JOURNAL OF OPTIMIZATION THEORY AND APPLICATIONS: Vol. 129, No. 2, pp. 293–308, May 2006 (© 2006)
DOI: 10.1007/s10957-006-9061-4

# Return Policy Model of Supply Chain Management for Single-Period Products

J. HE,[1] K. S. CHIN,[2] J. B. YANG,[3] AND D. L. ZHU[4]

Communicated by Q. C. Zhao

Published Online: 29 November 2006

**Abstract.** A return policy is one of the major issues in supply chain management, particularly for managing single-period products that are characterized by short sales period and little salvage value. The value of the buyback price is important to ensure a stable supply chain. The role of the risk attitude of the retailer and supplier is also known as an essential factor to the decision determining a return policy. In this paper, we present the result of our investigation into this problem. The aim of our work is to develop a model to determine optimal return policies for single-period products based on uncertain market demands and in the presence of risk preferences. The impact of the wholesale price and selling price is also investigated to determine the optimal order quantities and optimal buyback price for different types of risk attitudes.

**Key Words.** Supply management, operations strategy, decisions, risks.

## 1. Introduction

In recent years, there has been a growing trend toward the globalization of markets, which drives increasingly individual companies to become involved in a value-added process as part of a supply chain. Each company in a supply chain must execute a particular set of actions to achieve optimal performance. However, decisions that one company makes by considering its own revenue alone almost always create conflicts with other companies in the supply chain. Hence,

[1] Research Associate, Department of Manufacturing Engineering and Engineering Management, City University of Hong Kong, Hong Kong, PRC.
[2] Associate Professor, Department of Manufacturing Engineering and Engineering Management, City University of Hong Kong, Hong Kong, PRC.
[3] Chair Professor, School of Business Management, University of Manchester, Manchester, UK.
[4] Professor and Head of Management Science Department, School of Management, Fudan University, Shanghai, PRC.



0022-3239/06/0500-0293/0 © 2006 Springer Science+Business Media, Inc.

among both academics and practitioners (Refs. 1–9), there is upcoming interest to develop new methodologies to achieve optimal performance in supply chain management.

A significant proportion of the revenues and profits of general merchandise retailers like department stores and specialty shops are derived from style goods, such as fashion clothing and apparels, sport goods, etc., which are characterized with highly uncertain demands and little salvage value at the end of their short sale season usually a few weeks or months. Such goods are called single-period products or commodities (Refs. 10, 11). Due to tough competition, retailers have to confirm their entire seasonal-order quantities well before knowing the sale performance in their shops. There is no question that retailers are seeking various forms of monetary relief and guarantees of credit from their suppliers when the actual demands of single-period products do not match the original expectations or order quantities. In particular, the provision of a return policy and the commitment of the suppliers to buy back unsold inventory from the retailers at agreed prices is now a critical issue in the negotiation of purchase contracts between independent retailers and suppliers in many categories of style goods (Ref. 12). In such negotiations, the suppliers must properly treat their buyback price offer and wholesale price that they charge retailers as a joint decision.

A return policy is a fundamental competition strategy in supply chain management. A number of researchers have developed models to study the effectiveness of return polices in various settings. For instance, the authors of Refs. 13–16 discussed return policy pricing suing a multistage approach which assumes that both manufacturers and retailers are risk neutral and are committed to return policies in supply chain coordination. They found that the total chain profits under a supplier's return policy, which offers retailers partial credits for all unsold stocks, are approximately the same as the profits of a vertically integrated firm. Pasternack (Ref. 16) considered also the structure of the newsboy problem for a seasonal product, but the effect of the risk attitudes of the suppliers and retailers was not addressed. Kandel (Ref. 17) discussed the optimal relation between buyback prices and wholesales prices under the condition of cost dependence and derived the same result. Emmons and Gilbert (Ref. 18) pointed out that both the buyback prices and wholesales prices set by the suppliers can affect the order quantities of the retailers. They investigated the expected profits of suppliers and retailers and found that, at any given wholesale price, a return policy will affect the total combined profits of the two parties. Lau and Lau (Ref. 7) investigated the relationship between pricing and return policy for a monopolistic manufacturer of a single-period commodity, but the effect of the risk attitudes of suppliers and retailers was not fully addressed.

It is clear from the literature review that little past research has been devoted to studying the role of the risk attitudes of retailers and suppliers, though it is a well-known essential factor to decisions determining the return policies. Based on the work conducted in Refs. 7 and 18, our research focuses on the analysis of the optimal return policies under different demand patterns and risk preferences of the parties involved. In this paper, a model will be presented to determine the optimal return policies for a two-echelon supply chain that consists of one pair of an independent supplier and a retailer with risk preferences. Given the production costs and selling prices, the supplier has to determine the wholesale prices, the amount to be charged against the retailer, the buyback prices, and the amount to be given to the retailer for products returned. An approximation procedure will be developed to analyze the optimal policies and discussions will be provided about the relationship between the policies and how the policies are influenced by the changes of selling prices and wholesale prices.

The rest of the paper is organized as follows. In Section 2, assumptions and notations for the model development are described. The optimization models are developed in Section 3. The analysis of the developed model is given in Section 4. Numerical studies showing insightful results are provided in Section 5. Concluding remarks and discussions are provided in Section 6.

## 2. Assumptions and Notation

A supply chain with one supplier and one retailer in a single-period product setting is analyzed first for examples, newspapers, Christmas cards, etc., which have a long delivery lead time and a short sale season. It is assumed that the supplier and the retailer have different risk attitudes and are independent. The supply chain is thus a decentralized one in which the supplier or manufacturer produces and sells the goods to the independent retailer. The latter in turn sells the items to the ultimate consumers until the inventory is depleted or the sale season for the product ends.

It is assumed that the supplier offers the wholesale price as well as the buyback price to the retailer; then, the retailer orders the quantity of products based on the consideration of the two prices. Similar to early return policy discussions (Refs. 6, 19), we suppose that the wholesale price is prespecified and both the supplier and the retailer act in a self-interested manner and decide the wholesale price, the buyback price, and the order quantity respectively in order to maximize their own profits. The profits depend on the revenues generated, cost functions, and buyback prices. The retailer is assumed to bear all the inventory carrying and stock out costs. It is assumed that, throughout their interactions, the supplier and the retailer have equal and symmetric information about demand distributions, prices, and inventory cost parameters. Thus, the supplier has all the necessary information to estimate the retailer's order quantity in response to any combination of wholesale and buyback prices; then, the optimal decisions can be made.

The notation used in the models is as follows:
$p$ = selling price per item,
$c$ = production cost per item,
$r$ = buyback price per item(traditional model),
$c_{er}, c_{es}$ = holding cost per unit excess inventory incurred by the retailer and supplier separately,
$c_{ur}, c_{us}$ = goodwill cost per unit due to stock out incurred by retailer and supplier separately,
$q$ = order quantity of retailer,
$w$ = unit wholesale price,
$x$ = random variable for single-period demand,
$f(x)$ = probability density function (pdf) of single-period demand,
$F(x)$ = cumulative distribution function (cdf) of single-period demand,
$k_s$ = supplier risk-taking or risk-aversion measure,
$k_r$ = retailer risk-taking or risk-aversion measure,
$U_s$ = supplier expected profit,
$U_r$ = retailer expected profit.

## 3. Theoretical Model

The modeling framework for the single-period product model with risk-preferences is developed based on the classical newsboy problem structure (Refs. 7, 20). The expected profit structures of the retailer and the supplier are represented as the objective functions of the model. In order to facilitate the analysis, let $c_{er} = c_{es} = c_{ur} = c_{es} = 0$, which does not affect the essence of the research problem. For the retailer and the supplier, the expected profits, the difference between their total expected revenue and their total costs, are given as follows:

$$U_r = (p - r)E([x, q]) - (w - r)q - k_r(p - r)^2 \mathrm{var}([x, q]^-), \tag{1}$$

$$U_s = rE([x, q]^-) + (w - c - r)q - k_s r^2 \mathrm{var}([x, q]^-), \tag{2}$$

where

$$[y, z]^- = \min\{y, z\}$$

In this paper, the return polices with risk preferences under demand uncertainty will be discussed. In some papers on supply chain management with uncertainty demand, researchers (Ref. 21) use a uniform distribution in theoretical problems because of the complexity of the supply chain management. This assumption is in real existence for example in the fashion clothing and apparels market, new products market etc. (Ref. 22). In our paper, we suppose that the random demand variable $x$ follows a uniform distribution over the range $[0, a]$, where

$a$ represents the possible maximum market demand. Thus, the density function $f(x)$ is given by

$$f(x) = \begin{cases} 1/a, & \text{if } 0 \leq x \leq a, \\ 0, & \text{otherwise.} \end{cases} \tag{3}$$

Based on the density function, we get

$$E([x, q]^-) = \int_0^q xf(x)dx + \int_q^\infty qf(x)dx = q - q^2/2a, \tag{4a}$$

$$E(([x, q]^-)^2) = \int_0^q x^2 f(x)dx + \int_q^\infty q^2 f(x)dx = q^2 - 2q^3/3a, \tag{4b}$$

$$\text{var}([x, q]^-) = E(([x, q]^-)^2) - (E([x, q]^-))^2 = q^3/3a - q^4/4a^2. \tag{5}$$

Substituting (4) and (5) into (1) and (2), the expected profits of the retailer and the supplier are given by

$$U_r = (p - w)q - [(p - r)/2a]q^2 - k_r(p - r)^2(q^3/3a - q^4/4a^2), \tag{6}$$
$$U_s = (w - c)q - (r/2a)q^2 - k_s r^2(q^3/3a - q^4/4a^2). \tag{7}$$

A general model can be established in light of two-stage game theory. After the supplier sets the buyback price $r$, the retailer determines the order quantity $q(r)$ that maximizes the retailer's profit $U_r$. In turn, $q(r)$ determines the supplier's expected profit $U_s$. The basic model is designed to find the optimal buyback price $r^*$ that gives

$$U_s(r^*) = \max_r\{U_s(r, q^*(r)), r \geq 0\}, \tag{8}$$

where $q^*(r)$ is the $q$-value that yields

$$U_r(q^*(r)) = \max_q\{U_r(q, r), q \geq 0\}. \tag{9}$$

The model will be analyzed in the following sections with different types of risk-aversion, risk-neutral, or risk-taking shown by the supplier and the retailer.

## 4. Equilibrium Analysis of the Model

**4.1. Risk-Neutral Case ($k_s = k_r = 0$).** From Equation (6), we get

$$\partial^2 U_r/\partial^2 q = -(p - r)/a < 0,$$

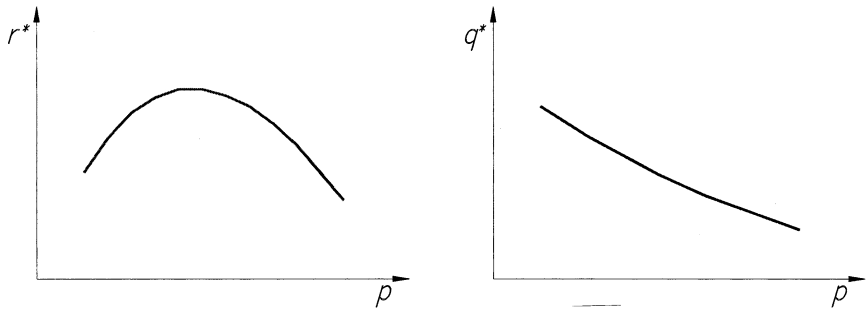


Fig. 1. Impact of the selling price on optimal order quantity and optimal buyback price.

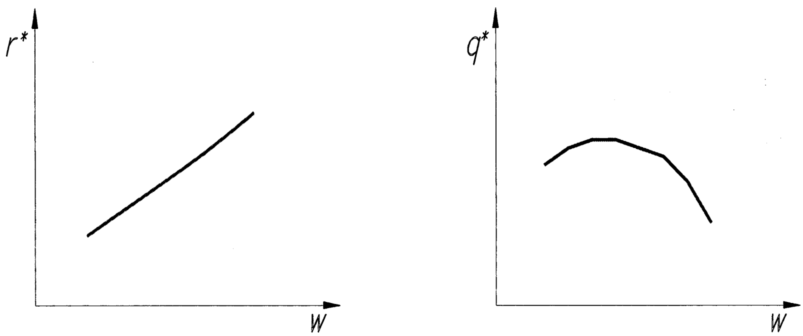


Fig. 2. Impact of the wholesale price on optimal order quantity and optimal buyback price.

so $U_r(q)$ is concave with regard to $q$. The optimal reaction function $q^*(r)$ that maximize $U_r(q)$ exists and can be found by setting the first-order derivative of (6) to zero, leading to

$$q^*(r) = a(p - w)/(p - r). \tag{10}$$

The second-stage equilibrium analysis can then be conducted as follows.

**Proposition 4.1.** If $4w < 2p + 2c + 3r(p - w)/(p - r)$, then $U_s(r, q^*(r))$ is strictly concave in $r$ and there exist anoptimal solution. Otherwise, the optimal value of the buyback price is $r_{\max}$, which is the upper bound of the buyback price set by the supplier.

**Proof.** Substituting the reaction function $q^*(r)$ [see (10)] and $k_s = 0$ into Equation (7), and considering its first derivative and second derivative on the buyback price, we get

$$\begin{aligned} &dU_s(r, q^*(r))/dr \\ &= [(3w - p - 2c)/2 - r(p - w)/p - r]a(p - w)/(p - r)^2, \end{aligned} \tag{11}$$

$$\begin{aligned} &d^2U_s(r, q^*(r))/d^2r \\ &= [4w - 2p - 2c - 3r(p - w)/(p - r)]a(p - w)/(p - r)^3. \end{aligned} \tag{12}$$

From Equation (12), we could prove the first part of Proposition 4.1. If

$$4w \geq 2p + 2c + 3r(p - w)/(p - r),$$

we see that

$$3w - p - 2c - 2r(p - w)/(p - r) = p(p - w)/(p - r) > 0.$$

Then,

$$dU_s(r, q^*(r))/dr > 0. \tag{13}$$

Therefore, $U_s(r, q^*(r))$ is increasing in $r$; the upper boundary $r_{\max}$ of the buyback price maximizes $U_s(r, q^*(r))$. □

Based on the above analysis, we have the following results.

**Proposition 4.2.** When $4w < 2p + 2c + 3r(p - w)/(p - r)$ and $3w > p + 2c$, the optimal buyback price $r^*$ is given by

$$r^* = p(3w - p - 2c)/(p + w - 2c). \tag{14}$$

**4.2. Risk Taking Case ($k_s \neq 0$ or $k_r \neq 0$).** Suppliers and retailers may be either risk-taking or risk-averse, depending upon actual situations. In the following analysis, the equilibrium solutions are generated under the condition of risk-propensity or risk-aversion. According to the same analytical method as stated in Section 3, we have

$$\partial U_r/\partial q = p - w - [(p - r)/a]q - k_r(p - r)^2(q^2/a - q^3/a^2), \tag{15}$$

$$\partial^2 U_r/\partial^2 q = [(p - r)/a]q - k_r(p - r)^2(q^2/a - q^3/a^2). \tag{16}$$

It is clear from Equation (16) that, if $k_r = 0$, then the retailer expected profit function is strictly concave in $q$ but that, if $k_r \neq 0$, it is difficult to estimate the characteristics of the optimal solutions. To generate a unique optimal solution, the following condition is required.

**Proposition 4.3.** When the retailer expected profit function is strictly concave in $q$, its reaction function exists and is unique. For $k_r \neq 0$, the necessary condition to guarantee that $q^*(r)$ exists is given by

$$q < \min\{2a/3, a/3[1 + \sqrt{1 + 3/ak_r(p - r)}]\}. \tag{17}$$

Since the value of $a$, the possible maximum market demand, is much larger than the order quantities, the above necessary condition is of general significance. Therefore, the retailer's reaction function must exist. The equilibrium reaction function can then be generated from Equation (15) by solving the following equation:

$$p - w - [(p - r)/a]q^*(r) - k_r(p - r)^2[(q^*(r))^2/a - (q^*(r))^3/a^2] = 0. \tag{18}$$

In order to get the finally optimal solutions, it is necessary to find whether the supplier's objective function is concave in the buyback price. Generally speaking, the optimal order quantities would increase for higher buyback prices, but its rate of increase will reduce as the buyback price increases. Therefore, it could be assumed that $dq^*(r)/dr > 0$ and $d^2q^*(r)/d^2r < 0$. Then substituting Equation (18) into the supplier's objective function leads to the first derivative and the second derivative of the supplier objective function at the buyback price as follows:

$$\begin{aligned}
&dU_s(r, q^*(r))/dr \\
&= \{w - c - rq^*(r)/a - k_s r^2[a - q^*(r)](q^*(r))^2/a^2\}dq^*(r)/dr \\
&\quad - (q^*(r))^2/2a - 2k_s r[(q^*(r))^3/a^2][a/3 - q^*(r)/4], \qquad (19)\\
&d^2U_s(r, q^*(r))/d^2r \\
&= \{w - c - rq^*(r)/a - k_s r^2[a - q^*(r)](q^*(r))^2/a^2\}d^2q^*(r)/d^2r \\
&\quad - 2k_s[(q^*(r))^3/3a - (q^*(r))^4/4a^2] - \{r/a + 2k_s r[(q^*(r))^2/a \\
&\quad - (q^*(r))^3/a^2]\}[dq^*(r)/dr]^2 - 2\{q^*(r)/a \\
&\quad + 2k_s r[a - q^*(r)](q^*(r))^2/a^2\}dq^*(r)/dr, \qquad (20)
\end{aligned}$$

where

$$dq^*(r)/dr = \frac{q^*(r) + 2k_r(p - r)[1 - q^*(r)/a](q^*(r))^2}{p - r + k_r(p - r)^2[2q^* - 3(q^*(r))^2/a]}.$$

The above deduction results in the following proposition.

**Proposition 4.4.** If $w > c + rq^*(r)/a + k_s r^2[(q^*(r))^2/a - (q^*(r))^3/a^2]$ and $4a > 3q^*(r)$, then the supplier expected profit function is strictly concave with respect to the buyback price and the optimal solution for the buyback price exists and is unique. The optimal value of the buyback price is given by

$$[w - c - r^* q^*/a - k_s r^2 (a - q^*)(q^*)^2/a^2] \frac{dq}{dr}\bigg|_{r^*}$$
$$- (q^*)^2/2a - 2k_s r^* [(q^*)^3/a^2](a/3 - q^*/4) = 0, \tag{21}$$

where $q^* = q^*(r^*)$.

## 5. Comparative Static Analysis of the Model

In this section, the sensitivity of the model to the changes of both the wholesale price and the selling price will be analyzed. First of all, the relationships between the order quantities and the different risk attitudes of suppliers and retailers are revealed as follows.

**Proposition 5.1.** Given that the buyback price is kept unchanged, we get

$$q^*{}_{rt} > q^*{}_{rn} > q^*{}_{ra},$$

where $q^*{}_{rt}, q^*{}_{rn}, q^*{}_{ra}$ express the order quantities of the retailer in the risk-taking, risk-neutral, and risk-averse cases, respectively.

**Proof.** Suppose that the buyback price is set at the level $r$. From (18), the order quantity from a risk taking retailer can be generated as follows:

$$q^*(r) = q(p - w)/(p - r) - k_r q^*(r)^2 (p - r)[1 - q^*(r)/a]. \tag{22}$$

The second term in the right-hand side describes the effect of the risk-taking measure on order quantity. From Proposition 4.3, as $k_{rt} < k_{rn} < k_{ra}$ (because $k_{rt} < 0$, $k_{rn} = 0$, and $k_{ra} > 0$), we have

$$q^*(r)^2 (p - r)[1 - q^*(r)/a] > 0.$$

When the retailers tend to take risk, they would likely have an optimistic attitude for future market demands and profits. Therefore, they may order and keep more goods before the sale season. If the retailers are risk-averse, they always consider problems and makes decisions from a pessimistic point of view. The order quantity from the risk-averse retailer is naturally the smallest among the three types of retailer. The conclusion of Proposition 5.1 is in accordance with the actual market situation. □

As seen in Section 3 the optimal solutions of the model are simple when both the suppliers and retailers are risk-neutral. We now turn to discuss how sensitive the optimal order quantity and optimal buyback price are to the selling price $p$ and the wholesale price $w$ under the case of both the retailer and supplier being risk-neutral. Substituting (14) into (10), we can get the optimal order quantity as follows:

$$q^* = (a/2p)(p + w - 2c). \tag{23}$$

**Proposition 5.2.** For a decentralized supply chain where both the independent supplier and retailer are risk-neutral, the optimal buyback price rises as the selling price decreases and vice versa. If $w < 2c$, the change of the optimal order quantity is positively proportional to the change of the selling price. But when $w \geq 2c$, the relationship is reversed.

**Proof.** From (14) and (23), we obtain that

$$\partial q^*/\partial p = a(w - 2c)/2p^2, \tag{24}$$

$$\partial r^*/\partial p = [(w - 2c)(3w - 2p - 2c) - p^2]/(p + w - 2c)^2. \tag{25}$$

Note that

$$-p < 3w - 2p - 2c < p,$$

for

$$3w > p + 2c.$$

So, we get

$$\partial r^*/\partial p < 0.$$

According to Equation (24), it is straightforward to prove the latter part of Proposition 5.2. □

**Proposition 5.3.** If the supplier and the retailer are risk-neutral, then both the optimal buyback price and the optimal order quantity will increase (decrease) as the wholesale price rises (drops).

**Proof.** It is similar to the proof for Proposition 5.2. □

Compared with the above analysis, it is relatively difficult to conduct a comparative static analysis for a supply chain with risk-taking or risk-averse supplier and retailer. The main reason is that its optimal solutions are normally functions with explicit from unknown. In order to conduct the analysis, it is therefore to make necessary certain simplifying assumptions. Suppose that $p \gg r$, $q \gg 1$ and $q/a \ll 1$. Then, we have

$$dq^*(r)/dr = \frac{q^*(r) + 2k_r(p - r)[1 - q^*(r)/a](q^*(r))^2}{p - r + k_r(p - r)^2[2q^*(r) - 3(q^*(r))^2/a]} \approx q^*(r)/(p - r), \tag{26}$$

Inserting (26) into (21) leads to the following equation:

$$[w - c - r^*q^*/a - k_s r^2(a - q^*)(q^*)^2/a^2]1/(p - r^*)$$
$$-q^*/2a - 2k_s r^*[(q^*)^2/a^2](a/3 - q^*/4) = 0. \tag{27}$$

For simplicity, the small values of $\vartheta(\frac{(q^*)^2}{a^2})$ are neglected when $a \gg q$. Obviously, this results in the following set of approximate equations:

$$w - c - r^*q^*/2a - k_s(r^*)^2[(q^*)^2/3a] - p[q^*/2a + 2k_s r^*(q^*)^2/3a] = 0, \tag{28a}$$

$$p - w - [(p - r^*)/a]q^* - k_r(p - r^*)^2(q^*)^2/a = 0. \tag{28b}$$

Differentiating (28) with respect to w, the unit wholesale price, and expressing the equations in matrix form lead to the following matrix equation:

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} \partial q^*/\partial w \\ \partial r^*/\partial w \end{bmatrix} = \begin{bmatrix} a \\ a \end{bmatrix}, \tag{29}$$

where

$$A = (r^* + p)/2 + [2k_s r^* q^*/3](r^* + 2p),$$
$$B = q^*/2 + [2k_s(q^*)^2/3](r^* + p),$$
$$C = r^* - p - 2k_r(p - r^*)^2 q^*,$$
$$D = q^* + 2k_r(p - r^*)(q^*)^2.$$

then we have

$$\begin{bmatrix} \partial q^*/\partial w \\ \partial r^*/\partial w \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix}^{-1} \begin{bmatrix} a \\ a \end{bmatrix} = [a/(AD - BC)] \begin{bmatrix} D - B \\ A - C \end{bmatrix}. \tag{30}$$

For the comparative static analysis about the wholesale price, the research is focused on how the changes of the wholesale price influence the optimal order quantity and the optimal buyback price. Note that, from Equation (30), the research question relates to the parameters $k_s$ and $k_r$. To simplify the discussion, the right-hand side of Equation (30) is decomposed into three main terms: $AD - BC$, $D - B$, $A - C$, and five different kinds of cases with different combinations of the parameters $k_s$ and $k_r$ are taken into account. The analysis is shown in Table 1.

Among the above fives cases in Table 1, Case 3 and Case 4 are the special situations of Case 5. Having generated the above values, an in-depth sensitivity analysis about the wholesale price can be conducted as summarized in Table 2.

To understand the results of Table 2, note that there are three effects of a wholesale price change. We explore how the optimal buyback price and the optimal order quantity for each case change with different parameters of the model.

Table 1. Values of $AD - BC$, $D - B$, $A - C$.

| | |
|---|---|
| Case 1 | |
| $k_s \neq 0, k_r = 0$ | $AD - BC = pq^* + [2k_s(q^*)^2/3](p^2 + 2pr^*)$ |
| | $D - B = q^*/2 - [2k_s(q^*)^2/3](r^* + p)$ |
| | $A - C = (3p - r^*)/2 + (2k_s r^* q^*/3)(r^* + 2p)$ |
| Case 2 | |
| $k_s = 0, k_r \neq 0$ | $AD - BC = pq^* + 2k_r p(q^*)^2(p - r^*)$ |
| | $D - B = q^*/2 + 2k_r(q^*)^2(p - r^*)$ |
| | $A - C = (3p - r^*)/2 + 2k_r q^*(p - r^*)^2$ |
| Case 3 | |
| $k_s \neq 0, k_r \neq 0$ | $AD - BC = pq^* + [2kp(q^*)^2/3](4p - r^*) + [4k^2(q^*)^3/3](p^2 + 2pr^*)(p - r^*)$ |
| $k_s = k_r = k$ | $D - B = q^*/2 + [4k(q^*)^2/3](p - 2r^*)$ |
| | $A - C = (3p - r^*)/2 + (2kr^*q^*/3)(r^* + 2p) + 2kq^*(p - r^*)^2$ |
| Case 4 | |
| $k_s \neq 0, k_r \neq 0$ | $AD - BC = pq^* + [2kp(q^*)^2/3](2p - 5r^*) + [4k^2(q^*)^3/3](p^2 + 2pr^*)(p - r^*)$ |
| $k_s = k_r = k$ | $D - B = q^*/2 + [4k(q^*)^2/3](2p - r^*)$ |
| | $A - C = (3p - r^*)/2 + (2kq/3)[3p^2 + 2(r^*)^2 - 8pr^*]$ |
| Case 5 | |
| $k_s \neq 0,\ k_r \neq 0$ | $AD - BC = pq^* + [2tkp(q^*)/3](p + 2r^*) + 2kp(q^*)^2(p - r^*)$ |
| $k_s = tk_r = tk,$ | $+[4tk^2(q^*)^3/3](p^2 + 2pr^*)(p - r^*)$ |
| $t \neq -1, 0, 1$ | $D - B = q^*/2 + [2k(q^*)^2/3][(3 - t)p - (3 + t)r^*]$ |
| | $A - C = (3p - r^*)/2 + (2kq/3)[3p^2 + (3 + t)(r^*)^2 + (2t - 6)pr^*$ |

Case 1. if $k_s > 0$, as the wholesale price $w$ increases, the optimal buyback price $r^*$ goes up, but when $3 - 4k_s q^*(r^* + p) > 0$, as $w$ increases, the optimal order quantity $q^*$ increases, otherwise the optimal order quantity $q^*$ reduces; if $k_s < 0$, when $9p - 3r^* + 4k_s r^* q^*(r^* + 2p)/[3 + 2k_s(p + 2r^*)] > 0[< 0]$ as $w$ increases, $r^*$ increases [decreases], but when $3 + 2k_s q^*(2r^* + p) > 0[< 0]$ as $w$ increases, $q^*$ increases [decreases].

Table 2. Results of comparative static analysis about the wholesale price.

| | |
|---|---|
| Case 1 | |
| $k_s \neq 0, k_r = 0$ | $\partial r^*/\partial w > 0$ and $\mathrm{sign}(\partial r^*/\partial w) = \mathrm{sign}[3 - 4k_s q^*(r^* + p)]$ if $k_s > 0$, |
| | $\mathrm{sign}(\frac{\partial r^*}{\partial w}) = \mathrm{sign}\{[(9p - 3r^* + 4k_s r^* q^*(r^* + 2p)]/[3 + 2k_s q^*(p + 2r^*)]\}$, |
| | $\mathrm{sign}(\frac{\partial q^*}{\partial w}) = \mathrm{sign}[3 + 2k_s q^*(2r^* + p)]$ if $K_s < 0$. |
| Case 2 | |
| $k_s \neq 0, k_r = 0$ | $\partial r^*/\partial w > 0$ and $\partial q^*/\partial w > 0$, if $k_r > 0$ |
| | $\mathrm{sign}(\partial r^*/\partial w) = \mathrm{sign}\{[3p - r^* + 4k_r q^*(p - r^*)^2]/[1 + 2k_r(p - r^*)]\}$, |
| | $\mathrm{sign}(\partial r^*/\partial w) = \mathrm{sign}\{[1 + 4k_r q^*(p - r^*)]/[1 + 2k_r(p - r^*)]\}$, if $k_r < 0$. |
| Cases 3, 4, 5 | |
| $k_s \neq 0, k_r \neq 0$ | $\mathrm{sign}(\partial r^*/\partial w) = \mathrm{sign}\{\frac{9p-3r^*+4kq^*[3p^2+(3+t)(r^*)^2+(2t-6)pr^*]}{3+2tkq^*(p+2r^*)+6kq^*(p-r^*)+4tk^2(q^*)^2(p+2r^*)(p-r^*)}\}$ |
| $k_s = tk_r = tk,$ | $\mathrm{sign}(\partial r^*/\partial w) = \mathrm{sign}\{\frac{3+4kq^*[(3-t)p-(3+t)r^*]}{3+2tkq^*(p+2r^*)+6kq^*(p-r^*)+4tk^2(q^*)^2(p+2r^*)(p-r^*)}\}$ |
| $t \neq 0$ | |

Case 2. If $k_r > 0$ as $w$ increases, both $r^*$ and $q^*$ increase; if $k_r < 0$, when $3p - r^* + 4k_r q^*(p - r^*)^2/[1 + 2k_r(p - r^*)] > 0[< 0]$ as $w$ increases, $r^*$ increases [decreases], but when $1 + 4k_r q^*(p - r^*)/[1 + 2k_r(p - r^*)] > 0[< 0]$, as $w$ increases, $q^*$ increases (decreases).

Case 3. If

$$\frac{9p - 3r^* + 4kq^*[3p^2 + (3+t)(r^*)^2 + (2t-6)pr^*]}{3 + 2tkq^*(p + 2r^*) + 6kq^*(p - r^*) + 4tk^2(q^*)^2(p + 2r^*)(p - r^*)} > 0\ [< 0],$$

as $w$ increases, $r^*$ increases [decreases], but when

$$\frac{3 + 4kq^*[(3-t)p - (3+t)r^*]}{3 + 2tkq^*(p + 2r^*) + 6kq^*(p - r^*) + 4tk^2(q^*)^2(p + 2r^*)(p - r^*)} > 0\ [< 0],$$

$q^*$ increases [decreases].

From the above, the effects of a wholesale price are different with different cases; the changing degree of the optimal buyback price and the optimal order quantity is related to both the stable parameters $k_s$, $k_r$, $p$ and the varying parameters $r^*$, $q^*$. From the viewpoint of the supplier and retailer, they must arrive at a decision according to different cases and conditions.

The selling price is also an important parameter, which is affected directly by marketing fluctuations. A small change of the selling price can break the old equilibrium state, lead to a new state, which in turn will affect the optimal order quantity and the optimal buyback price. Under the same conditions as assumed before, the comparative static analysis about the selling price is given as follows:

$$\begin{bmatrix} \partial q^*/\partial p \\ \partial r^*/\partial p \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix}^{-1} \begin{bmatrix} X \\ Y \end{bmatrix} = \frac{1}{AD - BC} \begin{bmatrix} DX - BY \\ AX - CY \end{bmatrix}, \tag{31}$$

where

$$X = -(q^*/2 + 2k_s r^*(q^*)^2),\ Y = -a + q^* + 2k_r(q^*)^2(p - r^*).$$

Note, however, that Equation (31) is a very close approximation to Equation (30). Therefore, the same analytic method can be applied to the comparative static analysis about the selling price.

## 6. Numerical Analysis of the Impact of Risk Preference

The previous section presented the results of the theoretical analysis for the impact of risk attitudes of the retailer and supplier. Since the optimal solutions in the presence of risk preferences are quite complex and in particular the comparative static analyses result in functions of the risk parameters which are interdependent, it is very difficult, if not impossible, to obtain closed-form analytical results. As

Table 3. Impact of the selling price on optimal order quantities and optimal buyback price.

| $p$ | 20.0 | 21.0 | 22.0 | 23.0 | 24.0 | 25.0 | 26.0 | 27.0 | 28.0 | 29.0 | 30.0 | 31.0 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| $r^*$ | 3.570 | 3.586 | 3.598 | 3.605 | 3.609 | 3.609 | 3.606 | 3.601 | 3.593 | 3.583 | 3.570 | 3.557 |
| $q^*$ | 25.1 | 24.4 | 23.7 | 23.1 | 22.5 | 21.9 | 21.4 | 20.9 | 20.5 | 20.1 | 19.7 | 19.3 |

such, a numerical analysis is conducted to gain further insights into the impact of risk preferences.

In the study, several scenarios are formulated first by taking into account different combinations of the risk parameters. Then, with all other parameters fixed, the effects of the selling price and the wholesale price on the optimal order quantity and the optimal buyback price in Case 3 are investigated. The fixed parameters and their selected values used in the study include the demand distribution parameter ($a = 100$), the risk-preference measures of the supplier and the retailer ($k_s = k_r = 1.0$), and the producing cost ($c = 1.0$). Given these parameter values, the optimal order quantity and the optimal buyback price are evaluated in two scenarios. The first scenario is that, if the wholesale price is fixed at 8.0, the effects of the selling price on the selling price on the optimal order quantity and the optimal buyback price are examined, with $p$ changing from 20.0 to 31.0 with steps of 1.0. The optimal order quantity and optimal buyback price are shown in Table 3 and depicted in Figure 1.

It is observed from Table 3 that:

(i) The optimal order quantity decreases as the selling price increases.

(ii) The effects of the selling price on the optimal buyback price are closely interconnected with the selling price. For small selling prices (in this sample, $p < 24$ or 25), the optimal buyback price increases as the selling price rises. However, for large selling prices (in this sample, $p \geq 24$ or 25), it decreases as the selling price increases.

The second scenario is designed to evaluate the effects of the wholesale price on the optimal order quantity and the optimal buyback price. To do so, $p$ is fixed at 20.0 and $w$ changes from 9.0 to 16.0 in a step of 1.0. The results are shown in Table 4 and depicted in Figure 2.

It is clear from Table 4 that the optimal buyback price increases as the wholesale price increases. If the wholesale price is small, however, the optimal order quantity increases as the wholesale price increases, otherwise it will decreases.

Table 4. Impact of the wholesale price on optimal order quantity and optimal buyback price.

| $w$ | 9.0 | 10.0 | 11.0 | 12.0 | 13.0 | 14.0 | 15.0 | 16.0 |
|---|---|---|---|---|---|---|---|---|
| $r^*$ | 8.511 | 9.121 | 9.720 | 10.320 | 10.931 | 11.557 | 12.243 | 12.968 |
| $q^*$ | 28.8 | 29.0 | 29.1 | 29.1 | 29.0 | 28.9 | 28.6 | 28.1 |

In this sample, from Table 1 and Table 2, one can see that, in Case 3, when $k = 1.0 > 0$, then $\partial r^*/\partial w > 0$ and

$$\text{sign}(\partial q^*/\partial w) = \text{sign}(3 + 8q^*(p - 2r^*)).$$

From Table 4, it is clear that the results of the numerical analysis are consistent with the comparative static analysis.

## 7. Conclusions

In this paper, a two-echelon supply chain of single-period products was investigated, which consists of one supplier and one retailer who have risk-preferences and operate in an environment where demand is uncertain. The contributions of this research are multifold. First, an analysis model was established and analytical optimal solutions were generated by assuming a uniform distribution. A Comparative static analysis was then conducted.

Second, the impact of risk-preferences on the supplier's return policies and the retailer's order quantities was investigate; theoretical results under different risk-preferences were generated. These results revealed the specific effects of the selling price and the wholesale price on the optimal order quantity and the optimal buyback price. This analysis approach provides a flexible means for allocating market risks between suppliers and retailers in a supply chain.

This research is focused primarily on developing a generic analytical approach, which is based on the assumption that the demand distribution is uniform. However, it can be shown that the return policy is also applicable to a realistic situation where the parties of a supply chain have risk-preferences. In this situation, supplier and retailer decisions are interrelated and interact with each other. It is difficult to generate closed-form solutions for general demand distributions; this needs to be investigated in further research.

## References


1. WEISS, H. J., and ROSENTHAL, E. C., *Optimal Ordering Policies When Anticipating a Disruption in Supply or Demand*, European Journal of Operational Research, Vol. 59, pp. 370–382, 1992.
2. LEVY, M., and WEITZ, B. A., *Retailing Management*, Irwin, Homewood, Town State, 1995.
3. PARLAR, M., and PERRY, D., *Optimal Policies in Deterministic and Random Yield Models with Uncertain Future Supply*, European Journal of Operational Research, Vol. 84, pp. 431–443, 1995.
4. ANUPINDI, R., and BASSOK, Y., *Supply Contracts with Quantity Commitments and Stochastic Demand*, Quantitative Models for Supply Chain Management, Edited by S. Tayur, R. Ganeshan, and M. Magazine, Kluwer Academic Publishers, Dordrecht, Holland, pp. 198–232, 1999.

5. LARIVIERE, M. A., *Supply Chain Contracting and Coordination with Stochastic Demand*, Quantitative Models for Supply Chain Management, Edited by S. Tayur, R. Ganeshan, and M. Magazine, Kluwer Academic Publishers, Dordrecht, Holland, pp. 233–268, 1999.
6. MANTRALA, M. K., and RAMAN, K., *Demand Uncertainty and Supplier's Return Policies for a Multistore Style-Good Retailer*, European Journal of Operational Research, Vol. 115, pp. 270–284, 1999.
7. LAU, H. S., and LAU, A. H. L, *Manufacturer's Pricing Strategy and Return Policy for a Single-Period Commodity*, European Journal of Operational Research, Vol. 116, pp. 291–304, 1999.
8. MARQUEZ, A. C., and BLANCHAR, C., *The Procurement of Strategic Parts: Analysis of a Portfolio of Contracts with Suppliers Using a System Dynamics Simulation Model*, International Journal of Production Economics, Vol. 88, pp. 29–49, 2004.
9. LAU, K. W., and YAM C. M., *A Case Study of Product Modularization on Supply Chain Design and Coordination in Hong Kong and China*, Journal of Manufacturing Technology and Management, Vol. 16, No. 4, pp. 432–446, 2005.
10. PASHIGIAN, B. P., *Demand Uncertainty and Sales: A Study of Fashion and Markdown Pricing*, American Economic Review, Vol. 78, pp. 936–953, 1988.
11. PADHIGIAN, B. P., BOWEN, B., and GOULD, E., *Fashion, Styling, and the Within-Season Decline in Automobile Prices*, Journal of Law and Economics, Vol. 38, pp. 281–309, 1995.
12. PADMANABHAN, V., and PNG, I. P. L., *Returns Polices: Make Money by Making Good*, Sloan Management Review, Vol. 37, pp. 65–72, 1995.
13. ZUSMAN, P., and ETGAR, M., *The Marketing Channel as an Equilibrium Set of Contracts*, Management Science, Vol. 27, pp. 124–136, 1981.
14. JEULAND, A. P., and SHUGAN, S. M., *Managing Channel Profits*, Marketing Science, Vol. 2, pp. 239–272, 1985.
15. MONAHAN, J. P., *A Quantity Discount Pricing Model to Increase Vendor Profits*, Management Science, Vol. 30, pp. 720–726, 1984.
16. PASTERNACK, B. A., *Optimal Pricing and Return Policies for Perishable Commodities*, Marketing Science, Vol. 4, pp. 166–176, 1985.
17. KANDEL, E., *The Right to Return*, Journal of Law and Economics, Vol. 39, pp. 329–356, 1996.
18. EMMONS, H., and GILBERT, S. M., *The Role of Return Polices in Pricing and Inventory Decisions for Catalogue Goods*, Management Science, Vol. 44, pp. 277–283, 1998.
19. PADMANABHAN, V., and PNG, I. P. L., *Manufacturer's Return Policies and Retail Competition*, Marketing Science, Vol. 16, pp. 81–94, 1997.
20. KHOUJA, M., *The Newsboy Problem under Progressive Multiple Discounts*, European Journal of Operational Research, Vol. 84, pp. 458–466, 1995.
21. ZIMMER, K., *Supply Chain Coordination with Uncertain Just-in-Time Delivery*, International Journal of Production Economics, Vol. 77, pp. 1–15, 2002.
22. VIROLAINEN, V. M., *A Survey of Procurement Strategy Development in Industrial Companies*, International Journal of Production Economics, Vols. 56–57, pp. 677–688, 1998.