Evidential Reasoning Approach for Multiattribute Decision Analysis Under Both Fuzzy and Interval Uncertainty

Min Guo, Jian-Bo Yang, Kwai-Sang Chin, Hong-Wei Wang, and Xin-Bao Liu

***Abstract*—Many multiple attribute decision analysis (MADA) problems are characterized by both quantitative and qualitative attributes with various types of uncertainties. Incompleteness (or ignorance) and vagueness (or fuzziness) are among the most common uncertainties in decision analysis. The evidential reasoning (ER) and the interval grade ER (IER) approaches have been developed in recent years to support the solution of MADA problems with interval uncertainties and local ignorance in decision analysis. In this paper, the ER approach is enhanced to deal with both interval uncertainty and fuzzy beliefs in assessing alternatives on an attribute. In this newly developed fuzzy IER (FIER) approach, local ignorance and grade fuzziness are modeled under the integrated framework of a distributed fuzzy belief structure, leading to a fuzzy belief decision matrix. A numerical example is provided to illustrate the detailed implementation process of the FIER approach and its validity and applicability.**

***Index Terms*—Fuzzy sets, multiple attribute decision analysis (MADA), evidential reasoning (ER) approach, uncertainty modeling, utility.**

I. INTRODUCTION

MANY real-world multiple attribute decision analysis (MADA) problems are characterized with both quantitative and qualitative attributes. In many circumstances, the attributes, especially qualitative ones, could only be properly assessed using human judgment, which is subjective in nature and is inevitably associated with uncertainties caused due to the following two phenomena.

1) a human being's inability to provide complete judgments, or the lack of information, which is referred to as "ignorance" (incompleteness);
2) the vagueness of meanings about attributes and their assessments, which is referred to as "fuzziness" (vagueness).

For decades, many MADA methods have been developed, such as the well-known analytical hierarchy process (AHP) [1] and multiple attribute utility theory [2]–[4], as well as their extensions, such as the interval-valued assessments approaches, which try to apply the interval arithmetic analysis and algorithms [5]–[7] to MADA problems, especially in the weight-evaluation process [8]–[12]. In these methods, MADA problems are modeled using decision matrices, in which an alternative is assessed on each attribute by either a single real number or an interval value. Unfortunately, in many decision situations using a single number or interval to represent a judgment proves to be difficult and may be unacceptable. Information may be lost or distorted in the process of preaggregating different types of information, such as a subjective judgment, a probability distribution, or an incomplete piece of information.

Fuzziness or vagueness can be well treated using the fuzzy set theory [13]–[18]. Concerning the fuzziness of MADA problems, a large amount of fuzzy MADA methods have been proposed in the literature, such as fuzzy hierarchical aggregation methods [19], conjunction implication methods [20]–[22], weighted average aggregation methods [23]–[26], and weighted average aggregation with criteria-assessment methods [27]. Nevertheless, these pure fuzzy MADA approaches are essentially based on traditional evaluation methods and are unable to handle probabilistic uncertainties such as ignorance.

Different from the traditional MADA methods, the evidential reasoning (ER) approach, which is the combination of the Dempster–Shaffer (D–S) theory [28], [29], with a distributed modeling framework, sheds new light on modeling complex MADA problems. The ER approach uses a distributed modeling framework, in which each attribute is accessed using a set of collectively exhaustive and mutually exclusive assessment grades. Probabilistic uncertainty, including local and global ignorance, is characterized by a belief structure in the ER approach, which can both model precise data and capture various types of uncertainties, such as probabilities and vagueness in subjective judgments. For example, in order to compare the handling performance of cars, experts usually give their assessments by means of a set of evaluation grades, e.g., {Worst (W), Poor (P), Average (A), Good (G), Excellent (E)}, and some evaluations given by experts may be expressed as follows.

Manuscript received November 16, 2007; revised March 7, 2008; accepted April 21, 2008. First published July 16, 2008; current version published June 11, 2009. This work was supported in part by the Research Grant Council of the Hong Kong SAR, China, under CERG Grant 9040786, in part by the U.K. Engineering and Physical Science Research Council under Grant EPSRC EP/F024606/1, in part by the Natural Science Foundation of China under Grant 70572033, Grant 60674085, and Grant 70631003, and in part by the Research Grants Council of the Hong Kong Special Administrative Region, China under Project CityU 1203/04E and Project CityU 111906.

M. Guo and H.-W. Wang are with the Institute of Systems Engineering, Key Laboratory of Image Processing and Intelligent Control, Huazhong University of Science and Technology, Wuhan 430074, China (e-mail: guomin_like@sina.com.cn; hwwang@mail.hust.edu.cn).

J.-B. Yang is with Manchester Business School, The University of Manchester, Manchester M13 9PL, U.K., and also with the School of Management, Hefei University of Technology, Hefei 230009, China (e-mail: jian-bo.yang@umist.ac.uk).

K.-S. Chin is with the Department of Manufacturing Engineering and Engineering Management, City University of Hong Kong, Kowloon, Hong Kong (e-mail: mekschin@cityu.edu.hk).

X.-B. Liu is with the School of Management, Hefei University of Technology, Hefei 230009, China (e-mail: lxinbao@mail.hf.ah.cn).

Digital Object Identifier 10.1109/TFUZZ.2008.928599

Statement 1: The handling of car 1 is evaluated to be "Average" with a confidence degree of 80% and to be "Good" with a confidence degree of 20%.

Statement 1 can be captured exactly by the ER assessment framework as $\{$(Average, 0.8); (Good: 0.2)$\}$.

Statement 2: The handling of car 2 is 80% "Average."

Statement 2 is incomplete with a degree of incompleteness (or global ignorance) of 20%, and can be expressed by ER format as $\{$(Average, 0.8)$\}$.

The distributed assessment framework of ER has a great advantage to express the "true" meanings of experts especially when the evaluations must be given to qualitative attributes. With the evidence of combination rule of the D-S theory, the ER's distributed assessment framework has since been applied to MADA problems, such as supplier assessment [30]–[33], business performance assessment [34]–[36], marine system safety analysis and synthesis [37], [38], software safety synthesis [39], [40], general cargo ship design [41], retrofit ferry design [42], product selections [43], and customer satisfaction survey and result analysis [44].

Extensive research dedicated to the ER approach has been conducted in recent years. Along with the application of ER modeling, experiences show that a decision maker may not always be confident enough to provide subjective assessments to individual grades only but at times wishes to be able to assess beliefs to subsets of adjacent grades. For example, for the handling of cars, some experts may have a following evaluation.

Statement 3: The handling of car 3 is mostly (with about 90% possibility) between "Average" and "Good," and with 10% possibility it is known to be "Excellent." However, the distributions between "Average" and "Good" are not known.

In Statement 3, the unknown or ignorance between grades "Average" and "Good" is referred to as local ignorance or interval uncertainty. It is to deal with the local ignorance that the interval grade ER (IER) approach is proposed [43]. In the IER approach, the individual evaluation grades are extended to include interval grades, such as "Average–Good" (i.e., the union of the grade "Average" and "Good"), "Poor–Average," "Average–Good–Excellent," etc., which can give the evaluation basis for the correspondent local ignorance. For Statement 3, we have the IER assessment as $\{$(Average–Good, 0.9); (Excellent, 0.1)$\}$.

Another extension to the original ER approach is to take account of vagueness or fuzzy uncertainty, i.e., the assessment grades are no longer clearly distinctive crisp sets, but are defined as dependent fuzzy sets. For example, some experts may fail to distinguish the adjacent grades, such as "Average" and "Good" precisely. Yang *et al.* [46] proposed the fuzzy ER approach to extend the original ER individual grades to fuzzy grades to capture fuzziness caused by the fuzzy evaluation grades.

Along with the further applications of the ER-related approaches, it is found that both distributed assessments with local ignorance and fuzzy evaluation grades may appear in real MADA problems, such as the marine system safety analysis and synthesis [37], [38], supplier assessment and customer satisfaction survey and result analysis [44], product selection and screening [47], [48], etc. Due to the subjective nature of these problems, evaluations given by experts may not be accurate enough to capture by the basic ER assessment format. In fact, assessments based on interval grades are common, and the fuzziness of grades themselves cannot be ignored. It is important to investigate uncertainties caused by both interval and fuzzy grade evaluations. As such, the aim of this paper is to integrate the main features of the fuzzy ER [46] and the IER [45] approaches and develop a general ER modeling framework and an attribute aggregation process, which is referred to as the fuzzy IER (FIER) algorithm, in order to deal with both fuzzy and interval grade assessments and provide a more powerful means to support the solution of complex MADA problems.

The paper is organized as follows. In Sections II and III, brief outlines of the original ER algorithm and the IER aggregation algorithm are introduced. Then, the FIER is developed in detail in Sections IV and V. In Section VI, a numerical study is provided to illustrate the methodology. The paper is concluded in Section VII.

II. Outline of the Original ER Algorithm

The ER algorithm is developed for aggregating multiple attributes based on a belief decision matrix and the evidence combination rule of D-S theory [28], [29], [43], [49]–[52]. Different from traditional MADA approaches that describe a MADA problem using a single average number to assess each alternative on every attribute summarized in a decision matrix, the ER approach uses the belief decision matrix, in which an alternative is assessed on each attribute by a distribution using a belief structure. The advantage of doing so is that, using a distribution assessment, it can both model precise data and capture various types of uncertainties, such as probabilities and vagueness in subjective judgments.

Suppose a MADA problem has M alternatives a_l, $l = 1, \ldots, M$, one upper level attribute v, which is referred to as a general attribute, and L the lower level attributes e_i, $i = 1, \ldots, L$, which are called basic attributes. The relative weights of the L basic attributes are denoted by $W = (w_1, \ldots, w_L)$, which are given and satisfy the conditions $0 \le w_i \le 1$, $i = 1, \ldots, L$, and $\sum_{i=1}^{L} w_i = 1$.

Suppose M alternatives are all assessed using the same set of N assessment grades $H_p, p = 1, \ldots, N$, which are mutually exclusive and collectively exhaustive for the assessment of all attributes. The N assessment grades as well as the whole set H_{1N} form the frame of discernment in the D-S theory of evidence:

$$H = \{H_1, H_2, \ldots, H_N, H_{1N}\}. \quad (1)$$

Suppose alternative a_l is assessed on the basic attributes e_i, $i = 1, \ldots, L$ using the N grades as well as the incompleteness assessment on the whole set H_{1N}. This assessment can be represented as follows:

$$S(a_l) = \{(C, \beta_i(C)),\ C \in H,\ i = 1, \ldots, L\} \quad (2)$$

where $S(a_l)$ is a distributed assessment, which means that there is a degree of belief $\beta_i(H_{1N})$ of unknown assessment that could be assigned to any grades of $H_1, H_2, \ldots, H_N$. Note that $\sum_{C \in H} \beta_i(C) = 1$ always holds in (2).

Suppose w_i are the normalized weight for attribute i. Then, the basic probability masses assigned to each element in set H and the unassigned mass are given by

$$m_i(C) = w_i\beta_i(C), \quad i = 1, \ldots, L, \quad C \neq \Phi, \quad C \in H \tag{3}$$

$$m_i(\Phi) = 0 \tag{4}$$

$$m_i(U) = 1 - w_i, \quad i = 1, \ldots, L \tag{5}$$

where $m_i(U)$ in (5) is the remaining probability mass that is unassigned to any individual evaluation grades or the whole set H_{1N} after only attribute i has been taken into account. In other words, $m_i(U)$ represents the remaining role that other attributes can play in the assessment. $m_i(U)$ should eventually be assigned to individual grades in a way that is dependent upon the importance of other attributes.

Based on the previous definition of mass functions, we can aggregate the L attributes recursively using the following ER combination algorithm, where we use s as the recursive number, and the combination results of the first $s = 1, 2, \ldots, L$ attribute are denoted as $m_{I(s)}(\cdot)$. Obviously, in the first recursion $s = 1$, we have

$$m_{I(1)}(C) = m_1(C), \quad C \neq \Phi, \quad C \in H \tag{6}$$

$$m_{I(1)}(U) = m_1(U). \tag{7}$$

In the next recursion $s + 1$, we have

$$m_{I(s+1)}(C) = \frac{1}{K_{I(s+1)}} \left[\sum_{A \cap B = C} m_{I(s)}(A) m_{s+1}(B) + m_{I(s)}(C) m_{s+1}(U) + m_{I(s)}(U) m_{s+1}(C) \right] \quad C \neq \Phi, \quad C \in H \tag{8}$$

$$m_{I(s+1)}(U) = \frac{1}{K_{I(s+1)}} [m_{I(s)}(U) m_{s+1}(U)] \tag{9}$$

$$K_{I(s+1)} = 1 - \sum_{A \cap B = \Phi} m_{I(s)}(A) m_{s+1}(B) \tag{10}$$

where $K_{I(s+1)}$ is the normalization factor that ensures $\sum_{C \in H} m_{I(s+1)}(C) + m_{I(s+1)}(U) = 1$ in the $(s + 1)$th combination.

The recursions are continued until $s = L$, i.e., all the L attributes have been combined, and then, the overall probability masses assigned to alternative a_l can be obtained as follows:

$$m(C) = m_{I(L)}(C), \quad C \neq \Phi, \quad C \in H \tag{11}$$

$$m(U) = m_{I(L)}(U). \tag{12}$$

The overall assessment of alternative a_l is given by

$$S(a_l) = \{(C, \beta(C)), C \in H\} \quad \text{with } \beta(C) = \frac{m(C)}{1 - m(U)}. \tag{13}$$

III. IER Algorithm

In the previous original ER algorithm, however, the decision makers are restricted to provide assessments to individual assessment grades only, and any ignorance is assigned to the whole set of grades H_{1N}. Such ignorance is referred to as global or total ignorance. Experience shows that the decision maker may not always be confident enough to provide subjective assessments to individual grades only but at times wish to be able to assess beliefs to subsets of adjacent grades. Such ignorance is referred to as local or partial ignorance or interval uncertainty. It is therefore desirable that the ER approach can be enhanced to model the assignment of beliefs to subsets of grades and subsequently process such assessments in decision analysis.

According to [45], in the IER algorithm, because the performances of alternatives can be assessed to an individual grade or a grade interval, the complete set of all individual grades and grade intervals is denoted by $\hat{H}$. For assessing each attribute, $\hat{H}$ can be represented by

$$\hat{H} = \{H_{pq}, p = 1, \ldots, N, q = p, \ldots, N\}$$

or equivalently

$$\hat{H} = \left\{ \begin{array}{ccccc} H_{11} & H_{12} & \cdots & H_{1(N-1)} & H_{1N} \\ & H_{22} & \cdots & H_{2(N-1)} & H_{2N} \\ & & \ddots & \vdots & \vdots \\ & & & H_{(N-1)(N-1)} & H_{(N-1)N} \\ & & & & H_{NN} \end{array} \right\} \tag{14}$$

where H_{pp} $(p = 1, \ldots, N)$ in (14) denotes an individual grade and is equivalent to H_p in formula (1). H_{pq} $(p = 1, \ldots, N, q = p + 1, \ldots, N)$ denotes the interval grade that is the union of individual grades $H_{pp}, H_{(p+1)(p+1)}, \ldots, H_{qq}$. Note that there is a difference between the sets given by (1) and (14). The former is used in the original ER algorithm and is a subset of the latter.

Based on the previous assumption, the assessment of alternative a_l on all attributes is then given by

$$S(a_l) = \{(C, \beta_i(C));\ C \in \hat{H},\ i = 1, \ldots, L\} \quad \text{where } \sum_{C \in \hat{H}} \beta_i(C) = 1 \text{ holds.} \tag{15}$$

Similarly to the original ER algorithm, the mass functions are defined as follows:

$$m_i(C) = w_i\beta_i(C), \quad i = 1, \ldots, L, \quad C \neq \Phi, \quad C \in \hat{H}$$

or equivalently

$$m_i(H_{pq}) = w_i\beta_i(H_{pq}), \quad p = 1, \ldots, N, \quad q = p, \ldots, N \tag{16}$$

$$m_i(\Phi) = 0 \tag{17}$$

$$m_i(U) = 1 - w_i, \quad i = 1, \ldots, L. \tag{18}$$

According to the D-S and the basic ER aggregation rules, the IER algorithms can be obtained as follows [45].

Let s be the recursive number and let the combination results of the first $s = 1, 2, \ldots, L$ attributes be denoted as $m_{I(s)}(\cdot)$. In the first recursion $s = 1$, we have

$$m_{I(1)}(C) = m_1(C), \quad C \neq \Phi, \quad C \in \hat{H},$$

or equivalently

$$m_{I(1)}(H_{pq}) = m_1(H_{pq}), \quad p = 1,\ldots,N, \quad q = p,\ldots,N \tag{19}$$

$$m_{I(1)}(U) = m_1(U). \tag{20}$$

In the next recursion $s+1$, we have

$$\begin{aligned} m_{I(s+1)}(H_{pq}) = \frac{1}{1-K_{I(s+1)}} \Bigg\{ & -m_{I(s)}(H_{pq})m_{s+1}(H_{pq}) \\ & + \sum_{k=1}^{p}\sum_{l=q}^{N}[m_{I(s)}(H_{kl})m_{s+1}(H_{pq}) \\ & + m_{I(s)}(H_{pq})m_{s+1}(H_{kl})] \\ & + \sum_{k=1}^{p-1}\sum_{l=q+1}^{N}[m_{I(s)}(H_{kq})m_{s+1}(H_{pl}) \\ & + m_{I(s)}(H_{pl})m_{s+1}(H_{kq})] \\ & + m_{I(s)}(U)m_{s+1}(H_{pq}) \\ & + m_{I(s)}(H_{pq})m_{s+1}(U) \Bigg\} \\ & p = 1,\ldots,N, \quad q = p,\ldots,N \end{aligned} \tag{21}$$

and the probability mass left to the set U is given by

$$m_{I(s+1)}(U) = \frac{m_{I(s)}(U)m_{s+1}(U)}{1-K_{I(s+1)}} \tag{22}$$

where $K_{I(s+1)}$ is the combined probability mass assigned to the empty set $\{\Phi\}$:

$$\begin{aligned} K_{I(s+1)} = \sum_{p=1}^{N}\sum_{q=p}^{N}\sum_{k=1}^{p-1}\sum_{l=k}^{p-1} & [m_{I(s)}(H_{kl})m_{s+1}(H_{pq}) \\ & + m_{I(s)}(H_{pq})m_{s+1}(H_{kl})]. \end{aligned} \tag{23}$$

The scaling factor $1/(1-K_{I(s+1)})$ is used to make sure that $\sum_{p=1}^{N}\sum_{q=p}^{N} m_{I(s+1)}(H_{pq}) + m_{I(s+1)}(U) = 1$.

Note that in (21) and (23), the summing up process $\sum_{i=i_1}^{i_2} f(i)$ will not be carried out when $i_1 > i_2$. Mathematically, we may say that $\sum_{i=i_1}^{i_2} f(i) = 0$ when $i_1 > i_2$, and this convention applies throughout the paper.

Similarly to the original ER algorithm, by applying the previous aggregation process recursively until all the L basic attribute assessments are aggregated, the overall assessment of alternative a_l can be expressed as

$$S(a_l) = \{(C, \beta(C)), C \in \hat{H}\} \quad \text{with } \beta(C) = \frac{m_{I(L)}(C)}{1-m_{I(L)}(U)}$$

or

$$S(a_l) = \{(H_{pq}, \beta(H_{pq})), p = 1,\ldots,N, q = p,\ldots,N\}$$
$$\text{with } \beta(H_{pq}) = \frac{m_{I(L)}(H_{pq})}{1-m_{I(L)}(U)}. \tag{24}$$

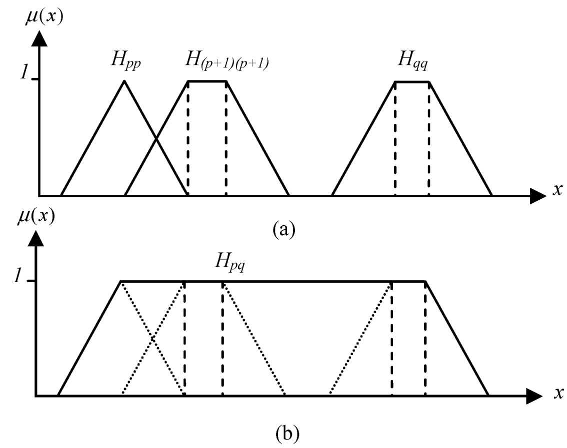


Fig. 1. Fuzzy sets definitions. (a) Individual fuzzy grade sets. (b) Interval fuzzy grade set.

IV. FIER Approach for MADA Under Fuzzy Uncertainty

A. New FIER Distributed Modeling Framework Using the Fuzzy Belief Structure

In the IER approach introduced earlier, all individual and interval assessment grades are assumed to be crisp and independent of each other. However, there are occasions where an assessment grade may represent a vague concept or standard and there may be no clear cut between the meanings of two adjacent grades. In this paper, we will drop the previous assumption and allow grades to be vague and adjacent grades to be dependent. To simplify the discussion and without loss of generality, fuzzy sets will be used to characterize vague assessment grades, and it is assumed that only two adjacent fuzzy grades have the overlap of meanings. This represents the most common features of fuzzy uncertainty in decision analysis. Note that the principle of the following method can be extended to more general cases.

In order to generalize the $\hat{H} = \{H_{pq}, p = 1,\ldots,N, q = p,\ldots,N\}$ to fuzzy sets, we assume that a general set of fuzzy individual assessment grades $\{H_{pp}\}, p = 1,\ldots,N$ are dependent on each other, which may be assumed to be either triangular or trapezoidal fuzzy sets or their combinations for simplifying the discussion and without loss of generality. Assuming that only two adjacent fuzzy individual assessment grades may intersect, we denote by $H_{p\Lambda(p+1)}$, $p = 1,\ldots,N-1$ the fuzzy intersection subset of the two adjacent fuzzy individual assessment grades H_{pp} and $H_{(p+1)(p+1)}$ [see Fig. 1(a)].

Furthermore, we define the sets $H_{pq}, p = 1,\ldots,N, q = p+1,\ldots,N$ as trapezoidal fuzzy sets that include fuzzy individual grades $H_{pp}, H_{(p+1)(p+1)},\ldots,H_{qq}$. If the individual assessment grades are triangular or trapezoidal fuzzy sets, every interval grade will be a trapezoidal fuzzy set [see Fig. 1(b)]. We also define $H_{p\Lambda(p+1)}$ as the fuzzy intersection subset of the two adjacent fuzzy interval assessment grades H_{kp} and $H_{(p+1)q}$, where $k = 1,\ldots,p$ and $q = p+1,\ldots,N$ [see Fig. 2(a) and (b)].

Note that the intersection $H_{p\Lambda(p+1)}$ of the two adjacent evaluation grades H_{kp} and $H_{(p+1)q}$, where $k = 1,\ldots,p$ and

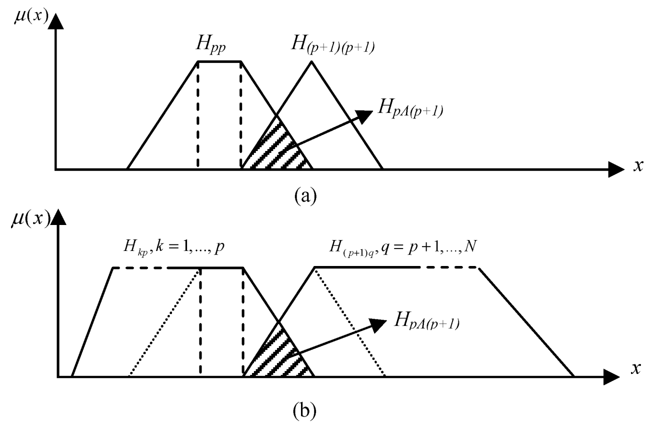


Fig. 2. Intersections between fuzzy assessment grades. (a) Intersection set between individual fuzzy sets. (b) Intersection set between interval fuzzy sets.

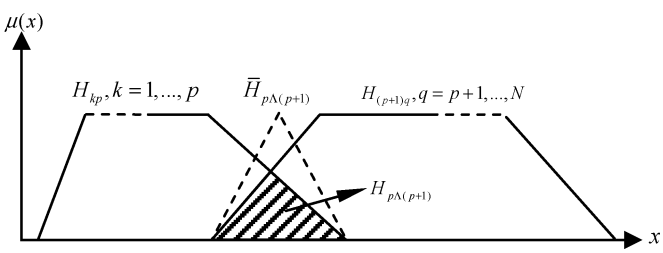


Fig. 3. Intersection of fuzzy sets and its normalized set.

$q=p+1,\ldots,N$, is not empty in general, as shown in Fig. 2. As in [46], the fuzzy intersection subset $H_{p\Lambda(p+1)}$, whose maximum degree of membership is represented by $\mu_{p\Lambda(p+1)}^{\max}$ and is usually less than one, will be normalized as a fuzzy subset $\bar{H}_{p\Lambda(p+1)}$, with the maximum membership degree being one, as shown in Fig. 3, so that $\bar{H}_{p\Lambda(p+1)}$ can be measured as a formal fuzzy set; therefore, it is assessed in the same scale as the other defined fuzzy evaluation grades, such as $H_{pq}, p=1,\ldots,N, q=p,\ldots,N$. If this were not done, the probability mass assigned to $H_{p\Lambda(p+1)}$ would have nothing to do with its shape or height. In other words, as long as the two fuzzy assessment grades H_{kp} and $H_{(p+1)q}$ intersect, the probability mass assigned to $H_{p\Lambda(p+1)}$ would always be the same, no matter how large or small the intersection subset may be.

By normalizing $H_{p\Lambda(p+1)}$ to $\bar{H}_{p\Lambda(p+1)}$, we can perform the D-S or ER combination rule in the generalized fuzzy set that are defined as follows:

$$\hat{H}_F = \{H_{pq}, p=1,\ldots,N, q=p,\ldots,N\} \cup \{\bar{H}_{p\Lambda(p+1)}, p=1,\ldots,N-1\} \tag{25}$$

where H_{pq} is a fuzzy set, and $\bar{H}_{p\Lambda(p+1)}$ is the normalized intersection of two adjacent fuzzy sets H_{kp} and $H_{(p+1)q}$, where $k=1,\ldots,p$, and $q=p+1,\ldots,N$.

Since fuzzy assessment grades and belief degrees are used, then $S(a_l)$ defined in Sections II and III can be extended to the following expression:

$$S(a_l) = \{(C, \beta_i(C)), C \in \hat{H}_F, i=1,\ldots,L\}. \tag{26}$$

B. New FIER Algorithm Under Both Interval Probabilistic and Fuzzy Uncertainties

In the derivation of the IER algorithms in (15)–(24), it was assumed that the evaluation grades are independent of each other. Due to the dependency of the adjacent fuzzy assessment grades on each other as shown in Figs. 1 and 2, the IER algorithm can no longer be employed without modification to aggregate attributes assessed using such fuzzy grades. However, the evidence theory provides scope to cope with such fuzzy assessments. The ideas similar to those used to develop the nonfuzzy ER algorithm [43], [49], [51] are used to deduce the new FIER algorithm.

We note that the nonfuzzy IER algorithm, which follows the basic ER combination rules, is given in recursive forms. In each recursive step, a normalization procedure is taken to ensure that the possibility mass assigned to an empty set is set to zero. However, with the normalization process, the nonfuzzy IER cannot be transformed directly to a fuzzy IER algorithm. Because the original ER and IER algorithms are derived from the D-S combination rules, in which the normalizations can be postponed to the end of the recursive algorithm, we can draw the following conclusion (the detailed proof is shown in Appendix A).

Theorem 1: In the original ER and IER recursive combination rules, the normalization can be postponed to the end of the recursive algorithms without changing the results.

In this section, an FIER algorithm will be developed using the similar technique used in [46] and [49] (the detailed proof is shown in Appendix B).

The algorithm FIER for an alternative $a_l$is as follows.

Step 1: Calculate the basic probability masses for alternative a_l

$$m_i(H_{pq}) = w_i\beta_i(H_{pq}), \quad i=1,\ldots,L, \quad p=1,\ldots,N \quad q=p,\ldots,N \tag{27}$$

$$m_i(\bar{H}_{p\Lambda(p+1)}) = 0 \quad \text{and} \quad m_i(H_{p\Lambda(p+1)}) = 0 \quad i=1,\ldots,L, \quad p=1,\ldots,N-1 \tag{28}$$

$$m_i(\Phi) = 0 \tag{29}$$

$$m_i(U) = 1 - w_i, \quad i=1,\ldots,L. \tag{30}$$

Step 2: Let the recursive number $s := 1$ and $\tilde{m}_{I(s)}(\cdot)$ denote the combination results of the first s attributes without normalization at each combination

$$\tilde{m}_{I(1)}(H_{pq}) = m_1(H_{pq}), \quad p=1,\ldots,N, \quad q=p,\ldots,N \tag{31}$$

$$\tilde{m}_{I(1)}(H_{p\Lambda(p+1)}) = m_1(H_{p\Lambda(p+1)}) = 0, \quad p=1,\ldots,N-1 \tag{32}$$

$$\tilde{m}_{I(1)}(U) = m_1(U). \tag{33}$$

Step 3: Calculate the combination results of the first $s+1$ attributes without normalization at each combination

$$\begin{aligned}\tilde{m}_{I(s+1)}(H_{pq}) = &-\tilde{m}_{I(s)}(H_{pq})m_{s+1}(H_{pq})\\ &+\sum_{k=1}^{p}\sum_{l=q}^{N}[\tilde{m}_{I(s)}(H_{kl})m_{s+1}(H_{pq})\\ &+\tilde{m}_{I(s)}(H_{pq})m_{s+1}(H_{kl})]\\ &+\sum_{k=1}^{p-1}\sum_{l=q+1}^{N}[\tilde{m}_{I(s)}(H_{kq})m_{s+1}(H_{pl})\\ &+\tilde{m}_{I(s)}(H_{pl})m_{s+1}(H_{kq})]\\ &+\tilde{m}_{I(s)}(U)m_{s+1}(H_{pq})\\ &+\tilde{m}_{I(s)}(H_{pq})m_{s+1}(U),\\ &p=1,\ldots,N,\quad q=p,\ldots,N \qquad (34)\end{aligned}$$

$$\begin{aligned}\tilde{m}_{I(s+1)}(H_{p\Lambda(p+1)}) = &\sum_{k=1}^{p}\sum_{q=p+1}^{N}[\tilde{m}_{I(s)}(H_{kp})m_{s+1}(H_{(p+1)q})\\ &+\tilde{m}_{I(s)}(H_{(p+1)q})m_{s+1}(H_{kp})]\\ &+\sum_{k=1}^{p+1}\sum_{\substack{l=p\\ l\geq k}}^{N}\tilde{m}_{I(s)}(H_{p\Lambda(p+1)})m_{s+1}(H_{kl})\\ &+\tilde{m}_{I(s)}(H_{p\Lambda(p+1)})m_{s+1}(U),\\ &p=1,\ldots,N-1 \qquad (35)\end{aligned}$$

$$\tilde{m}_{I(s+1)}(U) = \tilde{m}_{I(s)}(U)m_{s+1}(U) = \prod_{l=1}^{s+1} m_l(U). \qquad (36)$$

Step 4: Let $s := s+1$. If $s = L$, i.e., all the L attributes have been combined, continue to step 5; otherwise $s < L$, go to step 3.

Step 5: Conduct normalization at the end of L combinations. We denote $m_{I(L)}(\cdot)$ as the normalized combination results of the L attributes, (37), shown at the bottom of the page

$$m_{I(L)}(H_{pq}) = K\tilde{m}_{I(L)}(H_{pq}),\quad p=1,\ldots,N,\quad q=p,\ldots,N \qquad (38)$$

$$m_{I(L)}(\bar{H}_{p\Lambda(p+1)}) = K\mu^{\max}_{p\Lambda(p+1)}\tilde{m}_{I(L)}(H_{p\Lambda(p+1)}),\quad p=1,\ldots,N-1 \qquad (39)$$

$$m_{I(L)}(U) = K\tilde{m}_{I(L)}(U). \qquad (40)$$

Step 6: Calculate the overall assessments of alternative a_l

$$\beta(H_{pq}) = \frac{m_{I(L)}(H_{pq})}{1-m_{I(L)}(U)},\quad p=1,\ldots,N,\quad q=p,\ldots,N \qquad (41)$$

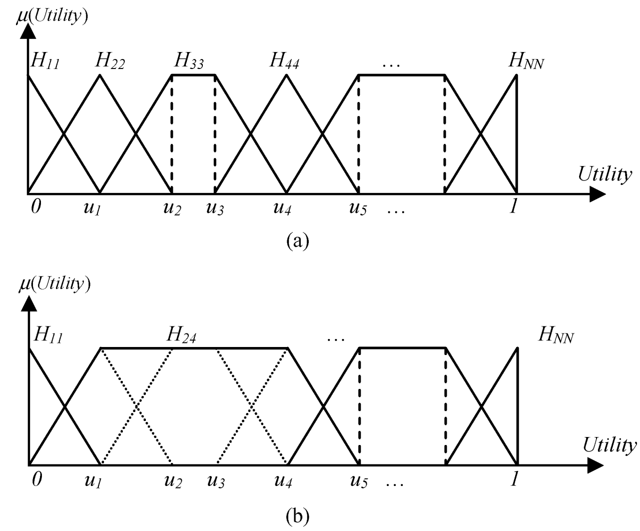


Fig. 4. Utility of fuzzy grades. (a) The utility values of individual fuzzy. (b) The utility values of interval fuzzy grade sets.

$$\beta(\bar{H}_{p\Lambda(p+1)}) = \frac{m_{I(L)}(\bar{H}_{p\Lambda(p+1)})}{1-m_{I(L)}(U)},\quad p=1,\ldots,N-1. \qquad (42)$$

V. Fuzzy Expected Utilities for Characterizing Alternatives

Utility is one of the most important concepts in decision analysis. It represents a decision maker's (DM) preferences for various values of a variable and measures the relative strength of desirability that the DM has for those values. A function that represents the DM's preferences is referred to as a utility function. In fuzzy MADA, however, utilities corresponding to fuzzy assessment grades can no longer be represented by singleton numerical values because the evaluation grades are fuzzy sets. If a fuzzy assessment grade is a triangular fuzzy number, its corresponding fuzzy grade utility should also be a triangular fuzzy number. If a fuzzy assessment grade is a trapezoidal fuzzy number, its corresponding fuzzy utility will also be a trapezoidal fuzzy number. If fuzzy assessment grades are the combinations of triangular and trapezoidal fuzzy numbers, their corresponding grade utilities will be trapezoidal fuzzy numbers. In other words, a fuzzy grade utility should have the same form as its corresponding fuzzy assessment grade. The fuzzy grade utilities corresponding to the fuzzy individual assessment grades in Fig. 1(a) are shown in Fig. 4(a). Note that in the FIER methodology, according to the definitions of fuzzy grades in Section IV-A, the utility values of an interval fuzzy grade can be calculated from the utility values of the corresponding fuzzy individual grades, as shown in Fig. 4(b).

According to the fuzzy grade utilities and the overall assessments of all alternatives shown in (41) and (42), fuzzy expected utilities are calculated for alternatives. They are employed to

$$K = \frac{1}{\left[\sum_{p=1}^{N}\sum_{q=p}^{N}\tilde{m}_{I(L)}(H_{pq}) + \sum_{p=1}^{N-1}\mu^{\max}_{p\Lambda(p+1)}\tilde{m}_{I(L)}(H_{p\Lambda(p+1)}) + \tilde{m}_{I(L)}(U)\right]} \qquad (37)$$

compare and rank alternatives. The fuzzy expected utility of an aggregated assessment $S(a_l)$ for alternative a_l is defined as follows:

$$u[S(a_l)] = \sum_{p=1}^{N}\sum_{q=p}^{N}\beta(H_{pq})u(H_{pq}) + \sum_{p=1}^{N-1}\beta(\bar{H}_{p\Lambda(p+1)})u(\bar{H}_{p\Lambda(p+1)}) \tag{43}$$

where $u(H_{pq})$ is the fuzzy grade utility of the assessment grade H_{pq}, $p = 1,\ldots,N$, $q = p,\ldots,N$, and $u(\bar{H}_{p\Lambda(p+1)})$, $p = 1,\ldots,N-1$ is the fuzzy grade utility of the intersection fuzzy grade set $\bar{H}_{p\Lambda(p+1)}$.

Without loss of generality, suppose $u(H_{pp})$ is the utility value of the grade H_{pp} and it is assumed that the grade $H_{(p+1)(p+1)}$ is preferred to H_{pp}. Suppose H_{11} is the least preferred fuzzy individual assessment grade, which has the lowest fuzzy grade utility, and H_{NN} is the most preferred fuzzy individual assessment grade, which has the highest fuzzy grade utility. Suppose $u(H_{pq})$ can take the lower bound value, the upper bound value, and the two most possible values (MPVs) as $u_{\min}(H_{pq})$, $u_{\max}(H_{pq})$, $u_{\mathrm{MPV1}}(H_{pq})$, and $u_{\mathrm{MPV2}}(H_{pq})$ ($u_{\mathrm{MPV1}}(H_{pq}) \leq u_{\mathrm{MPV2}}(H_{pq})$), respectively, if all grade sets are triangular or trapezoidal fuzzy sets. It is straightforward that the following equations hold according to the relationships of individual and interval grade sets:

$$u_{\min}(H_{pq}) = u_{\min}(H_{pp}) \tag{44}$$

$$u_{\max}(H_{pq}) = u_{\max}(H_{qq}) \tag{45}$$

$$u_{\mathrm{MPV1}}(H_{pq}) = u_{\mathrm{MPV1}}(H_{pp}) \tag{46}$$

$$u_{\mathrm{MPV2}}(H_{pq}) = u_{\mathrm{MPV2}}(H_{qq}) \tag{47}$$

where, in (45), the belief degree $\beta(H_{pq})$ could be assigned to the best individual grade in the interval grade H_{pq}, which is H_{qq}, and also can be assigned to the worst individual grade H_{pp}, as shown in (44).

Similarly, $u(\bar{H}_{p\Lambda(p+1)})$ can take the lower bound value, the upper bound value, and the two MPVs as $u_{\min}(\bar{H}_{p\Lambda(p+1)})$, $u_{\max}(\bar{H}_{p\Lambda(p+1)})$, $u_{\mathrm{MPV1}}(\bar{H}_{p\Lambda(p+1)})$, and $u_{\mathrm{MPV2}}(\bar{H}_{p\Lambda(p+1)})$ [note $u_{\mathrm{MPV1}}(\bar{H}_{p\Lambda(p+1)}) = u_{\mathrm{MPV2}}(\bar{H}_{p\Lambda(p+1)})$ if $\bar{H}_{p\Lambda(p+1)}$ have triangular membership functions], respectively, and the following equations hold:

$$u_{\min}(\bar{H}_{p\Lambda(p+1)}) = u_{\min}(H_{(p+1)(p+1)}) \tag{48}$$

$$u_{\max}(\bar{H}_{p\Lambda(p+1)}) = u_{\max}(H_{pp}). \tag{49}$$

Accordingly, the fuzzy expected utility $u[S(a_L)]$ is also a fuzzy number. From (43)–(49), the maximum utility value of alternative a_l could be calculated as

$$u_{\max}[S(a_l)] = \sum_{p=1}^{N}\sum_{q=p}^{N}\beta(H_{pq})u_{\max}(H_{qq}) + \sum_{p=1}^{N-1}\beta(\bar{H}_{p\Lambda(p+1)})u_{\max}(H_{pp}). \tag{50}$$

Similarly, in the worst case, if the uncertainty turned out to be against the assessed alternative, with the belief degree $\beta(H_{pq})$ being assigned to H_{pp} (the worst individual grade in the interval grade H_{pq}) and $H_{p\Lambda(p+1)}$ assigned to H_{pp}, then the minimum utility value would be given by

$$u_{\min}[S(a_l)] = \sum_{p=1}^{N}\sum_{q=p}^{N}\beta(H_{pq})u_{\min}(H_{pp}) + \sum_{p=1}^{N-1}\beta(\bar{H}_{p\Lambda(p+1)})u_{\min}(H_{(p+1(p+1)}). \tag{51}$$

We can also calculate the two most possible utilities and their average value as follows:

$$u_{\mathrm{MPV1}}[S(a_l)] = \sum_{p=1}^{N}\sum_{q=p}^{N}\beta(H_{pq})u_{\mathrm{MPV1}}(H_{pp}) + \sum_{p=1}^{N-1}\beta(\bar{H}_{p\Lambda(p+1)})u_{\mathrm{MPV1}}(\bar{H}_{p\Lambda(p+1)}) \tag{52}$$

$$u_{\mathrm{MPV2}}[S(a_l)] = \sum_{p=1}^{N}\sum_{q=p}^{N}\beta(H_{pq})u_{\mathrm{MPV2}}(H_{qq}) + \sum_{p=1}^{N-1}\beta(\bar{H}_{p\Lambda(p+1)})u_{\mathrm{MPV2}}(\bar{H}_{p\Lambda(p+1)}) \tag{53}$$

$$u_{\mathrm{AVG-MPV}}[S(a_l)] = \frac{u_{\mathrm{MPV1}}[S(a_l)] + u_{\mathrm{MPV2}}[S(a_l)]}{2}. \tag{54}$$

In order to generate the preference orders of all alternatives, we define the following dominance concepts.

1) Absolute Dominance: For alternatives A and B, A absolutely dominates B if and only if the lower bound of A's expected utility $u_{\min}[S(A)]$ is bigger than the upper bound of B's expected utility $u_{\max}[S(B)]$, i.e.,

$$A \succ B \Leftrightarrow u_{\min}[S(A)] \geq u_{\max}[S(B)]. \tag{55}$$

2) MPV Dominance: For alternatives A and B, A has MPV-dominance over B if and only if the value of $u_{\mathrm{MPV1}}[S(A)]$ is bigger than $u_{\mathrm{MPV2}}[S(B)]$, i.e.,

$$A \succ_{\mathrm{MPV}} B \Leftrightarrow u_{\mathrm{MPV1}}[S(A)] \geq u_{\mathrm{MPV2}}[S(B)]. \tag{56}$$

According to [53], the intersection fuzzy sets $H_{p\Lambda(p+1)}$ and $\bar{H}_{p\Lambda(p+1)}$ are different from what can be generated in traditional D-S combinations under nonfuzzy sets. These intersection sets capture the additional uncertainties caused by the fuzzy sets and are measured consistently by expected utility values as shown in (48) and (49). However, these intersection sets are beyond the basic evaluation grade sets defined by DMs. This explains why the possibility mass of these intersection sets was reassigned to the basic grade sets [46]. For example, the possibility mass assigned to $\bar{H}_{p\Lambda(p+1)}$, i.e., $m(\bar{H}_{p\Lambda(p+1)})$, is reassigned to the

TABLE I
EXPECTED UTILITIES BEFORE/AFTER REASSIGNMENT OF $m(\bar{H}_{p\Lambda(p+1)})$

	Before Reassignment	After Reassignment
$u[S(a_1)]$	$m(\bar{H}_{p\Lambda(p+1)})u(\bar{H}_{p\Lambda(p+1)})$	$\alpha_p m(\bar{H}_{p\Lambda(p+1)})u(H_{pp}) + (1-\alpha_p)m(\bar{H}_{p\Lambda(p+1)})u(H_{(p+1)(p+1)})$
$u_{\max}[S(a_l)]$	$m(\bar{H}_{p\Lambda(p+1)})u_{\max}(H_{pp})$	$\alpha_p m(\bar{H}_{p\Lambda(p+1)})u_{\max}(H_{pp}) + (1-\alpha_p)m(\bar{H}_{p\Lambda(p+1)})u_{\max}(H_{(p+1)(p+1)})$
$u_{\min}[S(a_l)]$	$m(\bar{H}_{p\Lambda(p+1)})u_{\min}(H_{(p+1)(p+1)})$	$\alpha_p m(\bar{H}_{p\Lambda(p+1)})u_{\min}(H_{pp}) + (1-\alpha_p)m(\bar{H}_{p\Lambda(p+1)})u_{\min}(H_{(p+1)(p+1)})$

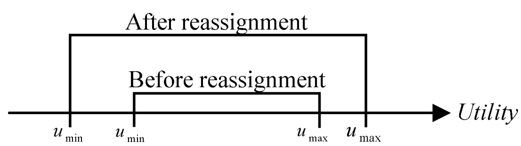


Fig. 5. Expected utility intervals before/after reassignment of $m(\bar{H}_{p\Lambda(p+1)})$.

grade sets H_{pp} and $H_{(p+1)(p+1)}$ as follows:

$$Bel(\bar{H}_{p\Lambda(p+1)} \subseteq H_{pp}) = \alpha_p \tag{57}$$

$$Bel(\bar{H}_{p\Lambda(p+1)} \subseteq H_{(p+1)(p+1)}) = 1 - \alpha_p \tag{58}$$

where α_p, $0 < \alpha_p < 1$, is determined according to the intersection properties of the two adjacent grade sets H_{pp} and $H_{(p+1)(p+1)}$ [46]. As a result, the possibility mass $m(\bar{H}_{p\Lambda(p+1)})$ in the final aggregation results is finally distributed to H_{pp} and $H_{(p+1)(p+1)}$, i.e., $\alpha_p m(\bar{H}_{p\Lambda(p+1)})$ is added to the possibility mass of H_{pp} and $(1-\alpha_p)m(\bar{H}_{p\Lambda(p+1)})$ to $H_{(p+1)(p+1)}$. However, this reassignment changes the basic expected utility value of $m(\bar{H}_{p\Lambda(p+1)})$. To show this, we can calculate the expected utilities before and after the reassignment of $m(\bar{H}_{p\Lambda(p+1)})$ in Table I. It is easy to get the following equations:

$$m(\bar{H}_{p\Lambda(p+1)})u_{\max}(H_{pp}) \leq \alpha_p m(\bar{H}_{p\Lambda(p+1)})u_{\max}(H_{pp}) + (1-\alpha_p)m(\bar{H}_{p\Lambda(p+1)})u_{\max}(H_{(p+1)(p+1)}) \tag{59}$$

$$m(\bar{H}_{p\Lambda(p+1)})u_{\min}(H_{(p+1)(p+1)}) \geq \alpha_p m(\bar{H}_{p\Lambda(p+1)})u_{\min}(H_{pp}) + (1-\alpha_p)m(\bar{H}_{p\Lambda(p+1)})u_{\min}(H_{(p+1)(p+1)}) \tag{60}$$

where exact inequalities hold in (59) and (60) if $m(\bar{H}_{p\Lambda(p+1)}) > 0$ and $0 < \alpha_p < 1$.

From the previous analysis, we can conclude that after the reassignment, the uncertainty interval between the maximum and minimum utility values is enlarged as shown in Fig. 5. Although the reassignment can simplify the analysis from the complex fuzzy grade set $\hat{H}_F = \{H_{pq}, p = 1, \ldots, N, q = p, \ldots, N\} \cup \{\bar{H}_{p\Lambda(p+1)}, p = 1, \ldots, N-1\}$ to a set of predefined assessment grades $\{H_{pq}, p = 1, \ldots, N, q = p, \ldots, N\}$, the reassignment procedure should be used with caution.

VI. APPLICATION OF THE FIER APPROACH TO A NEW PRODUCT SCREENING PROBLEM

The application concerned is a new product development (NPD) problem in an electronic manufacturer,[1] which manufactures a wide range of electronic entertainment products in its factories in southern China. Other than manufacturing, the company also markets its products in several countries including the United States, the United Kingdom, Canada, and Hong Kong. Every year, the company identifies market requirements and comes up with a list of potential product development projects. The selection of right projects is one of the key factors to determine a company's success. Thus, it is imperative to have a reliable system to evaluate potential projects. However, at a preliminary design phase, the assessment of a project on multiple criteria is mainly based on experts' subject judgments. Therefore, the NPD problem is characterized with a high level of uncertainty.

Suppose there are three projects available in this NPD.

Project 1 (*Motor Cycling*): It is a virtual 3-D interactive motor cycling game. The product is an extension of the existing product lines. The market is relatively mature and stable and there are limited risks in R&D, manufacturing, and distribution. However, the expected profit is not high.

Project 2 (*Sport Bass Fishing*)*:* It is a virtual fishing game that enables customers to feel the excitement of real fishing. With the product, customers can cast, feel the fish bite, set the hook with a jerk, and reel in the fish with a real handle. This is a brand new product that has high market potential. However, the product development project is highly risky because of high product price, uncertain distribution channel, high technological risk, and uncertainty of material/part availability.

Project 3 (*Play TV Baseball*)*:* It is an interactive game with which customers can play a virtual baseball game against a friend or a computer using a wireless bat. This is also a brand new product that has high market potential as well as risks.

In order to compare the three alternative projects, a detailed criteria hierarchy has been used by the company (similarly to [54]), which is shown in Table II.

Not surprisingly, there is a high level of uncertainty involved in such an NPD problem. Since all products are in their preliminary design phase, the assessment of the projects according to each criterion is mainly based on experts' subjective judgments. In general, most experts are willing to express their opinions by belief degrees (or possibility measures) based on a set of evaluation grades, i.e., {Bad, Poor, Average, Good, and Excellent}. However, we find that these evaluation grades may not be regarded as crisp sets. For example, it is difficult to separate the grade Bad from the grade Poor, especially if evaluations need to be given between these two grades. As such, the individual evaluation grade set can be defined as a fuzzy set H_F as follows:

$$H_F = \{H_{11}, H_{22}, H_{33}, H_{44}, H_{55}\} = \{\text{Bad, Poor, Average, Good, Excellent}\}.$$

[1]For confidentiality, the data presented in this section have been altered. However, the resulting qualitative relationships and insights drawn from this example are the same as they would be from using the actual data.

TABLE II
CRITERIA FOR EVALUATION OF THE PROJECTS

Level 1	Level 2
1. MKTFIT. Fit with firm's core marketing competencies.	1.1 TIMING. The product matches the desired launch timing needed by our target segment.
	1.2 PRICE. The product matches the desired price level for our target segment.
	1.3 LOGISTICS. The product fits with our logistics and distribution strengths.
	1.4 SALES. The product fits with our sales force coverage and strengths.
2. MANUFIT. Fit with firm's core manufacturing competencies.	2.1 MFGTECH. The product fits with our manufacturing technology.
	2.2 MFGCAP. The manufacturing capacity matches demands.
	2.3 SUPPLY. The product allows us to use our good suppliers.
3. CUSTFIT. Fit with customer's requirements.	3.1 DESIGN. The product is designed for the requirements needed by the customer.
	3.2 DIFFADV. The product gives the customer a differential advantage or benefit.
4. FINRISK. Financial risk of projects.	4.1 PAYOFFS. Expected payoffs.
	4.2 LOSSES. Potential losses.
5. TECUNCE. Uncertainty about project's outcomes.	5.1 R&DUNC. Technological uncertainties in research and product design.
	5.2 NONR&D. Uncertainties which are not related to research and product design (e.g. changes in government's regulations and markets).

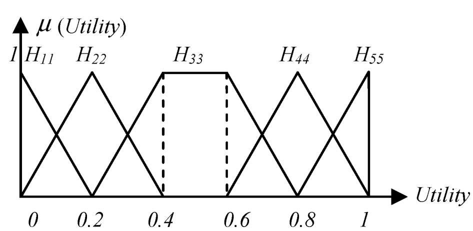


Fig. 6. Fuzzy utility of individual grade sets.

TABLE III
MEMBERSHIP FUNCTIONS OF THE INDIVIDUAL FUZZY GRADES AND THEIR FUZZY UTILITIES

Linguistic term	Membership functions of fuzzy grade utilities
Worst (W)	(0, 0, 0.2)
Poor (P)	(0, 0.2, 0.4)
Average (A)	(0.2, 0.4, 0.6, 0.8)
Good (G)	(0.6, 0.8, 1)
Excellent (E)	(0.8, 1, 1)

Based on the experts' opinions, we can approximate all the five individual assessment grades by either triangular or trapezoidal fuzzy numbers, and their membership function values can be determined according to the historical data and the detailed questionnaire answered by all experts, as shown in Fig. 6 and Table III. To simplify the calculation and without loss of generality, the maximum degree of membership for every fuzzy intersection set is 0.5, as shown in Table IV.

TABLE IV
MAXIMUM MEMBERSHIP DEGREES OF EACH FUZZY INTERSECTION

Fuzzy intersection subset	$H_{1\Lambda 2}$	$H_{2\Lambda 3}$	$H_{3\Lambda 4}$	$H_{4\Lambda 5}$
Maximum degree of membership ($\mu^{\max}_{p\Lambda(p+1)}$)	0.5	0.5	0.5	0.5

TABLE V
BELIEF MATRIX OF THE PERFORMANCE-ASSESSMENT PROBLEM

Criteria	Weights	Motor Cycling	Sport Bass Fishing	Play TV Baseball
TIMING	0.1	{ (H44, 1.0) }	{ (H34, 1.0) }	{ (H12, 1.0) }
PRICE	0.1	{ (H11, 1.0) }	{ (H44, 0.9), (H15, 0.1) }	{ (H45, 0.9), (H15, 0.1) }
LOGISTICS	0.05	{ (H44, 1.0) }	{ (H45, 1.0) }	{ (H45, 1.0) }
SALES	0.02	{ (H33, 1.0) }	{ (H33, 1.0) }	{ (H22, 1.0) }
MFGTECH	0.02	{ (H44, 1.0) }	{ (H33, 0.6), (H44, 0.4) }	{ (H22, 1.0) }
MFGCAP	0.02	{ (H44, 1.0) }	{ (H44, 1.0) }	{ (H45, 1.0) }
SUPPLY	0.05	{ (H34, 1.0) }	{ (H34, 1.0) }	{ (H45, 1.0) }
DESIGN	0.1	{ (H11, 1.0) }	{ (H44, 0.8), (H15, 0.2) }	{ (H45, 0.8), (H15, 0.2) }
DIFFADV	0.08	{ (H11, 1.0) }	{ (H55, 1.0) }	{ (H55, 1.0) }
PAYOFFS	0.08	{ (H11, 1.0) }	{ (H44, 0.8), (H15, 0.2) }	{ (H45, 0.8), (H15, 0.2) }
LOSSES	0.08	{ (H44, 1.0) }	{ (H22, 0.9), (H15, 0.1) }	{ (H23, 0.9), (H15, 0.1) }
R&DUNC	0.25	{ (H34, 1.0) }	{ (H34, 0.9), (H15, 0.1) }	{ (H12, 0.8), (H15, 0.2) }
NONR&D	0.05	{ (H44, 1.0) }	{ (H33, 0.8), (H15, 0.2) }	{ (H12, 0.8), (H15, 0.2) }

The evaluations provided by the experts based on the aforementioned criteria can be obtained as shown in Table V. Due to the high level of uncertainty involved in this NPD problem, it is not surprising that in some evaluations, experts prefer to give the belief degree measures on interval grades. For example, the TIMING for Sport Bass Fishing is {(H34, 1.0)}, which means that 100% belief is given to interval grade H_{34}, i.e., the worst assessment for Sport Bass Fishing on TIMING is Average, and the highest is Good. However, the exact belief degree to each of the two grades is not known. In a similar way, the incomplete opinions of experts in evaluating this NPD problem can be captured conveniently by the following fuzzy evaluation grades $\hat{H}_F$, as defined in Section IV-A:

$$\hat{H}_F = \left\{ \begin{array}{ccccc} H_{11} & H_{12} & H_{13} & H_{14} & H_{15} \\ & H_{22} & H_{23} & H_{24} & H_{25} \\ & & H_{33} & H_{34} & H_{35} \\ & & & H_{44} & H_{45} \\ & & & & H_{55} \end{array} \right\} \cup \{\bar{H}_{1\Lambda 2}, \bar{H}_{2\Lambda 3}, \bar{H}_{3\Lambda 4}, \bar{H}_{4\Lambda 5}\}.$$

TABLE VI
AGGREGATED PERFORMANCE DISTRIBUTIONS GENERATED BY THE FIER ALGORITHM

$\beta(H_{pq})$ p \ q	1	2	3	4	5
1	0.316	0	0	0	0
2	0	0	0	0	0
3	0	0	0.020	0.278	0
4	0	0	0	0.381	0
5	0	0	0	0	0
$\beta(\bar{H}_{q\Lambda(q+1)})$	0	0	0.005	0	

a) Motor Cycling

$\beta(H_{pq})$ p \ q	1	2	3	4	5
1	0	0	0	0	0.062
2	0	0.049	0	0	0
3	0	0	0.073	0.323	0
4	0	0	0	0.333	0.033
5	0	0	0	0	0.058
$\beta(\bar{H}_{q\Lambda(q+1)})$	0	0.016	0.016	0.037	

b) Sport Bass Fishing

$\beta(H_{pq})$ p \ q	1	2	3	4	5
1	0	0.334	0	0	0.094
2	0	0.074	0.058	0	0
3	0	0	0	0	0
4	0	0	0	0	0.330
5	0	0	0	0	0.094
$\beta(\bar{H}_{q\Lambda(q+1)})$	0	0	0.016	0	

c) Play TV Baseball

By using our proposed FIER methodology, the aggregated performance distribution of all the three alternative projects can be calculated and shown in Table VI. The expected maximum and minimum utilities can also be calculated according to formulae (50)–(54), as shown in Table VII. A final rank order can be obtained as follows. Sport Bass Fishing is possibly better than Motor Cycling and Play TV Baseball according to the average MPV values of all the three projects presented. However, it is obviously that Sport Bass Fishing does not absolutely dominate the other two projects. This is because

$$u_{\min}[\text{S(Sport Bass Fishing)}] = 0.3878 < u_{\max}[\text{S(Motor Cycling)}] = 0.7420$$

$$u_{\min}[\text{S(Sport Bass Fishing)}] = 0.3878 < u_{\max}[\text{S(Play TV Baseball)}] = 0.7406.$$

TABLE VII
FUZZY EXPECTED UTILITIES AND RANKING ORDER OF ALTERNATIVES

	Fuzzy expected utility u				
	Lower bound $u_{\min}[S(a_l)]$	Most possible value $u_{MPV1}[S(a_l)]$, $u_{MPV2}[S(a_l)]$		Upper bound $u_{\max}[S(a_l)]$	Avg. of MPV $u_{AVG-MPV}[S(a_l)]$
Motor Cycling	0.2910	0.4273	0.5425	0.7420	0.4849
Sport Bass Fishing	0.3878	0.5686	0.7809	0.9433	0.6747
Play TV Baseball	0.2831	0.3960	0.6458	0.7406	0.5209

While in the sense of MPV dominance, we can obtain

$$u_{\mathrm{MPV1}}[\text{S(Sport Bass Fishing)}] = 0.5686 > u_{\mathrm{MPV2}}[\text{S(Motor Cycling)}] = 0.5425.$$

This means that Sport Bass Fishing is preferred to Motor Cycling in the sense of MPV dominance, or

$$\text{Sport Bass Fishing} \succ_{\mathrm{MPV}} \text{Motor Cycling}.$$

While the relationships between Sport Bass Fishing and Play TV Baseball as well as Play TV Baseball and Motor Cycling are not clear even in the sense of MPV dominance due to the uncertainties in the initial assessment data. In order to generate clearer dominant relations, more information or more accurate evaluations are needed, and the similar techniques are given in [48].

It is clear that the IER approach is a special case of our FIER approach if only we let the maximum, minimum, MPV1, and MPV2 utilities of each individual grade take the same values, and all the $\mu^{\max}_{p\Lambda(p+1)}$, $p = 1, \ldots, N-1$, the maximum degree of membership of the fuzzy intersection subset $H_{p\Lambda(p+1)}$, are equal to zeros. To show this, we can redefine the fuzzy individual assessment grades as nonfuzzy grades by setting the utility values of the five individual grades as Worst (W): 0; Poor (P): 0.2; Average (A): 0.5; Good (G): 0.8; Excellent (E): 1.0. Then, the evaluations based on the same assessment data shown in Table V can be recalculated by our FIER algorithm, which have the same results of the IER approach.

Furthermore, it would be interesting if we compare the previous nonfuzzy results with the fuzzy ones listed in Table VII. As shown in Fig. 7, in the fuzzy grades case, the minimum utility value of each alternative $u_{\min}[S(a_l)]$ is denoted by F_Umin, the maximum utility value $u_{\max}[S(a_l)]$ by F_Umax, the lower most possible utility value $u_{\mathrm{MPV1}}[S(a_l)]$ by F_Umpv1, the higher most possible utility value $u_{\mathrm{MPV2}}[S(a_l)]$ by F_Umpv2, and the average of MPV utility value $u_{\mathrm{AVG-MPV}}[S(a_l)]$ by F_Uavg_mpv, while in the nonfuzzy grades case, $u_{\min}[S(a_l)]$ is denoted by NF_Umin, $u_{\max}[S(a_l)]$ by NF_Umax, and the average utility value $u_{\mathrm{avg}}[S(a_l)] = \{u_{\min}[S(a_l)] + u_{\max}[S(a_l)]\}/2$ by NF_Uavg. Due to the fuzziness of the five individual evaluation grades, the uncertainties of the final evaluations for the three alternatives are all enlarged apparently in comparison with the nonfuzzy results. In the case of

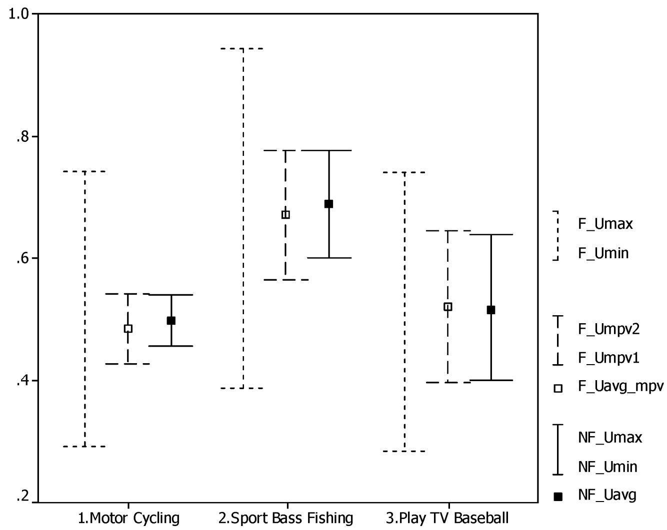


Fig. 7. Comparisons between fuzzy and nonfuzzy grade cases.

fuzzy grades, the intervals between the lower bound of the fuzzy expected utility $u_{\min}[S(a_l)]$ and the upper bound $u_{\max}[S(a_l)]$ are surprisingly large for the three alternatives, and only the intervals between $u_{\mathrm{MPV1}}[S(a_l)]$ and $u_{\mathrm{MPV2}}[S(a_l)]$ show that Motor Cycling has less uncertainty in evaluations than the other two alternatives. This can be explained by the fact that our definitions of the membership functions of the fuzzy assessment grades and their fuzzy utility values in Table III contain many uncertainties that may overwhelm that of the basic assessment values given by experts in Table V. These results shed light on the fact that we must pay much more attention to reduce the uncertainties in the determinations of the fuzzy utility values of each individual grade when these grades are defined as fuzzy sets.

VII. Concluding Remarks

Incompleteness and fuzziness are among the most common uncertainties in complex MADA problems. The new development, as reported in this paper, further extends the capability of the ER approach to utilize information with both local ignorance or interval uncertainty and fuzzy linguistic evaluation grades. Expert judgments can be captured by our proposed FIER method in such a convenient way that the evaluations made by experts, which are incomplete and fuzzy in nature, do not need to be converted to some strictly defined formats that may inevitably lead to the loss of important information, as shown in some classical MADA methods. In this sense, our FIER method can be used to deal with various types of uncertainties to help DMs in making more informative decisions.

Similarly to the previous ER approaches, this FIER method is aimed at generating the preference orders of alternatives without having to gather perfect or complete information, as is often done in real-life decision making. However, the results obtained using the new methods may be due to an incomplete or partial preference order as well due to the incompleteness and fuzziness in initial data, as illustrated in the example. In such cases, more information may be needed to support specific decision making such as finding a single winner in a performance-assessment problem. Further research is needed to investigate the process of information gathering for sensitivity analysis and develop our methodology to express more complex yet pragmatic fuzzy membership functions (other than the triangular or trapezoidal ones) that appear in real applications.

Appendix A
Proof of Theorem 1

Suppose L is the total number of the attributes in the MADA problem. It is easy to show that if $L = 1$, Theorem 1 is apparently true. Therefore, we only need to prove the case that $L \geq 2$.

We note that both the original ER and IER algorithms follow the D-S combination rules, and can be expressed consistently in (6)–(13), if we ignore the difference between $H = \{H_1, H_2, \ldots, H_N, H_{1N}\}$ and $\hat{H} = \{H_{pq}, p = 1, \ldots, N, q = p, \ldots, N\}$. Therefore, in the following proof, we use H as the assessment grade set to mean either $H = \{H_1, H_2, \ldots, H_N, H_{1N}\}$ or $\hat{H} = \{H_{pq}, p = 1, \ldots, N, q = p, \ldots, N\}$ without differing them.

In order to prove Theorem 1, we will first prove the following proposition.

Proposition A1: For a given integer s, $1 \leq s \leq L$, in the original ER and IER recursive combination algorithms, the aggregation results of the first s attributes can be obtained by postponing the normalization in each iteration to the end without changing the results.

If $s = 1$, Proposition A1 is apparently true.

For a given integer s, $s \geq 1$, suppose Proposition A1 is true, i.e., the normalization step in the first s combinations can be postponed in the end without changing the combination results. If we denote $m_{I(s)}(\cdot)$ as the combination results of the first s attributes in the original ER/IER approaches, which contain the normalization step in each combination, and $\tilde{m}_{I(s)}(\cdot)$ as the results of the s attributes without normalization in each combination, and $\tilde{K}_{I(s)}$ is the scale factor in the final normalization step, then

$$m_{I(s)}(C) = \frac{1}{\tilde{K}_{I(s)}} \tilde{m}_{I(s)}(C), \quad C \neq \Phi, \quad C \in H \tag{A1}$$

$$m_{I(s)}(U) = \frac{1}{\tilde{K}_{I(s)}} \tilde{m}_{I(s)}(U) \tag{A2}$$

$$\tilde{K}_{I(s)} = \sum_{C \in H} \tilde{m}_{I(s)}(C) + \tilde{m}_{I(s)}(U). \tag{A3}$$

For $s + 1$, we will prove that Proposition A1 remains correct.

According to the original ER and IER algorithm [45], [49], [51], the combination results of the first $s + 1$ attributes are

$$m_{I(s+1)}(C) = \frac{1}{K_{I(s)}} \left[\sum_{A \cap B = C} m_{I(s)}(A) m_{s+1}(B) + m_{I(s)}(C) m_{s+1}(U) + m_{I(s)}(U) m_{s+1}(C) \right] \tag{A4}$$

$$m_{I(s+1)}(U) = \frac{1}{K_{I(s)}} m_{I(s)}(U) m_{s+1}(U) \tag{A5}$$

$$K_{I(s)} = 1 - \sum_{A \cap B = \Phi} m_{I(s)}(A) m_{s+1}(B). \tag{A6}$$

We can substitute $m_{I(s)}(\cdot)$ with the right-hand sides of (A1) and (A2) as follows:

$$m_{I(s+1)}(C) = \frac{1}{K_{I(s)} \tilde{K}_{I(s)}} \left[\sum_{A \cap B = C} \tilde{m}_{I(s)}(A) m_{s+1}(B) + \tilde{m}_{I(s)}(C) m_{s+1}(U) + \tilde{m}_{I(s)}(U) m_{s+1}(C) \right] \tag{A7}$$

$$m_{I(s+1)}(U) = \frac{1}{K_{I(s)} \tilde{K}_{I(s)}} \tilde{m}_{I(s)}(U) m_{s+1}(U). \tag{A8}$$

From the properties of the ER combining rules [45], [49], [51], the equation $\sum_{C \in H} m_{I(s+1)}(C) + m_{I(s+1)}(U) = 1$ will always hold; therefore

$$K_{I(s)} \tilde{K}_{I(s)} = \sum_{C \in H} \left[\sum_{A \cap B = C} \tilde{m}_{I(s)}(A) m_{s+1}(B) + \tilde{m}_{I(s)}(C) m_{s+1}(U) + \tilde{m}_{I(s)}(U) m_{s+1}(C) \right] + \tilde{m}_{I(s)}(U) m_{s+1}(U). \tag{A9}$$

On the other hand, we will aggregate the first $s + 1$ attributes with the normalization step postponed in the end. In order to do this, first, we aggregate the $s + 1$ attributes without normalization in each combination:

$$\tilde{m}_{I(s+1)}(C) = \sum_{A \cap B = C} \tilde{m}_{I(s)}(A) m_{s+1}(B) + \tilde{m}_{I(s)}(C) m_{s+1}(U) + \tilde{m}_{I(s)}(U) m_{s+1}(C) \tag{A10}$$

$$\tilde{m}_{I(s+1)}(U) = \tilde{m}_{I(s)}(U) m_{s+1}(U). \tag{A11}$$

Then, we can normalize the sum of mass to one by multiplying $\tilde{m}_{I(s+1)}(U)$ and $\tilde{m}_{I(s+1)}(C)$ with $\tilde{K}_{I(s+1)}$:

$$\tilde{K}_{I(s+1)} = \sum_{C \in H} \tilde{m}_{I(s+1)}(C) + \tilde{m}_{I(s+1)}(U). \tag{A12}$$

Substituting $\tilde{m}_{I(s+1)}(\cdot)$ with the right-hand sides of (A10) and (A11), we get

$$\tilde{K}_{I(s+1)} = \sum_{C \in H} \left[\sum_{A \cap B = C} \tilde{m}_{I(s)}(A) m_{s+1}(B) + \tilde{m}_{I(s)}(C) m_{s+1}(U) + \tilde{m}_{I(s)}(U) m_{s+1}(C) \right] + \tilde{m}_{I(s)}(U) m_{s+1}(U). \tag{A13}$$

From (A9), we get

$$\tilde{K}_{I(s+1)} = K_{I(s)} \tilde{K}_{I(s)}. \tag{A14}$$

Therefore, the normalizations to $\tilde{m}_{I(s+1)}(U)$ and $\tilde{m}_{I(s+1)}(C)$ by $\tilde{K}_{I(s+1)}$ will yield $m_{I(s+1)}(U)$ and $m_{I(s+1)}(C)$, respectively. The results generated using the original ER/IER combination rules with normalization at each combination step are the same as the results with the normalization postponed to the end, i.e.,

$$m_{I(s+1)}(U) = \frac{\tilde{m}_{I(s+1)}(U)}{\tilde{K}_{I(s+1)}} \tag{A15}$$

$$m_{I(s+1)}(C) = \frac{\tilde{m}_{I(s+1)}(C)}{\tilde{K}_{I(s+1)}}. \tag{A16}$$

Next, we can continue the previous procedures until all the L attributes are aggregated. Therefore, Proposition A1 is true for any s, $s = 1, \ldots, L$.

It is easy to show that Theorem 1 is a special case of Proposition A1 if $s = L$. Q.E.D.

Appendix B
Proof of the FIER Aggregation Algorithm

From the combination rule of the D-S theory and the ER methodology, (27)–(33) can be obtained straightforwardly. Then, we need to prove formula (34)–(36) in which the assessments for the first $I(s)$ attributes are combined with that for attribute $s + 1$ to generate an assessment for the $I(s + 1)$ attributes.

We can separate the whole sets that take part in the combinations into three categories. The first category includes the individual or interval grade sets H_{pq} for $p = 1, \ldots, N$, $q = p, \ldots, N$, the second is the fuzzy intersection sets $H_{p\Lambda(p+1)}$ for $p = 1, \ldots, N - 1$, and the third is the set U.

It can be shown that these sets have the following relations:

$$H_{kq} \cap H_{pl} = H_{pq}, \quad q \geq p, \quad k = 1, \ldots, p \quad l = q, \ldots, N \tag{A17}$$

$$H_{kp} \cap H_{(p+1)q} = H_{p\Lambda(p+1)}, \quad k = 1, \ldots, p \quad q = p + 1, \ldots, N, \quad p = 1, \ldots, N - 1 \tag{A18}$$

$$H_{kl} \cap H_{p\Lambda p+1} = H_{p\Lambda(p+1)}, \quad k = 1, \ldots, p + 1 \quad l = p, \ldots, N, \quad k \leq l, \quad p = 1, \ldots, N - 1 \tag{A19}$$

$$H_{p\Lambda(p+1)} \cap U = H_{p\Lambda(p+1)}, \quad p = 1, \ldots, N - 1 \tag{A20}$$

$$H_{pq} \cap U = H_{pq}, \quad p = 1, \ldots, N, \quad q = p, \ldots, N. \tag{A21}$$

Regarding H_{pq}, only $H_{kq} \cap H_{pl}$ and $H_{pq} \cap U$ have the results of H_{pq}, although without any relation with the set $H_{p\Lambda(p+1)}$, and therefore, the combination results for H_{pq} are consistent with those of the IER algorithm, i.e.,

$$\begin{aligned}\tilde{m}_{I(s+1)}(H_{pq}) = & -\tilde{m}_{I(s)}(H_{pq})m_{s+1}(H_{pq}) \\ & + \sum_{k=1}^{p}\sum_{l=q}^{N}[\tilde{m}_{I(s)}(H_{kl})m_{s+1}(H_{pq}) \\ & + \tilde{m}_{I(s)}(H_{pq})m_{s+1}(H_{kl})] \\ & + \sum_{k=1}^{p-1}\sum_{l=q+1}^{N}[\tilde{m}_{I(s)}(H_{kq})m_{s+1}(H_{pl}) \\ & + \tilde{m}_{I(s)}(H_{pl})m_{s+1}(H_{kq})] \\ & + \tilde{m}_{I(s)}(U)m_{s+1}(H_{pq}) \\ & + \tilde{m}_{I(s)}(H_{pq})m_{s+1}(U). \end{aligned} \tag{A22}$$

Regarding $H_{p\Lambda(p+1)}$, from the combination rule of the D-S theory and the ER methodology, we only need to consider the cases $H_{kp} \cap H_{(p+1)q} = H_{p\Lambda(p+1)}$, $H_{kl} \cap H_{p\Lambda p+1} = H_{p\Lambda(p+1)}$, and $H_{p\Lambda(p+1)} \cap U = H_{p\Lambda(p+1)}$; therefore

$$\begin{aligned}\tilde{m}_{I(s+1)}(H_{p\Lambda(p+1)}) = & \sum_{k=1}^{p}\tilde{m}_{I(s)}(H_{kp})\sum_{q=p+1}^{N} m_{s+1}(H_{(p+1)q}) \\ & + \sum_{q=p+1}^{N}\tilde{m}_{I(s)}(H_{(p+1)q})\sum_{k=1}^{p} m_{s+1}(H_{kp}) \\ & + \sum_{k=1}^{p+1}\sum_{\substack{l=p \\ l\geq k}}^{N}[\tilde{m}_{I(s)}(H_{p\Lambda(p+1)})m_{s+1}(H_{kl}) \\ & + \tilde{m}_{I(s)}(H_{kl})m_{s+1}(H_{p\Lambda(p+1)})] \\ & + \tilde{m}_{I(s)}(H_{p\Lambda(p+1)})m_{s+1}(U) \\ & + \tilde{m}_{I(s)}(U)m_{s+1}(H_{p\Lambda(p+1)}). \end{aligned} \tag{A23}$$

Since $m_{s+1}(H_{p\Lambda(p+1)}) = 0$, we get

$$\begin{aligned}\tilde{m}_{I(s+1)}(H_{p\Lambda(p+1)}) = & \sum_{k=1}^{p}\tilde{m}_{I(s)}(H_{kp})\sum_{q=p+1}^{N} m_{s+1}(H_{(p+1)q}) \\ & + \sum_{q=p+1}^{N}\tilde{m}_{I(s)}(H_{(p+1)q})\sum_{k=1}^{p} m_{s+1}(H_{kp}) \\ & + \sum_{k=1}^{p+1}\sum_{\substack{l=p \\ l\geq k}}^{N}\tilde{m}_{I(s)}(H_{p\Lambda(p+1)})m_{s+1}(H_{kl}) \\ & + \tilde{m}_{I(s)}(H_{p\Lambda(p+1)})m_{s+1}(U) \end{aligned} \tag{A24}$$

and it is straightforward to obtain

$$\tilde{m}_{I(s+1)}(U) = \tilde{m}_{I(s)}(U)m_{s+1}(U) = \prod_{l=1}^{s+1} m_l(U). \tag{A25}$$

Finally, after all the L attributes are combined, we get

$$\tilde{m}(H_{pq}) = \tilde{m}_{I(L)}(H_{pq}) \tag{A26}$$

$$\tilde{m}(U) = \tilde{m}_{I(L)}(U) \tag{A27}$$

and

$$\tilde{m}(H_{p\Lambda(p+1)}) = \tilde{m}_{I(L)}(H_{p\Lambda(p+1)}). \tag{A28}$$

Since the fuzzy subset $H_{p\Lambda(p+1)}$ is the intersection of the two fuzzy assessment grades H_{kp} and $H_{(p+1)q}$, its maximum degree of membership is normally not equal to unity. It is necessary to normalize $H_{p\Lambda(p+1)}$ to $\bar{H}_{p\Lambda(p+1)}$ as defined in (25). From [53], the possibility mass assigned to $\bar{H}_{p\Lambda(p+1)}$ and that of $H_{p\Lambda(p+1)}$ should meet the following relation:

$$\tilde{m}(\bar{H}_{p\Lambda(p+1)}) = \mu_{p\Lambda(p+1)}^{\max}\tilde{m}(H_{p\Lambda(p+1)}) \tag{A29}$$

and according to [53] and Theorem 1, the combination results should be normalized at the end, as in (A30), shown at the bottom of the page, and

$$m(H_{pq}) = K\tilde{m}(H_{pq}) = K\tilde{m}_{I(L)}(H_{pq}) \tag{A31}$$

$$\begin{aligned} m(\bar{H}_{p\Lambda(p+1)}) &= K\tilde{m}(\bar{H}_{p\Lambda(p+1)}) \\ &= K\mu_{p\Lambda(p+1)}^{\max}\tilde{m}_{I(L)}(H_{p\Lambda(p+1)}) \end{aligned} \tag{A32}$$

$$m(U) = K\tilde{m}(U) = K\tilde{m}_{I(L)}(U). \tag{A33}$$

Therefore, (37)–(40) are also proved, and we can also get (41) and (42). Q.E.D.

REFERENCES

[1] T. L. Saaty, *The Analytic Hierarchy Process*. New York: McGraw-Hill, 1980.

[2] R. L. Keeney and H. Raiffa, *Decisions With Multiple Objectives-Preferences and Value Tradeoffs*, 2nd ed. Cambridge, U.K.: Cambridge Univ. Press, 1993.

[3] E. Jacquet-Lagreze and J. Siskos, "Assessing a set of additive utility functions for multicriteria decision making, the UTA method," *Eur. J. Oper. Res.*, vol. 10, no. 2, pp. 151–164, 1982.

[4] V. Belton and T. J. Stewart, *Multiple Criteria Decision Analysis—An Integrated Approach*. Boston, MA: Kluwer, 2002.

[5] R. E. Moore, *Method and Application of Interval Analysis*. Philadelphia, PA: SIAM, 1979.

[6] H. Ishibuchi and H. Tanaka, "Multiobjective programming in optimization of the interval objective function," *Eur. J. Oper. Res.*, vol. 48, pp. 219–225, 1990.

[7] S. Kundu, "Min-transitivity of fuzzy leftness relationship and its application to decision making," *Fuzzy Sets Syst.*, vol. 86, pp. 357–367, 1997.

[8] A. Arbel and L. G. Vargas, "The analytic hierarchy process with interval judgments," in *Multiple Criteria Decision Making*, A. Goicoechea,

$$\begin{aligned} K &= \frac{1}{\left[\sum_{p=1}^{N}\sum_{q=p}^{N}\tilde{m}(H_{pq}) + \sum_{p=1}^{N-1}\tilde{m}(\bar{H}_{p\Lambda(p+1)}) + \tilde{m}(U)\right]} \\ &= \frac{1}{\left[\sum_{p=1}^{N}\sum_{q=p}^{N}\tilde{m}_{I(L)}(H_{pq}) + \sum_{p=1}^{N-1}\mu_{p\Lambda(p+1)}^{\max}\tilde{m}_{I(L)}(H_{p\Lambda(p+1)}) + \tilde{m}_{I(L)}(U)\right]} \end{aligned} \tag{A30}$$

L. Duckstein, and S. Zoints, Eds., New York: Springer-Verlag, 1992, pp. 61–70.

[9] A. Arbel and L. G. Vargas, "Preference simulation and preference programming: robustness issues in priority deviation," *Eur. J. Oper. Res.*, vol. 69, pp. 200–209, 1993.

[10] A. Salo and R. P. Hämäläinen, "Processing interval judgments in the analytic hierarchy process," in *Multiple Criteria Decision Making*, A. Goicoechea, L. Duckstein, and S. Zoints, Eds., New York: Springer-Verlag, 1992, pp. 359–372.

[11] A. Salo and R. P. Hämäläinen, "Preference programming through approximate ratio comparisons," *Eur. J. Oper. Res.*, vol. 82, no. 3, pp. 458–475, 1995.

[12] R. Islam, M. P. Biswal, and S. S. Alam, "Preference programming and inconsistent interval judgments," *Eur. J. Oper. Res.*, vol. 97, pp. 53–62, 1997.

[13] L. A. Zadeh, "The concepts of a linguistic variable and its application to approximate reasoning (I), (II), (III)," *Inf. Sci.*, vol. 8, pp. 9, 43–80, 199–249, and 301–357, 1975.

[14] L. A. Zadeh, "Fuzzy sets as a basis for a theory of possibility," *Fuzzy Sets Syst.*, vol. 1, no. 1, pp. 3–28, 1978.

[15] R. R. Yager, "Fuzzy decision making including unequal objectives," *Fuzzy Sets Syst.*, vol. 1, pp. 87–95, 1978.

[16] R. R. Yager, "Generalized probabilities of fuzzy events from fuzzy belief structures," *Inf. Sci.*, vol. 28, pp. 45–62, 1982.

[17] T. Denoeux, "Modelling vague belief using fuzzy-valued belief structures," *Fuzzy Sets Syst.*, vol. 116, pp. 167–199, 2000.

[18] C. Carlsson and R. Fuller, "Fuzzy multiple criteria decision making: Recent developments," *Fuzzy Sets Syst.*, vol. 78, pp. 139–153, 1996.

[19] P. J. M. Laarhoven and W. Pedrycz, "A fuzzy extension of Saaty's priority theory," *Fuzzy Sets Syst.*, vol. 11, pp. 229–241, 1983.

[20] R. E. Bellman and L. A. Zadeh, "Decision-making in a fuzzy environment," *Manage. Sci.*, vol. 17, no. 4, pp. 141–164, 1970.

[21] R. R. Yager, "Multiple objective decision-making using fuzzy sets," *Int. J. Man-Machine Stud.*, vol. 9, pp. 375–382, 1977.

[22] R. R. Yager, "A new methodology for ordinal multi-objective decisions based on fuzzy sets," *Decis. Sci.*, vol. 12, pp. 589–600, 1981.

[23] S. Baas and H. Kwakernaak, "Rating and raking of multiple-aspect alternatives using fuzzy sets," *Automatica*, vol. 13, pp. 47–58, 1977.

[24] D. Dubois and H. Prade, *Fuzzy Sets and Systems: Theory and Applications*. New York: Academic, 1980.

[25] W. M. Dong and F. S. Wong, "Fuzzy weighted averages and implementation of the extension principle," *Fuzzy Sets Syst.*, vol. 21, pp. 183–199, 1987.

[26] T. Y. Tseng and C. M. Klein, "A new algorithm for fuzzy multicriteria decision making," *Int. J. Approx. Reason.*, vol. 6, pp. 45–66, 1992.

[27] R. R. Yager, "On ordered weighted averaging aggregation operators in multicriteria decision making," *IEEE Trans. Syst., Man, Cybern. B, Cybern.*, vol. 18, no. 1, pp. 183–190, Jan./Feb. 1988.

[28] A. P. Dempster, "Upper and lower probabilities induced by a multi-valued mapping," *Ann. Math. Stat.*, vol. 38, pp. 325–339, 1967.

[29] G. A. Shafer, *Mathematical Theory of Evidence*. Princeton, NJ: Princeton Univ. Press, 1976.

[30] J. Teng, "Development of a supplier pre-qualification model for Siemens UK," M.Sc. dissertation, Manchester School Manage., Univ. Manchester Inst. Sci. Technol. (UMIST), Manchester, U.K., 2002.

[31] I. Okundi, "An e-business risk assessment model for Siemens UK," M.Sc. dissertation, Manchester School Manage., Univ. Manchester Inst. Sci. Technol. (UMIST), Manchester, U.K., 2001.

[32] M. Sonmez, J. B. Yang, and G. D. Holt, "Addressing the contractor selection problem using an evidential reasoning approach," *Eng., Constr. Archit. Manage.*, vol. 8, no. 3, pp. 198–210, 2001.

[33] J. B. Yang and D. L. Xu, "Intelligent decision system for supplier assessment," presented at the DSS2004: The 2004 IFIP Conf. Decis. Support Syst., Prato, Italy.

[34] C. H. R. Siow, J. B. Yang, and B. G. Dale, "A new modeling framework for organizational self-assessment: Development and application," *Qual. Manage. J.*, vol. 8, no. 4, pp. 34–47, 2001.

[35] J. B. Yang, M. Deng, and D. L. Xu, "Nonlinear regression to estimate both weights and utilities via evidential reasoning for MADM," presented at the 5th Int. Conf. Optim.: Techn. Appl., Hong Kong, Dec. 15–17, 2001.

[36] D. L. Xu and J. B. Yang, "Intelligent decision system for self-assessment," *J. Multi-Criteria Decis. Anal.*, vol. 12, pp. 43–60, 2003.

[37] J. Wang, J. B. Yang, and P. Sen, "Safety analysis and synthesis using fuzzy sets and evidential reasoning," *Reliab. Eng. Syst. Saf.*, vol. 47, no. 2, pp. 103–118, 1995.

[38] J. Wang, J. B. Yang, and P. Sen, "Multi-person and multi-attribute design evaluations using evidential reasoning based on subjective safety and cost analysis," *Reliab. Eng. Syst. Saf.*, vol. 52, pp. 113–127, 1996.

[39] J. Wang, "A subjective methodology for safety analysis of safety requirements specifications," *IEEE Trans. Fuzzy Syst.*, vol. 5, no. 3, pp. 1–13, Aug. 1997.

[40] J. Wang and J. B. Yang, "A subjective safety based decision making approach for evaluation of safety requirements specifications in software development," *Int. J. Reliab., Qual. Saf. Eng.*, vol. 8, no. 1, pp. 35–57, 2001.

[41] P. Sen and J. B. Yang, "Multiple criteria decision making in design selection and synthesis," *J. Eng. Design*, vol. 6, no. 3, pp. 207–230, 1995.

[42] J. B. Yang and P. Sen, "Multiple attribute design evaluation of large engineering products using the evidential reasoning approach," *J. Eng. Design*, vol. 8, no. 3, pp. 211–230, 1997.

[43] J. B. Yang and P. Sen, "A general multi-level evaluation process for hybrid MADM with uncertainty," *IEEE Trans. Syst., Man, Cybern.*, vol. 24, no. 10, pp. 1458–1473, Oct. 1994.

[44] J. B. Yang and D. L. Xu, "An intelligent decision system based on evidential reasoning approach and its applications," presented at the 3rd Int. Conf. Decis. Support Telecommun. Inf. Soc., Warsaw, Poland, Sep. 3–6, 2003.

[45] D. L. Xu, J. B. Yang, and Y. M. Wang, "The evidential reasoning approach for multi-attribute decision analysis under interval uncertainty," *Eur. J. Oper. Res.*, vol. 174, no. 3, pp. 1914–1943, 2005.

[46] J. B. Yang, Y. M. Wang, D. L. Xu, and K. S. Chin, "The evidential reasoning approach for MADA under both probabilistic and fuzzy uncertainties," *Eur. J. Oper. Res.*, vol. 171, pp. 309–343, 2006.

[47] K. S. Chin, J. B. Yang, M. Guo, and J. P. K. Lam, "An evidential reasoning-interval based method for new product design assessment," *IEEE Trans. Eng. Manage.*, vol. 56, no. 1, pp. 142–156, 2009.

[48] M. Guo, J. B. Yang, K. S. Chin, and H. W. Wang, "Evidential reasoning based preference programming for multiple attribute decision analysis under uncertainty," *Eur. J. Oper. Res.*, vol. 182, pp. 1294–1312, 2007.

[49] J. B. Yang and M. G. Singh, "An evidential reasoning approach for multiple attribute decision making with uncertainty," *IEEE Trans. Syst., Man, Cybern.*, vol. 24, no. 1, pp. 1–18, Jan. 1994.

[50] J. B. Yang, "Rule and utility based evidential reasoning approach for multiple attribute decision analysis under uncertainty," *Eur. J. Oper. Res.*, vol. 131, no. 1, pp. 31–61, 2001.

[51] J. B. Yang and D. L. Xu, "On the evidential reasoning algorithm for multiattribute decision analysis under uncertainty," *IEEE Trans. Syst., Man, Cybern. A, Syst., Humans*, vol. 32, no. 3, pp. 289–304, May 2002.

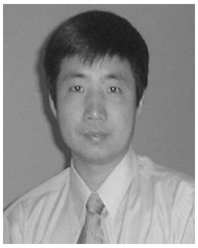
[52] J. B. Yang and D. L. Xu, "Nonlinear information aggregation via evidential reasoning in multiattribute decision analysis under uncertainty," *IEEE Trans. Syst., Man, Cybern. A, Syst., Humans*, vol. 32, no. 3, pp. 376–393, May 2002.

[53] J. Yen, "Generalizing the Dempster–Shafer theory to fuzzy sets," *IEEE Trans. Syst., Man, Cybern.*, vol. 20, no. 3, pp. 559–570, May/Jun. 1990.

[54] R. J. Calantone, C. A. D. Benedetto, and J. B. Schmidt, "Using the analytic hierarchy process in new product screening," *J. Prod. Innov. Manage.*, vol. 16, pp. 65–76, 1999.

Min Guo received the B.Eng. degree in automatic control from Taiyuan University of Technology, Taiyuan, China, in 1991 and the M.Eng. degree in control science and engineering and the Ph.D. degree in systems engineering from Huazhong University of Science and Technology, Wuhan, China, in 1998 and 2002, respectively.

He is currently an Associate Professor with the Department of Control Science and Engineering, Huazhong University of Science and Technology. His current research interests include decision makings with the evidential reasoning approach, optimization, and supply chain management.



Jian-Bo Yang received the B.Eng. and M.Eng. degrees in control engineering from the North Western Polytechnic University, Xi'an, China, in 1981 and 1984, respectively, and the Ph.D. degree in systems engineering from Shanghai Jiao Tong University, Shanghai, China, in 1987.

He was a faculty member at the University of Birmingham (1995–1997), Birmingham, U.K.; the University of Newcastle upon Tyne (1991–1995), Newcastle, U.K.; the University of Manchester Institute of Science and Technology (UMIST) (1990), Manchester, U.K.; and Shanghai Jiao Tong University (1987–1989). He is currently the Chair of Decision and System Sciences and the Director of the Decision Sciences Research Centre, Manchester Business School (MBS), The University of Manchester. He is also a Chang-Jiang Chair Professor with the School of Management, Hefei University of Technology, Hefei, China. He has been engaged in research on the areas of multiple criteria decision analysis under uncertainty, multiple objective optimization, intelligent decision support systems, hybrid quantitative and qualitative decision modeling using techniques from operational research, artificial intelligence and systems engineering, and dynamic system modeling, simulation and control for engineering and management systems. His current research interests include new product development, modeling and analysis of risk, safety, and security, quality modeling and evaluation, supply chain modeling and supplier assessment, performance measurement and assessment, and integrated evaluation of products, systems, projects, and policies.

Kwai-Sang Chin received the Ph.D. degree in industrial and manufacturing system engineering from the University of Hong Kong, Hong Kong, in 1996.

He is an Associate Professor and the Chair of Postgraduate Teaching with the Department of Manufacturing Engineering and Engineering Management, City University of Hong Kong, Kowloon, Hong Kong. He has authored or coauthored more than 100 papers published in international refereed journals in the fields of industrial engineering, quality management, and product development. His current research interests include decision making in new product development, quality management in the supply chain environment, and international technology transfer between West and East.

Hong-Wei Wang received the Ph.D. degree in systems engineering from Huazhong University of Science and Technology, Wuhan, China, in 1993.

He is a Professor with the Department of Control Science and Engineering, Huazhong University of Science and Technology, where he is also the Head of the Department of Control Science and Engineering and the Director of the Institute of Systems Engineering. His current research interests include supply chain management, decision making and simulation of logistics management, and decision support systems.

Xin-Bao Liu received the Ph.D. degree in management science and engineering from Hefei University of Technology, Hefei, China, in 2002.

He is a Professor with the School of Management, Hefei University of Technology, and he is also the Director of the Decision Support Systems Institute. His current research interests include decision theory, multiple criteria decision analysis, scheduling, process optimization, and decision support systems. His research is closely related to practical decision problems in industry.