



GA-based multiple objective optimisation for determining viscoplastic constitutive equations for superplastic alloys

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Abstract

A genetic algorithm (GA)-based multiple objective optimisation technique has been developed and used to determine unified viscoplastic constitutive equations for superplastic alloys. It has been found that the GA-based optimisation technique is very effective in solving this kind of problem. The difficulty of choosing correct starting values for the constants in the traditional optimisation techniques has been completely overcome and the GA technique provides a better chance to converge to the global minimum. It has also been found that the hardening due to grain growth plays a very important role in the superplastic deformation of Ti–6Al–4V at 927°C and simplified viscoplastic constitutive equations for the alloy have been determined to accurately model the superplastic flow of the material. © 1999 Elsevier Science Ltd. All rights reserved.

Keywords: Genetic algorithm; Multiple objective optimisation; Superplasticity; Constitutive modelling

1. Introduction

The superplastic behaviour of structural alloys, which include alloys of aluminium and titanium, are exploited increasingly for producing a range of light weight components with complex shapes, such as aircraft structural components, electronic equipment cabinets, etc. The superplasticity of a material is highly dependent on

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Nomenclature

d	grain size (μm)
d^c	computed grain size (μm)
d^e	experimental grain size (μm)
E	young's modulus (MPa)
$f(x)$	objective function for optimisation
F	total fitness of the parent population
$g(x)$	inequality constraint function
$h(x)$	equality constraint function
m	strain rate sensitivity ($n = 1/m$)
R, X	hardening variables (MPa)
w	weight functions
x	material constants to be optimised $x = [x_1, x_2, \dots, x_s]$
ε_p	true plastic strain
ε_T	true total strain
σ	stress (MPa)
σ^c	computed stress (MPa)
σ^e	experimental stress (MPa)
Ω	feasible decision space, $\Omega = [a, b]$

temperature and occurs only in a narrow range of strain rates with an optimum value that is unique to each material (Ghosh and Hamilton, 1979; Grimes and Butler, 1988; Hwang et al., 1997). Therefore, the control of deformation rate in a superplastic forming process is important (Lin et al., 1998). The finite element technique has been successfully used to simulate the superplastic forming process and optimize the forming parameters, such as temperature, loading history and final thickness (Hwang et al., 1997; Lin et al., 1998). In order to accurately simulate the superplastic forming process and thoroughly understand the superplastic deformation mechanisms of materials, unified viscoplastic constitutive equations for superplastic alloys need to be established (Khaleel et al., 1998; Khraisheh et al., 1997).

One of the most difficult tasks encountered in developing the viscoplastic constitutive equations is how to accurately determine material constants arising in the equations from experimental data. Lin and Hayhurst (1993) developed an optimisation technique and successfully determined the material constants for the constitutive equations where stress can be explicitly expressed as a function of strain. However, for a set of viscoplastic constitutive equations, the flow stress is related to the accumulated plastic strain, and the plastic strain rate is often a function of stress, hardening parameters, grain size, etc. (Lin et al., 1996; Zhou and Dunne, 1996). In addition, the equations are not continuous due to the existence of a yield stress parameter. This makes the constitutive models very complicated and strongly non-linear behaviour is exhibited.

Great efforts have been made for determining material constants for creep damage (Parrent and Hayhurst, 1996), cyclic plasticity (Lin et al., 1996) and superplasticity material models (Zhou and Dunne, 1996). Optimisation techniques were developed for obtaining the material parameters on the basis of minimising the sum of the errors between experimental and computed data. This work highlighted the difficulties associated with choosing poor starting values for the parameters leading to convergence to local minima producing poor parameter values. For example, Zhou and Dunne (1996) proposed a four-stage method to determine the material constants for a particular set of constitutive equations. Significant knowledge of material science, mathematics and computational mechanics is required for every stage of the optimisation. The starting values of the material constants have to be chosen carefully at each stage of the optimisation, otherwise convergence to a local minima is unavoidable and poor material parameters would be obtained.

This research develops a generic and easy-to-use technique to determine material constants for a range of constitutive equations, such as creep damage, cyclic plasticity and superplasticity. The optimisation can be single and multiple objectives. The choice of starting values for material parameters is completely avoided and a better chance to converge to the global minimum is provided. In the following sections, the development of unified viscoplastic constitutive equations for superplastic alloys and GA-based multiple objective optimisation methods for determining the material constants within the equations are described. The effectiveness of the optimisation technique is demonstrated by determining the constants in the material model for Ti–6Al–4V at 927°C.

2. Unified constitutive equations for superplastic alloys

2.1. Deformation mechanisms

Superplastic deformation has two major effects (Crooks, 1988; Pearce, 1989):

- i. grain growth occurs during superplastic straining, and
- ii. grain boundaries appear to be more sharply defined.

The latter is associated with high misorientation angles, which result in grain boundary sliding and grain accommodation by diffusion taking place during the deformation. Normally, the viscoplastic response is assumed to be given by (Chaboche and Novailhas, 1989; Dunne et al., 1992)

$$\sigma = K\dot{\epsilon}_p^m \quad \text{or} \quad \dot{\epsilon}_p = (\sigma/K)^n \quad (1)$$

where $n = 1/m$, m is strain rate sensitivity, $\dot{\epsilon}_p$ is true plastic strain rate and K is constant. Stress, $\sigma = P/A$, where P -load and A -cross-sectional area. The cross-sectional area shrinkage rate can be written as:

$$dA/dt = (P/K)^{1/m} / A^{(1-m)/m} \quad (2)$$

The shrinkage rate is related to the cross-sectional area and highly sensitive to m . Thus, increasing m adds stability to the diffuse neck and increases the uniform elongation. Obviously the ideal value of m is 1, in which case, $-dA/dt$ is independent of A and an irregular testpiece would maintain its irregularities during deformation.

The value of m , or n , is related to the deformation mechanisms of the material. For high temperature superplastic forming, the deformation mechanisms are mainly based on the following major processes (Pearce, 1989): diffusion creep, dislocation creep, diffusion-controlled dislocation glide and grain boundary sliding. Among these, grain boundary sliding plays a dominant role for alloys such as Ti–6Al–4V. The value of n should be within the region of 1 to 4 for most superplastic alloys.

2.2. Grain growth

The grain size is in general a function of deformation history as well as time at high temperature (Zhou and Dunne, 1996). By considering the interfacial energy associated with grain boundaries and the dependence of grain growth on diffusion processes, the grain growth rate can take the empirical form (Dunne, 1998):

$$\dot{d} = (\alpha + \beta|\dot{\epsilon}_p|)d^{-\gamma_0} \quad (3)$$

where α is a constant which specifies the isothermal grain growth kinetics and β is a constant which characterises the plastic strain rate dependence. γ_0 is a material constant, the value of which is greater than or equal to one.

2.3. Unified constitutive equations

Constitutive equations for viscoplasticity have been developed for many metal materials (Chaboche and Jung, 1997; Dunne et al., 1992; Lin et al., 1996). The equations enable a wide range of time dependent phenomena to be modelled, such as, strain hardening, stress relaxation and ratchetting, and in addition enable the important time-dependent effects, such as strain rates, recovery and creep, to be modelled. These equations mainly take the form of power laws and uniaxial constitutive equations for superplastic behaviour can be written as

$$\begin{aligned} \dot{\epsilon}_p &= ((\sigma - X) - R - k)/K)^n d^{-u} \\ \dot{X} &= C\dot{\epsilon}_p - \gamma X|\dot{\epsilon}_p| \\ \dot{R} &= b(Q - R)|\dot{\epsilon}_p| \\ \dot{d} &= (\alpha + \beta|\dot{\epsilon}_p|)d^{-\gamma_0} \\ \sigma &= E(\epsilon_T - \epsilon_p) \end{aligned} \quad (4)$$

where ϵ_T and ϵ_p are total and plastic strain; X and R the hardening variables and d the grain size. $K, k, n, u, C, \gamma, b, Q, \alpha, \beta, \gamma_0$ are material constants to be determined from experimental data. E is Young's modulus and $E = 1000$ MPa for Ti–6Al–4V at 927°C. The technique for determining the eleven material constants arising in Eq. (4) is detailed below.

3. Determination of material constants

3.1. Problem Description

Optimisation techniques for obtaining the material parameters in the constitutive equations are based on minimising the sum of the squares of the errors between the experimental and computational data. For the set of viscoplastic constitutive equations (4), two objective functions can be defined in terms of the square of the difference of the experimental and computed data for stress–strain and grain growth data

$$\begin{aligned}
 f_1(x) &= \sum_{j=1}^{n_1} \sum_{i=1}^{m_1} w_{ij} \left((\sigma_i^c)_j - (\sigma_i^e)_j \right)^2 \\
 f_2(x) &= \sum_{l=1}^{n_2} \sum_{k=1}^{m_2} w_{kl} \left((d_k^c)_l - (d_k^e)_l \right)^2
 \end{aligned}
 \tag{5}$$

where $f_1(x)$ and $f_2(x)$ are sums of residuals for stress and grain size, respectively, $x(x = [x_1, x_2, \dots, x_s])$ represents the material constants and s is the number of material constants to be determined. $(\sigma_i^c)_j$ and $(\sigma_i^e)_j$ are the computational and experimental stresses for the same strain level i and strain rate j , w is a weighting function, m_1 is the number of stress–strain data for each strain rate j , and n_1 is the number of strain rates considered. Similarly, $(d_k^c)_l$ and $(d_k^e)_l$ are computational and experimental grain-size data at the same time, k , and strain rate, l , m_2 is the number of grain size data for a strain rate l , and n_2 is the number of strain rates considered. The computed stress, $(\sigma_i^c)_j$, and grain size, $(d_k^c)_j$, are not available directly and have to be determined from the constitutive equations (4) by means of a numerical integration method. The objective functions, $f_1(x)$ and $f_2(x)$, require the global minimisation subject to determining the value of each constant within a region defined by the lower and upper bounds, that is, $x_i = \{a_i, b_i\}$ for $i = 1, 2, \dots, s$. The necessary conditions for the minima are given by

$$\begin{aligned}
 \partial f_1(x) / \partial x_i &= 0 \\
 \partial f_2(x) / \partial x_i &= 0 \quad i = 1, 2, \dots, s
 \end{aligned}
 \tag{6}$$

which lead to a set of $2s$ non-linear equations in x . For the set of constitutive equations (4), 11 material constants need to be determined from the 22 non-linear equations [Eq. (6)]. Obviously, using conventional optimisation methods, it is difficult to choose starting values for the constants such that the numerical process converges to the global minimum.

3.2. Non-linear multiple objective optimisation

In general, a non-linear optimisation problem may be defined as follows (Yang, 1999):

$$\begin{aligned}
 & \min f(x) \\
 & \text{s.t. } x \in \Omega \\
 & \quad x = [x_1, x_2, \dots, x_s]^T \\
 & \quad \Omega = \{x \mid g_j(x) \leq 0, h_l(x) = 0; \quad j = 1, \dots, m_1, l = 1, \dots, m_2\}
 \end{aligned} \tag{7}$$

where $f(x)$ is a non-linear objective function, x the decision variables (material constants) and Ω the feasible decision space. $g_i(x)$ and $h_l(x)$ are inequality and equality constraint functions. The problem discussed in Section 3.1 is a special case of problem (7).

Fig. 1 shows a typical non-linear one-variable minimisation problem where the objective function is strongly non-linear. In the feasible decision space $\Omega = [a, b]$, there may be many minima such as at x_2 and x_4 , most of which are local minima. The task of optimisation is to find a feasible global minimum such as the solution x_2 shown in Fig. 1.

If all the objective and constraint functions are continuously differentiable in the whole decision space Ω and there is only one feasible minimum, problem (7) may be solved using traditional mathematical programming techniques, for example sequential linear or quadratic programming and dual methods. If there are many local minima, however, most traditional techniques may only be capable of generating a local minimum instead of a global one unless a good starting point is provided a priori which is sufficiently close to the global minimum (Yang et al., 1990). For example in Fig. 1 the global minimum x_2 may not be identified using a traditional method unless a starting solution is in the interval $[x_1, x_3]$.

In many engineering optimisation problems, the objective function and/or constraint function may be non-smooth or even discontinuous. Many traditional methods based on some type of gradient search are not suitable to solve such problems. In

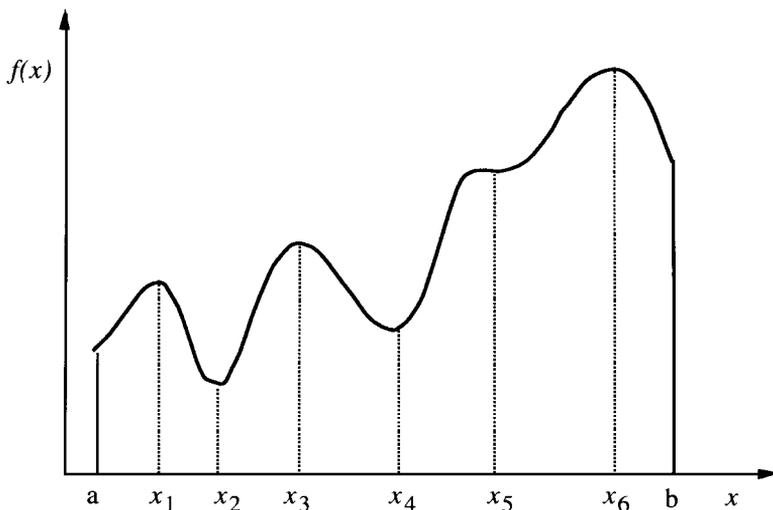


Fig. 1. Non-linear optimisation for one variable.

such circumstances, it is better to employ adaptive search techniques such as genetic algorithms which provide a better chance of locating a global optimum.

Most engineering optimisation problems involve multi-objectives. In general, a multi-objective optimisation problem is defined by

$$\begin{aligned} \min \quad & f(x) = [f_1(x), \dots, f_k(x)] \\ \text{s.t.} \quad & x \in \Omega \end{aligned} \tag{8}$$

where $f_k(x)$ is the k th objective function. Normally, there exists no single solution in Ω which could optimise all the objectives simultaneously. The task of multi-objective optimisation is to find a most preferred solution which should be pareto-optimal or efficient (South et al., 1993; Yang, 1996). A solution \hat{x} is efficient if there is no other solution in Ω , say \bar{x} , so that $f_i(\bar{x}) \leq f_i(\hat{x})$ for all $i = 1, \dots, k$ and $f_i(\bar{x}) < f_i(\hat{x})$ for at least one i .

Many methods have been developed for dealing with multi-objective optimisation problems (South et al., 1993; Yang, 1996). One class of methods, which are suitable for non-linear and non-smooth multi-objective optimisation, are based on the following p -norm formulation with p sufficiently large

$$\begin{aligned} \min \quad & \left\{ \sum_{i=1}^k [w_i(f_i(x) - f_i^*)]^p \right\}^{1/p} \\ \text{s.t.} \quad & x \in \Omega \end{aligned} \tag{9}$$

where $w_i (\geq 0)$ is the relative weight for an objective $f_i(x)$, f_i^* the minimum of $f_i(x)$ in Ω and p a positive integer.

Eq. (9) is equivalent to the following form when $p = \infty$ (Yang et al., 1990):

$$\min_{x \in \Omega} \max_i \{w_i(f_i(x) - f_i^*)\}, \quad i = 1, \dots, k \tag{10}$$

Eq. (10) is normally referred to as the minimax scheme or the ideal point method for multi-objective optimisation.

Fig. 2 is a schematic diagram showing non-linear two objective optimisation. Point A is the minimum for the objective f_1 , known as f_1^* , B the minimum for f_2 , known as f_2^* . Normally, f_1^* and f_2^* are not at the same point. The ideal solution is denoted by f^* but it is outside of the feasible solution space Ω . The optimisation process of the weighted minimax formulation acts as an equaliser to make all $w_i(f_i(x) - f_i^*) (i = 1, \dots, k)$ equal. The corresponding geometric interpretation is that the weighted minimax formulation searches an optimal feasible solution along the ray that starts from the point f^* and is specified by

$$w_1(f_1(x) - f_1^*) = \dots = w_k(f_k(x) - f_k^*)$$

The most preferred solution is the one on the ray which is a feasible solution closest to point f^* , such as point C in Fig. 2. It is also clear from Fig. 2 that changing weights (w_1 and w_2) will lead to another most preferred solution.

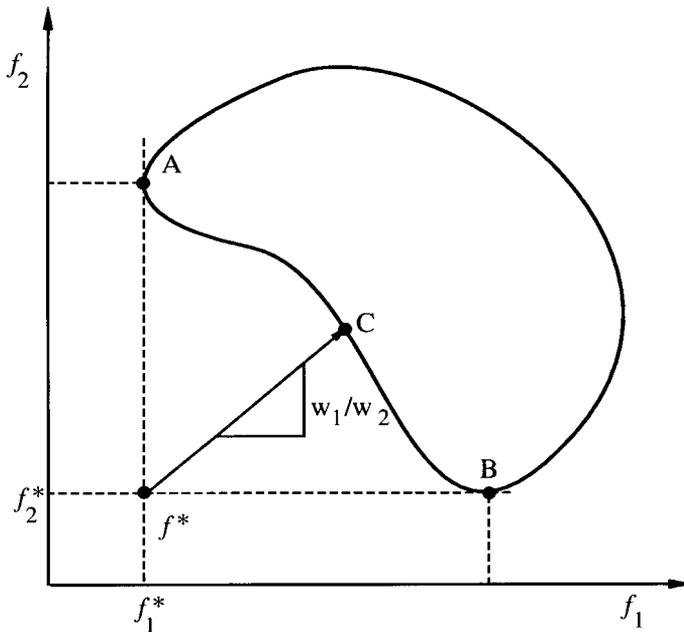


Fig. 2. Non-linear two objective optimisation.

3.3. Minimax genetic algorithm for complex optimisation

The problem presented in Section 3.1 is a special case of Eq. (8). It has two objectives of minimising grain size and stress residuals and is strongly non-linear and non-smooth. Eq. (10) could be adopted to solve this complex optimisation problem. Since it is strongly non-linear and non-smooth, the problem could be dealt with using a GA solution scheme based on Eq. (10) as discussed below.

The GA approach as used in this research employs three main genetic operators: reproduction, crossover and mutation. In reproduction, an initial parent population of solutions are generated randomly. The size of the population, or the number of solutions in the population, is denoted by N and may be fixed. Each solution has a performance measure or a fitness function f associated with it. The minimisation of the maximum weighted residual for a solution as defined by Eq. (10) may be used to define such a fitness function.

It is assumed that the fitness $f(x^j)$, $j = 1, \dots, N$, have been assigned for the N members (solutions) of the parent population, where x^j is the j th solution of the parent population. To produce the next generation of solutions, M members ($M \leq N$) are randomly sampled from the parent population. In this selection, $f(x^j)/F$ may be used as the probability of the j th member of the population being chosen, where F is the total fitness of the parent population, i.e.

$$F = \sum_{j=1}^N f(x^j) \quad (11)$$

Thus a member with high fitness is more likely to be selected for further genetic operations. Following reproduction, a mating pool of M solutions are obtained to which further genetic operations are applied.

The crossover transform allows the characteristics of the solutions in the mating pool to be altered, with the intent of representing the best characteristics in the next generation. In this transform, however, an offspring solution must be feasible. The crossover operation may be performed as follows. Given two parent solutions, S_1 and S_2 , each of which may be represented using a binary string, say

$$\begin{aligned} S_1 &= 11101110000111111 \\ S_2 &= 11000100011111111 \end{aligned}$$

from the strings, crossover points are randomly selected, for example,

$$\begin{aligned} S_1 &= 111|0111000|011111 \\ S_2 &= 110|0010001|111111 \end{aligned}$$

Exchanging the sub-strings between the two crossover points will generate two offspring solutions, O_1 and O_2 , as follows

$$\begin{aligned} O_1 &= 111|0010001|011111 \\ O_2 &= 110|0111000|111111 \end{aligned}$$

In this way, it is expected that the good characteristic of the parent strings may be transformed to the offspring, which may lead to the generation of better solutions. The number of solutions selected for crossover operation is governed by a given crossover rate.

Mutation prevents the genetic search process from a premature loss of genetic material due to reproduction and crossover and it compensates for sampling error. For a given solution, the process of mutation simply consists of randomly picking two distinct locations within the string representing the solution and exchanging the elements (jobs) at these locations to generate a single offspring solution. The process may be applied to a few members from the population pool according to a given probability (rate) of mutation. Obviously, the mutation process only involves swap transform. There are other ways of implementing mutation. For a more detailed discussion readers may refer to South et al. (1993).

The GA search process is generally governed by the size of population, the number of generations, the probabilities of crossover and mutation, and probably the generation gap or proportion to be replaced with new solutions in the next generation. These parameters could be adjusted to improve the quality of GA search. This GA approach for solution generation is task-independent and can be used for general purpose parameter estimation.

The set of GA parameters used for this research is as follows: population size 200, number of generations 500, crossover rate 0.95, mutation rate 0.01 and generation gap 1.

4. Modelling of superplastic behaviour for Ti–6Al–4V

The GA based multiple objective optimisation has been used for determining material constants arising in viscoplastic constitutive equations, and a software package, named MECHOPT, has been developed. The effectiveness of the technique and the power of the developed software are demonstrated by determining the material constants within Eq. (4) from experimental data for Ti-6Al-4V at 927°C (Ghosh et al., 1979). If the domains of the material constants are given, convergence to the global optimum point would be faster and physically meaningful results would be obtained.

The domains of the material constants can be determined from their physical significance. For example, according to superplastic deformation mechanisms the value of power n should be within the region of 1 to 4. Otherwise, physical significance would be lost. For some constants, e.g. C and γ , it is difficult to determine their variation regions, although they can not be negative according to their physical meanings. In this case, a relatively large variation range can be given, so that the optimum points are expected to be included within the region. For example, $1 \leq C \leq 40\,000$ and $0.5 \leq \gamma \leq 1000$. The domains of the material constants determined from their physical significance for the optimisation are given in Table 1.

4.1. Unified constitutive equations

The unified constitutive equations (4) discussed above are employed to model the superplastic behaviour of a titanium alloy, Ti–6Al–4V, at 927°C. The GA-based multiple objective optimisation technique is used to determine the material constants within the equations from uni-axial tensile test data for the material reported by Ghosh and Hamilton (1979). Strain rate-related grain-growth experimental data is available for strain rates 2.0×10^{-4} and $1.0 \times 10^{-3} \text{ s}^{-1}$ and stress–strain data available for 5.0×10^{-5} , 2.0×10^{-4} and $1.0 \times 10^{-3} \text{ s}^{-1}$ at the initial grain size of 6.4 μm . These sets of experimental data are used for determining the 11 material constants arising in Eq. (4).

The domains of the material constants listed in Table 1 are used for the optimisation, and the constants determined using the GA-based multiple objective optimisation technique are given in Table 2. Computed and experimental data are shown in Fig. 3. The symbols represent the experimental data due to Ghosh and Hamilton (1979) and the solid curves are obtained from the computations. Very

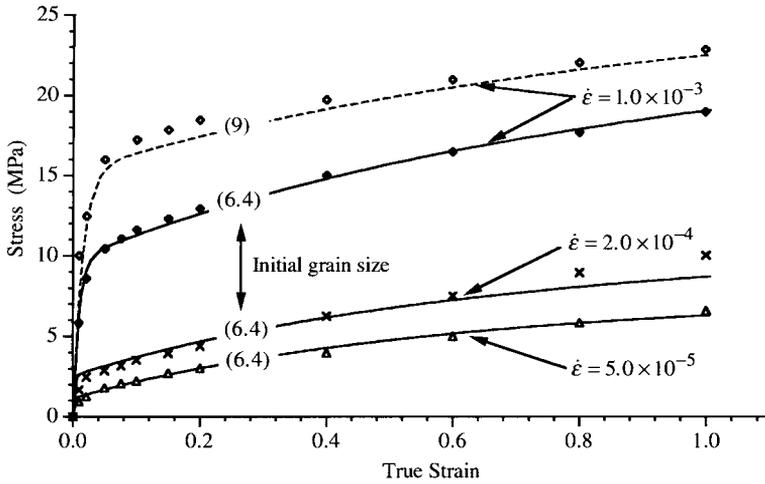
Table 1
Domains of the constants defined for the optimisation

$50.0 \leq K \leq 1000.0$	$0.5 \leq b \leq 10.0$
$0.1 \leq k \leq 4.0$	$0.5 \leq Q \leq 6.0$
$1.0 \leq n \leq 4.0$	$0.1 \times 10^{-20} \leq \alpha \leq 0.4$
$1.0 \leq u \leq 2.5$	$10.0 \leq \beta \leq 5000.0$
$1.0 \leq C \leq 40\,000.0$	$1.0 \leq \gamma_0 \leq 10.0$
$0.5 \leq \gamma \leq 1000.0$	

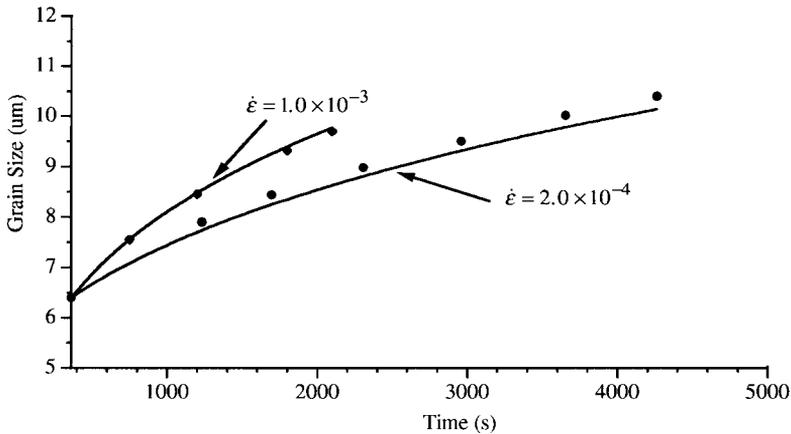
close agreements have been obtained for both stress-strain [Fig. 3(a)] and grain-growth [Fig. 3(b)] relationships. An independent check has also been carried out by comparing the experimental (symbols) and computed (dash curve) stress-strain relationship for $\dot{\epsilon} = 1 \times 10^{-3} \text{ s}^{-1}$ with the initial grain size $9 \mu\text{m}$, which is not included

Table 2
Optimised material constants for Eq. (4) for Ti-6Al-4V at 927°C

K (MPa)	k (MPa)	n	u	C	γ	b	Q	α	β	γ_0
645.8	0.229	1.020	1.400	349.3	842.0	1.780	4.985	0.268	386.2	2.767



(a) Stress - strain relationship.



(b) Variation of grain size with time.

Fig. 3. Comparison of experimental data (legends) with computed (solid curves) and predicted (dash curve) results for different strain rates.

in the optimisation. An accurate prediction has been observed. It can be seen that the unified constitutive equations (4) can be used to represent the superplastic behaviour of Ti–6Al–4V at 927°C.

4.2. Simplified constitutive equations

Eq. (4) accounts for material hardening due to both grain growth and dislocation pile-up, and material recovery through dislocation climb processes. Obviously, the constitutive equations developed for a material should be as simple as possible. This is not only for application purposes, but also for understanding the deformation mechanisms and finding out the predominated deformation mechanisms of the material under certain working conditions. This work can be done easily by removing certain equations from the equation set (4), for example excluding the grain growth or the hardening variable R equation. By doing this, the material constants in a new set of equations need to be re-determined using the GA optimisation technique. For the following optimisation processes, the same domains of the material constants defined in Table 1 are used.

4.2.1. Constitutive equations without the hardening variable R

With the hardening variable R removed from Eq. (4), a new set of constitutive equations can be written as:

$$\begin{aligned}\dot{\varepsilon}_p &= ((|\sigma - X| - k)/K)^n d^{-u} \\ \dot{X} &= C\dot{\varepsilon}_p - \gamma X|\dot{\varepsilon}_p| \\ \dot{d} &= (\alpha + \beta|\dot{\varepsilon}_p|)d^{-\gamma_0}\end{aligned}\quad (12)$$

The number of material constants have been reduced from 11 to nine. The constants in Eq. (12) have been re-determined using the established GA-based optimisation technique. Both the stress–strain relationships and grain growth are considered in the optimisation process. The optimised material constants for Eq. (12) are listed in Table 3.

In Fig. 4, the solid curves show the computed stress–strain relationships for the three different strain rates, which are compared with their corresponding experimental data (Ghosh and Hamilton, 1979), denoted as symbols. By comparing the results shown in Fig. 3(a), the overall error between the computed and experimental results presented in Fig. 4 is not significantly larger. Therefore, the simplified constitutive equations (12) are still capable of presenting the viscoplastic behaviour well.

Table 3
Optimised material constants for Eq. (12) for Ti–6Al–4V at 927°C

K (MPa)	k (MPa)	n	u	C	γ	α	β	γ_0
62.524	0.3285	1.3574	2.3110	25.032	953.78	0.2627	496.32	2.5117

4.2.2. Hardening due to grain growth only

The above constitutive equations are further simplified by removing the hardening variables, X and R :

$$\begin{aligned} \dot{\epsilon}_p &= ((|\sigma - X| - k)/K)^n \\ \dot{d} &= (\alpha + \beta|\dot{\epsilon}_p|)d^{-\gamma_0} \end{aligned} \tag{13}$$

In this case, the number of the material constants to be redetermined have been reduced to seven. Both the stress–strain relationships and the grain growth are considered in the optimisation process. The optimised material constants for Eq. (13) are given in Table 4.

Fig. 5 shows the comparison of the experimental and computed stress–strain relationships for the three strain rates. Although the overall error shown in Fig. 5 is bigger than both the results shown in Figs. 3(a) and 4, the simplified constitutive equation (13) with the optimised material constants listed in Table 4 can be used to express the material behaviour fairly closely. The accuracy is adequate for most engineering applications, especially for simulating the superplastic forming process using the finite element method.

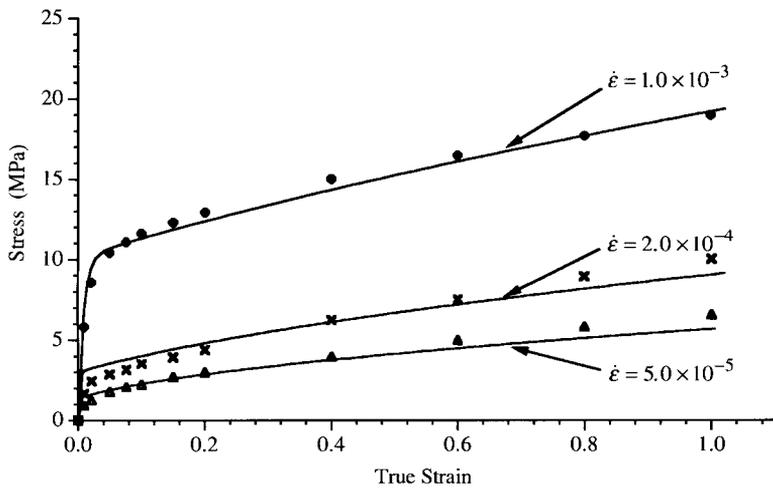


Fig. 4. Comparison of experimental (legends) and computed (solid curves) results using Eq. (12).

Table 4
Optimised material constants for Eq. (13) for Ti–6Al–4V at 927°C

K (MPa)	k (MPa)	n	u	α	β	γ_0
139.18	0.7132	1.1387	2.0767	0.2259	89.764	2.2832

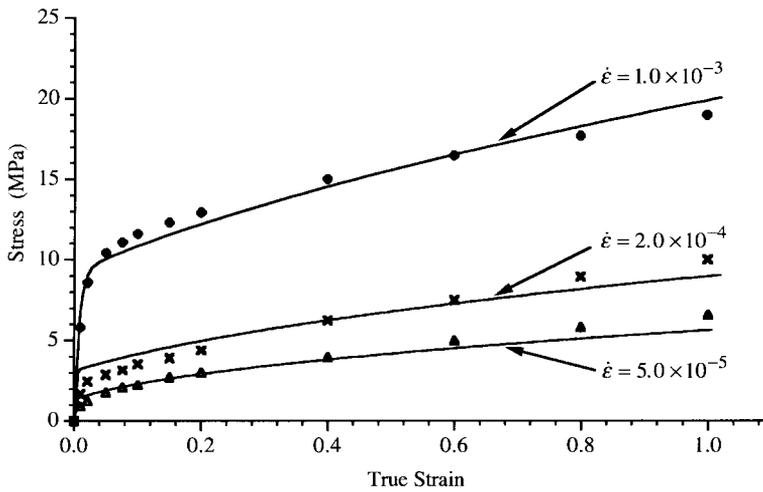


Fig. 5. Comparison of experimental (legends) and computed (solid curves) results using Eq. (13).

5. Discussion

Great efforts have been made previously for determining material constants arising in viscoplastic constitutive models, creep models, etc. (Lin et al., 1996; Parrent and Hayhurst, 1996). The common problem encountered in the optimisations is how to choose initial values of constants in a material model. If the number of the constants are more than five, it is almost impossible to choose correct initial values for all the material constants to get the global optimum. In order to avoid this Zhou and Dunne (1996) developed a four-stage optimisation technique to determine the constants for a particular set of superplastic equations. The number of constants to be optimised at each stage were reduced so that successful optimisation could be achieved.

Obviously, the conventional optimisation techniques can not be used to optimise the material constants automatically. A lot of human–computer interaction is required. The knowledge of mathematics, mechanics and material science is needed during the optimisation process to analyse the intermediate results, to determine the initial values of constants and to make a decision as to how to go to the next optimisation stage.

The GA-based multiple objective optimisation technique is suitable for this kind of problem. The difficulties of choosing proper starting values for the constants are overcome and the human–computer interaction during the optimisation process is almost eliminated. According to the theories of the GA-based optimisation technique, the determination of the domains of the constants can be ignored as well. Obviously, the convergence would be faster if the domains of the constants are given.

The GA-based optimisation technique is also easy to use and more suitable for industry. If a set of constitutive equations is given the material constants can be

determined easily according to experimental results. Little knowledge of mathematics and computational mechanics is required. Also the full set of constitutive equations can be simplified by eliminating certain mechanisms. This is very important for identifying the dominated deformation mechanisms. For example, the constitutive equation (4) can be simplified to Eq. (13) without significant loss of accuracy for Ti–6Al–4V at 927°C. The number of mechanism-based differential equations are reduced to two from four in this research. It is found that the hardening due to grain growth plays a very important role in the superplastic deformation.

6. Conclusions

1. The material constants arising in the unified viscoplastic constitutive equations have been determined for Ti–6Al–4V at 927°C. The material model can be used to model the hardening variables due to plastic deformation, grain growth with different initial grain sizes, and their influence on the viscoplastic flow of the material.
2. A GA-based multiple objective optimisation technique has been developed for determining the material constants arising in the viscoplastic constitutive equations. This generic technique can be used for determining a wide range of constitutive equations, such as creep damage and cyclic plasticity models, where the objective functions for optimisation are strongly non-linear. Choosing starting values for the constants in the equations is completely avoided and a better chance to converge to the global minimum is provided.
3. The GA-based optimisation technique provides an easy means for simplifying the unified constitutive equations. For example, the original four differential equations within the model can be reduced to two for Ti–6Al–4V at 927°C and the number of the material constants goes down to seven from 11. This is very important to industry for simulating the superplastic forming process for complex shaped structural components using an finite element method.

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