International Journal of Information Technology & Decision Making
Vol. 3, No. 3 (2004) 419–428

ESTIMATING THE ATTRIBUTE WEIGHTS THROUGH EVIDENTIAL REASONING AND MATHEMATICAL PROGRAMMING

MINGRONG DENG*

Institute of Decision and Optimisation, School of Management
Zhejiang University, Hangzhou, 310027, P.R. China
Dengmr@zju.edu .cn

WEIXUAN XU

Institute of Policy and Management, Chinese Academy of Sciences
Beijing, 100080, P.R. China
wxu@mail.casipm.ac.cn

JIAN-BO YANG

University of Manchester Institute of Science and Technology
Manchester, M60, 1QD, UK
Jian-bo.Yang@umist.ac.uk

An approach for estimating attribute weight is proposed, which is addressed to the problem inverse to the aggregation assessment based on a hierarchy of attributes. Evidential reasoning framework is employed since the assessment on some basic attributes may be uncertain and incomplete. As a by-product, a simple formula for evidence combination for our special case is obtained. The weights are estimated based on the preference given by the decision-maker, and represented as intervals. The more abundant preference the decision-maker shows, the clearer the relation among the basic attributes.

Keywords: Weight estimation; evidential reason; non-linear programming.

1. Introduction

Markets and manufacturers are quickly and enthusiastically going global. Customer information becomes more and more important.[8] One gains the competitive edge, if he understands the latent demand more sensitively than the competitor does. For example, if a car manufacturer discovered which attributes were most important to the Chinese customers, then he would have a chance to get more share of the Chinese market by implementing an efficient plan on how the reachable resources should be allocated to improve the crucial functions.

*Corresponding author.

More often than not, a customer cannot definitely articulate whether one attribute is more important than another, needless to say to what extent. However, he may be able to make a clear choice between two brands of cars. It deserves studying how to estimate the relative weight for each attribute among a sample of alternatives by investigating the preference of the decision-makers. The solution can also be used to design new product.

One may argue that regression technique applying to the sales volume can meet the above needs. But one should note that the sales volume only reflects the buyers' behaviour, not all of the latent customers. Besides, many uncertain factors influencing the sale should also be taken into account, most of which are difficult to quantify, such as the policy, trend of the energy source, public security, condition of traffic, insurance, and weather, etc. Furthermore, when a new product is to be assessed, there is no history data available.

Multiple-attribute decision analysis (MADA) is widely used in ranking the decision alternatives with respect to multiple, usually conflicting attributes. Some methods have been designed to weaken required information about the importance of the attributes, but most of them require definitions of quantitative weights for attributes. Hence, a number of methods for determining attribute weights in MADA have been developed.

A method of value trade-off was presented firstly by Keeney and Raiffa.[4] The approach requires the decision-maker to compare pairs of the alternatives with respect to each pair of the attributes under the assumption that both alternatives have identical values on the remaining attributes. The attribute weights are determined after numerous value trade-off processes.

The Analytic Hierarchy Process (AHP) is a well-known methodology,[7] with which a set of pair-wise comparison matrices based on the hierarchical structure are constructed, and attribute weights are obtained by various assessments. It is interesting that from certain two simple axioms, it seems that geometric mean is the only acceptable solution.[1]

However, it is not easy for the decision-maker to give certain and consistent estimation even for the comparison matrix. Some objective methods are therefore proposed. Deng *et al.*[2] developed a method where the attribute weights are measured by the average intrinsic information generated by the given alternatives. Diakoulaki *et al.*[3] suggested that the attribute weights be judged on both contrast intensity and conflict of the attributes. These methods are all in accordance with a principle affirming that an attribute is less important if all alternatives have similar performance ratings.

Some authors recognize the fact that the attribute weights are context-dependent and a task-oriented weight estimating technique is developed.[6,13] For a specified task, the attribute weights can be deducted from some fuzzy rules given by the decision-maker. These methods fall into subjective ones.

It is also an important topic to investigate that, for some given attribute weights, the extent of the ranking among the alternatives is dependent and sensitive to these estimates. Mareschal obtains the weight stability intervals in various senses, with the assumption that a weight is altered when the importance of the concerned pair of attributes is modified but the relative importance among the other attributes being kept constant.[5]

In cases of uncertainty, an evidential reasoning (ER) approach for MADA problem has been proposed.[9–12] By evidence combination and utility theory, with the attribute weights being known, assessment aggregation turns out to be more reasonable and fairly simple, especially for the case where some assessment for the basic attribute is incomplete or uncertain. The attribute involved can also be qualitative or quantitative.

In this paper, a weight estimating technique is developed through ER and solving mathematical programming. From a set of assessments (certain or uncertain, complete or incomplete) on the basic attributes and preference relations on the alternatives by the decision-maker, an interval for each attribute is estimated. The more information is included in the assessment and the preference, the more unequivocal estimation can be obtained. The interval is determined without the assumption made by Mareschal.[5] Since the preference given by the decision-maker can vary in different situation, the outcome is obviously context-dependent.

2. Problem Description

There is a set of alternatives $\{a_1, a_2, \ldots, a_M\}$ to be valued with respect to a set of basic attributes $\{e_1, e_2, \ldots, e_L\}$ (qualitative or quantitative) represented as a hierarchy. Each of the basic attributes can be assessed more directly by means of some objective or subjective way.

Although the grade sets for the attributes may be different, we can transform them into a unified one, e.g. $\mathbf{H} = \{H_1, H_2, \ldots, H_N\}$, using a rule-based technique for qualitative attributes or utility technique for quantitative attributes.[12]

For a basic attribute e_i, the assessment with uncertainty can be expressed as

$$S(e_i(a_l)) = \{(H_n, \beta_{n,i}^{(l)}), n = 1, \ldots, N\}, \qquad i = 1, \ldots, L; \quad l = 1, \ldots, M \tag{1}$$

where $\beta_{n,i}^{(l)}$ denotes the degree of attribute e_i being valued to grade H_n while alternative a_l is chosen. Hence,

$$\beta_{n,i}^{(l)} \geq 0 \quad \text{and} \quad \sum_{n=1}^{N} \beta_{n,i}^{(l)} \leq 1. \tag{2}$$

When $\sum_{n=1}^{N} \beta_{n,i}^{(l)} = 1$, the assessment of e_i at a_l is complete. Especially, when there is a some n such that $\beta_{n,i}^{(l)} = 1$, the assessment is precise.

As for assessing the alternatives, they can also be assessed as the same way as the attributes,

$$S(y(a_l)) = \{(H_n, \beta_n^{(l)}), n = 1, \ldots, N\}. \tag{3}$$

In practice, it is more convenient for the decision-maker to give the pair-wise preference relations such as one of the followings:

(a) a_i is indifferent to a_j;
(b) a_i is better than a_j;
(c) a_i is better than a some multiple of a_j, say, $L_{ij}a_j$; and
(d) a_i is better than $m_{ij}a_j$, and worse than $M_{ij}a_j$.

Based on the attribute assessments in forms of Eq. (1) and a sufficient number of preference relations in forms of (a)–(c) or (d), we are to estimate the important factor for every attribute contributing to the general value.

3. Evidence Combination

We first consider the aggregation assessment for an attribute hierarchy of two simple levels, namely, a general attribute y at the upper level and several basic attributes $\{e_1, e_2, \ldots, e_L\}$ at the lower level.

Let $m_{n,i}$ express a basic probability assignment to which e_i supports a hypothesis that the state of y is confirmed to H_n, and $m_{H,i}$ denotes the remaining probability unassigned to any of $H_n(i = 1, \ldots, L; n = 1, \ldots, N;)$

Since $\beta_{n,i}$ is the confidence degree associated with the attribute e_i being evaluated to H_n, it is natural that the weight ω_i of attribute e_i be defined as such so that,

$$m_{n,i} = \omega_i \beta_{n,i} \qquad n = 1, \ldots, N; \quad i = 1, \ldots, L \tag{4}$$

and

$$\sum_{i=1}^{L} \omega_i = 1, \quad 0 \le \omega_i \le 1. \tag{5}$$

The remaining basic probability, that is, those assignments to the whole grade set $\mathbf{H}$ amount to,

$$m_{H,i} = 1 - \omega_i B_i, \quad \text{where } B_i = \sum_{n=1}^{N} \beta_{n,i}.$$

According to the evidence combination principle of Demspter–Shafer theory, aggregated probability from basic probability assignment m_1 and m_2 can be obtained by the following equation,

$$(m_1 \oplus m_2)(X) = \begin{cases} 0, & X = \Phi \\ \dfrac{\sum_{A \cap B = X} m_1(A) m_2(B)}{1 - \sum_{A \cap B = \Phi} m_1(A) m_2(B)}, & X \neq \Phi. \end{cases} \tag{6}$$

Although a recursive method can be used for aggregated confidence assessment, for our special case, a simpler formula can be proven as follows.

Theorem. Let $M_n = (m_{n,1} + m_{H,1})(m_{n,2} + m_{H,2}) \cdots (m_{n,L} + m_{H,L})$ for every $n = 1, \ldots, N$, $M_0 = m_{H,1}m_{H,2} \cdots m_{H,L}$, then the confidence degree of the general attribute being assessed to each grade has the following form:

$$\beta_n = \left[\sum_{n=1}^{N} M_n - (N-1)M_0\right]^{-1} (M_n - M_0) \tag{7}$$

for $n = 1, \ldots, N$, and

$$\beta_H = \left[\sum_{n=1}^{N} M_n - (N-1)M_0\right]^{-1} (M_0). \tag{8}$$

Proof. Let us consider the following expression, which is obviously equal to 1, $(m_{1,1} + m_{2,1} + \cdots + m_{N,1} + m_{H,1})(m_{1,2} + m_{2,2} + \cdots + m_{N,2} + m_{H,2}) \cdots (m_{1,L} + m_{2,L} + \cdots + m_{N,L} + m_{H,L})$.

Among the items unfolded, the confidence degree of the general attribute assessed to H_n is the sum of the items which only contains $m_{n,i}$ (at least once) or $m_{H,i}$, divided by the sum of the items not including $m_{i,k1}m_{j,k2}$ where $i \neq j, i \neq H, j \neq H$. The former is $M_n - M_0$, and the latter is equal to

$$\sum_{n=1}^{N} (M_n - M_0) + M_0 = \sum_{n=1}^{N} M_n - (N-1)M_0. \tag{9}$$

The basic probability assigned to the whole set H, is equal to M_0 divided by (9), from (6).

Thus the theorem follows obviously. □

Assessment for a more general multi-level hierarchical evaluation can follow in the same way.

4. Weight Estimation

We estimate the relative weight for each basic attribute by calculating their value based on the general utilities of the alternatives satisfying the preference relation given by the decision-maker. From the relations given by the decision-maker among the "ideal" alternatives with pure grade, the utility of evaluation H_n can be estimated, denoted by μ_n. Since the utility is not a concrete concept from the view of the decision-maker, we treat them as variables, making the least assumptions,

$$\mu_1 = 0, \quad \mu_n = 1; \tag{10}$$

$$\mu_{n+1} > \mu_n + \delta, \quad (n = 1, \ldots, N-1). \tag{11}$$

Here δ is a small number that denotes the least difference between adjacent grades. It can be set to $1/(2(N-1))$, for example, if there is not any information available. Of course, if there is more information about the utility of the grade set of the general attribute, for example, $\mu_{n+1} > 0.5\,\mu_n + 0.1$, it should also be integrated into the constraint conditions.

The expected utility of $y(a_l)$ thereupon is,

$$\mu(S_y(a_l)) = \sum_{n=1}^{N} \mu_n \beta_n^{(l)}. \tag{12}$$

Since the weight of each attribute is unknown, we denote the total confidence degree of being assessed to H_n, for alternative a_l given by Eq. (7) by $\beta_n^{(l)}(\omega)$, then the preference described as (a)–(d) in Sec. 2 can be respectively described as

$$\sum_{n=1}^{N} \beta_n^{(i)}(\omega)\mu_n = \sum_{n=1}^{N} \beta_n^{(j)}(\omega)\mu_n \tag{13}$$

$$\sum_{n=1}^{N} \beta_n^{(i)}(\omega)\mu_n > \sum_{n=1}^{N} \beta_n^{(j)}(\omega)\mu_n \tag{14}$$

$$\sum_{n=1}^{N} \beta_n^{(i)}(\omega)\mu_n > L_{ij} \sum_{n=1}^{N} \beta_n^{(j)}(\omega)\mu_n \tag{15}$$

$$M_{ij} \sum_{n=1}^{N} \beta_n^{(j)}(\omega)\mu_n > \sum_{n=1}^{N} \beta_n^{(i)}(\omega)\mu_n > m_{ij} \sum_{n=1}^{N} \beta_n^{(j)}(\omega)\mu_n. \tag{16}$$

As the decision-maker may give the preference relation inconsistently, we add some inconsistency error variables for each of the above. Then (13)–(16) become,

$$s_{ij} \geq \sum_{n=1}^{N} \beta_n^{(i)}(\omega)\mu_n - \sum_{n=1}^{N} \beta_n^{(j)}(\omega)\mu_n \geq -s_{ij} \tag{17}$$

$$\sum_{n=1}^{N} \beta_n^{(i)}(\omega)\mu_n > \sum_{n=1}^{N} \beta_n^{(j)}(\omega)\mu_n + u_{ij} \tag{18}$$

$$\sum_{n=1}^{N} \beta_n^{(i)}(\omega)\mu_n > L_{ij} \sum_{n=1}^{N} \beta_n^{(j)}(\omega)\mu_n + v_{ij} \tag{19}$$

$$M_{ij} \sum_{n=1}^{N} \beta_n^{(j)}(\omega)\mu_n + p_{ij} > \sum_{n=1}^{N} \beta_n^{(i)}(\omega)\mu_n > m_{ij} \sum_{n=1}^{N} \beta_n^{(j)}(\omega)\mu_n + q_{ij}. \tag{20}$$

Now we have concluded a non-linear mathematical programming model in which there are three types of variable, i.e. utilities vector $\mu = (\mu_1, \ldots, \mu_N)^T$, weight vector $\omega = (\omega_1, \ldots, \omega_L)^T$, and deviation vector s.

There are three types of constraints, namely,

- weight constraints, (5)
- utility constraints, (10), (11)
- preference constraints, in forms of (17), (18), (19) and (20).

If we denote Ω the variable space satisfying all of the above constraints, we can estimate μ, ω and s by solving the following mathematical programming,

$$\begin{aligned} &\min \sum s_{ij}^2 \\ &\text{s.t. } (\mu, \omega, s) \in \Omega \end{aligned} \tag{21}$$

where s_{ij}^2 also represents u_{ij}^2, v_{ij}^2, or $p_{ij}^2 + q_{ij}^2$, depending on which kind of forms of (a)–(d) in Sec. 2 is preferred by the decision-maker.

If the inconsistency error of the solution is not equal to zero, then the decision-maker should be asked to amend the preference relation, for example, he/she can slacken some relation, drop an indefinite relation and so on. Once we get a group of preference relations within that there is no contradiction, a feasible weight vector is present. But this weight vector is not exactly what we want, since there may be numerous solutions satisfying all of the consistent constraints. We estimate the possible lowest and highest value of the weight of each attribute, $\bar{\omega}_i, \hat{\omega}_i$, by solving the following twin mathematical programming,

$$\begin{cases} \text{Min} \quad \omega_i \\ \text{ s.t. } (\mu,\omega,0) \in \Omega \end{cases} \quad \begin{cases} \text{Max} \quad \omega_i \\ \text{ s.t. } (\mu,\omega,0) \in \Omega. \end{cases} \tag{22}$$

Therefore, we obtain a desirable interval $[\bar{\omega}_i, \hat{\omega}_i]$ for every attribute e_i. We are sure that e_i is more important than e_j if $\bar{\omega}_i > \hat{\omega}_j$. When the two intervals overlap, the relation between them becomes complicated. To clarify their relation, there are two ways available. One is to acquire a weight value by simply averaging the interval, which is desirable when the interval is quite short. When the interval is longer, it needs to employ the other way, that is, to enhance the preference relation set, which is equivalent to having the decision-maker offering more information.

5. An Example

This example involves a performance assessment problem about six types of cars. There are seven basic attributes most concerned by the customers, namely, acceleration, braking, horsepower, handling, ride quality, power train and fuel economy. Four of them are quantitative, and three are qualitative. The general grade set is $\{W, P, A, G, E, T\}$, representing worst, poor, average, good, excellent and top respectively. More details about the assessment problem can be found in the references.[12] The idea developed there is to assess the six types of cars on the basis of importance of each attribute already known, whereas the problem here is to estimate the importance of each attribute.

The basic attributes assessment given by the decision-maker is listed in Table 1.

Although the assessments on the attributes are complete, one can see that it is not utilized in any means for the following process.

In order to estimate the weight for the seven attributes, the decision-maker is asked to give his preference among all the alternatives.

Table 1. Car assessment information.

Performance	Car 1	Car 2	Car 3	Car 4	Car 5	Car 6
Acceleration	$\{(P, 0.2), (A, 0.8)\}$	$\{(G, 0.5), (E, 0.5)\}$	$\{(E, 0.75), (T, 0.25)\}$	$\{(A, 0.4), (G, 0.6)\}$	$\{(G, 0.5), (E, 0.5)\}$	$\{(G, 0.25), (E, 0.75)\}$
Braking	$\{(G, 1.0)\}$	$\{(E, 0.333), (T, 0.667)\}$	$\{(G, 0.5), (E, 0.5)\}$	$\{(P, 0.75), (A, 0.25)\}$	$\{(P, 1.0)\}$	$\{(E, 1.0)\}$
Handling	$\{(A, 0.4), (G, 0.6)\}$	$\{(E, 0.6), (T, 0.4)\}$	$\{(A, 0.4), (G, 0.6)\}$	$\{(A, 1.0)\}$	$\{(G, 1.0)\}$	$\{(E, 0.6), (T, 0.4)\}$
Horsepower	$\{(E, 0.333), (T, 0.667)\}$	$\{(P, 0.533), (A, 0.467)\}$	$\{(G, 0.462), (E, 0.538)\}$	$\{(G, 0.385), (E, 0.615)\}$	$\{(W, 0.467), (P, 0.533)\}$	$\{(A, 0.267), (G, 0.733)\}$
Ride quality	$\{(G, 0.6), (E, 0.4)\}$	$\{(A, 1.0)\}$	$\{(A, 0.4), (G, 0.6)\}$	$\{(G, 1.0)\}$	$\{(G, 1.0)\}$	$\{(G, 0.6), (E, 0.4)\}$
Powertrain	$\{(A, 0.4), (G, 0.6)\}$	$\{(G, 1.0)\}$	$\{(E, 0.6), (T, 0.4)\}$	$\{(A, 0.4), (G, 0.6)\}$	$\{(G, 0.6), (E, 0.4)\}$	$\{(E, 0.6), (T, 0.4)\}$
Fuel economy	$\{(G, 1.0)\}$	$\{(G, 1.0)\}$	$\{(E, 1.0)\}$	$\{(G, 1.0)\}$	$\{(A, 1.0)\}$	$\{(G, 1.0)\}$

Firstly, provided that the preference given by the decision-maker is,

$$\text{Car 5} > \text{Car 4} > \text{Car 1} > \text{Car 2};$$
$$\text{Car 6} > \text{Car 3} > \text{Car 2}.$$

Through solving problem in Eq. (21) (e.g. the Solver in Excel), we get the weight vector,

$$(\omega_1, \omega_2, \omega_3, \omega_4, \omega_5, \omega_6, \omega_7)^T = (0.237, 0.01, 0.2596, 0, 0.5, 0, 0)^T.$$

The corresponding inconsistency error is zero. It does not mean that the fourth, the sixth and the last attribute are not-important, since there may be numerous feasible solutions from which it can also lead to non-inconsistency. Hence, we can go on with getting an interval for each attribute by searching the solution of the corresponding problem in Eq. (22). We have,

$$\omega_1 = 0 \sim 0.494, \quad \omega_2 = 0 \sim 0.238, \quad \omega_3 = 0 \sim 0.29, \quad \omega_4 = 0 \sim 0.283,$$
$$\omega_5 = 0.04 \sim 0.78, \quad \omega_6 = 0 \sim 0.734, \quad \omega_7 = 0 \sim 0.291.$$

The importance of any of the attributes is unclear. This means that the preference relation given by the decision-maker is not essential to the attribute weights. To obtain a more precise weight vector, one has to augment the preference relations.

Hence, the decision-maker is asked if he can expand the preference sets. He re-captures, for example,

$$\text{Car 5} > \text{Car 3}.$$

Then we can construct a new mathematical programming model, and obtain a more accurate estimation for each attribute weight with the first one unchanged,

$$\begin{aligned} &\omega_2 = 0 \sim 0.201, \quad &&\omega_3 = 0 \sim 0.285, \quad &&\omega_4 = 0 \sim 0.213, \\ &\omega_5 = 0.25 \sim 0.78, \quad &&\omega_6 = 0 \sim 0.635, \quad &&\omega_7 = 0 \sim 0.241. \end{aligned}$$

Now it will not take much risk to say that the fifth attribute is most important, but the other attributes remain vague.

Suppose the decision-maker gives an extra preference, for example,

$$\text{Car } 4 > \text{Car } 3.$$

The interval estimate for every attribute weight deducted from the mathematical programming will be,

$$\begin{aligned} &\omega_1 = 0.01 \sim 0.484, \quad &&\omega_2 = 0 \sim 0.169, \quad &&\omega_3 = 0 \sim 0.245, \quad &&\omega_4 = 0 \sim 0.214, \\ &\omega_5 = 0.388 \sim 0.78, \quad &&\omega_6 = 0 \sim 0.467, \quad &&\omega_7 = 0 \sim 0.241. \end{aligned}$$

It can be seen that the preference between Cars 1 and 3 will be crucial for determining the attribute weights. When faced with Cars 1 and 3, the decision-maker selects Car 3, for example. Hence, we can carry on with the process and obtain

$$\begin{aligned} &\omega_1 = 0.266 \sim 0.484, \quad &&\omega_2 = 0 \sim 0.169, \quad &&\omega_3 = 0 \sim 0.245, \quad &&\omega_4 = 0 \sim 0.214, \\ &\omega_5 = 0.388 \sim 0.678, \quad &&\omega_6 = 0 \sim 0.467, \quad &&\omega_7 = 0 \sim 0.241. \end{aligned}$$

At this time, we are quite definite that the importance of the attributes in the decision-maker's view has the following order:

$$\begin{aligned} \{\text{Ride quality}\} &\geq \{\text{Acceleration}\} > \{\text{Handling, Horsepower, Fuel economy}\} \\ &\geq \{\text{Braking}\}. \end{aligned}$$

To determine the importance of power train, the study should be carried on.

6. Conclusion

This paper proposed an approach for estimating attribute weight based on evidential reasoning and mathematical programming. The weight obtained is estimated as an interval. The more preference is given by the decision-maker, the more accurate the estimation is. In practice, the weight estimation can be an interactive iteration process.

The result of this paper can be used to analyze the customers' latent demand, and to develop new products as well.

Acknowledgement

The authors would like to acknowledge the valuable suggestions from the anonymous referees.

References

1. J. Barzilai, Deriving weight from pairwise comparison matrices, *J. Operational Research Society* **48**(12) (1997) 1226–1232.
2. H. Deng, C. H. Yeh and R. Willis, Inter-company comparison using modified TOPSIS with objective weights, *Computers & Operations Research* **27** (2000) 963–973.
3. D. Diakoulaki, G. Mavrotas and L. Papayannakis, Determining objective weights in multiple criteria problems: The CRITIC method, *Computers & Operations Research* **22** (1995) 963–770.
4. R. Keeney and H. Raiffa, *Decision with Multiple Objective, Preferences and Value Trade Offs* (Wiley, New York, 1976).
5. B. Mareschal, Weight stability intervals in multicriteria decision aid, *European J. Operational Research* **33** (1988) 54–64.
6. R. A. Ribeiro, Fuzzy multiple attribute decision making: A review and new preference elicitation techniques, *Fuzzy Sets and Systems* **78** (1996) 155–181.
7. T. L. Satty, *The Analytic Hierarchy Process* (McGraw Hill, New York, 1980).
8. G. Tyndall, C. Gopal, W. Partsch and J. Kamauff, *Supercharging Supply Chains* (John Wiley & Sons, Inc., 1998).
9. J. B. Yang and M. G. Singh, An evidential reasoning approach for multiple attribute decision making with uncertainty, *IEEE Transactions on Systems, Man, and Cybernetics* **24** (1994) 1–18.
10. J. B. Yang and P. Sen, A general multi-level evaluation process for hybrid MADM with uncertainty, *IEEE Transactions on Systems, Man and Cybernetics* **24** (1994) 1458–1471.
11. J. B. Yang and P. Sen, Preference modelling by estimating local utility functions for multiobjective optimization, *European J. Operational Research* **95** (1996) 115–138.
12. J. B. Yang, Rule and utility based evidential reasoning approach for multiattribute decision analysis under uncertainties, *European J. Operational Research* **131** (2001) 31–61.
13. C. H. Yeh, R. J. Willis, H. Deng and H. Pan, Task oriented weighting in multi-criteria analysis, *European J. Operational Research* **119** (1999) 130–146.