Review of Uncertainty Reasoning Approaches as Guidance for Maritime and Offshore Safety-Based Assessment

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ABSTRACT

Many different formal techniques have been developed over the past two decades for dealing with uncertain information for decision making. In this paper we review some of the most important ones, i.e., Bayesian theory of probability, Dempster-Shafer theory of evidence, and fuzzy set theory, describe how they work and in what ways they differ from one another, and show their strength and weakness respectively as well as their connection. We also consider hybrid approaches which combine two or more approximate reasoning techniques within a single reasoning framework. These have been proposed to address limitations in the use of individual techniques. The study is intended to provide guidance in the process of developing frameworks for safety-based decision analysis using different methods for reasoning under uncertainty.

1. Uncertainty in Decision Making

In conventional information processing techniques it is often assumed that problems are well structured, complete information is always available and information processing procedures can be clearly defined. However, in many real-world decision making problems, this is not always the case and decisions making is often associated with uncertainty. "Uncertainty" is a context dependent concept. There does not exist a comprehensive and unique definition of uncertainty. One definition of uncertainty is given as follows [Zimmermann 2000]:

"Uncertainty is a situation in which a person does not have the quantitatively and qualitatively appropriate information to describe, prescribe or predict deterministically and numerically a system, its behavior or other characteristics."

This definition focuses on the human related, subjective interpretation, which depends on the quantity and quality of information about a system that is available to a human observer.

Uncertainty has many different sources and different types. A review of the literature shows a variety of treatments of uncertainty. Uncertainty can be broadly classified into three types, namely randomness, fuzziness and incompleteness [Blockley and Godfrey 2001]. Uncertainty can be attributed to vagueness where there are ill-defined boundaries and ambiguity and where there are several choices for a given situation [Klir and Yuan 1995, Ibrahim and Ayyub 1992]. Uncertainties can be classified into two broad categories, namely probabilistic and cognitive [Gupta 1992]. They can also be referred to as epistemic and aleatory uncertainties, respectively [Pate'-Cornell 1996, Bonissone and Tong 1985, Kanal and Lemmer 1986, Krause and Clark 1993, Stefik 1995, Zimmermann 2000]. It was suggested in [Zimmermann 2000] that only a careful analysis of the contextual features of uncertainty will lead to the selection of the most suitable uncertainty theory.

Methods for dealing with uncertainty in many areas of artificial intelligence (AI) have received considerable attention for more than a decade. Several numerical and symbolic methods have been proposed for handling uncertain information [Clark 1990, Ibrahim and Ayyub 1992, Kruse et al. 1992, Krause and Clark 1993]. Three of the most common methods of representing and reasoning with uncertain knowledge are Bayesian probability theory [Pearl 1988], Dempster-Shafer theory of evidence [Dempster 1968 and 1969, Shafer 1976], and fuzzy set theory [Zadeh 1965].

2. Bayesian Theory of Probability

The classical approach to address uncertainty is the Bayesian theory of probability. Probability theory has been used to model precisely described, repetitive experiments with observable but uncertain outcomes. In this approach, uncertain variables are assumed to be described by statistical parameters which define the probability of the variable having a

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given value. The concept of probability has no univocal definition. The two main schools of thought in this field are the frequentist and the Bayesian. According to Paté-Cornell (1996) the frequentist school (including classical statisticians) defines probability as a limiting frequency, which applies only if one can identify a sample of independent, identically distributed observations of the phenomenon of interest. The Bayesian side, on the other hand, looks upon the concept of probability as a degree of belief. The Bayesian framework also provides methods for updating your probabilities when new data are introduced based on Bayesian rule. In theory, when there is sufficient data to estimate a probability, the subjectivist's assessment of his or her probability will converge to the frequentist's estimate of the probability of the event. In other words, they will tend to agree as information becomes available.

A basis for the application of probabilistic-oriented methods is the validity of statistical laws for stochastic input parameters. The heavy reliance on the probability theory as the only effective and reliable methodology to deal with uncertainty has historical roots. Probability theory has well-established and sound scientific foundations and has been widely used for centuries. There have been several adaptations of probability theory within the literature of artificial intelligence including the odds-likelihood formulation used in Prospector [Duda et al. 1976], the cautious approach adopted by Inferno [Quinlan 1983], diagnosis system application [Tawfik and Neufeld 1998], and the probabilistic logic approach by Nillson (1986).

It is recognized that in practical applications there are many subjective uncertainties, which are due to the lack of knowledge. To address these, subjective probability (Bayesian probability) is often applied. This approach uses the common probability approach, however expert judgments are used to generate probabilities or probability distribution functions representing the resulting states of knowledge. Subjective probability theory matches quite well to our knowledge base [Pearl 1988, Ng and Bruce 1990]. The Bayesian approach for uncertain reasoning is characterized by the following features ([Pearl 1988]):

1) Probability is interpreted as degree of belief, based on available evidence,
2) Current knowledge is represented by a (real-valued) probability distribution on a proposition space, and
3) New knowledge is learned by conditionalization.

Today, Bayesian networks represent a culmination of Bayesian probability theory and causal graphical representations for modelling causal and probabilistic applications [Charniak 1991, Heckerman et al. 1990, Roehrig 1996]. So we put emphasis on the investigation of Bayesian belief networks.

Before describing Bayesian belief networks the fundamentals of probability theory need to be discussed [Pearl 1988, Neapolitan 1990]. Let A be an event within the context of all possible events E within a domain, such that $A \in E$ and E is the event space. The probability of A occurring is denoted by $P(A)$. $P(A)$ is the probability assigned to A prior to the observation of any evidence and is also called the *a priori probability*. This probability must conform to certain laws. First, the probability must be non-negative and must also be less than or equal to one, therefore

$$\forall A \in E; 0 \leq P(A) \leq 1 \quad (1)$$

A probability of 0 means that the event will not occur while a probability of 1 means that the event will occur for sure. Secondly, the total probability of the event space is 1 or in other words, the sum of the probabilities of all of the events A_i in E must equal to 1.

$$\forall A \in E, \ \sum A_i = 1 \quad (2)$$

Finally, we consider the complement of A, $\neg A$, which includes all events in E except for A. From Eq. 2 the following can be obtained:

$$P(A) + P(\neg A) = 1 \quad (3)$$

Now consider another event B in E, or $B \in E$. The probability that event A will occur given that event B has occurred is called the *conditional probability* of A given B and is represented by $P(A|B)$. The probability that both A and B will occur is called the *joint probability* and is defined by $P(A \cap B)$.

$$P(A \cap B) = P(B)P(A|B)$$

From this law, a further result is derived as follows:

$$P(A) = \sum_{i=1,\dots n} P(A|B_i)P(B_i) \quad (4)$$

$P(A|B)$ is defined in terms of the joint probability of A and B by

$$P(A|B) = P(A \cap B)/P(B) \quad (5)$$

Eq. (5) can be further manipulated to yield Bayes Rule

$$P(A|B) = (P(B|A) \times P(A))/P(B) \quad (6)$$

and thus gives a means of computing one conditional probability relating two events from another conditional probability.

The Bayesian updating rule from *a priori* probability to posterior probability enables the probability method to update knowledge in light of new evidence. This lays the foundation for managing and manipulating uncertainty using probability theory in expert systems. Bayesian belief networks (BBN) use this mechanism in a graphical form, a

Directed Acyclic Graph (DAG), to represent and manipulate uncertain knowledge. Figure 1 presents a small example network with only four nodes.

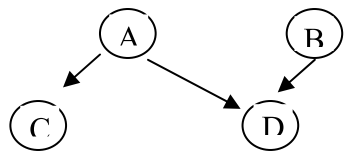

Fig. 1 A small Bayesian network

The interrelationships between the various elements can be expressed graphically. The nodes in the graph represent variables with domains of discrete, mutually exclusive values. Graph nodes that have interdependencies are connected graphically, but independent nodes are not connected. The connections are termed ‘‘edges’’ and are ordered pairs between nodes. Thus, the edges have direction. The connecting arrows represent the causal/relevance relationships between the variables. A random variable has a number of states (e.g. “yes” and “no” or multi-states) and a probability distribution for the states where the sum of the probabilities of all the states should be 1. In this way a BBN model is subject to the standard axioms of probability theory.

The structure of a Bayesian network is usually determined by consultation with experts. As the construction of Bayesian networks is guided by principles of causality and locality, most users find it easy to model a problem domain within this framework [Smith 1990]. Probabilities associated with nodes can either be estimated by experts or compiled from statistical studies. An important assumption of Bayesian networks is variable independence: a variable is independent (in the probabilistic sense) of all other non-descendant variables in the network except its parents.

A probability table [Pearl 1988] is determined for each node and provides the probabilities of each state of the variable for that node. For nodes with parents, the tables show conditional probabilities for each combination of parent state values; for nodes without parents, the tables provide just the unconditional probabilities (called marginal probabilities). The probability tables are built using a mixture of empirical data and expert judgments. Subjective degree of belief can be quantified as a subjective probability using for example the reference lottery method and there are also many other approaches for eliciting subjective probabilities [von Winterfeldt et al. 1986, Morgan 1990].

Considering the representation of the system, tree structures (e.g., fault tree or event tree) are often used, which have much more limited structures than networks, with a rigid hierarchical structure and a fixed set of relationships along the arcs. Information flows only between parent and child nodes. Networks can represent more relationships with a richer set of connectors and a more flexible set of allowable connections. Network representations are more suitable for the needs in that the intermediate nodes can be connected to each other. Smith (1990) identifies the following purposes of network models:

1) Efficient propagation of probabilities,
2) Help in eliciting model structure and the relationships between variables from clients, and
3) Help in understanding the model’s structure by the analyst/statistician.

However, the structures of Bayesian networks assume independence, which does not always capture all the possible situations and as such may act as a new source of error and information loss. Therefore, it appears that probabilities often carry too much information imposing approximate or inefficient solutions [Smith 1990]. The Bayesian network technique has been successfully used for creating consistent probabilistic representations of uncertain knowledge and providing computationally efficient algorithms for computing the posterior probabilities of specific nodes given evidence.

Some of the advantages of Bayesian belief networks (including Bayesian probability) are listed as follows:

1) The network representation is visual and easy to understand.
2) There are efficient algorithms available to incorporate new observations in the network and to predict the influence of possible future observations onto the results obtained so far [Heckerman 1990].
3) Probability theory is a well-refined method for dealing with knowledge of unknown certainty [Pearl 1988, Ng and Bruce 1990]. Hence, computational methods based on Bayesian rules have an axiomatic foundation and well-understood mathematical properties.
4) Bayesian methods generally require only a modest amount of computation time if the initial analysis to gather prior probabilities is given. Furthermore, they can incorporate diverse sources of information, including subjective opinions, historical observations and model outputs.

Bayesian belief networks (including Bayesian approaches) have the following disadvantages:

1) The major difficulty with the Bayesian approach is the large number of prior probabilities required. This computational explosion inevitably leads to the imposition of simplifying assumptions that may or may not be valid for

the given domain. For example, each edge of the DAG represents an independent concept, and probabilistic independence is important to ensure efficient and simple computation.

2) The assumptions about the independence of information/events may lead to counter intuitive, possibly incorrect results [Ng and Bruce 1990, Caudill 1993, Pednault 1981]. Zadeh (1965) suggested that the assumptions associated with Bayesian function, namely mutual exclusivity of events, conditional independence and exhaustivity of events do not always hold.

3) Another difficulty with the Bayesian approach is that prior probabilities are highly dependent on context.

4) Inaccuracies, unreliable data, or uncertainty which cannot be described or insufficiently described statistically can thus only be accounted for approximately. In view of this, probabilistic methods may only be applied to a limited extent.

5) Since probabilistic-oriented methods are based on the validity of statistical laws for stochastic input parameters, it is not possible to reliably define the required distribution functions and statistical parameters as the input data necessary for this purpose are only available to a limited extent.

6) The DAG cannot represent cycles (e.g. *A* implies *B* and *B* implies *A*) or infinite loops would occur in inferencing.

7) Additionally because the sum of all possible states must equal to 1, when evidence reinforces the belief in some possible world, it correspondingly decreases our belief in all other worlds. This is not necessarily the case in real life [Zadeh 1965]. Consider the case of medical diagnosis. A positive result on some test may increase our belief that the patient has some malady, however it does not necessarily decrease our belief that the patient has any other disease.

8) The Bayesian approach offers little opportunity to express incomplete information or partial belief, i.e., ignorance.

9) When a problem has multiple interdependencies embedded within it, Bayesian analysis can indeed be complicated, the graph may become cyclical and reasoning cannot be easily modeled using a Bayesian Belief Network. This is especially true if multiple information sources with correlated errors are to be considered

Because of these limitations, Bayesian Belief Networks have difficulty to model many reasoning processes. Items 1), 4), and 9) above have been some of the reasons why most developers of expert systems like medical and other applications do not adopt the full Bayesian philosophy [de Mantaras 1990]. Practitioners of the Bayesian approach admit that complexity is a problem, and are working hard at developing practical methods for more difficult problems [Varis et al. 1996]. More research and application of Bayesian approach can be found in [Oden 1977, Kahneman et al. 1982, Lindley 1987, Tsao et al. 1993, Nachitnebel et al. 1994, Tonn et al. 1994, Guan et al. 1997, Yager et al. 1999, Engemann et al. 1999 and 2000, Nayak et al. 2001].

More thorough introductions to Bayesian analysis and its applications can be found in [Raiffa 1968, Berger 1985, Gatsonis et al. 1992]. A detailed list of current applications of BBN is provided by Welman and Heckerman (1995). One of the most important features of BBN is the fact that this approach provides an elegant mathematical structure for modelling complicated relationships among random variables while keeping a relatively simple visualization of these relationships. However, this paper is not intended to provide an explanation of BBN.

3. Dempster-Shafer Theory (D-S Theory)

The Dempster-Shafer theory of evidence (DS theory) was formalized by Shafer (1976) for representing and reasoning with uncertain, imprecise and incomplete information [Smets 1988]. It is based on Dempster's original work on the modeling of uncertainty in terms of upper and lower probabilities that are induced by a multivalued mapping rather than as a single probability value [Dempster 1969]. Dempster felt that there was a need for a new system of dealing with uncertainty because of two shortcomings he saw with probability theory as discussed below.

One problem of probability theory is the difficulty of representing ignorance. In probability theory, ignorance is represented by assigning equal prior probabilities to all events, but this method is fraught with difficulties [Shafer 1976]. In fact, such representation means that there is no distinction between randomness and ignorance. To some extent, this approach seems to imply more information than was given, since equal prior beliefs can be attributed to either complete ignorance or to an equal belief in all hypotheses. It avoids the problem of having to assign non-available prior probabilities and makes no assumptions about non-available probabilities. D-S theory represents ignorance explicitly by working with the power set of all possible hypotheses within the domain. It does not fix the probability of the negation of a hypothesis either once the probability of the hypothesis itself is known.

The other problem Dempster recognized with probability theory was its requirement that the subjective belief in an event and its negation must sum to one. He claimed that in many situations evidence that supports one hypothesis should not necessarily decrease the belief in all others [Dempster 1969, Shafer 1976 and 1982]. In D-S theory, there is no requirement that belief not committed to a given proposition must be committed to its negation. This allows the construction and analysis of a "frame of discernment" in a more flexible way. The total allocation of belief can vary to suit the extent of our knowledge.

The Dempster-Shafer theory (DST) of evidence recognizes the distinction between uncertainty and ignorance by introducing belief functions that satisfy axioms that are weaker than those of probability functions [Peddle 1995, Denoeux 1999]. Thus, probability functions are a subclass of belief functions, and the theory of evidence reduces to probability theory when the probability values are known. Roughly speaking, the belief functions allow us to use our knowledge to put constraints or bounds on the assignment of probabilities to events without having to specify the probabilities themselves [Yager et al. 1994]. In addition, the theory of evidence provides appropriate methods for computing belief functions for combinations of evidence. DS theory has been popular since early 1980s when AI researchers were searching for different mechanisms to cope with those situations where Bayesian probability is powerless, with particular emphasis on combining evidence from different sources. Its relationships with related theories have been intensively discussed [Yager et al. 1994].

In the following we introduce the basic concepts of the D-S theory of evidence, briefly describing its origins and comparisons with the more traditional Bayesian theory. Following this we discuss recent developments of this theory including analytical and application areas of interest.

In D-S theory, a piece of information is usually described as a mass function on a frame of discernment. We first give some definitions of the theory [Shafer 1976].

Definition 3.1 (Frame of discernment): A set is called *a frame of discernment* (or simply a frame) if it contains mutually exclusive and exhaustive possible answers to a question. It is usually denoted as Θ. The set is required that at any time one and only one element in the set is true.

For instance, in diagnosis systems *a frame of discernment* generally represents diagnostic hypotheses. In classification systems, it in general represents the set of possible classes to which an item can belong.

Definition 3.2 (Mass function): A function m: $2^{\Theta} \rightarrow [0, 1]$ is called *a mass function* on frame Θ if it satisfies the following two conditions:

1) $m(\emptyset) = 0$; and

2) $\Sigma m(A) = 1$

where $\emptyset$ is an empty set and A is a subset of Θ.

The set of all subsets of Θ is denoted by 2^{Θ}. The notation 2^{Θ} is used because of the need to allow for the number of elements in the power set, i.e. all possible subsets of the frame of discernment Θ. All of the assigned probabilities sum to unity and there is no belief in the empty set (which can be thought of as a hypothesis known to be false).

A mass function is also called *a basic probability assignment*, denoted as *bpa*. Note that $m(A)$ measures an assignment of belief exactly to A, not the total assignment of belief to A. The total assignment of belief to A is measured by the following belief function $Bel(A)$.

Definition 3.3 (Belief function): A function $Bel : 2^{\Theta} \rightarrow [0; 1]$ is called *a belief function* if Bel satisfies

1) $Bel(\Theta) = 1$;

2) $Bel(\bigcup_{i=1}^{n} A_i) \geq \Sigma_i Bel(A_i) - \Sigma_{i>j} Bel(A_i \cap A_j) + \cdots + (-1)^{-n} Bel(\cap_i A_i)$.

It is easy to see that $Bel(\emptyset) = 0$ for any belief function. A belief function is also *called a support function*. The difference between $m(A)$ and $Bel(A)$ is that $m(A)$ is our belief committed to the subset A excluding any of its subsets while $Bel(A)$ is our degree of belief in A as well as all of its subsets.

In general, if m is a mass function on frame Θ then Bel defined above is a belief function on Θ:

$$Bel(B) = \sum_{A \subseteq B} m(A)$$

Recovering a mass function from a belief function is as follows [Yager et al. 1994]:

$$m(A) = \Sigma_{B \subseteq A} (-1)^{|B|} Bel(B)$$

For any finite frame, it is always possible to get the corresponding mass function from a belief function and the mass function is unique.

A subset A with $m(A) > 0$ is called *a focal element* of this belief function and represents the exact belief in the proposition depicted by A. If all focal elements of a belief function are the singletons of Θ, then the corresponding mass function is exactly a probability distribution on Θ. Thus mass functions are generalized probability distributions in this sense.

If there is only one focal element for a belief function and the focal element is the whole frame Θ, the belief function is called *a vacuous belief function*. It represents total ignorance.

Definition 3.4 (Plausibility function): A function Pls defined below is called a plausibility function:

$$Pls(A) = 1 - Bel(\neg A)$$

Pls(*A*) represents the degree to which the evidence fails to refute *A*. From a mass function, we can get its plausibility function as follows:

$$Pls(B) = \sum_{A \cap B \neq \phi} m(A)$$

These measures are clearly related to one another, i.e.,

$Bel(A)=1-Pls(\neg A)$ and $Pls(A)=1-Bel(\neg A)$

where $\neg A$ is referring to its complement "not *A*" and $Bel(\neg A)$ is often called the doubt in *A*. Other notable relationships include:

$Bel(A)+Bel(\neg A)\leq 1$, and $Pls(A) + Pls(\neg A)\geq 1$

The above two inequalities show a major departure from the more traditional simple probability function, used within the Bayesian approach. In the case of each of the focal elements being singletons, we return back to traditional Bayesian analysis incorporating normal probability theory, since in this case $Bel(A)=Pls(A)$.

Collectively the above measures provide DST with an explicit measure of ignorance about event *A* and its complement $\neg A$. The measure is defined as the length of the interval [*Bel*(*A*), *Pls*(*A*)] and can be used to specify an *evidential interval (EI)* that provides a lower and upper bound for the degree of belief in a set:

$EI(A) =[Bel(A), Pls(A)]$.

Table 1 provides one of the interpretations of the evidential interval *EI* [Garvey et al. 1981], which shows that this approach can describe and handle uncertainties by using the concept of the degrees of belief.

Table 1. Interpretation of uncertainty intervals

Evidential interval [*Bel*(*A*), *Pls*(*A*)]	Interpretation
[0, 1]	Total ignorance about proposition *A*
[0.6, 0.6]	A definite probability of 0.6 for proposition *A*
[0, 0]	Proposition *A* is false
[1, 1]	Proposition *A* is true
[0.25, 0.85]	Probability of *A* is between 0.25 and 0.85, i.e., the evidence simultaneously provides support for both *A* and $\neg A$ (the complement of *A*)

When more than one mass function is given on the same frame of discernment, the combined impact of these pieces of evidence is obtained using a mathematical formula called *Dempster's combination rule*. If m_1 and m_2 are two mass functions on frame Θ, then $m = m_1 \oplus m_2$ is the mass function after combining m_1 and m_2:

$$m(C) = \begin{cases} 0 & C = \varnothing, \\ \dfrac{\sum_{A \cap B = C} m_1(A) m_2(B)}{1 - \sum_{A \cap B = \varnothing} m_1(A) m_2(B)} & C \neq \varnothing. \end{cases}$$

$\oplus$ means that Dempster's combination rule is applied on two (or more) mass functions. The condition of using the rule is stated as `two or more pieces of evidence are based on distinct bodies of evidence' [Shafer 1976]. Dempster's rule does not result from the same basic axioms as those based on which conventional probability theorems are derived [Parsons 1994]. It does, however, possess the necessary commutative and associative properties. Bayes rule of conditioning is a special case of this rule [Caselton et al. 1992]. The independence required by Dempster's rule is simply probabilistic independence.

An important feature in the above formula is in the denominator i.e., $\sum_{A \cap B = \varnothing} m_1(A) m_2(B)$, denoted by *k*, which can be interpreted as a measure of conflict between the sources and is directly taken into account in the combination as a normalisation factor. The measure represents the mass which would be assigned to the empty set if masses were not normalised. It is very important to take this value into account for the appropriate use of the combination rule. When two pieces of evidence are in complete conflict and if either of them could decide the truth or false of the hypothesis in question, there would be k=1 and the denominator would be zero. This would lead to irrational combination of evidence if not treated carefully in particular reasoning situations. More discussion on this issue can be found in [Bloch 1996, Dubois and Prade 1988, Zadeh 1984]. A rational treatment of conflicting pieces of evidence was dealt with in [Yang and Xu 2002a and 2002b] by taking into consideration the relative importance of evidence in the framework of multiple criteria decision analysis.

Beyesian theory requires a more explicit formulation of conditioning and the prior probabilities of events. D-S theory embeds conditioning information into its belief function and does not rely on prior knowledge, making it appropriate for situations where it is difficult to either collect or posit such probability, or isolate their contributions [Yager et al. 1994].

In summary, the advantages of the Dempster-Shafer theory are listed as follows:

1) It has the ability to model information in a flexible way without requiring a probability (or *a prior*) to be assigned to each element in a set, and it provides a convenient and simple mechanism (Dempster's combination rule) for combining two or more pieces of evidence under certain conditions. The former allows an agent to describe ignorance because of lack of information, and the latter allows an agent to narrow down the possible solution space as more evidence is accumulated.
2) It can model ignorance explicitly.
3) Belief functions of Dempster–Shafer are set functions rather than point values.
4) Rejection of the law of additivity for belief in disjoint propositions.

The disadvantages of the D-S theory include the following.

1) The theory assumes that pieces of evidence are independent, as in the Baysian methods it is not always reasonable to assume independent evidence.
2) The computational complexity of reasoning within the Dempster-Shafer theory could be one of the major points of criticism if the combination rule is not used properly. In fact, Orponen (1990) showed that the combination of mass functions or basic probability assignments (*bpa*'s) using Dempster's rule is #P-complete (the class #P is a functional analogue of the class NP of decision problems). Given a frame of discernment of size $|\Theta| = N$, a mass function *m* can have up to 2^N focal elements all of which have to be represented explicitly in order to capture the complete information encoded in *m*, if all subset of Θ could be given non-zero basic probability assignments. Furthermore, the combination of two mass functions requires the computation of up to 2^{N+1} intersections. To overcome this difficulty various approximation algorithms have been suggested that aim at reducing the number of focal elements in the belief functions involved. Bauer (1997) reviewed a number of approximation algorithms and also proposed his new method. In multiple criteria decision analysis, however, specific decision analysis frameworks can be designed so that the computational complexity of reasoning using Dempster's rule becomes linear rather than #P-complete [Yang and Singh 1994, Yang and Sen 1994, Yang 2001].
3) Dempster-Shafer theory only works on exclusive and exhaustive sets of hypotheses. However, not all sets of competing hypotheses have the two properties, due to insufficient knowledge and resources.

4 Fuzzy Set Theory

In many human activities, the use of linguistic terms rather than precise numbers is necessary. A linguistic variable differs from a numerical one in that its values are not precise numbers, but words or sentences in a natural or artificial language. Words, in general, are less precise than numbers. The concept of a linguistic variable serves the purpose of providing a means of approximated characterization of phenomena, which are too complex, or too ill-defined to be amenable to their description in conventional quantitative terms. The use of words or sentences rather than precise numbers is, in general, a less specific, more flexible, direct, realistic and adequate form to express the qualitative aspects and it has been very widespread. This state of facts has motivated the introduction of fuzzy sets [Zadeh 1965]. It provides a systematic way to interpret linguistic variables in a natural decision-making procedure. Therefore, it can be viewed as complementary to traditional methods and can be a powerful tool to deal with imprecise information, especially linguistic information [Zadeh 1978, Duckstein 1994].

Fuzzy Set theory uses linguistic variables and membership functions with varying grades to model uncertainty inherent in natural language. Fuzzy logic starts with a set of objects, *U*. If *A* is a fuzzy subset of *U* then there is a function $\mu_A(u)$ which maps the elements of *U* into *A* by numbers between 0 and 1. These numbers represent the degrees of membership of the elements in set A. The difference of fuzzy set theory from crisp set theory is that rather than completely belonging or not completely belonging to a set, the elements of *U* can partially belong to a set. If $\mu_A(u) = 1$ then membership is absolute and $\mu_A(u)= 0$ indicates non-membership.

In a typical knowledge base, with their antecedents and consequents, approximate matching of facts to a rule's antecedent is difficult or impossible with conventional two valued logic but becomes natural for multivalued fuzzy logic. Unlike classical logic which requires a deep understanding of a system, exact equations, and precise numeric values, fuzzy logic incorporates an alternative way of thinking, which allows modelling complex systems using a higher level of abstraction originating from our knowledge and experience. Fuzzy logic resembles human reasoning in its use of imprecise information to generate decisions.

Fuzzy logic systems or Fuzzy Inference Systems (FIS) are schemes that (among other capabilities) allow the mapping of a number of fuzzy inputs into a number of fuzzy outputs [Mendel 1995]. The mapping is done by a set of fuzzy rules that relates inputs to outputs in an “if ... then” fashion. Inputs and outputs can be represented by means of fuzzy variables capable of containing language terms and fuzzy hedges. The collected expert knowledge can be condensed in a fuzzy inference system. A fuzzy system can be understood as a non-linear classifier that transforms several inputs into a unique output. The most successful examples of dealing with imprecision using fuzzy set theory come from the field of fuzzy control. The next diagram illustrates the core of a fuzzy inference system:

$$\text{Inputs} \Rightarrow \text{Fuzzifier} \rightarrow \text{Inference Engine} \rightarrow \text{Defuzzifier} \Rightarrow \text{Outputs}$$

$$\Uparrow$$

$$\text{Knowledge Bases}$$

The system's inputs go through a fuzzifier. The inference engine works with attribute values, which have fuzzy memberships attached. They may be created from real-valued attributes which are partitioned into individual fuzzy sets. The inference engine provides a fuzzy output which may need to be defuzzified. In control domain, the output must be defuzzified so that there is a single, well defined control action taken. Fuzzy control is attractive in situations where system behaviour is very difficult to describe using traditional quantitative models. However, it may not be an easy task to create a good inference engine.

In the most general form, the encoded knowledge of a multi-input-multi-output (MIMO) system [Mendel 1995] can be interpreted by a fuzzy model consisting of IF-THEN rules with multi-antecedent and multi-consequent variables as follows (with r antecedents, s consequents, and n rules):

IF U_1 is B_{11} AND U_2 is B_{12} AND… AND U_r is B_{1r}
THEN V_1 is D_{11} AND V_2 is D_{12} AND… AND V_s is D_{1s}
ALSO
…
ALSO
IF U_1 is B_{n1} AND U_2 is B_{n2} AND… AND U_r is B_{nr}
THEN V_1 is D_{n1} AND V_2 is D_{n2} AND… AND V_s is D_{ns};

where $U_1, U_2, \ldots, U_r$ are input variables, and $V_1; V_2, \ldots, V_s$ are output variables; B_{ij} (i=1,…, n; j=1,…, r) and D_{ik} (i=1,…, n; k =1,…, s) are fuzzy sets of the universes of discourse $X_1, X_2, \ldots, X_r$ and $Y_1, Y_2, \ldots, Y_s$ of input and output variables, respectively.

One major step in fuzzy-logic modeling is to decide the reasoning mechanism. The process of reasoning in fuzzy modeling proceeds through the following steps:

a) A crisp input is fuzzified by input membership functions.
b) Fuzzy aggregation of antecedents in each rule (AND connective).
c) Implication relation for each individual rule (IF-THEN connective).
d) Aggregation of the rules (ALSO connective).
e) Inference from the set of rules, using the crisp input to obtain the fuzzy output.
f) Defuzzification of the output.

There are a number of different ways to implement a fuzzy inference engine. Mamdani et al. (1974) described an inference engine in terms of a fuzzy relation matrix and uses the compositional rule of inference to arrive at the output fuzzy set for a given input fuzzy set. The output fuzzy set is subsequently defuzzified to arrive at a crisp control action. Other techniques include sum-product and threshold inferencing. Kosko (1990) provided a review on the different ways of implementing fuzzy inference engines.

Some of the characteristics of fuzzy logic are given as follows.

- In fuzzy logic, knowledge is interpreted as a collection of elastic or equivalently fuzzy constraints on a collection of variables.
- Inference is viewed as a process of propagation of elastic constraints.
- Fuzziness is identified with lack of sharp distinctions. Fuzzy sets are usually used to formalize this kind of uncertainty.

The advantages of fuzzy set theory include the following:

1) Fuzzy set techniques have low requirements about the precision of information (depending on the definition of the operators and the particular method used).

2) Fuzzy set techniques seem to be a good solution for some of the problems that arise due to lexical impression.

3) Fuzzy logic provides an alternative way to map an input space to an output space. It is also tolerant of imprecise data and therefore provides a simple way of obtaining relationships based on experimental data [Mendel 1995].

4) It is capable of dealing with incomplete data since there are no absolute rules for data requirements. (This may be a potential drawback as well)

The disadvantages of fuzzy set theory include the following:

1) It is not always clear how to construct reasonable membership functions. Various methods have been proposed including the use of statistical data, and the composition of simpler functions, but no completely general approach seems to exist yet. According to Zadeh, membership functions are subjective and context dependent, therefore, there is no general method to determine them either by experiment or analysis [Zadeh 1975].

2) The choice of appropriate definitions for the operators can be problematic. As Zadeh himself has acknowledged, different definitions are needed in different situations; however it is not always clear as to what definitions should be used.

3) The inherent flexibility of fuzzy set theory can also be a disadvantage since there is little guidance as to which methods to use to solve a given problem.

4) There is the inherent lack of formal definitions for functional modifier rules. This can lead to inconsistencies between knowledge bases [Giles 1982, Ng and Bruce 1990].

However, these problems have not stopped many from creating successful expert systems [Adlassnig 1982] particularly in Japan where fuzzy logic is widely used in a variety of fields [Ishizuka et al. 1982a, Ng and Bruce 1990, etc.].

Since fuzzy set theory was proposed by Zadeh, some methods based on the fuzzy set theory for approximate (fuzzy) reasoning have been proposed, such as in [Chen 1994, 1995 and 1997, Gorzalczany 1987, Chang 1991, Cao et al. 1990, Bien et al. 1994, Mendel 1995, etc.]. The approximate reasoning scheme has been discussed by many researchers [Negoita 1985, Zadeh 1975].

Zadeh introduced both the concept of fuzzy set and the concept of possibility measure [Zadeh, 1978,]. General introductions to fuzzy sets and possibility theory can be found in [Dubois and Prade 1980 and 1988, Dubois and Prade, 1986, Klir and Yuan 1995, Kruse et al. 1994, Dubois et al 1997 and 1998, Bezdek 1993]. An anthology of basic papers in fuzzy logic was given by Dubois et al. [Dubois et al. 1993]. Two anthologies of Zadeh's personal contributions were published in [Yager et al. 1987, Klir and Yuan 1996]. An organized overview of applications of fuzzy set theory was given under the form of an anthology by Dubois et al. (1997).

Applications of possibilities are mainly within the spectrum of fuzzy set applications. Fuzzy sets and logic have been used in representation and approximate reasoning [Yager et al. 1987], pattern recognition [Dubois et al. 1993, Pal et al. 1986], operations research [Dubois et al. 1993], and modeling uncertainty and control [Dubois et al. 1993]. Kraslawski et al. (1989 and 1993) applied fuzzy set theory to study uncertainty associated with incomplete and subjective information in process engineering. Ayyub et al. (1992 and 1998) studied structural reliability assessment by using fuzzy theory to treat the uncertainty associated with ambiguity. Juang et al. (1992) demonstrated the applicability of fuzzy set theory in the modeling and analysis of non-random uncertainties in geotechnical engineering. Further literature on industrial applications of fuzzy theory is available for the interested reader [Yen et al. 1995]. Specifically, it has been applied to characterize uncertainty in engineering design [Kraslawski et al 1993], in quality control [Quin et al. 1996], in sludge land selection [Crump et al. 1993], and in solute transport modeling [Dou et al. 1995 and 1997]. However, the drawbacks in its applicability to uncertainty analysis have been noted by many researchers [Spiegelhalter 1989, Walley 1991]. Fuzzy set theory appears to be more suitable for qualitative reasoning, and classification of elements into a fuzzy set, than for quantitative estimation of uncertainty.

5. Summary of methods

5.1. Comparison of the three approaches

Although the three approaches described previously all use the same scale as probabilities, fuzzy set membership functions are subject to different manipulation procedures.

Considering two main uncertainty forms in *ambiguity* and *fuzziness* [Klir 1995], Dempster-Shafer Theory provides an ideal framework for the study of ambiguity uncertainty, as it enlarges the scope of traditional probability theory. Fuzziness is identified as lack of sharp distinctions. Fuzzy sets are usually used to formalize this kind of uncertainty.

Each of the three approaches views uncertainty from a different perspective. In the Bayesian theory uncertainty is modelled by probability, in the D-S theory by the degree of belief, and in fuzzy set theory by the degree of set membership. Uncertainty treated in all the three approaches is specified by a numerical value in the range [0,1]. They differ form each other in terms of how and when they acquire uncertainty values and also in terms of how these uncertainty values are manipulated.

In the Bayesian theory it is difficult to represent ignorance since each entity must be assigned probability. In the D-S theory, ignorance is represented by assigning belief to larger subsets (i.e., given more knowledge, the belief would be assigned to a smaller subset, or even a singleton). In addition, the magnitude of the belief interval (plausibility-Bel) also reflects the degree of ignorance.

There are several papers in the literature which discuss the intuitive, philosophical, and theoretical connection between the three theories. For example, Grosoff (1986) discussed some of the relationships between the Bayesian and D-S theories. Prade (1983) presented a uniform framework for the Bayesian, D-S and fuzzy set theories, while Thompson (1985) investigated the structural similarities of these theories.

5.2 Appropriateness of the approaches

Based on the comparative analysis of the uncertainty theories, we make the following suggestions:

Probability is useful when dealing with serial events that require an enumeration notion of uncertainty but is not so useful when the uncertainty is associated with the degree of accomplishment of a known situation [Laviolette et al. 1994]. The Bayesian approach is well suited in situations where probability is known (or can be acquired with reasonable effort). This approach is especially attractive because of its strong theoretical foundation. Probabilistic approaches are best suited to mechanistic systems where the accuracy and precision are considered important.

The Dempster-Shafer theory is a good choice for applications where uncertainty is best thought of as being distributed in sets rather than just single items, i.e., properly representing ignorance. The Dempster-Shafer theory is most useful when one can use evidence to iteratively focus attention on smaller subsets of the frame of discernment. This has been particularly useful in domains in which a hierarchical structure can be imposed on the hypotheses so that groups of hypotheses form classes in the hierarchy [Pearl 1988].

There are three main characteristics of fuzzy systems that give them good performance for specific applications.

- A fuzzy system is well suited for applications where evidence is itself fuzzy in nature.
- Fuzzy systems are suitable for uncertain or approximate reasoning, especially for systems where mathematical models are difficult to derive.
- Fuzzy logic allows decision making with estimated values under incomplete or uncertain information.

5.3 Connections

The combination of different approaches to formulate a proposition that can express more than one kind of uncertainty has been increasingly seen as a necessary premise for the design of reliable and accurate procedures for classifying and combining multi-source heterogeneous data. Hence, it is necessary to clarify the connection among these strategies.

- *Fuzzy set and Possibility*

First we consider the relationship between fuzzy set and possibility. The degree of membership expresses the strength to which a given event belongs to an ill-defined set. The degree of possibility expresses the strength of one's opinion about the exact nature of the actual event given that it belongs to an ill-defined set. If S is a set and if s is an element of S, a fuzzy subset F of S is defined by a membership function $\mu_F(s)$ that measures the degree to which s belongs to F. To use a standard example, if S is the set of positive integers and F is the fuzzy subset of small integers, then we might have μ_F (1) = 1, μ_F (2) = 1, μ_F (3) =0.8,…, μ_F (20) = 0.1, and so on. Let X be a variable that can take values in S. The statement "X is F" (for example, the statement "X is a small integer"), induces a possibility distribution on X, and the possibility that $X = s$ is taken to be μ_F (s).

For most of the concepts of probability theory there is a corresponding concept in possibility theory. For example, it is possible to define multivariate possibility distributions, marginal possibility distributions, and conditional possibility distributions [Zadeh 1978]. Thus, in principle one can use fuzzy possibility theory much like probability theory to quantify the uncertainty introduced by vagueness, regardless of whether the vagueness comes from the data or from the rules.

Although possibility theory is a subject of great interest, it has yet to be exploited in work on knowledge-based system and engineering application. This is partly due to the fact that most of the problems that limit probability theory also arise in possibility theory -- such as the problem of prior possibilities and the problem of dependence in multivariate possibility distributions. Furthermore, as with certainty theory, possibility theory suffers from the problem that the semantics of its measure are not objectively defined. However, the distinction between uncertainty due to randomness and uncertainty due to vagueness is both valid and important.

- *Probabilities, Possibilities, and Fuzzy sets*

Probabilities, possibilities, and fuzzy sets are all measures used to formalize and quantify uncertainty. A brief presentation and qualitative comparison of these measures can be found in [Henkind and Harrison 1988]. A quantitative comparison of those measures in terms of both efficiency and expressiveness is also necessary in order to evaluate and characterize those measures. Some attempts in this direction are focused on consistency between probabilities and possibilities [Dubois and Prade 1983, Delgado et al. 1987] and transformations from probabilities and possibilities to Dempster-Shafer theory [Klir and Parviz 1992]. However, those transformations cannot tell us about the exact relationship of probabilities and possibilities. Furthermore, the transformations from possibilities to Dempster-Shafer theory as found in [Klir and Parviz 1992] require the elements of the universal set to be ordered in descending possibility values. However, such an ordering does not always exist when the universal set is infinite. This limits the applicability of the transformation to finite universal sets. Finally, there has been no study so far of the extension of the universal sets that are necessary in order to create maps between probabilities, possibilities, and fuzzy sets.

A formal quantitative analysis of probabilities, possibilities, and fuzzy sets is presented in [Drakopoulos 1995], where a number of theorems were presented in order to completely specify the relationship between probabilities, possibilities and fuzzy sets, e.g., from the viewpoint of relative expressiveness i.e. the ability of one to simulate the others. It was concluded that probabilities are proved to be more expressive than both possibilities and fuzzy sets.

Probabilities can simulate possibilities without any extra space requirements while the opposite is not true, in general. Probabilities can simulate fuzzy set formulation without any extra space requirements. However, fuzzy sets cannot simulate probability estimation without exponentially greater requirements. It was also shown that fuzzy sets can simulate possibility estimation without extra space requirements. This comes at no surprise as fuzzy sets have been defined in a possibilistic way.

Possibilities and fuzzy sets can in principle simulate probability estimation but, in that case, their space requirements are exponential when compared to those of probabilities. It was indicated that both probabilistic and possibilistic approaches are of benefit in different contexts. A choice should be made for each application depending on its demands in terms of accuracy and efficiency [Drakopoulos 1995].

The limitation of probabilities is their efficiency. Possibilistic measures like possibilities and fuzzy sets are easier to compute than probabilities since the former are extensional [Drakopoulos 1995] while the latter are intensional measures. This practical advantage of possibilities is very noticeable as one can easily compute the possibility value of a compound logical expression or relation among events or entities by simply applying the definition.

Many realistic design situations involve different types of uncertainties, which can be classified as subjective, objective and hybrid type of uncertainties. For example, stochastic uncertainty is associated with the geometry or material properties of a system (based on experimental measurements). At the same time, design imprecision may be involved in the subjective selection of a parameter values among design alternatives based on personal knowledge and desirability. This requires the development of a combined uncertainty-based decision making model using probability and fuzzy theories.

The assessment of system parameters may be both objective and subjective in character. Depending on the type and extent of information available, it is possible to apply fuzzy set theory or the theory of fuzzy random variables that provides a mathematical basis for taking account of uncertainties due to randomness and fuzziness simultaneously [Kwakernaak 1978 and 1979, Puri and Ralescu 1986, Wang and Zhang 1992]. In individual random variables possess (besides their randomness) fuzzy uncertainty, they may be described by means of fuzzy set theory. This approach leads to uncertain probability density, probability distribution functions, and uncertain limit state functions.

Uncertainty with the characteristic fuzzy randomness is described, quantified and processed on the basis of the theory of fuzzy random variables. This includes, as also in the case of fuzziness, both objective and subjective information. The theory of fuzzy random variables permits the modelling of uncertain structural parameters, which partly exhibit randomness, but which cannot be described using random variables without an element of doubt. The randomness is "disturbed" by a fuzziness component. The reasons for the existence of fuzzy randomness were given by Bothe (1993) as follows:

1) Although samples are available for a structural parameter, these are only limited in number. No further information exists concerning the statistical properties of the universe.

2) The statistical data possess fuzziness, i.e., the sample elements are of doubtful accuracy; or they were obtained under unknown or non-constant reproduction conditions.

On the other hand, fuzzy set-theoretic operations can be justified from several probabilistic points of view. Conversely the possibilistic nature of likelihood seems to be in accordance with the way statisticians have used them, and fuzzy events can be considered as relevant evidence in statistics. There are already several domains of application where fuzzy sets and probability are used jointly.

Finally fuzzy sets can model linguistic probabilities as verified experimentally [Wallsten et al. 1986]. Linguistic probabilities can be handled in inference processes, as done by Zadeh (1985), Dubois and Prade (1988), Jain and Agogino (1990), or in decision analysis as done by Watson et al. (1979) and Nau (1992). These examples show that instead of considering probability and fuzzy sets as rivals, it sounds more promising to build bridges and take advantage of the enlarged framework for modelling uncertainty and vagueness they can bring us to [Dubois et al. 1988].

- *D-S theory, possibility theory and probability*

The D-S theory has strong links with probability theory where a Dempster's probability space can always be translated into a normal probability space [Fagin and Halpern 1989]. Therefore a probability space is more general than a Dempster's probability space. The belief function theory is closely related to probability theory and comes out of probability theory.

Dempster-Shafer theory has been shown to be a broad theory that subsumes both probabilities and possibilities. When the focal elements (e.g. elements of non-zero basic probability density or assignment *m*) are singletons or nested then Dempster-Shafer theory reduces to probability or possibility theory, respectively [Dempster 1968b and 1969]. As iscussed in [Klir 1989 and 1992], however, possibilities apply only to finite universal sets, as their elements must be ordered in descending possibility values. Such an ordering is not always possible for infinite sets. Of course, a transformation similar to the one in probability transformation can be used to reduce possibilities to Dempster-Shafer theory in the general case. However, the opposite transform, in general, requires exponentially larger spaces. This is due to the fact that Dempster-Shafer theory subsumes probability theory. In short, the structure of the focal elements (singletons or nested) will determine whether the defined measures will be probability or possibility measures (respectively). However, they cannot be used at the same time.

- *Fuzzy set theory and D-S theory*

Although a large number of studies on evidence reasoning have been conducted, most of these studies are based on two-valued logic system. In practice, the true value of a proposition may not be simply true or false. It can be any number in the unit interval [0, 1]. Hence, it is desirable to combine the D-S theory with fuzzy set approach.

In classification or decision problems, intrinsically vague information may often coexist with conditions of ‘‘lack of specificity’’ originating from evidence not strong enough to induce knowledge, but only degrees of belief, or *credibility* regarding a hypothesis [Binaghi et al 1999]. It seems reasonable to extend the fuzzy logic framework to cover credibility uncertainty as well. The Dempster-Shafer theory based on the concept of *belief function* may be used to model and quantify the subjective credibility induced by partial evidence. Several researchers have investigated the relationship between fuzzy sets and belief functions, and suggested different ways of integrating them [Yager 1982]. The basic idea is to allow the focal elements of a belief structure to be fuzzy sets. Yager (1995) proposed a “smooth normalization procedure” (SNP) for converting a subnormal fuzzy belief structure into a normal one. In other words, if at least one of the focal elements is not normal, then a fuzzy belief function *m* is called a subnormal fuzzy belief structure. This method generalises both fuzzy set normalization and Dempster’s normalization of crisp belief structures.

Several researchers have investigated the relationships between fuzzy sets and Dempster-Shafer theory and suggested different ways of integrating them. Ishizuka et al (1982a, 1982b) firstly extended the D-S theory to include fuzzy knowledge for structural damage assessment. They developed a rule-based inference procedure by using fuzzy sets to represent imprecise data and employing the Dempster-Shafer theory to aggregate evidence for structural damage assessment. Ogawa et al (1985) subsequently proposed an inexact inference procedure on the base of the D-S theory of evidence and fuzzy sets to make the structural damage assessment more general and practical. Chen (1997) extended Ishizuka’s rule-based inference for more general decision-making problems. Yager (1995 and 1982) also considered fuzzy sets in the D-S belief structure to deal with probability uncertainty.

The benefit of combining fuzzy and Dempster-Shafer models may become substantial when conditions of lack of specificity in data are prevalent. These conditions may originate from the situations in which domains are potentially unknown and there may be no expert with enough experience to provide heuristics required in a decision making process. In these cases, experts may have difficulty in structuring and articulating causal relationships or the assessment. By using fuzzy evidential reasoning approach, this problem could be solved.

As mentioned before, it is hard to rank the approaches in general, because each of them is usually aimed at a special application environment. The choice of an appropriate "uncertainty" calculus may depend on [Zimmermann 2000]:

- The causes of "uncertainty", which influence the information flow between the observed system and the "uncertainty" model (paradigm chosen by the observer).
- Quantity and quality of information available, which implies that a selected "uncertainty" model or theory has to be appropriate to the available quantity and quality of input information.
- The type of information processing required by the respective "uncertainty" calculus whilst a chosen "uncertainty" theory also determines the type of information processing applied to available data or information.

- Language required by the final observer, which means that for pragmatic reasons the information offered to the observer (human or other) by the "uncertainty" model should be in an adequate language.

Simonovic (1997) pointed out that it is not the type of uncertainty that determines the appropriate way of modelling, but data sufficiency and their availability. If sufficient data are available to fit a probability density distribution then the use of stochastic variables will be the best way to quantify the uncertain values. On the other hand, if the requirements of sustainability are to be addressed, such as needs of future generations, expanded spatial and temporal scales and long-term consequences, then the information available is scarce. In this case the fuzzy set approach may make better use of information available [Simonovic 1997].

6. Possible Application in Maritime and Offshore Safety-Based Assessment

Offshore safety analysis involves examining the proposed design of an offshore installation or a well to identify potentially hazardous situations and assess associated risks in order to provide a rational basis for determining where risk reduction measures are required. Qualitative and quantitative safety assessment approaches or their mixture are often carried out to study the risk of a system in term of the probability of occurrence of each hazard and its possible consequences. Generally, the process of risk assessment is initially performed qualitatively and later extended quantitatively to include data when it becomes available.

Probabilistic safety/risk analysis relies on statistical methods and databases that can be used to identify numerical probability and consequence values for risk assessment. Quantitative risk assessment (QRA) is a means of estimating and evaluating numerical risks from a particular hazardous activity. However, the lack of data for risk assessment may present a problem if past experience and historical information is not available [Wang and Ruxton 1997].
It is worth noting that many typical safety assessment approaches may be difficult to use in situations where there is a lack of information, past experience, or ill-defined situation in risk analysis [Wang and Ruxton 1997]. In certain circumstances, probability theory can be a powerful tool. However, very often the type of uncertainty encountered in offshore projects does not fit the axiomatic basis of probability theory, simply because uncertainty in these projects is usually caused by the inherent fuzziness of the parameter estimate rather than randomness.

The operation of a ship, an offshore installation or a port is also associated with a high level of uncertainty because it usually operates in a very changeable environment while human errors and organisational malfunctions play an important role in many possible accidents. To facilitate safety based design/operation decision making, it is required to model risks in various situations with confidence. This certainly needs flexible risk modelling and decision making techniques to be developed and applied.
The use of fuzzy production rules in a fuzzy inference system, where the conditional part and/or the conclusions contain linguistic variables, can handle such uncertainty well. This greatly reduces the need for an expert, or a safety analyst, to know the precise point at which a risk factor exists. Linguistic variables are commonly used to represent risk factors in risk analysis [Karwowski and Mital 1986, Keller et al. 1989,Bell and Badiru 1996a, Bowles and Pelaez 1995, An et al. 2000, Wang et al. 1995 and 1996, Wang 1997, Sii et al. 2001,]. As in traditional risk analysis, the assessment is based on the failure rate or frequency of occurrence, the severity of an item failure and the failure consequence probability (the probability that consequences happen given the occurrence of the item failure). The three fundamental parameters *failure rate, consequence severity* and *failure consequence probability* are represented by natural languages, which can be further described by membership functions. These parameters can be represented as members of fuzzy sets, combined by matching them against rules in a rule base, evaluated with the Mamdani-type inference system [Mamdani et al. 1974], and then normalised to assess the safety estimates at the bottom level of a hierarchical structure (individual element level), e.g., the following is a fuzzy *IF-THEN* rule in a rule base:

IF the likelihood of a hazard is *frequent AND severity of occurrence* is *catastrophic, THEN risk level* is *high.*

The *frequent, catastrophic* and *high* are characterised by membership functions. A fuzzy system is constructed from a collection of such fuzzy *IF-THEN* rules.

The safety estimates associated with a failure event can be evaluated in the following form:

$$\{(\beta_1,'poor'),(\beta_2,'fair'),(\beta_3,'average'),(\beta_4,'good')\}$$

where β_4, for example, is the degree of belief that the safety is evaluated as “good”.

The safety of a structure is often determined by all the associated failure events/modes of each individual component that makes up the structure. A component usually has several failure events/modes. Problem may then arise as to how uncertain evaluations of safety analyses of all the failure events/modes of a component may be synthesized in a rational way so as to attain an (often uncertain) evaluation of the safety of the component. The problem may be ultimately generalized to estimate the safety of a system with a hierarchy. In such a hierarchical synthesis process, the evidential reasoning (ER) approach [Yang and Singh 1994, Yang 2001] may be one of the most useful methods for aggregation of multiple subjective factors (criteria). A generic framework for modelling system safety using fuzzy sets and the evidential reasoning approaches is depicted in Figure 2 [Siiet al.2000 and 2001b].

This conceptual framework of a hybrid process for safety analysis was initially investigated in [Wang et al. 1995] and has been further discussed recently in [Sii et al. 2000 and 2001b]. The ER approach for multiple criteria decision analysis (MCDA) under uncertainty has been developed and evolved over a number of years [Yang and Singh 1994, Yang and Sen 1994, Yang 2001, Yang and Xu 2002a, 2002b]. Recently, the ER approach has been combined with fuzzy sets, leading to a hybrid fuzzy evidential reasoning approach [Yang, Wang and Xu 2002b]. This hybrid approach resolves problems in traditional MCDA methods and has the following features. 1) It allows the analyst to evaluate risks associated with item failure modes by directly using linguistic terms that are employed in making the criticality assessment. 2) Ambiguous, qualitative, or imprecise information, as well as quantitative data, can be used in the assessment under the same framework and handled in a consistent manner. 3) It provides a more flexible structure for combining severity, occurrence, and detectability parameters. And 4) it provides a systematic and rigorous procedure for aggregating assessment information to arrive at final conclusions.

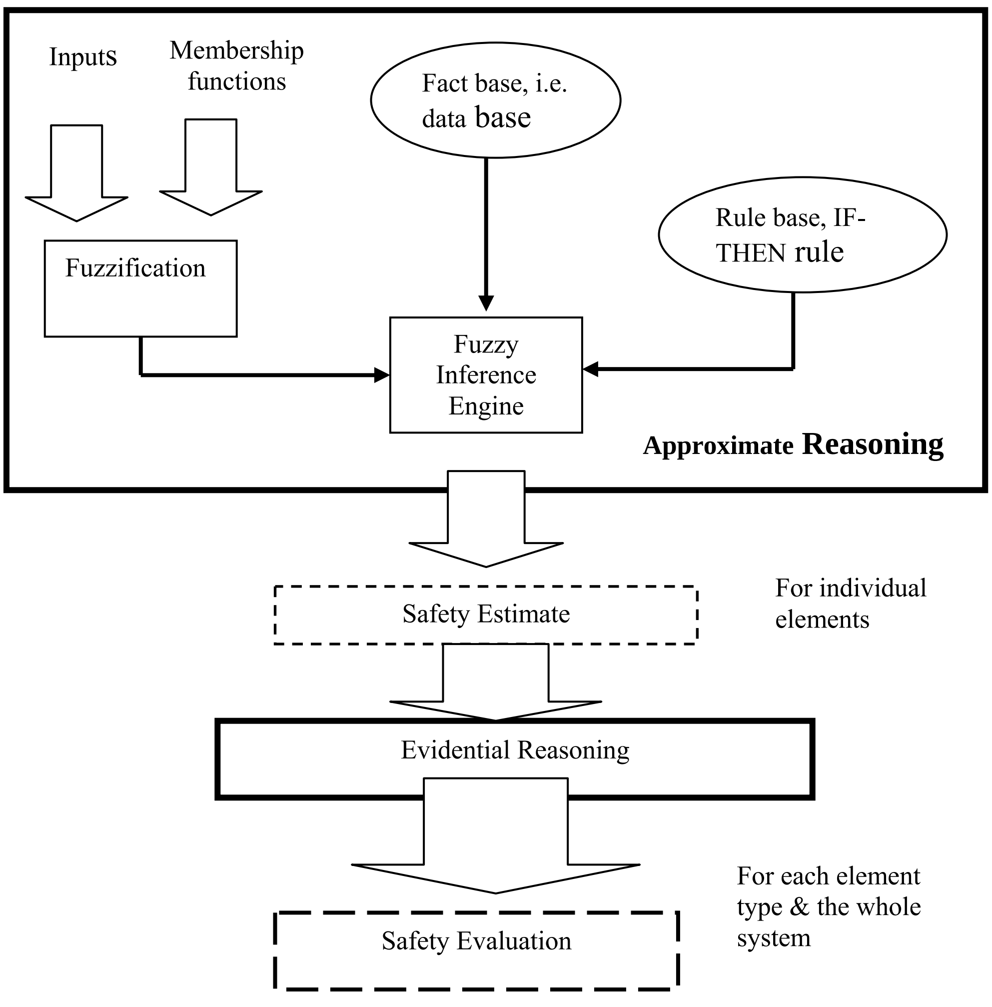

Fig.2 A Generic Qualitative Safety Assessment Framework

7. Conclusion

This paper presented a review of the commonly used uncertainty handling theories, i.e., Bayesian theory of probability, Dempster-Shafer theory of evidence and fuzzy set theory, This includes the discussions about how they work, in what ways they differ from one another, what are their strengths and weaknesses, and how they could be used jointly.

Moreover the possible application of these approaches in marine and offshore safety assessment is also outlined. The study is intended to provide guidance in the process of developing frameworks for safety-based decision analysis using different methods for reasoning under uncertainty.

It should be necessary and potentially beneficial to apply the three kinds of uncertainty theories to offshore and marine safety-based assessment and decision-making. The selection of uncertainty handling approaches depends on the availability of failure data (quantitative and qualitative information), the indenture level of the analysis required, the degree of complexity of the interrelationships in a design, the level of innovation in the design, the causes of "uncertainty", and languages required by the final observer [Zimmermann 2000].

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