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The network energy and environment efficiency analysis of 27 OECD countries: A multiplicative network DEA model

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A R T I C L E I N F O

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A B S T R A C T

This research enhances the ability of the data envelopment analysis (DEA) model to analyse regional energy and environmental efficiency in three aspects. The first is a networked efficiency analysis of the regional economy-energy-environment model, which allows us to analyse internal conflicts between different departments in the region. Secondly, we use a multiplicative function instead of a linear function for frontier efficiency analysis. Multiplicative function frontier efficiency analysis model has two advantages: (1) The multiplicative function can allow both increasing marginal productivity and diminishing marginal productivity, while linear functions can only deal with diminishing marginal productivity. (2) The variables involved in regional production activities in the multiplicative function are no longer independent of each other, but have synergistic effects. (3) There is no inconsistency between internal evaluation and external evaluation when using the multiplicative model to measure the efficiency of the network structure. Thirdly, we found friction efficiency and conflict efficiency by comparing the network model with the traditional single model. Friction efficiency represents the DMU internal conflicts impact and conflict represents the degree of efficiency of the process is affected by the internal conflict. They can help decision makers understand more clearly the reasons for the inefficiency within DMU. Finally, we use 27 OECD countries as examples to conduct case studies. The results show that the multiplicative model is more reasonable in calculating regional energy and environmental efficiency than the traditional DEA model. On the other hand, the networked analytical structure can give policymakers more detailed analysis results than single process method.

1. Introduction

Energy and environmental efficiency analysis is one of the mainstream models for measuring regional sustainable development. Traditional energy and environmental efficiency analysis models, especially the total factor energy efficiency (TFEE) model, always assume that the regional production department is in a perfect competitive external environment. In fact, this assumption is unreasonable, and many variables in the production department are not freely disposable. Regional energy and environmental efficiency are affected by two unavoidable factors. (1) The efficiency of the production department is restricted by other departments. (2) The efficiency of the production department is influenced by the interaction between the variables. In order to solve the above research gap, we conducted this study. In order to solve the issue of interaction between variables, we use a multiplicative function instead of a linear function for efficiency analysis. Because the multiplicative function allows interactions between variables. On the other hand, we have constructed a network structure with production, energy, humanities, and economics departments to study the impact of conflicts between different departments on regional energy and environmental efficiency.

The structure of this article is as follows: Section 2 We reviewed the existing energy and environmental efficiency analysis models in detail and pointed out the problems they encountered. Section 3 we describe the detail of our multiplicative network data envelopment analysis (MNDEA) and discuss structure, links, and their characteristics. In Section 4, compare MNDEA with single process multiplicative DEA, and discover their different performance in calculating department efficiency and overall efficiency. Section 5 shows the special network for regional sustainable development and the special MNDEA to take efficiency assessment. Section 6

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shows the results of the MNDEA model in the study of 27 OECD countries. Section 7 based on the results of empirical research, we discuss the difference between MNDEA and network SBM-DEA and explain why MNDEA is more suitable for analyzing regional energy and environmental efficiency than network SBM-DEA. Section 8 summarizes the contents of whole paper.

2. Literature review

In 1987, Färe et al. first proposed the measure method for eco-efficiency, and since then, it has been measured and reviewed eco-efficiency at different levels, including project, sector, economy and country. At the country level especially, there have been several studies about the eco-efficiency measurement [1–3]; [46]). In addition, there have been many discussions on the measurement of eco-efficiency in other fields – for example, Lansink and Silva used DEA calculations to measure the carbon dioxide emissions and energy use efficiency of Dutch vegetable production organisations based on the technical efficiency of inputs-direction [5]. Reinhard and Lovell used the DEA model to assess the eco-efficiency of Dutch dairy farms and compared them with the results of the SFA method [6], while Dyckhoff and Allen proposed a new system approach, allowing the eco-efficiency DEA model to measure environmental efficiency [7]. A mature eco-efficiency evaluation system already exists in the above literature, which is ideal for DMUs with multiple environmental impacts. Input variables in these studies typically include labour force, actual capital stock, coal intake, gas oil intake, and electricity intake, and the output variable is actual gross domestic product (GDP). In this stage of research, the efficiency of energy use is valued, and the emission of pollutants such as carbon dioxide is neglected. Following this, Tone and Sahoo discussed the elasticity of productivity scale in the frontier model [8], and Lansink and Reinhard established a more comprehensive evaluation system that simultaneously assesses the technical efficiency, eco-efficiency and economic efficiency of the DMU [9].

Total factor energy efficiency (TFEE) is the most important model in the field of eco-efficiency assessment. It was proposed by Hu and Wang in 2006 to measure the energy efficiency of 29 regions in China. The core concept of the TFEE framework is that energy alone cannot produce any output, and that it must be combined with other inputs to produce an output. Therefore, a multi-input model should be applied to properly assess the energy efficiency of a region. In our study, the regional target for energy input can be found through a multi-input frontier efficiency analysis model. Outdated technology and inefficient production processes create redundant parts of energy consumption, which is a source of inefficiency in DMU. Total adjustments, including slack and radial adjustment, are calculated from multivariate frontier efficiency analysis models. Total factor energy efficiency is constructed as the ratio of the energy input of the target DMU suggested by the frontier efficiency assessment model to the actual energy input of a region.Fig. 1

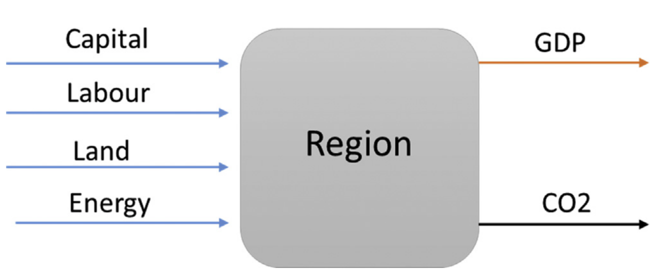

Fig. 1. The input-output structure of TFEE model.

The TFEE index uses energy consumption, labour and capital stock as inputs, and economic productivity (GDP) as output. In contrast, the traditional energy efficiency index only considers energy as a single input to generate GDP output while ignoring other key inputs such as capital and labour [11]. Therefore, in the definition of TFEE, energy efficiency improvement depends on the improvement of total factor productivity. We usually use an economic production function constructed by a multivariate frontier efficiency model to analyse the energy efficiency of DMU from the perspective of total factor productivity. Energy and resources consumption, including land area, is considered in conjunction with conventional inputs – labour and capital stock – which are normally used in an economic productivity analysis as the multiple inputs to produce economic output (GDP). For a country or a region, it is preferable that the GDP increases, and that energy consumption is saved in order to approach production efficiency. Thus, the objective for economic growth and efficiency of energy consumption should be put together in order to achieve sustainable economic development.

The total factor energy efficiency framework has been widely accepted by ecological efficiency studies over the past decade. With the continuous improvement of the frontier efficiency measurement model, the total factor energy efficiency framework has developed a number of different extension models. One of the most widely accepted model is the addition of an undesired output (usually CO2) to the original TFEE framework to reflect the environmental impact of total factor production.

The table below shows the different improvements in the TFEE framework in existing literature.Table 1

It is clear that the current regional energy and environmental efficiency studies are based on the single-process TFEE model and DEA. However, the existing models have two research gaps. First, the linear DEA model assumes that the variables of the DMU are in independent markets, yet the TFEE model believes that energy needs to collaborate with other variables to produce any output. Second, regional sustainable development is the result of cooperation between multiple departments. It is unreasonable to only study the energy and environment efficiency of the production department and to then unilaterally assume that the variables of the production department are completely competitive. In recent years, research on networked efficiency analysis of regional energy and environmental development has gradually received attention. In 2014, Song et al. [33] used sewage discharge as an intermediate variable to connect the production department and the sewage treatment department, and in 2018, Li et al. introduced an environmental efficiency analysis model with an approximate network structure – the intermediate variables were COD, SO2 and solid waste [36]. In 2016, Bian et al. [32] divided the regional production department into three parallel sub-departments in industry, agriculture and service for efficiency analysis, while in 2018, Iftikhar et al. [34] used GDP as an intermediate variable to connect the production department and income distribution department. Regional energy and environmental efficiency analysis are increasingly considering the impact of economic and human factors in the region; however, their research is not comprehensive and cannot fully explain the development performance of a region.

In this paper, we extend the network model within the generalized multiplicative directional distance function (GMDDF) framework proposed by Mehdiloozad in 2014 [37]. Unlike linear functions, multiplicative functions assume that variables are cooperative rather than independent. It is more in line with the TFEE model's assumptions about the relationship between energy and other variables of production factors. On the other hand, we use the network model to express the sustainable development of the region as the result of the collaboration of multiple departments.

Table 1
Literature list of energy and environmental efficiency.

Paper	Input	Output	Undesired Output	Methodology
[45]	TPES	GDP		Ratio Analysis
[13]	Resource Consumption	GDP	CO2, CH4, NO	Ratio Analysis
[10]	Energy, Capitical, Population	GDP		CCR-DEA
[12]	Energy, Population	GDP	CO2	SBM-DEA
[15]	Labour	GDP	CO2,SO2,NO	SBM-DEA
[20]	Water, Raw, Energy	IVA	CO2,SO2,NO,COD,water, dust,solid	SBM-DEA
[17]	Capitical, Labour, Coal, oil, gas	GDP	CO2	BCC-DEA
[19]	Capitical, Energy, Labour, WGF	IVA	CO2,SO2,NO,PM10	BCC-DEA
[20]	Capitical, Energy	GDP		DEA with Windows Analysis
[21]	Labour, Quay Length, Energy, land	Cargo, Vessel	CO2	SBM-DEA
[22]	Capitical, Energy, Labour	GDP	CO2,SO2	DEA with Windows Analysis
[23]	Capitical, Energy, Labour	GDP	SO2,Water, Solid	SBM-DEA with Network
[24]	Capitical, Energy, Labour, land	GDP	COD,SO2,Water, Dust,Solid	SBM-DEA
[25]	Capitical, Labour, Coal,Electricity	GDP	CO2	SBM-DEA with Malmquist Index
[44]	Capitical, Labour, Water,Energy	GDP	Solid, Water, Gas	SBM-DEA
[22]	Capitical, Labour, Water,Energy, Tech, GI	IVA	Dust, Solid, Water, Gas	SBM-DEA

The production department obtains production factors from the economic department, the humanities department, and the energy department such as capital, labor, and energy. Compared to previous research, our network structure can more fully describe the development performance of a region.

3. The multiplicative network DEA model

3.1. Network structure

The traditional single process method regards a DMU as an inseparable whole. The internal structure of the DMU is a black box. The internal structure is considered to be always in an optimal state, and internal conflicts do not exist. Obviously, the above ideal state is unreasonable in actual economic activities. DMUs often have complex internal structures, and internal conflicts are inevitable. Therefore, we need to consider the impact of the internal structure of the DMU when analysing the efficiency of the DMU. Based on data the DMU may be described as a network model. We observe n DMUs with Q processes. In each DMU, each process q has its respective inputs and outputs along with the intermediate product to the other process. The DMU concerned is examined as a system including all processes.

The network DEA differs from the traditional single process in that it has intermediate products that can connect different processes. There are two different assumptions about the properties of the intermediate product. In 2009, Tone and Tsutsui proposed that intermediate products have two different properties, free and fixed, but there is no strict regulation on how to use the two properties. This study defines the two properties of the intermediate product in detail and clarifies their conditions of use. The two properties of the intermediate product are as follows:

(1) Free link (Overall efficiency)

Free link treats intermediate products as a variable that the DMU can change. A DMU can achieve higher overall performance by adjusting free intermediate products. The adjustment of the intermediate product always has a benefit and a process but damages another process, so the adjustment of the intermediate product does not exist from the perspective of maximising the benefit of the process. Following the idea of Tone and Tsutsui [29]; the model for measuring the overall efficiency is only based on exogenous inputs and outputs. Therefore, the assumption of free link only applies to the pursuit of maximum overall efficiency.

(2) Fixed link (Process efficiency)

Compared to the free link, the fixed link considers the intermediate product to be part of the DMU's process environment and is not controlled by the DMU, because for any process, it is to pursue the maximization of its own interests. Fixed link guarantees the maximum output or minimum input of the process without harming any process. In measuring the process efficiency, Tone and Tsutsui [38] did not allow the process to be super-efficient, and assigned an efficiency score of one to the super-efficient intermediate product. Therefore, the fixed link is suitable for obtaining the technical efficiency of each process in the DMU with the internal conflict environment. However, it is not suitable for analysing the DMU's overall efficiency, because the value of the intermediate product is endogenous to the entire DMU, so it should be controlled by the DMU.

3.2. Production possibility set (PPS) and general models

The general network has n DMUs (j = 1…n), where Q processes (q = 1…Q) are linked by intermediate products. We denote X = {1, 2,…, m}, Y = {1, 2,…, s}, and M = {1, 2,…, t} as the index sets of the input, output, and intermediate, respectively; and similarly, $X^q \in X$ $Y^q \in Y$, and $M^{q,h} \in M$ as the corresponding subsets for Process q. m^q, s^q, and $t^{q,h}$ represent the number of inputs, outputs, and intermediate products in the corresponding subset. The Process q utilises inputs X^q_{ij} $i \in X^q$ and intermediate products $Z^{h,q}_{lj}$ $l \in M^{h,q}$, to produce outputs Y^q_{rj} $r \in Y^q$, and intermediate products $Z^{q,h}_{lj}$ $l \in M^{q,h}$. The intermediate $Z^{q,h}_{lj}$ $l \in M^{q,h}$ means the link as the output of process q and the input of process h.

The production possibilities of DMU_o (X^q_{io}, Y^q_{ro}, $Z^{q,h}_{lo}$) is defined by

$$
\begin{aligned}
&\prod_{j=1}^{n}\left(X^q_{ij}\right)^{\lambda^q_j} \le X^q_{io} \quad i \in X^q, q = 1,2,\dots Q\\
&\prod_{j=1}^{n}\left(Y^q_{rj}\right)^{\lambda^q_j} \le Y^q_{ro} \quad r \in Y^q, q = 1,2,\dots Q\\
&\prod_{j=1}^{n}\lambda^q_j = 1 \quad q = 1,2,\dots Q\\
&\lambda^q_j \ge 0 \quad j = 1,2\dots n, q = 1,2,\dots Q
\end{aligned}
\tag{1}
$$

The continuity of free link flows between process q and h can be guaranteed by the following condition:

$$\prod_{j=1}^{n}\left(Z_{ij}^{q,h}\right)^{\lambda_j^q}=\prod_{j=1}^{n}\left(Z_{ij}^{q,h}\right)^{\lambda_j^h}\quad i\in M^{q,h}, q=1,2,\ldots Q, h\neq q \tag{2}$$

The fixed link is not controlled by the DMU, so it is the same for any process. Therefore, there is no need to add redundant constraints.

$$\prod_{j=1}^{n}\left(Z_{ij}^{q,h}\right)^{\lambda_j^q}=Z_{io}^{q,h}=\prod_{j=1}^{n}\left(Z_{ij}^{q,h}\right)^{\lambda_j^h}\quad i\in M^{q,h}, q=1,2,\ldots Q, h\neq q \tag{3}$$

Using these expressions for production, we can express DMU_o as follows:

$$\prod_{j=1}^{n}\left(X_{ij}^{q}\right)^{\lambda_j^q}=X_{io}^{q}*w_{io}^{-}\quad i\in X^q, q=1,2,\ldots Q$$
$$\prod_{j=1}^{n}\left(Y_{rj}^{q}\right)^{\lambda_j^q}=Y_{ro}^{q}*w_{ro}^{+}\quad r\in Y^q, q=1,2,\ldots Q$$
$$0\le w_{io}^{-}\le 1\quad i=1,2\ldots m$$
$$1\le w_{ro}^{+}\quad r=r=1,2\ldots s$$
$$\prod_{j=1}^{n}\lambda_j^q=1\quad q=1,2,\ldots Q$$
$$\lambda_j^q\ge 0\quad j=1,2\ldots n, q=1,2,\ldots Q$$

For the free link:

$$\prod_{j=1}^{n}\left(Z_{ij}^{q,h}\right)^{\lambda_j^q}=\prod_{j=1}^{n}\left(Z_{ij}^{q,h}\right)^{\lambda_j^h}\quad i\in M^{q,h}, q=1,2,\ldots Q, h\neq q$$

Or for a fixed link:

$$\prod_{j=1}^{n}\left(Z_{ij}^{q,h}\right)^{\lambda_j^q}=Z_{io}^{q,h}=\prod_{j=1}^{n}\left(Z_{ij}^{q,h}\right)^{\lambda_j^h}\quad i\in M^{q,h}, q=1,2,\ldots Q, h\neq q \tag{4}$$

For the convenience of calculation, a logarithmic transformation can be performed on the production probability set. Let $x_{ij}^{q}=\ln(X_{ij}^{q})$, $y_{rj}^{q}=\ln(Y_{rj}^{q})$, $z_{ij}^{q,h}=\ln(Z_{ij}^{q,h})$, $s_{io}^{-}=-\ln(w_{io}^{-})$, $s_{ro}^{+}=\ln(w_{ro}^{+})$ The log formula can be written as:

$$\sum_{j=1}^{n}\lambda_j^q*x_{ij}^{q}=x_{io}^{q}-s_{io}^{-}\quad i\in X^q, q=1,2,\ldots Q$$
$$\sum_{j=1}^{n}\lambda_j^q*y_{rj}^{q}=y_{ro}^{q}+s_{ro}^{+}\quad r\in Y^q, q=1,2,\ldots Q$$
$$0\le s_{io}^{-}\quad i=1,2,\ldots m$$
$$0\le s_{ro}^{+}\quad r=1,2\ldots s^q$$
$$\prod_{j=1}^{n}\lambda_j^q=1\quad q=1,2,\ldots Q$$
$$\lambda_j^q\ge 0\quad j=1,2\ldots n, q=1,2,\ldots Q$$

For the free link:

$$\prod_{j=1}^{n}\left(Z_{ij}^{q,h}\right)^{\lambda_j^q}=\prod_{j=1}^{n}\left(Z_{ij}^{q,h}\right)^{\lambda_j^h}\quad i\in M^{q,h}, q=1,2,\ldots Q, h\neq q$$

Or for the fixed link:

$$\prod_{j=1}^{n}\left(Z_{ij}^{q,h}\right)^{\lambda_j^q}=Z_{io}^{q,h}=\prod_{j=1}^{n}\left(Z_{ij}^{q,h}\right)^{\lambda_j^h}\quad i\in M^{q,h}, q=1,2,\ldots Q, h\neq q \tag{5}$$

3.3. Objective functions and efficiency

From the GMDDF framework we learned that efficiency is made up of slack ratios. Thus we measure the overall efficiency of DMU_o taking $(\lambda_j^t, w_{io}^{-}, w_{ro}^{+})$ as variables. In log formula the variables change to the $(\lambda_j^t, s_{io}^{-}, s_{ro}^{+})$. Like the traditional DEA model, the MNDEA model can be divided into three orientations, namely, input-orientation, output-orientation, and non-orientation. Input-orientation focuses on reducing the possibility of input while keeping the output unchanged. The output-orientation focuses on maximising the output while keeping the input constant. The non-orientation is designed to ensure that the DMU gets the maximum virtual profit, which allows both input decrease and output increase.

The non-oriented overall efficiency θ_o^* is written as:

$$\theta_o^*=\min \sqrt[m]{\prod_{i=1}^{m}w_{io}^{-}}\Big/\sqrt[s]{\prod_{r=1}^{s}w_{ro}^{+}} \tag{6}$$

Subject to model (4).

The log non-oriented overall efficiency ρ_o^* is written as:

$$\rho_o^*=-max\frac{1}{m}\left(\sum_{r=1}^{m}s_{io}^{-}\right)+\frac{1}{s}\left(\sum_{r=1}^{s}s_{ro}^{+}\right) \tag{7}$$

Subject to model (5).

Let optimal process efficiency subject to model (4) be $(\lambda_j^{t*}, w_{io}^{-*}, w_{ro}^{+*})$. The non-oriented process efficiency θ_{oq}^* may be defined by

$$\theta_{oq}^*=\min \sqrt[m^q]{\prod_{i\in X^q}w_{io}^{-}}\Big/\sqrt[s^q]{\prod_{r\in Y^q}w_{ro}^{+}}\quad q=1,2,\ldots Q \tag{8}$$

Definition 1. Process efficient.

If all optimal solutions of model (8) can satisfy the $\theta_{oq}^*=1$, DMU_o is called input-oriented processes efficient for the process q. It means all the slack ratios for the process q are one, $w_{io}^{-*}=1\ (\forall\, i\varepsilon X^q)$, $w_{ro}^{+*}=1\ (\forall\, r\varepsilon Y^q)$, $wm_{io}=1\ (\forall\, i\varepsilon M^{q,h})$, *and* $wm_{io}=1\ (\forall\, i\varepsilon M^{h,q})$.

Definition 2. Over efficient.

If the solution of model (7) $\theta_o^*=1$, DMU_o is called input-oriented overall efficient. It means that all $w_{io}^{-*}=1\ (\forall \in X)$ *and* $w_{ro}^{+*}=1\ (\forall\, r\varepsilon Y)$.

From the above definitions derive:

Theorem 1. *DMU_o is overall efficient if, and only if, all processes are efficient separately.*

Proof:

If DMU_o is process efficient for all periods $\theta^*_{oq} = 1$ (∀ q).

It means all slack ratios for the are equal to one $w^{-*}_{io} = 1$ (∀i ∈X) and $w^{+*}_{ro} = 1$ (∀rεY)

Based on the model (6), it follows that:

$$\theta^*_o = \prod_{i=1}^{m} w^-_{io} \Big/ \prod_{r=1}^{s} w^+_{ro} = 1$$

Looked at from another perspective, if DMU_o has a process is inefficient $\theta^*_{oq} < 1$

It means that some input slack ratios in process q are less than 1 $w^{-*}_{io} < 1$, or output slack ratio in process q is larger than 1 $w^{+*}_{ro} > 1$

Due to there is a $w^{-*}_{io} \le 1$ or $w^{+*}_{ro} > 1$, thus

$$\theta^*_o = \prod_{i=1}^{m} w^-_{io} \Big/ \prod_{r=1}^{s} w^+_{ro} < 1$$

Theorem 2. *There are two DMUs a and b, If, all processes efficiency for DMU_a is higher than periods efficiency for DMU_b, $\theta^*_{aq} > \theta^*_{bq}$ (∀q), then the overall efficiency for DMU_a must be higher than overall efficiency for DMU_b, $\theta^*_a > \theta^*_b$.*

Proof:

$\theta^*_{at} > \theta^*_{bt}$ (∀q) implies that the product of slack ratios of DMU_a in each process is greater than DMU_b. If we set there is not slack ratio different in output variable, then we get $\prod_{i\in M^q} w^-_{ia} > \prod_{i\in M^q} w^-_{ib}$ (∀q). Thus

$$\prod_{i=1}^{m} w^-_{ia} > \prod_{i=1}^{m} w^-_{ib}$$

Then

$$\theta^*_a > \theta^*_b$$

Theorem 3. *The model in GMDDF is dimension free. That is, changes in the units used to express the input variables X^q_{ij} and output variables Y^q_{rj} will not affect the solution set or alter the value of efficiency.*

Proof:

Let

$$X^{q'}_{ij} = C^q_i X^q_{ij} \quad i = 1,2,\ldots m, q = 1,2,\ldots Q$$
$$Y^{q'}_{rj} = K^q_r Y^q_{rj} \quad r = 1,2,\ldots s, q = 1,2,\ldots Q$$

where C^q_i and K^q_r are any collection of positive constants. *By substitution in the constraints for model* (1)

$$\begin{aligned}
&\prod_{j=1}^{n}\left(X^{q'}_{ij}\right)^{\lambda^q_j} \le X^{q'}_{io} \quad i\in X^q, q = 1,2,\ldots Q\\
&\prod_{j=1}^{n}\left(Y^{q'}_{rj}\right)^{\lambda^q_j} \le Y^{q'}_{ro} \quad r\in Y^q, q = 1,2,\ldots Q\\
&\prod_{j=1}^{n}\lambda^q_j = 1 \quad q = 1,2,\ldots Q\\
&\lambda^q_j \ge 0 \quad j = 1,2\ldots n, q = 1,2,\ldots Q
\end{aligned} \tag{9}$$

Then the formula can be rewritten as:

$$\begin{aligned}
&\prod_{j=1}^{n}\left(C^q_i X^q_{ij}\right)^{\lambda^q_j} \le C^q_i X^q_{ij} \quad i\in X^q, q = 1,2,\ldots Q\\
&\prod_{j=1}^{n}\left(K^q_r Y^q_{rj}\right)^{\lambda^q_j} \le K^q_r Y^q_{rj} \quad r\in Y^q, q = 1,2,\ldots Q\\
&\prod_{j=1}^{n}\lambda^q_j = 1 \quad q = 1,2,\ldots Q\\
&\lambda^q_j \ge 0 \quad j = 1,2\ldots n, q = 1,2,\ldots Q
\end{aligned} \tag{10}$$

However $\prod_{j=1}^{n}\lambda^q_j = 1$ so

$$\prod_{j=1}^{n}\left(C^q_i\right)^{\lambda^q_j} = C^q_i \quad \forall i, q$$

And

$$\prod_{j=1}^{n}\left(K^q_r\right)^{\lambda^q_j} = K^q_r \quad \forall r, q$$

Thus, we can elimination of common terms C^q_i and K^q_r on the right and left of formula. It follows that the efficiency of model (1) *also the efficiency of model* (9).

3.4. The relation between overall efficiency and process efficiency

3.4.1. Review the relation in the linear model

Linear Network DEA has two mainstream intermediate product frameworks. Kao in 2009 Kao [28] proposed a relational model that requires the same intermediate product need to have the same virtual price, regardless of the process it corresponds. On the other hand, Tone and Tsutsui in 2009 proposed a cooperative model, the intermediate product flowing out of the preceding process is required to be equal to that flowing into the succeeding process to satisfy the equal condition. However, both the relational model and the cooperative model face problems that cannot measure both the overall efficiency and the process efficiency at the same time. External evaluation prefers to describe overall efficiency as an efficiency analysis based on exogenous variables. Internal evaluation method preference for describing overall efficiency as the weighted average of process efficiencies.

Tone and Tsutsui [29] measure the overall efficiency as the artificially weighted average of the process efficiencies. Kao [30] proposed using a weighted average of the process efficiencies in the aggregation, with the weight associated with a process equal to the proportion of the output process efficiency in the sum of the output overall efficiency. Kao [27] re-discussed the relationship between overall efficiency and process efficiency. For example, under SBM, Kao define the following process efficiencies

$$\theta_{oq} = \frac{1 - \frac{1}{m^q}\left(\sum_{i \in X^q} \frac{s_{io}^-}{x_{io}}\right)}{1 + \frac{1}{s^q}\left(\sum_{r \in Y^q} \frac{s_{ro}^+}{y_{ro}}\right)} \quad q = 1, 2, \ldots Q \tag{11}$$

The overall efficiency is

$$\theta_o = \frac{1 - \frac{1}{Q}\left(\sum_{q=1}^{Q}\left(\frac{1}{m^q}\left(\sum_{i \in X^q} \frac{s_{io}^-}{x_{io}}\right)\right)\right)}{1 + \frac{1}{Q}\left(\sum_{q=1}^{Q}\left(\frac{1}{s^q}\left(\sum_{r \in Y^q} \frac{s_{ro}^+}{y_{ro}}\right)\right)\right)} \tag{12}$$

Note that the overall efficiency can be written as a weights average of process efficiencies.

$$\theta_o = \sum_{q=1}^{Q} \omega_q \theta_{oq} \tag{13}$$

where the weights are $\omega_q = \frac{\phi_q}{\sum_{i=1}^{Q} \phi_i}$. They are functions of slacks. We do not know their values before the models are calculated. Obviously, in a linear model, the relationship between overall efficiency and process efficiencies is unstable for DMUs with the same network structure. Their relation is affected by input and output variables.

3.4.2. The relation in the multiplicative model

Based on the model (6) and (8) we can describe the process efficiencies in MNDEA as:

$$\theta_{oq} = \min \sqrt[m^q]{\prod\nolimits_{i \in X^q} w_{io}^-} \Big/ \sqrt[s^q]{\prod\nolimits_{r \in Y^q} w_{ro}^+} \quad \text{q} = 1, 2, \ldots \text{Q} \tag{14}$$

and the overall efficiency in MNDEA is

$$\theta_o = \min \sqrt[m]{\prod\nolimits_{i=1}^{m} w_{io}^-} \Big/ \sqrt[s]{\prod\nolimits_{r=1}^{s} w_{ro}^+} \tag{15}$$

when m^q and s^q have the same amount in each process. The overall efficiency can easy to be a geometric weighted average of process efficiencies.

$$\theta_o = \prod_{q=1}^{Q} \left(\theta_{oq}\right)^{\frac{m^q + s^q}{m+s}} \tag{16}$$

when m^q and s^q have a different amount, the overall efficiency still has approximate relation with process efficiencies in model (16). Note that, in MNDEA the relation between overall efficiency and process only affected by the structure of network. We can know the weight values before the models are calculated. In contrast to linear models, MNDEA is more stable when calculating Internal evaluation overall efficiency.

4. Network efficiency and single efficiency

When the traditional single process DEA model analyses the multi-input multi-output DMU, the internal structure of the DMU is usually assumed to be a black box, and the external environment of the DMU is assumed to be a completely competitive market. Therefore, the efficiency value of the single process DEA can be interpreted as the ratio of the ideal DMU to the real DMU in the case where the internal structure is optimal, and the external market is effective. However, in the actual environment, the internal structure of the DMU always has inefficiencies, and the external environment also limits the possibility that the DMU can obtain a better performance.

4.1. The difference in overall efficiency

In the single process DEA model, when analysing the overall efficiency, it is necessary to assume the internal structure of the DMU is a black box. The PPS of the overall efficiency in DMU_o can be written as:

$$\begin{aligned}
&\prod_{j=1}^{n} \left(X_{ij}^q\right)^{\lambda_j} = X_{io}^q * w_{io}^- \quad i \in X^q, q = 1, 2, \ldots Q \\
&\prod_{j=1}^{n} \left(Y_{rj}^q\right)^{\lambda_j} = Y_{ro}^q * w_{ro}^+ \quad r \in Y^q, q = 1, 2, \ldots Q \\
&0 \le w_{io}^- \le 1 \quad i = 1, 2 \ldots . m \\
&1 \le w_{ro}^+ \quad r = r = 1, 2 \ldots . s \\
&\prod_{j=1}^{n} \lambda_j = 1 \\
&\lambda_j \ge 0 \;\; j = 1, 2 \ldots . n
\end{aligned} \tag{17}$$

The efficiency function of the single non-oriented overall efficiency is:

$$\theta_o^* = \min \sqrt[m]{\prod\nolimits_{i=1}^{m} w_{io}^-} \Big/ \sqrt[s]{\prod\nolimits_{r=1}^{s} w_{ro}^+} \tag{18}$$

The single process DEA does not impose any constraints on the internal structure of the DMU when computing the overall efficiency. It implies that the intermediate product is a free link in the overall efficiency calculation. Maximising DMU performance may need to be achieved by changing intermediate variables. In order to make the efficiency result of the single process DEA comparable to the efficiency result of the network DEA, it is necessary to assume that the network DEA also uses the free link intermediate product when calculating the overall efficiency. The PPS of network overall efficiency is:

$$\begin{aligned}
&\prod_{j=1}^{n} \left(X_{ij}^q\right)^{\lambda_j^q} = X_{io}^q * w_{io}^- \quad i \in X^q, q = 1, 2, \ldots Q \\
&\prod_{j=1}^{n} \left(Y_{rj}^q\right)^{\lambda_j^q} = Y_{ro}^q * w_{ro}^+ \quad r \in Y^q, q = 1, 2, \ldots Q \\
&\prod_{j=1}^{n} \left(Z_{ij}^{q,h}\right)^{\lambda_j^q} = \prod_{j=1}^{n} \left(Z_{ij}^{q,h}\right)^{\lambda_j^h} \quad i \in M^{q,h}, q = 1, 2, \ldots Q, h \ne q \\
&0 \le w_{io}^- \le 1 \quad i = 1, 2 \ldots . m \\
&1 \le w_{ro}^+ \quad r = r = 1, 2 \ldots . s \\
&\prod_{j=1}^{n} \lambda_j^q = 1 \quad q = 1, 2, \ldots Q \\
&\lambda_j^q \ge 0 \quad j = 1, 2 \ldots . n, q = 1, 2, \ldots Q
\end{aligned} \tag{19}$$

The efficiency function of the network non-oriented overall efficiency is:

$$\theta_o^* = \min \sqrt[m]{\prod_{i=1}^{m} w_{io}^-} \Big/ \sqrt[s]{\prod_{r=1}^{s} w_{ro}^+} \tag{20}$$

Comparing model (23) with the model (25) it is evident that models based on network structures have higher evaluation criteria. Since network DEA considers the impact of internal structure on DMU efficiency, it can achieve better performance by improving the internal structure when generating an ideal DMU. Therefore, the network DEA efficiency calculation introduces inefficiency information brought by the structure. Hence, the ratio of network overall efficiency to single overall efficiency represents the efficiency of collaboration between different DMU process.

4.2. The difference in process efficiency

Another situation is that when solving the process efficiency by single process DEA, it is assumed that the external environment in which each process is located is completely competitive. Based on the above assumptions, the PPS for individual process efficiency is:

$$\begin{aligned}
&\prod_{j=1}^{n}\left(X_{ij}^{q}\right)^{\lambda_j^q} = X_{io}^{q} * w_{io}^{-} \quad i \in X^q, q = 1,2,\ldots Q \\
&\prod_{j=1}^{n}\left(Y_{rj}^{q}\right)^{\lambda_j^q} = Y_{ro}^{q} * w_{ro}^{+} \quad r \in Y^q, q = 1,2,\ldots Q \\
&\prod_{j=1}^{n}\left(Z_{ij}^{q,h}\right)^{\lambda_j^q} \geq Z_{io}^{q,h} \quad i \in M^{q,h}, q = 1,2,\ldots Q, h \neq q \\
&\prod_{j=1}^{n}\left(Z_{ij}^{q,h}\right)^{\lambda_j^h} \leq Z_{io}^{q,h} \quad i \in M^{q,h}, q = 1,2,\ldots Q, h \neq q \\
&\prod_{j=1}^{n}\lambda_j^q = 1 \quad q = 1,2,\ldots Q \\
&0 \leq w_{io}^{-} \leq 1 \quad i = 1,2\ldots m \\
&1 \leq w_{ro}^{+} \quad r = r = 1,2\ldots s \\
&\lambda_j^q \geq 0 \quad j = 1,2\ldots n, q = 1,2,\ldots Q
\end{aligned} \tag{21}$$

The efficiency function of the individual non-oriented process efficiency is:

$$\theta_{oq}^* = \min \sqrt[m^q]{\prod_{i \in X^q} w_{io}^-} \Big/ \sqrt[s^q]{\prod_{r \in Y^q} w_{ro}^+} \quad \mathrm{q} = 1,2,\ldots \mathrm{Q} \tag{22}$$

But in fact, from the process level, the intermediate products between different processes are uncontrolled. Intermediate products can only be described as part of the process's external environment, not as one of the variables. So it is assumed that the intermediate products are all fixed links when calculating network process efficiency because they are not controlled by the DMU. The PPS of network process efficiency is:

$$\begin{aligned}
&\prod_{j=1}^{n}\left(X_{ij}^{q}\right)^{\lambda_j^q} = X_{io}^{q} * w_{io}^{-} \quad i \in X^q, q = 1,2,\ldots Q \\
&\prod_{j=1}^{n}\left(Y_{rj}^{q}\right)^{\lambda_j^q} = Y_{ro}^{q} * w_{ro}^{+} \quad r \in Y^q, q = 1,2,\ldots Q \\
&\prod_{j=1}^{n}\left(Z_{ij}^{q,h}\right)^{\lambda_j^q} = Z_{io}^{q,h} = \prod_{j=1}^{n}\left(Z_{ij}^{q,h}\right)^{\lambda_j^h} \quad i \in M^{q,h}, q = 1,2,\ldots Q, h \neq q \\
&0 \leq w_{io}^{-} \leq 1 \quad i = 1,2\ldots m \\
&1 \leq w_{ro}^{+} \quad r = r = 1,2\ldots s \\
&\prod_{j=1}^{n}\lambda_j^q = 1 \quad q = 1,2,\ldots Q \\
&\lambda_j^q \geq 0 \quad j = 1,2\ldots n, q = 1,2,\ldots Q
\end{aligned} \tag{23}$$

The efficiency function of the network non-oriented process efficiency is:

$$\theta_{oq}^* = \min \sqrt[m^q]{\prod_{i \in X^q} w_{io}^-} \Big/ \sqrt[s^q]{\prod_{r \in Y^q} w_{ro}^+} \quad \mathrm{q} = 1,2,\ldots \mathrm{Q} \tag{24}$$

In the network structure, we believe that there is a conflict of interest between processes. This conflict of interest is caused by intermediate products. In model (23) it is assumed that all links are fixed, which means that all links are not controlled by the DMU. This does not affect the efficiency of the measurement, but it does affect the evaluation criteria. It brings new constraints to the calculation of efficiency. The efficiency value can be interpreted as finding the ratio of the maximum possible output to the actual output while keeping the input constant and the link constant. It is obvious that the addition of new constraints will make the efficiency of network process efficiency higher than the individual process efficiency because it limits the ability of the DMU to achieve a higher ideal output. In other words, the ideal DMU of the process must be obtained under current market conditions, not optimal market conditions. Therefore, the ratio of individual process efficiency to network process efficiency can be seen as representing the effectiveness of the current process link variables trading market.

5. Regional sustainable development framework

The traditional DEA-based TFEE model must be based on the assumption that the DMU variables are all in a perfectly competitive environment. In other words, we are free to add or subtract any variable in the DMU to achieve higher efficiency. DMU external conditions that are completely competitive in practice are nonexistent. Productivity has become the most important target of the TFEE model. However In the 2012 United Nations Conference on Sustainable Development that Rio+20 proposed 17 SDGs. It was noted that ten of them involve human rights issues, one involves economic development, three concern environmental protection, one involves energy consumption, another relates to design industry innovation, with one remaining. It is clear that addressing human rights issues has become the main direction of sustainable development, but the TFEE model did not consider. Therefore, a new analytical framework should be proposed to meet the needs of existing sustainability research.

The human development index (HDI) is a summary measure of average achievement in key dimensions of human development: a long and healthy life, being knowledgeable, and having a decent standard of living. In fact, the above three key dimensions of human development are almost coincident with the 10 SDGs related to human rights issues. Their common goal is to reduce poverty, reduce hunger, improve the education of citizens, improve health, and ensure the equality of human rights. Therefore, HDI can replace 10 SDGs as an important output variable in regional sustainable development.

Rio+20 also ranks energy issues as one of 17 SDGs. But unlike traditional models that only care about reducing energy consumption, Rio+20's energy sustainability goals are affordable, reliable, and modern energy supplies. Therefore, simply reducing energy consumption is no longer the main goal of sustainable development. Efficient, clean energy supply can be a new goal for sustainable development. Therefore, in the new model, efficient energy supply serves as a major analytical direction.

5.1. *The network of sustainable development performance*

First and foremost, the model has a complete and comprehensive economic operating structure. Compared with the single DEA model without any internal structure and the simple two-stage DEA model with one-sided structure, the model can fully analyse the details of an economy's eco-efficiency. It does not need to be based on the assumption that the internal allocation of the system is always optimal. Fig. 2

The development of productivity and economic activities are inseparable. Both the Solow Growth Model and the Translog production model consider the economic activity to be one of the main factors affecting production [35,40]. Economic activity is mainly to provide capital input for production, but also to provide a market for output. This study focuses on economic activities to provide capital input for the production process. In classical economic theory, the model tells us that investment in the economy will be equal to the total amount income minus consumption spending. Thus, in our new model, we employed income as the input variable of economic activities, and investment and consumption are output variables of economic activities. Fig. 3

The most important element of regional development is human

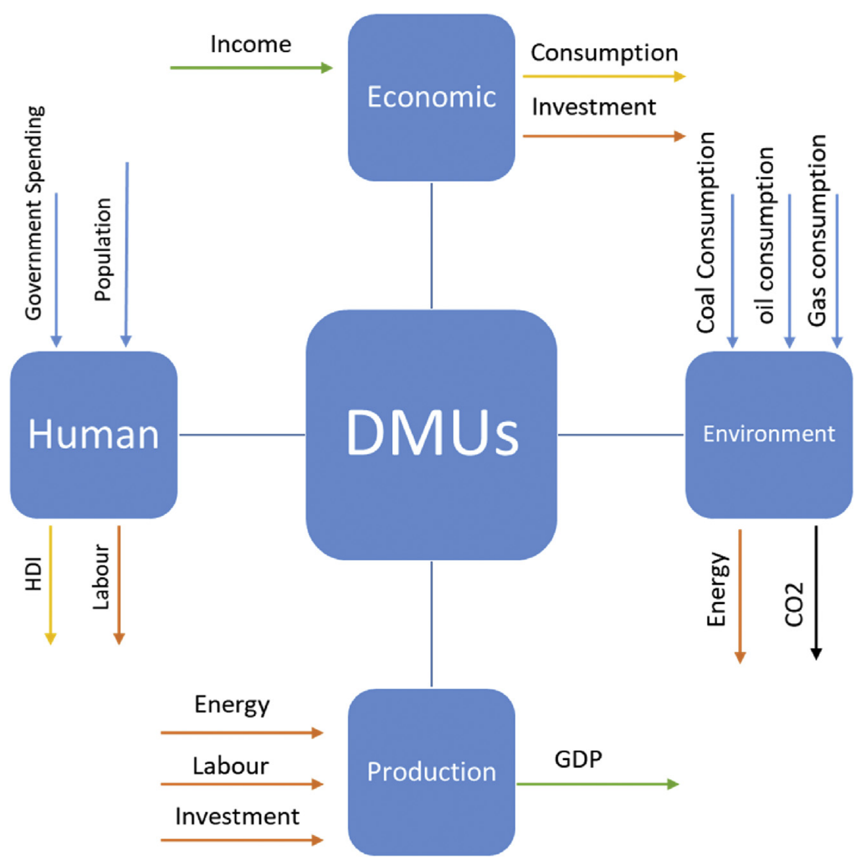

Fig. 2. Input-output structure of four department.

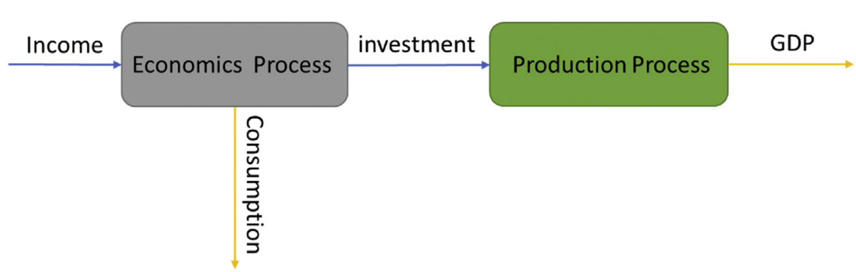

Fig. 3. The connect between economic and production.

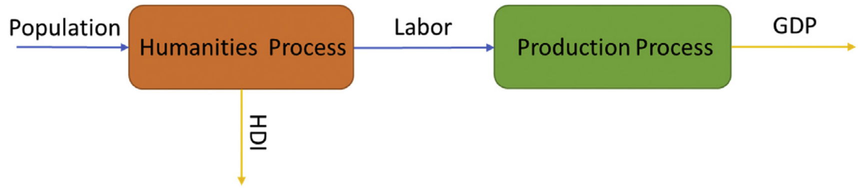

Fig. 4. The connect between humanities and production.

development, so in this study, the humanities department is one of the departments that focus on the discussion. Human development is not only an important indicator of regional development but also provides the necessary labor support for the production process. Therefore, we believe that the humanities department is the process of transforming the population into a labor force. On the other hand, the Human Development Index (HDI) is a summary measure of average achievement in key dimensions of human development: a long and healthy life, being knowledgeable and have a decent standard of living. In fact, the above three key dimensions of human development are almost coincident with the 10 SDGs related to human rights issues. Their common goal is to reduce poverty, reduce hunger, improve the education of residents, improve the health of residents, and ensure the equality of human rights. Therefore, HDI can represent the regional human development status as the output of the humanities department. Fig. 4

The pollution generated by the energy in the production process has always been the mainstream thinking of energy efficiency analysis (R.C [39]. However, with the massive use of clean energy, the inevitable relationship between energy consumption and carbon dioxide emissions is being broken. For example, an electric car does not produce any pollution at all when driving. In fact, several documents prove that industrial production is more dependent on indirect energy than direct energy [41,42]. The use of indirect energy does not directly generate CO2 emissions. Thus, CO2 emissions should be separated from the production department. We believe that CO2 occurs in the process of burning fossil fuels to generate energy. Therefore, we believe that the generation and use of energy should belong to two different processes. The process of generating energy also produces pollution. Energy is consumed for the production process. Fig. 5

The production department has always been the main research object of regional efficiency research. For example, the TFEE model only discusses the production department. In this study, the productivity department and the TFEE model have the same structure. So, the model of this study is an extension of TFEE. Fig. 6

In this complex network model, we set up seven input variables such as Income (X1), population (X2), government spending (X3), coal consumption (X4), oil consumption (X5), natural gas consumption (X6) and land (X7). At the same time, it also has three output variables that GDP (Y1), human development index (Y2) and consumption (Y3), and an undesired output CO2 emission (B1). The complex internal structure of the model consists of three intermediate variable investment (M1), labor (M2), and energy (M3). The intermediate variable investment is the channel connecting the production department with the economic department. Investment is both the output variable of the economic department and the input variable of the production department. The same, labor is an intermediate variable linking the human department to the production department. It is both the output of the population department and the input of the production department. Final, energy is an intermediate variable linking the energy department to the production department. It is both the output of the energy department and the input of the production department.Fig. 7

5.2. *The network DEA model for sustainable development*

First of all, we believe that the entire DMU seeks to maximise the efficiency of the system. Therefore, the objective function of the model concerns the overall system rather than the internal structure. But in the time dimension, it is hoped that the system efficiency of each process can reach the optimal value. The objective function is as follows:

For overall efficiency

$$E_k = Min \sqrt[m]{\prod_{i=1}^{m} w_{io}^-} \Big/ \sqrt[s]{\prod_{r=1}^{s} w_{ro}^+}$$

For processes efficiency

$$\theta_{oq}^* = \min \sqrt[m^q]{\prod_{i\in X^q} w_{io}^-} \Big/ \sqrt[s^q]{\prod_{r\in Y^q} w_{ro}^+} \qquad q=1,2,\ldots Q \qquad (25)$$

Regarding constraints, each sub-sector in the system should satisfy the condition that the input utility is greater than the output utility. Therefore the constraints are as follows:

For human:

$$\prod_{j=1}^{n} \left(X_{ij}^{Pop}\right)^{\lambda_j^{Pop}} = X_{io}^{Pop} * w_{io}^- \quad i=2,3$$

$$\prod_{j=1}^{n} \left(Y_{rj}^{Pop}\right)^{\lambda_j^{Pop}} = Y_{ro}^{Pop} * w_{ro}^+ \quad r=2$$

For economic:

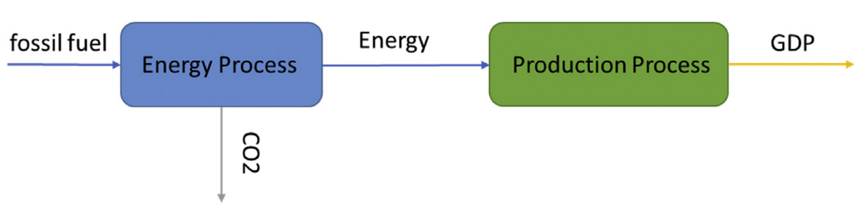

Fig. 5. The connect between energy and production.

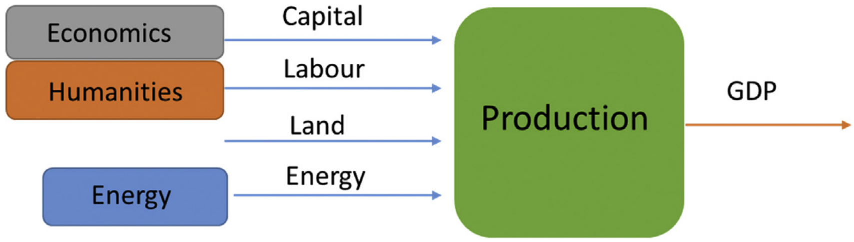

Fig. 6. The relation in four department.

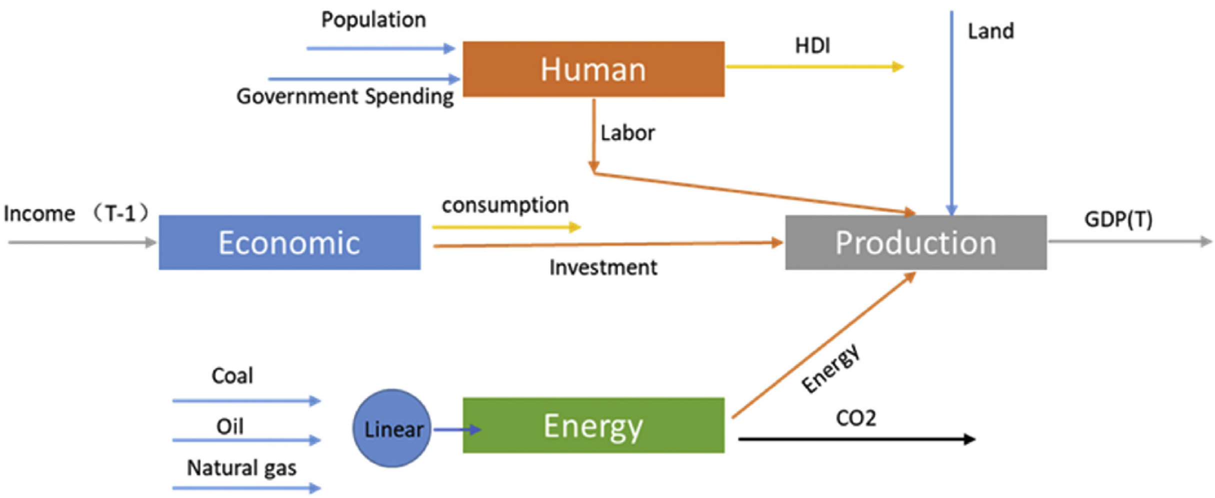

Fig. 7. The relation map for regional Energy and environmental efficiency.

$$\prod_{j=1}^{n}\left(X_{ij}^{Eco}\right)^{\lambda_j^{Eco}}=X_{io}^{Eco}*w_{io}^{-}\quad i=1$$

$$\prod_{j=1}^{n}\left(Y_{rj}^{Eco}\right)^{\lambda_j^{Eco}}=Y_{ro}^{Eco}*w_{ro}^{+}\quad r=3$$

For energy:

$$\prod_{j=1}^{n}\left(X_{ij}^{Energy}\right)^{\lambda_j^{Energy}}=X_{io}^{Energy}*w_{io}^{-}\quad i=4,5,6$$

$$\prod_{j=1}^{n}\left(B_{bad,j}^{Energy}\right)^{\lambda_j^{Energy}}=B_{bad,o}^{Energy}*wb_{bad,o}^{-}\quad bad=1$$

For production:

$$\prod_{j=1}^{n}\left(Y_{rj}^{Pro}\right)^{\lambda_j^{Pro}}=Y_{ro}^{Pro}*w_{ro}^{+}\quad r=1$$

$$\prod_{j=1}^{n}\left(X_{ij}^{Pro}\right)^{\lambda_j^{Pro}}=X_{io}^{Pro}*w_{io}^{-}\quad i=7$$

Free link:

$$\prod_{j=1}^{n}\left(Z_{ij}^{Eco,Pro}\right)^{\lambda_j^{Pop}}=\prod_{j=1}^{n}\left(Z_{ij}^{Eco,Pro}\right)^{\lambda_j^{Pop}}\quad \mathrm{i}=1$$

$$\prod_{j=1}^{n}\left(Z_{ij}^{Pop,Pro}\right)^{\lambda_j^{Pop}}=\prod_{j=1}^{n}\left(Z_{ij}^{Pop,Pro}\right)^{\lambda_j^{Pop}}\quad \mathrm{i}=2$$

$$\prod_{j=1}^{n}\left(Z_{ij}^{Energy,Pro}\right)^{\lambda_j^{Pop}}=\prod_{j=1}^{n}\left(Z_{ij}^{Energy,Pro}\right)^{\lambda_j^{Pop}}\quad \mathrm{i}=3$$

Fix link:

$$\prod_{j=1}^{n}\left(Z_{ij}^{Eco,Pro}\right)^{\lambda_j^{Pop}}=Z_{io}^{Eco,Pro}=\prod_{j=1}^{n}\left(Z_{ij}^{Eco,Pro}\right)^{\lambda_j^{Pop}}\quad \mathrm{i}=1$$
$$\prod_{j=1}^{n}\left(Z_{ij}^{Pop,Pro}\right)^{\lambda_j^{Pop}}=Z_{io}^{Pop,Pro}=\prod_{j=1}^{n}\left(Z_{ij}^{Pop,Pro}\right)^{\lambda_j^{Pop}}\quad \mathrm{i}=2$$
$$\prod_{j=1}^{n}\left(Z_{ij}^{Energy,Pro}\right)^{\lambda_j^{Pop}}=Z_{io}^{Energy,Pro}=\prod_{j=1}^{n}\left(Z_{ij}^{Energy,Pro}\right)^{\lambda_j^{Pop}}\quad \mathrm{i}=3$$
(26)

This study is a substantial improvement in the regional eco-efficiency assessment model. This model completely abandons the simple one-sided analysis of the traditional eco-efficiency assessment model. Regional eco-efficiency has been reinterpreted with a complex and systematic model to replace the single stage model. It can yield more abundant and detailed results; it can analyse each sub-structure in the regional system, and it offers more than a one-sided independent analysis. For policy makers, the

Table 2
The data description in 2014.

Variable	Unit	Max	Min	S.D	Mean
Input					
income	Millions dollars	14,454,861	87365.6	2,479,624	1369S17.296
Population	Thousand people	31649S	4442	70,796	512S25S4
Government Spending	Millions dollars	2,565,199	17799.3	490310.5	307S85.6064
Oil	Thousand Tons	907,600	7300	172,000	75732.41
Coal	Thousand Tons	340,600	100	68,200	35,385.S
Natural Gas	Thousand Tons	645,100	800	122,000	4523.08
land	Km2	4,349,250	5045	1,146,094	529724.5287
Output					
Consumption	Millions dollars	143S9232	94435.1	249S0S0	1343111.043
GDP	Millions dollars	16,242,526	130,070	2,966,750	1674825.903
HDI	Score	94.6	42.4	9.S67814	85.39722222
Undesired					
CO2	Million Tons	56S6.7	29.7	976.19S2	440.5902778
Intermediate					
Labor	Thousand people	1596S4.41	2694.6	33753.04	24,467.43S03
Energy	Million toe	2337.47	16.9	417.6596	196.9603704
Inferotemporal					
Investment	Million US dollars	3,556,283	26,545	595540.7	359535.6296

model will no longer be used only for tagging and reference evaluation models, but as a tool that can assist in detailed project policy planning.

6. Illustrative example and results

6.1. Data description

In this study, there are a total of 14 observed variables, including 7 input variables, 3 output variables, 3 intermediate products, and 1 undesired output variable.

The summary of data is shown in Table 2:

GDP, consumption, investment, and government spending data come from OECD Aggregate National Accounts. The population and labour data come from OECD Labour Force Statistics. The CO2 emission data come from IEA CO2 Emissions from Fuel Combustion Statistics. The energy consumption data come from IEA World Energy Statistics and Balances. And the oil, coal, natural gas consumption data come from the BP Statistical Review of World Energy. Finally, the HDI data come from the United Nations Development Programme (UNDP) Human Development Reports. A DMU entry was deleted owing to a missing value between the different data resources. There are 27 DMUs which have a complete set of 14 variables. The 27 countries left in our analysis contributed up to 60% of the global GDP; moreover, they shared 65% of global energy demand and 67% of CO2 emissions worldwide. With the availability of further data, more countries could be included in the analysis to expand the span of analysis to include a higher proportion of total global GDP, energy consumption, and CO2 emissions. It should be noted that, since the multiplicative DEA model cannot measure accurate efficiency when dealing with elements with complementary properties, the fuels coal, gas, and oil with

Table 3
The efficiency result based on single process DEA.

	Overall	Economic	Humanities	Energy	production
Austria	0.76430343	0.84975924	0.73781054	03,799,048	0.2760609
Belgium	1	0.87342399	0.6141821	0.18244264	0.66092965
Finland	1	0.91713845	0.74738449	1	1
France	1	0.88658355	0.54885321	1	0.09443229
Germany	1	0.85538639	1	0.35482352	0.19363348
Italy	0.72503295	0.91363424	0.56948925	0.26470891	0.28511998
Netherlands	0.61211895	0.79215856	0.70843929	0.16115948	1
Norway	1	0.69635308	1	0.9154724	1
Portugal	1	1	0.86677103	0.57015808	1
Spain	0.58727966	0.90324293	0.74444439	0.26721761	0.05932391
Sweden	1	0.76611059	0.66738467	1	0.29798129
Switzerland	1	0.75653623	1	1	1
United Kingdom	1	1	0.8586541	0.25673126	1
Hungary	1	1	1	0.66956602	0.05079612
Turkey	1	1	0.76181765	0.16916912	0.03105721
Greece	1	1	0.79478457	0.2970814	1
Poland	0.32485128	0.935684	0.92969431	0.15057304	0.04925921
Czech Republic	1	1	0.49238133	0.20616961	0.1047953
United States	1	1	1	1	1
Brazil	0.29742085	0.91625282	1	0.59589199	0.00794961
Chile	1	0.95941038	1	0.28310708	0.02726647
Mexico	1	1	1	0.23125552	0.01133385
Israel	1	0.8965177	0.73397026	0.28529657	1
Japan	1	0.8924806	1	0.33744544	1
Korea	1	0.85313811	1	0.3307894	0.79778162
Australia	0.25588592	0.81607476	0.87409872	0.14335156	0.00426946
New Zealand	1	0.95643664	1	1	1

Table 4
The efficiency result based on network DEA with free link.

	Overall	Economic	Humanities	Energy	production
Austria	0.52701856	0.83937141	0.71359936	0.33261845	0.352514
Belgium	0.50795655	0.86522206	0.63587788	0.12713661	1
Finland	0.64916271	0.92892476	0.82147471	0.64204092	0.32345649
France	0.4557039	0.88225837	0.48096316	0.36254076	0.31215564
Germany	0.57420749	0.85230269	0.87909889	0.19070182	0.64891283
Italy	0.35651943	0.91808835	0.41238559	0.11613118	0.4331044
Netherlands	0.54803197	0.79215856	0.70843929	0.16115948	1
Norway	0.8948086	0.69635308	1	0.9154724	1
Portugal	0.67404412	1	0.76692343	0.60863873	0.43648763
Spain	0.24989528	0.89799047	0.39129361	0.06124263	0.17161863
Sweden	0.43315819	0.76514744	0.59839607	0.73022401	0.08490364
Switzerland	0.93262582	0.75653623	1	1	1
United Kingdom	0.69131834	1	0.8586541	0.25673126	1
Hungary	0.62792936	1	1	0.71207495	0.17661201
Turkey	0.13057671	0.80181978	0.22032098	0.01948009	0.08369412
Greece	0.68665417	1	0.79478457	0.2970814	1
Poland	0.15423666	0.88431976	0.2814576	0.02086536	0.11051239
Czech Republic	0.35617869	1	0.43443902	0.15958056	0.19454433
United States	1	1	1	1	1
Brazil	0.13949932	0.89294557	0.19246929	0.11279169	0.01804083
Chile	0.27882445	0.98322274	0.70131835	0.23825024	0.0222938
Mexico	0.09620843	0.96653206	0.18048495	0.01147057	0.03820352
Israel	0.64242356	0.8965177	0.73397026	0.28529657	1
Japan	0.75848659	0.8924806	1	0.33744544	1
Korea	0.21647878	0.80765021	0.4128591	0.00842847	0.79417365
Australia	0.15046968	0.80990319	0.77547597	0.02608229	0.01047477
New Zealand	1	1	1	1	1

complementary characteristics cannot be directly measured by the current model. Therefore, we use the thermal values of the fuels to obtain the total fuel supply of coal equivalent by linear weighting.

6.2. The single overall efficiency and individual process efficiency

The overall efficiency in Table 3 shows the single overall efficiency based on models (17) and (18). It represents the efficiency value of the DMU's internal structure as a black box. The process efficiency in Table 3 shows the individual process efficiency based on models (21) and (22). It represents the individual efficiency of each process in the DMU without considering any link existence.

Obviously, we can see that in many DMUs, process efficiency cannot be efficient, but overall efficiency is efficient. This is because a single overall efficiency and individual process efficiency are based on two completely different assumptions, so there is no relationship between them.

6.3. The results of the network model with free link

The results of the network model with free link are shown in Table 4 based on models (19) and (20). Its overall efficiency represents the efficiency gained after considering the internal structure of the DMU. This means that the DMU can achieve better results by adjusting the internal structure, so the overall efficiency obtained by the network DEA contains the inefficiency information caused by the structure.

It is obvious that there is a relationship between process efficiency and overall efficiency because internal structures and connections are considered in the model. Therefore, the overall efficient DMU must satisfy the condition that all the processes are efficient.

6.4. The results of the network model with the fixed link

The results of the network model with fixed link are shown in Table 5 based on models (23) and (24). Its overall efficiency represents the efficiency that the DMU can achieve when all links in the DMU internal structure are known to be fixed one-way transmissions. Its process efficiency represents the ratio of the minimum input to the actual input that each process can achieve in the case where the intermediate products between processes are completely determined. Since the fixed link means that the link is completely uncontrolled by the DMU, the efficiency can also be considered as the conditional efficiency of the process under the current DMU internal conditions.

6.5. Comparing the efficiency from different models

To clarify favourable features of the network model, results are compared with the single processes model and two categories of network models. Fig. 8. Shows the overall efficiency in single, free link and fixed link models (see Fig. 8).

It is evident that, because the assumptions of the fixed link model are essentially different from the single model, there is no relationship between their overall efficiency. The gap between the free link model and the single model is only in the presence of the internal structure of the DMU, so the efficiency of the free link is strictly less than or equal to the single model. In section 4.1 it was explained that the ratio of overall efficiency to single overall efficiency based on the free link assumption could represent the efficiency of DMU internal structure operation, which we call structural friction efficiency, as shown in Fig. 9.

On the other hand, we propose in section 4.2 that the efficiency value of the individual process efficiency is less than or equal to the process efficiency of the fixed link model, because the fixed link model imposes stricter restrictions on the external environment of the process. The process efficiency based on the fixed link model represents the ratio of an ideal to actual performance of the process in the current internal environment. Its ratio to the efficiency of the single model reflects the efficiency of the environment in which the process is located. In other words, it can be interpreted as the market efficiency of the process's intermediate product. In the sustainable development network model, investment, labour, and

Table 5
The efficiency result based on network DEA with fixed link.

	Overall	Economic	Humanities	Energy	production
Austria	0.55720513	0.84975924	0.73781054	0.3799048	0.37904181
Belgium	0.54481608	0.87342399	0.6141821	0.18244264	1
Finland	0.87101645	0.91713845	0.74738449	1	1
France	0.62232144	0.88658355	0.54885321	1	0.36292517
Germany	0.68636056	0.85538639	1	0.35482352	0.61417878
Italy	0.50433257	0.91363424	0.56948925	0.26470891	0.51847586
Netherlands	0.54803197	0.79215856	0.70843929	0.16115948	1
Norway	0.8948086	0.69635308	1	0.9154724	1
Portugal	0.83175642	1	0.86677103	0.57015808	1
Spain	0.50720892	0.90324293	0.74444439	0.26721761	0.32626014
Sweden	0.51465749	0.76611059	0.66738467	1	0.11310315
Switzerland	0.93262582	0.75653623	1	1	1
United Kingdom	0.69131834	1	0.8586541	0.25673126	1
Hungary	0.64024718	1	1	0.66956602	0.20327148
Turkey	0.60086601	1	0.76181765	0.16916912	1
Greece	0.68665417	1	0.79478457	0.2970814	1
Poland	0.47840198	0.935684	0.92969431	0.15057304	0.33966898
Czech Republic	0.40771843	1	0.49238133	0.20616961	0.2206913
United States	1	1	1	1	1
Brazil	0.87491821	0.91625282	1	0.59589199	1
Chile	0.46047201	0.95941038	1	0.28310708	0.10801965
Mexico	0.62638999	1	1	0.23125552	0.57087886
Israel	0.64242356	0.8965177	0.73397026	0.28529657	1
Japan	0.75848659	0.8924806	1	0.33744544	1
Korea	0.74936519	0.85313811	1	0.3307894	1
Australia	0.27266971	0.81607476	0.87409872	0.14335156	0.02366191
New Zealand	1	1	1	1	1

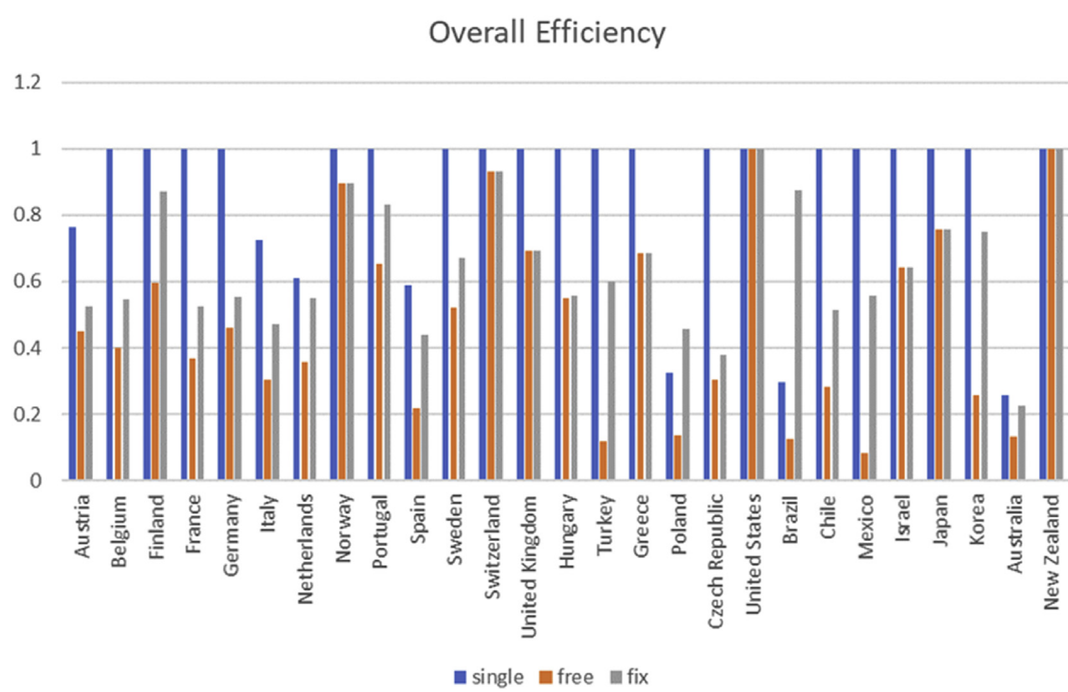

Fig. 8. The Overall efficiency in 27 OECD countries.

energy supply correspond to intermediate products in the economic, human, and energy departments, respectively. Therefore, the conflict efficiency of the economic department can be regarded as the demand efficiency of the regional financial market; the conflict efficiency of the human department can be regarded as the demand efficiency of the labour market; and the conflict efficiency of the energy department can be regarded as the demand efficiency of the energy market. Fig. 10

We can see in the conflict efficiency of the Economic department that only New Zealand is inefficient. All 27 OECD countries are conflict efficient in the energy department and humanities department, it means that there is no redundancy in energy supply and labour supply in all countries. In the production sector, it was mainly affected by internal conflicts. Brazil, Mexico, and Turkey have very low conflicts in production department conflict efficiency. This shows that the efficiency of the production department in the above three countries is significantly constrained by other departments (see Fig. 10).

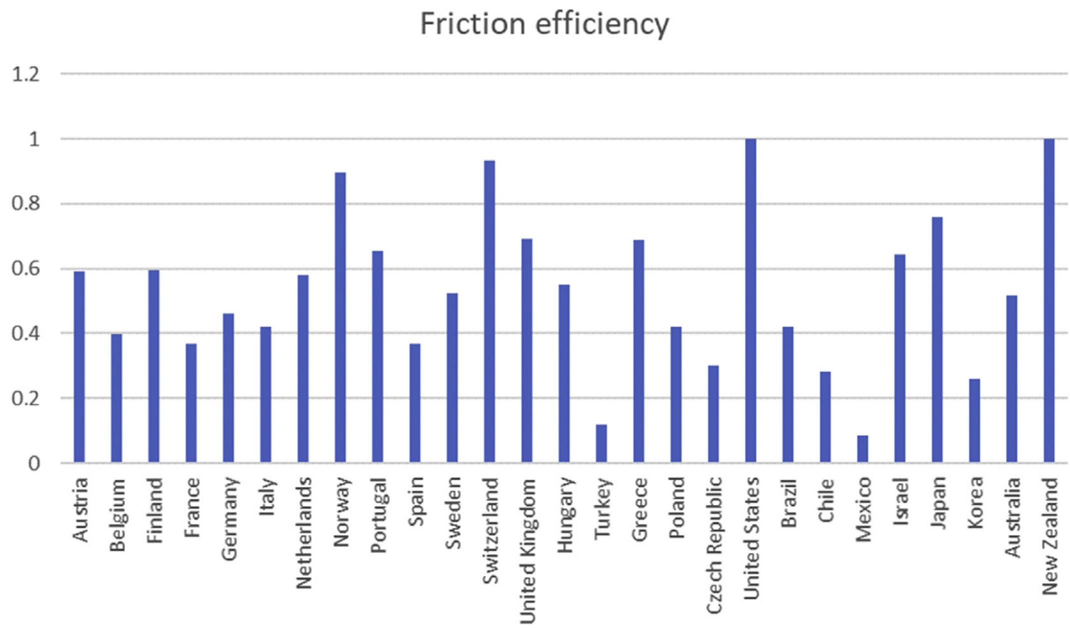

Fig. 9. The friction efficiency in 27 OECD countries.

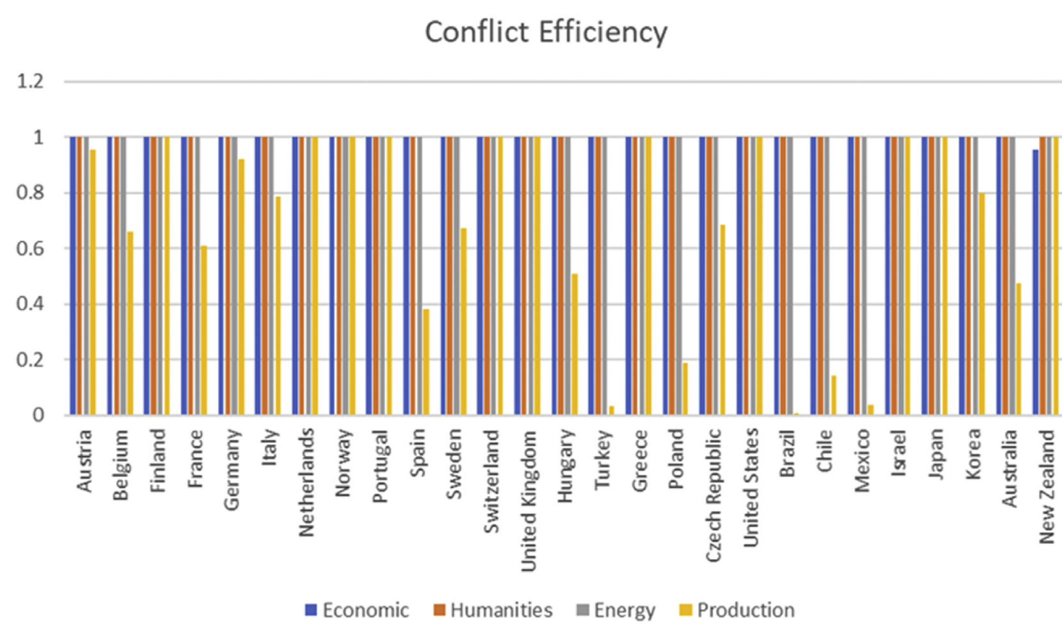

Fig. 10. The conflict process efficiencies in 27 OECD countries.

7. Discussion

This article discusses the overall efficiency and process efficiencies of 27 OECD countries in the MNDEA model. Please note that Australia's production department efficiency is very low (around 2.3%) when analyzing process efficiencies based on the fix link. This result seems unreasonable, but actually reflects the actual production efficiency. The biggest inefficiency comes from the extremely low utilization rate of land in Australia. Fig. 11 shows the Scatter plot of GDP vs land area.Fig. 11

We find that the US GDP is more than 10 times that of Australia, with almost the same land input. Therefore, it is reasonable for Australia's production department efficiency to be below 10%.

On the other hand, we noticed the difference between the MNDEA model and the traditional network SBM model. The following table shows the results of the overall efficiency calculation of the network SBM model and the MNDEA model based on different perspectives. We believe that MNDEA with the free link has a similar perspective to the network SBM in external evaluation, and MNDEA with fix link has a similar perspective to the network SBM in internal evaluation.Table 6

We found that the overall efficiency of the MNDEA model is always less than the overall efficiency of the SBM, whether it is internal evaluation or external evaluation. In fact, these gaps are ubiquitous in the multiplicative efficiency model and the linear efficiency model. In fact, this is caused by the theoretical difference

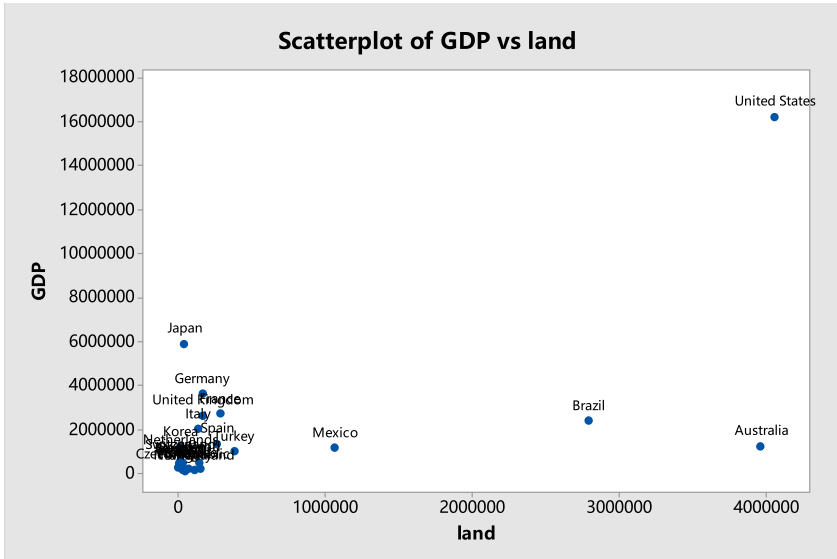

Fig. 11. The scatterplot of GDP vs Arable land.

Table 6
The overall efficiency from SBM-DEA and MNDEA.

	SBM-External	SBM-Internal	MNDEA-Free	MNDEA-Fix
Austria	0.642672035	0.681471173	0.527018565	0.557205133
Belgium	0.687674756	0.725773742	0.507956551	0.544816084
Finland	0.828844189	0.923266409	0.649162713	0.871016448
France	0.62870744	0.820040208	0.455703904	0.622321442
Germany	0.774964173	0.834963407	0.57420749	0.68636056
Italy	0.566952044	0.664453354	0.35651943	0.504332567
Netherlands	0.720949039	0.724130643	0.548031969	0.548031969
Norway	0.927960628	0.909950785	0.894808602	0.894808602
Portugal	0.733002752	0.921300804	0.674044122	0.831756422
Spain	0.464143123	0.694699516	0.249895276	0.50720892
Sweden	0.582005067	0.67967575	0.433158194	0.514657491
Switzerland	0.952640097	0.940800121	0.932625817	0.932625817
United Kingdom	0.860692106	0.856669434	0.691318336	0.691318336
Hungary	0.861953635	0.828209979	0.627929363	0.640247177
Turkey	0.318551781	0.808465197	0.130576712	0.6Q0866013
Greece	0.847850902	0.858580718	0.686654172	0.686654172
Poland	0.343248327	0.8456664	0.154236664	0.478401985
Czech Republic	0.644422459	0.782337381	0.356178687	0.407718428
United States	1	1	1	1
Brazil	0.355117957	0.919997422	0.139499319	0.874918212
Chile	0.565774875	0.689682357	0.278824448	0.460472007
Mexico	0.319734917	0.875254242	0.096208434	0.626389988
Israel	0.79026057	0.806984992	0.642423562	0.642423562
Japan	0.937045397	0.921306747	0.758486593	0.758486593
Korea	0.509599366	0.869921966	0.216478777	0.749365186
Australia	0.55247548	0.779435632	0.150469682	0.272669711
New Zeal a nd	1	1	1	1

between the multiplicative model and the linear model. SBM is a linear weighting model whose goal is to maximise the virtual profit (K [31]. In this study, based on the SBM model, Australia's land variable slack accounted for 96.3% of the total land. In other words, Australia's inefficiency value is only 3.7% from the perspective of the land. Due to the linear weighted nature of the SBM model, the inefficiency of land can be converted to 19.26% of the total inefficiency. Further, even if Australia's efficiency in the land is 0, it will only reduce the overall efficiency by 20%. Thus, the overall efficiency in the SBM model is higher than MNDEA model. It is obviously unreasonable because the efficiency of land is not an independent 20% part of the overall efficiency. No country can produce any output when land efficiency is zero. The multiplicative model assumes that variables are related and that they cooperate or repel each other. So, if Australia's efficiency in the land is 0, then its overall efficiency will be 0 as well. This is more in line with the economic significance of actual production activities. Therefore, we believe that the multiplicative model should be superior to the linear model, where variables need to cooperate with each other to produce an output.

8. Conclusion

This chapter mainly discusses the network improvement of multiplicative DEA. We found that multiplicative DEA can better explain the efficiency assignment of sequence structure and parallel structure relative to linear DEA. This feature allows us to decompose more easily the overall efficiency of the network DEA. The relationship between the efficiency of network DEA and the efficiency of single process DEA is also discussed. The results are meaningful, and technical efficiency has been further broken down to obtain conflict efficiency and friction efficiency. The latter represents the impact of a DMU's internal conflicts on its overall efficiency. In management theory of the firm, an inefficient DMU is allowed to exist in the market. Therefore, the single process DEA will be affected by these inefficient DMUs for error evaluation. In 1980, Fama [43] believed that the inefficiency of a DMU was more due to internal conflicts than technical flaws. Friction efficiency can readily distinguish the inefficiency information caused by internal conflicts existing in the DMU. However, conflict efficiency represents the department's influence on overall performance. The low efficiency of the conflict means that the performance of the department is wasted. The high efficiency of conflicts means that the performance of the department is being used. From another perspective, the high conflict efficiency of a department means that overall efficiency gains are mainly hindered by this department.

Possible avenues for future research include: (1) Further exploring the relationship between conflict efficiency and friction efficiency and understanding why the gap between them is generated. (2) The results ratio between free link network DEA and single process DEA is worthy of attention. We hope to explore the economic significance of this ratio. (3) Combining network DEA technology to form a DEA model with dynamic network dual structure. (4) Applying the MNDEA model to more areas such as economics, factories, resource management, and organisational performance. (5) Discussing the role of price factors in network DEA assessment.\

Declaration of interest

1. confirm that there's no financial/personal interest or belief that could affect my objectivity
2. This is an article that relies entirely on personal research interests, all from world bank open data. This article does not accept any funding sources

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