International Journal of General Systems, April–June 2004 Vol. 33 (2–3), pp. 183–204

FUZZY RULE-BASED EVIDENTIAL REASONING APPROACH FOR SAFETY ANALYSIS

JUN LIU[a,*], JIAN-BO YANG[a,†], JIN WANG[b,‡], HOW-SING SII[b,¶] and YING-MING WANG[c]

[a]*Manchester School of Management, UMIST, PO Box 88, Manchester M60 1QD, UK;*
[b]*School of Engineering, Liverpool John Moores University, Liverpool L3 3AF, UK;*
[c]*School of Public Administration, Fuzhou University, Fuzhou 350002, The People's Republic of China*

(Received 15 December 2002; In final form 9 September 2003)

This paper aims at proposing a framework for modelling the safety of an engineering system with various types of uncertainties using a fuzzy rule-based evidential reasoning (FURBER) approach. In the framework, parameters used to define the safety level, including failure rate, failure consequence severity and failure consequence probability, are described using fuzzy linguistic variables; a fuzzy rule-base designed on the basis of a belief structure is used to capture uncertainty and non-linear relationships between these parameters and the safety level; and the inference of the rule-based system is implemented using the evidential reasoning algorithm. A numerical study of collision risk analysis for a kind of offshore platform is used to illustrate the application of the proposed approach.

Keywords: Safety analysis, decision-making, uncertainty, fuzzy rule-base, evidential reasoning

1. INTRODUCTION

The growing technical complexity of large engineering systems such as offshore platforms and offshore support vessels, together with the intense public concern over their safety has stimulated the research and development of novel safety analysis methods and safety assessment procedures.

The safety of a large engineering system is affected by many factors regarding its design, manufacturing, installation, commissioning, operation and maintenance. Consequently, it may be extremely difficult to construct an accurate and complete mathematical model for the system in order to assess the safety because of inadequate knowledge about the basic failure evens. This leads inevitably to problems of uncertainty in representation (Wang *et al.*, 1995).

It is worth noting that many typical safety assessment approaches (such as the probabilistic risk assessment approach) have been widely used, but often fall short in their ability to permit the incorporation of subjective and/or vague terms as they rely heavily on supporting

*Corresponding author. E-mail: j.liu@umist.ac.uk
†E-mail: jian-bo.yang@umist.ac.uk
‡E-mail: j.wang@livjm.ac.uk
¶E-mail: h.s.sii@livjm.ac.uk

ISSN 0308-1079 print/ISSN 1563-5104 online
DOI: 10.1080/03081070310001633536

statistical information that may not be available (Wang and Ruxton, 1997). They may not be well suited for dealing with systems in situations having a high level of uncertainty, particularly in the feasibility and concept design stages of an engineering system or a system with a high level of innovation, where there is often inadequate data or imprecise information available when carrying out safety assessments for the system.

Novel safety analysis methods are therefore required to provide a basis and tool for safety analysis in complex engineering systems, in particular to deal with information that may be imprecise, ill-defined, and incomplete, for which traditional quantitative approaches (e.g. statistical approach) do not give an adequate answer.

One realistic way to deal with imprecision is to use linguistic assessments instead of numerical values. Fuzzy logic approaches (Zadeh, 1965) employing fuzzy IF–THEN rules (where the conditional part and/or the conclusions contain linguistic variables (Zimmerman, 1991)) can model the qualitative aspects of human knowledge and the reasoning process without employing precise quantitative analysis. It does not require an expert to provide a precise point at which a risk factor exists. This actually provides a tool for working directly with the linguistic information, which is commonly used in representing risk factors and carrying out safety assessments (Karwowski and Mital, 1986; Keller *et al.*, 1989; Duckstein, 1994; Bowles and Pelaez, 1995; Wang *et al.*, 1995; 1996; Bell and Badiru, 1996; Wang, 1997; An *et al.*, 2000; Sii *et al.*, 2001). In this context, a safety model using a fuzzy rule-based inference system can be more appropriately used to carry out risk analysis associated with incomplete safety information in the initial design stages or a system with a high level of innovation.

In view of the increasing complexity of many knowledge-based systems, the knowledge representation power of fuzzy rule-based systems will be severely limited if only fuzziness is taken into account in representing uncertain knowledge. Some other theories of uncertainty in fuzzy rule-base have been proposed to model various types of uncertainty, such as type 2 fuzzy sets theory (Zadeh, 1975; Mizumoto and Tanaka, 1976; Karnik *et al.*, 1999; Mendel, 2001; Mendel and John, 2002) and intuitionistic fuzzy sets theory (Atanassov, 1986; 1999; De *et al.*, 2001). Both are the extension of classical fuzzy set theory aiming at dealing with additional uncertainty in rule-bases, e.g. type-2 fuzzy sets are used to deal with uncertainty in antecedents and consequents of rules, measurement noise, and training date noisy, etc. The non-membership function is introduced in intuitionistic fuzzy sets to deal with situations where there is a fair chance of the existence of a non-zero hesitation part at each moment of evaluation of any unknown object (De *et al.*, 2001). Their rule-based inference schemes are also an extension of classical fuzzy inference based on type-2 fuzzy set or intuitionistic fuzzy sets.

In engineering safety analysis, intrinsically vague information may coexist with conditions of "lack of specificity" originating from evidence not strong enough to completely support a hypothesis but only with degrees of belief or *credibility* (Binaghi and Madella, 1999). Dempster-Shafer (D-S) theory of evidence (Dempster, 1968; Shafer, 1976) based on the concept of *belief function* is well suited to modeling subjective credibility induced by partial evidence (Smets, 1988). The D-S theory enlarges the scope of traditional probability theory, describes and handles uncertainties using the concept of the degrees of belief, which can model incompleteness and ignorance explicitly. It also provides appropriate methods for computing belief functions for combination of evidence (Pearl, 1988). Besides, the D-S theory also shows great potentials in multiple attribute decision analysis (MADA) under uncertainty, where an evidential reasoning (ER) approach for MADA under uncertainty has been developed, on the basis of a distributed assessment framework and the evidence combination rule of the D-S theory (Yang and Singh 1994; Yang and Sen 1994, 1997; Yang, 2001; Yang and Xu, 2002a, b).

Accordingly, it seems reasonable to extend the fuzzy logic framework to cover credibility uncertainty as well. The present work aims at combining fuzzy logic and D-S models to deal with fuzziness and incompleteness in safety analysis. The combination may become substantial when a lack of specificity in data is prevalent. In these cases, experts may have difficulty in structuring and articulating causal relationships (Liu *et al.*, 2003). Several researchers have investigated the relationships between fuzzy sets and D-S theory and suggested different ways of integrating them (Ishizuka *et al.*, 1982a, b; Yager, 1982; Ogawa *et al.*, 1985; Yager and Filev, 1995; Chen, 1997). A generic rule-base inference methodology using the evidential reasoning approach—RIMER approach was proposed recently in (Yang *et al.*, 2003) using fuzzy logic, decision theory and the evidential reasoning approach introduced in (Yang and Singh, 1994; Yang and Sen, 1994; Yang, 2001; Yang and Xu, 2002a, b).

In this paper, we propose a framework for modelling safety of engineering systems based on fuzzy logic and the evidential reasoning approach, referred to as a *fuzzy rule-based evidential reasoning* (FURBER) approach, which is based on the RIMER approach proposed recently in (Yang *et al.*, 2003). In the FURBER approach, safety-related parameters are described using fuzzy linguistic variables, and a fuzzy rule-base with a belief structure, i.e. fuzzy rules with belief degrees for all possible safety consequents, is used to capture uncertain causal relationships between these parameters and the safety level. Moreover, the antecedent of each IF–THEN rule forms an overall attribute, called *a packet antecedent attribute*. The activation weight of a rule can be generated by aggregating the degrees to which all antecedents in the rule are activated. In this context, an IF–THEN rule can be considered as an evaluation problem of a packet antecedent attribute being assessed to an output term in the consequent of the rule with certain degrees of belief. Finally, the inference of a rule-based system is implemented using the evidential reasoning approach in (Yang and Xu, 2002a, b). In other words, the inference is formulated as a multi-attribute decision-analysis problem under uncertainty. A case study of collision risk between a floating production storage offloading (FPSO) and a shuttle tanker due to technical failure during tandem offloading operations is used to illustrate the application of the proposed approach.

2. SAFETY ANALYSIS FRAMEWORK USING FUZZY RULE-BASED EVIDENTIAL REASONING APPROACH

The proposed framework for modelling system safety consists of four major components, which outline all the necessary steps required for safety evaluation using the fuzzy rule-based evidential reasoning approach.

2.1 Identify Causes/Factors

In this component, all anticipated causes/factors to the technical failure of an engineering system are identified. This can be done by a panel of experts during a brainstorming session at the early concept design stages of the system.

2.2 Identify and Define Fuzzy Input and Fuzzy Output Variables (i.e. Safety Estimates)

The three fundamental parameters used to assess the safety level of an engineering system on a subjective basis are *failure rate* (FR), *consequence severity* (CS) and *failure consequence probability* (FCP). Subjective assessments (using linguistic variables instead of precise numbers in probabilistic terms) are more appropriate for analysis using these three

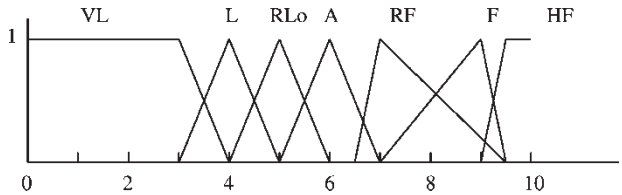

FIGURE 1 Fuzzy FR set definition (Sii and Wang, 2002).

parameters as they are always associated with great uncertainty, especially in the early design stage of an engineering product or for a novel system with a high level of innovation. These linguistic assessments can become the criteria for measuring safety levels.

The granularity of linguistic term sets used for describing each fundamental parameter is decided according to the situation of the case of interest. The recent literature survey indicates that the granularity from four to seven labels is commonly used to represent risk factors in risk analysis (Karwowski and Mital, 1986; Bowles and Pelaez, 1995; Bell and Badiru, 1996; Wang, 1997; An *et al.*, 2000).

The second step in this component is to select the types of fuzzy membership functions used to define each input variable. It is possible to have some flexibility in the definition of membership functions to suit different situations. The application of categorical judgments has been quite positive in several practical situations (Schmucker, 1984). It is also common and convenient for safety analysts to use categories to articulate safety information. The typical linguistic variables used to describe FR, CS, FCP of a particular element may be defined and characterized as follows (Sii and Wang, 2002).

FR describes failure frequencies in a certain period, which directly represents the number of failures anticipated during the design life span of a particular system or an item. To estimate FR, one may choose to use such linguistic terms as *very low* (VL), *low* (Lo), *reasonably low* (RLo), *average* (A), *reasonably frequent* (RF), *frequent* (F) and *highly frequent* (HF). Table A.1 in the Appendix describes the possible range of the frequencies of failure occurrence and defines the linguistic terms of FR. In Table A.1 of the Appendix, some numerical values for FR are given. Such values may vary with different engineering systems. If such numerical values are not available at all, then the modelling of FR can also be carried out only based on subjective judgements. Figure 1 shows the fuzzy FR set definition.

CS describes the magnitude of possible consequences, which is ranked according to the severity of failure effects. One may choose to use such linguistic terms as *negligible* (N), *marginal* (Ma), *moderate* (Mo), *critical* (Cr) and *catastrophic* (Ca). Table A.2 in the Appendix shows the possible criteria used to define the linguistics terms for describing and ranking the CS of failure effects. Figure 2 shows the fuzzy CS set definition. It is worth noting that this qualitative parameter is assessed using a subjective scale against which the range of the linguistic values is mapped in domains defined by the model builder. A subjective scale is called a *psychometric scale*, since it comes from the model builder's

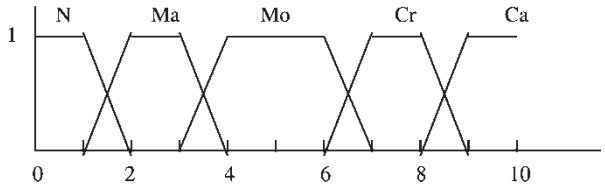

FIGURE 2 Fuzzy CS set definition (Sii and Wang, 2002).

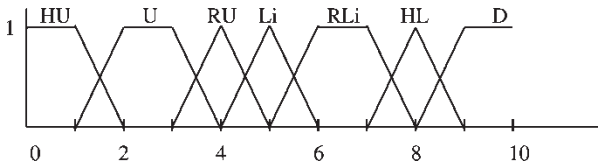

FIGURE 3 Fuzzy FCP set definition (Sii and Wang, 2002).

mind (Cox, 1998). The range of the psychometric scale is determined by the level of granularity and fines detail in the model. The fuzzy CS set definition shown in Fig. 2 is just an example of this kind of psychometric scale (the domain runs from 0 to 10 indicating the degree to which the concept CS is *negligible*, *marginal*,. . ., or *catastrophic*).

FCP defines the probability that consequences happen given the occurrence of the event. For FCP, one may choose to use such linguistic terms as *highly unlikely* (HU), *unlikely* (U), *reasonably unlikely* (RU), *likely* (Li), *reasonably likely* (RLi), *highly likely* (HL) and *definite* (D). Table A.3 in the Appendix shows the possible criteria used to define the linguistics terms for describing and ranking the FCP of failure effects. Figure 3 shows the FCP set definition.

Here note that the straight-line membership functions are used due to their advantage of simplicity, such as the triangular membership function and trapezoidal membership function. Both of these memberships are commonly used to describe risks in safety assessment (Wang, 1997). It is noted that they can be modified according to different requirements in codes and standards (e.g. safety/risk guidelines, regulations, laws etc.) and different aspects of engineering systems such as fire, explosions, structure, safety system etc.

In addition, the belief distribution assessment scheme proposed in sub-section 2.4 provides other alternative ways for inference while no membership function is available.

Safety estimate is the only output fuzzy variable used in this study to produce safety evaluation for a particular cause of technical failure. This variable is also described linguistically, which is described and determined by the above parameters. In safety assessment, it is common to express a safety level by degrees to which it belongs to such linguistic variables as *poor*, *fair*, *average* and *good* that are referred to as safety expressions.

2.3 Construct a Fuzzy Rule-base with the Belief Structure

Fuzzy logic systems are knowledge-based or rule-based systems constructed from human knowledge in the form of fuzzy *IF-THEN* rules. For example, a fuzzy *IF-THEN* rule for safety analysis is:

IF FR is *frequent* AND CS is *catastrophic* AND FCP is *likely*, THEN *safety estimate* is *Poor*.

As we stated in the first section, besides the fuzziness used to represent uncertain knowledge in a rule-based system, there is another kind of uncertainty in representing knowledge when the expert is unable to establish a strong correlation between premise and conclusion. That is, evidence available is not enough or experts are not 100% certain to believe in a hypothesis but only to certain degrees of belief or with credibility. For example, we may have the fuzzy rules with belief degrees for all possible consequent terms as follows:

R: IF FR is *frequent* AND CS *is critical* AND FCP *is unlikely* THEN *safety estimate is* {(*Good*, 0), (*Average*, 0), (*Fair*, 0.7), (*Poor*, 0.3)}.

Here {(*Good*, 0), (*Average*, 0), (*Fair*, 0.7), (*Poor*, 0.3)} is a belief distribution representation of the safety consequent, representing that we are 70% sure that safety level is *Fair*, and 30% sure that safety level is *Poor*.

To take into account belief degrees in a rule, the relative weight of each rule among all rules (the rule weight), as well as the relative weight of each antecedent attribute (the attribute weight), fuzzy rules for safety can be extended in the following way. In general, assume that the three antecedent parameters, $U_1 = \text{FR}$, $U_2 = \text{CS}$ and $U_3 = \text{FCP}$ can be described by J_i linguistic terms $\{A_{ij}, j = 1, \ldots, J_i\}$, $i = 1, 2, 3$, respectively. One consequent variable *safety estimates* can be described by N linguistic terms, i.e. D_1, D_2,...,D_N. Let A_i^k be a linguistic term corresponding to the ith attribute in the kth rule, with $i = 1, 2, 3$. Thus the kth rule in a rule-base can be written as follows:

$$\begin{aligned} &R_k: \text{ IF FR is } A_1^k \text{ AND CS is } A_2^k \text{ AND FCP is } A_3^k \text{ THEN } \textit{safety estimates} \text{ is} \\ &\{(D_1, \bar{\beta}_{1k}),\ (D_2, \bar{\beta}_{2k}), \ldots, (D_N, \bar{\beta}_{Nk})\},\ \left(\textstyle\sum_{i=1}^{N} \bar{\beta}_{ik} \le 1\right), \text{ with a rule weight } \theta_k, \\ &\text{and the attribute weights } \delta_1,\ \delta_2,\ \delta_3 \end{aligned} \tag{1}$$

where $\bar{\beta}_{ik}(i \in \{1, \ldots, N\}; k \in \{1, \ldots L\}$, L is the total number of the rules in the rule-base) is a belief degree measuring the subjective uncertainty of the consequent "*safety estimates* is D_i" drawn due to the antecedent "FR is A_1^k AND CS is A_2^k AND FCP is A_3^k in the kth rule. If $\sum_{i=1}^{N} \bar{\beta}_{ik} = 1$, the output assessment or the kth rule is said to be complete; if $\sum_{i=1}^{N} \bar{\beta}_{ik} = 1$ for all $k = 1, \ldots, L$, then the rule-base is a complete rule-base; otherwise, it is incomplete. Note that ($\sum_{i=1}^{N} \bar{\beta}_{ik} = 0$) denotes total ignorance about the output given the input in the kth rule. The rule-base with the rules in the form Eq. (1) is referred to as *a fuzzy rule-base with belief structures*. Note that a common fuzzy rule is the special cases of rule Eq. (1) with $\{\bar{\beta}_{1k}, \bar{\beta}_{2k}, \ldots, \bar{\beta}_{NK}\}$ being given special values. In fact, if we assign $\bar{\beta}_{ik} = 1$, $\bar{\beta}_{jk} = 0, j \neq i$, $j = 1, \ldots, N$ in rule Eq. (1), we get a common fuzzy rule.

A rule base given in the form shown in Eq. (1) represents functional mappings between antecedents and consequents with uncertainty. It provides a more informative and realistic scheme for uncertain knowledge representation. Note that the degrees of belief $\bar{\beta}_{ik}$ could be assigned directly by safety experts or more generally they may be trained and updated using dedicated learning algorithms if *a priori* or *up-to-date* information regarding the input and output of a rule-based system is available.

2.4 Fuzzy Rule-base Inference Mechanism Based on the Evidential Reasoning Approach

Once a rule-base is established, the knowledge contained can be used to perform inference for the given input. The inference procedure is basically composed of five steps, summarized in the following subsections.

2.4.1 Input Transformation

This is to discretize the input into the distributed representation of linguistic values in antecedents using belief degrees.

Corresponding to the rule-base (1), the general input form corresponding to the antecedent attribute in the kth rule is given as follows:

$$\left(A_1^*, \varepsilon_1\right) \text{ AND } \left(A_2^*, \varepsilon_2\right) \text{ AND } \left(A_3^*, \varepsilon_3\right) \tag{2}$$

where ε_i expresses the degree of belief assigned by an expert to the association of $A_i^*(i = 1, \ldots, 3)$, which reflects the uncertainty of the input data. For example, $(A_1^*, 0.9)$ means we are 90% certain that the input can take the value A_1^*.

By using the distribution assessment approach introduced in (Yang and Sen, 1994), in general, we may consider a linguistic term in the antecedent as an evaluation grade,

the input (A_i^*, ε_i) for an antecedent attribute $U_i \in \{\mathrm{FR}, \mathrm{CS}, \mathrm{FCP}\}$ can be assessed to a distribution representation of the linguistic terms using belief degrees as follows:

$$S\left(\left(A_i^*, \varepsilon_i\right)\right) = \{(A_{ij}, \alpha_{ij});\ j = 1, \ldots, J_i\},\ \ i = 1, 2, 3 \tag{3}$$

where $A_{ij}(j \in \{1, \ldots, J_i\})$ is the jth linguistic term of the ith attribute, α_{ij} the degree to which the input (A_i^*, ε_i) for U_i belongs to the linguistic term A_{ij} with $\alpha_{ij} \geq 0$ and $\sum_j^{J_i} \alpha_{ij} \leq 1 (i = 1, 2, 3)$. α_{ij} in Eq. (3) could be generated in various ways depending on the nature of an antecedent attribute and the available data, which is simply described in the following three cases. For more details one can refer to (Yang *et al.*, 2003).

(1) *Matching function method while the input is in numerical form and the linguistic value used to represent the antecedent is characterized by its fuzzy membership functions (for both quantitative and qualitative attribute).*

Note that normally attributes in the antecedent of a rule are different in nature. For example, FR and FCP are quantitative in nature, but CS is qualitative in nature. Due to the possible uncertainty in the inputs, for the purpose of safety modelling, it is assumed that each input parameter (i.e. FR, CS and FCP) may be fed to the proposed safety modelling in terms of fuzzy membership function (if it is available) in any one of the following forms based on history data and experts' experiences (Sii and Wang, 2002):

- A single deterministic value with 100 % certainty.
- A closed interval defined by an equally likely range.
- A triangular distribution defined by a most likely value, with lower and upper least likely values.
- A trapezoidal distribution defined by a most likely range, with lower and upper least likely values.

As for qualitative parameters, as we mentioned in subsection 2.2, a subjective numerical scale can be used against which the range of the CS parameters is mapped. It may come from the designer's experience and history records, i.e. here the numerical input for CS is based on that subject numerical scale.

Finally α_{ij} in Eq. (3) could be formulated in the following way:

$$\alpha_{ij} = \frac{\tau(A_i^*, A_{ij}) \cdot \varepsilon_i}{\sum_{j=1}^{J_i} [\tau(A_i^*, A_{ij})]},\ \ i = 1, 2, 3;\ \ j = 1, \ldots, J_i \tag{4}$$

where (A_i^*, ε_i) is the actual input corresponding to the ith antecedent, τ is a matching function, $\tau = (A_i^*, A_{ij}) = \tau_{ij}$ is a matching degree to which A_i^* belong to A_{ij}. Note that $\alpha_{ij} \geq 0$ and $\sum_j^{J_i} \alpha_{ij} \leq 1$. If A_i^* completely belongs to the jth linguistic expression, i.e. $\tau(A_i^*, A_{ij}) = 1$, α_{ij} may not be equal to 1 due to ε_i. In this framework, $S((A_i^*, \varepsilon_i))$ can be generated in various ways depending on the selection of the matching function. One possible matching function τ is given as follows and which is used in our case study in section 3:

$$\tau(A_i^*, A_{ij}) = \max_x [\min(A_i^*(x) \wedge A_{ij}(x))]. \tag{5}$$

We use the max-min operation in this definition for illustration purposes because it is a classical tool to set the matching degree between fuzzy sets (Zimmerman, 1991). However, other functions could be chosen. More details are referred to in (Yang *et al.*, 2003).

(2) *Rule-based or utility-based transformation methods while the input is in numerical forms but the fuzzy membership function of linguistic value for the antecedent is not*

available (only for the quantitative attribute) (Yang, 2001; Yang *et al.*, 2003). For quantitative attributes described by linguistic values, another way to get α_{ij} in Eq. (3) is using the rule or utility-based equivalence transformation techniques proposed in (Yang, 2001). The basic idea is that numerical data can be expressed as belief distributions using equivalence transformation techniques.

(3) *Subjective assessment method (for quantitative and qualitative attribute).* In this case the subjective judgements α_{ij} in Eq. (3) can be assessed based on the historical data, statistical distributions or expert experience. This subjective assessment can be taken as an alternative solution due to lack of information, e.g. when neither the membership function of each linguistic term nor numerical forms of the input is available at all, and is especially useful for qualitative attribute assessment, which sometimes is totally subjective. In assessment of qualitative parameter CS, for example, an expert may provide the following assessment: 30% sure that CS is at *moderate* level and 70% sure that it is *critical*.

2.4.2 Activation Weight for the Packet Antecedent of a Rule

Considering an input given by Eq. (2) corresponding to the kth rule defined as in (1),

$$\text{FR is } (A_1^k, \alpha_1^k) \text{ AND CS is } (A_1^k, \alpha_2^k) \text{ AND FCP is } (A_1^k, \alpha_3^k) \tag{6}$$

where $A_i^k \in \{A_{ij}, j = 1, \ldots, J_i\}$ and $\alpha_i^k \in \{\alpha_{ij}, j = 1, \ldots, J_i\}$ is the individual belief degree that the input belongs to A_i^k of the individual antecedent U_i appearing in the kth rule, the total degree α_k to which the input matches to the packet antecedent A^k in the kth rule is determined by combining the individual degrees $\alpha_i^k (i = 1, 2, 3)$.

Because "AND" connective is used for all antecedents in a rule. In other words, the consequent of a rule is not believed to be true unless all the antecedents of the rule are activated. Also considering that α_k is a belief degree (or a subjective probability), the "probability-product" aggregation functions may be used for generating such subjective probabilities. Hence α_k can be calculated using the following formula:

$$\alpha_k = \prod_{i=1}^{3} \left(\alpha_i^k\right)^{\bar{\delta}_i} \tag{7}$$

where

$$\bar{\delta}_i = \frac{\delta_i}{\max\limits_{i=1,2,3}\{\delta_i\}} \quad \text{so} \quad 0 \le \bar{\delta}_i \le 1.$$

$\delta_i (i = 1, 2, 3)$ is the weight of the ith antecedent attribute, L is the number of rules in the rule-base. Note that $0 \le \alpha_k \le 1$, $\alpha_k = 1$ if $\alpha_i^k = 1$ for all $i = 1, 2, 3$ and $\alpha_k = 0$ if $\alpha_i^k = 0$ for any $i = 1, 2, 3$. Also, the contribution of an antecedent attribute towards α_k is positively related to the weight of the attribute. In other words, an important attribute plays a greater role in determining α_k, which is further explained as follows.

(a) If $\bar{\delta}_l = 0\,(l \in \{i = 1, 2, 3\})$, then $\left(\alpha_l^k\right)^{\bar{\delta}_l} = 1$, which shows that an attribute with zero importance does not have any impact on the aggregation process; if $\bar{\delta} = 1_{,l}$, then $\left(\alpha_l^k\right)^{\bar{\delta}_l} = \alpha_l^k$, which shows that the most important antecedent has significant impact on the aggregation process proportional to the degree to which it is matched by the input.

(b) Note that if $a > b$ then $(a)^\delta \ge (b)^\delta$ for $\delta > 0$, which means that the function $(\alpha)^\delta$ is monotonically non-decreasing in the argument α if $\delta > 0$. In other words, if the individual belief with regard to the antecedent attribute is increased the overall belief shouldn't decrease.

(c) Furthermore $(\alpha)^\delta$ is non-increasing in δ for $0 \le \alpha \le 1$. In other words, if $\delta_1 < \delta_2$, then $(\alpha)^{\delta_1} \ge (\alpha)^{\delta_2}$ for $0 \le \alpha \le 1$. Therefore, as the ith antecedent attribute in the kth rule becomes more important, δ_i increases, which will increase the possibility of α^k being dominated by α_i^k.

Finally, the activation weight w_k of the packet antecedent A^k in the kth rule is generated by weighting and normalizing the matching degree α_k given by Eq. (7)) as follows:

$$w_k = (\theta_k \cdot \alpha_k) \Big/ \left(\sum_{i=1}^{L} \theta_i \alpha_i \right) \tag{8}$$

where θ_k is the relative weight of the kth rule. Note that $0 \le w_k \le 1 (k = 1, \ldots, L)$ and $\sum_{i=1}^{L} w_i = 1$, and $w_k = 0$ if the kth rule is not activated.

2.4.3 Degree of Belief in the Consequent of a Rule

An incomplete input for an attribute will lead to an incomplete output in each of the rules in which the attribute is used. In the inference procedure, such incompleteness should be considered. The original belief degree in the ith consequent term of the kth rule in (1) is updated based on the actual input information as follows:

$$\beta_{ik} = \bar{\beta}_{ik} * \left[\sum_{t=1}^{3} (\tau(t,k) * \sum_{j=1}^{J_t} \alpha_{ij} \right] \Big/ \left[\sum_{t=1}^{3} \tau(t,k) \right] \tag{9}$$

where $\tau(t,k) = \begin{cases} 1 & \textit{if } U_t \text{ is used in defining } R_k \\ 0 & \text{otherwise} \end{cases}$ $(t = 1,2,3)$, α_{tj} is given in Eq. (3)) with $\alpha_{tj} \ge 0$ and $\sum_{j}^{J_t} \alpha_{tj} \le 1$. $\bar{\beta}_{ik}$ is given in (1) with $0 \le \sum_{i=1}^{N} \bar{\beta}_{ik} \le 1$. Note that $0 \le \sum_{i=1}^{N} \beta_{ik} \le 1$ for all k and $1 - \sum_{i=1}^{N} \beta_{ik}$ denotes both the ignorance incurred in establishing R_k and the incompleteness that may exist in the input information.

2.4.4 Fuzzy Rule Expression Matrix for a Rule-base

Suppose a fuzzy rule-base with the belief structure is given by $R = \{R_1, R_2, \ldots, R_L\}$. The kth rule in Eq. (1)) can be represented as follows:

$$R_k : \text{IF } U \text{ is } A^k \text{ THEN } \textit{safety estimate} \text{ is } D \text{ with belief degree } \beta_k \tag{10}$$

where U represents the antecedent attribute vector (FR, CS, FCP), A^k the packet antecedents $\{A_1^k, A_2^k, A_3^k\}$, D the consequent vector $(D_1, D_2, \ldots, D_N)$, β_k the vector of the belief degrees $(\beta_{1k}, \beta_{2k}, \ldots, \beta_{Nk})$ and $k \in \{1, \ldots, L\}$. Each fuzzy rule with belief structure can be explained as follows.

The packet antecedent A^k of an IF–THEN rule can be considered as a global attribute, which is considered as being assessed to a consequent D_i with a belief degree of β_{ik} $(i \in \{1, \ldots, N\})$. This assessment can be represented by

$$S(A^k) = \{(D_i, \beta_{ik}); i = 1, \ldots, N\} \tag{11}$$

which is obviously a distributed assessment and is referred to as *a belief structure*, where β_{ik} measures the degree to which D_i is the consequent if the input activates the antecedent A^k in the kth rule, which is given using Eq. (9) with $0 \le \sum_{i=1}^{N} \beta_{ik} \le 1$, $i = 1, \ldots, N$, $k = 1, \ldots, L$. L is the number of rules in the rule-base and N is the number of the possible consequent terms in the kth rule.

A fuzzy rule-base with belief structure established using rules given by Eq. (10) can be summarized using the following rule expression matrix shown in Table I).

In the matrix, w_k is the activation weight of A^k, which measures the degree to which the kth rule is weighted and activated.

TABLE I Rule expression matrix for a fuzzy rule-base

Belief	Output					
Input	D_1	D_2	...	D_i	...	D_N
$A^1(w_1)$	β_{11}	β_{21}	...	β_{i1}	...	β_{N1}
⋮	⋮	⋮	...	⋮	...	⋮
$A^k(w_k)$	β_{1k}	β_{2k}	...	β_{ik}	...	β_{Nk}
⋮	⋮	⋮	...	⋮	...	⋮
$A^L(w_L)$	β_{1L}	β_{2L}	...	β_{iL}	...	β_{NL}

2.4.5 Rule Inference using the Evidential Reasoning (ER) Approach

Having represented each rule in a rule expression matrix, the ER approach (Yang and Xu, 2002a) can be used to combine rules and generate final conclusions, which can be directly implemented as follows. First, transform the degrees of belief β_{jk} for all $j = 1, \ldots, N$, $k = 1, \ldots, L$ into basic probability masses using the following equations (Yang and Xu, 2002a):

$$\begin{aligned} m_{j,k} &= w_k \beta_{j,k}, j = 1, \ldots, N; \\ m_{D,k} &= 1 - \sum\nolimits_{j=1}^{N} m_{j,k} = 1 - w_k \sum\nolimits_{j}^{N} \beta_{j,k}, \\ \bar{m}_{D,k} &= 1 - w_k \ \text{ and } \ \tilde{m}_{D,k} = w_k \left(1 - \sum\nolimits_{j=1}^{N} \beta_{j,k}\right). \end{aligned}$$

For all $k = 1, \ldots, L$, with $m_{D,k} = \bar{m}_{D,k} + \tilde{m}_{D,k}$ for all $k = 1, \ldots, L$ and $\sum_j^L w_j = 1$. The probability mass assigned to the consequent D, which is unassigned to any individual output terms D_j, is split into two parts, one caused by the relative importance of the kth packet antecedent A^k or $\bar{m}_{D,k}$, the other by the incompleteness of the kth packet antecedent A^k or $\tilde{m}_{D,k}$.

Then, aggregate all the packet antecedents of the L rules to generate the combined degree of belief in each possible consequent term D_j in D. Suppose $m_{j,I(k)}$ is the combined degree of belief in D_j by aggregating the first k packet antecedents ($A^1, \ldots, A^k$) and $m_{D,I(k)}$ is the remaining degree of belief unassigned to any output term. Let $m_{j,I(1)} = m_{j,1}$ and $m_{D,I(1)} = m_{D,1}$. Then the overall combined degree of belief β_j in D_j is generated as follows:

$$\{D_j\}: \ m_{j,I(k+1)} = K_{I(k+1)}[m_{j,I(k)} m_{j,k+1} + m_{j,I(k)} m_{D,k+1} + m_{D,I(k)} m_{j,k+1}]$$

$$m_{D,I(k)} = \bar{m}_{D,I(k)} + \tilde{m}_{D,I(k)}, \quad k = 1, \ldots, L$$

$$\{D\}: \tilde{m}_{D,I(k+1)} = K_{I(k+1)}[\tilde{m}_{D,I(k)} \tilde{m}_{D,k+1} + \tilde{m}_{D,I(k)} \bar{m}_{D,k+1} + \bar{m}_{D,I(k)} \tilde{m}_{D,k+1}]$$

$$\{D\}: \bar{m}_{D,I(k+1)} = K_{I(k+1)}[\bar{m}_{D,I(k)} \bar{m}_{D,k+1}]$$

$$K_{I(k+1)} = \left[1 - \sum_{j=1}^{N} \sum_{t=1, t \neq j}^{N} m_{j,I(k)} m_{t,k+1}\right]^{-1}, \quad k = 1, \ldots, L-1$$

$$\{D_n\}: \ \beta_j = (m_{j,I(L)})/(1 - \bar{m}_{D,I(L)}), \quad j = 1, \ldots, N$$

$$\{D\}: \ \beta_D = (\tilde{m}_{D,I(L)})/(1 - \bar{m}_{D,I(L)}).$$

β_D represents the remaining belief degrees unassigned to any D_j. It has been proved that $\sum_{j=1}^{N}\beta_j + \beta_D = 1$ (Yang and Xu, 2002a). For more details about the ER algorithm, its background and theoretical foundation one can refer to (Yang and Singh, 1994; Yang and Sen 1994, 1997; Yang, 2001; Yang and Xu, 2002a,b).

The final conclusion generated by aggregating the L rules, which are activated by the actual input vector $A^* = \{A^{*k}, k = 1, \ldots L\}$ for $U = (\mathrm{FR}, \mathrm{CS}, \mathrm{FCP})$ can be represented as follows:

$$S(A^*) = \{(D_j, \beta_j), j = 1, \ldots, N\}. \tag{12}$$

The inference procedure is based on fuzzy rule-base and the evidential reasoning approach, referred to as a *fuzzy rule-based evidential reasoning approach*—FURBER approach. The final result is still a belief distribution on safety expression, which gives a panoramic view about the safety level for a given input.

The logic behind the approach is that if the output in the kth rule includes D_i and the kth rule is activated, then the overall output must be D_i to a certain degree. The degree is measured by both the degree to which the kth rule is important to the overall output and the degree to which the antecedents of the kth rule are activated by the actual input.

The computational complexity of reasoning using Dempster's rule based on the above specific ER decision analysis framework becomes linear rather than #P-complete (Yang and Singh, 1994; Yang and Sen, 1994; Yang, 2001). It should also be noted that conflicting information can be explicitly modelled using the framework shown in Table I using the normalized activation weight w_k and can be logically processed using the ER algorithm described earlier in this subsection, thereby overcoming another drawback of the original combination rule of the Dempster-Shafer theory in dealing with conflicting evidence.

3. A NUMERICAL STUDY IN MARINE AND OFFSHORE ENGINEERING

3.1 Problem Description

FPSO (Floating production, storage and offloading) vessels are by far the most popular platform for floating production systems in offshore oil and gas fields worldwide. Since the tandem configuration is dominant in the North Sea, collision between FPSO and shuttle tanker in tandem offloading operation has caused a growing concern in the North Sea (McCaul, 2001). In this section, safety assessment is carried out on risk introduced by the collision of FPSO and shuttle tanker during tandem offloading operation. Only risk caused by technical failures is assessed here.

According to the literature survey, the technical failures that might cause collisions between an FPSO and a shuttle tanker during tandem offloading operations are malfunctions of propulsion systems (Chen and Moan, 2002). The four major causes of these technical failures are:

(1) Controllable pitch propeller (CPP) failure
(2) Thruster failure
(3) Position reference system (PRS) failure
(4) Dynamics positioning (DP) system failure

In this case study, for illustration purposes, we only consider the safety assessment related to CPP failure to demonstrate the procedure involved in the FURBER inference engine.

Seven levels of linguistic variables are used for FR; five levels for CS, seven levels for FCP. The definition of their linguistic terms and the corresponding membership functions were given in Tables A1–A3 in the Appendix and Figs. 1–3 and four linguistic expressions for safety mentioned in subsection 2.2.

3.2 Safety Estimate for CPP using FURBER Approach

In the following subsections, the evaluation made by the expert on collision risk caused by the CPP failure is discussed in detail to demonstrate the procedure involved in the proposed FURBER scheme for the safety model.

3.2.1 Rule-base with Belief Structure Construction

According to the number of linguistic terms used for describing the antecedent, the rule-base with a total number of 245 fuzzy rules with belief structures is used in the case study, a part of which (32 rules) is listed in Table A.4 of the Appendix (here the weights of rules and the attributes have not been considered in this rule-base). This fuzzy rule-base with belief structures is constructed on the basis of the fuzzy rule-base without considering the belief structure as described in Sii and Wang (2002). For instance, some original fuzzy rules in Sii and Wang (2002) are:

R_{198}: IF FR is *frequent* AND CS *is critical* AND FCP *is unlikely* THEN *safety estimate is Fair*
R_{199}: IF FR *is frequent* AND CS *is critical* AND FCP *is reasonably unlikely* THEN *safety estimate is Fair*
R_{200}: IF FR *is frequent* AND CS *is critical* AND FCP *is likely* THEN *safety estimate is Fair*.

They are further revised into the fuzzy rules with belief structures as follows:

R^*_{198}: IF FR *is frequent* AND CS *is critical* AND FCP *is unlikely* THEN *safety estimate is* {(*Good*, 0), (*Average*, 0.2), (*Fair*, 0.7), (*Poor*, 0.1)}
R^*_{199}: IF FR *is frequent* AND CS *is critical* AND FCP *is reasonably unlikely* THEN *safety estimate is* {(*Good*, 0), (*Average*, 0), (*Fair*, 0.8), (*Poor*, 0.2)}
R^*_{200}: IF FR *is frequent* AND CS *is critical* AND FCP *is likely* THEN *safety estimate is* {(*Good*, 0), (*Average*, 0), (*Fair*, 0.5), (*Poor*, 0.5)}.

Comparing the original rules with the revised rules, one can see the difference between the former and the latter, also the advantage of the latter rules. Actually, note that in the former rules, the different linguistic terms of antecedents lead to the same output *Fair*, which seems not entirely reasonable. But from the latter rules, the different linguistic terms of antecedents lead to different outputs. The latter rules actually provide a more flexible and rational way to construct fuzzy rule-bases. Here we need to mention that the belief degrees of the rules of the rule-base in Table A.4 of the Appendix are assumed only for illustration purposes. The actual degrees of belief depend on the context of applications.

3.2.2 Input Transformation and the Fuzzy Rule Expression Matrix

Suppose the experts use triangular distribution input form to address the inherent uncertainty associated with the information available while carrying out assessment on the three input

parameters for CPP. FR is described triangularly as (6.5, 8.0, 9.5), CS as (7.5, 8.5, 9.5), and FCP as (5.5, 7.0, 8.5). As shown in Table A.5 of the Appendix, these inputs are transformed into the distributed representation of linguistic terms in the antecedent using Eqs. (4) and (5) (ε_i is supposed to be 1 in Eq. (4)) based on the membership function defined in Figs. 1–3 in the Appendix.

In the rule-base, 245 rules have been established, of which only 32 rules are fired due to the CPP failure in this particular case for expert #1, i.e. Rules #130–133, #137–140, #165–168, #172–175, #200–203, #207–210, #235–238, and #242–245 which are all listed in Table A.4 in the Appendix.

Based on the individual belief degrees, the activation weight $w_k (k = 1, \ldots, 32)$ of each rule in the fired sub-rule-base shown in Table A.4 is calculated using Eqs. (7) and (8), where we suppose that the attribute weight $\delta_i = 1 (i = 1, 2, 3)$ and the rule weight $\theta_k = 1 (k = 1, \ldots, 32)$. Together with the fired rule-base with belief structures, the fuzzy rule expression matrix for the sub-rule-base with the fired 32 rules is shown in Table A.6 in the Appendix. Taking the Rule #132 as an example, the assessment can be represented by

$$S(A^{132}) = \{(Good, 0), (Average, 0), (Fair, 0.3), (Poor, 0.7)\}$$

with the activation weight $w_{132} = 0.015$.

3.2.3 Fired Rule Combination using the ER Algorithm

Based on Table A.6, the Window-based and graphically designed intelligent decision system (IDS) (Yang and Xu, 1999), which has been developed based on the ER approach in (Yang and Xu, 2002a), is used to implement the combination of 32 rules and generate safety estimates. The final assessment result for CPP is obtained as follows and shown in Fig. 4.

$$\text{Obtained Result}: \quad \{(Good, 0), (Average, 0.0057), (Fair, 0.3735), (Poor, 0.6208)\}.$$

This result can be interpreted in such a way that the safety estimate of CPP to technical failure is *Average* with a belief degree of 0.0057, *Fair* with a belief degree of 0.3735 and *Poor* with a belief degree of 0.6208.

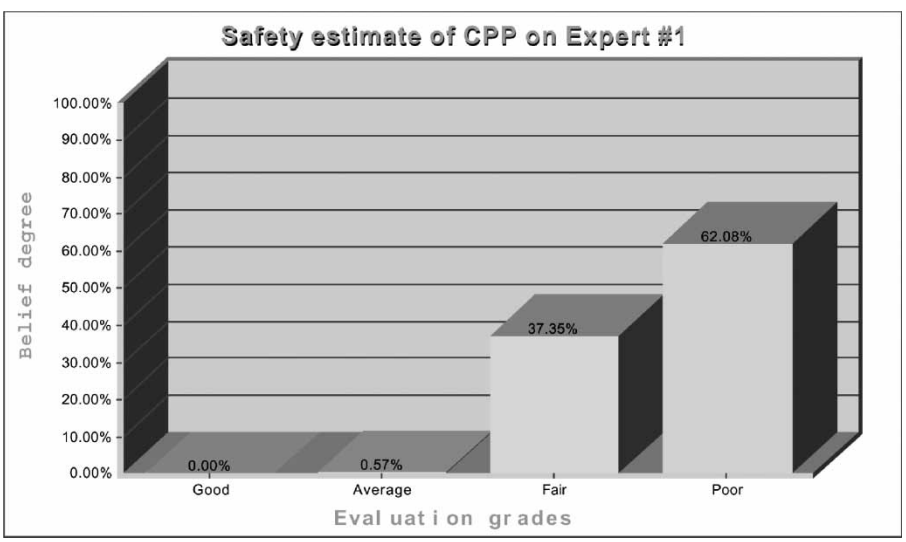

FIGURE 4 The safety estimate of CPP.

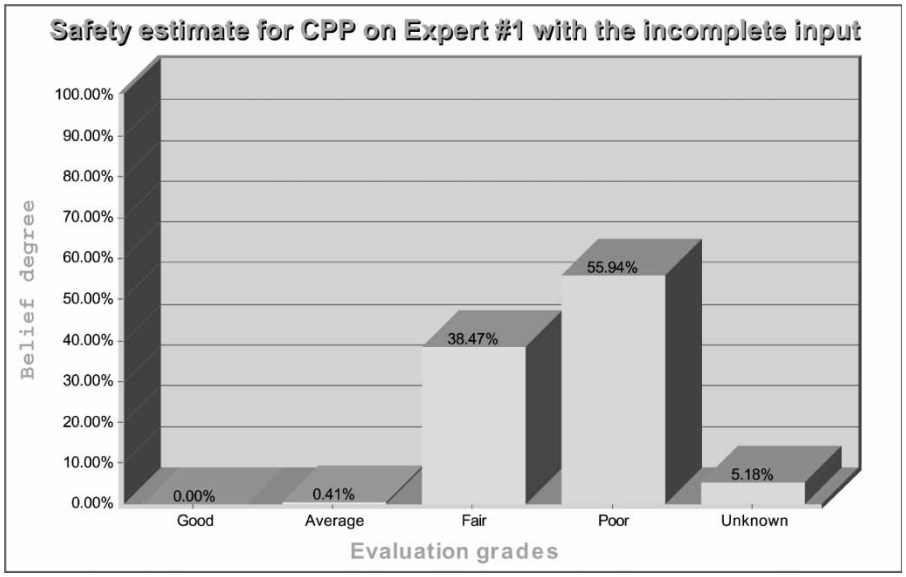

FIGURE 5 The final safety estimate of CPP based on the incomplete input.

This final distribution assessment result gives a panoramic view about the output status, from which one can see the variation between the original output and the revised output on each linguistic term. A distribution is easy to understand and flexible to represent input information than a single average value.

3.3 Illustration of Safety Estimate Based on the Incomplete Input

To illustrate how the incompleteness of input can be reflected and dealt with in the FURBER inference engine, we use another example by modifying the input for CPP, i.e. we use the same input for FR and FCP, but modify the input for CS so that it is incomplete. Suppose that the discretization of the input is $\{(Critical, 0.2), (Catastrophic, 0.6)\}$ (as shown in Table A.5 using bold fonts in the parentheses of the fourth column). It is possible in the case that no numerical value is available and only subjective judgements can be used with the incomplete belief assessment due to lack of information.

Due to the change in the input, the activation weight of each rule is also changed using Eqs. (7) and (8), and the degrees of belief of the rule are updated using Eq. (9), respectively. Because the input of CS is incomplete, i.e. $\sum_j^5 \alpha_{2j} = 0.8 < 1$, together with $\sum_j^7 \alpha_{1j} = \sum_j^7 \alpha_{3j} = 1$, then the degree of belief $\bar{\beta}_{ik}$ for the fired 32 rules ($i = 1, \ldots, 4$; $k = 1, \ldots, 32$) is updated into $\beta_{ik} = \bar{\beta}_{ik} * \frac{2.8}{3} = \beta_{ik} = \bar{\beta}_{ik} * 0.9333$ using Eq. (14). Hence, $0 < \sum_{i=1}^4 \beta_{ik} < 1$ for all k. Table A.7 in the Appendix is the fuzzy rule expression matrix for the sub-rule-base, which is generated using the fired 32 rules based on this new input. The final assessment result is obtained using IDS as follows and also shown in Fig. 5:

$$\{(Good, 0), (Average, 0.0041), (Fair, 0.3847), (Poor, 0.5594), (\text{unknown}, 0.0518)\}.$$

"Unknown" in the above result means that the output is also incomplete due to the incomplete input. The final output still gives an overall assessment about the safety level with the possible incompleteness, i.e. the degree of belief for the unknown factor.

The similar computation may be performed for safety assessment using the proposed fuzzy-logic-based evidential reasoning approach for the other three potential causes (Thruster, PRS and DP) of technical failure.

Based on the new generic framework for modelling system safety, the evidential reasoning approach can be used not only to aggregate fuzzy rules for safety analysis but to assess the safety of the whole system as well. This step is concerned with safety synthesis for a system using various configurations such as:

- Multi-cause safety synthesis—the synthesis of safety estimates of various causes to a technical failure done by an expert, or
- Multi-expert safety synthesis—the synthesis of safety estimates of a specific cause to a technical failure done by a panel of experts, or
- Multi-cause-multi-expert—a combination of the above two.

The work for these configurations will be discussed in detail in another paper.

4. CONCLUSION

A fuzzy rule-based safety analysis framework using the evidential reasoning approach was proposed in this paper. In this approach, a fuzzy rule-base with the belief structures was designed to capture uncertainty and nonlinear causal relationships in safety assessment. The inference process of such a rule-based system was characterized by a rule expression matrix and implemented using the ER approach. One of the unique features of the approach is its ability to handle both vague information and ignorance or incompleteness caused due to evidence not strong enough to make simple true or false judgments but with degrees of belief. This feature is quite useful for safety assessment of engineering systems, especially in the initial conceptual design stages or a system with a high level of innovation. The proposed approach can provide a flexible and effective way to represent and a rigorous procedure to deal with such hybrid uncertain assessment information to arrive at rational conclusions.

Different from most conventional rule-base inference methods, also from type-2 fuzzy model or intuitionistic fuzzy set theory, the FURBER approach is characterized with other unique features. Firstly, each input can be represented as a distribution on the linguistic value for the antecedent using the belief structure. The main advantage of doing so is that precise data, random numbers and subjective judgments with uncertainty can be consistently modelled under the unified framework. Secondly, the ER approach provides a novel procedure for aggregating rules, which can preserve the original features of various types of information. Moreover, this new methodology provides scope and flexibility for rule training and self-learning/updating in a rule-base.

The proposed framework offers great potential in safety assessment of engineering systems, especially in the initial conceptual design stages or a system with a high level of innovation where the related safety information is scanty or with various types of uncertainty involved.

Acknowledgements

This work forms part of the projects supported by the UK Engineering and Physical Sciences Research Council (EPSRC) under Grant Numbers GR/R30624 and GR/R32413; The National Natural Science Foundation of China (NSFC) under the Grant No: 70171035, and the Fujian Natural Science Foundation under the Grant No: A0010002.

References

An, M., Wang, J. and Ruxton, T. (2000) "The development of fuzzy linguistic risk levels for risk analysis of offshore engineering products using approximate reasoning approach", *Proceedings of OMAE 2000: 19th International Conference on Offshore Mechanics and Arctic Engineering* (14–17 February, New Orleans, USA).

Atanassov, K.T. (1986) "*Intuitionistic fuzzy sets*", *Fuzzy Set. Syst.* **20**, 87–96.
Atanassov, K.T. (1999) *Intuitionistic Fuzzy Sets* (Physica-Verlag, Heidelberg, New York).
Bell, P.M. and Badiru, A.B. (1996) "Fuzzy modelling and analytic hierarchy processing to quantify risk levels associated with occupational injuries—Part I: The development of fuzzy-linguistic risk levels", *IEEE Trans. Fuzzy Syst.* **4**(2), 124–131.
Binaghi, E. and Madella, P. (1999) "Fuzzy Dempster-Shafer reasoning for rule-based classifiers", *Int. J. Intellt Syst.* **14**, 559–583.
Bowles, J.B. and Pelaez, C.E. (1995) "Fuzzy logic prioritisation of failures in a system failure mode, effects and criticality analysis", *Reliab. Eng. Syst. Saf.* **50**, 203–213.
Chen, L.-H. (1997) "An extended rule-based inference for general decision-making problems", *Inf. Sci.* **102**, 247–261.
Chen, H. and Moan, T. (2002) "Collision risk analysis of FPSO-tanker offloading operation", *21st International Conference on Offshore Mechanics & Arctic Engineering* (23–28 June 2002, Oslo, Norway)
Cox, E.D. (1998) *The Fuzzy Systems Handbook: A practitioner's Guide to Building: A practitioner's Guide to Building, Using, and Maintaining Fuzzy Systems*, 2nd Edn. (Academic Press).
De, S.K., Biswas, R. and Roy, A.R. (2001) "An application of intuitionistic fuzzy sets in medical diagnosis", *Fuzzy Sets Syst.* **117**, 209–213.
Dempster, A.P. (1968) "A generalization of Bayesian inference", *Journal of the Royal Statistical Society (Series B)* **30**, 205–247.
Duckstein, L. (1994) "Elements of fuzzy set analysis and fuzzy risk", In: Nachtnebel, H.P., ed, *Decision Support Systems in Water Resources Management* (UNESCO Press, Paris), pp 410–430.
Ishizuka, M., Fu, K.S. and Ya, J.T.P. (1982a) "A rule-based inference with fuzzy sets for structural damage assessment", In: Gupta, M.M. and Sanchez, E, eds, *Approximate Reasoning in Decision Analysis* (North Holland, Amsterdam), pp 261–268.
Ishizuka, M., Fu, K.S. and Ya, J.T.P. (1982b) "Inference procedure under uncertainty for the problem-reduction method", *Inf. Sci.* **28**, 179–206.
Karnik, N., Mendel, J. and Liang, Q. (1999) "Type-2 fuzzy logic systems", *IEEE Trans. Fuzzy Sys.* **7**(6), 643–658.
Karwowski, W. and Mital, A. (1986) "Potential applications of fuzzy sets in industrial safety engineering", *Fuzzy Sets Sys.* **19**, 105–120.
Keller, A.A. and Kara-Zaitri (1989) "Further applications of fuzzy logic to reliability assessment and safety analysis", *Micro. Reliab.* **29**, 399–404.
Liu, J., Yang, J.B., Wang, J. and Sii, H.S. (2003) "Review of uncertainty reasoning approaches as guidance for maritime and offshore safety-based assessment", *J. UK Safety Reliab. Soc.* **23**(1), 63–80.
McCaul, J.R. (2001) "Special report—floating production systems", *Oil Gas J.*, June 11.
Mendel, J. (2001) *Uncertain Rule-Based Fuzzy Logic Systems: Introduction and New Directions* (Prentice-Hall, NJ).
Mendel, J. and John, R. (2002) "Type-2 fuzzy sets made simple", *IEEE Trans. Fuzzy Sys.* **10**(2), 117–127.
Mizumoto, M. and Tanaka, K. (1976) "Some properties of fuzzy sets of type 2", *Inf. Control* **31**, 312–340.
Ogawa, H., Fu, K.S. and Yao, J.T.P. (1985) "An inexact inference for damage assessment of existing structures", *Int. J. Man-Machine Studies* **22**, 295–306.
Pearl, J. (1988) *Probabilistic Reasoning in Intelligent Systems* (Morgan Kaufmann, San Mateo, CA).
Schmucker, K.J. (1984) *Fuzzy sets, Natural Language Computations, and Risk Analysis* (Computer Science Press).
Shafer, G. (1976) *A Mathematical Theory of Evidence* (Princeton University Press, Princeton, NJ).
Sii, H.S. and Wang, J. (2002) Safety assessment of FPSOs—*The process of modelling system safety and case studies*, Report of the project—"*The Application of Approximate Reasoning Methodologies to Offshore Engineering Design*" (EPSRC GR/R30624 and GR/R32413) (Liverpool John Moores University, UK)
Sii, H.S., Wang, J. and Ruxton, T. (2001) "A fuzzy-logic-based approach to subjective safety modelling for maritime products", *J. UK Safety Reliability Soc.* **21**(2), 65–79.
Smets, P. (1988) "Belief function", In: Smet, P., Mamdani, E.H., Dubois, D. and Prade, H., eds, *Non-Standard Logics for Automated Reasoning* (Academic Press, London), pp 253–277.
Wang, J. (1997) "A subjective methodology for safety analysis of safety requirements specifications", *IEEE Trans. Fuzzy Sys.* **5**(3), 418–430.
Wang, J. and Ruxton, T. (1997) "A review of safety analysis methods applied to the design process of large engineering products", *J. Eng. Design* **8**(2), 131–152.
Wang, J., Yang, J.B. and Sen, P. (1995) "Safety analysis and synthesis using fuzzy set modelling and evidential reasoning", *Reliability Eng. Sys. Safety* **47**(3), 103–118.
Wang, J., Yang, J.B. and Sen, P. (1996) "Multi-person and multi-attribute design evaluations using evidential reasoning based on subjective safety and cost analyses", *Reliab. Eng. Sys. Safety* **52**(2), 113–129.
Yager, R.R. (1982) "Generalized probabilities of fuzzy events from belief structures", *Inf. Sci.* **28**, 45–62.
Yager, R.R. and Filev, D.P. (1995) "Including probabilistic uncertainty in fuzzy logic controller modelling using Dempster-Shafer theory", *IEEE Trans. Syst., Man Cybernet.* **25**, 1221–1230.
Yang, J.B. (2001) "Rule and utility based evidential reasoning approach for multi-attribute decision analysis under uncertainties", *Eur. J. Operational Res.* **131**, 31–61.
Yang, J.B. and Sen, P. (1994b) "A general multi-level evaluation process for hybrid MADM with uncertainty", *IEEE Trans. Sys. Man Cybernet.* **24**(10), 1458–1473.
Yang, J.B. and Singh, M.G. (1994a) "An evidential reasoning approach for multiple attribute decision making with uncertainty", *IEEE Trans. Syst. Man Cybernet.* **24**(1), 1–18.

Yang, J.B. and Sen, P. (1997) "Multiple attribute design evaluation of large engineering products using the evidential reasoning approach", *J. Eng. Des.* **8**(3), 211–230.
Yang, J.B., Xu, D.L., (1999) *Intelligent Decision System via Evidential Reasoning* (Version 1.1, IDSL, Cheshire, England).
Yang, J.B. and Xu, D.L. (2002a) "On the evidential reasoning algorithm for multiple attribute decision analysis under uncertainty", *IEEE Trans. Syst. Man Cybernet. Part A* **32**(3), 289–304.
Yang, J.B. and Xu, D.L. (2002b) "Nonlinear information aggregation via evidential reasoning in multi-attribute decision analysis under uncertainty", *IEEE Trans. Sys., Man Cybernet. Part A: Sys. Hum.* **32**(3), 376–393.
Yang, J.B., Liu, J., Wang, J. and Sii, H.S. (2003) "A generic rule-bases inference methodology using the evidential reasoning approach—RIMER", *IEEE Trans. Sys. Man cybernet.*, (submitted).
Zadeh, L.A. (1965) "Fuzzy sets", *Information Control* **8**, 338–353.
Zadeh, L.A. (1975) "The concept of a linguistic variable and its application to approximate reasoning -1", *Inf. Sci.* **8**, 199–249.
Zimmerman, H.J. (1991) *Fuzzy Set Theory and Its Application* (Kluwer, Norwell, MA).

APPENDIX:

TABLE A.1 FR description (Sii and Wang, 2002)

Rank	*FR*	*Meaning (general interpretation)*	*FR (1/year)*
1,2,3	Very low	Failure is unlikely but possible during lifetime	$< 10^{-6}$
4	Low	Likely to happen once during lifetime	0.25×10^{-5}
5	Reasonably low	Between low and average	0.25×10^{-4}
6	Average	Occasional failure	10^{-3}
7	Reasonably frequent	Likely to occur from time to time	0.25×10^{-2}
8,9	Frequent	Repeated failure	0.125×10^{-1}
9,10	Highly frequent	Failure is almost inevitable or likely to exist repeatedly	$> 0.25 \times 10^{-1}$

TABLE A.2 CS description (Sii and Wang, 2002)

Rank	*CS*	*Meaning (generic marine and offshore structure/system interpretation)*
1	Negligible	At most a single minor injury or unscheduled maintenance required (service and operations can continue)
2,3	Marginal	Possible single or multiple minor injuries or/and minor system damage. Operations interrupted slightly, and resumed to its normal operational mode within a short period of time (say less than 2 h)
4,5,6	Moderate	Possible multiple minor injuries or a single severe injury, moderate system damage. Operations and production interrupted marginally, and resumed to its normal operational mode within, say, no more than 4 h
7,8	Critical	Possible single death, probable multiple severe injuries or major system damage. Operations stopped, platform closed, shuttle tanker's failure to function. High degree of operational interruption due to the nature of the failure such as an inoperable platform (e.g. drilling engine fails to start, power system failure, turret mooring system failure) or an inoperable convenience subsystem (e.g. DP, PRS)
9,10	Catastrophic	Possible multiple deaths, probable single death or total system loss. Very high severity ranking when a potential failure mode (e.g. collision between FPSO and shuttle tanker, blow-out, fire and explosion) affects safe platform operation and/or involves non-compliance with government regulations

TABLE A.3 FCP description (Sii and Wang, 2002)

Rank	*FCP*	*Meaning*
1	Highly unlikely	The occurrence likelihood of possible consequences is highly unlikely given the occurrence of the failure event (extremely unlikely to exist on the system or during operations)
2,3	Unlikely	The occurrence likelihood of possible consequences is unlikely but possible given that the failure event happens (improbable to exist even on rare occasions on the system or during operations)
4	Reasonably unlikely	The occurrence likelihood of possible consequences is reasonably unlikely given the occurrence of the failure event (likely to exist on rare occasions on the system or during operations)
5	Likely	It is likely that consequences happen given that the failure event occurs (a programme is not likely to detect a potential design or operations procedural weakness)
6,7	Reasonably likely	It is reasonably likely that consequences occur given the occurrence of the failure event (i.e. exist from time to time on the system or during operations, possibly caused by a potential design or operations procedural weakness)
8	Highly likely	It is highly likely that consequences occur given the occurrence of the failure event (i.e. often exist somewhere on the system or during operations due to a highly likely potential hazardous situation or design and/or operations procedural drawback)
9,10	Definite	Possible consequences happen given the occurrence of a failure event (i.e. likely to exist repeatedly during operations due to an anticipated potential design and operations procedural drawback)

TABLE A.4 Sub-rule-base including 32 fired safety rules with belief structure

	Belief				
		Safety estimate			
Item	*Antecedent attribute*	*good*	*average*	*fair*	*poor*
R#130	(*average*, *critical*, *likely*)			1	
R#131	(*average*, *critical*, *reasonably likely*)			0.5	0.5
R#132	(*average*, *critical*, *highly likely*)			0.3	0.7
R#133	(*average*, *critical*, *definite*)			0.2	0.8
R#137	(*average*, *catastrophic*, *likely*)			0.5	0.5
R#138	(*average*, *catastrophic*, *reasonably likely*)			0.4	0.6
R#139	(*average*, *catastrophic*, *highly likely*)			0.2	0.8
R#140	(*average*, *catastrophic*, *definite*)			0.15	0.85
R#165	(*reasonably frequent*, *critical*, *likely*)			0.8	0.2
R#166	(*reasonably frequent*, *critical*, *reasonably likely*)			0.5	0.5
R#167	(*reasonably frequent*, *critical*, *highly likely*)			0.4	0.6
R#168	(*reasonably frequent*, *critical*, *definite*)			0.3	0.7
R#169	(*reasonably frequent*, *catastrophic*, *highly unlikely*)		0.6	0.4	
R#172	(*reasonably frequent*, *catastrophic*, *likely*)		0.1	0.8	0.1
R#173	(*reasonably frequent*, *catastrophic*, *reasonably likely*)			0.9	0.1
R#174	(*reasonably frequent*, *catastrophic*, *highly likely*)			0.4	0.6
R#175	(*reasonably frequent*, *catastrophic*, *definite*)			0.3	0.7
R#200	(*frequent*, *critical*, *likely*)			0.5	0.5
R#201	(*frequent*, *critical*, *reasonably likely*)			0.3	0.7
R#202	(*frequent*, *critical*, *highly likely*)		0.1	0.1	0.8
R#203	(*frequent*, *critical*, *definite*)			0.2	0.8
R#207	(*frequent*, *catastrophic*, *likely*)			0.5	0.5
R#208	(*frequent*, *catastrophic*, *reasonably likely*)			0.4	0.6

TABLE A.4 – *continued*

	Belief				
		Safety estimate			
Item	*Antecedent attribute*	*good*	*average*	*fair*	*poor*
R#209	(*frequent*, *catastrophic*, *highly likely*)			0.3	0.7
R#210	(*frequent*, *catastrophic*, *definite*)			0.2	0.8
R#235	(*highly frequent*, *critical*, *likely*)			0.4	0.6
R#236	(*highly frequent*, *critical*, *reasonably likely*)			0.2	0.8
R#237	(*highly frequent*, *critical*, *highly likely*)			0.1	0.9
R#238	(*highly frequent*, *critical*, *definite*)				1
R#242	(*highly frequent*, *catastrophic*, *likely*)			0.2	0.8
R#243	(*highly frequent*, *catastrophic*, *reasonably likely*)			0.15	0.85
R#244	(*highly frequent*, *catastrophic*, *highly likely*)			0.05	0.95

Remarks: here (*average*, *critical*, *likely*) represents "FR is *average* AND CS is *critical* AND FCP is *likely*". Other rules have the same meaning. The blank for the belief in the table means "0".

TABLE A.5 The input transformation for CPP

Input parameter	*Linguistic term*	τ	α_{ij}
FR	*Average*	0.2	0.108
	Reasonably frequent	0.70	0.378
	Frequent	0.70	0.378
	Highly frequent	0.25	0.135
FCP	*Likely*	0.2	0.1
	Reasonably likely	1.0	0.5
	Highly likely	0.6	0.3
	Definite	0.2	0.1
CS	*Critical*	0.75	0.49 (**0.2**)
	Catastrophic	0.78	0.51(**0.6**)

Here τ is calculated using Eq. (5) and α_{ij} is calculated using Eq. (4). We have not listed the corresponding linguistic term with $\tau = 0$ for each input parameter.

TABLE A.6 Fuzzy rule expression matrix of 32 fired rules

Belief	*Output*			
Input	D_2 (*Good*)	D_2 (*Average*)	D_2 (*Fair*)	D_4 (*Poor*)
A_{130} (0.005)			1	
A_{131} (0.024)			0.5	0.5
A_{132} (0.015)			0.3	0.7
A_{133} (0.010)			0.2	0.8
A_{137} (0.005)			0.5	0.5
A_{138} (0.028)			0.4	0.6
A_{139} (0.015)			0.2	0.8
A_{140} (0.010)			0.15	0.85
A_{165} (0.017)			0.8	0.2
A_{166} (0.084)			0.5	0.5
A_{167} (0.050)			0.4	0.6
A_{168} (0.034)			0.3	0.7
A_{172} (0.017)		0.1	0.8	0.1
A_{173} (0.087)			0.9	0.1
A_{174} (0.052)			0.4	0.6
A_{175} (0.035)		0	0	3 0.7
A_{200} (0.017)			0.5	0.5
A_{201} (0.084)			0.3	0.7
A_{202} (0.050)		0.1	0.1	0.8

TABLE A.6 – *continued*

Belief	*Output*			
Input	D_2 (*Good*)	D_2 (*Average*)	D_2 (*Fair*)	D_4 (*Poor*)
A_{203} (0.034)		0	0.2	0.8
A_{207} (0.017)			0.5	0.5
A_{208} (0.087)			0.4	0.6
A_{209} (0.052)		0	0.3	0.7
A_{210} (0.035)			0.2	0.8
A_{235} (0.006)			0.4	0.6
A_{236} (0.031)			0.2	0.8
A_{237} (0.019)			0.1	0.9
A_{238} (0.012)			0	1
A_{242} (0.006)			0.2	0.8
A_{243} (0.032)			0.15	0.85
A_{244} (0.019)			0.05	0.95
A_{245} (0.013)				1

Here the values in the parentheses of the first column of input are the weights of the packet antecedent attributes A_k generated using Eq. (7) and (8), where the weights of rules and the weights of the antecedents are assumed to be equal. Moreover, because the inputs are complete, then the degrees of belief need not be updated.

TABLE A.7 Fuzzy rule expression matrix of 32 fired rules for the incomplete input

Belief	*Output*			
Input	D_2 (*Good*)	D_2 (*Average*)	D_2 (*Fair*)	D_4 (*Poor*)
A^{130} (0.002)			0.933333	0
A^{131} (0.011)			0.466667	0.466667
A^{132} (0.006)			0.28	0.653333
A^{133} (0.002)			0.186667	0.746667
A^{137} (0.006)			0.466667	0.466667
A^{138} (0.032)			0.373333	0.56
A^{139} (0.019)			0.186667	0.746667
A^{140} (0.006)			0.14	0.793333
A^{165} (0.008)			0.746667	0.186667
A^{166} (0.038)			0.466667	0.466667
A^{167} (0.023)			0.373333	0.56
A^{168} (0.008)			0.28	0.653333
A^{172} (0.023)		0.09333	0.74667	0.09333
A^{173} (0.113)			0.84	0.093333
A^{174} (0.068)			0.373333	0.56
A^{175} (0.023)		0	0.28	0.653333
A^{200} (0.008)			0.466667	0.466667
A^{201} (0.038)			0.28	0.653333
A^{202} (0.023)		0.093333	0.093333	0.746667
A^{203} (0.008)		0	0.186667	0.746667
A^{207} (0.023)			0.466667	0.466667
A^{208} (0.113)			0.373333	0.56
A^{209} (0.068)		0	0.28	0.653333
A^{210} (0.023)			0.186667	0.746667
A^{235} (0.003)			0.373333	0.56
A^{236} (0.014)			0.186667	0.746667
A^{237} (0.008)			0.093333	0.84
A^{238} (0.003)			0	0.933333
A^{242} (0.008)			0.186667	0.746667
A^{243} (0.041)			0.14	0.793333
A^{244} (0.024)			0.046667	0.886667
A^{245} (0.008)			0	0.933333

Jun Liu (B.Sc and M.Sc in Applied Mathematics in 1993 and 1996, respectively, and Ph.D. in Information Engineering and Control in 1999 from Southwest Jiaotong University, Chengdu, P.R. China) is currently a Postdoctoral Research Fellow in decision and system science at the Manchester School of Management of UMIST, Manchester, UK. Focusing research on algebraic and non-classical logic (many-valued logic, fuzzy logic), automated reasoning theory and methods, fuzziness and uncertainty theory and applications, intelligent decision support systems and evaluation analysis, information fusion and data combinations. His current applied research covers intelligent decision analysis and support under uncertainties for design decision-making, risk and safety analysis in engineering fields.

Jian-Bo Yang is Chair Professor of Decision and System Sciences at the Manchester School of Management (MSM) of UMIST, and is currently Head of the MSM Operations Management Group. He received his BEng and MEng degrees in Control Engineering at the North Western Polytechnic University, Xi'an, China in 1981 and 1984, respectively, and a Ph.D. degree in Systems Engineering at Shanghai Jiao Tong University, Shanghai, China in 1987. His current main research interests include intelligent decision analysis and support under uncertainties, multiobjective optimisation, system modelling, simulation and control with applications in both engineering and management systems.

Jin Wang is Professor of Marine Technology in the School of Engineering at Liverpool John Moores University. He received his BSc in Marine Automation from Dalian Maritime University, P. R. China, his M.Sc in Marine Engineering and Ph.D. (via staff registration) in Marine Safety Engineering from the University of Newcastle upon Tyne in 1989 and 1994, respectively. He has been involved in marine and offshore safety research for the past 13 years with support from the EPSRC, EU, HSE, etc. He is a member of the Council of the UK Safety and Reliability Society (SaRS) and the Technical Committee of the Institute of Marine Engineering, Science and Technology (IMarEST). He is also a Chartered Engineer (CEng) and a Fellow of the IMarEST. Professor Wang's major research interests include safety and reliability based design and operations of large marine and offshore systems, probabilistic and non-probabilistic safety analysis and decision making, port safety assessment and analysis of safety-critical systems in the software domain.

How Sing Sii obtained his Bachelor Degree in Mechanical Engineering from the University of South Australia, a Masters Degree in Engineering Science from the University of Malaya, Malaysia, and a Ph.D. Degree in Marine and Mechanical Engineering from Staffordshire University, UK. He is a Chartered Mechanical & Marine Engineer, a member of IMarEST and IMechE, and an associate member of UK SaRS. He has more than 15 years' industrial experience in various disciplines. Currently he is a Postdoctoral Research Fellow at Liverpool John Moores University working on an offshore safety research project funded by the UK EPSRC. He is also a safety consultant in the rail industry. His research and consultation interests are quality, reliability and safety engineering and management.

Ying-Ming Wang is a Professor of Management Sciences at the School of Public Administration of Fuzhou University in China. He received his BEng degree in Industrial Electric Automation from Jiangsu University of Science and Technology in 1984, his MEng and Ph.D. degrees both in Systems Engineering, respectively, from Huazhong University of Science and Technology in 1987 and Southeast University in 1991. Before joining Fuzhou University in June 2002, he was a Professor of Management Science at the School of Management of Xiamen University. His research interests include multiple attribute decision making (MADM), evidential reasoning (ER), analytic hierarchy process (AHP), data envelopment analysis (DEA), combined forecasting and rough sets (RS), etc.