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ORIGINAL ARTICLE

Generalised probabilistic linguistic evidential reasoning approach for multi-criteria decision-making under uncertainty

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ABSTRACT

As a multi-criteria decision-making (MCDM) method, the evidential reasoning (ER) approach can deal with uncertainties that are resulted from the limited knowledge and experience of experts. Due to the lack of information in the decision-making process, experts usually cannot give quantitative evaluations, but can only express their views with linguistic terms. It is usually impossible for experts to give accurate linguistic terms since they may hesitate among several linguistic terms or interval ones. In addition, experts may have different preferences for different linguistic evaluations. To fully express the evaluations, the probability can be introduced to model the preferences of experts. In this study, we propose the generalised probabilistic linguistic term set (G-PLTS) to represent the evaluation information with various linguistic forms. Then, the ER approach is investigated in the environment with G-PLTSs. Besides, a gained and lost dominance score (GLDS) method is utilised to rank the alternatives, forming an integrated method, which we call the generalised probabilistic linguistic evidential reasoning (GPLER) approach, to solve the MCDM problems with several uncertainties. Finally, we apply this method to the screening of high-risk population of lung cancer to verify the effectiveness of the proposed method.

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General probabilistic linguistic term set; evidential reasoning approach; gained and lost dominance score method; screening of high-risk population for lung cancer

1. Introduction

Decision-making is an important part of human social practice. With the continuous progress of human practice, decision-making problems become more and more complex (Rodríguez, Labella & Martínez, 2016). There are many factors that affect the outcomes of decision-making problems at the same time. To simplify and standardise the decision-making process, decision-making problems can be divided into multiple criteria to be evaluated separately. These kinds of decision-making problems are called the multi-criteria decision-making (MCDM) problems. Because of the nature of qualitative criteria and the high cost of obtaining precise numerical values, people usually have to express their views in linguistic terms (Zadeh, 1975, 1996). In recent decades, decision-making under the linguistic environment has been extensively studied (Liao, Wu, Mi, & Herrera, 2019; Liao, Xu, Herrera-Viedma, & Herrera, 2018; Mi et al., 2019; Pang, Wang, & Xu, 2016; Rodriguez, Martinez, & Herrera, 2012).

Due to the complexity and uncertainty of practical problems, it is difficult for experts to express their evaluations with only one linguistic term since they may hesitate among several possible linguistic terms. Considering this fact, Rodriguez, Martinez, and Herrera (2012) proposed the hesitant fuzzy linguistic term set (HFLTS), which enables experts to express their opinions with several possible linguistic variables. However, in HFLTSs, all linguistic terms are supposed to be of equal importance. This does not match practical cases well since experts may prefer to some linguistic terms and have different degrees of importance over the possible linguistic terms. In this regard, Pang, Wang, and Xu (2016) introduced the probabilistic linguistic term sets (PLTSs) to extend HFLTSs by adding the probability of each linguistic term. Moreover, considering the fuzziness of human thinking and the lack of sufficient knowledge, experts may prefer to give uncertain linguistic terms (Xu, 2004) as evaluation information. To model this situation, Li and Wang (2018) extended the single linguistic terms under certain probabilities in the original PLTS to several linguistic terms, and Lin et al. (2018) further extended it to interval linguistic terms. In addition, to model the fuzziness of evaluation information, many new decision-making methods have been proposed in recent years. For example, Rouyendegh (2018) proposed an intuitionistic fuzzy ELECTRE model; Luo et al. (2019) proposed a probabilistic fuzzy analytic network process approach with probability information; Ocampo (2018) applied the probabilistic linguistic preference relationships to sustainable manufacturing strategy infrastructural decisions, which verified the practicability of the decision-making method.

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In fact, due to the complexity and uncertainty of actual decision-making problems, it is difficult for experts to use only one linguistic term or several adjacent linguistic terms for linguistic assessment modelling. Experts may use both several adjacent or non-adjacent linguistic terms and interval linguistic terms to make evaluations. For example, suppose that a panel of four experts are invited to diagnose a lung cancer patient who had just undergone chemotherapy and soon became mentally better. If the linguistic term set $S = \{s_0 =$ very bad, $s_1 =$ bad, $s_2 =$ fair, $s_3 =$ good, $s_4 =$ very good$\}$ is utilised by experts, suppose that the first expert thinks that the condition of the patient is between "good" and "very good"; the second expert thinks that the patient is in "fair" condition; the other two experts hesitate between "fair" and "good". Then, the patient's diagnostic results can be expressed as $\{\langle[s_3, s_4], 0.25\rangle, \langle s_2, 0.25\rangle, \langle\{s_2, s_3\}, 0.5\rangle\}$. As far as we know, the existing linguistic representation models cannot model and process such uncertain linguistic information. So, it is necessary to develop a new linguistic representation method. On this basis, a generalised probabilistic linguistic term set (G-PLTS) is proposed to model such MCDM problems with uncertainties, which allows experts to express their opinions with not only several discrete linguistic terms but also interval terms.

Moreover, although the G-PLTS can solve the problem of linguistic representation well, in the actual decision-making process, it is usually difficult to obtain the complete information of probability distribution. To model the uncertainty in actual evaluations, Yang (2001) and Yang and Xu (2002) took into account the local ignorance in the framework of evidential reasoning (ER) approach in which the probability information is interpreted as belief degrees. However, the ER approach can only be applied to decision-making problems with independent linguistic evaluation grades, which requires high accuracy of evaluations provided by experts. To enhance the original ER approach proposed by Yang (2001) and Yang and Xu (2002), we extend the form of evaluation grades in the ER approach from the original collectively exhaustive and mutually exclusive linguistic terms to continuous linguistic terms, forming a new ER approach framework based on G-PLTSs.

In addition, ranking methods are also essential in solving MCDM problems. Basically, the ranking methods fall into two categories: utility value-based methods and outranking methods (Liao, Xu, Herrera-Viedma, & Herrera, 2018). For the first category, different aggregation functions are used to combine the criterion values with the related weights of criteria to obtain the overall values of alternatives (Liao & Wu, 2019; Opricovic & Tzeng, 2004; Pang, Wang, & Xu, 2016; Wu et al., 2018; Zhang & Xing, 2017). The reference point-based methods are widely applied to solve MCDM problems with PLTSs, such as the PL-TOPSIS (Pang, Wang, & Xu, 2016) and PL-VIKOR (Zhang & Xing, 2017). The main idea of these two methods is to choose a compromise solution which is closest to the ideal solution. However, the solution closest to the ideal one may not always dominate other solutions. The second type of MCDM methods are outranking methods which are based on pairwise comparisons of alternatives under each criterion (Behzadian, Kazemzadeh, Albadvi, & Aghdasi, 2010; Qin, Liu & Pedrycz, 2017; Wu & Liao, 2018). But these methods ignored the values of "group utility" and "individual regret". To overcome these shortcomings, Wu and Liao (2019) proposed the gained and lost dominance score (GLDS) method and developed a new aggregation formula to obtain the comprehensive score of each option. However, since some evaluations of criteria may be outliers, there is a possibility that the results generated by the GLTS method may go to be one-sided. Besides, the GLDS method cannot be applied to solve the MCDM problems with uncertainties that are represented by G-PLTSs. Therefore, inspired by the original GLDS method, we propose a novel ranking method considering the distribution of ignorance.

Based on the above analysis, this study proposes an integrated ER approach based on G-PLTSs and the GLDS method to handle the MCDM problems with multiple uncertainties, which we refer to the generalised probabilistic linguistic evidential reasoning (GPLER) approach. The innovative contributions of this paper are highlighted as follows:

1. The original concept of PLTS with single linguistic terms is generalised to multiple (adjacent or not adjacent) linguistic terms and interval linguistic terms, forming the G-PLTS.
2. The traditional ER approach is enhanced from the original discrete independent linguistic evaluation grades to continuous ones.
3. An integrated GPLER approach based on G-PLTSs and the GLDS method is developed for dealing with MCDM problems under uncertainty. The GLDS method is applied to rank G-PLTSs.
4. We apply the GPLER approach to screen high-risk population of lung cancer for reducing the burden of doctors and increasing the effective screening rate.

The rest of this paper is organised as follows. Section 2 reviews some basic concepts, including the

PLTS, three different linguistic scale functions, and the ER approach. Section 3 presents the GPLER approach for MCDM problems under uncertainty. In Section 4, the proposed method is applied to help screen high-risk population of lung cancer. Finally, Section 5 concludes the paper with some comments.

2. Preliminaries

This section briefly reviews the concept of PLTS and three different linguistic scale functions before recalling steps and operations of the ER approach.

2.1. Probabilistic linguistic term set

Since experts may have different preferences for different linguistic evaluations, Pang, Wang, and Xu (2016) introduced the concept of PLTS which uses probabilities to express the preference degrees of linguistic terms of experts. Suppose that $A = \{a_i | i = 1, 2, \ldots, M\}$ is a set of M alternatives and $S = \{s_\alpha | \alpha = 0, 1, \ldots, 2\tau\}$ is a linguistic term set (LTS). Then, a PLTS on S is defined as follows (Pang, Wang, & Xu, 2016):

$$H_S(p) = \{\langle a_i, h_S(p)\rangle | a_i \in A\} \tag{1}$$

with $h_S(p) = \{s_t(p_t) | s_t \in S, \quad p_t \geq 0, t = 1, 2, \ldots, T, \sum_{t=1}^{T} p_t \leq 1\}$, where $s_t(p_t)$ is the tth linguistic term s_t associated with the probability p_t, and T is the number of all different linguistic terms in $h_S(p)$ arranged in ascending order.

The PLTS not only can indicate the grades of alternatives corresponding to the given evaluation scale but also can indicate the preference degrees of experts for the evaluations. In this sense, it is a good way to include all evaluation information of experts, and thus has attracted the attention of a large number of scholars. Many scholars have extended the PLTS to various forms. For example, Zhai, Xu and Liao (2016) generalised the probability to vector form; Lin et al. (2018) extended the linguistic evaluations to interval form; Bai et al. (2018) extended the probability to interval form; Malik et al. (2018) extended the PLTS to intuitionistic fuzzy linguistic environment. However, few of these models can deal with the ignorance and uncertainty in the evaluation very well. In this study, we shall introduce the belief degree of ER approach to model the uncertain linguistic evaluation and make information fusion.

2.2. Linguistic scale functions

Given that different experts may have different cognitions and preferences about the semantics of linguistic judgements, it is necessary to standardise different forms of linguistic evaluations before using the ER approach for information fusion. In addition, if the subscripts of linguistic terms are calculated directly, the results may exceed the boundary values. To solve these problems, three linguistic scale functions were introduced to transform the linguistic terms to corresponding semantic values and maintain the integrity of information in calculation process (Liao et al., 2019):

1. If the semantic values are evenly distributed, then

$$\begin{cases} f(s_\alpha) = (\alpha + \tau)/2\tau \\ f^{-1}(\theta_\alpha) = 2\tau \cdot \theta_\alpha - \tau \end{cases}, \text{ if } \alpha \in [-\tau, \tau] \tag{2}$$

2. If the deviation of the semantic values of two adjacent linguistic terms increases, respectively, from the median to both sides, then

$$\begin{cases} f(s_\alpha) = \dfrac{t^\tau - t^{-\alpha}}{2t^\tau - 2} \times 1_{\{\alpha \in [-\tau, 0]\}} + \dfrac{\mu^\tau + \mu^\alpha - 2}{2\mu^\tau - 2} \times 1_{\{\alpha \in (0, \tau]\}} \\ f^{-1}(\theta_\alpha) = -\log_t(t^\tau - (2t^\tau - 2)\theta_\alpha) \times 1_{\{\alpha \in [-\tau, 0]\}} + \log_\mu((2\mu^\tau - 2)\theta_\alpha - (\mu^\tau - 2)) \times 1_{\{\alpha \in (0, \tau]\}} \end{cases}, \text{ if } \alpha \in [-\tau, \tau] \tag{3}$$

where $1_{\{\alpha \in (0, \tau]\}} = \begin{cases} 1, \alpha \in (0, \tau] \\ 0, \alpha \in [-\tau, 0] \end{cases}$. t and μ are two parameters which imply different risk preferences evaluated as "*bad*" and "*good*", satisfying $t > 1$ and $\mu > 1$ to guarantee f a monotonically increasing function.

3. If the deviation of the semantic values of two adjacent linguistic terms decreases, respectively, from the median to both sides, then

$$\begin{cases} f(s_\alpha) = \dfrac{\tau^\gamma - (-\alpha)^\gamma}{2\tau^\gamma} \times 1_{\{\alpha \in [-\tau, 0]\}} + \dfrac{\tau^\psi + \alpha^\psi}{2\tau^\psi} \times 1_{\{\alpha \in (0, \tau]\}} \\ f^{-1}(\theta_\alpha) = -\tau\sqrt[\gamma]{1 - 2\theta_\alpha} \times 1_{\{\alpha \in [-\tau, 0]\}} + \tau\sqrt[\psi]{2\theta_\alpha - 1} \times 1_{\{\alpha \in (0, \tau]\}} \end{cases}, \text{ if } \alpha \in [-\tau, \tau] \tag{4}$$

where γ and ψ are parameters satisfying $\gamma, \psi \in (0,1]$. γ represents the risk attitude parameter of the expert when the scheme is in negative evaluation, and ψ represents the risk preference parameter in positive evaluation. Such two parameters are determined according to specific problems. For conservative experts, the values of γ and ψ are small, while for radical experts, the values are relatively large.

Through the aforementioned semantic functions, the qualitative linguistic terms can be transformed into quantitative semantic values, which provides a basis for subsequent information fusion with the ER approach.

2.3. The ER approach

To deal with ignorance in decision-making process is a vital problem. In the traditional Bayesian reasoning method, it is impossible to distinguish "ignorance" and "equal possibility", and it is necessary to know the prior probability. However, "ignorance" and "equal possibility" are obviously different, and in many cases, it is not easy to obtain a priori probability due to the limitation of resources and time. In this regard, the Dempster–Shafer (DS) theory (Dempster, 1967; Shafer, 1976) was put forward by generalising the Bayesian reasoning method to solve these weaknesses. However, the DS theory cannot fuse evidences with high conflict. To avoid the counter-intuitive results of high-conflict evidence fusion, Yang et al. (2006) proposed the ER approach. As an information fusion theory, the ER approach not only can well model the uncertainties and incompleteness of information in decision-making process but also can be applied to fuse evidences with high conflict.

Suppose that M alternatives $a_i (i = 1, 2, ..., M)$ are assessed over L criteria $c_j (j = 1, 2, ..., L)$ with the weight vector $(\omega_1, \omega_2, ..., \omega_L)^T$ satisfying $0 \le \omega_j \le 1$ and $\sum_{j=1}^{L} \omega_j = 1$. The alternatives are evaluated under the frame of discernment $H = \{H_1, H_2, ..., H_N\}$, where H_n (for $n = 1, 2, ..., N$) are collectively exhaustive and mutually exclusive. The belief degree $\beta_{n,j}(a_i)$ to grade H_n for alternative a_i under criterion e_j satisfies $\beta_{n,j}(a_i) \ge 0$ and $\sum_{n=1}^{N} \beta_{n,j}(a_i) \le 1$.

By the ER approach (Yang et al., 2006), the belief degrees of alternatives can be transformed into basic probability assignments:

$$m_{n,j} = \omega_j \beta_{n,j}(a_i) \tag{5}$$

$$m_{H,j}(a_i) = \tilde{m}_{H,j}(a_i) + \bar{m}_{H,j}(a_i) \tag{6}$$

$$\tilde{m}_{H,j}(a_i) = \omega_j \left(1 - \sum_{n=1}^{N} \beta_{n,j}(a_i)\right) \tag{7}$$

$$\bar{m}_{H,j}(a_i) = 1 - \omega_j \tag{8}$$

where $m_{n,j}(a_i)$ refers to the probability mass assessed to grade H_n on criterion e_j. $m_{H,j}$ refers to the remaining probability mass unassigned to any individual grade. $m_{H,j}(a_i)$ is composed of two parts, with one part $\tilde{m}_{H,j}(a_i)$ caused by the ignorance in criterion j and the other part $\bar{m}_{H,j}(a_i)$ produced by the importance of criterion j.

The criterion assessments are aggregated by combining the calculated basic probability masses:

1. For $\{H_n\}$, we obtain

$$\begin{aligned} m_{n,I(j+1)}(a_i) = K_{I(j+1)}(a_i)\big[&m_{n,I(j)}(a_i) m_{n,j+1}(a_i) \\ &+ m_{H,I(j)}(a_i) m_{n,j+1}(a_i) \\ &+ m_{n,I(j)}(a_i) m_{H,j+1}(a_i)\big] \end{aligned} \tag{9}$$

2. For $\{H\}$, we obtain

$$m_{H,I(j)}(a_i) = \tilde{m}_{H,I(j)}(a_i) + \bar{m}_{H,I(j)}(a_i) \tag{10}$$

$$\begin{aligned} \tilde{m}_{H,I(j+1)}(a_i) = K_{I(j+1)}(a_i)\big[&\tilde{m}_{H,I(j)}(a_i) \tilde{m}_{H,j+1}(a_i) \\ &+ \bar{m}_{H,I(j)}(a_i) \tilde{m}_{H,j+1}(a_i) \\ &+ \tilde{m}_{H,I(j)}(a_i) \bar{m}_{H,j+1}(a_i)\big] \end{aligned} \tag{11}$$

$$\bar{m}_{H,I(j+1)}(a_i) = K_{I(j+1)}(a_i)\left(\bar{m}_{H,I(j)}(a_i) \bar{m}_{H,j+1}(a_i)\right) \tag{12}$$

where $K_{I(j+1)}(a_i) = \left[1 - \sum_{l=1}^{N} \sum_{\substack{n=1 \\ n \ne l}}^{N} m_{l,I(j)}(a_i) m_{n,j+1}(a_i)\right]^{-1}$, $m_{n,I(1)}(a_i) = m_{n,1}(a_i)$ and $m_{H,I(1)}(a_i) = m_{H,1}(a_i)$.

Finally, the belief degree of alternative a_i on grade H_n can be obtained by

$$\beta_n(a_i) = m_{n,I(L)}(a_i) / (1 - \bar{m}_{H,I(L)}(a_i)) \tag{13}$$

$$\beta_H(a_i) = \tilde{m}_{H,I(L)}(a_i) / (1 - \bar{m}_{H,I(L)}(a_i)) \tag{14}$$

where $\beta_H(a_i)$ is the belief degree caused by various uncertainties in the MCDM problem.

Suppose that the utility values of the evaluation grades are known. To rank the alternatives, we can calculate the utility of each alternative as

$$u(a_i)_{\max} = \sum_{n=1}^{N} u(H_n)\beta_n(a_i) + u(H_N)\beta_H(a_i) \tag{15}$$

$$u(a_i)_{\min} = \sum_{n=1}^{N} u(H_n)\beta_n(a_i) + u(H_1)\beta_H(a_i) \tag{16}$$

$$u(a_i)_{avg} = (u(a_i)_{\max} + u(a_i)_{\min})/2 \tag{17}$$

The distributed results of ER approach can provide not only the grades that the alternatives belong to and their corresponding preferences but also the belief degrees of uncertainties. Tackling uncertainties with different principles, we can obtain different comprehensive scores of alternatives so as to select the optimal alternative. For example, with the optimistic principle, we can assign the belief degree of ignorance to the best evaluation grade to obtain the highest score of the alternative, while with the pessimistic principle, we can assign the belief degree of ignorance to the worst evaluation grade to obtain the lowest score.

3. Generalised probabilistic LTS

In MCDM problems, experts tend to use linguistic terms to express views. To further express preference information, experts assign corresponding probabilities to the given linguistic terms. Therefore, the PLTS has been introduced to model the linguistic evaluations of experts (Pang, Wang, & Xu, 2016). Although it is not excluded that some experts have the ability to provide exact single linguistic terms when evaluating alternatives, in general cases, most experts can only use interval or multiple discrete linguistic terms to make evaluations. The PLTS and its extensions do not include the evaluation with non-adjacent linguistic terms, and most of them are evaluated by adjacent discrete or interval linguistic terms. There is a high demand for the accuracy of evaluations. In this section, we shall propose the concept of G-PLTS to allow experts to use discrete or continuous linguistic terms to well model uncertainties and preferences information in MCDM problems. Then, linguistic scale functions are used to standardise the different semantics of linguistic terms.

3.1. Generating evaluation information with G-PLTSs

Suppose that $S_0 = \{s_t | t \in [-\tau, \tau]\}$ is a continuous set of linguistic terms with τ being a positive integer. Experts can use continuous interval-valued linguistic terms (Liao et al., 2018) such as $[s_{1.3}, s_{2.5}]$ to evaluate alternatives. However, in practical cases, due to the limited knowledge and experience of experts, the continuous interval-valued linguistic evaluations cannot be accurately given. Therefore, to facilitate the evaluations of experts, we choose $2\tau + 1$ dividing points whose subscripts are integer in the original continuous set S_0 to form a set of discrete linguistic terms $S = \{s_\alpha | \alpha = -\tau, \dots, 0, \dots, \tau\}$ for making evaluations.

In practice, experts are flexible to adopt continuous interval-valued linguistic form $[s_{t_{k_1}}, s_{t_{k_2}}] \subset [s_{-\tau}, s_\tau]$ or discrete linguistic form $\{s_{\alpha_k}\}$ with $\alpha_k \in \{-\tau, \dots, 0, \dots, \tau\}$ to evaluate alternatives, and give the corresponding probabilities of them to generate the evaluation information. To model the information comprehensively, we introduce the concept of G-PLTS.

Definition 1. Let $S = \{s_\alpha | \alpha = -\tau, \dots, 0, \dots, \tau\}$ be an LTS. Then,

$$h_{ij}(p) = \{\{s_{\alpha_k}\}(p_{n_1,j}), [s_{t_1}, s_{t_2}](p_{n_2,j}) | \alpha_k \in \{-\tau, \dots, 0, \dots, \tau\}, k = 1, 2, \dots, K, -\tau \le t_1 \le t_2 \le \tau, n_1 = 1, \dots, N_1, n_2 = 1, 2, \dots, N_2, \sum_{n_1=1}^{N_1} p_{n_1,j} + \sum_{n_2=1}^{N_2} p_{n_2,j} \le 1\} \tag{18}$$

is a G-PLTS, where K is the number of linguistic terms under probability $p_{n_1,j}$, N_1 is the number of discrete linguistic evaluations $\{s_{\alpha_k}\}(p_{n_1,j})$, and N_2 is the number of interval-valued linguistic evaluations $[s_{t_1}, s_{t_2}](p_{n_2,j})$.

Example 1. Suppose that ten experts are invited to assess the health status of a lung cancer patient. The status vary from *very bad* (s_{-2}) to *very good* (s_2) according to CT images (c_j). In this sense, we can establish the relevant linguistic evaluation range $S_0 = \{s_t | t \in [-2, 2]\}$ and then the linguistic evaluation grades are set as $S = \{s_\alpha | \alpha = -2, -1, 0, 1, 2\}$= {very bad, bad, fair, good, very good}. On this premise, six experts hesitate among s_{-1}, s_0 and s_1, three experts believe that the status of the patient should be evaluated from s_{-1} to s_2, and the remaining one expert dose not express his/her opinion. In this case, the evaluation results can be represented by a G-PLTS as $\{\langle \{s_{-1}, s_0, s_1\}, 0.6\rangle, \langle [s_{-1}, s_2], 0.3\rangle\}$.

In general, the linguistic terms in the evaluation of experts should be continuous or consecutive, but in some non-expert evaluation or some special cases, there are also jumps in evaluation (Wang, 2015). For example, in Example 1, suppose that five experts are invited to evaluate the status of the patient as a group. One expert gives an evaluation as s_{-1} and the other as s_2. Thus, the evaluation result of such a group can be shown as $\langle \{s_{-1}, s_2\}, 0.2\rangle$. The proposed G-PLTS method is also applicable to such case.

When $\sum_{n_1=1}^{N_1} p_{n_1,j} + \sum_{n_2=1}^{N_2} p_{n_2,j} < 1$, some probabilities are not assigned to any linguistic term. The original PLTS representation does not take into account these uncertainties, which is unreasonable (Jiang & Liao, 2019). To model ignorance and uncertainty lying in evaluations, motivated by the belief degrees in ER approach, we can introduce the belief-based G-PLTS.

Definition 2. For a G-PLTS $h_{ij}(p)$, if $\sum_{n_1=1}^{N_1} p_{n_1,j} + \sum_{n_2=1}^{N_2} p_{n_2,j} < 1$, then, the belief-based G-PLTS is defined as

$$h_{ij}(p) = \{\langle s_{\alpha_k}, \beta_{n_1,j}\rangle, \langle [s_{t_1}, s_{t_2}], \beta_{n_2,j}\rangle, \langle S_0, \beta_{S_0,j}\rangle\} \tag{19}$$

where $\beta_{n_1,j} = p_{n_1,j}, \beta_{n_2,j} = p_{n_2,j}$ and $\beta_{S_0,j} = 1 - (\sum_{n_1=1}^{N_1} p_{n_1,j} + \sum_{n_2=1}^{N_2} p_{n_2,j})$.

According to Equation (19), the generated evaluation $\{\langle \{s_{-1}, s_2\}(0.6)\rangle, \langle [s_{-1}, s_2](0.3)\rangle\}$ in Example 1 can be converted into a belief-based G-PLTS as $\{\langle \{s_{-1}, s_2\}, 0.6\rangle, \langle [s_{-1}, s_2], 0.3\rangle, \langle S_0, 0.1\rangle\}$.

By introducing the concepts of G-PLTS and belief-based G-PLTS, the uncertainty and incompleteness of evaluation information can be well modelled. Then, the decision-making matrix $D_g = \{h_{ij}(p)\}_{M\times L}$ is transformed into $D_{bg} = \{h_{ij}(p)\}_{M\times L}$.

3.2. Standardising the G-PLTSs with linguistic scale functions

The G-PLTSs and belief-based G-PLTSs provide a preparation for making an accurate decision. However, it is difficult to directly use the ER approach to fuse such information with different forms. Thus, there is a need to standardise the decision matrix with G-PLTSs.

In the continuous set of linguistic terms, the relationship between linguistic terms and semantic values is one-to-one. Different linguistic terms have different semantic values (Liao et al., 2018). When a continuous set of linguistic terms is divided into small enough linguistic terms, these linguistic terms can be regarded as independent with each other. Besides, since experts have different cognition for different judgements, the corresponding semantics of different linguistic terms are also different. In this regard, we can use different semantic functions to model such difference. Moreover, there are uneven semantic values and the possibility that the subordinate calculation of linguistic terms may exceed the boundaries of a given LTS (Wu & Liao, 2019).

To solve these shortcomings, three different semantic functions were introduced in Section 2.2 to transform the linguistic terms to corresponding semantic values (Liao et al., 2019). On this basis, we can develop a novel method to standardise the belief-based G-PLTSs by calculating the semantics of linguistic terms. The specific procedure is as follows:

At first, we translate different forms of linguistic evaluations in a belief-based G-PLTS into single linguistic terms through linguistic scale functions:

1. For s_{α_k} associated with $\beta_{n_1,j}$, we have
$$\{s_{\alpha_k,j}\} = f^{-1}\left(\sum_{k=1}^{K} f(s_{\alpha_k,j})/K\right) = s_{n_1,j} \tag{20}$$
2. For $[s_{t_1}, s_{t_2}]$ associated with $\beta_{n_2,j}$, we have
$$[s_{t_1}, s_{t_2}] = f^{-1}\left(\int_{t_1}^{t_2} f(s_t)d_t/(t_2 - t_1)\right) = s_{n_2,j} \tag{21}$$
3. For S_0 associated with $\beta_{S_0,j}$, we keep it the same.

Then, since experts cannot make accurate evaluations with linguistic terms, it is necessary to standardise the transformed single linguistic terms into the linguistic terms in the original LTS S, which can make a better understanding. This goal can be achieved in the following way:

1. If n_l is an integer, i.e., $n_l \in \{-\tau, \ldots, 0, \ldots, \tau\}$, then, we have $\langle \hat{s}_{\alpha,j}, \hat{\beta}_{\alpha,j}\rangle = \langle s_{n_l,j}, \beta_{n_l,j}\rangle$, for $l = 1, 2$;
2. If n_l is not an integer, i.e., $n_l \in (\alpha_0, \alpha_0 + 1)$ with $\alpha_0 \in \{-\tau, \ldots, 0, \ldots, \tau-1\}$, then, we have
$$\langle \hat{s}_{\alpha,j}, \beta_{\alpha,j}\rangle = \left\{\left\langle s_{\alpha_0,j}, \frac{f(s_{\alpha_0+1}) - f(s_{n_l})}{f(s_{\alpha_0+1}) - f(s_{\alpha_0})}\beta_{n_l,j}\right\rangle, \left\langle s_{\alpha_0+1,j}, \frac{f(s_{n_l}) - f(s_{\alpha_0})}{f(s_{\alpha_0+1}) - f(s_{\alpha_0})}\beta_{n_l,j}\right\rangle\right\}, \text{ for } l = 1, 2 \tag{22}$$

Through the above steps, the following standardised matrix can be obtained:
$$D_{bg} = \{\langle \hat{s}_{\alpha,j}, \beta_{\alpha,j}\rangle, \langle S_0, \beta_{S_0,j}\rangle | \alpha = -\tau, \ldots, 0, \ldots, \tau; \; j = 1, 2, \ldots, L\}_{M\times L} \tag{23}$$

Example 2. For the belief-based G-PLTS $\{\langle\{s_{-1}, s_0, s_1\}, 0.6\rangle, \langle[s_{-1}, s_2], 0.3\rangle, \langle S_0, 0.1\rangle\}$ given in Example 1, assume that the semantics of the linguistic terms is balanced and the semantic function f is given as Equation (2). The standardisation process of this G-PLTS is as follows:
$$\begin{aligned}\{s_{-1}, s_0, s_1\} &= f^{-1}\left(\sum_{k=1}^{3} f(s_{\alpha_k})/2\right)\\ &= f^{-1}((f(s_{-1}) + f(s_0) + f(s_1))/3)\\ &= s_0 = s_{n_1}\end{aligned}$$
$$[s_{-1}, s_2] = f^{-1}\left(\int_{-1}^{2} f(s_t)d_t/(2 - (-1))\right) = s_{0.5} = s_{n_2}$$

Since $n_1 = 0 \in \{-2, -1, 0, 1, 2\}$ and $n_2 = 0.5 \in (0, 1)$, then the standardised evaluation can be generated as
$$\begin{aligned}&\{\langle \hat{s}_{\alpha,j}, \beta_{\alpha,j}\rangle, \langle S_0, \beta_{S_0,j}\rangle\}\\ &= \left\{\langle s_{n_1,j}, \beta_{n_1,j}\rangle, \left\langle s_{0,j}, \frac{f(s_1) - f(s_{0.5})}{f(s_1) - f(s_0)}\beta_{n_2,j}\right\rangle, \left\langle s_{1,j}, \frac{f(s_{0.5}) - f(s_0)}{f(s_1) - f(s_0)}\beta_{n_2,j}\right\rangle\right\}\\ &= \{\langle s_0, 0.6\rangle, \langle s_0, 0.15\rangle, \langle s_1, 0.15\rangle, \langle S_0, 0.1\rangle\}\\ &= \{\langle s_0, 0.75\rangle, \langle s_1, 0.15\rangle, \langle S_0, 0.1\rangle\}\end{aligned}$$

4. The GPLER approach for MCDM problems

To deal with various uncertainties in MCDM problems, in this section, we introduce the ER approach to fuse the belief-based G-PLTSs in the standardised decision matrix. Besides, the ranking method in the original ER approach only considers the gained scores of alternative but not consider the lost scores from "regret". To this point, in Section 4.2, we shall propose a new method to rank alternatives, which considers both the gained and lost scores. For the facility of dealing with uncertainties lying in MCDM problems, we give the algorithm of the GPLER approach in Section 4.3.

4.1. Aggregating the standardised evaluations of alternatives by the ER algorithm

Considering the relationship between linguistic terms and their semantic values, it is possible to accurately model continuous qualitative evaluations. The standardised belief-based decision matrix provides clear and distributed evaluation information of alternatives under each criterion. However, it is not easy to rank the alternatives based on the evaluations over criteria. In this regard, it is necessary to integrate the evaluations of alternatives over all criteria to obtain the collective assessment of each alternative.

The ER approach (Yang et al., 2006; Yang & Xu, 2002), as a method to fuse information, can well model ignorance and uncertainty in MCDM problems and generate distributed evaluation results of alternatives, so as to provide complete evaluation information for decision-makers. Thus, below we use the ER approach to aggregation the belief-based generalised probabilistic linguistic evaluation information of alternatives over all criteria.

By Equations (6)–(14), the comprehensive evaluations of alternative a_i can be generated as

$$S(a_i) = \{\langle \hat{s}_{\alpha,i}, \beta_{\alpha,i}\rangle, \langle S_0, \beta_{S_0,i}\rangle | \alpha = -\tau, \ldots, 0, \ldots, \tau\},\quad i = 1, 2, \ldots, M \tag{24}$$

Since $\beta_{S_0,i}$ can be assigned to any linguistic term in S_0, the score of the alternative a_i is uncertain. The lowest score of a_i can be obtained by assigning $\beta_{S_0,i}$ to the worst linguistic term $s_{-\tau}$ while the highest score of a_i can be obtained by assigning $\beta_{S_0,i}$ to the best linguistic term s_τ, i.e.,

$$S(a_i)_{\min} = \{\langle \hat{s}_{\alpha,i}, \beta_{\alpha,i}\rangle, \langle s_{-\tau}, \beta_{S_0,i}\rangle | \alpha = -\tau, \ldots, 0, \ldots, \tau\},\quad i = 1, 2, \ldots, M \tag{25}$$

$$S(a_i)_{\max} = \{\langle \hat{s}_{\alpha,i}, \beta_{\alpha,i}\rangle, \langle s_{\tau}, \beta_{S_0,i}\rangle | \alpha = -\tau, \ldots, 0, \ldots, \tau\},\quad i = 1, 2, \ldots, M \tag{26}$$

4.2. A GLDS-based method to rank the belief-based G-PLTSs

After obtaining the collective evaluations, the next is to rank the alternatives in a reasonable way based on these evaluations. Since the calculated values are still belief-based G-PLTSs, how to rank the belief-based G-PLTSs turns to be a critical issue. In this subsection, we shall introduce a novel method to rank the belief-based G-PLTSs.

When ranking alternatives, we often process evaluation information directly from an objective perspective (Wang et al., 2018), while in some complex situations, we sometimes need to consider the assessing attitude of decision-makers (Thuong et al., 2018) and their psychological preferences (Tian et al., 2019). The ranking method in the original ER approach (Yang & Xu, 2002) is to calculate the score directly based on the overall evaluation of each alternative, without considering the "regret" value or the negative utilities of evaluation grades. The GLDS method (Wu & Liao, 2019), as an MCDM method to rank alternatives, does a good job as it considers both gains and losses by comparing the alternatives under different criteria. Inspired by the idea of the GLDS method, below we develop a method to rank the belief-based G-PLTSs.

For the final collective belief-based G-PLTSs $\hat{S}(a_i)$ and $\hat{S}(a_v)$ of alternatives a_i and a_v, to operate them correctly, we need to adjust them by decomposing the properties of different linguistic terms (Wu et al., 2018). Then we can obtain

$$S^*(a_i) = \{\langle s_\alpha^{i(q)}, \beta^{*(q)}\rangle | q = 1, 2, \ldots, Q\},\quad S^*(a_v) = \{\langle s_\alpha^{v(q)}, \beta^{*(q)}\rangle | q = 1, 2, \ldots, Q\},\ \text{for } i, v = 1, 2, \ldots, M \tag{27}$$

where Q is the number of $\beta^{*(k)}$. In this way, the evaluations with different belief distributions can be adjusted to the same distribution without losing any information. For details about the adjusting process, readers can refer to Wu et al. (2018).

Example 3. Let $S(a_i) = \{<s_{-2}, 0.4>, <s_{-1}, 0.4>, <s_0, 0.2>\}$ and $S(a_v) = \{<s_0, 0.2>, <s_1, 0.5>, <s_0, 0.3>\}$. Then, we can adjust them to $S^*(a_i) = \{\langle s_{-2}, 0.2\rangle, \langle s_{-2}, 0.2\rangle, \langle s_{-1}, 0.3\rangle, \langle s_{-1}, 0.1\rangle, \langle S_0, 0.2\rangle\}$ and $S^*(a_v) = \{\langle s_0, 0.2\rangle, \langle s_1, 0.2\rangle, \langle s_1, 0.3\rangle, \langle S_0, 0.1\rangle, \langle S_0, 0.2\rangle\}$. The adjusting process is illustrated in Figure 1.

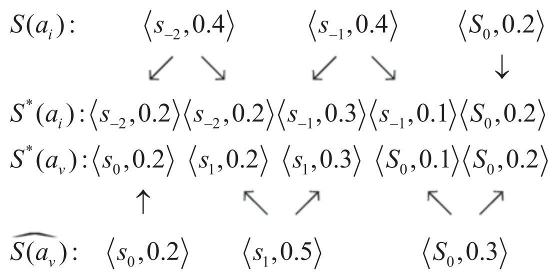

Figure 1. The process of adjusting the evaluation.

For the adjusted belief-based G-PLTSs $S^*(a_i)$ and $S^*(a_v)$, let

$$g_{iv}^{(q)}(a_i) = \max\{f(s_\alpha^{i(q)}) - f(s_\alpha^{v(q)}), 0\}, \text{ for } q = 1, 2, \ldots, Q \tag{28}$$

where f is a linguistic scale function. $g_{iv}^{(q)}(a_i)$ implies the positive semantic difference between the linguistic terms of $S^*(a_i)$ and $S^*(a_v)$ under each belief $\beta^{*(q)}$. Furthermore, the dominance flow of $S^*(a_i)$ over $S^*(a_v)$ can be defined as

$$df(a_i, a_v) = \sum_{q=1}^{Q} (g_{iv}^{(q)}(a_i)\beta^{*(q)}), \text{ for } i, v = 1, 2, \ldots, M \tag{29}$$

The calculated dominance flow should be normalised by

$$df^N(a_i, a_v) = \frac{df(a_i, a_v)}{\sqrt{\sum_{v=1}^{M}\sum_{i=1}^{M}(df(a_i, a_v))^2}}, \quad \text{for } i, v = 1, 2, \ldots, M \tag{30}$$

Then, the net gained dominance score of $S^*(a_i)$ can be obtained by

$$DS_1(a_i) = \sum_{v=1}^{M} df^N(a_i, a_v), \text{ for } i = 1, 2, \ldots, M \tag{31}$$

The lost dominance score of $S^*(a_i)$ can be obtained by

$$DS_2(a_i) = \max_v df^N(a_i, a_v), \text{ for } i = 1, 2, \ldots, M \tag{32}$$

On this basis, the collective belief-based G-PLTSs can be ranked in ascending order of $DS_1(a_i)$ and $DS_2(a_i)$ for $i = 1, 2, \ldots, M$, and then we obtain two rank sets $R_1 = \{r_1(a_1), r_1(a_2), \ldots, r_1(a_M)\}$ and $R_2 = \{r_2(a_1), r_2(a_2), \ldots, r_2(a_M)\}$, respectively.

Next, $DS_1(a_i)$ and $DS_2(a_i)$ should be normalised by vector normalisation:

$$DS_y{}^N(a_i) = \frac{DS_y(a_i)}{\sqrt{\sum_{i=1}^{M} DS_y(a_i)^2}}, y = 1, 2 \tag{33}$$

Considering the dominance scores and the obtained rank sets simultaneously, we can calculate the collective score of each belief-based G-PLTS by

$$BS(a_i) = DS_1{}^N(a_i)\frac{M - r_1(a_i) + 1}{M(M+1)/2} - DS_2{}^N(a_i)\frac{r_2(a_i)}{M(M+1)/2} \tag{34}$$

At the same time, since ignorance can be assigned to the best or worst linguistic term, it leads to the best evaluation $S(a_i)_{\max}$ and worst evaluation $S(a_i)_{\min}$ for each alternative. As a result, the maximum and minimum collective scores $BS(a_i)_{\max}$ and $BS(a_i)_{\min}$ can be obtained by Equation (33). Thus, we can obtain the average scores by

$$BS(a_i)_{avg} = \frac{BS(a_i)_{\max} + BS(a_i)_{\min}}{2}, \text{ for } i = 1, 2, \ldots, M \tag{35}$$

Therefore, the final rank set $R = \{r(a_1), r(a_2), \ldots, r(a_M)\}$ is determined in descending order of $BS(a_i)_{avg}$ $(i = 1, \ldots, M)$.

The above GLDS-based method is efficient to rank the belief-based G-PLTSs. It calculates the gained and lost scores of different linguistic terms directly based on different semantic functions, and then obtain the final ranking order. In addition, the ignorance and uncertainty in the belief-based G-PLTSs are well handled.

4.3. Algorithm of the GPLER approach

To clarify the proposed GPLER approach and simplify its application, we summarise the specific procedures as Algorithm 1.

Algorithm 1 (The GPLER approach for MCDM)

Step 1. (Formalisation) Experts evaluate alternatives with interval or multiple discrete linguistic terms with corresponding probability information. Then, the decision-making matrix $D_g = \{h_{ij}(p)\}_{M\times L}$ can be generated. Go to the next step.

Step 2. (Transformation) By Equation (19), we can transform the decision matrix $D_g = \{h_{ij}(p)\}_{M\times L}$ in the form of G-PLTSs into $D_{bg} = \{h_{ij}(p)\}_{M\times L}$ in the form of belief-based G-PLTSs. Go to the next step.

Step 3. (Standardisation) Since the forms of linguistic evaluations are different and different linguistic terms have different semantics, to operate correctly, we then transform the belief-based decision matrix D_{bg} into the standardised decision matrix D_{bg} by Equations (20)–(23). Go to the next step.

Step 4. (Aggregation) By the ER approach given by Equations (5)–(14), we can obtain the comprehensive evaluation of each alternative shown as Equation (24). Then, considering that the global ignorance $\beta_{S_0,i}$ can be distributed to any linguistic term, we can obtain the best and worst evaluations of each alternative, i.e., $S(a_i)_{\max}$ and $S(a_i)_{\min}$. Go to the next step.

Step 5. (Ranking) Adjust the final collective belief-based G-PLTSs $\hat{S}(a_i)$ (for $i = 1, 2, \ldots, M$) and calculate the net gained dominance scores $DS_1(a_i)$ and the net lost dominance scores $DS_2(a_i)$ of each alternative corresponding to the optimism and pessimism principles by Equations (31)–(32), respectively. Normalise $DS_1(a_i)$ and $DS_2(a_i)$ by Equation (33), and integrate the subordinate rank sets and the scores by Equation (34), respectively. Then, we

Table 1. The information of the candidates.

	c_1	c_2	c_3
a_1	47	Smoking for 18 years, 20 packs per year, 10 years since quitting smoking	Excellent air quality; long-term exposure to cooking fumes
a_2	62	No smoking	High sulphur dioxide content in the air
a_3	56	Smoking for 20 years, 30 packs per year, 15 years since quitting smoking	The concentration of PM2.5 in atmosphere is higher in recent ten years; long-term exposure to second-hand smoke
a_4	36	Smoking for 16 years, 30 packs per year, not quitting smoking	The concentration of PM2.5 in atmosphere is higher in recent ten years
a_5	68	Smoking for 48 years, 20 packs per year, not quitting smoking	Excellent air quality

	c_4	c_5	c_6	c_7
a_1	None	Lymphoma was diagnosed three years ago and is in good condition	None	None
a_2	Long-term exposure to asbestos	None	Grandpa died of lung cancer	None
a_3	None	Gastric cancer eight years ago, cured	None	15 years of chronic obstructive pulmonary disease
a_4	None	None	Great grandmother died of lung cancer	None
a_5	Long-term exposure to radioactive substances	None	None	20 years of bronchial asthma

can gain two different rank sets. Obtain the average score of each alternative $BS(a_i)_{avg}$ by Equation (35), and then determine the final rank set of alternatives.

Below we highlight the contributions of the GPLER approach for MCDM problems:

1. The GPLER approach allows experts to evaluate alternatives in various forms, including single or multiple discrete linguistic terms or continuous interval linguistic terms. This greatly reduces the difficulty of experts in evaluating alternatives and is more in line with the habits of human thinking and expression.
2. The evaluation grades in the original ER approach can be extend to the form of G-PLTSs. The concept of semantics is introduced to define the corresponding semantic value of each linguistic term, and the transformation between qualitative linguistic evaluations and quantitative linguistic evaluations is accomplished without losing information. Thus, it is possible to standardise various forms of linguistic evaluations into a single linguistic term. The realisation of standardisation reduces the difficulty of linguistic information fusion. At the same time, the one-to-one relationship between linguistic terms and semantic values ensures the integrity of information in the process of transformation, ensuring the accuracy of the transformation results.
3. To express expert evaluations accurately and comprehensively, the belief degree is introduced to well model the uncertainty and incompleteness in decision-making. Then, the ER approach is used to fuse the evaluations under each criterion to obtain the comprehensive distributed evaluations of each alternative. The fused evaluations not only can express experts' preferences for various evaluations, but also can express uncertainties.
4. The GLDS-based method can synthetically rank the collective belief-based G-PLTSs, taking into account both the gained and lost scores and the uncertainties in evaluations. Since the global ignorance is considered in the ranking process, the accuracy of ranking is guaranteed, and more accurate auxiliary information can be provided for decision-makers.

5. A case study: the screening of high-risk population for lung cancer

In this section, to validate the proposed approach, we apply the GPLER method to solve a practical case concerning the screening of high-risk population for lung cancer.

5.1. Case description

According to the Global Cancer Report 2018 (Bray et al., 2018), lung cancer is the most common diagnostic cancer for both men and women (11.6% of total cases) and the leading cause of cancer death (18.4% of total cancer deaths). Because the early symptoms of lung cancer are not easy to find (Hamilton et al., 2005), once clinical symptoms occur, most patients often reach the middle or late stage of the disease with high mortality rates. It has been found that low-dose spiral CT can detect and treat 20% of the high-risk population of lung cancer early, thus significantly reducing lung cancer mortality (Aberle et al., 2011). Although low-dose spiral CT can effectively detect lung cancer patients, since CT examination will cause certain radiation damage to the human body and the cost is high, experts do

Table 2. The decision-making matrix with G-PLTSs.

	c_1	c_2	c_3	c_4
a_1	$(s_1(0.3), [s_0, s_1](0.4))$	$(s_0(0.3), \{s_1, s_2\}(0.5))$	$(s_{-1}(0.1), s_0(0.2), \{s_{-1}, s_0\}(0.5))$	$(s_{-2}(0.2), s_{-1}(0.4), [s_{-1}, s_0](0.3))$
a_2	$(s_0(0.3), s_1(0.4), \{s_0, s_1\}(0.1))$	$(s_{-1}(0.6), [s_{-1}, s_0](0.4))$	$(s_1(0.4), [s_1, s_2](0.6))$	$(s_0(0.4), s_1(0.4))$
a_3	$(s_{-1}(0.2), s_0(0.4), [s_{-1}, s_0](0.2))$	$(s_0(0.2), s_1(0.4), \{s_0, s_1\}(0.2))$	$(s_0(0.3), s_1(0.5))$	$(s_{-2}(0.2), s_{-1}(0.4), [s_{-1}, s_0](0.3))$
a_4	$(s_{-1}(0.6), s_0(0.2))$	$(s_0(0.2), s_1(0.5), [s_1, s_2](0.3))$	$(s_0(0.4), [s_0, s_1](0.4))$	$(s_{-2}(0.2), s_{-1}(0.4), [s_{-1}, s_0](0.3))$
a_5	$(s_0(0.3), s_1(0.2), s_2(0.2))$	$(s_0(0.1), s_1(0.1), s_2(0.5))$	$(s_{-1}(0.7), \{s_{-1}, s_0\}(0.1))$	$(s_0(0.3), \{s_0, s_1\}(0.3), s_2(0.3))$

	c_5	c_6	c_7
a_1	$(s_0(0.2), s_1(0.6))$	$(s_{-1}(0.6), \{s_{-1}, s_0\}(0.2))$	$(s_0(0.2), s_1(0.2), [s_1, s_2](0.3))$
a_2	$(s_{-1}(0.7), [s_{-1}, s_0](0.2))$	$(s_1(0.3), s_2(0.5))$	$(s_0(0.2), s_1(0.2), [s_1, s_2](0.3))$
a_3	$(s_{-1}(0.3), s_0(0.3), \{s_1, s_2\}(0.2))$	$(s_{-1}(0.6), \{s_{-1}, s_0\}(0.2))$	$(s_1(0.3), s_2(0.3), \{s_1, s_2\}(0.2))$
a_4	$(s_{-1}(0.7), [s_{-1}, s_0](0.2))$	$(s_0(0.2), s_1(0.2), [s_1, s_2](0.3))$	$(s_0(0.2), s_1(0.2), [s_1, s_2](0.3))$
a_5	$(s_{-1}(0.7), [s_{-1}, s_0](0.2))$	$(s_{-1}(0.6), \{s_{-1}, s_0\}(0.2))$	$(s_0(0.2), s_1(0.3), \{s_1, s_2\}(0.2))$

not recommend routine CT examination to general population, only recommend the high-risk group of lung cancer to participate in CT screening. Therefore, to improve the effective screening rate, it is particularly important to identify the high-risk population of lung cancer.

Before identifying the high-risk population of lung cancer, relevant screening indicators need to be determined. Marcus et al. (2015) pointed out that the prediction model based on smoking history and age can be used to identify the high-risk population of lung cancer economically and effectively. Wood et al. (2000) and Carr et al. (2015) also pointed out that lung cancer excluding smoking may be related to family genes. In 2015, China issued an expert consensus that the risk factors with important reference significance for early screening and diagnosis of lung cancer are (Jing, 2017): (1) smoking; (2) environmental pollution, such as indoor local air pollution, indoor domestic fuels, soot, cooking oil fumes, urban industry, heating and automobile exhaust; (3) occupational exposure, such as long-term exposure to arsenic, chromium, asbestos, nickel, cadmium and beryllium; (4) history of malignant tumours; (5) family history of lung cancer; (6) history of chronic lung diseases. Based on the above analysis, seven key indicators related to the screening of high-risk groups for lung cancer are set as: c_1: age, c_2: smoking state, c_3: long-term residential environment, c_4: occupational exposure, c_5: past history of malignant tumours, c_6: family history of lung cancer and c_7: history of chronic lung disease with the weight vector $(\omega_1, \omega_2, \ldots, \omega_7)^T = (0.1, 0.15, 0.1, 0.1, 0.15, 0.2, 0.2)^T$.

To determine the degree to which five candidates $a_i (i = 1, 2, \ldots, 5)$ are at high risk for lung cancer, a panel of 10 experts are invited to make evaluations. The information of the candidates is listed in Table 1.

In addition, to describe the possibility of belonging to high-risk groups, we introduce a discrete LTS $S = \{s_\alpha | \alpha = -2, \ldots, 0, \ldots, 2\} =$ {Impossible, Slightly possible, Possible, Quite possible, Must be} and the continuous LTS $S_0 = \{s_t | t \in [-2, 2]\}$. Experts need to judge the possibility of candidates being at high risk for lung cancer using $[s_{t_{k_1}}, s_{t_{k_2}}] \subset S_0$ or $\{s_\alpha\} \subset S$ on the seven criteria according to the personal circumstances of candidates, and give the corresponding probability. Since the probabilistic linguistic evaluations given by the experts have several forms, the existing PLTS and its extension cannot deal with such situation. So, we use the G-PLTS to model the evaluation information of experts.

The decision-making matrix $D_g = \{h_{ij}(p)\}_{M \times L}$ with G-PLTSs is shown in Table 2.

5.2. Rank the candidates by the GPLER approach

Below we use the GPLER method to solve this problem. Since the problem formalisation has been done in Section 4.1, we start the algorithm from Step 2.

Step 2. We transform the decision matrix $D_g = \{h_{ij}(p)\}_{M \times L}$ with G-PLTSs into the decision-making matrix D_{bg} with belief-based G-PLTSs. Due to the space limit, here we do not list the transformed matrix.

Step 3. We choose the linguistic scale function shown as Equation (2) for criteria c_1 and c_6, Equation (3) for criteria c_2, c_3 and c_4 with $t = \mu = 1.5$, and Equation (4) for criteria c_5 and c_7 with $\gamma = \psi = 0.5$. Then, by Equations (20)–(23), we can obtain the normalised decision-making matrix D_{bg}, as shown in Table 3.

Step 4. By Equations (5)–(14), the collective evaluation $\hat{S}(a_i)$ of each alternative can be calculated as follows:

$$\hat{S}(a_1) = \left((s_{-2}, 0.1), (s_{-1}, 0.4), (s_0, 0.2), (s_1, 0.1), (S_0, 0.2)\right)$$

$$\hat{S}(a_2) = ((s_{-2}, 0.1), (s_{-1}, 0.4), (s_0, 0.1), (s_1, 0.2), (s_2, 0.1), (S_0, 0.1))$$

$$\hat{S}(a_3) = ((s_{-1}, 0.3), (s_0, 0.2), (s_1, 0.2), (s_2, 0.1), (S_0, 0.2))$$

$$\hat{S}(a_4) = ((s_{-2}, 0.1), (s_{-1}, 0.4), (s_0, 0.2), (s_1, 0.2), (S_0, 0.1))$$

$$\hat{S}(a_5) = ((s_{-1}, 0.4), (s_0, 0.2), (s_1, 0.1), (s_2, 0.1), (S_0, 0.2))$$

Step 5. The above results can clearly show the degree to which each candidate belongs to each evaluation grade and the degree of ignorance in

Table 3. The normalised decision-making matrix.

	c_1	c_2	c_3	c_4
a_1	$((s_0,0.2),(s_1,0.5),(S_0,0.3))$	$((s_0,0.3),(s_1,0.25),(s_2,0.25),(S_0,0.2))$	$((s_{-1},0.35),(s_0,0.45),(S_0,0.2))$	$((s_{-2},0.2),(s_{-1},0.55),(s_0,0.15),(S_0,0.1))$
a_2	$((s_0,0.35),(s_1,0.45),(S_0,0.2))$	$((s_{-1},0.8),(S_0,0.2))$	$((s_1,0.8),(s_2,0.2))$	$((s_0,0.4),(s_1,0.4),(S_0,0.2))$
a_3	$((s_{-1},0.3),(s_0,0.5),(S_0,0.2))$	$((s_0,0.3),(s_1,0.5),(S_0,0.2))$	$((s_0,0.3),(s_1,0.5),(S_0,0.2))$	$((s_{-2},0.2),(s_{-1},0.55),(s_0,0.15),(S_0,0.1))$
a_4	$((s_{-1},0.6),(s_0,0.2),(S_0,0.2))$	$((s_0,0.2),(s_1,0.7),(s_2,0.1))$	$((s_0,0.6),(s_1,0.2),(S_0,0.2))$	$((s_{-2},0.2),(s_{-1},0.55),(s_0,0.15),(S_0,0.1))$
a_5	$((s_0,0.3),(s_1,0.2),(s_2,0.2),(S_0,0.3))$	$((s_0,0.1),(s_1,0.1),(s_2,0.5),(S_0,0.3))$	$((s_{-1},0.75),(s_0,0.05),(S_0,0.2))$	$((s_0,0.4),(s_1,0.1),(s_2,0.3),(S_0,0.2))$

	c_5	c_6	c_7
a_1	$((s_0,0.2),(s_1,0.6),(S_0,0.2))$	$((s_{-1},0.7),(s_0,0.1),(S_0,0.2))$	$((s_{-2},0.31),(s_{-1},0.59),(S_0,0.1))$
a_2	$((s_{-1},0.84),(s_0,0.06),(S_0,0.1))$	$((s_1,0.3),(s_2,0.5),(S_0,0.2))$	$((s_{-2},0.31),(s_{-1},0.59),(S_0,0.1))$
a_3	$((s_{-1},0.3),(s_0,0.3),(s_1,0.1),(s_2,0.1),(S_0,0.2))$	$((s_{-1},0.7),(s_0,0.1),(S_0,0.2))$	$((s_1,0.4),(s_2,0.4),(S_0,0.2))$
a_4	$((s_{-1},0.84),(s_0,0.06),(S_0,0.1))$	$((s_0,0.2),(s_1,0.35),(s_2,0.15),(S_0,0.3))$	$((s_{-2},0.31),(s_{-1},0.59),(S_0,0.1))$
a_5	$((s_{-1},0.84),(s_0,0.06),(S_0,0.1))$	$((s_{-1},0.7),(s_0,0.1),(S_0,0.2))$	$((s_0,0.2),(s_1,0.39),(s_2,0.11),(S_0,0.3))$

the evaluations. They provide comprehensive and accurate auxiliary information for decision-makers. Besides, since the global ignorance $\beta_{S_0,i}$ can be distributed in each linguistic term, the ranking of candidates is not always the same. Here, we consider two extreme cases by assigning $\beta_{S_0,i}$ to the best and worst linguistic terms, respectively.

1. Ordering by the optimism principle
   For the optimism principle, $\beta_{S_0,i}$ can be assigned to the best linguistic term s_{-2}, which means that the candidate has the lowest probability of being at high risk of lung cancer. Then, we can obtain the best evaluation $S(a_i)_{\max}$ of each candidate.
   By Equations (31)–(33), we can obtain the net gained dominance scores $DS_1(a_i)$ and the net lost dominance scores $DS_2(a_i)$, and then the normalised scores can be obtained as
   $$\{DS_1{}^N(a_i)|i=1,2,\dots,5\}=\{0,0.51,0.77,0.19,0.32\}$$
   $$\{DS_2{}^N(a_i)|i=1,2,\dots,5\}=\{0,0.54,0.65,0.32,0.43\}$$
   On this basis, we can obtain the subordinate rank sets. By Equation (34), we can obtain the score set $BS(a_i)_{\max}=\{0,0.07,0.21,-0.06,-0.02\}$, and then determine the final rank set corresponding to the optimism principle as $R_{\max}=\{r(a_i)|i=1,\dots,5\}=\{3,1,2,5,4\}$, which means $a_3>a_2>a_1>a_5>a_4$. The results show that a_3 is the most likely to be at high risk of lung cancer, followed by a_2, and a_4 is the least likely.
2. Ordering by the pessimism principle
   For the pessimism principle, $\beta_{S_0,i}$ can be assigned to the worst linguistic term s_2, which means that the candidate has the highest probability of being at high risk of lung cancer. Then, we obtain the worst evaluation $S(a_i)_{\min}$.
   By Equations (31)–(33), we can obtain the normalised scores as
   $$\{DS_1{}^N(a_i)|i=1,2,\dots,5\}=\{0.04,0.13,0.85,0,0.51\}$$
   $$\{DS_2{}^N(a_i)|i=1,2,\dots,5\}=\{0.11,0.23,0.79,0,0.56\}$$
   By Equation (34), we can obtain $BS(a_i)_{\min}=\{-0.02,-0.02,0.23,0,0.06\}$ and $R_{\max}=\{r(a_i)|i=1,2,\dots,5\}=\{4,2,1,5,3\}$, i.e., $a_3>a_5>a_4>a_2>a_1$. The results show that a_3 is still the most likely to be at high risk of lung cancer, followed by a_5, and a_1 is with the least likely.
   By Equation (35), we can obtain $BS(a_i)_{avg}=\{-0.02,0.05,0.44,-0.06,0.04\}$, and then obtain $a_3>a_2>a_5>a_1>a_4$.
3. Ordering by the maximum possibility principle
   According to the existing evaluations given by experts, the most possible allocation of

Table 4. The gained dominance score of each candidate.

Alternatives	Gained dominance scores c_1	c_2	c_3	c_4	c_5	c_6	c_7	Net gained dominance scores
a_1	0.03	0.6	0.11	0	2.45	0	0.00	0.47
a_2	1.06	0	1.31	1.1	0	2.35	0.00	0.82
a_3	0.2	0.32	0.67	0	0.91	0	3.00	0.87
a_4	0	0.53	0.39	0	0	1.55	0.00	0.43
a_5	1.41	1.54	0	1.65	0	0	2.31	1.00

Table 5. The lost dominance score of each candidate.

Alternatives	Lost dominance scores c_1	c_2	c_3	c_4	c_5	c_6	c_7	Net lost dominance scores
a_1	0.03	0.42	0.11	0.00	0.68	0.00	0.00	0.10
a_2	0.40	0.00	0.51	0.37	0.00	0.72	0.00	0.14
a_3	0.11	0.32	0.34	0.00	0.30	0.00	0.94	0.19
a_4	0.00	0.40	0.25	0.00	0.00	0.52	0.00	0.10
a_5	0.49	0.67	0.00	0.51	0.00	0.00	0.77	0.15

uncertainty is to redistribute according to the allocation proportion of the original evaluations (Pang, Wang & Xu, 2016). Thus, by Equations (31)–(33), we can obtain the normalised score of each alternative under the maximum possibility principle as follows:

$$\{DS_1{}^N(a_i)|i = 1, 2, ..., 5\} = \{0, 0.23, 0.68, 0.14, 0.68\}$$

$$\{DS_2{}^N(a_i)|i = 1, 2, ..., 5\} = \{0, 0.36, 0.63, 0.27, 0.63\}$$

By Equation (34), we can obtain $BS(a_i) = \{0, 0.01, 0.19, -0.03, 0.19\}$ and thus $a_3 = a_5 > a_2 > a_1 > a_4$.

To sum up, we can conclude that a_3 is the most likely candidate to be a high-risk group for lung cancer, whether under the optimistic, pessimistic principle or maximum possibility principle. Therefore, candidate a_3 should be advised to go to the hospital for further lung cancer diagnosis to obtain timely treatment.

5.3. Comparative analysis with the GLDS method and the original ER approach

To verify the advantages of the proposed GPLER approach, below we use the original GLDS method (Wu and Liao, 2019) and the original ER approach (Yang & Xu, 2002) to solve the above case.

5.3.1. Rank the candidates by the GLDS method

Steps 1–3 are the same as those in Section 4.2.

By Equations (36)–(39), we obtain the gained and lost dominance scores shown in Tables 4 and 5, respectively.

$$gd_j(a_i) = \sum_{v=1}^{M} df_j{}^N(a_i, a_v) \tag{36}$$

$$ld_j(a_i) = \max_v df_j{}^N(a_i, a_v) \tag{37}$$

$$DS_1(a_i) = \sum_{j=1}^{L} \omega_j gd_j(a_i) \tag{38}$$

$$DS_2(a_i) = \max_j \omega_j ld_j(a_i) \tag{39}$$

Then, similar to Equations (33)–(35), we can obtain the score of each alternative as $BS(a_1) = 0.02$, $BS(a_2) = 0.04$, $BS(a_3) = 0.01$, $BS(a_4) = -0.01$ and $BS(a_5) = 0.11$, which implies that $a_5 > a_2 > a_1 > a_3 > a_4$.

5.3.2. Rank the candidates by the original ER approach

Steps 1–4 are the same as those in Section 4.2.

By assigning the global ignorance $\beta_{S_0,i}$ to s_{-2} or s_2, we can use the following equations to calculate the best and worst utilities of each alternative (Yang, 2001):

$$u(a_i)_{\max} = \sum\nolimits_{\alpha=-2}^{2} u(s_\alpha)\beta_{s_\alpha,i} + u(s_{-2})\beta_{S_0,i}$$

$$u(a_i)_{\min} = \sum\nolimits_{\alpha=-2}^{2} u(s_\alpha)\beta_{s_\alpha,i} + u(s_2)\beta_{S_0,i}$$

where $u(s_\alpha)$ is the utility of the linguistic term s_α. Suppose that the decision-maker is neutral, so $\{u(s_\alpha)|\alpha = -2, -1, 0, 1, 2\} = \{0, 0.25, 0.5, 0.75, 1\}$. On this basis, we can obtain $a_3 > a_2 > a_5 > a_4 > a_1$ by $u(a_i)_{\max}$ and $a_3 > a_5 > a_2 > a_1 > a_4$ by $u(a_i)_{\min}$. We can also calculate the average utility of each alternative by Equation (17) and obtain $a_3 > a_5 > a_2 > a_4 > a_1$. According to the maximum possibility principle, the standardised evaluation information can be obtained through the proportional distribution of uncertainty. Then, by Equations (5)–(14) and $u(a_i) = \sum_{n=1}^{N} u(H_n)\beta_n(a_i)$, we can calculate the utility score of each alternative as $u(a_i) = \{0.3, 0.475, 0.55, 0.45, 0.55\}$ and obtain $a_3 = a_5 > a_2 > a_4 > a_1$.

In summary, from the results deduced by the above methods, we can obtain the ranks shown as Table 6.

According to Table 6, whether through optimism principle, pessimism principle, maximum possibility principle or average principle in the GPLER approach and the original ER approach, the ranking of a_3 is the first, indicating that the degree of a_3 belonging to the high-risk group of lung cancer is the highest. Comparing the GPLER approach with the original ER approach under each principle, we can find that the ranking result of a_1 changes the most. While ranking alternatives, the gained and lost scores can balance the negative effects of some outlier scores, and thus the ranking results will be affected. In this sense, it is necessary to apply the GLDS method to rank alternatives considering both the gained and lost scores.

By the original GLDS method, a_3 ranks the fourth. The main reason is that the net lost score of a_3 under criterion c_7 is too high, resulted in the decline of the overall ranking. In the original GLDS method, the candidates are ranked directly

Table 6. Comparisons of relevant ranking methods.

Methods	Principles	Rankings	Advantages	Disadvantages
GPLER	Optimism Pessimism Average Maximum possibility	$a_3 > a_2 > a_1 > a_5 > a_4$ $a_3 > a_5 > a_4 > a_2 > a_1$ $a_3 > a_2 > a_5 > a_1 > a_4$ $a_3 = a_5 > a_2 > a_1 > a_4$	• Consider the different distributions of ignorance • Represent the comprehensive evaluations of alternatives • The gained and lost scores are considered.	N/A
GLDS	N/A	$a_5 > a_2 > a_1 > a_3 > a_4$	Both gained and lost scores are considered.	• Not applicable to the situations where ignorance exists • The calculation of gains and losses from the evaluations of alternatives may lead to inaccurate ranking of schemes due to the existence of outliers.
Original ER	Optimism Pessimism Average Maximum possibility	$a_3 > a_2 > a_5 > a_4 > a_1$ $a_3 > a_5 > a_2 > a_1 > a_4$ $a_3 > a_5 > a_2 > a_4 > a_1$ $a_3 = a_5 > a_2 > a_4 > a_1$	• Considering different distributions of ignorance • Represent the comprehensive evaluations of alternatives	Consider only the gained scores, but not the lost scores.

according to the evaluations of candidates. However, because of the outliers of individual evaluations, the ranking of the whole alternatives will be problematic. In addition, since the global ignorance in the evaluations is proportionally assigned to each evaluation grade, the ranking results are also different due to the different ways of dealing with ignorance. Under the influence of the above two reasons, the rankings of the candidates are different.

It can be seen from the sorting results obtained by the GPLER approach or the original ER approach that different allocation methods of ignorance will produce different rankings. Decision-makers can rank the candidates according to different principles and then consider the actual situation of each candidate to give suggestions. Since a_3 is most likely to be at high risk for lung cancer, the suggestion for candidates a_3 may be *go to the third-level hospital for further lung cancer examination as soon as possible*. The ranking results of other candidates are not obvious, and the decision-maker can make further suggestions to the candidates in combination with the collective evaluation obtained in Step 4.

In the process of identifying high-risk population of lung cancer, due to the diversity and complexity of personal conditions, the existing technology cannot accurately use artificial intelligence to help identify, but mainly make judgements by experienced doctors. Usually, the linguistic terms are more in line with the habits of human thinking and expression than numerical values. Moreover, in many cases, experts cannot accurately give only a few discrete linguistic terms or continuous interval linguistic terms, and there may be several different expressions occurring at the same time. However, the existing PLTS and its extensions cannot model such diverse expressions. Therefore, the proposed GPLER approach can model such MCDM problems with uncertainties well and provide more accurate and comprehensive decision-making information for the identification of high-risk groups of lung cancer.

6. Conclusion

With the developments of science and technology, decision-making problems in human practice become increasingly complex. When experts evaluate alternatives, they often choose linguistic terms to express their opinions, which are more in line with the habits of human thinking and expression than numerical values. At the same time, since experts have different preferences for different linguistic terms, they usually give corresponding probabilities to express such preferences. Due to the limitation of human knowledge and the complexity of decision-making problems, experts may give a variety of linguistic terms to make evaluations in many cases. To solve the MCDM problems with complex linguistic expressions, in this paper, we proposed a GPLER approach to model such MCDM problems, which fills in the research gap. The G-PLTS and belief-based G-PLTS were introduced to model the MCDM problem with uncertainties, and then the ER approach was used to fuse the information to obtain the overall evaluations of alternatives. In addition, we extended the ER method to the situation with continuous linguistic evaluation grades. Given that the utility-based sorting method in ER approach only considers the gained scores of alternatives but neglects the lost scores caused by "regret", we proposed a GLDS-based method to rank the collective belief-based G-PLTSs. We applied the GPLER approach to screen high-risk population of lung cancer. Because of the limitations of current developments of science, technology and medicine, we mainly use the judgements of experienced experts to make the screening. Experts' judgements are mostly expressed in different forms of linguistic terms, but the existing methods cannot deal with such situations well. In this sense, the introduced GPLER approach is useful in solving the problem. It not only can model MCDM problems with uncertainties well, but also can rank candidates according to different principles, providing accurate and comprehensive auxiliary information for decision-makers.

In the future, we will implement the proposed GPLER approach to solve other practical decision-making problems, such as auxiliary diagnosis of lung cancer, rehabilitation evaluation of lung cancer patients, and follow-up of lung cancer patients. To investigate the group decision-making problem with G-PLTSs is also an interesting research topic.

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