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How to evaluate one's behavior toward 'bad' individuals? Exploring good social norms in promoting cooperation in spatial public goods games

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Abstract. Cooperation in modern society is based on mutual trust and reputation information can well reflect people's social image. Good social norms encourage people to cooperate with high-reputation (or called 'good') individuals and severely punish those who defect 'good' individuals. However, the existing research lacks a unified criterion of how to evaluate the behavior when interacting with low-reputation (or called 'bad') people. Based on public goods games, we focus on exploring second-order social norms that can improve the evolution of cooperation in this study. Parameters to control the reward and punishment intensity through increasing or decreasing one's reputation when interacting with 'bad' individuals are introduced. Simulation results show that this reward and punishment mechanism through reputation can effectively promote the evolution of cooperation in both well-mixed and latticed populations. Under certain high-reputation threshold environments, a well-mixed population structure can even promote cooperation more significantly than a lattice network. However, increasing reward intensity for cooperating with 'bad' individuals cannot further improve cooperation, but in a high-reputation threshold environment, increasing punishment intensity for defecting 'bad' individuals can further improve cooperation. This research extends the use of statistical physics to study the evolution of cooperation from the perspective of reputation-based dynamics.

Keywords: agent-based models, evolutionary game theory

Contents

1. Introduction

Cooperation maximizes social welfare, but people need to lose their own interests for cooperative behavior. Darwin's natural selection theory favors the fittest and the most successful individuals, which implies the selection of the innate selfishness that greatly challenges the emergence of cooperation [1–3]. There is tension between what is good for single individuals and what is good for the population, which leads to the occurrence of social dilemmas [4]. Understanding cooperative behavior in human societies has been declared as one of the grand scientific challenges of the 21st century. Evolutionary game theory provides an effective framework to explore the origin of cooperation [5, 6]. Public goods game (PGG) is a typical theoretical model to study cooperation issues within group-interaction. In the PGG, cooperators contribute to the public goods whereas defectors do not. All contributions are summed up and multiplied by an enhancement factor and then equally divided among all players irrespective of their strategies. Thus, defectors bear no costs when collecting identical benefits as cooperators, which ultimately results in the 'tragedy of commons' [7].

Previous studies have identified five categories of mechanisms to promote the evolution of cooperation [8], among which network reciprocity is a relatively new development

[9–14]. Network reciprocity assumes individuals interacting in a structured population, in which the PGG is also known as the spatial PGG (SPGG). Statistical physics provides a powerful tool in this direction [15, 16]. Along this line of research, many micro-mechanisms for promoting cooperation have been proposed, such as introducing reward and punishment [17–27], social exclusion [28–34], considering individual or group heterogeneity [35–39], intelligent learning [40–43], and so on. Essentially, these mechanisms change the payoff structure of strategies or the rules of strategy updating.

In recent years, with the development of internet technology, the frequency of interaction between strangers has gradually increased. In this context, reputation information that can reflect people's history behavior becomes extremely important. In fact, the role of reputation in promoting cooperation has been explored in some research [44–51]. It is generally assumed that one's reputation can affect the strategy choice of his partners and thus the benefits he can obtain in the future. For instance, Nowak and Sigmund [44] innovatively linked personal image scores with strategy selection in the games, proving that individual image had a great influence on the evolution of cooperation. Milinski *et al* [45] further proposed that reputation can solve the tragedy of commons. In the spatial dilemma games, there are also studies assuming that reputation affects the strategy update rule [52–57] or link weight between partners [58–61]. Concerning the reputation evaluation, first-order information that only considers the focused individual's strategy is widely adopted [62–66]. Recently, the second-order reputation evaluation norms are introduced in the PGG, in which both one's behavior and the reputation of his opponents are considered [67–69]. Specifically, individuals are divided into two categories of high reputation and low reputation (which are called 'good' and 'bad' individuals), based on which different social norms can be constructed.

The existing social norms clarify the evaluation criteria for interacting with 'good' individuals. That is, it is reasonable to reward those who cooperate with high-reputation people and severely punish those who defect 'good' individuals. However, there still lacks a unified criterion of how to evaluate the behavior when interacting with 'bad' people. Such as whether cooperation with 'bad' individuals should be rewarded and defecting them should be punished. Moreover, if so, what punishment and reward intensity is appropriate that can improve cooperation? We target to answer these questions in this study. Specifically, based on the SPGG model, second-order reputation evaluation norms are introduced in this paper. We focus on exploring good social norms that can improve the evolution of cooperation. We introduce two parameters to adjust the reward and punishment intensity through increasing of decreasing their reputation when they act toward 'bad' individuals. Thus, the effect of different evaluation criteria or social norms in promoting cooperation can be compared. Through simulation, it can be concluded that appropriate reward and punishment toward the behaviors of cooperation and defection with 'bad' individuals are conducive to the emergence of cooperation, but further increasing the reward intensity has no significant effect on cooperation. The effect of further increasing the punishment intensity for defecting 'bad' individuals depends on the reputation threshold. Under high reputation thresholds, the cooperation level can be significantly enhanced; while under low reputation thresholds, it can not promote cooperation any further.

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The remainder of the article is arranged as follows. The evolutionary SPGG model based on the second-order reputation is introduced in detail in section 2, including the reward and punishment mechanism through reputation toward the interaction behavior with 'bad' individuals. Then, the specific process and details of the simulation, as well as the results and discussion of the experiments are presented in section 3. Finally, we summarize the full context in section 4.

2. Model

2.1. Spatial PGG

In this study, the SPGG is conducted on a regular lattice network with periodic boundaries. The size of the lattice is $L \times L$. Each individual is placed at the intersection of the lattice with $G-1$ direct neighbors, which forms a PGG group of size G centered on the focal individual. Thus, each player simultaneously participates in G groups of PGGs which are centered on himself and his $G-1$ neighbors, respectively.

Inside every PGG group, each cooperator contributes a fixed amount of investment $c=1$ to the public pool, and defectors contribute nothing. Then the cumulative contributions of all players are multiplied by an enhancement factor r $(r>1)$. The total amount is allocated equally among all individuals in the group. After a round of PGG, the payoff of player i with strategy s_i (as a cooperator or a defector) in group g $(g=1,2,\ldots,G)$ is

$$\pi_{s_i}^{\mathrm{g}} = \begin{cases} \dfrac{r(N_C^{\mathrm{g}}+1)}{G} - 1 & \text{if } s_i = C \\ \dfrac{rN_C^{\mathrm{g}}}{G} & \text{if } s_i = D \end{cases}, \tag{1}$$

where N_C^{g} is the number of cooperators in group g excluding player i. After participating in G rounds of games, the total payoff of player i is

$$\Pi_i = \sum_{g=1}^{G} \pi_{s_i}^{\mathrm{g}}. \tag{2}$$

2.2. Second-order reputation evaluation

Second-order reputation evaluation norms are adopted in this study. Let $R_i(t)$ denote the reputation of individual i at time t. For individual i, his reputation at time $t+1$ is updated based on his behavior at time $t+1$ and the reputation of his neighbors at time t. At each time, player i needs to participate in G rounds of PGG, and the reputation of player i's neighbors is defined as the average reputation of all individuals that player i is interacting. First, we calculate the average reputation R_i^g of i's neighbors in group

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Table 1. Second-order reputation update rules.

	Reputation of i's neighbors	
Strategy of i	G	B
C	$+1$	$+P_1$
D	-5	$-P_2$

g. Then the average reputations of i's neighbors from all groups are averaged as follows.

$$R_{i'\mathrm{nebr}} = \frac{1}{G}\sum_{g=1}^{G} R_i^g \tag{3}$$

Moreover, we introduce a reputation threshold Z. When one's reputation $R \geqslant Z$, it is regarded as a 'good' individual (denoted as G), whereas when $R < Z$, it is regarded as a 'bad' individual (denoted as B). The rules for player i updating his reputation based on the two aspects of information are shown in table 1. Inside the table, C/D in the column indicates the strategy of the focal player i, and G/B in the row means the average reputation type of player i's neighbors.

The evaluation rules we set up are based on the common values generally accepted by different societies. That is, 'cooperation when interacting with good neighbors' is what we should do and should be appropriately rewarded (+1), and 'defection when interacting with good neighbors' is unethical which should be severely punished (−5). In order to investigate how to evaluate the cooperation and defection behavior toward 'bad' individuals, we introduce two variables, P_1 and P_2 in the table. P_1 is an integer from 0 to 5, which represents the degree of reward for cooperating with 'bad' individuals (denoted as C/B). P_2 is an integer from 0 to 3, which represents the degree of punishment for defecting with 'bad' individuals (denoted as D/B). Each combination of P_1 and P_2 corresponds to a different social norm. By adjusting the two variables, we can control the intensity of reward and punishment for these two behaviors when interacting with 'bad' individuals. We set boundary conditions of 0 and 100 for the reputation. If a player's reputation exceeds 100, his reputation will remain at 100. Similarly, if the player's reputation is below 0, his reputation will remain at 0.

2.3. The strategy update rule

For each time iteration, one individual i is randomly selected from the population. Then, individual i randomly selects a neighbor j from his direct neighbors. As the Fermi function indicates, individual i imitates the strategy of his neighbor j with the following probability.

$$p(s_j \rightarrow s_i) = \omega_j \frac{1}{1 + \exp[(\Pi_i - \Pi_j)/K]} \tag{4}$$

Among them, Π_i and Π_j represent the cumulative payoff of individual i and j, respectively; ω_j is a strategy imitation coefficient based on the reputation of individual j, such

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that when $R_j \geqslant Z$, then $\omega_j = 1$; and *when* $R_j < Z$, then $\omega_j = \omega$ ($\omega < 1$). This setting can ensure that the high-reputation individual's strategy is more likely to be imitated. According to the previous research [52, 68], ω is set to 0.001 as a default value in this paper. Z is the reputation threshold. K denotes noise intensity quantifying uncertainty in the strategy imitation. $K \to \infty$ corresponds to a completely random strategy adoption. On the contrary, $K \to 0$ leads to a deterministic strategy choice of the higher payoff. In consistence with the previous research [6], K is set to 0.5 in this study. Each time step consists of $L \times L$ iterations to ensure that every individual has a chance on average to update his strategy.

3. Results and discussion

Monte Carlo method is used to simulate the evolutionary process of strategies in the population. The simulations are carried out on a lattice with size $L = 100$. We focus on the frequency of cooperators ρ_c in the population when the system reaches stable under different social norms. Each time step in the simulation consists of some basic Monte Carlo steps (MCS). Initially, strategies C and D are randomly distributed on each node with equal probability, and each individual's reputation is randomly assigned in the interval [0,100]. For each MCS, $L \times L$ iterations are performed to ensure that all individuals on the network have an opportunity on average to update their strategy and reputation. For most parameter combinations, the system can be stabilized in 10^4 MCS. At some critical points, we prolong the process to 5×10^4 MCS. Moreover, all results of ρ_c are the average of the final 100 MCS when the system reaches an evolutionarily stable state.

First of all, we investigate whether the reward for the C/B and the punishment for the D/B can promote the evolution of cooperation. Specifically, should we reward cooperation behavior (by improving its reputation) and punish defection behavior (by reducing its reputation) when interacting with 'bad' individuals? To answer this question, we set up four corresponding social norms in figure 1 to compare their effect on promoting cooperation. Panel (a) represents no reward for C/B and no punishment for D/B when interacting with 'bad' individuals, whereas panel (d) represents the case of reward for C/B and punishment for D/B. In each panel, we plot the relationship between the frequency of cooperators ρ_c and the multiplication factor r at the stationary state for different values of Z. The results of the traditional model with no reputation evaluation mechanism are also provided in each panel.

By comparing the results as shown in the four panels, it is evident that the social norm we set in panel (d) can improve cooperation the most significantly, in which there is a reward for C/B and a punishment for D/B. Moreover, in this setting, when Z increases, the critical value of r for phase transition decreases, indicating that the more stringent for the high reputation criteria, the more conducive to the evolution of cooperation. Panel (b) brings no reward for C/B behavior, but a punishment for D/B behavior. Obviously, this setting hinders the emergence of cooperation. For most parameter combinations, the level of cooperation is even lower than that of the traditional model. Panel (a) shows

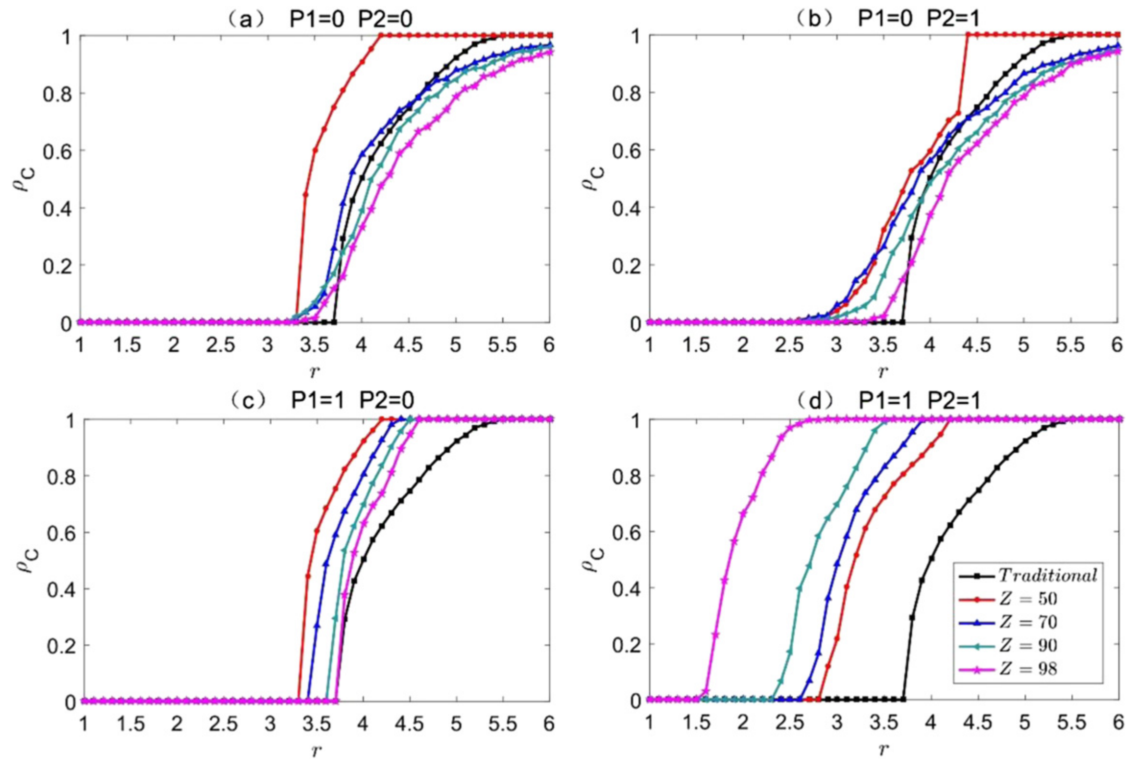

Figure 1. The relationship between the frequency of cooperators ρ_c at the stationary state and the enhancement factor r under different reputation thresholds. P_1 represents the intensity of reward toward C/B behavior and P_2 is the intensity of punishment toward D/B behavior. (a) No reward for C/B, no punishment for D/B; (b) no reward for C/B, punishment for D/B; (c) reward for C/B, no punishment for D/B; (d) reward for C/B, punishment for D/B.

the same characteristics as panel (b). In panel (c), there is a reward for C/B behavior but not a punishment for D/B behavior. When r is fixed, the rise of the threshold Z hinders the emergence of cooperation. This is because as the threshold increases, the proportion of 'bad' individuals in the group increases. Not to punish the behavior of D/B will increase the benefits of punishment and lead to more individuals choosing defection. In general, the implementation of rewards for C/B and punishment for D/B can effectively enhance the evolution of cooperation. In this setting, the higher values of threshold in reputation evaluation provide a better environment for the survival of cooperators.

The evolutionary processes of cooperation with time under different reputation thresholds are compared in figure 2(a), in which the enhancement factor r is fixed at 3.1 and other parameter settings are the same as in figure 1(d). The effect of reward and punishment through reputation on the evolutionary process when interacting with 'bad' individuals under different reputation criteria can be observed. Evidently, there are more 'bad' individuals in the initial stage when the reputation threshold is high ($Z = 90$, 98). But with the reward and punishment playing a role, the cooperation frequency continues to rise until it is stable. These results are consistent with those in

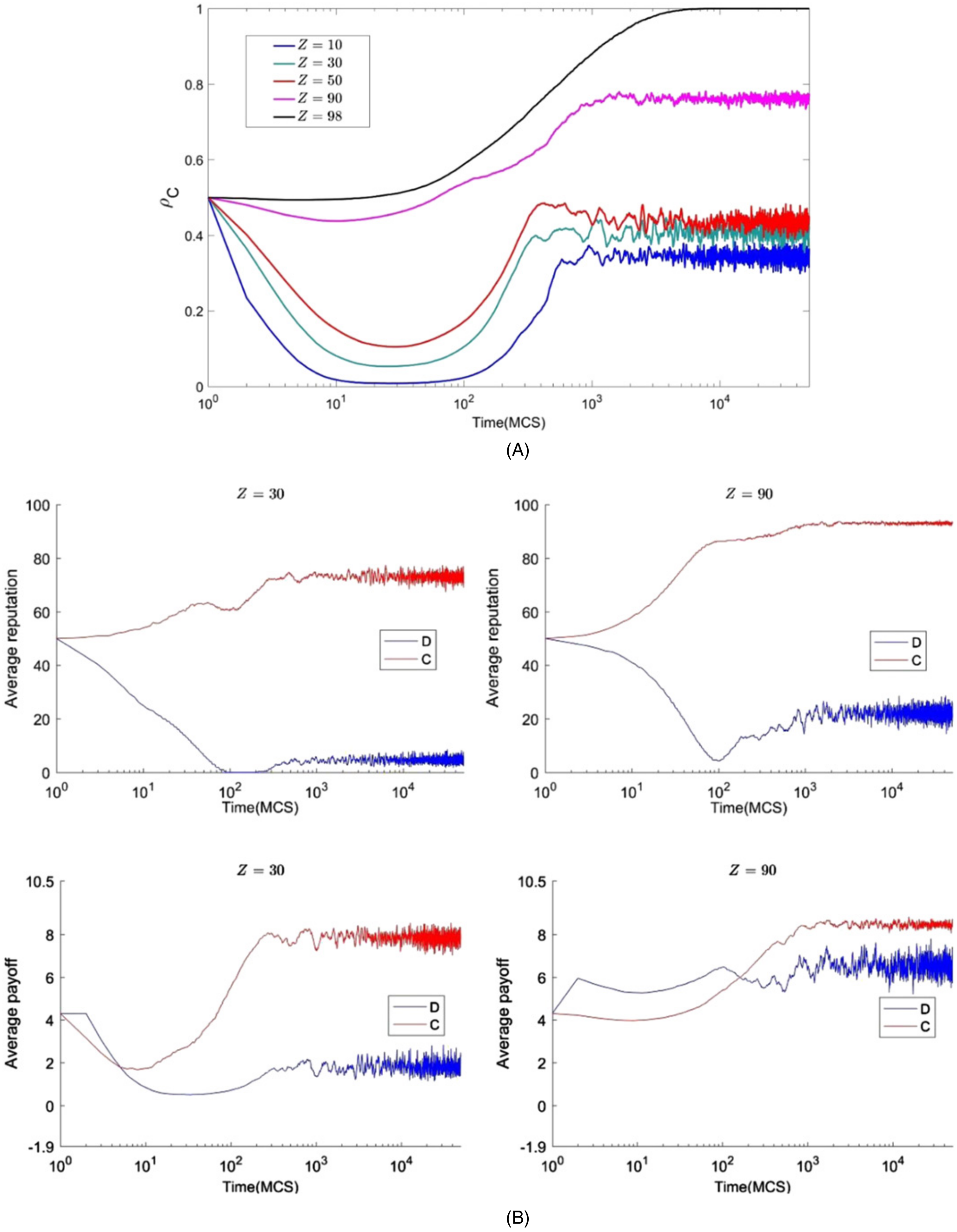

Figure 2. (a) Frequency of cooperators ρ_c varies with time for different reputation thresholds Z and fixed enhancement factor $r = 3.1$. For $Z = 10$, 30 and 50, the frequency of cooperators firstly falls and then rises gradually toward a dynamic stable and non-zero level, whereas $Z = 90$ and 98, cooperation frequency keeps rising and finally stabilizes at a high value. which implies that high-reputation thresholds are more conducive to the emergence of cooperation. (b) The evolutionary process of the average reputation and payoff per strategy for fixed $r = 3.1$. The results under two different values of reputation thresholds Z are compared. Other parameter settings are the same as in (a).

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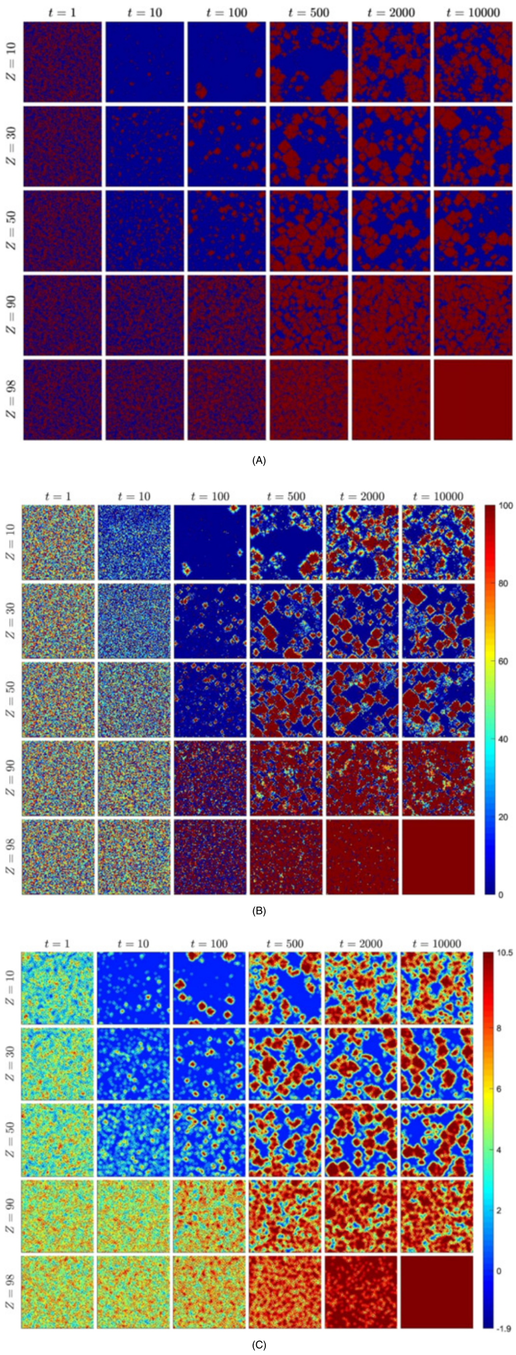

Figure 3. (a) Snapshots of individuals’ strategy distribution at different time steps for different z and fixed $r = 3.1$. From top to bottom, each row represents a situation corresponding to a different Z. From left to right, each column represents a different time step t. Among them, dark red stands for cooperators; dark blue stands for defectors. (b) Snapshots of individuals’ reputation distribution at different time steps. All parameter settings are the same as in (a). (c) Snapshots of individuals’ payoff distribution at different time steps. All parameter settings are the same as in (a).

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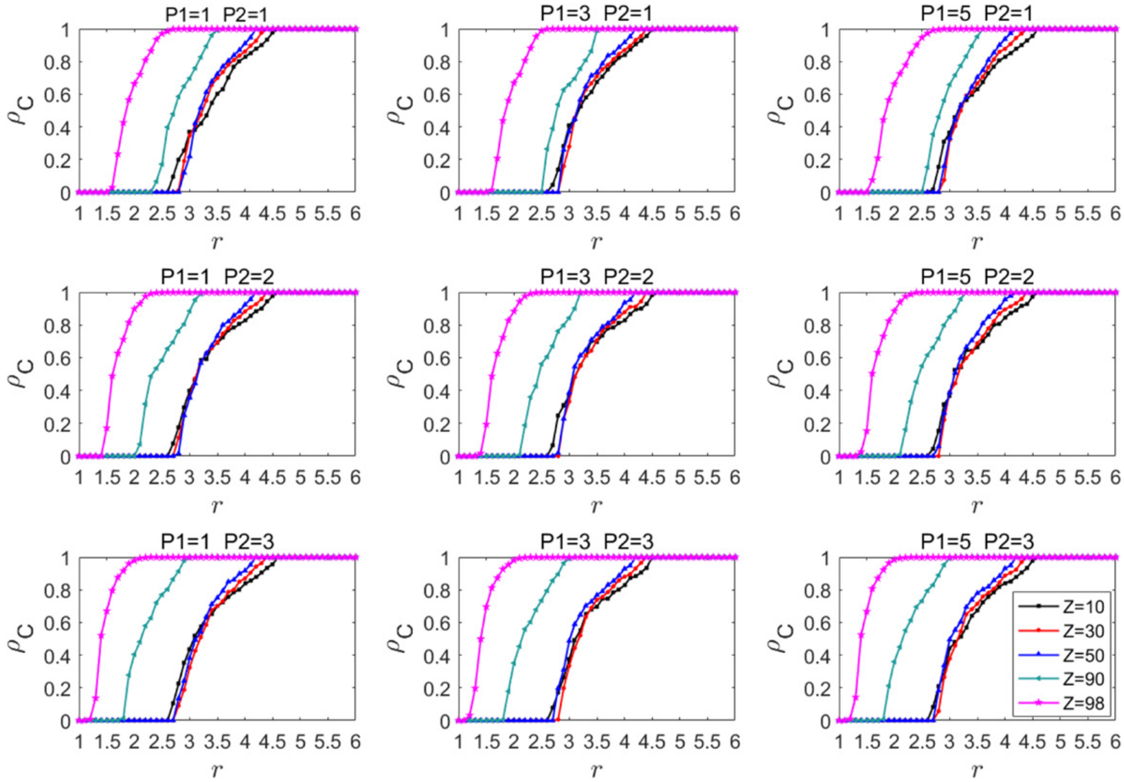

Figure 4. The relationship between the fraction of cooperators ρ_c at the stationary state and the enhancement factor r under different reputation thresholds. Each panel shows a specific intensity of reward and punishment for C/B and D/B behavior. From left to right, each row represents a situation corresponding to the degree of reward P_1. From top to bottom, each column represents a situation corresponding to the degree of punishment P_2. The reputation threshold Z is set to be 10, 30, 50, 90 and 98, respectively.

figure 1. When the reputation threshold is low ($Z < 50$), with the decrease of Z, the reduction of 'bad' individuals weakens the role of reward and punishment. The temptation of defection leads to a decrease in cooperation at the beginning. When $t = 10$, the reputation gap between high-reputation individuals and low-reputation individuals is gradually widened, and individuals with high-reputation begin to spread their strategies to the surrounding neighbors. At $t = 100$, the high-reputation cooperators begin to form clusters to resist the invasion of defectors, so that frequency of cooperation began to rise until the cooperation-defection spatial combinations reach an equilibrium. We further plot the evolutionary process of the average reputation and payoff per strategy. The results are shown in figure 2(b), with all parameter settings are the same as in figure 2(a). When $Z = 30$, the average reputation of cooperators and defectors is around 73 and 5 respectively, and the average payoffs of cooperators and defectors are around 7.9 and 1.9 respectively. When $Z = 90$, the average reputation of cooperators and defectors is around 93 and 22 respectively, and the average payoffs of cooperators and defectors are around 8.5 and 6.5 respectively. Interestingly, in both situations, cooperators not only have a higher average reputation but also a higher average payoff than

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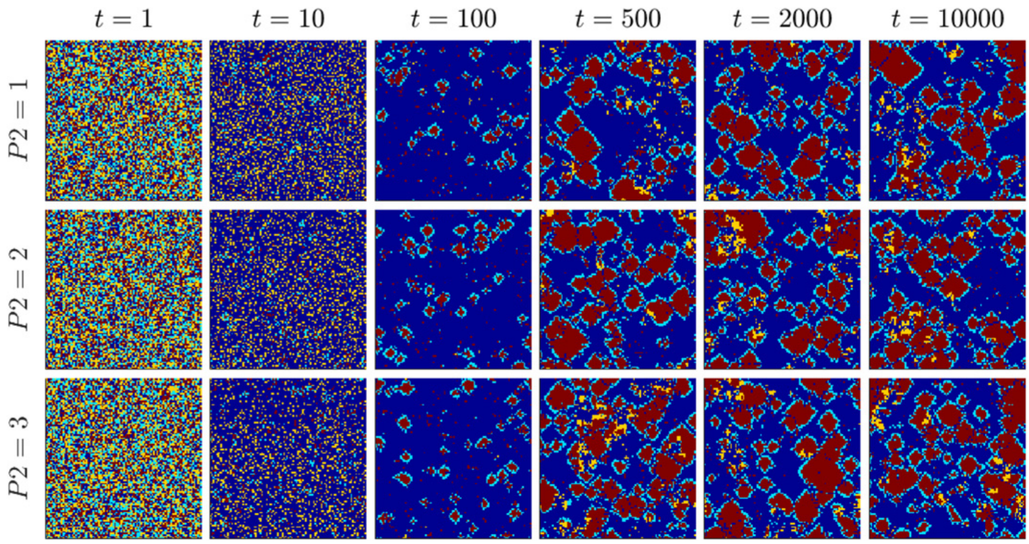

Figure 5. Snapshots of individuals' strategies distribution at different time steps for different values of P_2. Here $P_1 = 1$, $Z = 30$ and $r = 3.1$. From top to bottom, the punishment degree for D/B behavior P_2 increases from 1 to 3. Dark red stands for high-reputation cooperators; yellow stands for high-reputation defectors; dark blue stands for low-reputation defectors; and light blue stands for low-reputation cooperators.

defectors. Moreover, an increase in Z improves the average reputation and payoff of defectors significantly.

The situation discussed above can be demonstrated through the evolutionary patterns in the three subfigures of figure 3. They show the effect of reward and punishment on the evolution of the distributions of strategies, reputation, and payoff under different reputation thresholds Z. The parameter settings are the same as in figure 2(a). From top to bottom, five values of $Z = 10$, 30, 50, 90, 98 are taken, respectively, with each row corresponding to a different reputation threshold Z. As can be seen in the figures, when the system is stable, cooperators are distributed in the population by forming compact clusters. Moreover, the distributions of high-reputation and high-payoff individuals in the population are extremely similar. The three distributions of strategies, reputation, and payoffs show that cooperators not only have a higher reputation but also a higher payoff on average. With the increase in Z, the area of red parts expands in all three subfigures when the system is stable. Notably, when $Z = 98$, the strictly high threshold in reputation makes high-reputation and high-payoff individuals occupy the whole population finally.

In fact, for a low reputation threshold Z, most individuals are regarded as 'good' and the proportion of 'bad' individuals in the initial state is low. Although 'good' individuals are more likely to spread their strategies to the surrounding neighbors in our model, a population with a small number of 'bad' individuals can not have a significant effect on the imitation dynamics. 'Good' defectors occupy most of the network. As defectors keep their choice, their reputation continues to decrease which induces the number of 'bad' individuals to increase. When more and more defectors are regarded as 'bad', the

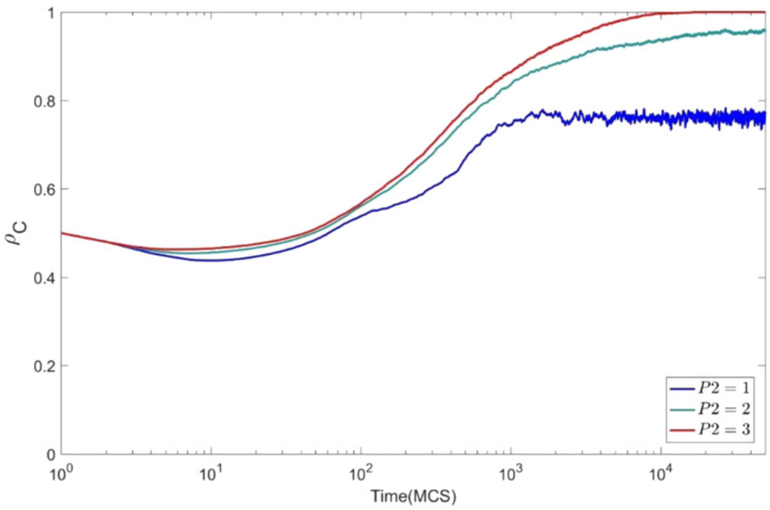

Figure 6. Frequency of cooperators ρ_c varies with time for different punishment degrees P_2. Here $P_1 = 1$, $Z = 90$ and $r = 3.1$. For all three values of $P_2 = 1$, 2 and 3, the frequency of cooperators keeps rising gradually toward a dynamic stable and non-zero level, and the system can reach evolutionarily stable in 10^4 MCS.

imitation dynamics which favors the strategy of 'good' individuals plays a role. Through imitation, many 'bad' defectors become 'bad' cooperators. Cooperative behavior can also improve their reputation levels making them become 'good' cooperators. On the other hand, more and more cooperative clusters are formed in the evolutionary process and the payoff of 'good' cooperators in the clusters continues to increase which further accelerates the transformation of defectors to cooperators. When the system is stable, 'good' defectors surround clusters of 'good' cooperators, separating 'good' cooperators from 'bad' defectors. Whereas when the reputation threshold Z is high, the proportion of 'bad' individuals in the initial state is also high. In this situation, the imitation dynamics which favors the spread of 'good' individuals' strategies plays a role. The mechanism of punishing D/B behavior and rewarding C/B behavior through reputation can make some 'bad' defectors turn into 'bad' cooperators. Cooperative behavior can also improve their own reputation levels making them become 'good' cooperators. At $t = 10$, 'good' cooperators have already formed a large number of cooperative clusters. With time goes, cooperative clusters gradually expand and the reputation and payoff of cooperators continue to increase. Defectors can only survive at the border of cooperative clusters. As can be seen in figures 3(b) and (c), when $t = 500$, the border of the dark red clusters is surrounded by individuals marked with light blue. From then on, the cooperative clusters continue to expand until the system is stable.

Next, we further explore the degree of reward and punishment on the evolution of cooperation. Specifically, when we interact with 'bad' individuals, whether increasing the intensity of reward for cooperation and punishment for defection can further promote cooperation. Figure 4 depicts the relationship between ρ_c at the stationary state and r for different combinations of reward and punishment intensity. In each subfigure, different reputation thresholds of $Z = 10$, 30, 50, 90, 98 are considered. From left to

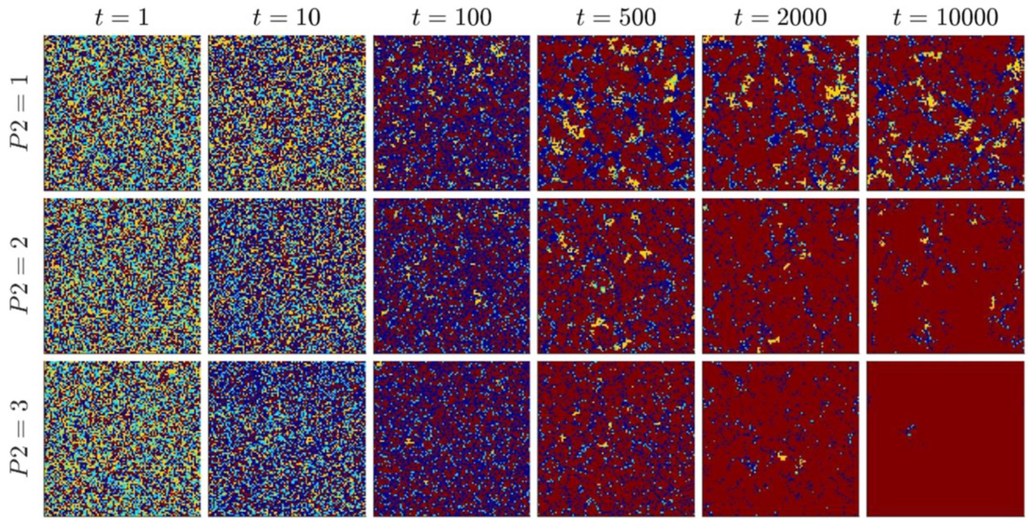

Figure 7. Snapshots of individuals' strategies distribution at different time steps for different values of P_2. Here $P_1 = 1$, $Z = 90$ and $r = 3.1$. From top to bottom, the punishment degree for D/B behavior P_2 increases from 1 to 3. Dark red stands for high-reputation cooperators; yellow stands for high-reputation defectors; dark blue stands for low-reputation defectors, and light blue stands for low-reputation cooperators.

right, only the degree of reward for C/B behavior is changed. It can be seen that when P_1 increases from 1 to 5, the critical value of r at which cooperation emerges does not change. Therefore, appropriate rewards should be given for cooperating with 'bad' individuals, but increasing the reward degree for C/B behavior cannot further improve cooperation. From top to bottom, we fix the value of P_1 in each column and only adjust the punishment degree P_2 for D/B behavior. However, we find that in the context of high and low thresholds, the change of P_2 has different effects. In a low reputation environment ($Z = 10/30$), an increase in P_2 has no significant effect on the emergence of cooperation. But in a high threshold environment ($Z = 90/98$), an increase in P_2 promotes cooperation. Especially, when $P_2 = 3$ and $Z = 98$, cooperation emerges at an extremely low critical factor of $r = 1.3$, and the full cooperation state can be reached when $r = 2.3$.

In order to understand the micro-mechanism of different effects of punishment in the low and high reputation threshold environments on cooperation, we present the spatiotemporal distributions of strategies for different punishment intensity P_2 and fixed $P_1 = 1$. Two reputation thresholds $Z = 30$ and 90 corresponding to low and high reputation environments are chosen. The results are shown in figures 5 and 7, respectively, in which the strategy distribution of the four types of individuals at different time steps can be observed. Intuitively, the low threshold $Z = 30$ in figure 5 results in a high proportion of 'good' individuals in the population. Thus, individuals have more opportunities to interact with 'good' individuals. Then increasing the degree of punishment for those who defect 'bad' opponents does not affect the strategic choices for most individuals.

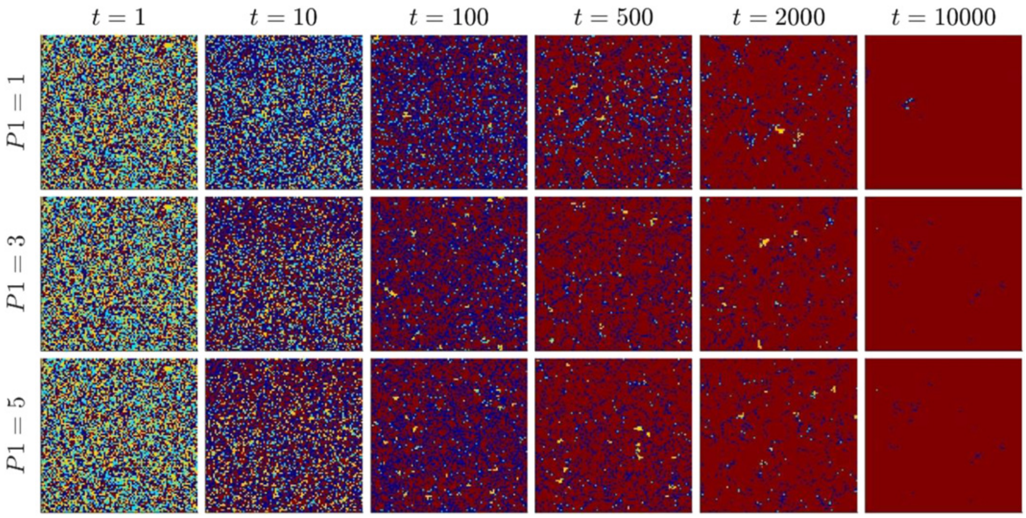

Figure 8. Snapshots of individuals' strategies distribution at different time steps for different values of P_1. Here $P_2 = 3$, $Z = 90$ and $r = 3.1$. From top to bottom, the reward degree for C/B behavior P_1 increases from 1 to 5. Dark red stands for high-reputation cooperators; yellow stands for high-reputation defectors; dark blue stands for low-reputation defectors, and light blue stands for low-reputation cooperators.

As the results show that an increase in P_2 has no significant effect on the emergence of cooperation.

As shown in figure 5, although the proportion of low-reputation individuals rises initially, cooperative clusters are gradually formed in the evolution process. At certain times, low-reputation cooperators start to initiate a self-protection mechanism. They gather around the cluster of high-reputation cooperators, and with the help of high-reputation cooperators in the neighborhood to reduce the loss of benefits from their cooperative behavior, which also quickly increase the average reputation of their neighbors. Low-reputation defectors are on the border of the clusters, and due to the high average reputation of their neighbors, their defection is defined as D/C behavior. The punishment intensity for D/C behavior is 5 units. Further increasing the penalty P_2 for D/B behavior can not induce a change of strategy. Therefore, a low-reputation threshold environment endows a special function for high-reputation individuals, which improves the average reputation of neighbors around the cluster to the 'good' level, and thus changing the degree of punishment for D/B behavior does not affect the strategy of low-reputation defectors. To some extent, it hinders the emergence of cooperation.

We also discuss the effect of increasing the punishment for D/B behavior on the emergence of cooperation in a high threshold environment. Figure 6 shows the evolution of the frequency of cooperators over time when $Z = 90$. Due to the high proportion of low-reputation individuals in the early stage, increasing the punishment degree promotes the transfer of low-reputation defectors to low-reputation cooperators, so that the cooperation level can be maintained in the early stage of evolution when there are many defectors. After about 1000 steps, the increasing cooperation behavior results in a high reputation level of the population. Thus, the average reputation of defectors' neighbors

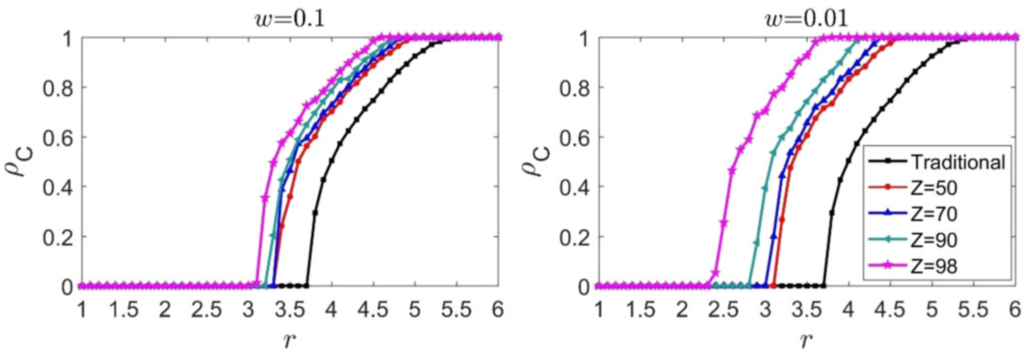

Figure 9. The relationship between the frequency of cooperators ρ_c at the stationary state and the enhancement factor r under different reputation thresholds for fixed $w = 0.1$ (left) and $w = 0.01$ (right), respectively. Here $P_1 = 1$ and $P_2 = 1$.

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is 'good', while the D/G behavior will be punished severely, which further promotes the cooperation frequency to a higher level. Therefore, the increase of punishment degree P_2 for D/B behavior is conducive to promote more defectors to become cooperators in the early stage. When $P_2 = 3$, the punishment intensity makes low-reputation defectors unable to survive and expand, and cooperators occupy the whole population finally.

The spatiotemporal distributions of strategies in the high threshold environment corresponding to figure 6 are shown in figure 7, from which we can clearly observe the spatial evolution patterns of four types of individual clusters. In the initial state, because strategy types and reputation values of individuals are randomly distributed on the network and $Z = 90$ is a high threshold environment, there are more individuals being regarded as 'bad'. As can be seen in the figure, when $t = 1$, there are more dark blue and light blue dots in the population. With the increase of P_2, those who defect 'bad' individuals will be punished more severely. Therefore, individuals tend to cooperate with low-reputation opponents, and some of them begin to switch to being cooperators gradually. When $t = 10$, the number of cooperators begins to increase, and the number of low-reputation cooperators (light blue) changes more significantly. With the increase of cooperative behavior, individuals begin to transfer from low-reputation cooperators to high-reputation cooperators, which results in the shrinking of the light blue parts and the expansion of the dark red parts at the 500 MCS. At this time, for most defectors, the average reputation of the surrounding neighbors is the 'good' type. As the punishment intensity for D/C behavior is 5 units, thus, the severe punishment will inhibit the expansion of defectors.

Notably, we observe that when P_2 is fixed at a non-zero level, increasing the reward degree P_1 for C/B behavior has no significant effect on the evolution of cooperation. The mechanisms behind this result are explored in figure 8, in which we set the punishment intensity $P_2 = 3$ for D/B behavior, and let the reward intensity P_1 for C/B behavior gradually increase. We set a high reputation threshold $Z = 90$ such that there is a high proportion of 'bad' individuals initially, which is more conducive to the function of the reward mechanism. With the increase of reward for C/B behavior, cooperation with 'bad' individuals becomes more attractive.

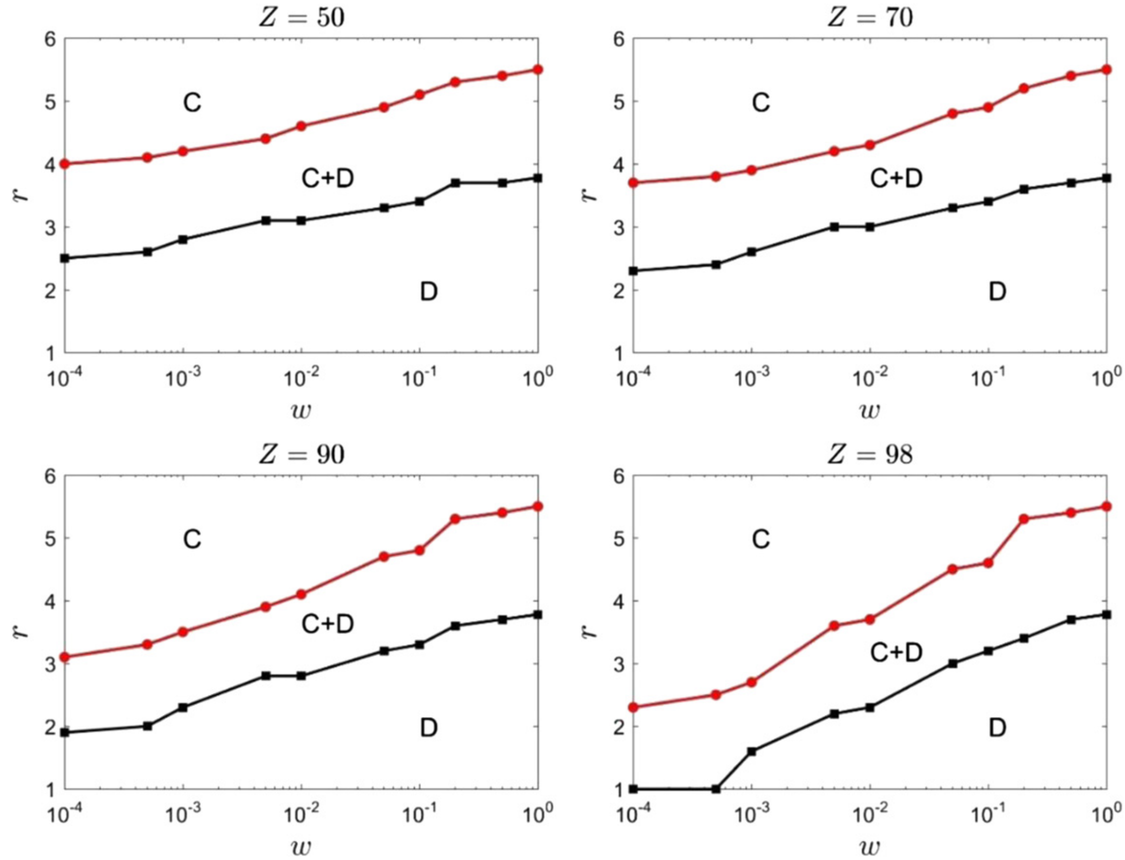

Figure 10. Two curves corresponding to two critical values of r at which cooperation emerges and defection disappears, respectively, as functions of w ($10^{-4} \leqslant w \leqslant 10^{0}$). The results under four different values of reputation threshold Z are compared.

It can be seen from figure 8 that the proportion of 'bad' individuals in the initial stage is high. At this time, the average reputation of most individuals' neighbors is 'bad'. Increasing the reward level for C/B behavior will have a bidirectional effect. Because the reward degree of C/B behavior is greater than C/G behavior, low-reputation cooperators choose to cooperate with low-reputation defectors, and the severe punishment for D/B behavior will result in a transfer of low-reputation defectors to low-reputation cooperators. Successive cooperation makes the reputation image of 'bad' individuals become better, and thus high-reputation individuals expand rapidly. When $t = 10$, the red parts expand, while the yellow parts decrease. When $t = 500$, individuals in the population enter a high reputation environment, in which cooperation is regarded as C/G behavior. Thus, further increasing the reward degree for C/D behavior has no significant effect on the evolution of cooperation. In a word, increasing the reward degree for C/D behavior only plays a role in the early stage (before 500 MCS) in a high threshold environment, helping cooperators to expand in short time steps. But in the later stage when individuals' reputation level is generally high, increasing the reward degree for C/D behavior does not work.

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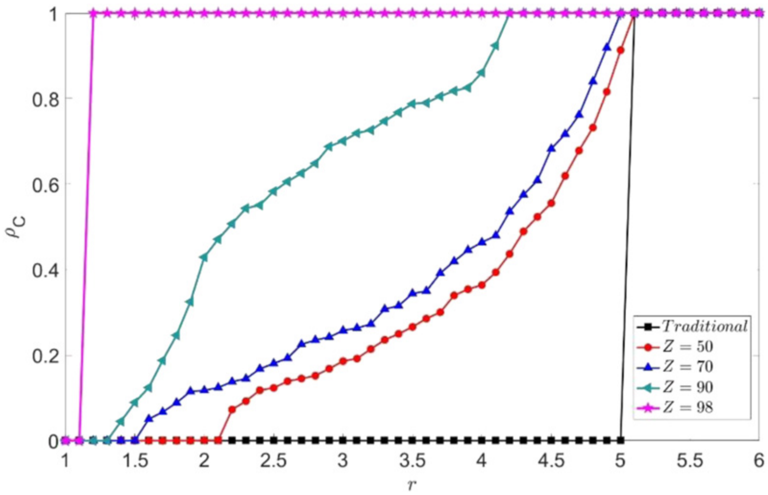

Figure 11. The relationship between the frequency of cooperators ρ_c and the enhancement factor r in a well-mixed population with size $N = 10\,000$. All parameter settings are the same as in figure 1(d).

Finally, we change some parameter settings that are fixed in the above to explore their influence on the results. Firstly, we conduct simulations for varying values of w. The results for $w = 0.1$ and 0.01 are presented in figure 9, with other parameter settings being the same as in figure 1(d). Moreover, two critical values of r at which cooperation emerges and defection disappears, respectively, as functions of w ($10^{-4} \leqslant w \leqslant 10^{0}$) are reported in figure 10. When $w = 1$, the model degenerates to the traditional version, and the reputation threshold Z has no effect on the results. In this situation, cooperation emerges at $r = 3.75$ and defection disappears at $r = 5.5$ for all cases. Evidently, the smaller the value of w, the more significant the effect of Z on the results.

Secondly, we test the robustness of our model in a well-mixed population. The results are presented in figure 11, with all parameter settings are the same as in figure 1(d). As can be seen in the figure, when $Z = 50$, 70, 90, and 98, the critical values of r above which cooperation emerges are 2.2, 1.6, 1.4, and 1.2, respectively. However, in the traditional model without reputation, cooperation can not emerge when $r \leqslant 5$. The results show that the reputation mechanism we introduced can also greatly promote the evolution of cooperation in a well-mixed population. Notably, when $Z = 50$, 70 and 90, the cooperation frequency increases more slowly with r, which makes the critical values of r for the system reaching a fully cooperative state be larger than those in a lattice network. However, for an extremely high level of Z ($Z = 98$), the situation is different, in which the two critical values of r for cooperation emerges and defection disappears are lower than those in the lattice network. The results show that under extremely high reputation thresholds, the reputation mechanism in a well-mixed population structure can promote cooperation more significantly than in the lattice network.

Thirdly, we test how the values in the first column of table 1 affect the results. We let the values of C/G and D/G in the first column change (one at a time), with $P_1 = 1$ and $P_2 = 1$ in the second column of table 1 being fixed. The results show that when the reward intensity for C/G and the punishment intensity for D/G are within a certain range ($1 \leqslant \mathrm{C/G} \leqslant 3$, $-5 \leqslant \mathrm{D/G} \leqslant -3$), it has little effect on the results.

4. Conclusions

In this paper, we have explored the second-order social norms that can promote the evolution of cooperation in SPGG. We focus on different criteria to evaluate one's behavior toward 'bad' individuals. The reputation evaluation is based on the information of the focal individual's behavior and his neighbors' reputation level. Two parameters to control the intensity of reward and punishment through increasing or decreasing individuals' reputation when they act toward 'bad' individuals are introduced, through which the effect of different social norms on the evolution of cooperation can be compared. The results show that appropriate rewards and punishments can significantly promote the evolution of cooperation. However, further increasing the intensity of reward for cooperation with 'bad' individuals cannot improve the level of cooperation. Concerning the optimal intensity of punishment, it depends on the reputation threshold which defines the 'good' and 'bad'. In a high-reputation threshold environment, increasing punishment intensity can inhibit the expansion of the defectors and improve the level of cooperation; whereas in a low-reputation threshold environment, increasing punishment intensity cannot improve the level of cooperation further. The robustness of the model is also tested in a well-mixed population, in which the reward and punishment mechanism through reputation can also work well. Moreover, we find that under certain high-reputation threshold environment, a well-mixed population structure can even promote cooperation more significantly than a lattice network. This research extends the use of statistical physics to study the evolution of cooperation from the perspective of reputation-based dynamics and the results also facilitate us to better understand the pervasive phenomenon of cooperation in society.

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