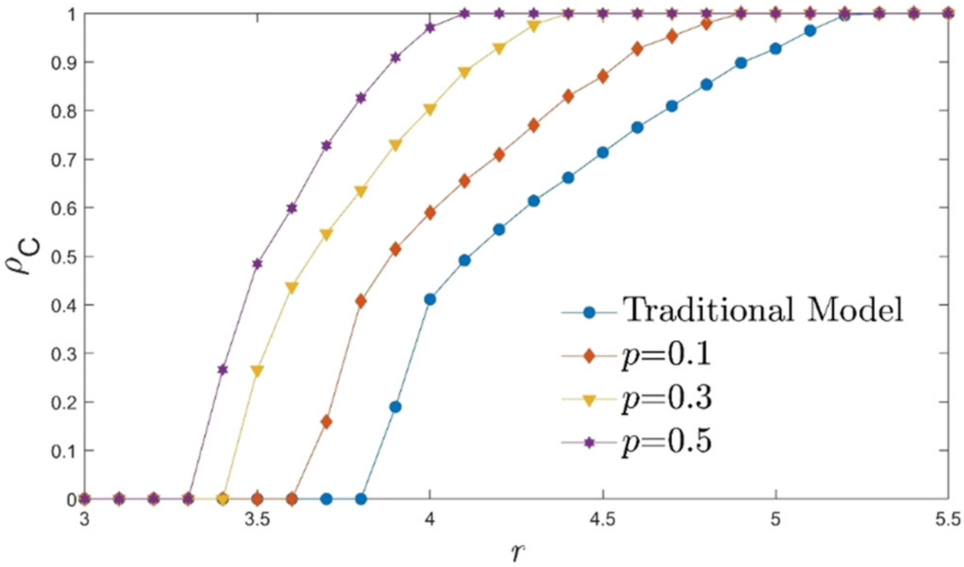

Figure 4. Comparison curves of cooperation density ρ_C with enhancement factor r when the proportion of payoff redistribution p is 0, 0.1, 0.3, and 0.5, respectively. We fix $\theta = 10\%$. Compared with $p = 0$, when $0 < p \leqslant 0.5$, a higher proportion of payoff redistribution can promote the emergence of cooperation, which indicates that the introduction of the compassion mechanism is conducive to cooperation. Furthermore, as the proportion of payoff redistribution p expands, cooperation can emerge at a smaller enhancement factor and can be maintained over a wider range of parameters.

where $R_i(t-1)$ is the reputation of individual i at time $t-1$ and $S_i(t)$ is the strategy of individual i at time t. Importantly, during the update process, the reputation value cannot continue to increase after reaching the upper boundary, similarly but opposite at the lower bound.

3. Results and discussion

Monte Carlo simulations are performed on a 100 × 100 square lattice network with periodic boundaries, hence $N = 10\,000$. The simulation begins with an initialization that every individual is randomly assigned a reputation and strategy. We focus on the cooperation rate of the population ρ_C which is defined as

$$\rho_C = \frac{1}{N}\sum_{i=1}^{N} S_i. \tag{8}$$

The proportion of cooperators can be stabilized by 5×10^4 MCSs for most parameters. In the following, the value of ρ_C is the average result of the last 1000 steps in an at least 5×10^4 steps simulation. In order to eliminate the influence of accidental errors in

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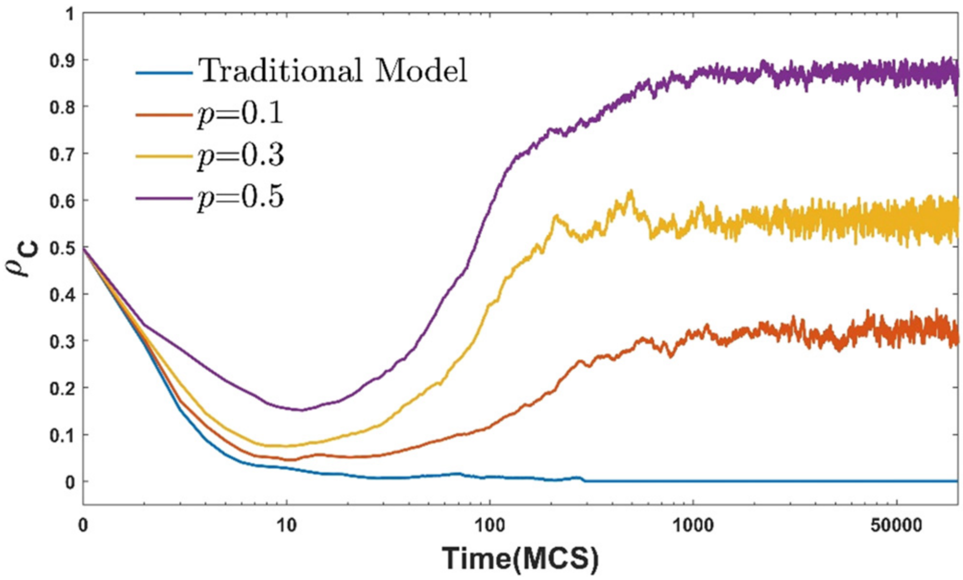

Figure 5. Evolutionary processes of the cooperation density when the proportion of payoff redistribution p is 0, 0.1, 0.3, and 0.5, respectively. Here $r = 3.8$. When $p = 0$, the proportion of cooperators reduces to zero finally and defectors occupy the entire population, whereas when $p > 0$, the trend of cooperation density first decreases and then increases, indicating that the cooperator clusters are expanding and can resist the invasion of defectors. Moreover, the larger the proportion of payoff redistribution p, the higher the proportion of cooperators when the system reaches a stable state.

experiments, all the following results were obtained with an average of 20 independent realizations. In order to verify the validity in the large system-size limit, we have also performed simulation tests on a larger-size system of 500 × 500 based on procedures mentioned in [75]. Under each set of parameter combinations, we find that there is no essential difference between 100 × 100 and 500 × 500 size systems on presenting our results and conclusions. Eliminating the effect of accidental errors in experiments, they have almost the same average cooperation level and almost identical distribution of strategies when the system reaches stability.

The maximum tolerance of the sympathizer to the reputation of the sympathized is fixed as $Tol = 20$ in the simulation. First, a comparative experiment is designed to compare the different effects between individuals' conditional compassion and random compassion on the evolution of cooperation. The control group is the typical model and we set up three experimental groups. We set 10% of the individuals with the highest payoff, with the highest reputation, or randomly selected, respectively, to endow them with sympathy in the three independent experiments. Figure 2 shows the comparison curves of cooperation density ρ_C varying with r in the three experiments. As shown in the figure, cooperation emerges at $r = 3.87$ in the typical model. When compassionate individuals are randomly selected, cooperation emerges at $r = 3.8$, and the cooperation rate of each ρ_C value is slightly higher than that of the typical model. However, when high-payoff or high-reputation individuals are compassionate, cooperation emerges at

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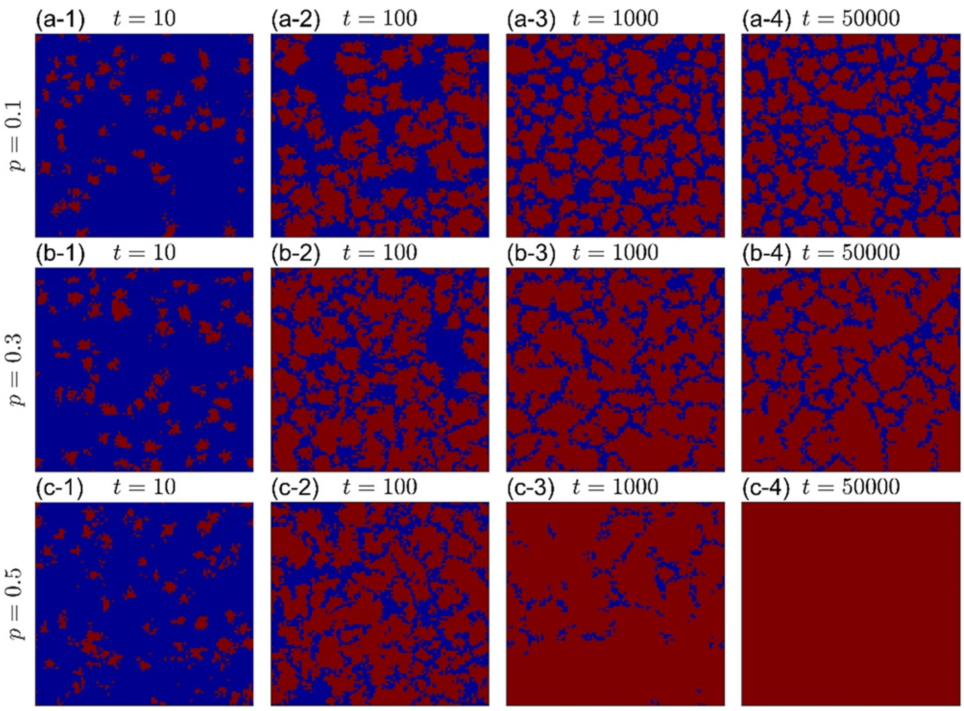

Figure 6. Snapshots of individuals' strategy distribution over time when (a) $p = 0.1$, (b) $p = 0.3$ and (c) $p = 0.5$, respectively. Here $r = 4.1$. Red parts represent cooperators while blue parts represent defectors. Looking horizontally, as time goes on, cooperators connect tightly to resist the invasion of defectors. From the vertical perspective, the red regions are expanding with the increase of the proportion of payoff redistribution for the same enhancement factor.

$r = 3.4$ and full cooperation is achieved and stability is maintained at $r = 4.1$. The results demonstrate that compared with compassionate individuals randomly selected, individuals in the top rank of payoff or reputation being compassionate is more likely to promote cooperation.

After that, the effect of the proportion of high-payoff sympathizers on cooperation is studied. Figure 3 shows the corresponding curves of cooperation density ρ_C varying with r for different values of θ. As shown in the figure, when all individuals are compassionate ($\theta = 100\%$), cooperation emerges at $r = 3.2$. Amazingly, we find that except for the traditional model and the unconditional model, the other five curves are of little difference. Obviously but slightly, there is a positive correlation between cooperation rate ρ_C and θ. When $\theta = 1\%$, the cooperation rate $\rho_C = 0.9679$ is little different from unconditional sympathy ($\theta = 100\%$, $\rho_C = 0.9859$). Therefore, we can conclude that when individuals with high payoff have compassion, even if the proportion of them is only 1% ($\theta = 0.01$), the level of cooperation can be significantly improved.

The following data and results are obtained by fixedly setting 10% of individuals with the highest payoff to be compassionate ($\theta = 10\%$). To study the effect of the proportion of payoff redistribution caused by sympathy on the evolution of cooperation, figure 4 illustrates the comparison curves of cooperation density ρ_C with enhancement factor r for four different values of p. As the figure shows, the proportion of payoff redistribution

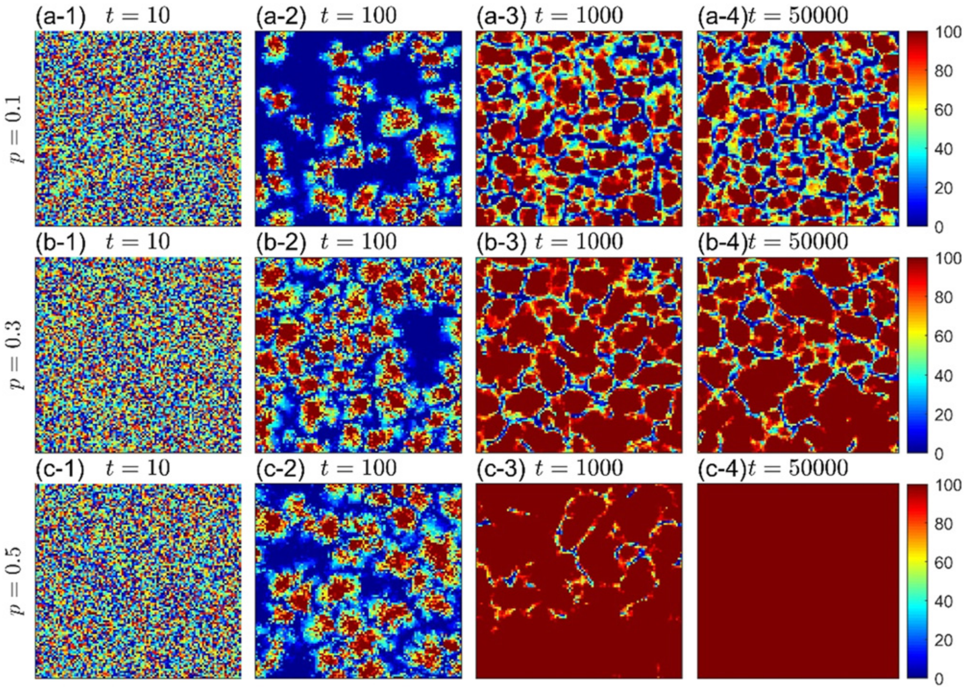

Figure 7. Snapshots of individuals' reputation distribution over time when (a) $p = 0.1$, (b) $p = 0.3$, (c) $p = 0.5$ at various time steps. Here $r = 4.1$. Different colors represent different reputation values defined by the color bar. It shows that with the evolution of cooperation, the proportion of people with high reputations has increased and stabilized, which is similar to the changing trend of cooperation rate.

has a positive influence on the evolution of cooperation. Especially, when $p = 0$, there is no change in the payoff gap between the sympathizer and the sympathized ones even if the sympathy mechanism is successfully triggered, at which the model degenerates into the basic vision. Conversely, when $p = 0.5$, the payoff gap between the sympathizers and the sympathized ones is minimized to the most extent. In this situation, we can notice that the cooperation level has been greatly promoted and cooperation emerges when $r > 3.3$. There is a significantly positive correlation between cooperation density ρ_C and the proportion of payoff redistribution p.

To compare the dynamics process and stabilization time of the system under different enhancement factors, figure 5 illustrates the evolutionary processes of cooperation density for four representative values of p and fixed $r = 3.8$. As shown by the four curves in figure 5, when $p = 0$, the proportion of cooperators reduces to zero and defectors occupy the entire population finally; whereas when $p > 0$, the trend of cooperation density first decreases and then increases, indicating that the cooperator clusters are expanding and can resist the invasion of defectors. Although the trend of the four curves is the same, when $p > 0$, the cooperation rate they could induce the population to achieve is different, which is also observed in figure 4.

To observe the evolution of strategies and reputations on the spatial network, figures 6 and 7 show the snapshots of the distribution of individuals' strategy and

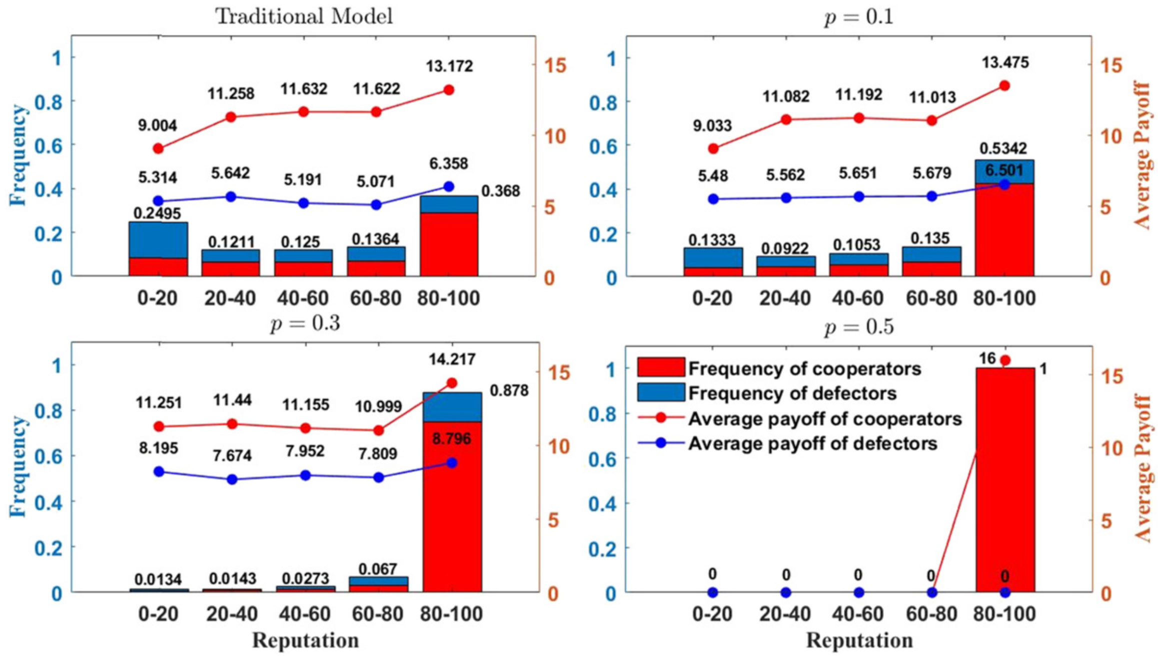

Figure 8. The proportion and the average payoff of cooperators and defectors in different reputation intervals for four different p and fixed $r = 4.2$. Comparing the four subgraphs, we can see that much bigger values of the proportion of payoff redistribution can promote the emergence of cooperation which is consistent with the above conclusions. In each subgraph, cooperators in high reputation intervals account for most of the proportion, and the average payoff of the cooperators is higher than that of the defectors in each reputation range, which makes them more likely to be compassionate in our mechanism setting.

reputation over time for three different p and fixed $r = 4.2$. Comparing the results corresponding to $p = 0.1$, $p = 0.3$ and $p = 0.5$, as shown in figures 4(a)–(c), we can notice that the proportion of red regions expands with the increase of p which implies the level of cooperation is increasing. The conclusions in figure 5 also illustrate this observation. Moreover, when $t = 10$, as shown in figures 6(a-1)–(c-1), red regions are scattered in the blue regions piecemeal, which implies that cooperators in the population are few. As time increases from 10 to 1000, the proportion of red regions expands and connects gradually. When $t = 20\,000$, the system has reached a stable state, and the clusters formed by cooperators can stably resist the invasion of defectors. Based on the above phenomenon, we believe that the reduction of the payoff gap among individuals caused by sympathy can promote group cooperation rapidly and effectively. Similarly, figure 7 shows the distribution of individuals' reputations over time. By comparing with figure 6, we can easily find that there is a highly positive correlation between the distribution of individuals' reputations and strategies.

To study the relationship between reputation, strategy, and payoff, figure 8 shows the proportion and the average payoff of cooperators and defectors in different reputation intervals for four different values of p and fixed $r = 4.2$. Obviously, in each subgraph, we can see that most of the high reputation groups are cooperators, while most of the

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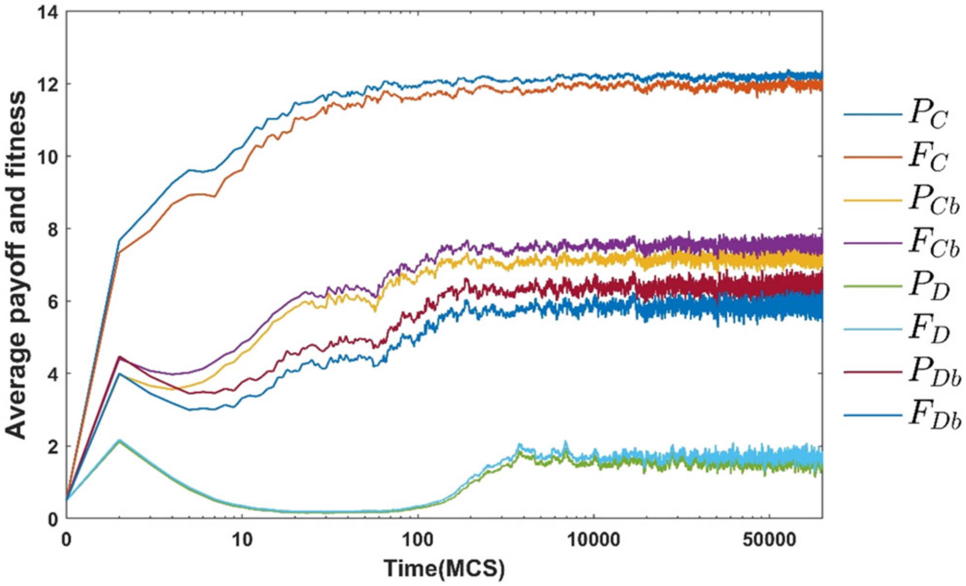

Figure 9. Time series of the four types of individual's average payoff (P) and fitness (F). By comparing P and F for central cooperators (C), boundary cooperators (C_b), central defectors (D), and boundary defectors (D_b), we can observe the payoff changes for different types of individuals by reputation-based conditional compassion. After the payoff redistribution, the average payoffs of C_b and D increase.

low reputation groups are defectors. Whether in general or in each reputation interval, the average payoff of the cooperator group is significantly higher than that of the defector group. This makes us convinced that cooperators constitute the main part of sympathizers. The payoff redistribution led by cooperators can have a great influence on strategy imitation.

To further explore the micromechanism of sympathy on cooperation, we observe the payoff redistribution which mainly occurs at the boundary of the cooperator clusters. According to the strategy types of an individual's four neighbors, we divide each individual into four types: central cooperators (C), boundary cooperators (C_b), central defectors (D), and boundary defectors (D_b). Here, a boundary cooperator (defector) is a cooperator (defector) with at least one defection (cooperation) neighbor. Accordingly, a central cooperator (defector) is a cooperator (defector) with four cooperation (defection) neighbors. Figure 9 shows the evolution of average payoff (P) and average fitness payoff (F) for four types of individuals when $p = 0.5$, $r = 3.6$, at which the density of cooperation after stabilization is $\rho_C = 0.5988$. In the initial state, the average payoff of the four types of individuals is the same. During the evolution, stable cooperative clusters can resist the invasion of defectors which will result in the average payoff ranking $P_C > P_{C_\mathrm{b}} > P_{D_\mathrm{b}} > P_D$ after stabilization. The average fitness ranking of the four types is the same, that is $F_C > F_{C_\mathrm{b}} > F_{D_\mathrm{b}} > F_D$, but the relationship between average payoff and average fitness depends on their types. As shown in figure 9, because of the sympathy, the payoffs of C_b and D after redistribution will increase while the payoff of D_b and C will decrease after distribution. Therefore, we can speculate that D_b and C

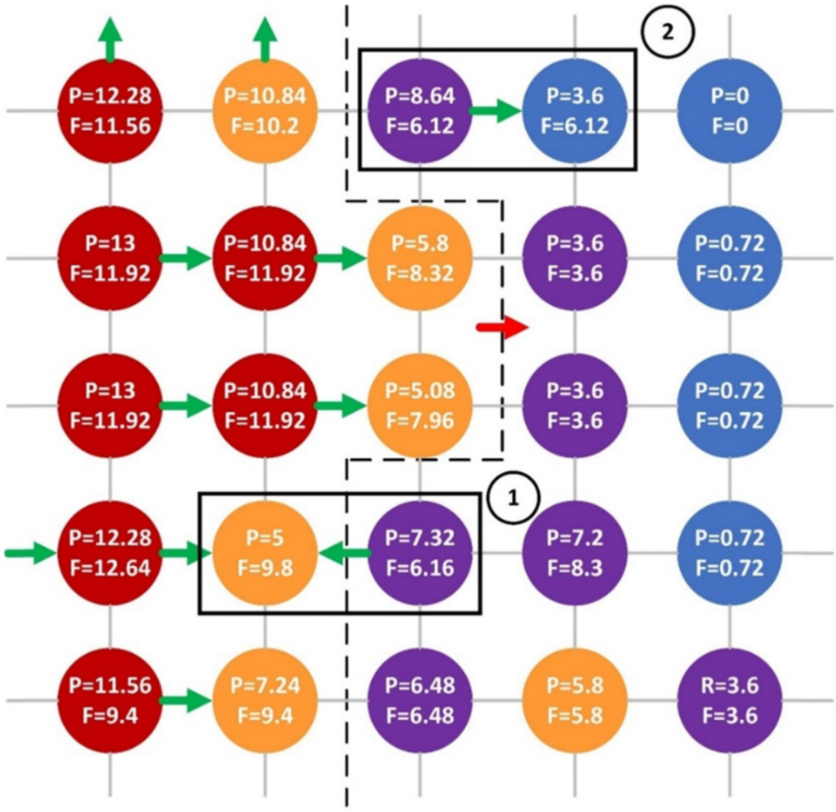

Figure 10. A toy model of a 5×5 part selected from the 100×100 square lattice network. When $r = 3.6$, after 150 MCSs, the cooperation rate is 0.1993. On the left side of the dotted line, there is a part of the boundary of a cooperator cluster. The red circle represents C and the orange circle represents C_b. On the right side, there are individuals around the cooperative cluster. The blue circle represents D and the purple circle represents D_b. The green arrows indicate the payoff transfer from the sympathizer to the sympathized one. The red arrow indicates the expanding trend of the cooperative cluster.

distribute part of their payoffs to C_b, which leads to the result of $F_{C_\mathrm{b}} > P_{C_\mathrm{b}}$, $F_{D_\mathrm{b}} < P_{D_\mathrm{b}}$ and $F_C < P_C$.

To confirm the above conjecture, we select a representative 5×5 part from the 100×100 square lattice network and depict the payoff and fitness distribution when the system evolves to time = 150, at which $\rho_C = 0.1993$. Figure 10 illustrates the process of payoff redistribution. Intuitively, we find that sympathy diffuses from the center of the cooperator cluster, and a large number of C_b are sympathized by C, while a small number of C_b and D are sympathized by D_b (see box ① and box ② in figure 8). The former phenomenon is because C has obtained high payoffs after cooperating with other cooperators; the latter is because D_b free rides cooperators in the games and obtains high payoffs without paying for the investment. Thus, sympathizers are mainly C and D_b who distribute part of their payoffs to C_b and D. We can explain the promotion of cooperation from the perspective of strategy imitation caused by payoff redistribution. In equation (6), assuming individual x is a defector and individual y is a cooperator,

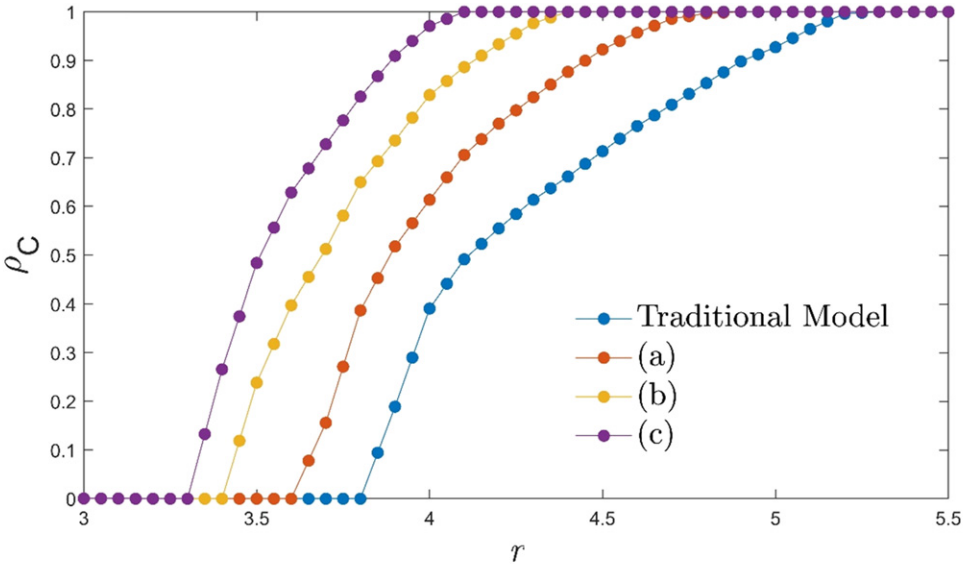

Figure 11. Comparison curves of cooperation density ρ_C with enhancement factor r when the probability of compassion is (a) 40%, (b) 70%, and (c) 100%, compared with the traditional model. We set the top 10% individuals with the highest payoff have compassion ($\theta = 10\%$) and the proportion of payoff redistribution is 0.5 ($p = 0.5$). Evidently, higher values of probability in compassion promote the emergence of cooperation. As the probability of compassion increases, cooperators emerge at a lower enhancement factor and maintain over a wider range of parameters.

through payoff redistribution, the probability of individual x changing his strategy from defection to cooperation increases, which can promote the expansion of cooperative clusters. By analyzing the results of figures 9 and 10 together, we can better understand how payoff redistribution caused by sympathy affects the strategy imitation among different individuals in the square lattice network.

Several works have shown that people are more sensitive to unfavorable inequality (i.e. behindness aversion) than favorable inequality [76, 77]. This means that those high-payoff individuals do not always have compassion for those who are both high-reputation and low-payoff in reality. Some errors and mistakes may also lead to deviation in their decision-making. Thus, we further discuss the situation of probabilistic help in the following. We set the case where the top 10% individuals with the highest payoffs have compassion ($\theta = 10\%$) and the proportion of payoff redistribution is 0.5 ($p = 0.5$). The probability that they sympathize individuals with low payoffs but good reputations is 40%, 70%, and 100%. The comparison curves of cooperation density are shown in figure 11. Evidently, when high-payoff individuals sympathize low-payoff and high-reputation individuals with a high probability, reputation-based compassion mechanism has a more obvious effect in promoting cooperation.

4. Conclusion

To sum up, a kind of reputation-based compassion mechanism is introduced in the SPGG model and the effect of model settings and parameters on the evolution of cooperation is explored in this study. Specifically, we first design a comparative experiment to compare the effects of high payoff, high reputation, and randomly selected individuals being as sympathizers on cooperation. Then, we study the influence of the proportion of sympathizers on the cooperation level. To explain the micro mechanism of compassion on payoff redistribution, we divide individuals into central cooperators (C), boundary cooperators (C_{b}), central defectors (D), and boundary defectors (D_{b}) based on the strategy of themselves and their neighbors. By analyzing the correlation between reputation, payoff, and strategy, we explore the composition of individual types being as sympathizers and the direction of payoff redistribution between different individual types. The simulation results on the two-dimensional square lattice network demonstrate that compared with randomly selecting individuals who are compassionate, individuals in the top rank of payoff or reputation being compassionate is more likely to promote cooperation. In this case, even if only one percent of individuals in the population are compassionate, the level of cooperation can be significantly improved. Moreover, C-type and D_{b}-type individuals have high payoff and reputation, thus being the main part of sympathizers. They distribute a part of their payoff to C_{b}-type individuals. The reduction of the payoff gap can affect strategy imitation. We observe that the payoff gap reduction caused by the compassion mechanism mainly occurs at the boundary of the cooperative cluster, which leads to the further expansion of cooperator clusters. These results expand our understanding of the social preference of compassion in promoting cooperation in the structured population.

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