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Decision Support

Evidential reasoning based preference programming for multiple attribute decision analysis under uncertainty

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Abstract

Multiple attribute decision analysis (MADA) problems having both quantitative and qualitative attributes under uncertainty can be modelled and analysed using the evidential reasoning (ER) approach. Several types of uncertainty such as ignorance and fuzziness can be consistently modelled in the ER framework. In this paper, both interval weight assignments and interval belief degrees are considered, which could be incurred in many decision situations such as group decision making. Based on the existing ER algorithm, several pairs of preference programming models are constructed to support global sensitivity analysis based on the interval values and to generate the upper and lower bounds of the combined belief degrees for distributed assessment and also the expected values for ranking of alternatives. A post-optimisation procedure is developed to identify non-dominated solutions, examine the robustness of the partial ranking orders generated, and provide guidance for the elicitation of additional information for generating more desirable assessment results. A car evaluation problem is examined to show the implementation process of the proposed approach.

Keywords: Multiple attribute decision analysis; The evidential reasoning approach; Uncertainty modelling; Interval evaluation; Nonlinear optimization

1. Introduction

Many complex multiple attribute decision analysis (MADA) problems involve both quantitative and qualitative attributes as well as various types of uncertainties such as local and global ignorance (incomplete or no information) and fuzziness (vague information). Such complex MADA problems can be consistently modelled using the evidential reasoning (ER) approach (Yang and Sen, 1994; Yang and Singh, 1994; Yang, 2001; Yang and Xu, 2002a,b; Wang et al., 2006b; Xu et al., in press; Yang et al., 2006). The ER approach models both

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quantitative and qualitative attributes using a distributed modelling framework, in which each attribute is assessed using a set of collectively exhaustive and mutually exclusive assessment grades, probabilistic uncertainty including local and global ignorance is characterized by a belief structure, and fuzzy uncertainty by fuzzy linguistic variables (Yang et al., 2006).

In certain decision situations, due to the complexity and uncertainty involved in real world decision problems and the inherent subjective nature of human judgments, it may not always be realistic or feasible to acquire exact judgments. In group decision making, for example, group members' different views could be better captured using interval judgments, which also leave rooms for discussion, negotiation and further analysis. In the ER approach and its extensions, both belief degrees and weight estimation may take interval values. In such cases, the aggregated evaluations of alternatives may no longer be single values but become interval judgments. According to the interval belief structures proposed in the ER models, Wang et al. (2006b) constructed pairs of non-linear optimisation models to estimate the upper and lower bounds of the combined belief degrees and to compute the maximum and the minimum expected utilities of each alternative. The purpose of this paper is to handle both the interval beliefs and interval weights in an integrated manner, in order to develop an enhanced ER approach for MADA under two types of interval uncertainties.

The recent literature reviews conducted by the authors and their colleagues (Wang et al., 2006b) show that the estimation of interval weights and their use in decision making processes have attracted a lot of attention and several weight evaluation methods have been developed. Among other frameworks, pairwise comparison matrices provide an easy-to-use framework to elicit preferences from decision makers and have been used in several weight generation methods such as the principal right eigenvector method (Saaty, 1977, 1980, 1988, 1994) and the logarithmic least squares method (Saaty and Vargas, 1984a,b; Crawford and Williams, 1985; Barzilai et al., 1987), which is also known as the geometric mean method (Crawford, 1987; Barzilai, 1997). For interval pairwise judgments, a number of techniques have been developed to generate a single weight vector from a feasible region. Saaty and Vargas (1987) proposed interval judgments for the AHP method as a way to model subjective uncertainty and used a Monte Carlo simulation approach to generate weight intervals from interval comparison matrices. Arbel (1989, 1991) interpreted interval judgments as linear constraints on local priorities and applied a linear programming approach to generate weights, which is simple to implement and can generate true weight intervals from a consistent interval comparison matrix.

The feasible region of a weight vector was also analysed by Arbel and Vargas (1992, 1993). They formulated maximization and minimization problems for establishing bounds for the components of principal right eigenvectors. They characterized weight intervals as solutions to non-linear programmes in which all local priorities in a hierarchy are included as decision variables. On the other hand, Salo and Hämäläinen (1992a,b, 1995), Salo (1993) developed the "preference programming" approach. They computed the maximum and minimum feasible values for each weight using linear programming techniques and incorporated the resultant intervals into further synthesis to obtain global weight intervals. They also defined the concept of local "dominance" given interval judgments. Islam et al. (1997) used lexicographic goal programming to generate weights from inconsistent pairwise interval judgment matrices and explored its properties and advantages as a weight estimation technique. Haines (1998) proposed a statistical approach to extract preferences from interval judgment matrices. Two specific distributions on a feasible region were examined and the mean of the distributions was used as a basis for assessment and ranking.

In this paper, the framework proposed by Wang et al. (2006b) for handling interval beliefs is extended to take into account interval weights together with interval beliefs. Based on the original evidential reasoning algorithm, several pairs of preference programming (referred to as ER-PP) models are developed to cater for incomplete or imprecise weight information either assigned directly or through pairwise comparisons. In the ER-PP models, both assessment and preference information is treated as constraints. Instead of generating interval weight vectors, preference information is used to generate combined interval beliefs and interval values (utilities) to support global sensitivity analysis and further information elicitation. Interval uncertainty may lead to local non-dominance between different options. In the paper, the dominance structure caused due to interval uncertainty will be defined and analysed. New procedures are designed and investigated to support the further elicitation of additional information for discriminating locally non-dominated options.

The paper is organized as follows. In Section 2, the original ER approach is first introduced. Section 3 investigates the relevant theoretical issues raised in the existing interval belief ER models (Wang et al.,

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2006b). Section 4 investigates the interval ER approach for MADA with both interval belief degrees and interval preference information, which may be taken as either direct interval weight assignments or pairwise comparisons. Several pairs of non-linear optimization models are constructed to generate the upper and lower bounds of the combined belief degrees in the ER framework. In Section 4 the definition and computation of the maximum and minimum expected utilities for each alternative are studied for given interval assessment information and two dominance concepts are also defined to support the further elicitation of preference information, and post-aggregation analysis is discussed based on variance indices. Section 5 reports a numerical study to illustrate the implementation of the proposed approach.

2. The evidential reasoning approach for MADA

The evidential reasoning (ER) algorithm is developed for aggregating multiple attributes based on a belief decision matrix and the evidence combination rule of the Dempster–Shafer (D–S) theory (Dempster, 1967; Shafer, 1976; Yang and Singh, 1994; Yang and Sen, 1994; Yang, 2001; Yang et al., 2001; Yang and Xu, 2002a,b; Yang et al., 2006; Xu et al., in press). Different from traditional MADA approaches that describe a MADA problem using a decision matrix, the ER approach uses the belief decision matrix, in which each attribute of an alternative is described by a distribution assessment using a belief structure. The advantages of doing so is that using a distribution assessment can both model precise data and capture various types of uncertainties such as ignorance and vagueness in subjective judgments (Wang et al., 2006b).

Suppose a MADA problem has M alternatives a_l, $l = 1, \ldots, M$, one upper level attribute, referred to as general attribute, and L lower level attributes e_i, $i = 1, \ldots, L$, called basic attributes. The relative weights of the L basic attributes are denoted by $W = (w_1, \ldots, w_L)$, which are supposed to be known and satisfy the conditions $0 \leqslant w_i \leqslant 1$ and $\sum_{i=1}^{L} w_i = 1$.

Suppose M alternatives are all assessed using the same set of N assessment grades H_n, $n = 1, \ldots, N$, which are required to be mutually exclusive and collectively exhaustive for the assessment of all attributes. The N assessment grades formulate the frame of discernment $H = \{H_1, \ldots, H_N\}$ in the D–S theory of evidence. If alternative a_l is assessed to a grade H_n on an attribute e_i to a belief degree of $\beta_{n,i}$, this assessment will be denoted by $S(e_i(a_l)) = \{(H_n, \beta_{n,i}(a_l)), n = 1, \ldots, N\}$.

The individual assessments of the M alternatives on the L basic attributes can be represented by the following belief decision matrix:

$$D_g = (S(e_i(a_l)))_{L \times M}. \tag{1}$$

Based on the above belief decision matrix and the evidence combination rule of the D–S theory, both the recursive and analytical ER algorithms have been developed to aggregate the L basic attributes. The ER approach provides a non-linear attribute aggregation process in nature. In the rest of this section, we briefly introduce the analytical ER algorithm to pave the way for the later development of the new ER-PP optimization models.

The ER algorithm first transforms the original belief degrees into basic probability masses by combining the relative weights and the belief degrees using the following equations:

$$m_{n,i} = m_i(H_n) = w_i \beta_{n,i}(a_l), \quad n = 1, \ldots, N, \; i = 1, \ldots, L, \tag{2}$$

$$m_{H,i} = m_i(H) = 1 - \sum_{n=1}^{N} m_{n,i} = 1 - w_i \sum_{n=1}^{N} \beta_{n,i}(a_l), \quad i = 1, \ldots, L, \tag{3}$$

$$\bar{m}_{H,i} = \bar{m}_i(H) = 1 - w_i, \quad i = 1, \ldots, L, \tag{4}$$

$$\tilde{m}_{H,i} = \tilde{m}_i(H) = w_i \left(1 - \sum_{n=1}^{N} \beta_{n,i}(a_l)\right), \quad i = 1, \ldots, L \tag{5}$$

with $m_{H,i} = \bar{m}_{H,i} + \tilde{m}_{H,i}$ and $\sum_{i=1}^{L} w_i = 1$.

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Note that the probability mass assigned to the whole set H, $m_{H,i}$, which is currently unassigned to any individual grades, is split into two parts: $\bar{m}_{H,i}$ and $\tilde{m}_{H,i}$, where $\bar{m}_{H,i}$ is caused by the relative importance of the attribute e_i and $\tilde{m}_{H,i}$ by the incompleteness of the assessment on e_i for a_l.

Next, the basic probability masses on the L basic attributes are aggregated into the combined probability assignments by using the following analytical formulae:

$$\{H_n\}: \quad m_n = k\left[\prod_{i=1}^{L}(m_{n,i} + \bar{m}_{H,i} + \tilde{m}_{H,i}) - \prod_{i=1}^{L}(\bar{m}_{H,i} + \tilde{m}_{H,i})\right], \quad n = 1, \ldots, N, \tag{6}$$

$$\{H\}: \quad \tilde{m}_H = k\left[\prod_{i=1}^{L}(\bar{m}_{H,i} + \tilde{m}_{H,i}) - \prod_{i=1}^{L}\bar{m}_{H,i}\right], \tag{7}$$

$$\{H\}: \quad \bar{m}_H = k\left[\prod_{i=1}^{L}\bar{m}_{H,i}\right], \tag{8}$$

where

$$k = \left[\sum_{n=1}^{N}\prod_{i=1}^{L}(m_{n,i} + \bar{m}_{H,i} + \tilde{m}_{H,i}) - (N-1)\prod_{i=1}^{L}(\bar{m}_{H,i} + \tilde{m}_{H,i})\right]^{-1}. \tag{9}$$

Finally, the combined probability assignments are normalised into overall belief degrees by using the following equations:

$$\{H_n\}: \quad \beta_n = \frac{m_n}{1 - \bar{m}_H}, \quad n = 1, \ldots, N, \tag{10}$$

$$\{H\}: \quad \beta_H = \frac{\tilde{m}_H}{1 - \bar{m}_H}, \tag{11}$$

where β_n and β_H represent the overall belief degrees of the combined assessments, assigned to the assessment grades H_n and H, respectively. The combined assessment is also a distribution assessment, which can be denoted by $S(y(a_l)) = \{(H_n, \beta_n(a_l)), n = 1, \ldots, N\}$.

The above formulae (1)–(11) together constitute a complete ER analytical algorithm. Compared with the evidence combination rule of the D–S theory, the ER algorithm has at least the following features: (1) taking into account the relative importance of evidence; (2) modeling ignorance clearly by breaking down unassigned probability mass into two parts and treating them differently; (3) generating rational conclusions in the combination of multiple pieces of conflict evidence (Murphy, 2000) by employing the two normalisation operations on both the attribute weights and the combined probability assignments.

The rationale of the ER methodology is demonstrated by many applications, such as business performance assessment (Sonmez et al., 2001; Siow et al., 2001; Yang et al., 2001; Teng, 2002; Xu and Yang, 2003; Yang and Xu, 2004, 2005), safety and risk analysis and synthesis (Wang, 1997; Wang et al., 1995, 1996, 2006a,b; Wang and Yang, 2001; Okundi, 2001; Wang and Elhag, in press), product design and selection (Sen and Yang, 1995; Yang and Sen, 1997), environmental impact assessment (Wang et al., 2006a), etc. In many real applications, it is more appropriate if assessments are given in interval formats. If the assessments of a qualitative attribute such as handling are provided by a human expert, for example, he may be "to a large extent sure that the handling of a car is 'Good' with a belief degree of 60–80%". In other words, given his experiences of assessing other cars' handling, he is not certain about the "good" assessment of this car's handling but is sure that his belief in the assessment is neither above 80% nor below 60%, or indeed within the interval [60%, 80%]. On the other hand, the assessment of a quantitative attribute may not be precise either. For example, the "acceleration" of a car can be measured by the time required (in seconds) to accelerate from standstill to 60 mph. A series of tests may be made for the same car at different times, or for different cars of the same type. The test results may differ. If the discrepancies between these test results are larger than the tolerance error, the acceleration performance for this type of cars should not be expressed as a single value, and an interval of values may better represent their true acceleration performances, for example with a lower bound

of 8.2 second and an upper bound of 8.6 second, or an interval of [8.2 second, 8.6 second], though an average value may be used as a performance indicator for simplicity, such as the middle point 8.4 second of the interval. However, the average value does not reflect the variation of the car's true acceleration performances.

In order to model interval assessments, the basic ER methodology can be extended to model interval-valued distribution assessments (Wang et al., 2006b) in capturing ambiguity and diversity in individual or group assessments. If alternative a_l is assessed to a grade H_n on an attribute e_i to a belief degree interval of $[\beta_{n,i}^-, \beta_{n,i}^+]$, or $\beta_{n,i} \in [\beta_{n,i}^-, \beta_{n,i}^+]$ with $\beta_{n,i}^+ \geqslant \beta_{n,i}^- \geqslant 0$, and ignorance is also given in interval values $[\beta_{H,i}^-, \beta_{H,i}^+]$, or $\beta_{H,i} \in [\beta_{H,i}^-, \beta_{H,i}^+]$ with $1 \geqslant \beta_H^+ \geqslant \beta_H^- \geqslant 0$, we denote this by $S(e_i(a_l)) = \{(H_n, [\beta_{n,i}^-(a_l), \beta_{n,i}^+(a_l)]), n = 1, \ldots, N; (H, [\beta_{H,i}^-(a_l), \beta_{H,i}^+(a_l)])\}$, which is an interval-valued distribution assessment vector. Note that precise belief degree is a special case of interval belief degree with $\beta_{n,i}^- = \beta_{n,i}^+$ for every $n = 1, \ldots, N$ and $\beta_{H,i}^- = \beta_{H,i}^+$, $i = 1, \ldots, L$.

Numerical interval data can also be modeled using the belief structure (Yang, 2001; Wang et al., 2006b). Suppose $Y_{n,i}$ $(n = 1, \ldots, N)$ are the typical values used as grades for assessing the ith attribute. If a numerical value $y_i \in [y_i^-, y_i^+]$ for the ith attribute lies within $(k_i + 1)$ grades with $k_i \geqslant 1$, or $y_i^- \in (Y_{n_i,i}, Y_{n_i+1,i})$ and $y_i^+ \in (Y_{n_i+k_i-1,i}, Y_{n_i+k_i,i})$, then y_i may be assessed to any of $Y_{n,i}$ for $n = n_i, \ldots, n_i + k_i$ with a belief degree of $\beta_{n,\ i}$, where $\beta_{n,i}$ may take values in an interval, or $\beta_{n,i} \in [\beta_{n,i}^-, \beta_{n,i}^+]$. The lower bound $\beta_{n,i}^-$ and upper bound $\beta_{n,i}^+$ of the belief interval are determined as follows.

First, the following 0–1 integer variables are introduced (Wang et al., 2006b):

$$I_{j,i} = \begin{cases} 1, & \text{if } y_i \text{ lies between } Y_{n_i+j-1,i} \text{ and } Y_{n_i+j,i}, \\ 0, & \text{otherwise}, \end{cases} \qquad j = 1, 2, \ldots, k_i, \quad k_i \geqslant 1.$$

Only one of the above 0–1 integer variables can be non-zero. So, there is

$$\sum_{j=1}^{k_i} I_{j,i} = 1. \tag{12}$$

Due to the difficulty in processing integer variables, we reformat the constraints $\sum_{j=1}^{k_i} I_{j,i} = 1$ and $I_{j,i} = 0$ or 1 as the following equivalent equations, where $I_{j,i}$ are continuous variables:

$$\sum_{j=1}^{k_i} I_{j,i} = 1, \quad \sum_{j=1}^{k_i} \left[I_{j,i} \left(\sum_{k=j+1}^{k_i} I_{k,i} \right) \right] = 0 \quad \text{and} \quad I_{j,i} \geqslant 0. \tag{13}$$

Then the upper bound $\beta_{n_i+j-1,i}^+$ and lower bounds $\beta_{n_i+j-1,i}^-$ for the belief degrees $\beta_{n_i+j-1,i}$ for all $j = 1, 2, \ldots, k_i + 1$ can be obtained as follows (Wang et al., 2006b):

$$\beta_{n_i,i}^- = 0 \quad \text{and} \quad \beta_{n_i,i}^+ = I_{1,i} \cdot \frac{Y_{n_i+1,i} - y_i^-}{Y_{n_i+1,i} - Y_{n_i,i}} \tag{14}$$

$$\beta_{n_i+1,i}^- = 0 \quad \text{and} \quad \beta_{n_i+1,i}^+ = I_{1,i} + I_{2,i} \tag{15}$$

$$\ldots$$

$$\beta_{n_i+j,i}^- = 0 \quad \text{and} \quad \beta_{n_i+j,i}^+ = I_{j,i} + I_{j+1,i} \tag{16}$$

$$\ldots$$

$$\beta_{n_i+k_i-1,i}^- = 0 \quad \text{and} \quad \beta_{n_i+k_i-1,i}^+ = I_{k_i,i} + I_{k_i-1,i} \tag{17}$$

$$\beta_{n_i+k_i,i}^- = 0 \quad \text{and} \quad \beta_{n_i+k_i,i}^+ = I_{k_i,i} \cdot \frac{y_i^+ - Y_{n_i+k_i-1,i}}{Y_{n_i+k_i,i} - Y_{n_i+k_i-1,i}}. \tag{18}$$

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Using the interval belief structures, y_i can be equivalently expressed as follows:

$$y_i \in [y_i^-, y_i^+] \Longleftrightarrow \{(H_{n_i}, [\beta_{n_i,i}^-, \beta_{n_i,i}^+]); (H_{n_i+1}, [\beta_{n_i+1,i}^-, \beta_{n_i+1,i}^+]); \ldots; (H_{n_i+k_i}, [\beta_{n_i+k_i,i}^-, \beta_{n_i+k_i,i}^+]); \} \quad \text{with} \sum_{j=1}^{k_i+1} \beta_{n_i+j-1,i} = 1 \text{ and } \sum_{j=1}^{k_i+1} I_{j,i} = 1, \ \sum_{j=1}^{k_i+1} I_{j,i} \cdot \left(\sum_{k=j+1}^{k_i+1} I_{k,i} \right) = 0, \tag{19}$$

where $\beta_{n_i,i} \in [\beta_{n_i,i}^-, \beta_{n_i,i}^+], \beta_{n_i+1,i} \in [\beta_{n_i+1,i}^-, \beta_{n_i+1,i}^+], \ldots, \beta_{n_i+k_i,i} \in [\beta_{n_i+k_i,i}^-, \beta_{n_i+k_i,i}^+]$.

According to Wang et al. (2006b), if the original belief decision matrix $D_g = (S(e_i(a_l)))_{L\times M}$ contains interval belief degrees, the following ER non-linear optimization models can be used to aggregate multiple interval belief structures:

$$\text{Max/Min} \quad \beta_n(a_l) \quad (\text{for each } n = 1, \ldots, N) \text{ and } \beta_H(a_l) \tag{20}$$

$$\text{s.t.} \quad \beta_{n,i}^- \leqslant \beta_{n,i} \leqslant \beta_{n,i}^+, \quad n = 1, \ldots, N; \ i = 1, \ldots, L, \tag{21}$$

$$\beta_{H,i}^- \leqslant \beta_{H,i} \leqslant \beta_{H,i}^+, \quad i = 1, \ldots, L, \tag{22}$$

$$\sum_{n=1}^{N} \beta_{n,i} + \beta_{H,i} = 1, \quad i = 1, \ldots, L, \tag{23}$$

where $\beta_n(a_l)$ (for $n = 1, \ldots, N$) and $\beta_H(a_l)$ are the functions of $\beta_{n,i}$ and $\beta_{H,i}$ for $n = 1, \ldots, N$; $i = 1, \ldots, L$ generated using the analytical ER algorithm as follows:

$$\beta_n(a_l) = \frac{\prod_{i=1}^{L}(w_i\beta_{n,i} + 1 - w_i + w_i\beta_{H,i}) - \prod_{i=1}^{L}(1 - w_i + w_i\beta_{H,i})}{\sum_{q=1}^{N}\prod_{i=1}^{L}(w_i\beta_{q,i} + 1 - w_i + w_i\beta_{H,i}) - (N-1)\prod_{i=1}^{L}(1 - w_i + w_i\beta_{H,i}) - \prod_{i=1}^{L}(1 - w_i)}, \tag{24}$$

$$\beta_H(a_l) = \frac{\prod_{i=1}^{L}(1 - w_i + w_i\beta_{H,i}) - \prod_{i=1}^{L}(1 - w_i)}{\sum_{q=1}^{N}\prod_{i=1}^{L}(w_i\beta_{q,i} + 1 - w_i + w_i\beta_{H,i}) - (N-1)\prod_{i=1}^{L}(1 - w_i + w_i\beta_{H,i}) - \prod_{i=1}^{L}(1 - w_i)}. \tag{25}$$

3. ER-based preference programming models using both interval weights and beliefs

3.1. ER-PP models using direct interval weight assignments

If a weight for an attribute is given not as a precise value but as an interval of values, it can be represented using an interval, for example $[w_i^-, w_i^+]$ for the ith attribute where w_i^- and w_i^+ are the lower and upper bounds of w_i with $w_i^- \leqslant w_i \leqslant w_i^+$ and $w_i^-, w_i^+ \geqslant 0$. Such interval weights could be assigned by an individual decision maker or in group decision situations where group members may have different views about the importance of attributes. The consequence of having interval weights is that overall assessment can no longer be certain. In the ER framework, this would lead to intervals of overall combined belief degrees. Therefore there is a need to generate the upper and lower bounds of such an interval for each combined belief degree.

Suppose the first q attributes are assessed using quantitative intervals, i.e., $y_i^- \leqslant y_i \leqslant y_i^+$ for $i \leqslant q$. Given direct interval weight assignments, the upper and lower bounds for the combined interval belief degree β_n and β_H can be generated by solving the following pairs of basic ER algorithm-based preference programming (ER-PP) models.

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$$\begin{aligned}
&\text{Min/Max} \quad \beta_n \text{ and } \beta_H \text{ for } n = 1, \ldots, N \\
&\text{s.t.} \quad w_i^- \leqslant w_i \leqslant w_i^+, \quad i = 1, \ldots, L, && (26)\\
&1 - \sum_{i=1}^{L} w_i = 0, && (27)\\
&\beta_{n,i}^- \leqslant \beta_{n,i} \leqslant \beta_{n,i}^+, \quad n = 1, \ldots, N;\ i = 1, \ldots, L, && (28)\\
&\beta_{H,i}^- \leqslant \beta_{H,i} \leqslant \beta_{H,i}^+, \quad i = 1, \ldots, L, && (29)\\
&\sum_{n=1}^{N} \beta_{n,i} + \beta_{H,i} = 1, \quad i = 1, \ldots, L, && (30)
\end{aligned}$$

where for each numerical interval assessment $y_i \in [y_i^-, y_i^+]$, if $[y_i^-, y_i^+]$ lies within $(k_i + 1)$ grades with $k_i \geqslant 1$, $y_i^- \in (Y_{n_i,i}, Y_{n_i+1,i})$ and $y_i^+ \in (Y_{n_i+k_i-1,i}, Y_{n_i+k_i,i})$, then

$$\beta_{n_i,i}^- = 0 \quad \text{and} \quad \beta_{n_i,i}^+ = I_{1,i} \cdot \frac{Y_{n_i+1,i} - y_i^-}{Y_{n_i+1,i} - Y_{n_i,i}}, \qquad i = 1, \ldots, q, \tag{31}$$

$$\beta_{n_i+j,i}^- = 0 \quad \text{and} \quad \beta_{n_i+j,i}^+ = I_{j,i} + I_{j+1,i}, \qquad j = 1, 2, \ldots, k_i - 1,\ i = 1, \ldots, q, \tag{32}$$

$$\beta_{n_i+k_i,i}^- = 0 \quad \text{and} \quad \beta_{n_i+k_i,i}^+ = I_{k_i,i} \cdot \frac{y_i^+ - Y_{n_i+k_i-1,i}}{Y_{n_i+k_i,i} - Y_{n_i+k_i-1,i}}, \qquad i = 1, \ldots, q, \tag{33}$$

$$\beta_{n,i}^- = \beta_{n,i}^+ = 0,\ n < n_i, \quad \text{or} \quad n > n_i + k_i,\ n = 1, 2, \ldots, N,\ i = 1, \ldots, q, \tag{34}$$

$$\sum_{j=1}^{k_i} I_{j,i} = 1, \quad i = 1, \ldots, q, \tag{35}$$

$$\sum_{j=1}^{k_i} \left[I_{j,i} \cdot \left(\sum_{k=j+1}^{k_i} I_{k,i} \right) \right] = 0, \quad i = 1, \ldots, q, \tag{36}$$

$$I_{j,i} \geqslant 0, \quad i = 1, \ldots, q. \tag{37}$$

If β_n^- and β_n^+ are the optimal solutions of the nth min and max problems for β_n, and β_H^- and β_H^+ the optimal solutions of the min and max problems for β_H, then we have $\beta_n(a_l) \in [\beta_n^-, \beta_n^+]$ and $\beta_H(a_l) \in [\beta_H^-, \beta_H^+]$. The combined assessment is an interval distribution assessment given by

$$\begin{aligned}
&S(a_l) = \{(H_n, \beta_n(a_l)), n = 1, \ldots, N;\ (H, \beta_H(a_l))\} \\
&\quad \text{with } \sum_{n=1}^{N} \beta_n(a_l) + \beta_H(a_l) = 1.
\end{aligned}$$

Note that β_H is zero if $\tilde{m}_{H,i} = 0$ for all $i = 1, \ldots, L$. This means that the combined assessment for the general attribute is complete if and only if all the original assessments for the basic attributes are complete.

3.2. ER-PP models for consistent interval pairwise comparisons

Alternatively, the relative weight assessments of attributes may be given in pairwise comparisons with interval ratios. Let $V = \{(v_{ij}^-, v_{ij}^+)\}$ for all $i,j = 1, 2, \ldots, L$ denote a consistent pairwise interval comparison matrix for L attributes, where v_{ij}^- and v_{ij}^+ denote the lower and upper bounds of the weight ratios for attribute i compared to attribute j. Each entry in the matrix V is non-negative (or $v_{ij}^-, v_{ij}^+ \geqslant 0$) with $v_{ij}^- \leqslant v_{ij}^+$, and the reciprocal condition (or $v_{ij}^- = 1/v_{ji}^+$, $v_{ij}^+ = 1/v_{ji}^-$ for all $i,j = 1, 2, \ldots, L$) is also satisfied.

Let Ω_V be the feasible set of weight vectors of the matrix V, or $\Omega_V = \{w = (w_1, w_2, \ldots w_L) | w_i \geqslant 0, \sum_{i=1}^{L} w_i = 1,$ and $v_{ij}^- w_j \leqslant w_i \leqslant v_{ij}^+ w_j\}$. If V is inconsistent, Ω_V will be empty and V should be revised according to the decision maker's preferences or transformed to a consistent one (Salo, 1993).

If weight evaluation information is given as in V, then the combined belief degree β_n for an assessment grade H_n will be an interval, which can be generated by solving the following optimization models.

$$\begin{aligned}
&\text{Min/Max} \quad \beta_n \text{ and } \beta_H \quad \text{for } n = 1, \ldots, N \\
&\text{s.t.} \quad 1 - \sum_{i=1}^{L} w_i = 0, && (38)\\
&\quad w_i - v_{ij}^{-} w_j \geqslant 0, \quad i > j,\ j = 1, \ldots, L,\ i = 1, \ldots, L, && (39)\\
&\quad w_i - v_{ij}^{+} w_j \leqslant 0, \quad i > j,\ j = 1, \ldots, L,\ i = 1, \ldots, L, && (40)\\
&\quad w_i \geqslant 0, \quad n = 1, \ldots, N,\ i = 1, \ldots, L, && (41)\\
&\quad \text{and}(28)\text{–}(37).
\end{aligned}$$

Similarly, if β_n^- and β_n^+ are the optimal solutions of the nth min and max problems for β_n and β_H^- and β_H^+ the optimal solutions of the min and max problems for β_H, then we have $\beta_n(a_l) \in [\beta_n^-, \beta_n^+]$ and $\beta_H(a_l) \in [\beta_H^-, \beta_H^+]$. The combined assessment is an interval distribution assessment given by

$$S(a_l) = \{(H_n, \beta_n(a_l)), n = 1, \ldots, N;\ (H, \beta_H(a_l))\}$$
$$\text{with } \sum_{n=1}^{N} \beta_n(a_l) + \beta_H(a_l) = 1.$$

4. ER non-linear optimization models for computing expected utilities

Different from other MADA approaches, the ER approach provides a combined interval distribution assessment for each alternative. In order to rank alternatives based on the combined distribution assessments, the ER approach employs the concept of expected utility, which is defined as follows (Yang, 2001):

$$u(S(y(a_l))) = \sum_{n=1}^{N} u(H_n)\beta_n(a_l), \quad l = 1, \ldots, M, \tag{42}$$

where $u(S(y(a_l)))$ or $u(a_l)$ in short is the expected utility of the combined distribution assessment $S(y(a_l))$, $u(H_n)$ the utility of the grade H_n and $\beta_n(a_l)$ the combined belief degree to which a_l is assessed to H_n. For simplicity, $u(S(y(a_l)))$ is referred to as the expected utility of a_l. Without loss of generality, suppose H_{n+1} is preferred to H_n for $n = 1, \ldots, N-1$.

In Eq. (42), it is assumed that $\sum_{n=1}^{N} \beta_n(a_l) = 1$, so $\beta_n(a_l)$ is used as probability. However, if there is ignorance or incompleteness, we have $\sum_{n=1}^{N} \beta_n(a_l) < 1$, or $\beta_H(a_l) \neq 0$ which may be assigned to any assessment grade in H. In this case, belief degrees should not be directly used as probabilities. Note that in the ER assessment framework defined in this paper the belief function $B(H_n)$ and the plausibility function $P(H_n)$ of an assessment grade H_n in the frame of discernment $H = \{H_1, \ldots, H_N\}$ are given by $B(H_n) = \beta_n$ and $P(H_n) = \beta_n + \beta_H$, respectively. $B(H_n)$ and $P(H_n)$ define the lower and upper bounds of the probability p_n to which a_l is assessed to H_n. A probability function over the frame of discernment $H = \{H_1, \ldots, H_N\}$ is given as follows:

$$B(H_n) \leqslant p_n \leqslant P(H_n) \quad \text{for } n = 1, \cdots, N \quad \text{and} \quad \sum_{n=1}^{N} p_n = 1.$$

The minimum and maximum utilities of an option a_l can then be found by solving the following optimisation problems:

$$\begin{aligned}
&\text{Min/Max} \quad u(a_l) = \sum_{n=1}^{N} p_n u(H_n)\\
&\text{s.t.} \quad \beta_n(a_l) \leqslant p_n \leqslant \beta_n(a_l) + \beta_H(a_l) \quad \text{for } n = 1, \ldots, N,\\
&\quad \sum_{n=1}^{N} p_n = 1.
\end{aligned}$$

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If H_{n+1} is preferred to H_n for $n = 1, \ldots, N-1$, then the above optimisation problem could be solved straightforward. In fact, it is easy to prove that when $\beta_H(a_l)$ is assigned to the most preferred assessment grade H_N, $u(a_l)$ achieves its maximum, given by

$$u_{\max}(a_l) = \sum_{n=1}^{N-1} \beta_n(a_l)u(H_n) + (\beta_N(a_l) + \beta_H(a_l))u(H_N), \quad l = 1, \ldots, M. \tag{43}$$

If $\beta_H(a_l)$ is assigned to the least preferred assessment grade H_1, then $u(S(y(a_l)))$ achieves its minimum given by

$$u_{\min}(a_l) = (\beta_1(a_l) + \beta_H(a_l))u(H_1) + \sum_{n=2}^{N} \beta_n(a_l)u(H_n), \quad l = 1, \ldots, M. \tag{44}$$

So, $u(a_l)$ is in fact an interval if $\beta_H(a_l) \neq 0$. The midpoint of the interval is defined as the average utility by

$$u_{\text{avg}}(a_l) = \frac{u_{\max}(a_l) + u_{\min}(a_l)}{2}, \quad l = 1, \ldots, M. \tag{45}$$

4.1. Utility estimation models for direct interval weight assignments

If weights and/or assessments are given as intervals, the above defined utility measures are no longer unique. In fact, they become intervals as well. In such cases, we are interested in finding the overall maximum and minimum utilities of each alternative rather than the upper and lower bounds of either $u_{\max}(a_l)$ or $u_{\min}(a_l)$. The overall maximum and minimum utilities for an alternative a_l can be determined by solving the following pair of non-linear optimization problems:

$$\begin{aligned} \text{Max} \quad & u_{\max}(a_l) = \sum_{n=1}^{N-1} u(H_n)\beta_n(a_l) + u(H_N)(\beta_N(a_l) + \beta_H(a_l)) \\ \text{s.t.} \quad & \text{Eqs. (26)–(37)} \end{aligned}$$

and

$$\begin{aligned} \text{Min} \quad & u_{\min}(a_l) = u(H_1)(\beta_1(a_l) + \beta_H(a_l)) + \sum_{n=2}^{N} u(H_n)\beta_n(a_l) \\ \text{s.t.} \quad & \text{Eqs. (26)–(37).} \end{aligned}$$

4.2. Utility estimation model for interval pairwise comparisons

If weight evaluation information is given in terms of consistent interval pairwise comparison matrix V, the expected utility measures of the combined distribution assessments will be intervals as well. Again, we are only interested in generating the overall maximum and minimum utilities, which are denoted as $U_{\max}(a_l)$ and $U_{\min}(a_l)$ and can be determined by solving the following pair of non-linear optimization models for $l = 1$ to M, respectively:

$$\begin{aligned} U_{\max}(a_l) = \text{Max} \quad & u_{\max}(a_l) = \sum_{n=1}^{N-1} u(H_n)\beta_n(a_l) + u(H_N)(\beta_N(a_l) + \beta_H(a_l)) \\ \text{s.t.} \quad & \text{Eqs. (28)–(41)} \end{aligned}$$

and

$$\begin{aligned} U_{\min}(a_l) = \text{Min} \quad & u_{\min}(a_l) = u(H_1)(\beta_1(a_l) + \beta_H(a_l)) + \sum_{n=2}^{N} u(H_n)\beta_n(a_l) \\ \text{s.t.} \quad & \text{Eqs. (28)–(41).} \end{aligned}$$

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And similarly we can also define the average overall utility:

$$U_{\text{avg}}(a_l) = \frac{U_{\max}(a_l) + U_{\min}(a_l)}{2}.$$

4.3. Dominance structures based on expected utilities

The expected utility approach in the ER models can be used to generate preference orders for all alternatives based on two dominance concepts: absolute dominance and pairwise dominance, which Salo and Hämäläinen (1992b) proposed in the interval weighted AHP process.

The absolute dominance is based on value intervals. That is, alternative a_1 is preferred to a_2 in the sense of absolute dominance if and only if the smallest utility of a_1 exceeds the largest utility of a_2, i.e.,

$$a_1 \succ_A a_2 \iff U_{\min}(a_1) > U_{\max}(a_2),$$

where $U_{\min}(a_1)$ and $U_{\max}(a_2)$ are the minimum and maximum expected utilities of a_1 and a_2 respectively, which are calculated as shown in Section 4.1 or Section 4.2 (Yang, 2001).

The set of dominated alternatives can also be determined by means of pairwise dominance. In pairwise dominance, a_1 is preferred to a_2 if and only if the utility of a_1 exceeds that of a_2 for each given set of weights in the entire feasible region, i.e.,

$$a_1 \succ_P a_2 \iff \text{Min}_\Omega[u_{\min}(a_1) - u_{\max}(a_2)] \geqslant 0,$$

where $\Omega = \{$the feasible region defined by Eqs. (26)–(37)$\}$ for direct interval weight assignments or $\Omega = \{$the feasible region defined by Eqs. (26)–(41)$\}$ for interval weight pairwise comparisons. Note that even though the utility interval of a_2 may overlap the utility interval of a_1, a_2 may still be pairwisely dominated by a_1 if $\text{Min}_\Omega[u_{\min}(a_1) - u_{\max}(a_2)] \geqslant 0$. In other words, pairwise dominance is less strict than absolute dominance.

4.4. Post-aggregation analysis on interval weights and beliefs

As discussed above, if there is overlap between the utility intervals of two alternatives, the precise ranking order between them may not be found, or they may not dominate each other. In such cases, it is desirable to develop supporting processes and methods for guiding post-aggregation analysis in an informed way. In this subsection, such guidance will be developed for the necessary elicitation of extra information in order to differentiate alternative decisions without wasting the DM's effort.

(1) Re-evaluation of interval weights

If weights are given as direct intervals, it should make sense to reevaluate a wide interval (thus highly uncertain) weight assigned to an important attribute first. It is therefore expected that such reevaluation would lead to significant reduction of the width of an overall utility interval. Thus, both weight value and interval width should be used to identify the most significant attribute for weight revision, for example in the following way.

$$\begin{aligned} i^* &= \arg\max_i \left\{ \left(\frac{w_i^+ + w_i^-}{2} \right)(w_i^+ - w_i^-) \middle| \cdot i = 1, 2, \ldots, L \right\} \\ &= \arg\max_i \left\{ \frac{(w_i^+)^2 - (w_i^-)^2}{2} \middle| \cdot i = 1, 2, \ldots, L \right\}. \end{aligned} \tag{46}$$

If the weight evaluations are given in a pairwise comparison matrix V as shown in Section 3.2, the following simple procedure may be used to identify which pairwise comparison should be reevaluated first.

Firstly we construct two new matrixes $VL = \{l_{ij}, i = 1, \ldots L, j = 1, \ldots, L\}$ and $VH = \{u_{ij}, i = 1, \ldots L, j = 1, \ldots, L\}$ corresponding to matrix V, and their items are defined as follows:

$$l_{ij} = \begin{cases} -1/v_{ij}^-, & \text{if } v_{ij}^- < 1, \\ v_{ij}^-, & \text{else.} \end{cases}$$

$$u_{ij} = \begin{cases} -1/v_{ij}^+, & \text{if } v_{ij}^+ < 1, \\ v_{ij}^+, & \text{else.} \end{cases}$$

Find the indices i^* and j^* of the attributes, with the lower and upper bounds having the biggest variances to the desired weight vector $W^* = (w_1^*, w_2^*, \ldots, w_L^*)$, or

$$(i^*, j^*) = \arg\max_{i,j}\{u_{ij} - l_{ij} | i = 1, 2, \ldots, L, j = i + 1, \ldots, L\}. \tag{47}$$

(2) Re-evaluation of interval beliefs

The non-dominance of alternatives may also result from interval beliefs. A wide belief interval may lead to large overlap of the expected utilities of non-dominated alternatives. So the belief intervals associated with every alternative should be measured and compared. We propose to use the variance of the expected utilities to measure the level of uncertainty as follows.

For an alternative a_l, if the minimum, maximum and average expected utilities are denoted by $U_{\min}(a_l)$, $U_{\max}(a_l)$ and $U_{\text{avg}}(a_l)$, respectively, the variance of the expected utilities of the alternative a_l is defined as follows:

$$\sigma(a_l) = \sqrt{(U_{\text{avg}}(a_l) - U_{\min}(a_l))^2 + (U_{\max}(a_l) - U_{\text{avg}}(a_l))^2} = \frac{U_{\max}(a_l) - U_{\min}(a_l)}{\sqrt{2}}. \tag{48}$$

If the l^*th alternative has the highest variance, or $l^* = \text{argmax}_l\{\sigma(a_l) | l = 1, 2, \ldots, M\}$, then it will be suggested for re-evaluation.

Furthermore, if we define the minimum, maximum and average expected utilities of the alternative a_l on every attribute $i = 1, 2, \ldots, L$ as follows:

$$U_{i,\min}(a_l) = \min \left\{ \sum_{n=1}^{N} \beta_{n,i} u(H_n) + \beta_{H,i} u(H_1) \right\}, \tag{49}$$

$$\text{s.t.} \quad \text{Eqs. (28)–(37).}$$

$$U_{i,\max}(a_l) = \max \left\{ \sum_{n=1}^{N} \beta_{n,i} u(H_n) + \beta_{H,i} u(H_N) \right\}, \tag{50}$$

$$\text{s.t.} \quad \text{Eqs. (28)–(37)}$$

and

$$U_{i,\text{avg}}(a_l) = \frac{u_{i,\min}(a_l) + u_{i,\max}(a_l)}{2}. \tag{51}$$

The variance of every attribute $i = 1, 2, \ldots, L$ for a_l can be defined as follows:

$$\sigma_i(a_l) = \sqrt{(U_{i,\max}(a_l) - U_{i,\text{avg}}(a_l))^2 + (U_{i,\text{avg}}(a_l) - U_{i,\min}(a_l))^2} = \frac{U_{i,\max}(a_l) - U_{i,\min}(a_l)}{\sqrt{2}}. \tag{52}$$

Then the i^*th attribute for the alternative a_l will be suggested for re-evaluation, which has the highest weighed variance, or $i^* = \arg\max_i\{w_i^* \cdot \sigma_i(a_l) | i = 1, 2, \ldots, L\}$, where $W^* = (w_1^*, w_2^*, \ldots w_L^*)$ is the most desirable weight in the weight feasible region that can be calculated by the models of Chandran et al. (2005) (the detailed modelling procedures are shown in Appendix).

5. A numerical study

In this section, a sport car evaluation problem (Yang, 2001) is revisited and examined to demonstrate the application of the extended ER approach in modelling MADA problems using interval weight judgments. The

proposed interval weight evidential reasoning approach is applied to analyze the performance of six types of cars: Car 1, Car 2, ..., Car 6 based on seven performance attributes including four quantitative attributes: acceleration (seconds, from 0 to 60 mph), Braking (ft, from 0 to 60 mph), Horsepower (hp) and Fuel economy (mpg), and three qualitative attributes, handling, ride quality, and powertrain.

The consistent interval pairwise weight comparison matrix V for the seven attributes is given in Table 1:

The following general grade set is defined to evaluate the cars:

$$\begin{aligned} H &= \{H_n, n = 1, \ldots, 6\} \\ &= \{\text{Worst}, \text{Poor}, \text{Average}, \text{Good}, \text{Excellent}, \text{Top}\} \\ &= \{\text{W}, \text{P}, \text{A}, \text{G}, \text{E}, \text{T}\}. \end{aligned}$$

The performances of acceleration, which is a minimization attribute, are given in quantitative intervals (unit: second from standstill to 60 mph) by

Car 1: [8.5, 8.9]; Car 2: 8.0; Car 3: 7.7; Car 4: 8.45; Car 5: [8.0, 8.8]; Car 6: 7.9.

The referential values used as assessment grades for assessing the attribute "acceleration" are defined by

$$H^{\text{acceleration}} = \{Y_{1,1}, Y_{2,1}, Y_{3,1}, Y_{4,1}, Y_{5,1}, Y_{6,1}\} = \{10, 9.2, 8.7, 8.2, 7.8, 7.4\}.$$

For Car 5, the acceleration time [8.0, 8.8] is within the four grades $\{Y_{2,1}, Y_{3,1}, Y_{4,1}, Y_{5,1}\}$, so

$$\begin{aligned} &\beta^-_{2,1} = 0 \quad \text{and} \quad \beta^+_{2,1} = I_{1,1} \cdot \frac{8.8 - 8.7}{9.2 - 8.7} = 0.2 I_{1,1}, \\ &\beta^-_{3,1} = 0 \quad \text{and} \quad \beta^+_{3,1} = I_{1,1} + I_{2,1}, \\ &\beta^-_{4,1} = 0 \quad \text{and} \quad \beta^+_{4,1} = I_{2,1} + I_{3,1}, \\ &\beta^-_{5,1} = 0 \quad \text{and} \quad \beta^+_{5,1} = I_{3,1} \cdot \frac{8.0 - 7.8}{8.2 - 7.8} = 0.5 I_{3,1}, \\ &\text{where } I_{1,1} + I_{2,1} + I_{3,1} = 1, \quad I_{1,1}(I_{2,1} + I_{3,1}) + I_{2,1} \cdot I_{3,1} = 0, \quad \text{and} \quad I_{1,1}, I_{2,,1}, I_{3,1} \geqslant 0. \end{aligned}$$

So $\beta_{2,1} \in [\beta^-_{2,1}, \beta^+_{2,1}]$, $\beta_{3,1} \in [\beta^-_{3,1}, \beta^+_{3,1}]$, $\beta_{4,1} \in [\beta^-_{4,1}, \beta^+_{4,1}]$, $\beta_{5,1} \in [\beta^-_{5,1}, \beta^+_{5,1}]$ with the extra constraint $\sum_{n=1}^{6} \beta_{n,1} = 1$, or equivalently $\beta_{H,1} = 0$.

The other quantitative assessments and the typical values used as assessment grades for each attribute are given as follows:

Braking: Car 1: 128; Car 2: 124; Car 3: 127; Car 4: 134; Car 5: 135; Car 6: 124.2, and $H^{\text{Braking}} = \{Y_{1,2}, Y_{2,2}, Y_{3,2}, Y_{4,2}, Y_{5,2}, Y_{6,2}\} = \{140, 135, 131, 128, 126, 123\}$;

Horsepower: Car 1: 196; Car 2: 152; Car 3: 182; Car 4: 183; Car 5: 138; Car 6: 171, and $H^{\text{Horsepower}} = \{Y_{1,4}, Y_{2,4}, Y_{3,4}, Y_{4,4}, Y_{5,4}, Y_{6,4}\} = \{130, 145, 160, 175, 188, 200\}$;

Fuel economy: Car 1: 20; Car 2: 20; Car 3: 21; Car 4: 20; Car 5: 19; Car 6: 20, with $H^{\text{Fuel economy}} = \{Y_{1,7}, Y_{2,7}, Y_{3,7}, Y_{4,7}, Y_{5,7}, Y_{6,7}\} = \{17, 18, 19, 20, 21, 22\}$.

Table 1
The interval pairwise weight comparison matrix V

Attribute	Attribute						
	Acceleration	Braking	Handling	Horsepower	Ride quality	Powertrain	Fuel economy
Acceleration	1	[3, 8]	1	[4, 6]	[1, 3]	[4, 9]	[5, 8]
Braking	[0.125, 0.3333]	1	[0.125, 0.3333]	[0.5, 2]	[0.25, 0.5]	[0.5, 2]	[0.5, 2]
Handling	1	[3, 8]	1	[3, 6]	[1, 2]	[4, 9]	[5, 8]
Horsepower	[0.1667, 0.25]	[0.5, 2]	[0.1667, 0.3333]	1	[0.25, 0.5]	[1, 2]	[1, 2]
Ride quality	[0.3333, 1]	[2, 4]	[0.5, 1]	[2, 4]	1	[2, 8]	[3, 4]
Powertrain	[0.1111, 0.25]	[0.5, 2]	[0.1111, 0.25]	[0.5, 1]	[0.125, 0.5]	1	[0.5, 2]
Fuel economy	[0.125, 0.2]	[0.5, 2]	[0.125, 0.2]	[0.5, 1]	[0.25, 0.3333]	[0.5, 2]	1

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Table 2
Assessment data for six cars

Attributes	Car 1	Car 2	Car 3	Car 4	Car 5	Car 6
Acceleration (e_1)	{(P, [0, 0.4 · $I_{1,1}$]), (A, [0, $I_{1,1}$ + $I_{2,1}$]), (G, [0, 0.4 · $I_{2,1}$]), (H, 0)} where $I_{1,1} + I_{2,1} = 1$, $I_{1,1} \cdot I_{2,1} = 0$	{(G, 0.5), (E, 0.5)}	{(E, 0.75), (T, 0.25)}	{(A, 0.5), (G, 0.5)}	{(P, [0, 0.2 · $I_{1,1}$]), (A, [0, $I_{1,1} + I_{2,1}$]), (G, [0, $I_{2,1} + I_{3,1}$]), (E, [0.5 · $I_{3,1}$]), (H, 0)} where $I_{1,1} + I_{2,1} + I_{3,1} = 1$, $I_{1,1} \cdot I_{2,1} + I_{2,1} \cdot I_{3,1} + I_{3,1} \cdot I_{1,1} = 0$	{(G, 0.25), (E, 0.75)}
Braking (e_2)	{(G, 1.0)}	{(E, 0.3333), (T, 0.6667)}	{(G, 0.5) (E, 0.5)}	{(P, 0.75) (A, 0.25)}	{(P, 1.0)}	{(E, 0.6), (T, 0.4)}
Handling (e_3)	{(E, [0.3, 0.4]), (T, [0.4, 0.6]), (H, [0, 0.1])}	{(A, [0.4, 0.5]), (G, [0.5, 0.6]), (H, [0, 0.1])}	{(G, [0.3, 0.6]), (E, [0.3, 0.7]), (H, [0, 0.2])}	{(E, [0.2, 0.3]), (T, [0.5, 0.6]), (H, [0, 0.2])}	{(A, [0.3, 0.4]), (G, [0.5, 0.6]), (H, [0, 0.2])}	{(P, [0.5, 0.6]), (A, [0.4, 0.5]), (H, [0, 0.1])}
Horsepower (e_4)	{(E, 0.3333), (T, 0.6667)}	{(P, 0.5333), (A, 0.4667)}	{(G, 0.4615), (E, 0.5385)}	{(G, 0.3846), (E, 0.6154)}	{(W, 0.4667), (P, 0.5333)}	{(A, 0.2667), (G, 0.7333)}
Ride quality (e_5)	{(E, [0.1, 0.2]), (T, [0.8, 0.9]), (H, [0, 0.1])}	{(G, [0.2, 0.4]), (E, [0.6, 0.7]), (H, [0, 0.2])}	{(G, [0.4, 0.5]), (E, [0.5, 0.6]), (H, [0, 0.1])}	{(E, [0.1, 0.2]), (T, [0.6, 0.9]), (H, [0, 0.2])}	{(A, [0.5, 0.6]), (G, [0.4, 0.5]), (H, [0, 0.1])}	{(P, [0.6, 0.7]), (A, [0.3, 0.4]), (H, [0, 0.1])}
Powertrain (e_6)	{(A, 0.4), (G, 0.6)}	{(G, 1.0)}	{(G, 0.7), (E, 0.3)}	{(A, 0.4)}, (G, 0.6)}	{(G, 0.6), (E, 0.4)}	{(E, 0.6), (T, 0.4)}
Fuel economy (e_7)	{(G, 1.0)}	{(G, 1.0)}	{(E, 1.0)}	{(G, 1.0)}	{(A, 1.0)}	{(G, 1.0)}

Table 3
The combined interval belief degrees for each car

Grade	W	P	A	G	E	T
Car 1	0	[0, 0.1401]	[0.1757, 0.3671]	[0.0763, 0.1610]	[0.1162, 0.1874]	[0.32, 0.45]
Car 2	0	[0.0178, 0.0314]	[0.1184, 0.1899]	[0.4391, 0.5728]	[0.239, 0.3244]	[0.0171, 0.0495]
Car 3	0	0	0	[0.211, 0.3267]	[0.5763, 0.686]	[0.0924, 0.1184]
Car 4	0	[0.0208, 0.0587]	[0.1344, 0.17]	[0.2586, 0.3193]	[0.0697, 0.122]	[0.333, 0.4642]
Car 5	[0.0152, 0.0284]	[0.0447, 0.1077]	[0.1755, 0.3042]	[0.4177, 0.7090]	[0.0083, 0.1860]	0
Car 6	0	[0.2373, 0.3523]	[0.18, 0.2587]	[0.1423, 0.182]	[0.2628, 0.3288]	[0.0221, 0.0529]

Similarly, These quantitative assecements are transformed (Yang, 2001) and presented using the belief structures as shown in Table 2. The assessment information on qualitative attributes including handling, ride quality, and powertrain is provided by the decision maker directly and also shown in Table 2.

Using the conventional constrained non-linear optimization algorithms such as the augmented Lagrangian method and the feasible direction method (Miller, 2000), or any non-linear optimization tools such as LINGO and Matlab optimization toolbox, we can construct and solve the Min/Max models proposed in Section 3.2. The upper and lower bounds of the combined belief degrees for each assessment grade are generated as in Table 3 and plotted in Fig. 1.

5.1. Dominance calculations based on expected utilities

Fig. 1 provides the decision maker with a panoramic view of the performances of the cars. The performance diversity for each car is clearly illustrated in the figures. However, it is not easy to directly compare the combined interval distribution assessments of the six cars for ranking purpose. In order to rank the cars, it is useful to compute their expected utilities. The ER utility calculation equations investigated in Section 4.2 can be used to derive the expected utility intervals, which are shown in Table 4 and plotted in Fig. 2.

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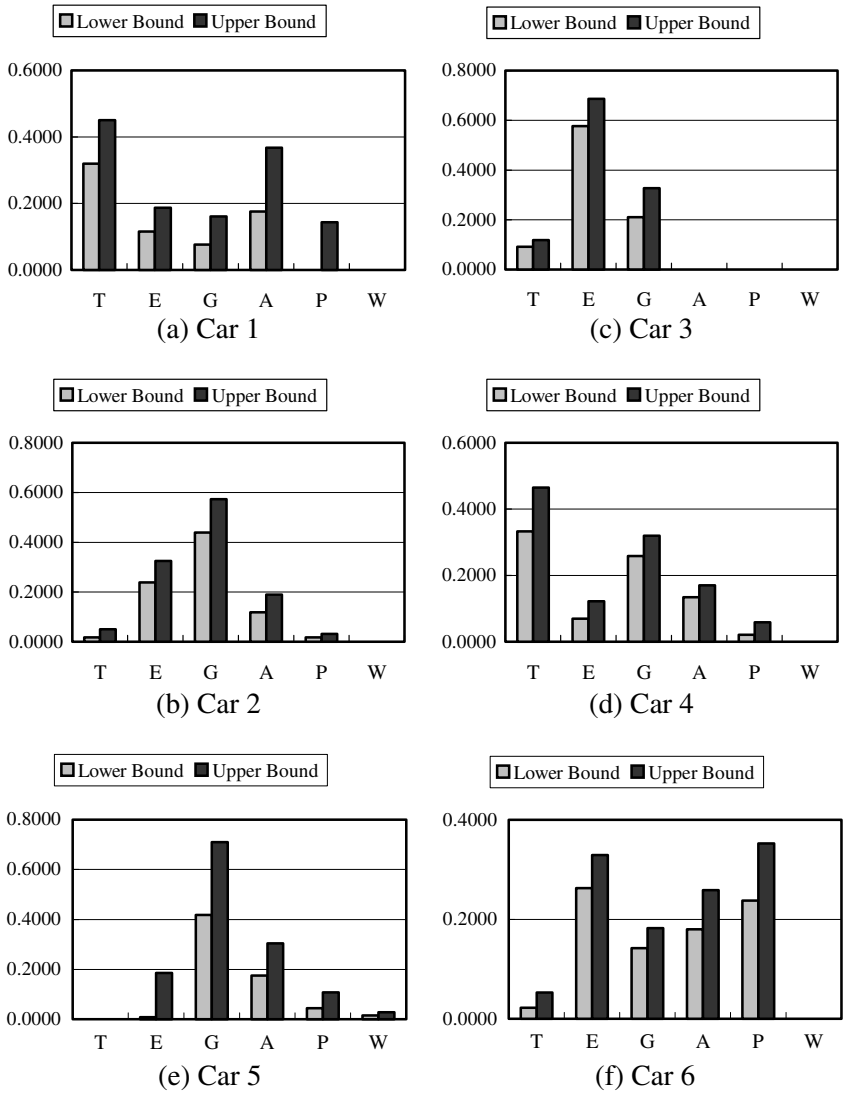

Fig. 1. The aggregated interval belief degrees for the six cars (a) Car 1; (b) Car 2; (c) Car 3; (d) Car 4; (e) Car 5; (f) Car 6.

The maximum utility of Car 5 is smaller than the minimum utility of Car 1, so Car 5 is absolutely dominated by Car 1. i.e.: Car 1 $\succ_A$ Car 5. Similarly, we can generate the following absolute dominance relations for the cars.

Car 1 $\succ_A$ Car 5 and Car 6;
Car 2 $\succ_A$ Car 5 and Car 6;
Car 3 $\succ_A$ Car 2, Car 5 and Car 6;
Car 4 $\succ_A$ Car 2, Car 5 and Car 6.

On the other hand, since the utility intervals of several cars overlap, they do not absolutely dominate each other. For example, the maximum utility of Car 5 (ranked no. 6) is 0.5669, larger than the minimum utility of Car 6 (ranked no. 5) which is 0.4939. So, it is not appropriate to say that Car 6 is absolutely better than Car 5, or the former dominating the latter, though one could argue that Car 6 may be better than Car 5 based on the average utilities. We denote this non-dominance relation by Car 5 $\prec\succ_A$ Car 6. Similarly, we have

Table 4
The expected utilities of the six cars and their ranking order

Expected utilities	Car 1	Car 2	Car 3	Car 4	Car 5	Car 6
Minimum expected utility $U_{\min}(a_l)$	0.6454	0.5973	0.7619	0.6706	0.4655	0.4939
Maximum expected utility $U_{\max}(a_l)$	0.7802	0.6636	0.8139	0.7658	0.5669	0.5558
Average expected utility $U_{\text{avg}}(a_l)$	0.7128	0.6304	0.7879	0.7182	0.5162	0.5248
Rank on average expected utility	3	4	1	2	6	5
Variance of expected utility	0.0953	0.0469	0.0368	0.0673	0.0717	0.0438

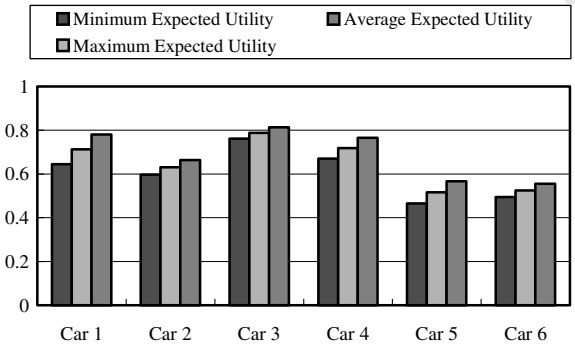

Fig. 2. The expected utilities of the six cars.

Car 1 $\prec\succ_A$ Car 2, Car 3 and Car 4;
Car 3 $\prec\succ_A$ Car 4.

However, if we investigate the pairwise dominance based on the model investigated in Section 4.3, the following pairwise dominance or non-pairwise dominance preference can be obtained:

$$\text{Min}[u_{\min}(\text{Car 3}) - u_{\max}(\text{Car 4})] = 0.0013 > 0 \Rightarrow \text{Car 3} \succ_P \text{Car 4}.$$

Although Car 3 does not absolutely dominate Car 4, Car 3 pairwise dominates Car 4. So we can conclude that Car 3 is strictly better than Car 4.

Due to the uncertainty contained in the initial data, the above analysis cannot differentiate between Car 3 and Car 1, or between Car 6 and Car 5. If the ranking orders between these cars need to be identified, the initial data, including pairwise weight comparison matrix and interval belief assessments for these cars should be revised.

5.2. Weight re-evaluation

The index value $(u_{ij} - l_{ij})$ for $i = 1, 2, \ldots, 7$, $j = i + 1, \ldots, 7$ can be calculated as shown in Table 5. Then the element $(5, 6)$ in matrix V has the largest index value among others. So it has the priority to be suggested for re-evaluation, which means that the pairwise comparison of relative importance between *Fuel economy* and *Powertrain* needs to be reevaluated first.

5.3. Interval belief re-evaluation

From Table 4 Car 1 and Car 5 have the largest variances than other cars, so these two cars have the priority to be re-evaluated. Using the data given in Table 2 and the models shown by Eq. (49) to Eq. (51), for Car 1 the variance of belief evaluations for the third attribute "Handling" can be calculated using the following model:

Table 5
The index value $(u_{ij} - l_{ij})$ for the elements in matrix V

Row	Column						
	Acceleration	Braking	Handling	Horsepower	Ride quality	Powertrain	Fuel economy
Acceleration	–	5	0	2	2	5	3
Braking	–	–	5	4	2	4	4
Handling	–	–	–	3	1	5	3
Horsepower	–	–	–	–	2	1	1
Ride quality	–	–	–	–	–	**6**	1
Powertrain	–	–	–	–	–	–	4
Fuel economy	–	–	–	–	–	–	–

$$\begin{aligned} \text{Min} \quad & u_{3,\min}(\text{Car 1}) = \beta_{5,3}u(H_5) + \beta_{6,3}u(H_6) + \beta_{H,3}u(H_1) \\ \text{s.t.} \quad & 0.3 \leqslant \beta_{5,3} \leqslant 0.4, && (53) \\ & 0.4 \leqslant \beta_{6,3} \leqslant 0.6, && (54) \\ & 0 \leqslant \beta_{H,3} \leqslant 0.1, && (55) \\ & \beta_{5,3} + \beta_{6,3} + \beta_{H,3} = 1. && (56) \end{aligned}$$

The optimal objective value of the above model is given by $u_{3,\min}(\text{Car 1}) = 0.84$.

From the following model:

$$\begin{aligned} \text{Max} \quad & u_{3,\max}(\text{Car 1}) = \beta_{5,3}u(H_5) + \beta_{6,3}u(H_6) + \beta_{H,3}u(H_6) \\ \text{s.t.} \quad & \text{Eqs. (53)–(56).} \end{aligned}$$

We can obtain $u_{3,\max}(\text{Car 1}) = 0.955$. So $u_{3,\text{avg}}(\text{Car 1}) = (0.84 + 0.955)/2 = 0.8975$. Then the variance of the third attribute for Car 1 is given by

$$\sigma_3(\text{Car 1}) = \sqrt{(0.955 - 0.8975)^2 + (0.8975 - 0.84)^2} = 0.0813.$$

Similarly, for Car 1, the variances as well as weighted variances for all the attributes can be calculated as shown in Table 6. Note that the most desirable weight vector can be generated as $W^* = (w_1^*, w_2^*, \ldots, w_7^*) = (0.275, 0.051, 0.275, 0.068, 0.206, 0.068, 0.055)$.

So the first attribute "acceleration" has the largest weighted variance for Car 1 and the assessment of car 1 on acceleration needs to be revised first, which makes sense as this assessment is an interval assessment.

5.4. Dominance results after re-evaluation

If the DM agrees to re-evaluate the comparisons in the weight pairwise matrix V from the interval value $[2, 8]$ located in $(5, 6)$ to the interval value $[7, 8]$, and the interval assessment of car 1 on "acceleration" from $[8.5, 8.9]$ to $[8.65, 8.75]$. In other words, for car 1 the acceleration time $[8.65, 8.75]$ is within the grades $\{Y_{2,1}, Y_{3,1}, Y_{4,1}\}$. Then, we have

Table 6
The variances on all attributes for Car 1

	Acceleration	Braking	Handling	Horsepower	Ride quality	Powertrain	Fuel economy
Variance	0.1131	0	0.0813	0	0.0707	0	0
Weighted variance	0.0311	0	0.0224	0	0.0145	0	0

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Table 7
The expected utilities of the six cars after re-evaluations

Expected utilities	Car 1	Car 2	Car 3	Car 4	Car 5	Car 6
Minimum expected utility $U_{\min}(a_l)$	0.6937	0.61	0.7665	0.6938	0.472	0.4939
Maximum expected utility $U_{\max}(a_l)$	0.7649	0.6573	0.8052	0.7658	0.5457	0.5126
Average expected utility $U_{\text{avg}}(a_l)$	0.7293	0.6337	0.7859	0.7298	0.5089	0.5033

$$\beta_{2,1}^{-} = 0, \quad \beta_{2,1}^{+} = 0.1 I_{1,1},$$
$$\beta_{3,1}^{-} = 0 \quad \text{and} \quad \beta_{3,1}^{+} = I_{1,1} + I_{2,1},$$
$$\beta_{4,1}^{-} = 0 \quad \text{and} \quad \beta_{4,1}^{+} = 0.1 I_{2,1},$$

where $I_{1,1} + I_{2,1} = 1$, $I_{1,1} \cdot I_{2,1} = 0$, and $I_{1,1}, I_{2,1} \geqslant 0$.

We can generate the new utilities of the cars as shown in Table 7. Their new dominance relations can be found as follows.

Car 3 $\succ_A$ Car 1, Car 2, Car 4, Car 5 and Car 6;
Car 4 $\succ_A$ Car 2, Car 5 and Car 6;
Car 1 $\succ_A$ Car 2, Car 5 and Car 6;
Car 2 $\succ_A$ Car 5 and Car 6.

So we can find the following new preference orders:

Car 3 $\succ$ {Car 1, Car 4} $\succ$ {Car 2} $\succ$ {Car 5, Car 6},

where $\succ$ denotes either $\succ_A$ or $\succ_P$.

In the above ranking order, the precise preference order between Car 4 and Car 1 and that between Car 5 and Car 6 are still unknown. If necessary, the proposed re-evaluation process could be repeated until the exact preference orders of all alternatives are found.

6. Concluding remarks

In this paper we investigated the evidential reasoning approach for dealing with both interval beliefs and interval weights. The latter may take the formats of either direct assignments or pairwise comparisons. Several non-linear optimization models were developed to handle these types of interval uncertainty for multiple attribute decision analysis using the ER approach. In these models, the upper bounds and lower bounds of interval weights were considered as constraints and incorporated into the ER-based non-linear optimization process. The interactive processes to re-evaluate interval beliefs and weights were developed to help improve the quality of the original information for achieving required preference ranking orders with as little extra effort from the DM as possible. A numerical study about a car evaluation problem was conducted using the proposed interval ER approach. The study shows that the interval ER approach provides a flexible way to model complex MADA problems. The application of this method to the assessment of new product designs is reported in another paper.

7. Uncited reference

Wang et al. (2004).

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Appendix. The calculation procedures of the most desirable weight vector (Chandran et al., 2005)

Model 1:

$$\text{Min} \quad \sum_{i=1}^{L}\sum_{j=i+1}^{L} z_{ij}$$

$$\text{s.t} \quad x_i - x_j - y_{ij} = \ln\sqrt{v_{ij}^{-}\cdot v_{ij}^{+}}, \tag{57}$$

$$x_i - x_j \geqslant \ln v_{ij}^{-}, \tag{58}$$

$$x_i - x_j \leqslant \ln v_{ij}^{+}, \tag{59}$$

$$z_{ij} \geqslant y_{ij}, \tag{60}$$

$$z_{ij} \geqslant -y_{ij}, \tag{61}$$

$$x_1 = 0, \tag{62}$$

$$z_{ij} \geqslant 0, \tag{63}$$

$$x_i, y_{ij} \text{ unrestricted.}$$

If we denote the optimal objective value as z^*, then the second linear model can be formulated as follows:

Model 2:

$$\text{Min} \quad z_{\max}$$

$$\text{s.t.} \quad \sum_{i=1}^{L}\sum_{j=i+1}^{L} z_{ij} = z^*, \tag{64}$$

$$z_{\max} \geqslant z_{ij}, \quad i,j = 1,2\ldots L,\ i<j, \tag{65}$$

$$z_{\max} \geqslant 0, \tag{66}$$

and Eqs. (57)–(63).

If the optimal solution of the second model is $x_i = x_i^*$, $i = 1,2,\ldots,L$, then the most desirable weight vector $W^* = (w_1^*, w_2^*, \ldots w_L^*)$ can be obtained as follows:

$$w_i^* = \mathrm{e}^{x_i^*} \Big/ \left(\sum_{i=1}^{L} \mathrm{e}^{x_i^*}\right), \quad \text{for } i = 1,2,\ldots,L.$$

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