Knowledge-Based Systems 253 (2022) 109559

Contents lists available at ScienceDirect

Knowledge-Based Systems

journal homepage: www.elsevier.com/locate/knosys

A linguistic belief-based evidential reasoning approach and its application in aiding lung cancer diagnosis

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ARTICLE INFO

Article history:
Received 25 April 2022
Received in revised form 13 July 2022
Accepted 23 July 2022
Available online 1 August 2022

Keywords:
Evidential reasoning
Linguistic belief structure
Multiple criteria decision-making
Lung cancer diagnosis
MEMCDM

ABSTRACT

Evidential Reasoning (ER) approach is a widely used information aggregation method to deal with uncertain information in decision making. However, as decision-making problem becomes complicated, it is usually difficult for experts to provide accurate belief degrees for each evaluation grade. In this regard, the linguistic belief structure allows experts to give belief degrees with linguistic terms. In this study, we extend the classical ER approach to the linguistic belief-based ER (LB-ER) approach in which the hesitancy degrees are introduced to determine the weights of experts. Afterwards, the LB-ER approach is further enhanced to deal with multi-expert multi-criteria decision-making (MEMCDM) problems, where the best worst method (BWM) is introduced to generate the weights of criteria. Finally, to verify the practicability of the proposed method, we implement the method in lung cancer diagnosis.

1. Introduction

As a cancer with increasing morbidity and mortality, lung cancer has become a great threat to human life [1]. Since the symptoms of lung cancer in early stage are not obvious, when clinical symptoms appear, most patients with lung cancer are at the middle or late stage, which greatly reduces the cure rate [2]. NLST (National Lung Screening Trial) has shown that the low-dose CT can effectively identify early lung cancer patients, so as to make sure that patients obtain timely treatment and reduce mortality.[1] Most existing researches [1,2] focused on computer-aided diagnosis systems to identify CT images, and strived to replace doctors. However, due to the diversity of personal conditions, using a unified recognition program of computer-aided diagnosis system to judge all patients is likely to increase the rate of misdiagnosis. Considering this, the role of doctors is still irreplaceable. This paper is devoted to exploring the decision-making model to assist doctors in diagnosing lung cancer patients, so as to provide reference for inexperienced doctors.

The diagnosis of lung cancer can be divided into two stages: (1) multiple radiologists screen high-risk groups of lung cancer according to the characteristics of low-dose CT images; (2) the attending physician of individual patients determines the possibility of lung cancer according to the patient's own conditions [3]. Both the radiologists in the first stage and the attending physician in the second stage need to comprehensively consider several criteria when making judgments. Therefore, the first stage can be regarded as a multi-expert multi-criteria decision-making (MEMCDM) problem, and the second stage can be regarded as an MCDM problem [4–6]. In addition, in the first stage, the characteristics of CT images are difficult to be accurately characterized by numerical values, and in the second stage, the information provided by patients is usually vague, so radiologists and attending physician tend to use linguistic terms to give diagnostic results, for example, "*the patient's grandfather died of lung cancer, so from a family history perspective, the patient is quite possible to suffer from lung cancer*". The MCDM is a special case when there is only one expert in MEMCDM, so the same MEMCDM method can be applied to deal with the decision-making problems in the two stages, which can reduce decision-making time and improve treatment efficiency. Besides, due to the fuzziness of linguistic terms and the incompleteness of information, the diagnosis of lung cancer can be regarded as dealing with uncertain MEMCDM problems. In this regard, how to solve such uncertain MEMCDM problems with linguistic information is the main research issue of this paper.

As an MCDM method, evidential reasoning (ER) approach can deal with linguistic information well by introducing linguistic evaluation grades and the belief structure [7], which makes the ER approach and its extensions have been implemented to deal with MCDM problems in several fields ranging from risk analysis [8],

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[1] https://www.cancer.gov/types/lung/research/nlst.

https://doi.org/10.1016/j.knosys.2022.109559
0950-7051/

data classification [9,10], to cancer diagnosis [11,12]. The original ER approach [7] allows experts to use linguistic evaluation grades, but requires them to provide precise belief degrees, which cannot always be satisfied due to the complexity of decision-making. In this regard, some efforts have been made to extend the ER approach. Lee et al. [13] proposed the interval value belief structure, which has been developed by some researchers [14–17]. In addition, Chen et al. [18] introduced a discrete belief structure, allowing experts to hesitate between multiple belief values. However, when some known information is fuzzy and insufficient, it is not easy for experts to provide quantitative belief degrees and tend to give linguistic forms, which is general but has not been considered in the existing ER approach and its extensions. Similar linguistic belief structures can be found in the dual linguistic term set proposed by Wang, Huang and Cai [19] based on the probabilistic linguistic term set (PLTS), where linguistic probability can be regarded as linguistic belief degree. However, this method cannot directly deal with incomplete belief structures. For example, a piece of diagnostic information as "*since the patient's family history is incomplete, there is only slightly possible to suffer from lung cancer for this patient*". It is obvious that the diagnostic information shows there is only a small possibility of suffering from lung cancer for the patient. But since there are no other possible situations in this diagnostic information, the dual linguistic term set normalized the information to "*must be suffering from lung cancer*", which obviously distorts the original evaluation information. ER approach can deal with this kind of incomplete evaluation information well by assigning the belief degree to the ignorant information directly by adding "*more other possibilities cannot be determined*" to the original incomplete information. Therefore, how to deal with linguistic belief degrees accurately in the ER approach is the first research challenge of this paper.

Since it can deal with fuzzy linguistic evaluation and incomplete belief structure, the ER approach has been widely used to solve MEMCDM problems [20–22]. How to objectively determine expert weights is a key problem in the application of ER approach to MEMCDM. To reflect the different professional backgrounds and knowledge of experts, Fu, Yang and Yang [20] and Zhou et al. [21] proposed the expert reliability to quantify the relative importance of experts. However, the methods proposed in [20,21] determine the weight of an expert by calculating the similarity between the evaluation given by the expert and that given by other experts, which requires that the group of experts is large enough and artificially reduces the influence of a few experts with different opinions. In addition, Ren, Liao and Fang [22] calculated the amount of information contained in the evaluation by introducing entropy, so as to determine the weights of experts. However, this method only considers the number of linguistic evaluation grades and quantitative belief degrees, ignoring that different linguistic terms have different semantic values. This issue can be well solved by introducing the hesitancy degrees of experts proposed by Liao et al. [23] in the probabilistic linguistic environment. However, this method also requires experts to provide quantitative belief structure, and cannot deal with linguistic belief degrees. Therefore, how to measure the weights of expert based on their hesitancy degrees in the ER approach with linguistic belief degrees is the second research challenge of this paper.

In the ER approach, the assumption that the weights of criteria are usually given directly by experts [7], requires experts to not only comprehensively understand all criteria, but also be rational enough and have strong judgment ability, which is difficult to achieve in actual decision-making. Considering this, to determine the weights of criteria based on the weight range given by experts in advance, Wang et al. [24] established two programming models to maximize the utility of alternatives and minimize the utility of alternatives respectively; Zhou et al. [25] established a programming model to minimize the differences between the alternatives and the ideal solution. However, these methods deduce the appropriate criteria weight to obtain a preset result, so the programming models established for different decision-making problems are also different, and the generality is insufficient. As a method of weight acquisition, the best worst method (BWM) method establishes a programming model by minimizing the difference between the determined weights and the limitations provided by experts in advance, which has strong generality [26]. In addition, since the BWM only requires experts to select the best and worst criteria and provide the paired comparison information between the two and other criteria, it is relatively simple and allows less time to improve the decision-making efficiency [27]. However, there is still a lack of research on applying BWM to the ER approach to determine the weights of criteria, which is the third research challenge of this paper.

To address the aforementioned research challenges, in this paper, we introduce a linguistic belief-based ER (LB-ER) approach to solve uncertain MEMCDM problems with linguistic information and apply it in the diagnosis of lung cancer. Considering that doctors tend to use uncertain linguistic expressions to express diagnosis results, we use the LB-ER approach to model uncertain linguistic information. To represent different perceptions of experts on linguistic terms, three linguistic scale functions are introduced to determine the hesitancy degrees of experts and then generate the weights of experts. In addition, we apply the BWM to determine criteria weights in the ER approach. The contributions of this study are highlighted as:

(1) The LB-ER approach is proposed to solve uncertain MEMCDM problems with linguistic information.

(2) The hesitancy degrees of experts are considered based on three linguistic scale functions when generating the weights of experts, which improves the accuracy of decision results.

(3) The BWM method is introduced to generate the weights of criteria in the ER approach.

(4) The LB-ER approach is implemented in the diagnosis of lung cancer to provide an accurate diagnosis report for the attending physician, based on which the final diagnosis of the patient can be obtained in accordance with external conditions.

This paper is arranged as follows: Section 2 reviews the ER approach and linguistic scale functions. Section 3 proposes the LB-ER approach for MEMCDM. Section 4 gives a numerical study. Section 5 ends the paper.

2. Preliminaries

This paper is carried out under the framework of the ER approach. In this approach, it is essential to determine experts' weights according to the linguistic scale functions of linguistic terms and the hesitancy degrees of experts. Therefore, this section recalls the advances of the ER approach and reviews linguistic scale functions.

2.1. A brief description of the ER approach

The ER approach, as an information aggregation method, can analyze MCDM problems with uncertainties in the forms of randomness, ignorance and fuzziness. Due to its distributed expression of belief degrees in evaluation, the ER approach has been widely applied in various fields in the past two decades. Table 1 lists 15 selected papers which applied the ER approach practically.

As can be seen from Table 1, scholars have made lots of efforts to implement the ER approach to practical applications ranging

Table 1
The ER approach and its variations.

Reference	Forms of evaluation grades	Forms of belief degree	Weight determining method	Group decision	Application
This paper	Independent linguistic terms	Linguistic terms	BWM	√	Diagnosis aiding of lung cancer
Dymova, Kaczmarek & Sevastjanov [28]	Interval-valued intuitionistic fuzzy sets	Interval values	Belief rule base	Expert system	Diagnosis of the type 2 diabetes
Wang & Yu [29]	Independent linguistic terms	Interval values	Subjective single values	×	Sustainable operation of Shanghai rail transit system assessment
Diao et al. [10]	Independent linguistic terms	Single values	Machine learning	×	Data classification
Fu et al. [9]	Independent linguistic terms	Single values	Machine learning	×	Data classification
Fu, Liu & Chang [12]	Independent linguistic terms	Single values	Machine learning	×	Diagnosis of thyroid cancer
Chen et al. [11]	Independent linguistic terms	Single values	Machine learning	×	Prediction of lymph node metastasis in head and neck cancer
Chen et al. [18]	Independent linguistic terms	Discrete value sets	Subjective single values	√	Doctoral dissertations assessment
Zhang et al. [30]	Independent linguistic terms	Interval values	Subjective interval values	×	Partner selection
Zhou et al. [25]	Hesitant fuzzy linguistic terms	Value sets	Subjective interval values	×	Engine assessment
Wang et al. [24]	Independent linguistic terms	Triangular intuitionistic fuzzy numbers	Subjective single values	×	Car assessment
Tang et al. [8]	Independent linguistic terms	Single values	Belief rule base	Expert system	Risk analysis
Guo et al. [31]	Independent linguistic terms	Interval values	Subjective interval values	×	Car assessment
Wang et al. [17]	Independent linguistic terms	Single values	Subjective single values	×	Environmental impact assessment
Liu et al. [32]	Independent linguistic terms	Single values	Belief rule base	Expert system	Safety analysis
Xu, Yang & Wang [7]	Independent linguistic terms	Single values	Subjective single values	Intelligent decision system	Self-assessment

from industrial fields to management fields over the past two decades. Originally, evaluation grades in the ER approach can only be expressed by independent linguistic terms. To expand the application scopes of the ER approach, evaluation grades were extended to be expressed by linguistic terms that intersect with each other [33]. Subsequently, to generalize the expression forms, evaluation grades were further developed to several linguistic terms [7], and then to hesitant fuzzy linguistic elements [25]. In addition, researchers extended the form of the belief degrees from numerical values to intervals [17,31], then to intuitionistic fuzzy numbers [24,34] and several discrete values [18]. Although scholars have extended the ER approach to various forms, there are still issues that cannot be modeled well.

To facilitate further presentation, we briefly review the original ER approach [7]. Suppose that there is an MCDM problem in which M alternatives $a_j\,(j=1,2,\ldots,M)$ are evaluated regarding L criteria $e_i\,(i=1,2,\ldots,L)$. The weight vector of criteria is $(\omega_1,\omega_2,\ldots,\omega_L)^T$, satisfying $0\le\omega_i\le 1\,(i=1,2,\ldots,L)$ and $\sum_{i=1}^{L}\omega_i=1$. Suppose that all alternatives are evaluated based on a frame of discernment $H=\{H_1,H_2,\ldots,H_N\}$ where $H_n(n=1,2,\ldots,N)$ are collectively exhaustive and mutually exclusive with each other. $\beta_{n,i}(a_j)$ is the belief degree assigned to grade H_n for alternative a_j on criterion e_i, satisfying $\beta_{n,i}(a_j)\ge 0$ and $\sum_{n=1}^{N}\beta_{n,i}(a_j)\le 1$. In the recursive ER algorithm [7], the basic probability assignments are calculated based on belief degrees:

$$m_{n,i}(a_j)=\omega_i\beta_{n,i}(a_j) \tag{1}$$

$$m_{H,i}(a_j)=\tilde{m}_{H,i}(a_j)+\bar{m}_{H,i}(a_j) \tag{2}$$

$$\tilde{m}_{H,i}(a_j)=\omega_i\left(1-\sum_{n=1}^{N}\beta_{n,i}(a_j)\right) \tag{3}$$

$$\bar{m}_{H,i}(a_j)=1-\omega_i \tag{4}$$

where $m_{n,i}(a_j)$ is the basic probability mass, representing the probability assigned to grade H_n under criterion e_i for alternative a_j. $m_{H,i}(a_j)$ is the remaining probability mass, representing the probability unassigned to any evaluation grade, which includes two parts: $\tilde{m}_{H,i}(a_j)$ caused by ignorance in e_i and $\bar{m}_{H,i}(a_j)$ caused by the weight of e_i.

For $H_n(n=1,2,\ldots,N)$, the masses of the first $i+1$ criteria are combined by

$$m_{n,I(i+1)}=K_{I(i+1)}[m_{n,I(i)}m_{n,i+1}+m_{H,I(i)}m_{n,i+1}+m_{n,I(i)}m_{H,i+1}] \tag{5}$$

For H,

$$m_{H,I(i)}=\tilde{m}_{H,I(i)}+\bar{m}_{H,I(i)} \tag{6}$$

$$\tilde{m}_{H,I(i+1)}=K_{I(i+1)}[\tilde{m}_{H,I(i)}\tilde{m}_{H,i+1}+\bar{m}_{H,I(i)}\tilde{m}_{H,i+1}+\tilde{m}_{H,I(i)}\bar{m}_{H,i+1}] \tag{7}$$

$$\bar{m}_{H,I(i+1)}=K_{I(i+1)}\left(\bar{m}_{H,I(i)}\bar{m}_{H,i+1}\right) \tag{8}$$

where $K_{I(i+1)} = \left[1 - \sum_{t=1}^{N}\sum_{\substack{n=1\\n\neq t}}^{N} m_{t,I(i)}m_{n,i+1}\right]^{-1}$, $m_{n,I(1)} = m_{n,1}$ and $m_{H,I(1)} = m_{H,1}$. $K_{I(i+1)}$ is a normalization coefficient to make sure $\sum_{n=1}^{N} m_{n,I(i+1)} + \tilde{m}_{H,I(i+1)} + \bar{m}_{H,I(i+1)} = 1$.

The belief degree of a_j assigned to grade H_n and H can be generated by

$$\beta_n(a_j) = m_{n,I(L)}(a_j)\big/(1 - \bar{m}_{H,I(L)}(a_j)) \tag{9}$$

$$\beta_H(a_j) = \tilde{m}_{H,I(L)}(a_j)\big/(1 - \bar{m}_{H,I(L)}(a_j)) \tag{10}$$

The ER approach has been applied in a variety of fields owing to its ability in handling uncertain linguistic information. However, this method requires experts to provide precise numerical belief degrees, which is not easy for experts. In this regard, this paper considers to introduce linguistic beliefs to in the ER approach, which will be presented in Section 3.

2.2. A brief description of linguistic scale functions

Suppose that $S = \left\{s_\alpha \middle| \alpha = 1, 2, \ldots, 2\tau + 1\right\}$ is an LTS where τ is a positive integer, and $g(s_\alpha)$ is the semantic function of $s_\alpha (\alpha = 1, 2, \ldots, 2\tau + 1)$. There are three linguistic scale functions [23] considering balanced and unbalanced semantic distribution.

(1) If the semantics of linguistic terms are evenly distributed, then,

$$g(s_\alpha) = (\alpha - 1)\big/2\tau \tag{11}$$

where experts are neutral and have no bias on extreme and median values.

(2) If the deviation between the semantics of adjacent linguistic terms increases as the distance from $s_{\tau+1}$ increases, that is, experts are conservative in judging linguistic semantics and sensitive to extreme values, then,

$$g(s_\alpha) = \begin{cases} \dfrac{\psi^\tau - \psi^{-\alpha+\tau+1}}{2\psi^\tau - 2}, & \text{if } \alpha = 1, 2, \ldots, \tau + 1 \\ \dfrac{\psi^\tau + \psi^{\alpha-\tau-1} - 2}{2\psi^\tau - 2}, & \text{if } \alpha = \tau + 1, \ldots, 2\tau + 1 \end{cases} \tag{12}$$

where $\psi > 1$ is a threshold associated to specific problems.

(3) If the deviation between the semantics of adjacent linguistic terms decreases as the distance from $s_{\tau+1}$ increases, that is, experts are radical in judging linguistic semantics and sensitive to median values, then,

$$g(s_\alpha) = \begin{cases} \dfrac{\tau^\varsigma - (-\alpha + \tau + 1)^\varsigma}{2\tau^\varsigma}, & \text{if } \alpha = 1, \ldots, \tau + 1 \\ \dfrac{\tau^\xi + (\alpha - \tau - 1)^\xi}{2\tau^\xi}, & \text{if } \alpha = \tau + 1, \ldots, 2\tau + 1 \end{cases} \tag{13}$$

where $\varsigma, \xi \in (0, 1]$ are set based on different problems. ς and ξ respectively refers to the attitude parameter of "*bad*" and "*good*".

These three linguistic scale functions can reflect the psychology of experts when giving information. If experts are insensitive to semantic values, then choose Eq. (11) as the semantic scale function; if experts are sensitive to extreme values, then we can choose Eq. (12); if experts are sensitive to median values, then we can choose Eq. (13). In this paper, we introduce these linguistic scale functions to reflect the hesitancy degrees of expert so as to determine the weights of experts, which will be presented in Section 3.2.

3. The linguistic belief-based evidential reasoning approach for MCDM with multiple experts

The original ER approach and its extensions can model uncertain information in MCDM problems, but there is no relevant research for the uncertain MEMCDM problems with linguistic information. To fill this gap, the LB-ER approach for uncertain MEMCDM problems is proposed in this section. We model linguistic evaluation given by multiple experts, and form decision matrices with linguistic belief degrees (Section 3.1). Afterwards, we determine the weights of experts by considering hesitancy degrees of experts, and then aggregate the evaluations of experts to form a collective decision matrix with numerical belief degrees based on linguistic scale function (Section 3.2). The weights of criteria are obtained by the BWM (Section 3.3). Finally, the comprehensive evaluation results of alternatives are obtained by the ER approach (Section 3.4). The algorithm of the LB-ER approach for MEMCDM is given in Section 3.5. The framework of the LB-ER approach for MEMCDM with linguistic belief information can be demonstrated intuitively in Fig. 1.

3.1. Generating individual decision matrices

For an MCDM problem that Q experts $(\{c_q | q = 1, 2, \ldots, Q\})$ are invited to evaluate alternatives under different criteria, experts may have different standards for the distribution of belief degrees. Individual LTSs $S^{(q)} = \{s_\theta^{(q)} | \theta = 1, 2, \ldots, 2\delta + 1\}$ with δ being a positive integer for expert c_q $(q = 1, 2, \ldots, Q)$ are proposed to represent individual linguistic belief degrees. For example, expert c_1 may use $S^{(1)} = \{$*Impossible, Slightly possible, Possible, Quite possible, Must be*$\}$ to represent linguistic degrees, while expert c_2 may use $S^{(2)} = \{$*Impossible, Possible, Must be*$\}$. It is difficult to aggregate the values of criteria based on individual linguistic belief degrees with different standards. In this regard, there is a need to unify all linguistic belief degrees based on a unified scale. Motivated by [7], individual linguistic terms can be transformed to be covered in a unified LTS S by:

$$s_{\theta,i}{}^{(q)} = \{(s_{\alpha,i}{}^{(q)}, \gamma_{\alpha,i}{}^{(q)}) | \alpha = 1, 2, \ldots, \tau, \ldots, 2\tau + 1; s_{\alpha,i}{}^{(q)} \in S\} \tag{14}$$

where $s_{\theta,i}c \in S^{(q)}$, and $\gamma_{\alpha,i}{}^{(q)}$ represents the degree to which $s_{\theta,i}{}^{(q)}$ maps to $s_{\alpha,i}{}^{(q)}$ under criterion e_i for expert c_q, with $\gamma_{\alpha,i}{}^{(q)} \geq 0$ and $\sum_{\alpha=1}^{2\tau+1} \gamma_{\alpha,i}{}^{(q)} = 1$.

If all experts give evaluations based on the same LTS, we have

$$s_{\theta,i}{}^{(q)} = \{(s_{\alpha,i}{}^{(q)}, \gamma_{\alpha,i}^{(q)}) | \gamma_{\alpha,i}{}^{(q)} = 1, \alpha = \theta, \quad s_{\alpha,i}{}^{(q)} \in S\} \tag{15}$$

We denote an evaluation as

$$S(e_i(a_j))^{(q)} = \{(H_n, s_{\theta,i}{}^{(q)}) | n = 1, 2, \ldots, N; s_{\theta,i}{}^{(q)} \in S^{(q)}\} \tag{16}$$

By Eq. (14), the evaluation given as Eq. (16) can be transformed to

$$S(e_i(a_j))^{(q)} = \{(H_n, \gamma_{\alpha,i}{}^{(q)} s_{\alpha,i}{}^{(q)}) | n = 1, 2, \ldots, N; s_{\alpha,i} \in S\} \tag{17}$$

In this way, experts can express their belief degrees of evaluations by linguistic terms with personal preferences. Then, individual decision matrices with linguistic beliefs can be generated as $D_g^{(q)} = (S(e_i(a_j))^{(q)})_{L \times M}$ $(q = 1, 2, \ldots, Q)$.

3.2. Generating a collective decision-making matrix

To form a collective decision matrix, owing to different preferences for semantics and different professional degrees of experts, we need to consider linguistic scale functions and hesitancy degrees of experts.

The semantics of linguistic terms should be quantified to numbers that can be regarded as belief degrees. The linguistic scale functions for different LTSs mentioned in Section 2.2 provide a good way to develop the transformation. The semantic of linguistic terms in $S^{(q)}$ can be expressed as $g(s_{\theta,i}{}^{(q)})$ based on Eqs.

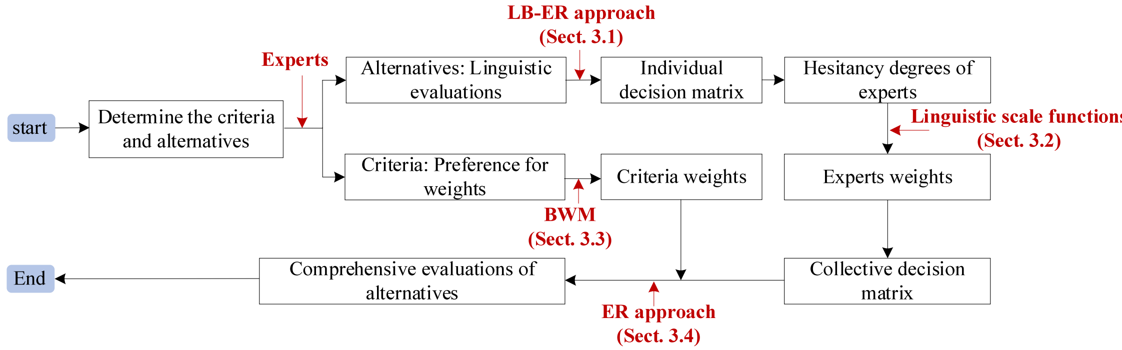

Fig. 1. Framework of the LB-ER approach for uncertain MEMCDM problems with linguistic information.

(11)–(13). Then, the semantics of the elements in the decision matrix $D_g{}^{(q)}$ can be determined by

$$g(\gamma_{\alpha,i}{}^{(q)}s_{\alpha,i}{}^{(q)}) = \gamma_{\alpha,i}{}^{(q)}g(s_{\theta,i}{}^{(q)}) \tag{18}$$

Since it is difficult for experts to give an accurate evaluation grade for each alternative, experts are usually hesitant among several evaluation grades. It is necessary to consider the hesitancy degree of experts when determining the weights of experts. Considering this point, this paper extends the hesitancy degree function proposed by Liao et al. [23] to calculate the hesitancy degree of a piece of evaluation as

$$HD(S(e_i(a_j))^{(q)}) = \frac{T(H_{n,i}{}^{(q)})\ln(T(H_{n,i}{}^{(q)}))}{N\ln N} \tag{19}$$

where $T(H_{n,i}{}^{(q)})$ is the cardinality of H_n in $S(e_i(a_j))^{(q)}$ with given $s_{\theta,i}{}^{(q)}$. For example, for $S(e_1(a_1)) = \{(H_1, s_2), (H_2, s_2)\}$, $T(H_{n,i}) = 2$.

$HD(S(e_i(a_j))^{(q)})$ satisfies: (1) $HD(S(e_i(a_j))^{(q)}) = 0$, if and only if there is a sole evaluation grade in $S(e_i(a_j))^{(q)}$; (2) $HD(S(e_i(a_j))^{(q)}) = 1$, if and only if all grades are included in $S(e_i(a_j))^{(q)}$; (3) $HD(S(e_{i_1}(a_{j_1}))^{(q_1)}) \leq HD(S(e_{i_2}(a_{j_2}))^{(q_2)})$, if $T(H_{n,i_1}{}^{(q_1)}) \leq T(H_{n,i_1}{}^{(q_1)})$.

Similar to the score of a hesitant fuzzy linguistic element (HFLE) [23], the score of the linguistic term can be generated as

$$E(s_{\alpha,i}{}^{(q)}) = (1 - HD(S(e_i(a_j))^{(q)})) \times g(s_{\alpha,i}{}^{(q)}) \tag{20}$$

Considering all the hesitancy degrees of evaluations given by expert c_q, the hesitancy degree of c_q can be obtained as

$$CHD^{(q)} = \frac{1}{M \times L}\sum_{i=1}^{M}\sum_{j=1}^{L} HD(S(e_i(a_j))^{(q)}) \tag{21}$$

Then, based on the hesitancy degree, we can obtain the weight of expert c_q by

$$w^{(q)} = (\mu - CHD^{(q)}) \Big/ \sum_{q=1}^{Q}(\mu - CHD^{(q)}) \tag{22}$$

where μ is a normalization parameter such that $\sum_{q=1}^{Q} w^{(q)} = 1$. The hesitancy degrees of experts obtained by Eqs. (19)–(21) are relatively small. To reflect the difference between experts' hesitancy degrees without over-amplifying the difference, we set $\mu = 0.5$ in this paper [23].

Example 1 shows how to obtain expert weights based on hesitancy degrees.

Example 1. Two doctors (c_1, c_2) are invited to diagnose two patients (a_1, a_2) under two criteria (e_1, e_2), with the evaluation values being provided as:

$$c_1: \begin{array}{cc} & \begin{array}{cc} e_1 & e_2 \end{array} \\ \begin{array}{c} a_1 \\ a_2 \end{array} & \begin{bmatrix} \{(H_1, s_1), (H_3, s_4)\} & \{(H_3, s_5)\} \\ \{(H_2, s_1)\} & \{(H_2, s_4), (H_3, s_2)\} \end{bmatrix} \end{array},$$

$$c_2: \begin{array}{cc} & \begin{array}{cc} e_1 & e_2 \end{array} \\ \begin{array}{c} a_1 \\ a_2 \end{array} & \begin{bmatrix} \{(H_3, s_3)\} & \{(H_3, s_3)\} \\ \{(H_4, s_2)\} & \{(H_2, s_3)\} \end{bmatrix} \end{array}.$$

By Eq. (20), the hesitancy degree $HD(S(e_i(a_j))^{(q)})$ of each evaluation can be obtained as:

$$c_1: \begin{array}{cc} & \begin{array}{cc} e_1 & e_2 \end{array} \\ \begin{array}{c} a_1 \\ a_2 \end{array} & \begin{bmatrix} 0.1290 & 0 \\ 0.1290 & 0.1290 \end{bmatrix} \end{array}, \quad c_2: \begin{array}{cc} & \begin{array}{cc} e_1 & e_2 \end{array} \\ \begin{array}{c} a_1 \\ a_2 \end{array} & \begin{bmatrix} 0 & 0.1290 \\ 0 & 0 \end{bmatrix} \end{array}.$$

By Eq. (21), the hesitancy degrees of experts (c_1, c_2) can be generated as $CHD^{(1)} = 0.0967$ and $CHD^{(2)} = 0.0322$. By Eq. (22), the weights of experts (c_1, c_2) can be determined as $w^{(1)} = 0.46$ and $w^{(2)} = 0.54$.

With expert weights, we can generate the collective linguistic score of alternative a_j on criterion e_i as

$$E(s_{\alpha,i}) = \sum_{q=1}^{Q} w^{(q)}E(s_{\alpha,i}^{(q)}) \tag{23}$$

Considering the weights of experts, we transform the decision matrix with linguistic belief degrees $D_g{}^{(q)} = (S(e_i(a_j))^{(q)})_{L\times M}$ to the decision matrix with numerical belief degrees as $D_g = (S(e_i(a_j)))_{L\times M}$, where

$$S(e_i(a_j)) = \{(H_n, \beta_{n,i}(a_j)), n = 1, 2, \ldots, N\} \tag{24}$$

For the evaluation $S(e_i(a_j))$ in D_g with $\sum_{n=1}^{N}\beta_{n,i}(a_j) > 1$, we normalize it to $S(e_i(a_j))$ with $\beta'_{n,i}(a_j) = \beta_{n,i}(a_j)\big/\sum_{n=1}^{N}\beta_{n,i}(a_j)$; for the evaluation $S(e_i(a_j))$ in D_g with $\sum_{n=1}^{N}\beta_{n,i}(a_j) < 1$, we regard it as incomplete value where the remaining belief is $1 - \sum_{n=1}^{N}\beta_{n,i}(a_j) > 0$.

3.3. Generating criteria weights by the BWM

The original ER approach does not give a specific method to determine criteria weights. In this section, the BWM [18] is introduced to generate the weights of criteria in the ER approach. In the BWM, only referred comparisons are needed, which reduces the difficulty of expert judgments and increases the accuracy of weight determination. By selecting the best and the worst criteria and comparing them with other criteria, a corresponding nonlinear programming model is formed, and then the weight vector of criteria can be obtained. The process is organized as follows:

Step 1. Experts are invited to select the best criterion e_B and the worst criterion e_W. If there are two or more best or worst criteria, the best and worst criterion can be selected arbitrarily.

Step 2. Compare the best criterion e_B with the other criteria respectively by the 1–9 scale. In this way, we establish the best to others vector as: $BO = \left(a_{B1}, a_{B2}, \ldots, a_{Bj}, \ldots, a_{BL}\right)^T$ with $a_{Bi} \geq 1$, where a_{Bi} is the preference degree of the best criterion e_B over criterion e_i.

Step 3. Compare the worst criterion e_W with the other criterion respectively by the 1–9 scale. In this way, we establish the others to the worst vector as: $OW = (a_{1W}, a_{2W}, \ldots, a_{iW}, \ldots, a_{LW})^T$ with $a_{iW} \geq 1$, where a_{iW} is the preference degree of criterion e_i over the worst criterion e_W. Here, $L-2$ pairwise comparisons are done since a_{BW} is obtained in BO.

Step 4. To generate the weight vector of criteria, a nonlinear programming model can be built as:

$$\begin{aligned} &\min\ \xi \\ &s.t.: \sum_{i=1}^{L}\omega_i = 1, \\ &\quad \omega_i \geq 0, i = 1, 2, \ldots, L \\ &\quad |\omega_B/\omega_i - a_{Bi}| \leq \xi \\ &\quad |\omega_i/\omega_W - a_{iW}| \leq \xi \end{aligned} \tag{25}$$

where ξ denotes the maximum absolute difference and can be determined for specific problems. By solving this model, the weight vector of criteria could be generated as $\omega = (\omega_1, \omega_2, \ldots, \omega_L)^T$

3.4. Calculating the values of alternatives

To rank all alternatives, we assign a value function [7] to evaluation grades, and aggregate final synthesis results to get the values of alternatives.

Based on the belief decision matrix and criteria weights, the combined belief degrees $\beta_n(a_j)$ and $\beta_H(a_j)$ can be calculated by the ER algorithm defined by Eqs. (1)–(10). Then, we can get the value of each alternative by

$$u(s(y(a_j))) = \sum_{n=1}^{N}\beta_n(a_j)u(H_n) \tag{26}$$

where $u(H_n)(n = 1, 2, \ldots, N)$ is the value of H_n, and $u(H_n) < u(H_{n+1})$ when H_{n+1} is preferred over H_n.

If $\beta_H(a_j) > 0$, it means that there is the global ignorance in the evaluation, and the belief degree $\beta_H(a_j)$ could be reassigned to any evaluation grade. Based on this, we can get an interval value of alternative a_j as $[u_{\min}(a_j), u_{\max}(a_j)]$ by assigning the remaining belief to the worst and the best evaluation grade respectively, where

$$u_{\min}(a_j) = \sum_{n=1}^{N}\beta_n(a_j)u(H_n) + \beta_H(a_j)u(H_1) \tag{27}$$

$$u_{\max}(a_j) = \sum_{n=1}^{N}\beta_n(a_j)u(H_n) + \beta_H(a_j)u(H_N) \tag{28}$$

To show the value of a_j directly, we calculate the average value as

$$u_{avg}(a_j) = \frac{u_{\min}(a_j) + u_{\max}(a_j)}{2} \tag{29}$$

If $\beta_H(a_j) = 0$, there lies $u_{\min}(a_j) = u_{\max}(a_j) = u_{avg}(a_j)$.

The three types of utilities given above represent three typical levels of optimism for the decision makers. In the decision-making problem in which the larger the utility value means the better the result, $u_{\max}$ is chosen for the extremely optimistic case, $u_{\min}$ for the extremely pessimistic case, and u_{avg} for the indifference case.

3.5. Algorithm of the LB-ER approach for MEMCDM

To concretize the application of the LB-ER approach for uncertain MEMCDM problems. We give the algorithm of the approach as Algorithm I.

Algorithm I (*The LB-ER Approach for MEMCDM*)**.**

Step 1. Invite Q experts to evaluate alternatives under criteria and then form individual decision-making matrices with linguistic belief degrees as $D_g{}^{(q)} = (S(e_i(a_j))^{(q)})_{L\times M}(q = 1, 2, \ldots, Q)$. Go to the next step.

Step 2. Determine the hesitancy degrees of experts by Eqs. (11)–(24), and generate the collective decision matrix as $D_g = (S(e_i(a_j)))_{L\times M}$. Go to the next step.

Step 3. Use the BWM to obtain the weight vector $\omega = (\omega_1, \omega_2, \ldots, \omega_L)^T$ of criteria by Eq. (25). Go to the next step.

Step 4. Apply Eqs. (1)–(10) to generate the combined belief degrees of alternative a_j as $\beta_n(a_j)$ and $\beta_H(a_j)$, which are assessed to H_n or H, respectively. Go to the next step.

Step 5. If there is a need to rank alternatives, go to the next step; if not, the evaluation grade with the maximum belief degree is what the alternative belongs to, and the algorithm ends.

Step 6. To rank alternatives, we obtain three different values of alternatives by Eqs. (26)–(28) and then terminate the algorithm by ranking alternatives according to utility values.

In the proposed LB-ER approach, the linguistic evaluation information given by experts can be modeled to form individual linguistic belief decision matrices. Then, considering the decision preference and hesitancy degrees of experts, the comprehensive decision matrix with numerical belief degrees is obtained. The weight vector of criteria is obtained by the BWM. Then alternatives are sorted according to the results by the ER approach. The LB-ER approach makes it possible to model linguistic belief information.

The hesitancy degrees of experts are considered to determine the weight vector of experts, through different sensitivity of experts denoted by three linguistic scale functions, which enhances the accuracy in the decision-making process. In addition, the BWM is introduced to determine the weights of criteria, which brings a fresh way to establish criterion weights in the ER approach.

4. Application to the diagnosis of lung cancer

In this section, the LB-ER approach is applied to aid the diagnosis of lung cancer, which can provide an accurate diagnosis report for attending physicians and an effective decision-making method for lung cancer diagnosis.

4.1. Background of the diagnosis of lung cancer

Lung cancer has become the most susceptible cancer to human beings in recent years. The Global Cancer Statistic 2018 [35] published by CA: A Cancer Journal for Clinicians, showed that lung cancer remains the main cause of cancer morbidity and mortality worldwide. The ELCAP (Early Lung Cancer Action Project) results [36] showed that early diagnosis and treatment of lung cancer plays a crucial role in improving cure rate and prognosis. The 5-year survival rate of patients with early lung cancer was 90%, that of patients with stage I lung cancer was 60%, and with stage II–IV decreased from 40% to less than 5%. Therefore, striving for "*early detection, early diagnosis and early treatment*" is an important measure to reduce the mortality rate of lung cancer [37]. As a result, more and more attention has been paid to the early diagnosis of lung cancer.

There are several clinical diagnostic methods for lung cancer, which can be divided into histological diagnosis and imaging diagnosis [3]. Histological diagnosis methods include sputum cytology, bronchoscopy, and needle biopsy. Imaging diagnosis methods include chest X-ray, CT, MR (Magnetic Resonance), and PET (Positron Emission Tomography) thoracic scanning imaging. Histological diagnostic methods except sputum cytology, and other examinations cause pain or infection of pulmonary haemorrhage. Not all types or stages of lung cancer cells can appear in sputum. In addition, the imaging diagnosis can make people directly observe the size, shape and location of the lesion tissue, and will not bring sufferings to patients like invasive examination. Therefore, although the results of histological diagnosis can be used as gold standard, most early lung cancer diagnosis methods are still imaging diagnosis methods. With the rapid development of imaging technology, its application is more and more wide. Moreover, the National Lung Cancer Screening Test (NLST) in the United States confirmed that, compared with routine chest X-ray, the low-dose CT screening in high-risk groups can detect and treat 20% of patients early, thus reducing the mortality of lung cancer [38–40]. Therefore, CT imaging has become the best method for early screening of lung cancer owing to its good density resolution for various lung diseases [41].

Thousands of CT images have been generated in CT imaging examination, and relevant and clear images of lungs can be selected for observation and then to generate diagnostic reports. Different image features can produce different diagnostic results, while the same features may be caused by different diseases. Therefore, it is not easy for doctors to fully determine the illness of patients only based on the CT image information. At the same time, due to the incompleteness of information, it is difficult for doctors to give a numerical value or numerical range of the possibility of diseases. Often, doctors provide diagnostic reports in the form of linguistic terms. It is reasonable to apply the LB-ER approach to model the generation process of diagnostic reports, and provide accurate diagnostic information for radiologists and the attending physician.

4.2. modeling the CT diagnostic process of lung cancer

There are two stages from detection in CT images to disease diagnosis. First of all, radiologists need to select clear and critical CT images of patients for detection, and write CT detection reports. Then, the patient's attending physician makes final diagnosis according to patient's own conditions and his/her CT test report obtained in the previous step. The whole process is fixed and full of uncertainty. Doctors cannot always and completely be in a rational state. Providing doctors with auxiliary diagnostic information can reduce the burden of doctors and increase the accuracy of diagnostic results. In this case study, we apply the LB-ER approach to model the two stages separately. First, we introduce the model of the first stage in the CT diagnosis in detail.

The key and clear CT images are first selected by radiologists for observation. In this paper, three images of patients are taken as $a_j (j = 1, 2, 3)$. According to the literature [4,5], four characteristic factors which have great significance in differentiating lung cancer in CT images are mediastinal lymph nodes, speculations, edge features and small bubble sign, which can be regarded as four criteria of CT image observation. They are recorded as $e_i (i = 1, 2, 3, 4)$. In addition, due to the complexity of human lung structure, lung health condition can be roughly divided into six possibilities, $H = \{H_n | n = 1, 2, \ldots, 6\} = \{$*Disease-free, Benign tumors, Pneumonia, Pulmonary tuberculosis, Lung cancer, Other pulmonary diseases*$\}$. All the belief degrees can be denoted by $S = \{s_\alpha | \alpha = 1, 2, \ldots, 5\} = \{$*Impossible, Slightly possible, Possible, Quite possible, Must be*$\}$. According to different image features, the diagnostic results are also different. To minimize the influence of individual subjective factors and provide accurate and objective diagnostic information, three radiologists $c_q\ (q = 1, 2, 3)$ are invited to observe three CT images respectively and give diagnostic information. The following are specific steps to solve this case study by our proposed method.

Step 1. Collect evaluations and build individual decision matrices as Table 2. For example, the radiologist c_1 give evaluation based on the edge features e_3 for the CT image a_3 as "*The margin features of the pulmonary nodule in this image show that the patient is slightly possible pulmonary tuberculosis, slightly possible lung cancer, and impossible other pulmonary disease*", which can be modeled by the LB-ER approach as $\{(H_4, s_2), (H_5, s_2), (H_6, s_1)\}$.

Step 2. Generate the collective decision matrix. To provide accurate diagnostic results through generating diagnostic reports, we consider the preference types in linguistic judgments, such as conservative, neutral or radical. In this case, to show the universality of the application of the proposed approach, it is assumed that the three radiologists are of three different types, and the semantic judgment values of each radiologist for linguistic terms can be calculated by Eqs. (11)–(13). The results are:

$$g(s_\alpha^{(1)}) = \{0, 0.25, 0.5, 0.75, 1\}; g(s_\alpha^{(2)}) = \{0, \frac{1}{3}, \frac{1}{2}, \frac{2}{3}, 1\};$$
$$g(s_\alpha^{(3)}) = \{0, 0.15, 0.5, 0.85, 1\}.$$

The decision matrices with linguistic belief degrees can be transformed into the ones with numerical belief degrees as Table 3.

To make decisions accurately, the weights of radiologists should be considered. Due to the influence of external factors and the complexity of human thoughts, the weights of radiologists vary from time to time. It will take a lot of time and energy to obtain radiologist weights through a series of complex tests and calculations before each decision. In medical diagnosis, time is life, and any delay by seconds may lead to catastrophic consequences in patient survival. Therefore, we introduce the concept of expert hesitancy degrees to obtain the weights of radiologists quickly and shorten the diagnosis time.

By Eqs. (14)–(21), the hesitancy degrees of radiologists can be obtained as:

$$CHD^{(1)} = 0.141; \quad CHD^{(2)} = 0.152; \quad CHD^{(3)} = 0.137$$

According to the hesitancy degrees of radiologists, their weights can be further generated by Eq. (22):

$$w^{(1)} = 0.335; \quad w^{(2)} = 0.325; \quad w^{(3)} = 0.339$$

By Eqs. (23)–(24), we can obtain the collective decision matrix as Table 4.

Step 3. Generate criteria weights by the BWM.

Based on the statistical analysis of historical data or experience, radiologists can compare and analyze the criteria, and choose e_4 (small bubble sign) as the best criterion e_B and e_1 (mediastinal lymph nodes) as the worst criterion e_W. In this case, the comparison vectors are given by the radiologists with a scale of 1–9. $BO = (4, 2, 2, 1)^T$ can be obtained by comparing the best criterion e_B with each criterion, and $OW = (1, 3, 2, 4)^T$ can be obtained by comparing each criterion with the worst criterion e_W.

According to Eq. (25), the following non-linear programming models can be constructed:

$$\begin{aligned} &\min\ \xi \\ &s.t.: \sum_{i=1}^{4} \omega_i = 1, \\ &\quad \omega_i \geq 0, i = 1, 2, 3, 4 \\ &\quad |\omega_4/\omega_1 - 4| \leq \xi, |\omega_4/\omega_2 - 2| \leq \xi \\ &\quad |\omega_4/\omega_3 - 2| \leq \xi, |\omega_2/\omega_1 - 3| \leq \xi \\ &\quad |\omega_3/\omega_1 - 2| \leq \xi \end{aligned}$$

Solving this linear programming model, we obtain the weight vector of criteria as $\omega = (0.1, 0.27, 0.19, 0.44)^T$.

Table 2
The individual decision matrices.

		e_1	e_2	e_3	e_4
c_1	a_1	$\{(H_1, s_1), (H_6, s_2)\}$	$\{(H_5, s_4)\}$	$\{(H_1, s_1), (H_4, s_2), (H_5, s_3)\}$	$\{(H_3, s_2), (H_4, s_2)\}$
	a_2	$\{(H_3, s_2), (H_4, s_2), (H_5, s_2)\}$	$\{(H_3, s_2), (H_4, s_3)\}$	$\{(H_6, s_2)\}$	$\{(H_3, s_3), (H_4, s_4)\}$
	a_3	$\{(H_6, s_3)\}$	$\{(H_5, s_4), (H_6, s_2)\}$	$\{(H_4, s_2), (H_5, s_2), (H_6, s_1)\}$	$\{(H_3, s_4), (H_4, s_3)\}$
c_2	a_1	$\{(H_5, s_2), (H_6, s_2)\}$	$\{(H_4, s_2), (H_5, s_2), (H_6, s_2)\}$	$\{(H_4, s_3), (H_5, s_2)\}$	$\{(H_3, s_3), (H_4, s_2)\}$
	a_2	$\{(H_5, s_3)\}$	$\{(H_3, s_3), (H_4, s_2)\}$	$\{(H_5, s_2), (H_6, s_2)\}$	$\{(H_3, s_3), (H_4, s_3), (H_5, s_2)\}$
	a_3	$\{(H_4, s_2), (H_5, s_3), (H_6, s_2)\}$	$\{(H_5, s_3)\}$	$\{(H_4, s_2), (H_5, s_3)\}$	$\{(H_3, s_4), (H_4, s_2)\}$
c_3	a_1	$\{(H_4, s_2), (H_5, s_4)\}$	$\{(H_4, s_3), (H_5, s_4)\}$	$\{(H_4, s_4), (H_5, s_2)\}$	$\{(H_3, s_4), (H_4, s_2), (H_5, s_3)\}$
	a_2	$\{(H_5, s_3), (H_6, s_2)\}$	$\{(H_5, s_2)\}$	$\{(H_5, s_3), (H_6, s_4)\}$	$\{(H_4, s_2), (H_5, s_3)\}$
	a_3	$\{(H_4, s_3), (H_5, s_3)\}$	$\{(H_5, s_3), (H_6, s_2)\}$	$\{(H_5, s_4)\}$	$\{(H_3, s_2), (H_4, s_3), (H_5, s_2)\}$

Table 3
The transformed individual decision matrices.

		e_1	e_2	e_3	e_4
c_1	a_1	$\{(H_6, 0.25)\}$	$\{(H_5, 0.75)\}$	$\{(H_4, 0.25), (H_5, 0.5)\}$	$\{(H_3, 0.25), (H_4, 0.25)\}$
	a_2	$\{(H_3, 0.25), (H_4, 0.25), (H_5, 0.25)\}$	$\{(H_3, 0.25), (H_4, 0.5)\}$	$\{(H_6, 0.25)\}$	$\{(H_3, 0.5), (H_4, 0.75)\}$
	a_3	$\{(H_6, 0.5)\}$	$\{(H_5, 0.75), (H_6, 0.25)\}$	$\{(H_4, 0.25), (H_5, 0.25)\}$	$\{(H_3, 0.75), (H_4, 0.5)\}$
c_2	a_1	$\{(H_5, 1/3), (H_6, 1/3)\}$	$\{(H_4, 1/3), (H_5, 1/3), (H_6, 1/3)\}$	$\{(H_4, 1/2), (H_5, 1/3)\}$	$\{(H_3, 1/2), (H_4, 1/3)\}$
	a_2	$\{(H_5, 1/2)\}$	$\{(H_3, 2/3), (H_4, 1/2)\}$	$\{(H_5, 1/3), (H_6, 1/3)\}$	$\{(H_3, 1/2), (H_4, 1/2), (H_5, 1/3)\}$
	a_3	$\{(H_4, 1/3), (H_5, 1/2), (H_6, 1/3)\}$	$\{(H_5, 1/2)\}$	$\{(H_4, 1/2), (H_5, 1/3)\}$	$\{(H_3, 2/3), (H_4, 1/3)\}$
c_3	a_1	$\{(H_4, 0.15), (H_5, 0.85)\}$	$\{(H_4, 0.5), (H_5, 0.85)\}$	$\{(H_4, 0.85), (H_5, 0.15)\}$	$\{(H_3, 0.85), (H_4, 0.15), (H_5, 0.5)\}$
	a_2	$\{(H_5, 0.5), (H_6, 0.15)\}$	$\{(H_5, 0.15)\}$	$\{(H_5, 0.5), (H_6, 0.85)\}$	$\{(H_4, 0.15), (H_5, 0.5)\}$
	a_3	$\{(H_4, 0.5), (H_5, 0.5)\}$	$\{(H_5, 0.5), (H_6, 0.15)\}$	$\{(H_5, 0.85)\}$	$\{(H_3, 0.15), (H_4, 0.5), (H_5, 0.15)\}$

Table 4
The collective decision matrix.

	e_1	e_2	e_3	e_4
a_1	$\{(H_4, 0.04), (H_5, 0.35), (H_6, 0.17)\}$	$\{(H_4, 0.22), (H_5, 0.58), (H_6, 0.08)\}$	$\{(H_4, 0.45), (H_5, 0.25)\}$	$\{(H_3, 0.41), (H_4, 0.20), (H_5, 0.12)\}$
a_2	$\{(H_3, 0.06), (H_4, 0.06), (H_5, 0.37), (H_6, 0.04)\}$	$\{(H_3, 0.26), (H_4, 0.29), (H_5, 0.05)\}$	$\{(H_5, 0.24), (H_6, 0.43)\}$	$\{(H_3, 0.26), (H_4, 0.38), (H_5, 0.22)\}$
a_3	$\{(H_4, 0.22), (H_5, 0.22), (H_6, 0.24)\}$	$\{(H_5, 0.53), (H_6, 0.12)\}$	$\{(H_4, 0.20), (H_5, 0.44)\}$	$\{(H_3, 0.44), (H_4, 0.36), (H_5, 0.04)\}$

Step 4. $\beta_n(a_j)$ and $\beta_H(a_j)$ can be generated by Eqs. (1)–(10). The results are

$$S(a_1) = \{(H_3, 0.19), (H_4, 0.25), (H_5, 0.31), (H_6, 0.03), (H, 0.22)\}$$

$$S(a_2) = \{(H_3, 0.21), (H_4, 0.29), (H_5, 0.21), (H_6, 0.07), (H, 0.22)\}$$
$$S(a_3) = \{(H_3, 0.23), (H_4, 0.25), (H_5, 0.26), (H_6, 0.05), (H, 0.22)\}$$

Step 5. Obviously, there is no need to sort the three CT images, where the combined results are the diagnostic results obtained from the three CT images. For the same clear and critical CT images, the importance should be the same. Therefore, the final diagnostic result of CT images can be obtained by calculating the arithmetic average of diagnostic results.

$$S(a) = \{(H_3, 0.21), (H_4, 0.26), (H_5, 0.26), (H_6, 0.05), (H, 0.22)\}$$

Finally, a corresponding CT examination report can be generated. From the CT images, the lung of the patient is problematic; the possibility is that he has "*Pulmonary tuberculosis*" or "*Lung cancer*", also has the possibility of "*Pneumonia*", and very small may belong to "*Other pulmonary lung diseases*". Next, the attending physician needs to make a final diagnosis based on the CT diagnosis results and report information, in accordance with the patient's external factors, such as smoking history, history of tumors or family history of lung cancer.

4.3. Modelling the diagnostic process of lung cancer

According to the CT test report in the first stage generated in Section 4.2, the attending physician needs to make a final diagnosis by considering the patient's conditions.

As mentioned above, the possible results of diagnosis are $H = \{H_n|n = 1, 2, \ldots, 6\} = \{$*Disease-free, Benign tumors, Pneumonia, Pulmonary tuberculosis, Lung cancer, Other pulmonary diseases*$\}$. Six factors of great significance [6] in accordance with the patient's conditions for differentiating lung cancer together with CT images can be regarded as seven criteria for diagnosis, which are recorded as $O = (o_p|p = 1, 2, \ldots, 7) = \{$*CT diagnosis report, smoking history, long-term residential air quality, occupational exposure, history of malignant tumors, family history of lung cancer, history of chronic lung diseases*$\}$. The specific steps to make a final diagnosis by the LB-ER approach are as follows:

Step 1. Generate the individual evaluation of the attending physician as $S(a)_{e_1} = \{(H_3, 0.21), (H_4, 0.26), (H_5, 0.26), (H_6, 0.05)\}$, $S(a)_{e_2} = \{(H_4, s_2), (H_5, s_4)\}$, $S(a)_{e_3} = \{(H_3, s_2), (H_4, s_3)\}$, $S(a)_{e_4} = \{(H_4, s_2), (H_5, s_4)\}$, $S(a)_{e_5} = \{(H_5, s_3)\}$, $S(a)_{e_6} = \{(H_4, s_3)\}$, and $S(a)_{e_7} = \{(H_3, s_2), (H_4, s_3)\}$.

Step 2. Generate the decision matrix with numerical belief degrees.

The attending physician is supposed to have enough experience, who can make objective judgments and is neutral in the expression of semantics. According to Eq. (11), we can obtain $g(s_\alpha) = \{0, 0.25, 0.5, 0.75, 1\}$.

In addition, we need to consider the hesitancy degree of the attending physician. According to Eqs. (19)–(20), we can get the corresponding semantic scores. For the convenience of calculation, the scores of linguistic belief degrees can be regarded as the numerical form of belief degrees, shown as $S(a)_{e_1} = \{(H_3, 0.21), (H_4, 0.26), (H_5, 0.26), (H_6, 0.05)\}$, $S(a)_{e_2} = \{(H_4, 0.22), (H_5, 0.65)\}$, $S(a)_{e_3} = \{(H_3, 0.22), (H_4, 0.44)\}$, $S(a)_{e_4} = \{(H_4, 0.22), (H_5, 0.65)\}$, $S(a)_{e_5} = \{(H_5, 0.5)\}$, $S(a)_{e_6} = \{(H_4, 0.5)\}$, and $S(a)_{e_7} = \{(H_3, 0.22), (H_4, 0.65)\}$.

Step 3. Generating the weights of criteria by the BWM.

Through statistical analysis of historical data or experience of attending physician, the criteria can be compared and analyzed, and then the best criterion e_1 (CT diagnostic report) and the worst criterion e_3 (long-term residential air quality) can be selected. In this case, the criterion comparison vectors are given by the attending physician with a scale of 1–9. $BO = (1, 3, 4, 2, 2, 3, 3)^T$

Table 5
The collective decision matrix without considering hesitancy degrees of experts.

	e_1	e_2	e_3	e_4
a_1	$\{(H_4, 0.05), (H_5, 0.39), (H_6, 0.19)\}$	$\{(H_4, 0.27), (H_5, 0.62), (H_6, 0.11)\}$	$\{(H_4, 0.53), (H_5, 0.03)\}$	$\{(H_3, 0.53), (H_4, 0.24), (H_5, 0.17)\}$
a_2	$\{(H_3, 0.08), (H_4, 0.08), (H_5, 0.42), (H_6, 0.05)\}$	$\{(H_3, 0.25), (H_4, 0.27), (H_5, 0.45), (H_6, 0.03)\}$	$\{(H_5, 0.28), (H_6, 0.48)\}$	$\{(H_3, 0.31), (H_4, 0.43), (H_5, 0.26)\}$
a_3	$\{(H_4, 0.28), (H_5, 0.28), (H_6, 0.28)\}$	$\{(H_5, 0.58), (H_6, 0.13)\}$	$\{(H_4, 0.25), (H_5, 0.48)\}$	$\{(H_3, 0.51), (H_4, 0.44), (H_5, 0.05)\}$

can be obtained by comparing the best criterion with each criterion, and $OW = (4, 2, 1, 2, 3, 2, 3)^T$ is obtained by comparing each criterion with the worst criterion.

According to Eq. (25), the following non-linear programming models can be established:

$$\min \ \xi$$
$$s.t.: \sum_{i=1}^{7} \omega_i = 1,$$
$$\omega_i \geq 0, i = 1, 2, \ldots, 7$$
$$|\omega_1/\omega_2 - 3| \leq \xi, |\omega_1/\omega_3 - 4| \leq \xi$$
$$|\omega_1/\omega_4 - 2| \leq \xi, |\omega_1/\omega_5 - 2| \leq \xi$$
$$|\omega_1/\omega_6 - 3| \leq \xi, |\omega_1/\omega_7 - 3| \leq \xi$$
$$|\omega_2/\omega_3 - 3| \leq \xi, |\omega_4/\omega_3 - 3| \leq \xi$$
$$|\omega_5/\omega_3 - 3| \leq \xi, |\omega_6/\omega_3 - 3| \leq \xi$$
$$|\omega_7/\omega_3 - 3| \leq \xi$$

Solving this nonlinear programming model, we can get the weight vector of criteria as:

$$\omega = (0.24, 0.11, 0.05, 0.19, 0.19, 0.11, 0.11)^T$$

Step 4. By Eqs. (1)–(10), the reasoning result can be generated by the ER approach as:

$$S(a) = \{(H_3, 0.09), (H_4, 0.31), (H_5, 0.36), (H_6, 0.01), (H, 0.23)\}$$

Step 5. According to the aggregation results, we can conclude that there are health problems in the lungs of this patient. It is very likely that he has "*Pulmonary tuberculosis*" and "*Lung cancer*", and more likely to have "*Lung cancer*". Relatively speaking, "*Other pulmonary diseases*" are less likely to occur. In addition, due to the limitation of knowledge and medical level at the present stage, as well as the diversity of human individuals and the complexity of lung structure, there is still a considerable amount of uncertainty in the process of diagnosis. But, with the progress of science and technology, many medical problems are getting solved.

4.4. Comparative analyses and discussions

(1) The first comparative analysis

In the first model, we explore the influence of hesitancy degrees on decision-making results. Since there is no special explanation in the case that radiologists have different degrees of importance, we compare the collective decision matrix generated by our proposed method and that generated by the method proposed in [42]. The only difference between these two methods is the way to aggregate experts' evaluations. The method proposed in [42] integrates all experts' evaluations without considering different importance of experts, that is, the weight vector of experts is $w^1 = \left(\frac{1}{3}, \frac{1}{3}, \frac{1}{3}\right)^T$. Then, by normalizing the evaluations with $\sum_{n=1}^{N} \beta > 1$ in the collective decision matrix through the transformation function $\beta'_n = \beta_n / \sum_{n=1}^{N} \beta_n$, the collective decision matrix without considering the hesitancy degrees of experts can be obtained as shown in Table 5.

Next, based on the weight vector of criteria, $\omega = (0.1, 0.27, 0.19, 0.44)^T$, generated in Section 4.3, the reasoning results can be generated by the ER approach as $S^1(a) = \{(H_3, 0.25), (H_4, 0.30), (H_5, 0.32), (H_6, 0.06), (H, 0.07)\}$, whose remaining belief degree (the belief degree assigned to H) is quite different from that in the original decision result $S(a) = \{(H_3, 0.21), (H_4, 0.26), (H_5, 0.26), (H_6, 0.05), (H, 0.22)\}$.

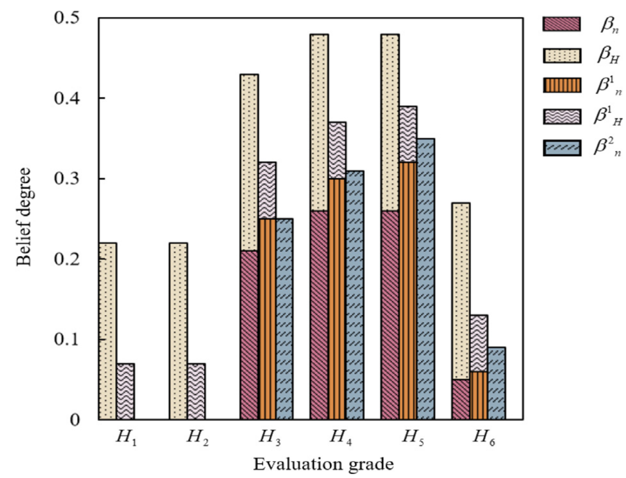

Fig. 2. The three decision results.

(2) The second comparative analysis

In the second model, to explore the influence of retaining remaining belief on decision-making results, we normalize the incomplete assessments, where $\sum_{n=1}^{N} \beta_n < 1$. Firstly, for the collective decision matrix shown as Table 4, which considers the hesitancy degrees of experts, we introduce the method of normalizing incompleteness in [42]. Through replacing all the belief degrees in each assessment via the transformation function $\beta''_n = \beta_n / \sum_{n=1}^{N} \beta_n$, the collective decision matrix without considering incompleteness can be obtained as Table 6.

Next, based on the weight vector of the criteria $\omega = (0.1, 0.27, 0.19, 0.44)^T$, the weighted average operator in [39] is introduced to aggregate assessments and we obtain the reasoning results as $S^2(a) = \{(H_3, 0.25), (H_4, 0.31), (H_5, 0.35), (H_6, 0.09)\}$, which is quite different from the original decision result $S(a) = \{(H_3, 0.21), (H_4, 0.26), (H_5, 0.26), (H_6, 0.05), (H, 0.22)\}$.

(3) Discussions

The decision results obtained by the proposed method and the first model are uncertain, where the remaining belief can be assigned to each evaluation grade respectively. The results are shown in Fig. 2. In this case study, the best result is reassigning all the belief degrees of HtoH_1 (*Disease-free*) and the worst result is reassigning all the belief degrees of HtoH_5 (*Lung cancer*). The original, best and worst results obtained by the proposed method and two comparative methods are shown in Table 7.

It is worth noting that the differences in the results of the three methods come from two aspects: (1) the original belief distributions are different; (2) the decision-making methods are different. Here, we are conducting different decision analysis in the same case study, that is, the original belief distributions are the same, and the decision methods are different, so the pros and cons of the result can reflect the pros and cons of decision-making methods. When screening high-risk groups for lung cancer, the

Table 6
The collective decision matrix without considering incompleteness.

	e_1	e_2	e_3	e_4
a_1	$\{(H_4, 0.07), (H_5, 0.63), (H_6, 0.30)\}$	$\{(H_4, 0.25), (H_5, 0.66), (H_6, 0.09)\}$	$\{(H_4, 0.64), (H_5, 0.36)\}$	$\{(H_3, 0.56), (H_4, 0.27), (H_5, 0.16)\}$
a_2	$\{(H_3, 0.11), (H_4, 0.11), (H_5, 0.70), (H_6, 0.08)\}$	$\{(H_3, 0.43), (H_4, 0.48), (H_5, 0.08)\}$	$\{(H_5, 0.36), (H_6, 0.64)\}$	$\{(H_3, 0.30), (H_4, 0.44), (H_5, 0.26)\}$
a_3	$\{(H_4, 0.32), (H_5, 0.32), (H_6, 0.35)\}$	$\{(H_5, 0.82), (H_6, 0.18)\}$	$\{(H_4, 0.31), (H_5, 0.69)\}$	$\{(H_3, 0.52), (H_4, 0.43), (H_5, 0.05)\}$

Table 7
The results of three methods.

	The proposed method	The first comparative method	The second comparative method
The original result	$S(a) = \{(H_3, 0.21), (H_4, 0.26), (H_5, 0.26), (H_6, 0.05), (H, 0.22)\}$	$S^1(a) = \{(H_3, 0.25), (H_4, 0.30), r(H_5, 0.32), (H_6, 0.06), (H, 0.07)\}$	$S^2(a) = \{(H_3, 0.25), (H_4, 0.31), (H_5, 0.35), (H_6, 0.09)\}$
The best result	$S(a)_{best} = \{(H_3, 0.43), (H_4, 0.26), (H_5, 0.26), (H_6, 0.05)\}$	$S^1(a)_{best} = \{(H_3, 0.32), (H_4, 0.30), (H_5, 0.32), (H_6, 0.06)\}$	$S^2(a)_{best} = \{(H_3, 0.25), (H_4, 0.31), (H_5, 0.35), (H_6, 0.09)\}$
The worst result	$S(a)_{worst} = \{(H_3, 0.21), (H_4, 0.26), (H_5, 0.48), (H_6, 0.05)\}$	$S^1(a)_{worst} = \{(H_3, 0.25), (H_4, 0.30), (H_5, 0.39), (H_6, 0.06)\}$	$S^2(a)_{worst} = \{(H_3, 0.25), (H_4, 0.31), (H_5, 0.35), (H_6, 0.09)\}$

worst results are usually considered to avoid delays in treatment. For this case study, if all the remaining belief is assigned to H_5 (*Lung cancer*) in $S(a)$, $S^1(a)$ and $S^2(a)$, the worst decision results can be obtained as $S(a)_{worst} = \{(H_3, 0.21), (H_4, 0.26), (H_5, 0.48), (H_6, 0.05)\}$, $S^1(a)_{worst} = \{(H_3, 0.25), (H_4, 0.30), (H_5, 0.39), (H_6, 0.06)\}$ and $S^2(a)_{worst} = S^2(a) = \{(H_3, 0.25), (H_4, 0.31), (H_5, 0.35), (H_6, 0.09)\}$. For the proposed method, it is obvious that the belief of suffering from "*Lung cancer*" in the worst result is much higher than those of other situations. Therefore, the patient is usually advised to go to the hospital for further examination as soon as possible. In contrast, there is little difference between the belief of "*Lung cancer*" and the belief of "*Pneumonia*" and "*Pulmonary tuberculosis*" in both the decision results generated by the comparative models. In this situation, the patient is advised to observe himself for one year before another CT examination. However, if the patient is actually suffering from lung cancer, the treatment may be delayed due to the lack of considering the expert hesitancy or uncertainty, which may seriously affect the survival rate of the patient.

To sum up, considering the hesitancy degrees of experts and the uncertainty of evaluation, the method proposed in this paper is more flexible and accurate than the two comparative methods. Different decision-making problems have different emphases, one may value the best result, some may value the worst result, and some may value the average result. Different methods to reassign the remaining belief match to different decision-making problems.

4.5. Practical management implications

From the managerial point of view, the introduction of the LB-ER approach can improve the efficiency of diagnosis. It is beneficial to optimize the allocation of limited medical resources.

(1) Firstly, based on the powerful historical database and the rich experience of professional doctors, the programmed assistant diagnosis process can quickly provide accurate and objective assistant diagnosis results for inexperienced doctors as a reference, so as to reduce the burden of doctors and improve the accuracy and comprehensiveness of diagnosis.
(2) Secondly, the time saved from the diagnosis process enables patients to be treated as early as possible to reduce mortality, and doctors can provide more patients with diagnostic opportunities.
(3) In addition, distributed diagnostic results can provide informative advice for patients. Those with serious illness are advised to be treated in large hospitals, while other patients can be diverted to smaller hospitals in order to optimize the allocation of limited medical resources and alleviate the phenomenon of "*overcrowding of third-class A hospitals and no one visiting county-level hospitals*" in society.

5. Conclusions

With the rapid development of artificial intelligence, decision-making with the ER approach has obtained more and more attention. Since experts tend to express their views with qualitative information, it is necessary to provide belief degrees with linguistic terms to extend the ER approach. In this regard, we introduced the LB-ER approach in this paper. Firstly, the hesitancy degrees of experts and three different linguistic scale functions were considered to determine experts' weights in uncertain MEMCDM problems with liguistic information. The BWM was introduced to generate the weights of criteria. The proposed method gave a good attempt to solve uncertain linguistic decision-making problems. Besides theoretically filling in the gap of the ER approach in solving MEMCDM problems with linguistic information, the application of the LB-ER approach in lung cancer diagnosis also confirmed the practicability, which provided accurate and effective auxiliary diagnosis, reduced the burden of doctors and improved the efficiency of diagnosis.

There are still some issues that need to be further studied. In this paper, we suppose that the set of criteria is complete enough to evaluate all alternatives and the criteria are not related to each other. However, it is difficult to determine such a set of criteria in practical decision-making problems. In addition, this paper only considered a simple case study for aiding the diagnosis of lung cancer. In the future, we will apply the LB-ER approach to deal with more practical problems, such as screening of high-risk population for other cancers and evaluating the rehabilitation of patients.

CRediT authorship contribution statement

Huchang Liao: Conceptualization, Funding acquisition, Supervision, Writing – review & editing. **Ran Fang:** Conceptualization, Data curation, Formal analysis, Writing – original draft. **Jian-Bo Yang:** Conceptualization, Writing – review & editing. **Dong-Ling Xu:** Conceptualization, Writing – review & editing.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Acknowledgments

The work was supported by the National Natural Science Foundation of China (71771156, 71971145, 72171158).

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