Demand Analysis with Aggregation Systems

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The demand is a fundamental variable in economic analysis that measures the needs of goods and services of the consumers. This paper presents a new approach for representing the demand by using aggregation systems that consider the attitudinal character of the consumers and their beliefs regarding the degree of importance of the different variables that may affect them. Several developments are introduced by using multiperson systems, different criterion, attributes, and states of nature. These developments are focused on the use of the weighted average, the ordered weighted average, and the ordered weighted averaging weighted average. Further aggregation systems are suggested by using the concept of the demand growth. The paper ends with an application of the new approach in a forecasting process that considers the attitude of the decision maker and its subjective beliefs.

1. INTRODUCTION

The demand is a fundamental variable in business and economics because it determines the needs of the consumers regarding a wide range of products or services. In the literature, there are a wide range of studies in this area that have focused in a lot of different perspectives[1,2] and applications.[3–5] A fundamental issue when studying the demand is that usually, when we analyze the expected demand for a future period, the demand is not clearly known so it is necessary to use techniques that are able to deal with imprecise information. A very common tool for doing so is the aggregation operator.[6,7]

A very well-known aggregation operator is the ordered weighted average (OWA).[8–10] It is an aggregation operator that provides a parameterized family of aggregation operators between the minimum and the maximum. An interesting extension of the OWA operator is the generalized OWA (GOWA) operator.[11] It generalizes the OWA operator by using generalized means and including a wide range

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of particular cases such as the quadratic and geometric OWA. Note that the GOWA operator can be further generalized by using quasi-arithmetic means forming the quasi-arithmetic OWA (Quasi-OWA) operator.[12,13]

Recently, Merigó[14] has suggested a new approach that integrates the OWA operator and the weighted average in the same formulation considering the degree of importance that each concept has in the formulation. This paper presents a new generalization that unifies the OWA operator with the weighted average by using generalized means. Thus, the generalized OWA weighted average (GOWAWA) operator is obtained. Its main advantage is that it includes a wide range of aggregation operators including the OWA operator, the GOWA operator, and the generalized weighted average. A further generalization is presented by using quasi-arithmetic means forming the quasi-arithmetic ordered weighted averaging weighted average (OWAWA) (Quasi-OWAWA) operator.

These new techniques can be implemented in a wide range of applications. This paper is focused on the analysis of the demand. Thus, it is analyzed several aggregation frameworks to reach the most efficient forecasts. Therefore, it is possible to aggregate the expected demand considering a wide range of scenarios from the minimum demand to the maximum one. The main advantage of this approach is that it provides a robust framework for analyzing the demand so it is correctly assessed without losing any information in the analysis. Several perspectives regarding the formation of the expected demand are studied including a wide range of criteria that can be used in the analysis, especially because in business and economics it is usually assumed that the demand depends on other variables such as the price and the quantity.

An illustrative example concerning the formation of the demand according to several criteria, expert opinions, and states of nature is presented. By using this approach, it is possible to consider a wide range of situations and deal with those that are most expected to occur. Note that this paper is strictly focused on the analysis and formation of the demand. However, to develop a complete analysis, it is necessary to consider many other variables such as the supply and other external factors. However, the aim of this paper is to understand how the demand can be studied in uncertain environments where the information is not clearly known and considering the attitudinal character of the experts or decision makers that are dealing with this problem.

The paper is organized as follows. Section 2 reviews some basic concepts regarding the demand and the aggregation operators. Section 3 analyzes the demand with a wide range of aggregation systems. Section 4 focuses on the use of aggregation processes in the analysis of the demand growth. Section 5 presents an illustrative example of the new approach, and Section 6 summarizes the main findings of the paper.

2. PRELIMINARIES

2.1. Demand Analysis

The demand is a very broad concept that has been used in a wide range of fields including macroeconomic theory,[15] monetary economics,[16] and microeconomics. In this section, the focus is in microeconomic analysis regarding consumer demand.[2]

In this context, the demand can be formulated in different ways. A very common approach is the following one:

$$Q_x^d = f(P_x, P_y, M, H), \tag{1}$$

where P_x is the price of the good, P_y the price of related goods, M the income, and H any other variable. A very practical way for representing this approach is as follows:

$$Q_x^d = \alpha_0 + \alpha_x P_x + \alpha_y P_y + \alpha_M M + \alpha_H H, \tag{2}$$

where α_0, α_x, α_y, α_M, and α_H are the parameters that determine the influence of each variable. A very interesting issue is to analyze the variations through time. Thus, it is possible to analyze the demand as follows:

$$\mathrm{DG} = \frac{Q_{xt}^d}{Q_{x(t-1)}^d} - 1. \tag{3}$$

If the demand is decomposed in its variables and we use a transformation $\alpha_{0(t\text{-}1)}/Q^d{}_{x(t\text{-}1)}$, and so on, it is formed the following equation:

$$\mathrm{DG} = \left(\begin{array}{l} \frac{\alpha_{0(t-1)}}{Q_{x(t-1)}^d}\left(\frac{\alpha_{0t}}{\alpha_{0(t-1)}} - 1\right) + \frac{\alpha_{x(t-1)}P_{x(t-1)}}{Q_{x(t-1)}^d}\left(\frac{\alpha_{xt}P_{xt}}{\alpha_{x(t-1)}P_{x(t-1)}} - 1\right) \\ + \frac{\alpha_{y(t-1)}P_{y(t-1)}}{Q_{x(t-1)}^d}\left(\frac{\alpha_{yt}P_{yt}}{\alpha_{y(t-1)}P_{y(t-1)}} - 1\right) + \frac{\alpha_{M(t-1)}M_{t-1}}{Q_{x(t-1)}^d}\left(\frac{\alpha_{Mt}M_t}{\alpha_{M(t-1)}M_{t-1}} - 1\right) \\ + \frac{\alpha_{H(t-1)}H_{t-1}}{Q_{x(t-1)}^d}\left(\frac{\alpha_{Ht}H_t}{\alpha_{H(t-1)}H_{t-1}} - 1\right) \end{array} \right). \tag{4}$$

Note that the first part of each subequation can be seen as the weight of each variable in the whole structure, forming a kind of weighted average. Thus, Equation (4) can be expressed as follows:

$$\mathrm{DG} = \left(\begin{array}{l} \mathrm{WA}_{\alpha_0}\left(\frac{\alpha_{0t}}{\alpha_{0(t-1)}} - 1\right) + \mathrm{WA}_{\alpha_x P_x}\left(\frac{\alpha_{xt}P_{xt}}{\alpha_{x(t-1)}P_{x(t-1)}} - 1\right) \\ + \mathrm{WA}_{\alpha_y P_y}\left(\frac{\alpha_{yt}P_{yt}}{\alpha_{y(t-1)}P_{y(t-1)}} - 1\right) + \mathrm{WA}_{\alpha_M M}\left(\frac{\alpha_{Mt}M_t}{\alpha_{M(t-1)}M_{t-1}} - 1\right) \\ + \mathrm{WA}_{\alpha_H H}\left(\frac{\alpha_{Ht}H_t}{\alpha_{H(t-1)}H_{t-1}} - 1\right) \end{array} \right), \tag{5}$$

where WA is the weighted average for each variable considered. Note that sometimes these weights have to be forecasted. In these situations, it is used some kind of beliefs regarding the importance that each variable should have. Moreover, if they are not known, it is possible to use OWA operators in the analysis. Note that when using the weighted average and the OWA at the same time because there is some partial information about the variables but it is incomplete and assessed with the degree of optimism of the decision maker, we are using the OWAWA.[14] Further information will be given in the following subsection.

The demand can also be represented by using other expressions. A very common one used in microeconomic theory is the analysis of products bought and their respective price. Note that there is the implicit assumption that the demand is equal to the income of the consumer, and thus the demand is restricted to the budget constraint.[2] It can be formulated as follows:

$$D = \sum_{i=1}^{n} p_i x_i = Y, \tag{6}$$

where p_i is the price and x_i the quantity of the ith product. Note that it can also be formulated in the following way:

$$D = p_1 x_1 + p_2 x_2 + \cdots + p_i x_i + \cdots + p_n x_n = Y. \tag{7}$$

As mentioned before, it is assumed that the demand is equal to the income. However, sometimes it is possible to find under consumption to save income and overconsumption using credits (debt). In the case of underconsumption, Equation (7) can be formulated as

$$D = p_1 x_1 + p_2 x_2 + \cdots + p_i x_i + \cdots + p_n x_n + S = Y, \tag{8}$$

where S refers to the savings. And with overconsumption, it becomes

$$D = p_1 x_1 + p_2 x_2 + \cdots + p_i x_i + \cdots + p_n x_n - Db = Y, \tag{9}$$

where Db refers to the debts. Usually, consumers combine both under- and overconsumption forming the following expression:

$$D = p_1 x_1 + p_2 x_2 + \cdots + p_i x_i + \cdots + p_n x_n + S - Db = Y. \tag{10}$$

An interesting issue is to analyze how the demand changes over time when using Equation (10). In this case, a similar expression as in Equations (4) and (5) of the demand growth can also be constructed. First, let us analyze the principal expression that represents the growth of each variable:

$$\mathrm{DG} = \left(\begin{array}{l} \frac{p_{1t} x_{1t}}{D_{t-1}} + \frac{p_{2t} x_{2t}}{D_{t-1}} + \cdots + \frac{p_{it} x_{it}}{D_{t-1}} + \cdots + \frac{p_{nt} x_{nt}}{D_{t-1}} \\ + \frac{S_t}{D_{t-1}} - \frac{Db_t}{D_{t-1}} = \frac{Y_t}{Y_{t-1}} \end{array} \right) - 1. \tag{11}$$

By using the transformation $p_{1(t-1)} x_{1(t-1)} / p_{1(t-1)} x_{1(t-1)}$ and so on, it becomes the following formula:

$$\mathrm{DG} = \begin{pmatrix} \frac{p_{1(t-1)}x_{1(t-1)}}{D_{t-1}}\left(\frac{p_{1t}x_{1t}}{p_{1(t-1)}x_{1(t-1)}} - 1\right) + \frac{p_{2(t-1)}x_{2(t-1)}}{D_{t-1}}\left(\frac{p_{2t}x_{2t}}{p_{2(t-1)}x_{2(t-1)}} - 1\right) + \cdots \\ + \frac{p_{i(t-1)}x_{i(t-1)}}{D_{t-1}}\left(\frac{p_{it}x_{it}}{p_{i(t-1)}x_{i(t-1)}} - 1\right) + \cdots + \frac{p_{n(t-1)}x_{n(t-1)}}{D_{t-1}}\left(\frac{p_{nt}x_{nt}}{p_{n(t-1)}x_{n(t-1)}} - 1\right) \\ + \frac{S_{t-1}}{D_{t-1}}\left(\frac{S_t}{S_{t-1}} - 1\right) + \frac{Db_{t-1}}{D_{t-1}}\left(\frac{Db_t}{Db_{t-1}} - 1\right) \end{pmatrix}. \quad (12)$$

From this, it is straightforward to construct Equation (12) by using weighted averages as follows:

$$\mathrm{DG} = \begin{pmatrix} \mathrm{WA}_{p_1x_1}\left(\frac{p_{1t}x_{1t}}{p_{1(t-1)}x_{1(t-1)}} - 1\right) + \mathrm{WA}_{p_2x_2}\left(\frac{p_{2t}x_{2t}}{p_{2(t-1)}x_{2(t-1)}} - 1\right) + \cdots \\ + \mathrm{WA}_{p_ix_i}\left(\frac{p_{it}x_{it}}{p_{i(t-1)}x_{i(t-1)}} - 1\right) + \cdots + \mathrm{WA}_{p_nx_n}\left(\frac{p_{nt}x_{nt}}{p_{n(t-1)}x_{n(t-1)}} - 1\right) \\ + \mathrm{WA}_S\left(\frac{S_t}{S_{t-1}} - 1\right) + \mathrm{WA}_{Db}\left(\frac{Db_t}{Db_{t-1}} - 1\right) \end{pmatrix}, \quad (13)$$

where WA represents the weighted average for each variable. In this formulation, it is also possible to assume that the weights are not known and use OWA aggregations such as the OWA, the OWAWA, and the POWAWA operators[8,17,18].

Note that there are other methodologies for studying and representing the demand[19–22] but in this paper the focus will be given to those explained in this section.

2.2. Aggregation Systems

Aggregation systems are statistical techniques very useful for dealing with sets of data providing summarized information. There exist a wide range of aggregation operators. A very common group is the averaging aggregation operators that include the classical average and the weighted average. Another very well-known aggregation operator is the OWA. It provides a parameterized family of aggregation operators between the minimum and the maximum. It can be defined as follows:

DEFINITION 1. *An OWA operator of dimension n is a mapping OWA:* $R^n \rightarrow R$ *that has an associated weighting vector W of dimension n with* $\sum_{i=1}^{n} w_i = 1$ *and* $w_j \in$ *[0, 1], such that*

$$\mathrm{OWA}(a_1, a_2, \ldots, a_n) = \sum_{j=1}^{n} w_j b_j, \quad (14)$$

where b_j *is the jth smallest of the* a_i.

It accomplishes a wide range of properties including the monotonicity property, idempotency, commutativity, and the boundary condition. A key issue is that it includes the minimum, the maximum, and the average as particular cases. Therefore, in decision theory it includes the classical methods for decision making under uncertainty as particular cases. In other words, it is a method that provides a unified framework for dealing with uncertainty. In the following, let us demonstrate that the OWA operator includes these cases as particular manifestations of the weighting vector:

- optimistic criteria (the maximum): $w_n = 1$ and $w_j = 0$ for all $j \neq n$,
- pessimistic criteria (the minimum): $w_1 = 1$ and $w_j = 0$ for all $j \neq 1$,
- Laplace criteria (the average): $w_j = 1/n$ for all j, and
- Hurwicz criteria: $w_1 = \alpha$ and $w_n = (1 - \alpha)$ and $w_j = 0$ for all $j \neq 1, n$.

The OWA operator has been extended and generalized by a wide range of authors.[10] A very practical extension is the generalized OWA (GOWA) operator[11] because it includes many other types of aggregation operators by using the concept of generalized mean.

Definition 2. *A GOWA operator of dimension n is a mapping GOWA: $R^n \to R$ that has an associated weighting vector W of dimension n with $w_j \in [0, 1]$ and $\sum_{j=1}^{n} w_j = 1$, such that*

$$\text{GOWA}(a_1, a_2, \ldots, a_n) = \left(\sum_{j=1}^{n} w_j b_j^{\lambda} \right)^{1/\lambda}, \tag{15}$$

where b_j is the jth largest of the a_i and λ is a parameter such that $\lambda \in (-\infty, \infty) - \{0\}$.

Furthermore, it is possible to generalize this approach by using quasi-arithmetic means.[6,12,13] The main difference is that instead of using a parameter between $(-\infty, \infty) - \{0\}$, it uses a strictly continuous monotonic function. Therefore, this formulation is more general because the use of functions may adapt to an extremely wide range of scenarios.

Next, it is worth noting those aggregation operators that integrates the OWA operator and the weighted average under the same framework. Among others, let us recall the work developed by Torra[23] regarding the weighted OWA (WOWA) operator, the hybrid average,[24] the importance OWA,[25] and the immediate weights.[26] Recently, Merigó[14] has suggested another approach that unifies both concepts considering the degree of importance that each concept has in the analysis. This is very relevant because it gives more flexibility to the aggregation being able to adapt more efficiently to any environment. This aggregation operator has been called the OWAWA operator. This operator has also been generalized by using generalized means forming the generalized OWAWA (GOWAWA) operator. It can be defined as follows:

Definition 3. *A GOWAWA operator of dimension n is a mapping GOWAWA: $R^n \to R$ that has an associated weighting vector W of dimension n such that $w_j \in [0, 1]$ and $\sum_{j=1}^{n} w_j = 1$, according to the following formula:*

$$\text{GOWAWA}(a_1, a_2, \ldots, a_n) = \beta \left(\sum_{j=1}^{n} w_j b_j^{\lambda} \right)^{1/\lambda} + (1 - \beta) \left(\sum_{i=1}^{n} v_i a_i^{\delta} \right)^{1/\delta}, \tag{16}$$

Table I. Families of GWA, GOWA, and GOWAWA operators.

	GWA	GOWA	GOWAWA
$\lambda = 1$	Weighted average	OWA	OWAWA
$\lambda \to 0$	Weighted geometric average	Geometric OWA	Geometric OWAWA
$\lambda = 2$	Weighted quadratic average	Quadratic OWA	Quadratic OWAWA
λ −1	Weighted harmonic average	Harmonic OWA	Harmonic OWAWA
$\lambda = 3$	Weighted cubic average	Cubic OWA	Cubic OWAWA

where b_j is the jth largest of the a_i, λ and δ are parameters such that $\lambda, \delta \in (-\infty, \infty) - \{0\}$, $\beta \in [0, 1]$ each argument a_i has an associated weight v_i with $\sum_{i=1}^{n} v_i = 1$ and $v_i \in [0, 1]$.

The GOWAWA operator can also be generalized by using quasi-arithmetic means forming the quasi-arithmetic OWAWA (Quasi-OWAWA) operator. Note that if $\beta = 1$, the GOWAWA becomes the GOWA operator and when $\beta = 0$, the generalized weighted average (GWA) that can be formulated as follows:

$$\mathrm{GWA}\,(a_1,\ a_2, \ldots,\ a_n) = \left(\sum_{i=1}^{n} v_i a_i^{\lambda}\right)^{1/\lambda}, \tag{17}$$

where λ is a parameter such that $\lambda \in (-\infty, \infty) - \{0\}$.

Observe that the GWA, GOWA, and GOWAWA, includes the following particular cases shown in Table I.

3. DEMAND ANALYSIS WITH MULTIPERSON AND MULTICRITERIA AGGREGATION PROCESSES

The demand is a key variable to determine the necessities of an economy either at macro- or microlevel. In microeconomics, it is very useful for expressing the demand of the individual consumers. In the following, let us analyze it by using simple aggregation operators. Note that when forecasting[27] the demand, many aspects may influence it including several attributes, criterion, states of nature, and the opinion of experts.[28] First, let us assume an analysis by using the opinion of several experts. Thus, the expect demand is different depending on the opinion considered and it is necessary to integrate this information. to do so, it can be used the following aggregation system:

$$\mathrm{D} - \mathrm{WA}(D_1, D_2, \ldots, D_n) = \sum_{e=1}^{n} v_e D_e, \tag{18}$$

where D_e is the demand according to the eth expert and v_e is the eth weight representing the importance of each expert in the aggregation such that $v_e \in [0, 1]$

and $\sum_{e=1}^{n} v_e = 1$. Note that if all the experts are equally important, this aggregation becomes a simple average (demand – arithmetic mean (D-AM)) as follows:

$$\mathrm{D-AM}(D_1, D_2, \ldots, D_n) = \frac{1}{n}\sum_{e=1}^{n} D_e. \tag{19}$$

Moreover, if only one expert is considered, $v_1 = 1$ and only one forecast is available being $D = D_1$.

Sometimes, the importance of the experts is not known. In these situations, an additional approach is needed such as the OWA operator[8–10] that represents the information according to the degree of optimism or pessimism of the decision maker. By using the OWA operator, the opinion of the experts can be used in the demand as

$$\mathrm{D-OWA}(D_1, D_2, \ldots, D_n) = \sum_{j=1}^{n} w_j D_j, \tag{20}$$

where D_j is the jth smallest of the D_e and w_j is the jth weight that represents the degree of optimism of the aggregation such that $w_j \in [0, 1]$ and $\sum_{j=1}^{n} w_j = 1$. By using the D-OWA operator, we can forecast the demand from the minimum to the maximum. Note that the most pessimistic demand (minimum) is formed when $w_1 = 1$ and $w_j = 0$ for all $j \neq 1$ and the most optimistic demand (maximum) when $w_n = 1$ and $w_j = 0$ for all $j \neq n$.

To integrate both approaches in the same formulation, it can be used the OWAWA operator:

$$\mathrm{D-OWAWA}(D_1, D_2, \ldots, D_n) = \beta \sum_{j=1}^{n} w_j D_j + (1-\beta) \sum_{e=1}^{n} v_e D_e, \tag{21}$$

where $\beta \in [0, 1]$. As we can see, if $\beta = 1$, the D-OWAWA becomes the D-OWA and if $\beta = 0$, the weighted demand D-WA.

More generally, it is possible to use the GOWAWA and the Quasi-OWAWA operator. By using the GOWAWA operator, it is formed a similar formulation than Equation (16) and with the Quasi-OWAWA operator, the following expression:

$$\mathrm{D-QOWAWA}(D_1, D_2, \ldots, D_n) = \beta f^{-1}\left(\sum_{j=1}^{n} w_j f(D_j)\right) + (1-\beta) g^{-1}\left(\sum_{e=1}^{n} v_e g(D_e)\right), \tag{22}$$

where f and g are strictly continuous monotonic functions. Note that if $f = D^{\lambda}$ and $g = D^{\delta}$, the D-QOWAWA becomes the D-GOWAWA operator. One of the main advantages of this approach is that it includes a wide range of particular cases that

Table II. Families of D-QOWAWA operators.

OWA	WA	D-QWA ($\beta = 0$)	D-QOWA ($\beta = 1$)	D-QOWAWA
$f = D$	$g = D$	D-WA	D-OWA	D-OWAWA
$f \to D0$	$g \to D0$	Geometric D-WA	Geometric D-OWA	Geometric D-OWAWA
$f = D^2$	$g = D^2$	Quadratic D-WA	Quadratic D-OWA	Quadratic D-OWAWA
$f = D^{-1}$	$g = D^{-1}$	Harmonic D-WA	Harmonic D-OWA	Harmonic D-OWAWA
$f = D^3$	$g = D^3$	Cubic D-WA	Cubic D-OWA	Cubic D-OWAWA
$w_1 = 1$	–	–	Min-D	Min-D-QWA
$w_n = 1$	–	–	Max-D	Max-D-QWA
$w_1 = 1$	$v_e = 1/n$	D-QAM	Min-D	Min-D-QAM
$w_n = 1$	$v_e = 1/n$	D-QAM	Max-D	Max-D-QAM
$w_j = 1/n$	$v_e = 1/n$	D-QAM	D-QAM	D-QAM
$f = D^2$	$g = D$	D-WA	Quadratic D-OWA	D-OWQAWA
$f = D^3$	$g = D^2$	Quadratic D-WA	Cubic D-OWA	D-OWCAQWA

represents different forms that an aggregation system may take. In Table II, some of the most relevant particular cases are presented.

Note this analysis has been developed only for one aggregation process that represents the opinion of a group of experts. However, more general structures can be constructed by adding more aggregation processes that includes attributes, states of nature, criterion, and opinions given by other experts. Next, let us assume the introduction of a set of criteria ($Cr_1, Cr_2, \ldots, Cr_m$). Thus, by using Equation (18) it is formed the following aggregation system:

$$\mathrm{D-WA}_2\left(\left[D_1^1, \ldots, D_1^m\right], \ldots, \left[D_n^1, \ldots, D_n^m\right]\right) = \sum_{e=1}^{n}\sum_{c=1}^{m} v_e u_e^c D_e^c, \quad (23)$$

where ${D_e}^c$ is the demand according to each eth expert and cth criteria, v_e is the eth weight for each expert such that $v_e \in [0, 1]$ and $\sum_{e=1}^{n} v_e = 1$, and ${u_e}^c$ is the weight for each expert e and criteria c such that ${u_e}^c \in [0, 1]$ and $\sum_{c=1}^{m} u_e^c = 1$. Note that the subindex 2 expresses the use of two weighted averages.

A similar formulation could be used with the OWA operator in both aggregation processes. In this case, it is obtained the D-OWA$_2$. Note that the subindex 2 indicates that two OWA aggregations are used in the process.

$$\mathrm{D-OWA}_2\left(\left[D_1^1, \ldots, D_1^m\right], \ldots, \left[D_n^1, \ldots, D_n^m\right]\right) = \sum_{j=1}^{n}\sum_{k=1}^{m} w_j x_j^k D_j^k, \quad (24)$$

where ${D_j}^k$ is the demand ordered from lowest to highest for each eth expert and cth criteria, w_j is the jth weight for each expert such that $w_j \in [0, 1]$ and $\sum_{j=1}^{n} w_j = 1$, and ${x_j}^k$ is the weight for each expert e and criteria c such that ${x_j}^k \in [0, 1]$ and $\sum_{k=1}^{m} x_j^k = 1$.

Furthermore, it is possible to integrate both expressions in the same formulation with the OWAWA operator as follows:

$$\mathrm{D-OWA_2WA_2}\left(\left[D_1^1,\ldots,D_1^m\right],\ldots,\left[D_n^1,\ldots,D_n^m\right]\right)=\beta\sum_{j=1}^{n}\sum_{k=1}^{m}w_jx_j^kD_j^k$$

$$+\,(1-\beta)\sum_{e=1}^{n}\sum_{c=1}^{m}v_eu_e^cD_e^c, \tag{25}$$

where $\beta \in [0, 1]$. Note that if $\beta = 1$, it becomes the D-OWA$_2$ and if $\beta = 0$, the D-WA$_2$. Additionally, it is also possible to consider more complex structures by mixing both aggregations. That is, using an OWA for the experts and a WA for the criteria and vice versa. Moreover, this approach can be generalized by using generalized and quasi-arithmetic means forming the D-GOWA$_2$WA$_2$ and the D-QOWA$_2$WA$_2$ operator.

Next, let us include a set of states of nature $(S_1, S_2, \ldots, S_l)$ in the aggregation process. By using Equation (23), it can be constructed the following aggregation structure:

$$\mathrm{D-WA_3}\left(\left[D_{11}^1,\ldots,D_{11}^m\right],\ldots,\left[D_{nl}^1,\ldots,D_{nl}^m\right]\right)=\sum_{e=1}^{n}\sum_{c=1}^{m}\sum_{h=1}^{l}v_eu_e^ct_{eh}^cD_{eh}^c, \tag{26}$$

where ${D_{eh}}^c$ is the demand according to each eth expert, cth criteria, and hth state of nature and ${t_{eh}}^c$ is the weight for each expert e, criteria c, and state of nature h such that ${t_{eh}}^c \in [0, 1]$ and $\sum_{h=1}^{l} t_{eh}^c = 1$. As it is shown, this aggregation system uses three weighted averages.

By extending this analysis to the OWA framework, the following aggregation system is obtained:

$$\mathrm{D-OWA_3}\left(\left[D_{11}^1,\ldots,D_{11}^m\right],\ldots,\left[D_{nl}^1,\ldots,D_{nl}^m\right]\right)=\sum_{j=1}^{n}\sum_{k=1}^{m}\sum_{p=1}^{l}w_jx_j^ky_{jp}^kD_{jp}^k, \tag{27}$$

where ${y_{jp}}^k$ is the weight for each expert e, criteria c, and state of nature h such that ${y_{jp}}^k \in [0, 1]$ and $\sum_{p=1}^{l} x_{jp}^k = 1$ and ${D_{jp}}^k$ is the demand ordered from lowest to highest for each eth expert, cth criteria, and hth state of nature. Recall that the main advantage of the OWA operator is the possibility of considering any scenario from the minimum (most pessimistic) to the maximum (most optimistic).

Under this framework it is also possible to integrate the previous equations with the OWAWA operator forming the D-OWA$_3$WA$_3$:

$$\mathrm{D-OWA_3WA_3}\left(\left[D_{11}^1,\ldots,D_{11}^m\right],\ldots,\left[D_{nl}^1,\ldots,D_{nl}^m\right]\right)=\beta(D-OWA_3)$$
$$+(1-\beta)(D-WA_3). \tag{28}$$

It is straightforward to demonstrate that if $\beta = 1$, it becomes the D-OWA$_3$ and if $\beta = 0$, the D-WA$_3$. Furthermore, many other generalizations and particular cases could be developed to this approach. For example, by using quasi-arithmetic means it could be formed the D-QOWA$_3$WA$_3$ operator and so on.

Finally, it is worth noting that more complex structures could be developed by adding new concepts in the analysis such as the use of attributes, additional experts, and criterion and so on. Note that the extension process follows a similar methodology as it has been explained for the three structures presented in this section by using, experts, criterion, and states of nature. That is, it consists in the addition of more weighting vectors that represents the additional concept. Obviously, this process can be represented with weighted averages, OWAs, OWAWAs, and any other type of aggregation operator.

As a very general example, these complex structures may appear in real-world problems when a lot of variables have to be considered. For example, when modeling the average demand of a consumer of a country, many additional aggregations may be studied such as a classification by regions, municipalities, and each of them with an individual structure that includes criterion, experts, states of natures, and attributes.

4. FURTHER GENERALIZATIONS OF DEMAND ANALYSIS WITH AGGREGATION OPERATORS

Demand analysis can be studied with aggregation operators by using a wide range of methodologies. In this section, it is shown how the approaches presented in Section 2 can be extended by using OWA and OWAWA operators. It is worth noting that many analyses could be developed such as the aggregation of the variables that compose the demand individually. But the focus is going to be on Equations (5) and (13). These equations study the demand growth for a period of time t. Obviously, this can be studied in the past, present, and future. Usually, when analyzing the present and the past, most of the information is available although some exceptional situations may also occur in these situations because there is missing information in the analysis. However, when forecasting the future, there are a lot of uncertainties and it clearly becomes necessary the use of a wide range of tools to represent the information in an appropriate way.

First, let us look into Equation (5). In this equation, it is weighted all the variables according to their degree of importance. These weights can be calculated exactly when all the information is known. However, many times it is not clear or because the analysis is for the future and a forecast is used. Additionally, it can happen that the weights are not known and cannot be forecasted. In these situations, it is needed another approach such as the use of the OWA operators. The main advantage of using OWA in this type of problems is that it permits to represent the information from the minimum to the maximum taking into account all the situations that may occur. It can be formulated as follows:

$$\mathrm{DG} = \begin{pmatrix} \mathrm{OWA}_{\alpha_0}\left(\frac{\alpha_{0t}}{\alpha_{0(t-1)}} - 1\right) + \mathrm{OWA}_{\alpha_x P_x}\left(\frac{\alpha_{xt} P_{xt}}{\alpha_{x(t-1)} P_{x(t-1)}} - 1\right) \\ + \mathrm{OWA}_{\alpha_y P_y}\left(\frac{\alpha_{yt} P_{yt}}{\alpha_{y(t-1)} P_{y(t-1)}} - 1\right) + \mathrm{OWA}_{\alpha_M M}\left(\frac{\alpha_{Mt} M_t}{\alpha_{M(t-1)} M_{t-1}} - 1\right) \\ + \mathrm{OWA}_{\alpha_H H}\left(\frac{\alpha_{Ht} H_t}{\alpha_{H(t-1)} H_{t-1}} - 1\right) \end{pmatrix}, \quad (29)$$

where OWA is the OWA weight for each variable that constitutes the demand growth. Note that to have the initial ordering, it is necessary to adapt the ordering of the OWA operator to the initial information. That is, the *j*th weights have to be reordered according to the *i*th variables instead of using Equation (14).

Furthermore, it is possible to extend and generalize this approach by using a wide range of aggregation operators including the OWAWA, the POWAWA,[17] and the QOWAWA operators. A very interesting type of OWAWA operator that could be used in the analysis is the OWAWA_2 because one weight could be used for representing the present knowledge and the other one as the forecast for the future. Thus, among the different aggregation operators available in the literature, it is recommended the following one:

$$\mathrm{DG} = \begin{pmatrix} \mathrm{OWAWA}_{2\alpha_0}\left(\frac{\alpha_{0t}}{\alpha_{0(t-1)}} - 1\right) + \mathrm{OWAWA}_{2\alpha_x P_x}\left(\frac{\alpha_{xt} P_{xt}}{\alpha_{x(t-1)} P_{x(t-1)}} - 1\right) \\ + \mathrm{OWAWA}_{2\alpha_y P_y}\left(\frac{\alpha_{yt} P_{yt}}{\alpha_{y(t-1)} P_{y(t-1)}} - 1\right) + \mathrm{OWAWA}_{2\alpha_M M}\left(\frac{\alpha_{Mt} M_t}{\alpha_{M(t-1)} M_{t-1}} - 1\right) \\ + \mathrm{OWAWA}_{2\alpha_H H}\left(\frac{\alpha_{Ht} H_t}{\alpha_{H(t-1)} H_{t-1}} - 1\right) \end{pmatrix}, \quad (30)$$

where the OWAWA_2 can be represented as $\mathrm{OWAWA}_2 = C_1 w_i + C_2 v_i + C_3 u_i$, where C_1, C_2, and C_3 represent the importance that the OWA, the WA, and the present WA have in the analysis such that C_1, C_2, and $C_3 \in [0, 1]$ and $C_1 + C_2 + C_3 = 1$. Note that the OWA weight w_j has been reordered according to the initial information i. Moreover, observe that this expression is equivalent to the POWAWA operator if it is assumed that the present weights can be understood as probabilities.

Next, let us look into Equation (13). In this case, the focus is on the price of each good and the quantity consumed. When analyzing the demand growth, it is also used a similar expression as Equation (5) but with an individual focus on the growth of each product. In Equation (13), it is used the weighted average as a representation of the importance that each variable has in the analysis. However, in a lot of situations, the weights are not known. Therefore, it can be used other aggregations such as the OWA and the OWAWA operator. As mentioned before, it seems that the most practical aggregation operator from a simplistic point of view is the OWAWA_2. However, note that in real-world problems, a lot of decompositions may appear, so more complex aggregation structures will be needed. By using the OWAWA_2 operator, it is formed the following aggregation structure:

$$\mathrm{DG} = \begin{pmatrix} \mathrm{OWAWA}_{2p_1x_1}\left(\frac{p_{1t}x_{1t}}{p_{1(t-1)}x_{1(t-1)}} - 1\right) + \mathrm{OWAWA}_{2p_2x_2}\left(\frac{p_{2t}x_{2t}}{p_{2(t-1)}x_{2(t-1)}} - 1\right) + \cdots \\ + \mathrm{OWAWA}_{2p_ix_i}\left(\frac{p_{it}x_{it}}{p_{i(t-1)}x_{i(t-1)}} - 1\right) + \cdots + \mathrm{OWAWA}_{2p_nx_n}\left(\frac{p_{nt}x_{nt}}{p_{n(t-1)}x_{n(t-1)}} - 1\right) \\ + \mathrm{OWAWA}_{2S}\left(\frac{S_t}{S_{t-1}} - 1\right) + \mathrm{OWAWA}_{2Db}\left(\frac{Db_t}{Db_{t-1}} - 1\right) \end{pmatrix}, \quad (31)$$

where the OWAWA_2 weights are represented in a similar way as Equation (30). Again, recall that many other aggregation operators could be used depending on the needs of the specific problem considered. The main objective is to use the

Table III. Expected demand provided by expert 1.

	Criteria 1					Criteria 2				
Country	S_1	S_2	S_3	S_4	S_5	S_1	S_2	S_3	S_4	S_5
Spain	60	40	50	40	50	30	70	60	50	40
France	70	60	50	30	40	70	60	80	70	30
UK	40	70	80	60	30	80	80	60	40	40
Italy	50	60	60	50	50	60	90	70	60	40
Germany	60	40	40	60	60	60	70	80	70	50

Table IV. Expected demand provided by expert 2.

	Criteria 1					Criteria 2				
Country	S_1	S_2	S_3	S_4	S_5	S_1	S_2	S_3	S_4	S_5
Spain	70	80	60	50	40	60	80	70	50	40
France	50	60	70	70	60	50	70	70	70	60
UK	80	70	70	50	60	80	60	60	70	50
Italy	60	70	60	70	70	60	30	40	30	50
Germany	60	80	90	60	70	70	50	60	70	80

aggregation system that is in closest accordance with the interests of the decision maker in the analysis.

5. FORECASTING THE DEMAND WITH AGGREGATION SYSTEMS

Demand forecasting is a fundamental issue to correctly assess a wide range of micro- and macroeconomic problems. This section focuses on an illustrative example by using the aggregation systems explained in the previous sections. The aim of this approach is to forecast the demand when it is necessary to consider the consumers attitude in the analysis. This is assessed with the OWA operator because it aggregates the information from the minimum to the maximum according to the degree of optimism shown by the consumers. Note that in the literature there are a wide range of approaches for demand forecasting including the demand for money, job demands, and the aggregate demand[1,2].

Assume a group of three experts that wants to analyze the expected demand of a good for the next year in five countries (Spain, France, United Kingdom, Italy, Germany). They assume that two main criterions may affect the demand:

- C_1: economy of the European Union.
- C_2: other variables.

The experts assume that five potential states of nature may occur in the general economic environment:

- S_1: very good economic environment.
- S_2: good economic environment.

Table V. Expected demand provided by expert 3.

	Criteria 1					Criteria 2				
Country	S_1	S_2	S_3	S_4	S_5	S_1	S_2	S_3	S_4	S_5
Spain	40	50	60	30	50	80	70	50	40	30
France	60	50	60	60	50	60	70	60	60	50
UK	40	60	70	70	50	90	70	60	50	50
Italy	60	40	50	50	50	60	40	70	60	50
Germany	80	70	60	50	50	70	80	60	70	60

Table VI. Aggregated demand: expert 1.

	WA		Min		OWA		Max		OWAWA	
Country	C_1	C_2	C_1	C_2	C_1	C_2	C_1	C_2	C_1	C_2
Spain	47	55	40	30	46	46	60	70	46.5	50.5
France	49	68	30	30	46	57	70	80	47.5	62.5
UK	65	60	30	40	51	56	80	80	58	58
Italy	56	68	50	40	53	59	60	90	54.5	63.5
Germany	48	71	40	50	50	63	60	80	49	67

- S_3: regular economic environment.
- S_4: bad economic environment.
- S_5: very bad economic environment.

After careful review of the information, they provide the following expected demand depending on the state of nature and criteria considered. The results are shown in Tables III–V.

Note that this information can be integrated by using several methodologies. Since this example is based on Equations (26) and (27), it will be followed these instructions. That is, first aggregate the states of nature, next the criterion, and finally the experts. However, it is also possible to follow the inverse direction. That is, first aggregate the experts, next the criterion, and finally the states of nature. With Equation (26), the result is always the same. However, with the OWA operator the results may change due to the reordering process. Therefore, with the OWA it is important to delimitate the information to be aggregated first and for what reasons. In this example, it is assumed that each expert works individually and at the end they meet and integrate their results. With the inverse approach, the interpretation is also different. In this case, it is assumed that the experts work collectively and from the beginning they integrate their information. Following Equations (26) and (27), the next step is the aggregation of the states of nature. This is done with the weighted average, the OWA operator that includes the minimum and the maximum, and the OWAWA operator. It is assumed the following expected degrees of importance for the states of nature $T = (0.1, 0.2, 0.4, 0.2, 0.1)$. With the OWA operator, it is assumed the following attitudinal character: $Y = (0.3, 0.2, 0.2, 0.2, 0.1)$. Note that throughout the process, it is considered that the OWA and the weighted average are equally important. Thus, $\beta = 0.5$. The results are shown in Tables VI–VIII.

Table VII. Aggregated demand: expert 2.

	WA		Min		OWA		Max		OWAWA	
Country	C_1	C_2	C_1	C_2	C_1	C_2	C_1	C_2	C_1	C_2
Spain	61	64	40	40	56	56	80	80	58.5	60
France	65	67	50	50	60	62	70	70	62.5	64.5
UK	66	63	50	50	63	61	80	80	64.5	62
Italy	65	39	60	30	65	39	70	60	65	39
Germany	77	63	60	50	69	64	90	80	73	63.5

Table VIII. Aggregated demand: expert 3.

	WA		Min		OWA		Max		OWAWA	
Country	C_1	C_2	C_1	C_2	C_1	C_2	C_1	C_2	C_1	C_2
Spain	49	53	30	30	43	49	60	80	46	51
France	57	61	50	50	55	58	60	70	56	59.5
UK	63	62	40	50	55	60	70	90	59	61
Italy	49	59	40	40	48	53	60	70	48.5	56
Germany	61	67	50	60	59	66	80	80	60	66.5

Table IX. Expected demand: expert 1.

Country	WA	Min	OWA	Max	OWAWA
Spain	51	30	46	70	48.5
France	58.5	30	50.4	80	54.45
UK	62.5	30	53	80	57.75
Italy	62	40	55.4	90	58.7
Germany	59.5	50	55.2	80	57.35

Table X. Expected demand: expert 2.

Country	WA	Min	OWA	Max	OWAWA
Spain	62.5	40	56	80	59.25
France	66	50	60.8	70	63.4
UK	64.5	50	61.8	80	63.15
Italy	52	30	49.4	70	50.7
Germany	70	50	66	90	68

Next, the experts proceed to aggregate the two criteria to form their collective results. With the weighted average, it is assumed that both criterion are equally important, that is, $U = (0.5, 0.5)$. With the OWA, it is assumed a bit pessimistic position by using $X = (0.6, 0.4)$. The results are shown in Tables IX–XI.

Finally, their opinions are integrated forming the final expected demand for each country of the product. It is assumed that both experts are equally important. Thus, with the weighted average it is used $V = (0.4, 0.3, 0.3)$. With the OWA operator, the attitude is the same as in the previous tables. That is, $W = (0.3, 0.5,$

Table XI. Expected demand: expert 3.

Country	WA	Min	OWA	Max	OWAWA
Spain	51	30	45.4	80	48.2
France	59	50	56.2	70	57.6
UK	62.5	40	57	90	59.75
Italy	54	40	50	70	52
Germany	64	50	61.8	80	62.9

Table XII. General expected demand.

Country	WA	Min	OWA	Max	OWAWA
Spain	54.45	30	47.82	80	51.135
France	60.9	30	55.38	80	58.14
UK	63.1	30	56.76	90	59.93
Italy	56.6	30	50.9	90	53.75
Germany	64	50	60.66	90	62.33

0.2). The expected demand by using the weighted average, the OWA, the minimum, the maximum, and the OWAWA are shown in Table XII.

As we can see, the weighted average provides a result based on the degrees of importance expected for each variable. However, since this information is imprecise and not known, the use of the OWA operator may be useful for representing the expected results according to different degrees of optimism. Moreover, it permits to analyze the most extreme cases formed with the minimum and the maximum demand. The last column presents the results with the OWAWA operator that represents an aggregation process that considers both the OWA and the weighted average in the analysis.

6. CONCLUSIONS

A new framework for modeling the demand by using aggregation systems has been presented. The main advantage of this approach is that it considers different sources of information in the analysis including the opinion of several experts, criterion, and states of nature. Thus, it is very flexible and can be adapted to the different needs shown in the specific problem considered. This is of great interest in real-world problems because the information is very complex and uncertain, and it is not easy to assess it analytically. Therefore, these structures can deal with complex information classifying it in several substructures to form the final expected results. The paper has focused on the use of the weighted average and the OWA operator in the analysis. By using the weighted average, it measures the degree of importance that each variable is expected to have. However, in many situations, these degrees are not known and only represent partial approximations. Therefore, an additional approach has been suggested by using OWA operators that aggregates the information considering the attitudinal character of the consumers. The key advantage of the OWA operator is that it represents the information from the most pessimistic result to the most optimistic one and selects the result in the closest accordance with the particular attitude of the specific decision maker considered. A further generalization that integrates the OWA and the weighted average in the same formulation has also been introduced to provide a more general approach that considers two different sources of information in the analysis.

This approach has been tested in an illustrative example regarding demand forecasting of a product in five different countries. The practicality of this approach has been demonstrated to deal with complex information and provide some simple expected results that do not loose information in the analysis. A key issue when dealing with the OWA operator is that it does not provide the same result between subaggregations if the ordering of the aggregations is not the same. Therefore, with the OWA operator, it is necessary to clearly decide the ordering of the aggregations according to the type of information that it is expected to provide. Note that with the weighted average this issue does not matter because the results are always the same independently of the ordering of the subaggregations. Furthermore, it is worth noting that the results provided with the OWA[29,30] have some connections with the results provided by a box-plot[31] since the information departs from the interval formed by the minimum and the maximum. Moreover, the use of the OWA operator permits to represent the attitude of the consumer in the problem by considering pessimistic and optimistic results that represent different scenarios with different positions taken by the consumers.

Another methodology by using the demand growth has also been introduced in the paper. In this case, the idea has been the development of a growth equation based on the division of each variable that constitutes the demand. Therefore, it has been possible to analyze all the variables individually and considering the degree of importance that they have in the aggregation. Moreover, when making forecasts in the future regarding the demand growth, it has been found that the degree of importance that each variable has is not known and a different approach is needed. For doing so, the use of the OWA operator has been suggested by assuming a specific optimistic or pessimistic attitude regarding the information. Furthermore, a more general framework has been considered that includes the OWA, the initial information, and the expected degrees of importance in the same formulation. This approach has been named the $OWAWA_2$ operator, clearly stating that it is using an OWA operator and two weighted averages in the aggregation process. Many other aggregation systems could be used under this framework although the focus has been on those that seem to be the most relevant ones.

This approach provides new methodologies for studying the demand and making efficient forecasts that consider the attitudinal character of the consumers. However, the research has some limitations. First, real-world problems are very complex and usually should be assessed with a lot of subaggregations and not only three levels as shown in the paper by using experts, criterion, and states of nature. It has been explained that it is straightforward to construct more complex structures by using additional substructures. However, this is not easy because sometimes these hierarchies are very complex and with a not well-defined process. Note that each particular real-world problem may have specific needs to be considered. Therefore, this paper provides some general ideas regarding the methodology to use in these problems but recall that the complexity of the real world may bring a lot of exceptional situations. Second, many other aggregation operators could have been studied such as induced aggregation operators,[32,33] Choquet integrals,[34–36] moving averages,[37] and other generalized aggregation operators.[38–40] The main idea is to use the type of aggregation operator that better fits the needs of the specific problem considered. Thus, the aggregation systems presented in the paper are assumed to be the basic ones when dealing with these types of problems. However, in many situations they may not be enough and other ones are needed. Finally, the demand is a very broad concept that can be studied from a wide range of perspectives. This paper has focused on the consumer's microeconomic perspective. But it is worth noting that similar studies could be developed in other contexts including the demand for money,[41] aggregated demand,[15] job demand,[42–44] and tourism demand.[45]

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