# A HIERARCHICAL ANALYSIS MODEL FOR MULTIOBJECTIVE DECISIONMAKING

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Abstract. In this paper we attempt to propose a method for preference degree analysis in multiobjective decisionmaking problems(MDMPs), called the multiple factor analysis method based on a hierarchical analysis model. The concept of preference degree(PD) is first discussed. Here a PD is used as a measurement for describing preference information of decision makers (DMs) to some value of an objective. A hierarchical model for analyzing the preference information is then deduced by defining objective domain of discourse (ODD), fuzzy degree domain of discourse (FDDD), factor domain of discourse (FDD) and sensitivity vector. Furthermore, some methods are explored for analyzing DMs' PDs of objective values according to a single factor and multiple factors, respectively. At last, an example is briefly introduced.

Keywords. Decision theory; hierarchical decision making; fuzzy set theory; multiobjective optimization; identification; expert systems.

## INTRODUCTION

Solving continuous MDMPs in interactive methods usually includes two basic steps (Chankong, 1983): (i) to generate efficient (non-inferior) solutions and (ii) to select a preferred solution from the generated efficient ones. The former step mainly deals with a computation problem of multiobjective mathematical programming. Many methods have been put forward for the purpose. In the later step, we are confronted with a multi-attribute decision analysis problem, which relates to how the preference information of DMs to objective functions can be obtained and quantified so as to direct interactive decision making processes.

Classical multi-attribute utility theory is undoubtedly one of the theoretical bases for decision analysis. Two reasons, however, make the theory difficult to be directly applied to solving practical MDMPs. One is complexity to construct multi-attribute utility functions, and the other is abstraction of the ways for acquiring preference information from DMs. A concept of preference degree was introduced by H.C.Wang(1986) to solve discrete MDMPs. The concept was used by J.B. Yang (1987b) to develop an interactive fuzzy decision making method for solving continuous MDMPs. As far as we know, however, it is still an unsolved problem to acquire, analyze and describe the preference information of DMs in order to generate PDs. Hence, it seems necessary to explore quantitative methods to analyze PDs of objective functions. Such research work may be significant to apply multi—objective decision making methods to practical decision problems, especially to the development of intelligent decision support systems(J.B.Yang, 1987b).

Two main reasons make it complex to acquire, analyze and describe preference information of DMs. At first, there may be many factors influencing DMs' preference information simultaneously, and these factors usually have different characteristics and are even conflicting with one another, which we call the multi-dimension property of the factors. Besides, different DMs may provide distinct preference information for one MDMP, and on the other hand a DM may provide distinct preference information for one MDMP in different situations. Such properties form the fuzziness of preference information. These two reasons interact on each other, and are characteristic of common practical decision making problems.

Therefore when analyzing the PDs of objectives, we have to consider two aspects: (i) how to synthesize the influence of these multiple factors on PD analysis and (ii) how to reflect the fuzziness of preference information. For the first aspect, we'll conduct PD analysis for a single factor and then for multiple factors. For the second aspect, it'll be necessary to define fuzzy degrees, to scale the degrees and to analyze the influence of the fuzziness on PDs. It can be shown that the basic principle of the analytic hierarchy process(AHP) may be used to construct analytical models for PDs. It can also be explained how to use the fuzzy set theory to treat the fuzziness of preference information. Besides, it seems that the parameter identification method may be helpful to synthesize the influence of multiple factors on PDs.

In this paper, a method is developed for analyzing the PDs of objectives on the basis of the fuzzy set theory, the analytic hierarchy process and the parameter identification method. The concept of PD and its roles in decision analysis are first introduced according to the characteristics of multiobjective decision making. Then, the concepts of ODD, FDDD and FDD are defined. The concepts of fuzzy scale and sensitivity vector are sequentially defined. Furthermore, we are in a position to propose a model for PD analysis: the hierarchical analysis model. By using the model, a method for PD analysis for both a single factor and multiple factors is proposed. At last, an application is simply illustrated.

## THE CONCEPT OF PD AND ITS ROLES

Continuous multiobjective decisionmaking problems can usually be expressed as follows:

$$\max \ F(X)=[F_1(X)\dots F_k(X)\dots F_K(X)] \quad (2.1)$$
$$\text{s.t. } X\in\Omega \quad \Omega=\{X|g_i(X)\geq 0,\ i=1,\dots,m\}$$

Normally, there are infinite efficient solutions in problem (2.1). Our purpose is to find from these solutions a preferred efficient solution. DMs may use qualitative linguistic variables to express their preference information. We can use the real numbers defined in the interval [-1, 1] to quantitatively describe the preference information. These real numbers are called preference degrees (or PDs), represented by p.

The values of an objective, $F_k$, may change within a feasible domain defined by certain constraints. If the objective reaches its best feasible value, $F_k^*$, the PD of the value is defined as 1, i.e. $p(F_k^*)=1$. On the contrary, if it reaches its worst feasible value, $F_k^-$, the corresponding PD is defined as -1, i.e. $p(F_k^-)=-1$. For an indifference case, let $p(F_k)=0$. For other cases, let $p(F_k(X))\in(-1,0)\cup(0,1)$, where $p(F_k(X))<0$ means that the value $F_k(X)$ is not preferred, $p(F_k(X))>0$ means preferred, and the value of $|p(F_k(X))|$ reflects preference degree.

The concept of preference degree can be understood as a kind of membership function. Generally, membership functions may be defined in the interval [0,1]. In our case, however, it is considered that "preferred" and "not preferred" are two opposite concepts. Hence, it seems more proper to express these opposite concepts by the values of the intervals [-1,0) and (0,1] respectively. We call $p(F_k(X))(\forall X\in\Omega)$ preference function, which has no dimension and satisfies

$$-1 \leq p(F_k(X)) \leq 1 \qquad \forall X\in\Omega \qquad (2.2)$$

Besides, we can also define a preference domain of $F_k(X)$ as $p(F_k(X))\geq\alpha\ (0\leq\alpha\leq 1)$. If the preference function is used to substitute $F_k(X)$ in problem (2.1) to form a fuzzy multiobjective decision making problem, then it will be convenient to obtain a preferred solution by using interactive methods(J.B.Yang, 1987a). Obviously, whether or not the preferred solution is rational by using such a fuzzy decision making method depends on whether or not the acquired preference functions and preference domain can reflect the real preference information of DMs.

## A HIERARCHICAL MODEL FOR ANALYZING PDs

### Discourse Domains of Objectives, Fuzzy Degrees and Factors

In problem (2.1), the feasible solution set is an infinite one. Therefore, the values of the K objective functions will construct K infinite sets. Although we are not in a position to analyze the PDs of all values of the objectives, we can select finite discrete points $\{F_k^r\}$ $(r=0,1,\dots,H_k)$ of $F_k(X)$ $(k=1,\dots,K)$ to conduct the analysis. $F_k^r$ will be called the rth assessed value(AV) of the objective function $F_k(X)$. The row vector $F_k$ composed of these AVs is called the AV subset of $F_k(X)$ where $F_k$ is defined by

$$F_k=\{\ F_k^0\ \ F_k^1\ \ \dots\ F_k^{H_k}\ \} \qquad (3.1)$$

We can then define objective domain of discourse(ODD) as follows.

Definition 3.1: The set composed of all the AV subsets of the K objective functions is called the discourse domain of objective, expressed by F, where

$$F = [\ F_1^T\ \ F_2^T\ \ \dots\ F_K^T\ ] \qquad (3.2)$$

When an objective varies within its feasible domain, or the same objective is assessed according to different factors, the preference information given by a DM can be classified into several fuzzy degrees. The set of such fuzzy degrees constitute a fuzzy degree domain of discourse(FDDD), which can be defined as follows:

$$\tilde{X} = \{\ \tilde{X}_1\ \ \tilde{X}_2\ \ \dots\ \tilde{X}_n\ \dots\ \tilde{X}_N\ \} \qquad (3.3)$$

where N is the number of the classified fuzzy degrees. Suppose N=7, for instance, the FDDD can be defined as follows:

$$\tilde{X} = \{\ \tilde{X}_1\ \ \tilde{X}_2\ \ \tilde{X}_3\ \ \tilde{X}_4\ \ \tilde{X}_5\ \ \tilde{X}_6\ \ \tilde{X}_7\ \}$$
= {the most unsatisfactory, very unsatisfactory, unsatisfactory, indifference, satisfactory, very satisfactory, the most satisfactory } (3.4)

Obviously the FDDD is the set of qualitative linguistic variables. By projecting the domain into the interval [-1,1], the membership function of the domain to PD can be produced, which may be called fuzzy scale as well, written as $\mu_P(\tilde{X}_n)$, $n=1,\dots,N$.

Definition 3.2: The fuzzy scale of the FDDD to PD can be written as

$$\mu_P:\ \tilde{X} \longrightarrow [-1\quad 1]$$
$$\tilde{X}_n \longrightarrow \mu_P(\tilde{X}_n) \qquad n=1,\dots N$$

Obviously, $\mu_P(\tilde{X}_n)$ is really the quantification of the FDDD, so that we can treat the qualitative linguistic variables by using quantitative methods. Take N=7 for example. Without loss of generation, we take the following fuzzy scales:

$$\mu_P(\tilde{X}) = [\mu_P(\tilde{X}_1)\ \mu_P(\tilde{X}_2)\ \mu_P(\tilde{X}_3)\ \mu_P(\tilde{X}_4)\ \mu_P(\tilde{X}_5)\ \mu_P(\tilde{X}_6)\ \mu_P(\tilde{X}_7)] = [-1,\ -0.8,\ -0.4,\ 0,\ 0.4,\ 0.8,\ 1] \qquad (3.5)$$

As mentioned in Introduction, the multi-dimension is one of the characteristics of practical factors influencing PD assessment. Although the practical factors possess different properties, they can generally be classified into two classes: quantitative factors and qualitative factors. Quantitative factors are defined as follows.

Definition 3.3: $v_{ki}$ is called the ith quantitative factor for assessing the PDs of $F_k(X)$ if $v_{ki}$ is one of the factors influencing the objective function $F_k(X)$, if it can be expressed by some numeric values with certain dimension, and if there exists some numeric relation between $v_{ki}$ and $F_k(X)$.

If there are $M_k$ quantitative factors influencing $F_k(X)$, then they constitute a quantitative factor subset $V_k$ for assessing the PDs of $F_k(X)$, written as

$$V_k = [v_{k1} \ \ v_{k2} \ \cdots \ v_{kM_k}] \tag{3.6}$$

Qualitative factors are defined as follows

Definition 3.4: $s_{ki}$ is called the ith qualitative factor for assessing the PDs of $F_k(X)$ if $s_{ki}$ is one of the factors influencing $F_k(X)$, if it cannot be expressed by any numeric value, and if there exist no numeric relations between $s_{ki}$ and $F_k(X)$ but only fuzzy relations involved in some qualitative rules.

If there are $I_k$ qualitative factors influencing $F_k(X)$, then they consist of a qualitative factor subset $S_k$ for assessing the PDs of $F_k(X)$, written as

$$S_k = [s_{k1} \ \ s_{k2} \ \cdots \ s_{kI_k}] \tag{3.7}$$

The set composed of $V_k$ and $S_k$ is called factor sub-domain of discourse for assessing the PDs of the objective function $F_k(X)$, written as $[V_k \ \ S_k]$. Let

$$L_k = M_k + I_k \tag{3.8}$$

The set composed of all the K factor subdomains of discourses is called factor domain of discourse(FDD).

Sensitivity Vector

Because different factors may influence an objective function to different extensions, the following definition will be useful when we synthetically consider a factor sub-domain of discourse to assess the PDs of an objective.

Definition 3.5: Let $w_{ki}$ express the sensitivity degree of an objective $F_k$ to a factor $v_{ki}$(or $s_{ki}$), and $w_{k1}+\ldots+w_{kL_k}=1$, $w_{ki} > 0$ $(i=1,\ldots,L_k)$. Then, all such $w_{ki}(i=1,\ldots,L_k)$ constitute a vector, written as

$$W_k = [w_{k1} \ \cdots \ w_{kM_k} \cdots \ w_{kL_k}]^T$$

$W_k$ is called the sensitivity vector of the objective $F_k$ to the factor sub-domain of discourse $[V_k \ \ S_k]$. $w_{ki}$ reflects the influence degree of factor $v_{ki}$ or $s_{ki}$ to objective $F_k$. $W_k$ is required to meet the following property.

Property 3.1: $W_k$ remains unchanged for all the AV $F_k^r(r=0,1,\ldots, H_k)$ of $F_k$.

Since $F_k^r$ is usually selected within the feasible domain of $F_k(X)$, the above property can be satisfied in common cases. By the way, Property 3.1 is very important to identify $W_k$.

A Hierarchical Analysis Model

To analyze the PDs of an objective function $F_k(X)$ is really to find such a transformation that can project $F_k(X)$ onto the interval [-1 1]. From the discussion of the above two sub-sections, the transformation should include ODD, FDDD and FDD. We propose a hierarchical analysis model, shown in Fig.3.1, to complete the projection. In Fig.3.1, the highest stage is the ODD, expressing the goals to be reached; the middle stage is the FDDD, reflecting the fuzziness; and the lowest stage includes all the factor sub-domains of discourse, reflecting the multi-dimension property of factors.

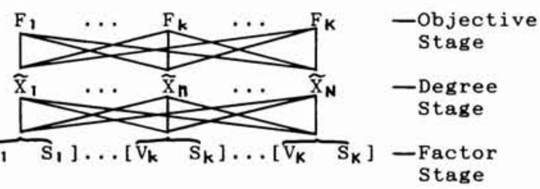


Fig.3.1 A Hierarchical Analysis Model

The PD of each AV of an objective $F_k$, $F_k^r$, will be assessed respectively. The assessment consists of two fundamental steps. First conduct the assessment according to a single factor $v_{ki}$(or $s_{ki}$), and then according to the sub-domain $[V_k \ \ S_k]$ of $F_k(X)$. Suppose the PD of $F_k^r$ to $v_{ki}$ (or $s_{ki}$) is $p_{kr}(v_{ki})$(or $p_{kr}(s_{ki})$), then the PD of $F_k^r$ to $[V_k \ \ S_k]$, $p_{kr}$, can be given by

$$p_{kr}=\sum_{i=1}^{M_k} w_{ki}\, p_{kr}(v_{ki})+\sum_{i=M_k+1}^{L_k} w_{ki}\, p_{kr}(s_{k(i-M_k)}) = W_k^T \begin{bmatrix} P_{kr}(V_k) \\ P_{kr}(S_k) \end{bmatrix} \tag{3.9}$$

where

$$P_{kr}(V_k)=[p_{kr}(v_{k1})\ldots p_{kr}(v_{kM_k})]^T \tag{3.10}$$

$$P_{kr}(S_k)=[p_{kr}(s_{k1})\ldots p_{kr}(s_{kI_k})]^T \tag{3.11}$$

## ANALYSIS METHOD FOR SINGLE FACTOR

There are two steps for analyzing the PDs of an objective according to a single factor. At first, determine the membership degrees of the factor to the FDDD according to the AVs of the objective and the relation between the objective and the factor. Then by using the membership degrees as weights, calculate the PD of an AV for the factor on the basis of fuzzy scales of the FDDD to preference degree.

Fuzzy Scales of Factors

Suppose the value of a factor $v_{ki}(s_{ki})$ relative to an AV $F_k^r$ can be written as $v_{ki}^r(s_{ki}^r)$. If DM can judge that the factor value $v_{ki}^r$(or $s_{ki}^r$) is just corresponding to a fuzzy degree $\tilde{X}_n$, then $\mu_P(\tilde{X}_n)$ will be the PD of $F_k^r$ assessed from $v_{ki}$(or $s_{ki}$), that is $p_{kr}(v_{ki})=\mu_P(\tilde{X}_n)$, or $P_{kr}(s_{ki}) = \mu_P(\tilde{X}_n)$. Generally speaking, however, that is not the case. DM may consider that $v_{ki}$ (or $s_{ki}$) is corresponding to both $\tilde{X}_n$ and $\tilde{X}_{n+1}$, or $\tilde{X}_{n-1}$ and $\tilde{X}_n$, though the membership degrees may be different. Therefore, it is useful to give the following definition.

Definition 4.1: The membership function of the factor sub-domain of discourse $V_K$(or $S_K$) to the FDDD is given by

$$\mu_{\tilde{X}}: V_K \rightarrow [0 \quad 1], \text{ or } S_K \rightarrow [0 \quad 1]$$

$$v_{ki} \rightarrow \mu_{\tilde{X}_n}(v_{ki}), \text{ or } s_{ki} \rightarrow \mu_{\tilde{X}_n}(s_{ki})$$

$$n=1,\ldots,N; \quad k=1,\ldots,K; \quad i=1,\ldots,L_K$$

Then the membership degree $\mu_{\tilde{X}_n}(v_{ki})$ is called the fuzzy scale of the factor $v_{ki}$ to the fuzzy degree $\tilde{X}_n$.

Analysis Method for Quantitative Factors

From Definition 3.3, we can obtain the feasible interval $[a_0 b_0]$ of a factor $v_{ki}$ if we know the feasible interval of $F_K$. In this case, $a_0$ can be considered to belong to the fuzzy degree of "the most unsatisfactory". In another word, if $v_{ki}=a_0$, then

$$\mu_{\tilde{X}_1}(v_{ki})=-1; \mu_{\tilde{X}_n}(v_{ki})=0, \ n=2,\ldots,N \quad (4.1)$$

Similarly, if $v_{ki}=b_0$, then

$$\mu_{\tilde{X}_N}(v_{ki})=1; \mu_{\tilde{X}_n}(v_{ki})=0, \ n=1,\ldots,N-1 \quad (4.2)$$

Suppose the FDDD is evenly distributed in the continuous interval $[a_0 \ b_0]$ of the factor $v_{ki}$, and for a fuzzy degree there exists a sine relation between $\mu_{\tilde{X}_n}(v_{ki})$ and $v_{ki}$, then we can construct the following diagram between $v_{ki}$ and $\mu_{\tilde{X}_n}(v_{ki})$.

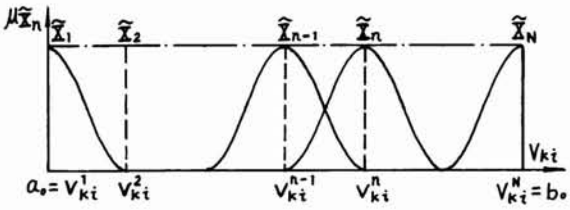

Fig.4.1 Fuzzy Scale Characteristics between $\mu_{\tilde{X}_n}(v_{ki})$ and $v_{ki}$

A factor value $v^n_{ki}$ just corresponding to a fuzzy degree $\tilde{X}_n$ can be given by following equations

$$v^n_{ki} = a_0 + (n-1)C_0$$

$$C_0 = \frac{b_0 - a_0}{N-1} \qquad (n=1,\ldots,N) \qquad (4.3)$$

Then, the membership function of a factor $v_{ki}(\in [a_0 \ b_0])$ to a fuzzy degree $\tilde{X}_n$ can be determined as follows

$$\mu_{\tilde{X}_n}(v_{ki})=(1/2)\cos[(v_{ki}-v^n_{ki})/C_0]+1/2 \qquad (4.4)$$

$$v_{ki} \in (v^{n-1}_{ki}, \ v^n_{ki}), \quad n=1,\ldots,N$$

When N=7, the equations (4.4) can be illustrated by Fig.4.2.

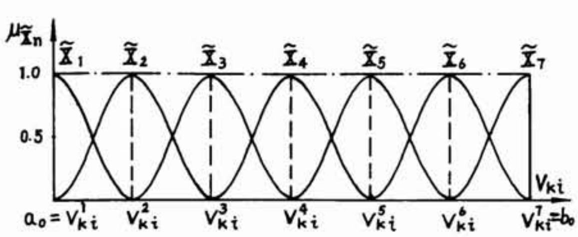

Fig.4.2 Illustration of Equations(4.4)

From Fig.4.2, it is obvious that only adjacent membership functions are dependent and at the points of intersection the fuzzy scales of $v_{ki}$ to the adjacent fuzzy degrees are all equal to 0.5. At any point there exists the following property:

Theorem 4.1: If $v_{ki} \in (v^{n-1}_{ki} \ \ v^n_{ki})$ and the fuzzy scales of $v_{ki}$ to $\tilde{X}_n$ are calculated from equations (4.4), then 1) $\mu_{\tilde{X}_{n-1}}(v_{ki}) \neq 0$; 2) $\mu_{\tilde{X}_n}(v_{ki}) \neq 0$; 3) $\mu_{\tilde{X}_j}(v_{kj})=0$, $j=1,\ldots,N$, $j \neq n-1$, n; and 4) $\mu_{\tilde{X}_{n-1}}(v_{ki}) + \mu_{\tilde{X}_n}(v_{ki})=1$, or $\mu_{\tilde{X}_1}(v_{ki})+\ldots+\mu_{\tilde{X}_N}(v_{ki})=1$.

The proof of Theorem 4.1 is obvious from equations (4.4) and Fig.4.2. By using the principle of the analytic hierarchy process, we can obtain the PD of an objective value $F^r_k$ for a quantitative factor $v_{ki}$ as follows.

$$p_{kr}(v^r_{ki})=\sum_{n=1}^{N}\mu_{\tilde{X}_n}(v^r_{ki})\,\mu_p(\tilde{X}_n) \qquad (4.5)$$

It is easy to prove that

$$-1 \leq p_{kr}(v^r_{ki}) \leq 1$$

Notice that $\mu_{\tilde{X}_n}(v_{ki})$ is obtained on the basis of two assumptions. 1)The FDDD is evenly distributed in the continuous interval $[a_0 \ \ b_0]$. 2) The membership function of a factor to any fuzzy degree is assumed to be a sine curve. Although these two assumptions possess typical significance, they may not always be satisfied. In real problems, the FDDD may not be evenly distributed. DMs may be sensitive to the change of $v_{ki}$ in some intervals but slow in others. The assumption of sine curve is obviously ideal. Therefore the intervals of fuzzy degrees and the shapes of membership functions should be independently determined for a real problem. The determined intervals and membership functions may be different for different problems, but the unit property similar to Theorem 4.1 should be satis-fied, which will be defined in the next sub-section.

Analysis Method for Qualitative Factors

Since there exist no numerical relations between qualitative factors and objective functions, analysis method for quantitative factors may not be proper for qualitative factors. However, the former idea is still useful. Here the problem is how to determine the membership degrees of qualitative factors to fuzzy degrees according to the AVs of objectives. One way is to determine the membership degrees through such approaches as experts' evaluation. Such approaches may involve certain subjectivity but can reflect experts' experiences and knowledge. The approaches may be made more perfect through learning. Of course, the evaluation process may also be completed by expert systems. The process can be called the expert evaluation approach of membership degrees.

The membership degrees obtained by the expert evaluation approach should possess the following unit property similar to Theorem 4.1.

Definition 4.1: Let the membership degree of a qualitative factor $s_{ki}$ to a fuzzy degree $\tilde{X}_n$ be $\mu_{\tilde{X}_n}(s^r_{ki})$. If $\mu_{\tilde{X}_1}(s^r_{ki}) +\ldots+ \mu_{\tilde{X}_N}(s^r_{ki})=1$, then it is defined that the membership degree of $s^r_{ki}$ to $\tilde{X}_n$ satisfies unit property.

If $\mu_{\tilde{X}_n}(s^r_{ki})$ $(n=1,\ldots,N)$ satisfies the unit property, from the similar discussion to the former subsection we can obtain the PD $p_{kr}(s^r_{ki})$ of a single factor $s_{ki}$ to $F^r_k$:

$$p_{kr}(s^r_{ki}) = \sum_{n=1}^{N} \mu_{\tilde{X}_n}(s^r_{ki})\,\mu_P(\tilde{X}_n) \tag{4.6}$$

## SYNTHETICAL ANALYSIS METHOD FOR MULTIPLE FACTORS

If there is only one factor influencing the PDs of $F_k$, then $p_{kr}(v_{ki})$ (or $p_{kr}(s_{ki})$) generated in the above two sub-sections is the PD of $F_k$, that is $p_{kr}=p_{kr}(v_{ki})$ (or $p_{kr}(s_{ki})$). If there are $L_k$ factors, however, $p_{kr}$ should be solved by equation (3.9). Consequently, sensitivity vector $W_k$ has to be identified.

### Identification of Sensitivity Vector $W_k$

The scalars $w_{ki}$ $(i=1,\ldots,L_k)$ of the sensitivity vector $W_k$ are a kind of estimated values. It seems necessary to combine DMs' subjective judgments with optimization methods in order to identify the value of $w_{ki}$. If $w_{ki}$ and $w_{kj}$ are known, let $a_{ij}=w_{ki}/w_{kj}$, which expresses the relative importance of $p_{kr}(v_{ki})$ and $p_{kr}(v_{kj})$ on $p_{kr}$. Then, we can construct the following comparative matrix $\bar{A}_k$:

$$\bar{A}_k = \begin{bmatrix} \bar{a}_{11} & \bar{a}_{12} & \cdots & \bar{a}_{1L_k} \\ \bar{a}_{21} & \bar{a}_{22} & \cdots & \bar{a}_{2L_k} \\ \vdots & \vdots & & \vdots \\ \bar{a}_{L_k1} & \bar{a}_{L_k2} & \cdots & \bar{a}_{L_kL_k} \end{bmatrix} = (\bar{a}_{ij})_{L_k\times L_k} \tag{5.1}$$

Obviously,

$$\bar{a}_{ii}=1,\quad \bar{a}_{ij}=1/\bar{a}_{ji},\quad \bar{a}_{ij}=\bar{a}_{ik}/\bar{a}_{jk}$$

$$(i,j=1,2,\ldots,L_k) \tag{5.2}$$

and

$$\bar{A}_kW_k=L_kW_k \quad \text{or} \quad \bar{A}_kW_k-L_kW_k=0 \tag{5.3}$$

On the contrary, suppose $w_{ki}$, $w_{kj}$ are unknown but DMs can judge the relative importance of $p_{kr}(v_{ki})$ and $p_{kr}(v_{kj})$ on $p_{kr}$, that is, the value of $\bar{a}_{ij}$ can be judged, written as $a_{ij}$. The matrix, produced by substituting $\bar{a}_{ij}$ in (5.1) with the judged value $a_{ij}$, is called the judged matrix, which reflects the subjective judgment of DMs. In the judged matrix $A_k$, the former two equations in (5.2) can normally be satisfied, but generally the third one in (5.2) and the equation(5.3) may not be satisfied. Let

$$A_kW_k - L_kW_k=\varepsilon_1 \tag{5.4}$$

where $\varepsilon_1$ is an error vector. If the values of $p_{kr}$, $P_{kr}(V_k)$ and $P_{kr}(S_k)$ are estimated at a special AV of an objective, written as $\hat{p}_{kr}$, $\hat{P}_{kr}(V_k)$ and $\hat{P}_{kr}(S_k)$ respectively, then error may appear by putting the estimated values into (3.10), i.e.

$$\varepsilon_2 = \hat{p}_{kr} - W_k^T \begin{bmatrix} \hat{P}_{kr}(V_k) \\ \hat{P}_{kr}(S_k) \end{bmatrix} \tag{5.5}$$

Suppose $\eta$ is a given weight, then an error function can be defined as follows

$$\varepsilon = \varepsilon_1^T \varepsilon_1 + \eta(\varepsilon_2)^2 \tag{5.6}$$

The purpose to identify $W_k$ is to find a $\hat{W}_k$ minimizing $\varepsilon$. So, construct the following optimization problem with a constraint:

$$\min \ \varepsilon = \varepsilon_1^T\varepsilon_1 + \eta(\varepsilon_2)^2$$
$$\text{s.t.} \ \sum_{i=1}^{L_k} w_{ki}=1 \tag{5.7}$$

Let I be a $L_k$-dimensional unit vector (the elements are all 1) and E a $L_k$-dimensional unit matrix. By solving (5.7), the optimal estimated value of $W_k$ can be written as

$$\hat{W}_k=G_k\left(\frac{1-\eta\,\hat{p}_{kr} I^T G_k \begin{bmatrix}\hat{P}_{kr}(V_k)\\ \hat{P}_{kr}(S_k)\end{bmatrix}}{I^T G_k I}\, I + \eta\,\hat{p}_{kr}\begin{bmatrix}\hat{P}_{kr}(V_k)\\ \hat{P}_{kr}(S_k)\end{bmatrix}\right) \tag{5.8}$$

where

$$G_k=\left[(A_k-L_kE)^T(A_k-L_kE) + \eta\begin{bmatrix}\hat{P}_{kr}(V_k)\\ \hat{P}_{kr}(S_k)\end{bmatrix}[\hat{P}_{kr}(V_k)\ \ \hat{P}_{kr}(S_k)^T]\right]^{-1} \tag{5.9}$$

From (5.8) and (5.9), $\hat{W}_k$ can be generated as long as estimated values $\hat{p}_{kr}$, $\hat{P}_{kr}(V_k)$ and $\hat{P}_{kr}(S_k)$ are known.

### Consistency Examination

From the above sub-section, the judged matrix $A_k$ may not always satisfy the properties of the comparative matrix $\bar{A}_k$. Hence, it is required to take consistency examination for $A_k$. The consistency index CI can be defined as follows

$$CI = (\lambda_{max}-L_k)/(L_k-1) \tag{5.10}$$

$\lambda_{max}$ is calculated by

$$\lambda_{max} = \sum_{i=1}^{L_k} \frac{(A_k\hat{W}_k)_i}{L_k\hat{w}_{ki}} \tag{5.11}$$

where $(A_k\hat{W}_k)_i$ is the ith scalar of the vector $(A_k\hat{W}_k)$. When the judged matrix possesses complete consistency ((5.2) and (5.3) are all satisfied), then CI=0. The larger $(\lambda_{max}-L_k)$, the worse the consistency of the judged matrix. Define a stochastic consistency index CR=CI/RI, where RI is average stochastic consistency index, which changes with the order $L_k$ of the judged matrix, shown in Table 5.1. When CR<0.1, the consistency of the judged matrix is satisfactory. Otherwise, the judged matrix is required to be regulated and the weight may be changed as well.

TABLE 5.1

| $L_k$ | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| RI | 0.0 | 0.0 | 0.58 | 0.9 | 1.12 | 1.24 | 1.32 | 1.41 |

### Steps of The Synthetical Analysis for Multiple Factors

As a result of the above discussion, we can obtain the following steps for synthetically analyzing the PDs of objective functions for multiple factors.

Step 1: Define the decisionmaking problem and construct the relative hierarchical analysis model;
Step 2: Determine fuzzy scales of fuzzy degrees;
Step 3: Select sub-domain of discourse of

the kth objective function $F_k(X)$, $\{F_k^0, \ldots, F_k^r, \ldots, F_k^{H_k}\}$. Let k=1, r=1;

Step 4: According to (4.2)—(4.4), solve the PD of $F_k^r$ for single quantitative factor $v_{ki}$, $p_{kr}(v_{ki})$. Let i=1;

Step 5: Let i=i+1. If $i \leq M_k$, go to step 4; if $i > M_k$, go to next step.

Step 6: By (4.5), solve the PD of $F_k^r$ for single qualitative factor $s_{ki}$, $p_{kr}(s_{ki})$. Let i=1;

Step 7: Let i=i+1. If $i \leq I_k$, go to step 6; if $i > I_k$, continue;

Step 8: Calculate $\hat{W}_k$ from (5.8) and (5.9);

Step 9: By (3.9) solve the PD of $F_k^r$ for the factor sub-set $[V_k\ S_k]$, $p_{kr}$. Let r=r+1. If $r \leq H_k$, go to step 3; otherwise, go on;

Step 10: Construct the PD vector of the objective function $F_k(X)$ from (3.11). Let k=k+1. If k<K, go to Step 3; otherwise, stop.

## AN APPLICATION

The above-mentioned model and analysis methods for PD analysis have been extended and applied to the multiobjective decision making problem for production planning of Shanghai Oil Refinery(J.Yang, 1988). The practical problem includes four linear objective functions(i.e. economic profits, energy consumption, total production costs and light oil), over three hundred variables, over two hundred linear constraints and thirty-five factors influencing the PD analysis of the four objectives.

Since the relations among these factors possess hierarchical structure, a factor-relation graph is designed. In the graph there are several layers, including an objective layer(OL), two synthetical factor layers(SFL) and a basic factor layer(BFL). The graph is briefly demonstrated in Fig. 5.1, where there are some basic factors in SFL.

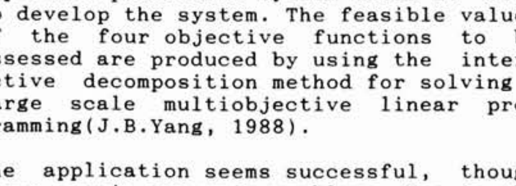


Fig.5.1 Factor-Relation Graph

A fuzzy inference expert system is developed to conduct the PD analysis. OPS5—an important production system shell is used to develop the system. The feasible values of the four objective functions to be assessed are produced by using the interactive decomposition method for solving a large scale multiobjective linear programming(J.B.Yang, 1988).

The application seems successful, though there remain two main problems. One is the transformations between the two computer languages, OPS5 for fuzzy inference and FORTRAN for optimization computation of the linear programming. The other is that because there are no available Chinese versions of OPS5 or FORTRAN, we have not designed a Chinese man-machine interface to make a convenience of those users who are not familiar with English.

## CONCLUDING REMARKS

Summarily, the methods proposed in the paper are feasible and have the following characteristics. 1> The hierarchical analysis model can clearly reflect the essence of decision analysis process of real multiobjective decisionmaking problems. 2> The analysis methods for a single factor can properly consider the fuzziness of preference information. 3> The synthetical method for multiple factors can involve the subjective judgment of DMs. 4> The proposed methods can also be used to treat discrete multiobjective decision making problems and to establish intelligent decision support systems(J.B.Yang, 1987b).

However, it is a practical problem to calculate the value $v_{ki}^r$ of a quantitative factor $v_{ki}$ and to judge the value $s_{ki}^r$ of a qualitative factor $s_{ki}$ according to $F_k^r$, which may be required to be explored further.

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