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An evidential reasoning based approach for quality function deployment under uncertainty

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ABSTRACT

Quality function deployment (QFD) is a methodology for translating customer wants (WHATs) into relevant engineering design requirements (HOWs) and often involves a group of cross-functional team members from marketing, design, quality, finance and production and a group of customers. The QFD team is responsible for assessing the relationships between WHATs and HOWs and the interrelationships between HOWs, and the customers are chosen for assessing the relative importance of each customer want. Each member and customer from different backgrounds often demonstrates significantly different behavior from the others and generates different assessment results, complete and incomplete, precise and imprecise, known and unknown, leading to the QFD with great uncertainty. In this paper, we present an evidential reasoning (ER) based methodology for synthesizing various types of assessment information provided by a group of customers and multiple QFD team members. The proposed ER-based QFD methodology can be used to help the QFD team prioritize design requirements with both customer wants and customers' preferences taken into account. It is verified and illustrated with a numerical example.

1. Introduction

Quality function deployment (QFD) is a cross-functional planning methodology commonly used to ensure that customer expectations or requirements, often referred to as the voice of the customer (VoC) or WHATs, are deployed through product planning, part development, process planning and production planning. It is a team-based and disciplined approach to product design, engineering and production and provides in-depth evaluation of a product. An organization that correctly implements QFD can improve engineering knowledge, productivity and quality and reduce costs, product development time and engineering changes (Besterfield, Besterfield-Michna, Besterfield, & Besterfield-Sacre, 2003). QFD has now become a standard practice by most leading organizations and has been successfully implemented world widely. A comprehensive literature review of QFD and its extensive applications is provided by Chan and Wu (2002).

The successful implementation of QFD requires a significant number of subjective judgments from both customers and QFD team members. Customers are selected for assessing the relative importance of customer expectations or requirements (WHATs). The QFD team is set up to identify customer wants, map them into relevant engineering requirements, which are often called the HOWs, meaning how the WHATs are to be met, develop the relationship matrix between WHATs and HOWs and the interrelationship matrix between HOWs, and prioritize the HOWs.

As two of the key issues of QFD, prioritization methods for WHATs and HOWs have been extensively researched and quite a number of approaches have been suggested in the QFD literature. For example, the analytic hierarchy process (AHP), a well-known and commonly used multi-criteria decision making method, and its variants:fuzzy AHP, analytic network process (ANP) and fuzzy ANP have been suggested and widely applied to prioritize customer requirements (Akao, 1990; Armacost, Componation, Mullens, & Swart, 1994; Büyüközkan, Ertay, Kahraman, & Ruan,

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2004; Ertay, Büyüuközkan, Kahraman, & Ruan, 2005; Fung, Popplewell, & Xie, 1998; Hanumaiah, Ravi, & Mukherjee, 2006; Kahraman, Ertay, & Büyüközkan, 2006; Karsak, Sozer, & Alptekin, 2003; Kim, Yoon, & Yun, 2005; Kwong & Bai, 2003; Kwong & Bai, 2002; Lu, Madu, Kuei, & Winokur, 1994; Park & Kim, 1998; Partovi, 2007; Wang, Xie, & Goh, 1998). Fuzzy and entropy method (Chan, Kao, Ng, & Wu, 1999; Chan & Wu, 2005) have also been proposed to rate the importance of customer needs. The weighted sum method (Chen & Weng, 2003; Wasserman, 1993), fuzzy weighted average (FWA) (Chen, Fung, & Tang, 2006; Chen & Weng, 2006; Liu, 2005; Vanegas & Labib, 2001), fuzzy outranking approach (Wang, 1999) and grey model (Wu, 2006) have all been suggested for prioritizing engineering design requirements. Fuzzy logic and fuzzy inference have been extensively applied to assess the importance of WHATs and prioritize HOWs (Bottani & Rizzi, 2006; Chen, Chen, & Lin, 2004; Chen, Fung, & Tang, 2005; Karsak, 2004; Karsak, 2004; Khoo & Ho, 1996; Ramasamy & Selladurai, 2004; Shen, Tan, & Xie, 2001; Temponi, Yen, & Tiao, 1999).

To reduce the heavy burden of customers and QFD team members in making their judgments, Franceschini and Rossetto (2002) develop an interactive algorithm for prioritizing product's technical design characteristics, called IDCR (interactive design characteristic ranking), which allows the QFD team to determine a ranking order for design characteristics without using subjective rating scales and explicitly knowing the relative degree of importance of customer requirements. The IDCR algorithm avoids an inappropriate conversion from qualitative information to a relationship matrix. Han, Kim, and Choi (2004) suggest a linear partial ordering approach for assessing the information in QFD and prioritizing engineering characteristics. The linear partial information is used to extract the weights of customer wants and the relationship values between WHATs and HOWs. Four types of dominance relations that are frequently used in multi-attribute decision making with incomplete information are used to determine the priorities of engineering characteristics when the linear partial orderings of customers and QFD team members are given. The dominance relations between engineering characteristics are established through the solution of a number of linear programming models.

Considering the fact that people contributing to the QFD process may give their preferences in different formats, numerically or linguistically, depending on their backgrounds, Büyüközkan and Feyzioğlu (2005) present a fuzzy logic based group decision making approach with multiple expression formats for QFD with the hope to better capture and analyze the demand of customers, where different expressions are aggregated into one collaborative decision by using fuzzy set theory. Their approach is illustrated with a software development example. Ho, Lai, and Chang (1999) also discuss group behaviors in QFD and present an integrated group decision making approach for aggregating team members' opinions in the case where some members in a team have an agreed criteria set while others prefer individual criteria sets. By using voting and linear programming techniques, their integrated approach consolidates individual preferences into a group consensus and is used for determining the relative importance of customer requirements.

The above literature review clearly shows that quite a lot of efforts have been made to deal with fuzziness in the process of QFD. However, no attempt has been made to address the issue of how to deal with incomplete, imprecise and missing (ignorance) information in QFD, which is essentially inherent and sometimes inevitable in human being's subjective judgments. Fuzzy logic based approaches have been extensively used to model vagueness and ambiguity, but it cannot deal with such uncertainties as incomplete, imprecise and missing information. The purpose of this paper is to develop a rigorous and systematic methodology, on the basis of the ER approach (Wang, Yang, & Xu, 2006b; Wang, Yang, Xu, & Chin, 2006a; Xu, Yang, & Wang, 2006; Yang & Singh, 1994; Yang & Sen, 1994; Yang, 2001; Yang & Xu, 2002; Yang & Xu, 2002; Yang, Wang, Xu, & Chin, 2006), for synthesizing various types of assessment information provided by a group of customers and multiple QFD team members, which is referred to as the evidential reasoning (ER) based QFD methodology, in order to handle various types of possible uncertainties that may occur in the implementation process of QFD. The proposed ER-based QFD methodology can be used to help the QFD team to prioritize design requirements with both customer wants and customers' preferences taken into account. It is capable of modeling various types of uncertainties using a unified belief structure in a pragmatic, rigorous, reliable, systematic, transparent and repeatable way.

The rest of the paper is organized as follows:in Section 2, we develop the ER-based QFD methodology and describe in detail its modeling mechanism and steps. The methodology is then verified and illustrated with a numerical example in Section 3. Comparisons with other QFD methodologies are provided in Section 4. The paper is concluded in Section 5.

2. The methodology

The ER approach developed for multiple attribute decision analysis (MADA) has found an increasing number of applications in recent years (Wang et al., 2006a; Wang et al., 2006b; Xu et al., 2006; Yang & Singh, 1994; Yang & Sen, 1994; Yang, 2001; Yang & Xu, 2002; Yang & Xu, 2002; Yang et al., 2006). In this section, we develop an ER-based QFD methodology to deal with various types of uncertainties in QFD. The methodology allows customers and QFD team members to express their subjective judgments using belief structures developed on the basis of the theory of evidence (Shafer, 1976). It also allows customer wants to be aggregated in a rigorous yet nonlinear rather than linear manner. The methodology is described in detail as follows.

2.1. Modeling subjective judgments using belief structures

QFD begins with the identification of customer requirements and their mapping into relevant engineering design requirements, as shown in Fig. 1, where $CR_1 \sim CR_m$ are the m identified customer wants (WHATs), $DR_1 \sim DR_n$ are the n relevant engineering design requirements (HOWs), $w_1 \sim w_m$ are the relative weights (also called the degrees of importance) of the customer wants with $\sum_{i=1}^{m} w_i = 1$ and $w_i > 0$ for $i = 1,\ldots,m$, $R = (R_{ij})_{m\times n}$ is the relationship matrix between WHATs and HOWs, and $r = (r_{jk})_{n\times n}$ is the interrelationship matrix (also called correlation matrix) between HOWs, satisfying $r_{jk} \equiv r_{kj}$ for $j, k = 1,\ldots,n$. The figure looks similar to a house and is thus often referred to as the house of quality (HOQ).

2.1.1. Modeling the relative importance of customer wants

The relative importance of the identified customer wants is usually assessed by customers rather than by QFD team members because only customers know what they really want and what are more important or less important to them. Suppose there are L customers selected for assessing the relative importance of the m customer wants, each customer associated with a relative weight $\lambda_l > 0$ ($l = 1,\ldots,L$) with $\sum_{l=1}^{L} \lambda_l = 1$.

To help the customers express their opinions on the relative importance of the customer wants, rating scales can be defined and adopted. Table 1 shows one of the possible rating scale definitions. Other rating scales can also be defined. It is not our purpose to explore which rating scale is the best or more appropriate for a specific situation, which is beyond the scope of this paper. Our purpose is to provide a pragmatic yet simple way to elicit customers' preferences and produce an estimate for the relative importance of the customer wants.

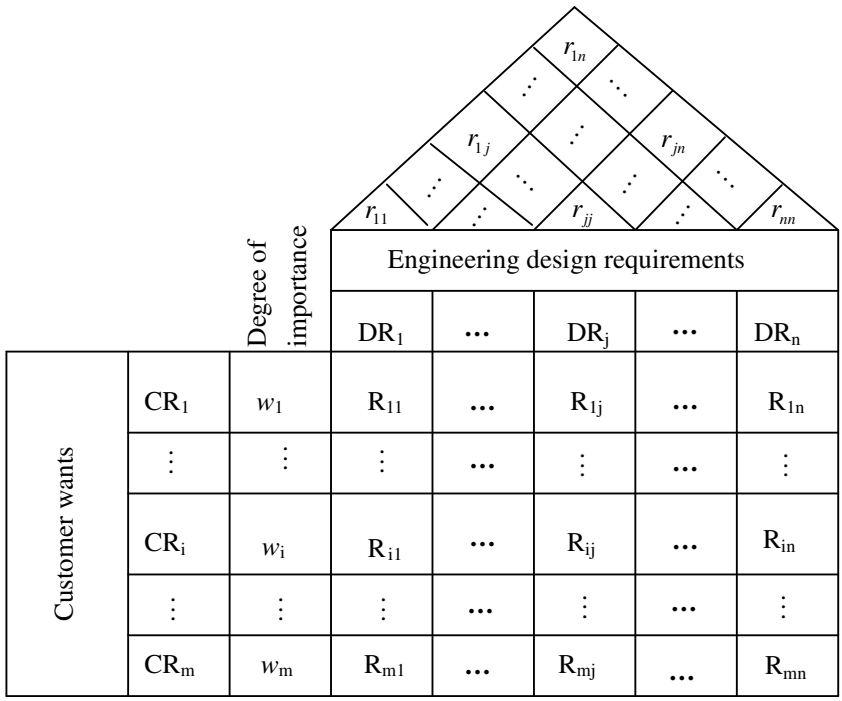

Fig. 1. The house of quality in QFD.

Table 1
Rating scales for relative importance, relationship and interrelationship assessments

Rating	Definition for		
	Relative importance	Relationship matrix	Interrelationship matrix
9	Extremely important	Very strong relationship	Very strong positive correlation
7	Very important	Strong relationship	Strong positive correlation
5	Moderately important	Moderate relationship	Moderate positive correlation
3	Weakly important	Weak relationship	Weakly positive correlation
1	Very weakly important	Very weak relationship	Very weakly positive correlation
0	Not important	No relationship	No correlation
−1	–	–	Very weakly negative correlation
−3	–	–	Weakly negative correlation
−5	–	–	Moderate negative correlation
−7	–	–	Strong negative correlation
−9	–	–	Very strong negative correlation

In terms of the defined rating scales, customers can express their opinions using belief structures. A belief structure is a distribution of probabilistic assessment. Take the assessment of CR_1 for example. Customer 1 may assess it to be very important to the belief degree of 60% and moderately important to the degree of 40%. Such an assessment can be modeled as {(7, 60%), (5, 40%)}, with the belief degrees added to 100%. If a customer rates the importance of CR_1 between very important and extremely important with a belief degree of 20% and moderately important to a belief degree of 70%, such an assessment can be modeled as {(7–9, 20%), (5, 70%)}, leading to a total belief degree of 90%. If the total belief degree of an assessment is less than 100%, the assessment is said to be incomplete; otherwise it is said to be complete. Note that the total belief degree cannot be larger than 100%; otherwise, the assessment makes no sense. For an incomplete assessment, the remaining belief degree represents the probability that has not been assigned to any rating, but it could be assigned to any one of the ratings or their combinations by the theory of evidence (Shafer, 1976). Therefore, the remaining belief degree could be assigned to any of the rating scale 0–9. If a customer has no idea about the relative importance of a customer want or cannot provide any information about the assessment, such an assessment is called total ignorance and can be characterized by the belief structure {(0–9, 100%)}.

For a given belief structure, it can be characterized by an expected score no matter whether it is complete or not. For example, the above three belief structures can be characterized by expected scores in the following way:

$$\{(7,60\%),(5,40\%)\} \Rightarrow 7\times 60\% + 5\times 40\% = 6.2,$$
$$\{(7\text{–}9,20\%),(5,70\%)\} \Rightarrow [7\text{–}9]\times 20\% + 5\times 70\% + [0\text{–}9]\times 10\% = 4.9\text{–}6.2,$$
$$\{(0\text{–}9,100\%)\} \Rightarrow [0\text{–}9]\times 100\% = 0\text{–}9,$$

where the 10% in the second belief structure is the remaining belief degree which could be assigned to any rating of 0–9.

Let $E(S_i^{(l)})$ be the expected score obtained from the belief structure of Customer l in assessing the relative importance of customer want CR_i. The total expected score for the relative importance of CR_i can be expressed as the weighted sum of the expected scores of the L customers. That is

$$E(S_i) = \sum_{l=1}^{L} \lambda_l E(S_i^{(l)}), \quad i = 1,\ldots,m, \tag{1}$$

where λ_l is the relative weight of Customer l. Based on Eq. (1), the relative importance of CR_i can be defined as

$$w_i = \frac{E(S_i)}{\sum_{k=1}^{m} E(S_k)}, \quad i = 1,\ldots,m. \tag{2}$$

In the case that some $E(S_i)$ is interval, w_i (i = 1,…,m) will be normalized intervals satisfying $\sum_{i=1}^{m} w_i = 1$.

2.1.2. Modeling the relationship matrix between WHATs and HOWs

The relationship matrix between WHATs and HOWs reflects the impact of the fulfillment of HOWs on the satisfaction of WHATs. The matrix should be developed by QFD team members. To help the team members assess the relationships and express their opinions, ratings such as 1–3–9 or 1–5–9 may be used to denote weak, medium and strong relationships between WHATs and HOWs. In this paper, rating scale 0–9 is defined to characterize different strengths of the relationships between WHATs and HOWs, as shown in the third column of Table 1.

Different from the traditional QFD, which requires the team members to provide a consensus assessment for each relationship, the ER-based QFD methodology allows the team members to express their opinions using belief structures individually and independently. Each belief structure may be complete or incomplete, precise or imprecise. Take the assessment of the relationship R_{11} for example. A team member may provide a belief structure of {(9, 60%), (7, 40%)}, which is a complete assessment and represents that R_{11} is assessed by the team member as very strong relationship to a belief degree of 60% and strong relationship to a degree of 40%. If a team member assesses R_{11} to be at least moderate relationship with a belief degree of 80%, such an assessment can be modeled as {(5–9, 80%)}, which is an incomplete and imprecise belief structure. To sum up, QFD team members can express their opinions freely, truly and independently.

Suppose there are M team members and each of them is assigned a weight $\theta_l > 0$ ($l = 1,\ldots,M$) with $\sum_{l=1}^{M}\theta_l = 1$. Let $\{(H_{pq}, \beta_{pq}^{(l)}), p = 0,\ldots,N; q = p,\ldots,N\}$ be the belief structure provided by team member l on the assessment of relationship R_{ij}, where H_{pp} for $p = 0$ to N are the crisp ratings defined for relationship assessment, H_{pq} for $p = 0$ to N and $q = p + 1$ to N are intervals between H_{pp} and H_{qq}, and $\beta_{pq}^{(l)}$ are the belief degrees to which the relationship R_{ij} is assessed to interval rating H_{pq}. For the rating scale 0–9 defined in Table 1, we have six crisp ratings inclusive of zero, which are 0, 1, 3, 5, 7, 9, and fifteen possible intervals that are 0–1, 0–3, 0–5, 0–7, 0–9, 1–3, 1–5, 1–7, 1–9, 3–5, 3–7, 3–9, 5–7, 5–9 and 7-9. Therefore, we have $N = 5$ and

$$H = \begin{Bmatrix} H_{00} & H_{01} & H_{02} & H_{03} & H_{04} & H_{05} \\ & H_{11} & H_{12} & H_{13} & H_{14} & H_{15} \\ & & H_{22} & H_{23} & H_{24} & H_{25} \\ & & & H_{33} & H_{34} & H_{35} \\ & & & & H_{44} & H_{45} \\ & & & & & H_{55} \end{Bmatrix} = \begin{Bmatrix} 0 & 0-1 & 0-3 & 0-5 & 0-7 & 0-9 \\ & 1 & 1-3 & 1-5 & 1-7 & 1-9 \\ & & 3 & 3-5 & 3-7 & 3-9 \\ & & & 5 & 5-7 & 5-9 \\ & & & & 7 & 7-9 \\ & & & & & 9 \end{Bmatrix}, \tag{3}$$

which constitutes a frame of discernment in the terminology of the theory of evidence. The collective assessment of the M team members for each relationship is also a belief structure, which is denoted as $\{(H_{pq}, \beta_{pq}), p = 0,\ldots,N;\ q = p,\ldots,N\}$ and determined by

$$\beta_{pq} = \sum_{l=1}^{M}\theta_l\beta_{pq}^{(l)}, \quad p = 0,\ldots,N;\ q = p,\ldots,N. \tag{4}$$

All the assessments for the relationships between WHATs and WHOs form a belief relationship matrix $R = (R_{ij})_{m\times n}$, where R_{ij} is characterized by a belief structure. The belief relationship matrix will be aggregated using the interval ER algorithm, which will be discussed later.

2.1.3. Modeling the interrelationship matrix between HOWs

The interrelationship matrix measures the interrelationships or called correlation relationships between HOWs and is also assessed by the QFD team members. To distinguish between positive and negative interrelationships between HOWs, positive and negative ratings are both adopted. Table 1 shows one of the possible rating scale definitions for the assessment of interrelationships. Note that the rating scales defined for the assessments of the relative importance of WHATs, relationship and interrelationship matrices may be different. The ER-based QFD methodology does not require them to be the same. In addition, if there are no negative correlation relationships, then negative ratings can be dropped from Table 1. As such, if there is no positive correlation relationship, positive ratings can be dropped as well.

Each QFD team member can assess the interrelationship matrix using belief structures independently and the collective assessment of the interrelationship matrix by the M team members is also a belief interrelationship matrix $r = (r_{jk})_{n\times n}$, where r_{jk} are belief structures determined by

$$\begin{aligned} r_{jk} &= \sum_{l=1}^{M}\theta_l r_{jk}^{(l)} = \sum_{l=1}^{M}\theta_l\{(H_{pq}, \alpha_{pq}^{(l)}), p = -N,\ldots,N; q = p,\ldots,N\} \\ &= \left\{\left(H_{pq}, \sum_{l=1}^{M}\theta_l\alpha_{pq}^{(l)}\right), p = -N,\ldots,N; q = p,\ldots,N\right\}, \quad j,k \\ &= 1,\ldots,n, \end{aligned} \tag{5}$$

in which $r_{jk}^{(l)} = \{(H_{pq}, \alpha_{pq}^{(l)}), p = -N,\ldots,N; q = p,\ldots,N\}$ is the belief structure on r_{jk} provided by team member l and $\alpha_{pq}^{(l)}$ is the belief degree to which r_{jk} is assessed to the interval H_{pq}. Due to the fact that each design requirement is always very strongly positively correlated to itself, r_{jj} is thus always identical to {(9, 100%)} for any $j = 1,\ldots,n$.

The above belief interrelationship matrix is then converted into an expected score matrix $E(r) = (E(r_{jk}))_{n\times n}$, where $E(r_{jk}) = \sum_{p=-N}^{N}\sum_{q=p}^{N}\sum_{l=1}^{M}\theta_l\alpha_{pq}^{(l)}H_{pq}$ is the expected score of belief structure r_{jk} and can be computed using interval arithmetic.

Not that the expected score matrix can also be generated by first transforming the belief structures provided by the team members into the expected scores and then weighting them together. The result will be the same.

2.2. Aggregating the belief relationship matrix using the interval ER algorithm

Different from the existing QFD methodologies which utilize the weighted sum or fuzzy weighted average as the ratings of the technical importance of HOWs, the ER-based QFD methodology provides a systematic yet rigorous way of aggregating the relationships between WHATs and HOWs. The aggregation is based on the belief relationship matrix and the combination rule of the Dempster–Shafer theory of evidence (Shafer, 1976). Different ER algorithms have been developed to handle different types of belief structures and provide flexibility for their aggregation. If the belief structures to be aggregated contain no interval ratings, or $\beta_{pq} = 0$ for $p \neq q$, then a recursive or analytical ER algorithm can be adopted (Wang et al., 2006b; Yang, 2001; Yang & Xu, 2002). In this paper, the interval ER algorithm (Xu et al., 2006) will be employed to aggregate the belief relationship matrix because the belief structures in the matrix may contain interval ratings such as 7–9, 5–7, 0–9 and so on. The interval ER algorithm includes the original recursive ER algorithm as a special case and is also carried out recursively.

Let $R_{i_1j} = \{(H_{pq}, \beta_{pq}(R_{i_1j})), p = 0,\ldots,N; q = p,\ldots,N\}$ and $R_{i_2j} = \{(H_{pq}, \beta_{pq}(R_{i_2j})),\ p = 0,\ldots,N;\ q = p,\ldots,N\}$ be two belief structures which characterize the relationships between the customer

wants CR_{i_1} and CR_{i_2} and the design requirement DR_j, respectively, and w_{i_1} and w_{i_2} be the normalized weights for CR_{i_1} and CR_{i_2}. The interval ER algorithm first transforms the belief structures into basic probability masses by considering their weights and using the equations below:

$$m_{pq} = w_{i_1}\beta_{pq}(R_{i_1j}), \quad p = 0,\ldots,N;\ q = p,\ldots,N, \tag{6}$$

$$m_H = 1 - \sum_{p=0}^{N}\sum_{q=p}^{N} w_{i_1}\beta_{pq}(R_{i_1j}) = 1 - w_{i_1}\sum_{p=0}^{N}\sum_{q=p}^{N}\beta_{pq}(R_{i_1j}) = 1 - w_{i_1}, \tag{7}$$

$$n_{pq} = w_{i_2}\beta_{pq}(R_{i_2j}), \quad p = 0,\ldots,N;\ q = p,\ldots,N, \tag{8}$$

$$n_H = 1 - \sum_{p=0}^{N}\sum_{q=p}^{N} w_{i_2}\beta_{pq}(R_{i_2j}) = 1 - w_{i_2}. \tag{9}$$

The above probability masses are viewed as two pieces of evidence and combined to produce a set of joint probability masses: c_{pq} ($p = 0,\ldots,N$; $q = p,\ldots,N$) and c_H, which are computed by the following equations:

$$c_{pq} = \frac{1}{1-K}\left[\sum_{s=0}^{p}\sum_{t=q}^{N}(m_{st}n_{pq} + m_{pq}n_{st}) + \sum_{s=0}^{p-1}\sum_{t=q+1}^{N}(m_{sq}n_{pt} + m_{pt}n_{sq})\right] + \frac{1}{1-K}\left[m_H n_{pq} + m_{pq}n_H - m_{pq}n_{pq}\right], \tag{10}$$

$$c_H = \frac{m_H n_H}{1-K}, \tag{11}$$

$$K = \sum_{p=0}^{N}\sum_{q=p}^{N}\sum_{s=0}^{p-1}\sum_{t=s}^{p-1}(m_{st}n_{pq} + m_{pq}n_{st}), \tag{12}$$

where the summing up process $\sum_{h=h_1}^{h_2} f(h)$ will not be carried out if $h_1 > h_2$. That is to say, $\sum_{h=h_1}^{h_2} f(h) = 0$ for $h_1 > h_2$. The combined probability masses are then combined further with the basic probability masses transformed from another belief structure in the same column of the belief relationship matrix. The above aggregation process is carried out recursively until the m belief structures in the same column are all aggregated. Such a recursive process is easy to implement on a Microsoft Excel worksheet. In Appendix A, Table 11 shows how three pieces of evidence can be recursively combined on a Microsoft Excel worksheet. If there are more pieces of evidence, they can be recursively combined in the same way.

Let x_{pq} ($p = 0,\ldots,N$; $q = p,\ldots,N$) and x_H be the final combined probability masses. Then the overall assessment for DR_j will be $\{(H_{pq}, \delta_{pq}), p = 0,\ldots,N;\ q = p,\ldots,N\}$, which is an aggregated belief structure and δ_{pq} are determined by

$$\delta_{pq} = \frac{x_{pq}}{1-x_H}, \quad p = 0,\ldots,N;\ q = p,\ldots,N. \tag{13}$$

The overall assessment can finally be characterized by an expected interval:

$$E(DR_j) = \sum_{p=0}^{N}\sum_{q=p}^{N}\delta_{pq}H_{pq} = \left[\sum_{p=0}^{N}\sum_{q=p}^{N}\delta_{pq}H_p, \sum_{p=0}^{N}\sum_{q=p}^{N}\delta_{pq}H_q\right], \tag{14}$$

which represents the interval rating $[E_j^L, E_j^U]$ of the technical importance of design requirement DR_j. All the other design requirements can be rated in the same way.

2.3. Optimizing the technical importance ratings of HOWs

The interval ER algorithm requires the weights of customer wants to be given precisely and normalized, i.e. $\sum_{i=1}^{m} w_i = 1$ with $w_i > 0$ ($i = 1,\ldots,m$). In this case, the technical importance ratings of the HOWs can be directly determined by Eq. (14). However, if the weights of customer wants cannot be precisely determined by the selected customers, they will be imprecise and uncertain. This is often the case in QFD applications. In the previous Section 2.1.1, we described an approach for assessing the relative importance of customer wants. If one or more customers provide an incomplete assessment or interval belief structure, then the final weights will be intervals determined by Eqs. (1) and (2). In this case, the technical importance ratings cannot be uniquely determined by Eq. (14) since precise weights are not known. They have to be optimized by solving the following pair of preference programming models:

$$\text{Minimize} \quad E_j^L = \sum_{p=0}^{N}\sum_{q=p}^{N}\delta_{pq}H_p \tag{15}$$
$$\text{Subject to} \quad w_i^L \leqslant w_i \leqslant w_i^U, \quad i = 1,\ldots,m, \quad \sum_{i=1}^{m} w_i = 1,$$

$$\text{Maximize} \quad E_j^U = \sum_{p=0}^{N}\sum_{q=p}^{N}\delta_{pq}H_q \tag{16}$$
$$\text{Subject to} \quad w_i^L \leqslant w_i \leqslant w_i^U, \quad i = 1,\ldots,m, \quad \sum_{i=1}^{m} w_i = 1,$$

where δ_{pq} are determined by Eq. (13) and are functions of $w_1,\ldots,w_m$. To solve the models, δ_{pq} need to be written out recursively. If the interval ER algorithm discussed in Section 2.2 is implemented on Microsoft Excel worksheets, then the above pair of models can be solved on the same worksheet by Excel Solver without the need to write the expressions of δ_{pq} separately. By solving the above pair of models for each design requirement, the technical importance ratings for all the design requirements can be generated. However, such ratings still need to be incorporated further with the interrelationship matrix between HOWs into the final ratings of technical importance.

2.4. Incorporating the interrelationship matrix into the final ratings of technical importance

Existing QFD methodologies either take no account of the interrelationships between HOWs (Büyüközkan & Feyzioğlu, 2005; Chan & Wu, 2005; Chen et al., 2006) or consider them at the very beginning (Wasserman, 1993). The most commonly used approach is to incorporate the impact of interrelationships into the relationship matrix between WHATs and HOWs and modify its elements by the equation below:

$$R'_{ij} = \sum_{k=1}^{n} R_{ik}r_{kj}, \quad i = 1,\ldots,m;\ j = 1,\ldots,n, \tag{17}$$

where R'_{ij} is the adjusted relationship strength between CR_i and DR_j and r_{kj} is the interrelationship between DR_k and DR_j. Based upon Eq. (17), the technical importance ratings of DR_j can be obtained as

$$TR'_j = \sum_{i=1}^{m} w_i R'_{ij} = \sum_{i=1}^{m} w_i\left(\sum_{k=1}^{n} R_{ik}r_{kj}\right) = \sum_{k=1}^{n}\left(\sum_{i=1}^{n} w_i R_{ik}\right)r_{kj} = \sum_{k=1}^{n} TR_k r_{kj}, \quad j = 1,\ldots,n, \tag{18}$$

where TR'_j and TR_j are respectively the technical importance ratings with and without the consideration of the interrelationships between HOWs. As can be seen from (18), the interrelationships can also be considered at the end of computational process after the initial technical importance ratings TR_k ($k = 1,\ldots,n$) are all obtained.

The ER-based QFD methodology considers the influence of the interrelationship matrix on the final technical importance ratings along this guideline.

Let $E(\mathrm{DR}_j) = [E_j^L, E_j^U]$ ($j = 1,\ldots,n$) be the initial technical importance ratings obtained by solving models (15) and (16) and $E(r) = (E(r_{ij}))_{n\times n}$ be the expected score matrix obtained in Section 2.1.3. Obviously, the initial ratings $E(\mathrm{DR}_j)$ for $j = 1$ to n have not considered the interrelationships between HOWs. After considering the interrelationship matrix characterized by the expected score matrix $E(r) = (E(r_{ij}))_{n\times n}$, the final technical importance ratings can be computed by Eq. (18) as

$$\mathrm{TR}'_j = \sum_{k=1}^{n} E(\mathrm{DR}_k)E(r_{kj}), \quad j = 1,\ldots,n. \tag{19}$$

2.5. Normalizing and ranking technical importance ratings

The technical importance ratings determined by Eq. (19) are usually intervals due to the presence of uncertainty in subjective judgments, which are non-normalized and can be normalized by the following equation:

$$\begin{aligned}\mathrm{TR}_j &= \frac{\mathrm{TR}'_j}{\sum_{i=1}^{n}\mathrm{TR}'_i} \\ &= \left[\frac{(\mathrm{TR}'_j)^L}{(\mathrm{TR}'_j)^L + \sum_{i\neq j}(\mathrm{TR}'_i)^U}, \frac{(\mathrm{TR}'_j)^U}{(\mathrm{TR}'_j)^U + \sum_{i\neq j}(\mathrm{TR}'_i)^L}\right], \quad j \\ &= 1,\ldots,n,\end{aligned} \tag{20}$$

where $(\mathrm{TR}'_j)^L$ and $(\mathrm{TR}'_j)^U$ are the lower and upper bounds of TR'_j, respectively.

The normalized technical importance ratings can then be utilized to determine the priority of each design requirement or maximize customer satisfaction. To prioritize the design requirements, one way is to compare the average value (i.e. midpoint value) of each interval rating to generate an average ranking order with a degree of preference indicating the extent to which one interval rating is on average preferred to another, though in general this does not provide an absolute ranking order for the design requirements. The degree of preference is calculated using the equation below developed by Wang, Yang, and Xu (2005):

$$P(a > b) = \frac{\max(0, a_2 - b_1) - \max(0, a_1 - b_2)}{(a_2 - a_1) + (b_2 - b_1)}, \tag{21}$$

where $a = [a_1, a_2]$ and $b = [b_1, b_2]$ are two positive interval numbers.

3. An illustrative example

In this section, we present an illustrative example to show how the ER-based QFD methodology can be used to model uncertainty in QFD. The example is adapted from a classic QFD example about a hypothetical writing instrument (Wasserman, 1993), where *easy to hold*, *does not smear*, *point lasts*, and *does not roll* are the four identified important customer wants based on a market survey, and *length of pencil*, *time between sharpening*, *lead dust generated*, *hexagonality*, and *minimal erasure residue* are the five important engineering design requirements. Different from reference (Wasserman, 1993) which assumes that the relative importance of customer wants is known precisely and the relationship and interrelationship matrices are only judged by one team member or a design team with consensus opinion, this paper assumes that the relative importance of the four customer wants will be assessed by multiple (e.g. five) equally important customers and the relationship and interrelationship matrices will be evaluated by multiple (e.g. four) team members of different importance.

For illustration purpose, the rating scale defined in Table 1 is used in this numerical study. Table 2 shows the assessment information on the relative importance of the four customer wants provided by five hypothetical customers, from which it can be seen that except for customer 1 who is assumed to provide precise assessments, the other four customers are all assumed to give imprecise assessments. More specifically, customers 2 and 4 provide complete assessments, whilst customers 3 and 5 offer incomplete assessments. In particular, customer 5 provides nothing about the relative importance of CR_1 and CR_4.

According to the method described in Section 2.1.1, all the belief structures in Table 2 can be represented by expected scores, as shown in Table 3. The expected scores obtained from the belief structures provided by the five customers are then weighted and averaged by Eq. (1). Since the five customers are assumed to be equally important, their relative weights are therefore determined as $\lambda_1 = \lambda_2 = \cdots = \lambda_5 = 1/5$. The weighted and averaged results are presented in the last but one column of Table 3 and are finally normalized by Eqs. (2) and (20) to generate the relative importance weights of the customer wants. The normalized weights of the four customer wants are shown in the last column of Table 3 and will be used later to rate the technical importance of the five design requirements.

Tables 4 and 5 show the assessment information provided by four hypothetical design team members on the relationships between the four customer wants and the five engineering design requirements and the interrelationships between the five engineering design requirements, respectively. The four team members (TM_1–TM_4) are assumed to be of different importance. The weights

Table 2
Assessment information on the relative importance of four customer wants

Customer wants (CRs)	Importance rating				
	Customer 1 (20%)	Customer 2 (20%)	Customer 3 (20%)	Customer 4 (20%)	Customer 5 (20%)
Easy to hold (CR_1)	3	3:80% 5:20%	3:90%	3–5	Unknown
Does not smear (CR_2)	5	5:70% 7:30%	5	5–7	5
Point lasts (CR_3)	9	9:90% 7:10%	9	7–9	9
Does not roll (CR_4)	3	3:80% 5:20%	3:90%	3–5	Unknown

Table 3
Expected ratings for the relative importance of the four customer wants

CRs	Expected ratings obtained from the five customers (CMs)					Weighted average rating	Normalized expected rating
	CM_1 (20%)	CM_2 (20%)	CM_3 (20%)	CM_4 (20%)	CM_5 (20%)		
CR_1	3	$3\times 80\% + 5\times 20\%$	$3\times 90\% + [0–9]\times 10\%$	3–5	0–9	2.42–4.80	0.11–0.23
CR_2	5	$5\times 70\% + 7\times 30\%$	5	5–7	5	5.12–5.52	0.22–0.29
CR_3	9	$9\times 90\% + 7\times 10\%$	9	7–9	9	8.56–8.96	0.36–0.47
CR_4	3	$3\times 80\% + 5\times 20\%$	$3\times 90\% + [0–9]\times 10\%$	3–5	0–9	2.42–4.80	0.11–0.23

Table 4
Assessments on the relationships between the four customer wants and five design requirements

Customer wants		Engineering design requirements				
		Length of pencil (DR_1)	Time between sharpening (DR_2)	Lead dust generated (DR_3)	Hexagonality (DR_4)	Minimal erasure residue (DR_5)
CR_1	TM_1 (20%)	3:80% 5:20%	0	0	7–9:90% 5–7:10%	0
	TM_2(30%)	3:75%	0	0	9	0
	TM_3 (30%)	Unknown	0	0	9	0
	TM_4 (20%)	3:80%	0	0	9	0
CR_2	TM_1 (20%)	0	3	7–9:80%	0	7–9
	TM_2 (30%)	0	3	7–9:90%	0	9:80% 7:20%
	TM_3 (30%)	0	3	7–9:80%	0	9
	TM_4 (20%)	0	3:80% 1:20%	Unknown	0	9
CR_3	TM_1 (20%)	1	3	9	0	9:80%
	TM_2 (30%)	1	3:80%	9	0	7–9:75% 5–7:25%
	TM_3 (30%)	1	3:60% 1:40%	9	0	9:80%
	TM_4 (20%)	1	3	7–9:75% 5:25%	0	9:80%
CR_4	TM_1 (20%)	Unknown	0	0	9:80%	0
	TM_2 (30%)	1	0	0	9	0
	TM_3 (30%)	1	0	0	9:90% 7:10%	0
	TM_4 (20%)	1	0	0	7–9:85% 5–7:15%	0

Table 5
Assessments on the interrelationships between the five engineering design requirements

Engineering design requirements		DR_1	DR_2	DR_3	DR_4	DR_5
DR_1	TM_1 (20%)	9	0	0	0	0
	TM_2 (30%)	9	0	0	0	0
	TM_3 (30%)	9	0	0	0	0
	TM_4 (20%)	9	0	0	0	0
DR_2	TM_1 (20%)	0	9	3–5	0	3–5
	TM_2 (30%)	0	9	5	0	5
	TM_3(30%)	0	9	3	0	5
	TM_4 (20%)	0	9	5–7	0	3
DR_3	TM_1 (20%)	0	3–5	9	0	7:80% 9:20%
	TM_2 (30%)	0	5	9	0	9
	TM_3 (30%)	0	3	9	0	9
	TM_4 (20%)	0	5–7	9	0	7
DR_4	TM_1 (20%)	0	0	0	9	0
	TM_2 (30%)	0	0	0	9	0
	TM_3 (30%)	0	0	0	9	0
	TM_4(20%)	0	0	0	9	0
DR_5	TM_1 (20%)	0	3–5	7:80% 9:20%	0	9
	TM_2 (30%)	0	5	9	0	9
	TM_3 (30%)	0	5	9	0	9
	TM_4 (20%)	0	3	7	0	9

of the four team members are given in the parentheses after the team members' names.

Each assessment in Tables 4, 5 provided by the four team members is a belief structure and needs to be weighted and averaged by Eqs. (4) and (5) to generate a collective assessment for each relationship and interrelationship. Tables 6 and 7 show the collective assessment results for the relationships and interrelationships, which form the belief relationship matrix and the belief interrelationship matrix, respectively. Note that the belief degrees assigned to the rating interval 0–9 in Table 6 represent "ignorance" information. That is to say, they have not been assigned to any ratings by the four team members.

Table 6
Belief relationship matrix between the four customer wants and the five design requirements

Customer wants	Engineering design requirements				
	DR_1	DR_2	DR_3	DR_4	DR_5
CR_1	3:54.5% 5:4% 0–9:41.5%	0	0	9:80% 5–7:2% 7–9:18%	0
CR_2	0	1:4% 3:96%	7–9:67% 0–9:0.33%	0	7:6% 9:74% 7–9:20%
CR_3	1	3:82% 1:12% 0–9:6%	5:5% 9:80% 7–9:15%	0	9:56% 5–7:7.5% 7–9:22.5% 0–9:14%
CR_4	1:80% 0–9:20%	0	0	9:73% 7:3% 5–7:3% 7–9:17% 0–9:4%	0

Table 7
Belief interrelationship matrix between the five design requirements

Engineering design requirements	DR_1	DR_2	DR_3	DR_4	DR_5
DR_1	9	0	0	0	0
DR_2	0	9	3:30% 5:30% 3–5:20% 5–7:20%	0	3:20% 5:60% 3–5:20%
DR_3	0	3:30% 5:30% 3–5:20% 5–7:20%	9	0	7:36% 9:64%
DR_4	0	0	0	9	0
DR_5	0	3:20% 5:60% 3–5:20%	7:36% 9:64%	0	9

Table 8
Technical importance ratings without considering the correlations between design requirements

Engineering design requirements		DR_1	DR_2	DR_3	DR_4	DR_5
Technical importance ratings	Inf	0.7614	1.6702	4.4806	1.2383	4.6236
	Suf	1.8750	2.5541	7.3924	3.3172	7.3831

Table 9
Expected score correlation matrix transformed from Table 7

Engineering design requirements	DR_1	DR_2	DR_3	DR_4	DR_5
DR_1	9	0	0	0	0
DR_2	0	9	4–4.8	0	4.2–4.6
DR_3	0	4–4.8	9	0	8.28
DR_4	0	0	0	9	0
DR_5	0	4.2–4.6	8.28	0	9

For the belief relationship matrix, by implementing the interval ER algorithm, Eqs. (6)–(14), and optimizing preference programming models (15) and (16) for each design requirement, we obtain the initial technical importance ratings of the five design requirements, which are shown in Table 8, where inf and suf represent the lower and upper bounds, respectively.

To consider further the impact of the interrelationships between design requirements on the initial technical importance ratings, we first transform the belief interrelationship matrix in Table 7 into the expected score matrix by the method described in Section 2.1.3, as shown in Table 9, then calculate the final technical importance ratings by Eq. (19), as shown in Table 10, and finally normalize them by Eq. (20). The normalized technical importance ratings are also shown in Table 10 and visualized in Fig. 2, from which the ranking order of the five design requirements can be generated as $DR_5 \overset{50.04\%}{\succ} DR_3 \overset{78.05\%}{\succ} DR_2 \overset{100\%}{\succ} DR_4 \overset{71.41\%}{\succ} DR_1$, where the degrees of preference are computed by Eq. (21). It is clear that DR_5 and DR_3 are nearly indifferent from each other and therefore can be seen as equally important.

Table 10
The final technical importance ratings considering the interrelationships between design requirements and their ranking order

Engineering design requirements		DR_1	DR_2	DR_3	DR_4	DR_5
Technical importance ratings	Inf	6.8522	52.3737	85.2899	11.1444	85.7270
	Suf	16.8747	92.4324	139.9226	29.8549	139.4051
Normalized technical importance ratings	Inf	0.0168	0.1384	0.2344	0.0279	0.2350
	Suf	0.0671	0.3284	0.4727	0.1148	0.4725
	Average	0.0419	0.2334	0.3535	0.0713	0.3537
Ranking order		5	3	2	4	1

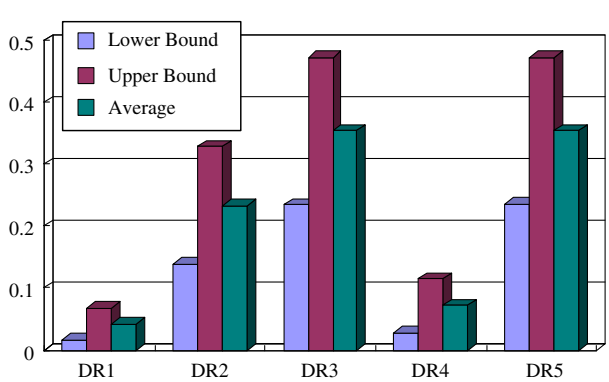

Fig. 2. Normalized technical importance ratings of the five design requirements.

4. Comparisons with other QFD methods

According to our literature review in Section 1, there have been no QFD methods so far that can be used for dealing with incomplete, imprecise and ignorance information in QFD. So, from the methodological point of view, the ER-based QFD methodology has significant advantages over existing QFD methods in modeling uncertainties in QFD associated with incompleteness, imprecision and ignorance.

To provide further numerical comparisons between the ER-based QFD methodology and other QFD methods, we remove uncertainties from the numerical example in Section 3 and reconvert it into a deterministic QFD example. Tables 11, and 12 show, respectively a deterministic relationship matrix between the four

Table 11
Deterministic relationship matrix between the four customer wants and the five design requirements

Customer wants	Engineering design requirements				
	DR_1	DR_2	DR_3	DR_4	DR_5
CR_1	3	0	0	9	0
CR_2	0	3	5	0	9
CR_3	1	3	5	0	9
CR_4	1	0	0	9	0

Table 12
Deterministic interrelationship matrix between the five design requirements

Engineering design requirements	DR_1	DR_2	DR_3	DR_4	DR_5
DR_1	9	0	0	0	0
DR_2	0	9	3	0	3
DR_3	0	3	9	0	9
DR_4	0	0	0	9	0
DR_5	0	3	9	0	9

Table 13
Belief structures for the deterministic relationship matrix in Table 11

Customer wants	Engineering design requirements				
	DR_1	DR_2	DR_3	DR_4	DR_5
CR_1	{(3, 1.0)}	{(0, 1.0)}	{(0, 1.0)}	{(9, 1.0)}	{(0, 1.0)}
CR_2	{(0, 1.0)}	{(3, 1.0)}	{(5, 1.0)}	{(0, 1.0)}	{(9, 1.0)}
CR_3	{(1, 1.0)}	{(3, 1.0)}	{(5, 1.0)}	{(0, 1.0)}	{(9, 1.0)}
CR_4	{(1, 1.0)}	{(0, 1.0)}	{(0, 1.0)}	{(9, 1.0)}	{(0, 1.0)}

Table 14
Technical importance ratings of the five design requirements and their ranking orders

	Methods	DR_1	DR_2	DR_3	DR_4	DR_5
QFD	Rating	6.2%	13.1%	28.0%	24.5%	28.0%
	Ranking	5	4	1	3	1
ER-based QFD	Initial rating (IR)	1.0119	2.3628	3.9380	1.9116	7.0884
	Final rating (FR)	9.1070	54.3446	106.3264	17.2042	106.3264
	Normalized FR	3.1%	18.5%	36.3%	5.9%	36.3%
	Ranking	5	3	1	4	1

Table 15
Recursive combination of three pieces of evidence

$\mathbf{m} \oplus \mathbf{n}$			Evidence: $\boldsymbol{m}$								
			H_{00}	…	H_{0N}	H_{11}	…	H_{1N}	…	H_{NN}	H
			m_{00}	…	m_{0N}	m_{11}	…	m_{1N}	…	m_{NN}	m_H
Evidence: $\mathbf{n}$	H_{00}	n_{00}	$n_{00}m_{00}$ $\{H_{00}\}$	…	$n_{00}m_{0N}$ $\{H_{00}\}$	$n_{00}m_{11}$ $\{\Phi\}$	…	$n_{00}m_{1N}$ $\{\Phi\}$	…	$n_{00}m_{NN}$ $\{\Phi\}$	$n_{00}m_H$ $\{H_{00}\}$
	⋮	⋮	⋮	…	⋮	⋮	…	⋮	…	⋮	⋮
	H_{0N}	n_{0N}	$n_{0N}m_{00}$ $\{H_{00}\}$	…	$n_{0N}m_{0N}$ $\{H_{0N}\}$	$n_{0N}m_{11}$ $\{H_{11}\}$	…	$n_{0N}m_{1N}$ $\{H_{1N}\}$	…	$n_{0N}m_{NN}$ $\{H_{NN}\}$	$n_{0N}m_H$ $\{H_{0N}\}$
	H_{11}	n_{11}	$n_{11}m_{00}$ $\{\Phi\}$	…	$n_{11}m_{0N}$ $\{H_{11}\}$	$n_{11}m_{11}$ $\{H_{11}\}$	…	$n_{11}m_{1N}$ $\{H_{11}\}$	…	$n_{11}m_{NN}$ $\{\Phi\}$	$n_{11}m_H$ $\{H_{11}\}$
	⋮	⋮	⋮	…	⋮	⋮	…	⋮	…	⋮	⋮
	H_{1N}	n_{1N}	$n_{1N}m_{00}$ $\{\Phi\}$	…	$n_{1N}m_{0N}$ $\{H_{1N}\}$	$n_{1N}m_{11}$ $\{H_{11}\}$	…	$n_{1N}m_{1N}$ $\{H_{1N}\}$	…	$n_{1N}m_{NN}$ $\{H_{NN}\}$	$n_{1N}m_H$ $\{H_{1N}\}$
	⋮	⋮	⋮	…	⋮	⋮	…	⋮	…	⋮	⋮
	H_{NN}	n_{NN}	$n_{NN}m_{00}$ $\{\Phi\}$	…	$n_{NN}m_{0N}$ $\{H_{NN}\}$	$n_{NN}m_{11}$ $\{\Phi\}$	…	$n_{NN}m_{1N}$ $\{H_{NN}\}$	…	$n_{NN}m_{NN}$ $\{H_{NN}\}$	$n_{NN}m_H$ $\{H_{NN}\}$
	H	n_H	n_Hm_{00} $\{H_{00}\}$	…	n_Hm_{0N} $\{H_{0N}\}$	n_Hm_{11} $\{H_{11}\}$	…	n_Hm_{1N} $\{H_{1N}\}$	…	n_Hm_{NN} $\{H_{NN}\}$	n_Hm_{NN} $\{H\}$
Non-normalized probability masses			Sum for $\{H_{00}\}$	…	Sum for $\{H_{0N}\}$	Sum for $\{H_{11}\}$	…	Sum for $\{H_{1N}\}$	…	Sum for $\{H_{NN}\}$	n_Hm_{NN} $\{H\}$
Normalized probability masses			c_{00}	…	c_{0N}	c_{11}	…	c_{1N}	…	c_{NN}	c_H
Evidence: $\mathbf{s}$	H_{00}	s_{00}	$s_{00}c_{00}$ $\{H_{00}\}$	…	$s_{00}c_{0N}$ $\{H_{00}\}$	$s_{00}c_{11}$ $\{\Phi\}$	…	$s_{00}c_{1N}$ $\{\Phi\}$	…	$s_{00}c_{NN}$ $\{\Phi\}$	$s_{00}c_H$ $\{H_{00}\}$
	⋮	⋮	⋮	…	⋮	⋮	…	⋮	…	⋮	⋮
	H_{0N}	s_{0N}	$s_{0N}c_{00}$ $\{H_{00}\}$	…	$s_{0N}c_{0N}$ $\{H_{0N}\}$	$s_{0N}c_{11}$ $\{H_{11}\}$	…	$s_{0N}c_{1N}$ $\{H_{1N}\}$	…	$s_{0N}c_{NN}$ $\{H_{NN}\}$	$s_{0N}c_H$ $\{H_{0N}\}$
	H_{11}	s_{11}	$s_{11}c_{00}$ $\{\Phi\}$	…	$s_{11}c_{0N}$ $\{H_{11}\}$	$s_{11}c_{11}$ $\{H_{11}\}$	…	$s_{11}c_{1N}$ $\{H_{11}\}$	…	$s_{11}c_{NN}$ $\{\Phi\}$	$s_{11}c_H$ $\{H_{11}\}$
	⋮	⋮	⋮	…	⋮	⋮	…	⋮	…	⋮	⋮
	H_{1N}	s_{1N}	$s_{1N}c_{00}$ $\{\Phi\}$	…	$s_{1N}c_{0N}$ $\{H_{1N}\}$	$s_{1N}c_{11}$ $\{H_{11}\}$	…	$s_{1N}c_{1N}$ $\{H_{1N}\}$	…	$s_{1N}c_{NN}$ $\{H_{NN}\}$	$s_{1N}c_H$ $\{H_{1N}\}$
	⋮	⋮	⋮	…	⋮	⋮	…	⋮	…	⋮	⋮
	H_{NN}	s_{NN}	$s_{NN}c_{00}$ $\{\Phi\}$	…	$s_{NN}c_{0N}$ $\{H_{NN}\}$	$s_{NN}c_{11}$ $\{\Phi\}$	…	$s_{NN}c_{1N}$ $\{H_{NN}\}$	…	$s_{NN}c_{NN}$ $\{H_{NN}\}$	$s_{NN}c_H$ $\{H_{NN}\}$
	H	s_H	s_Hc_{00} $\{H_{00}\}$	…	s_Hc_{0N} $\{H_{0N}\}$	s_Hc_{11} $\{H_{11}\}$	…	s_Hc_{1N} $\{H_{1N}\}$	…	s_Hc_{NN} $\{H_{NN}\}$	s_Hc_{NN} $\{H\}$
Non-normalized probability masses			Sum for $\{H_{00}\}$	…	Sum for $\{H_{0N}\}$	Sum for $\{H_{11}\}$	…	Sum for $\{H_{1N}\}$	…	Sum for $\{H_{NN}\}$	n_Hm_{NN} $\{H\}$
Normalized probability masses			x_{00}	…	x_{0N}	x_{11}	…	x_{1N}	…	x_{NN}	x_H
Aggregated belief degrees β_{pq}			$\frac{x_{00}}{1-x_H}$	…	$\frac{x_{0N}}{1-x_H}$	$\frac{x_{11}}{1-x_H}$	…	$\frac{x_{1N}}{1-x_H}$	…	$\frac{x_{NN}}{1-x_H}$	-

Note: Φ represents the empty/null set and the sum for $\{H_{pq}\}$ represents the sum of all the probability masses assigned to the rating $\{H_{pq}\}$, seeing $\{H_{00}\}$ for example, whose probability masses are highlighted and shaded.

customer wants and the five design requirements and a deterministic interrelationship matrix between the five design requirements. The relative weights of the four customer wants are reconverted to 15%, 25%, 45%, and 15%, which are also deterministic. In order to solve such a deterministic QFD problem using the ER-based QFD methodology, we first transform the deterministic relationship matrix in Table 11 into a belief relationship matrix, as shown in Table 13. Obviously, the deterministic relationship matrix in Table 11 is a special case of the belief relationship matrix described in Section 2.1.2. We then implement the interval ER algorithm to aggregate the belief relationships in Table 13 for each of the five design requirements. Since there are no interval ratings in this deterministic QFD example, the interval ER algorithm is reduced to an ordinary recursive ER algorithm (Xu et al., 2006; Yang & Xu, 2002). The aggregated results are shown in the fourth row of Table 14 and serve as the initial ratings (IR) of the technical importance of the five design requirements, which are further coupled with the deterministic interrelationship matrix in Table 12 by Eq. (18) or (19) to generate the final ratings (FR) of the technical importance of the five design requirements. The results are shown in the fifth row of Table 14 and further normalized to add up to one. For the purpose of comparison, we also show in Table 14 the technical importance ratings of the five design requirements obtained by Wasserman's method for prioritizing design requirements in QFD (Wasserman, 1993).

As can be seen from Table 14, both QFD (Wasserman's method) and the ER-based QFD methodology evaluate DR_3 and DR_5 to be the most important design requirements and of the same importance, while DR_1 to be the least important design requirement. The differences between the two methodologies mainly lie in the ranking order between DR_2 and DR_4. Wasserman's method ranks DR_2 ahead of DR_4, while the ER-based QFD methodology evaluates DR_2 to be less important than DR_4. The ranking orders between them are reversed. The reason for this is because the two methodologies aggregate the relationships between customer wants (WHATs) and engineering design requirements (HOWs) in different manners. Wasserman's method aggregates the relationships using a simple additive weighting method, which is a linear aggregation function of the relationships; while the interval ER algorithm in the ER-based QFD methodology proves to be highly nonlinear (Wang et al., 2006b; Yang & Xu, 2002). So, the ER-based QFD methodology aggregates the relationships in a nonlinear manner. It is very natural that different aggregation manners may lead to different results and ranking orders.

From Table 12 it can be observed that DR_1 and DR_4 are two independent design requirements, whose fulfillment has no impact on that of the other design requirements, while DR_2, DR_3 and DR_5 are three positively correlated design requirements, whose fulfillment is positively influenced by one another. In another word, the fulfillment of DR_2 can promote the fulfillment of the two most important design requirements DR_3 and DR_5. From this point of view, the fulfillment of DR_2 is of great benefit to the engineering design and it should be more important than DR_4, whose fulfillment has no impact on the fulfillment of any other design requirements.

Another slight difference between the two methodologies lies in the numerical values of the technical importance ratings of the five design requirements. That is, the most important design requirements have higher technical importance rating by the ER-based QFD methodology than that by Wasserman's method, and the least important design requirement has lower technical importance rating by the former than that by the latter. This is because the ER-based QFD methodology can highlight the most important and the least important design requirements very effectively.

Finally, we point out that the ER-based QFD methodology can be further extended to model fuzziness in QFD and offer some advantages over fuzzy QFD. However, this extension and comparison requires the use of a fuzzy ER algorithm instead of the interval ER algorithm, which is beyond the scope of the current paper and will not be further discussed here. The interested reader may refer to Yang et al. (2006) for more details on the fuzzy ER algorithm.

5. Conclusions

Uncertainty is inherent in human being' subjective judgments and needs to be taken into account properly in human decision making. In this paper, we developed an ER-based QFD methodology for dealing with various types of uncertainties such as incomplete, imprecise and missing information that may occur in the implementation process of QFD. The proposed methodology allows customers and QFD team members to express their opinions using a unified belief structure independently, can accommodate judgments that may be complete or incomplete, precise or imprecise like intervals, and can also handle situations where customers and QFD team members do not provide any assessment information either because they feel difficult to make proper judgment or simply do not want to make any judgment due to whatever reasons they may have. This provides desirable flexibility for customers and QFD team members to make their true judgments and freely express their opinions yet still within a rigorous and systematic framework.

A numerical example was also examined to illustrate the implementation process of the ER-based QFD methodology. It is shown that uncertainties associated with incompleteness, imprecision or ignorance can all be well modeled using the belief structures. The interval ER algorithm provides a rigorous, reliable and systematic way to aggregate multiple belief structures. Perhaps, a drawback to use the ER-based QFD methodology is its complexity. However, such a drawback can be overcome when the interval ER algorithm is implemented recursively on a Microsoft Excel worksheet.

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Appendix A. Recursive combination manner of multiple pieces of evidence

For the convenience of the readers to understand and implement the interval ER algorithm, we provide Table 15 to show how three pieces of evidence can be combined on a Microsoft Excel worksheet in a recursive way. If there are more pieces of evidence to be combined, they can be recursively combined in the same way.

In the above Table, the first two pieces of evidence $\boldsymbol{m}$ and $\boldsymbol{n}$ are first combined to produce a new piece of evidence $\boldsymbol{c}$, which is then combined with the piece of evidence $\boldsymbol{s}$ to generate the final evidence $\boldsymbol{x}$, where the piece of evidence $\boldsymbol{c}$ and $\boldsymbol{x}$ are respectively characterized by

$$\begin{aligned} c(H_{00}) &= c_{00}, \ldots, c(H_{0N}) = c_{0N}, c(H_{11}) = c_{11}, \ldots, c(H_{1N}) \\ &= c_{1N}, \ldots, c(H_{NN}) = c_{NN}, c(H) = c_H \end{aligned}$$

and

$$\begin{aligned} x(H_{00}) &= x_{00}, \ldots, x(H_{0N}) = x_{0N}, x(H_{11}) = x_{11}, \ldots, x(H_{1N}) \\ &= x_{1N}, \ldots, x(H_{NN}) = x_{NN}, x(H) = x_H. \end{aligned}$$

The final piece of evidence $\boldsymbol{x}$ is finally converted into a belief structure $\{(H_{pq}, \beta_{pq}), p = 0, \ldots, N; q = p, \ldots, N\}$ by using the formulas in the last row of the table.

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