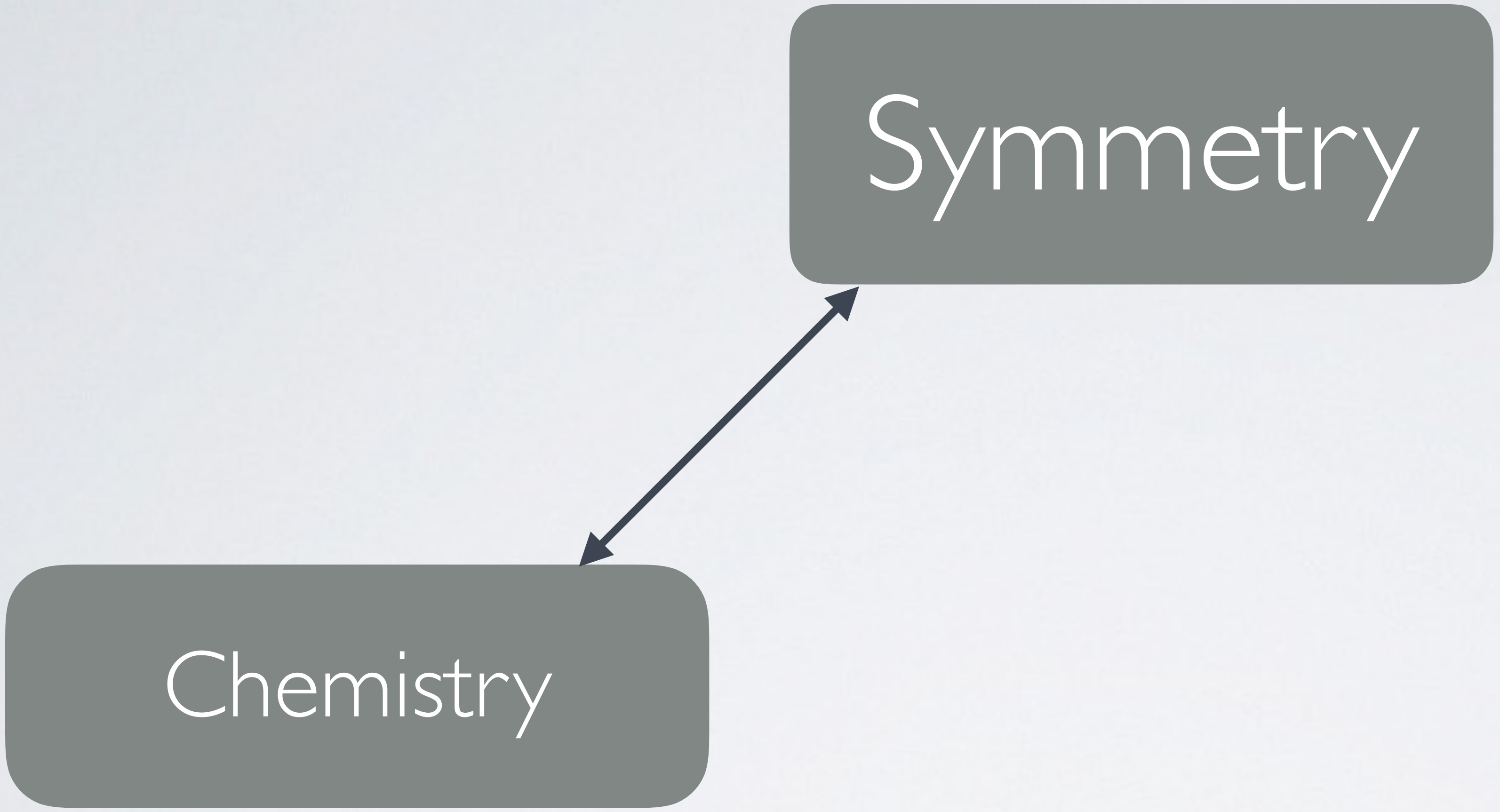
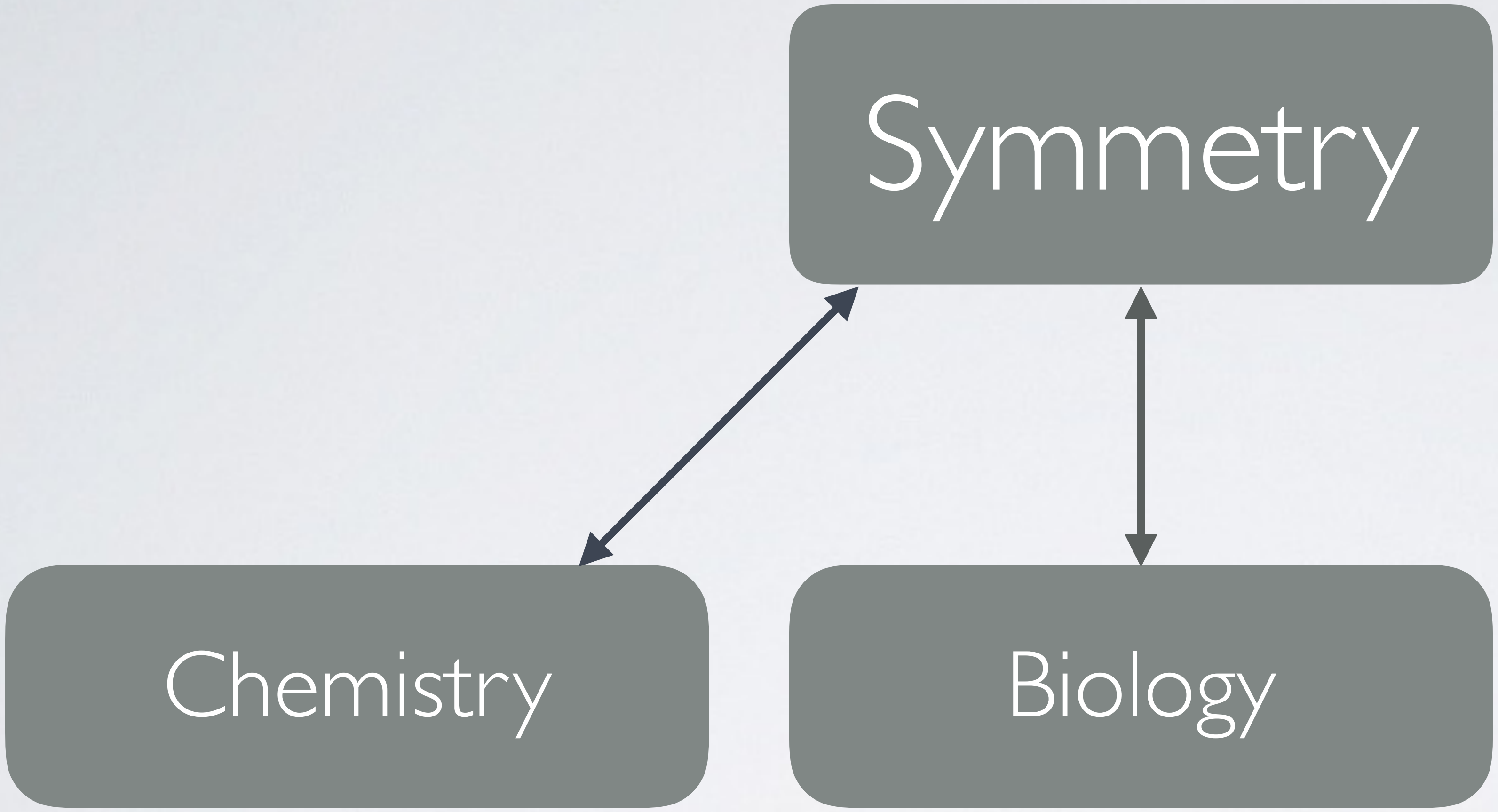
COMPUTING CHARACTER TABLES OF FINITE GROUPS

Jay Taylor
(Università degli Studi di Padova)

Symmetry


Symmetry
Chemistry


Symmetry
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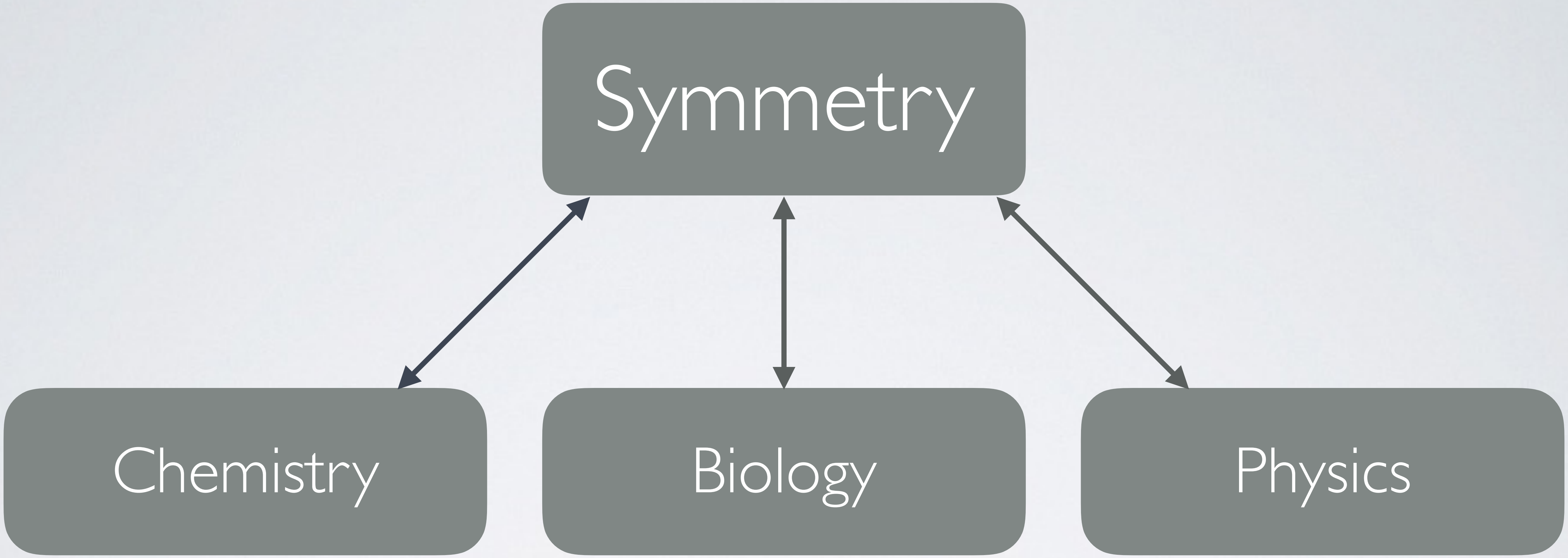
Symmetry
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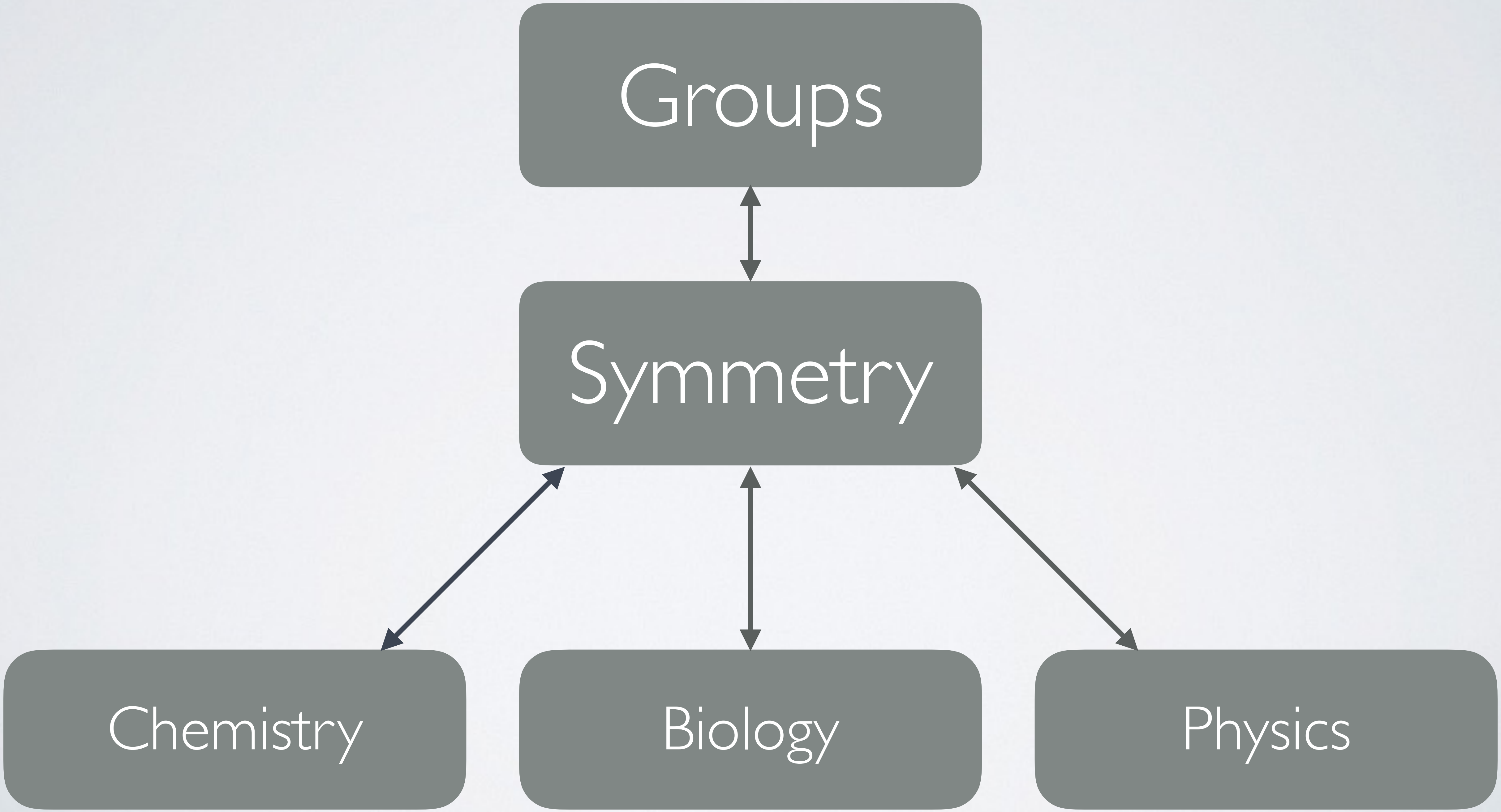
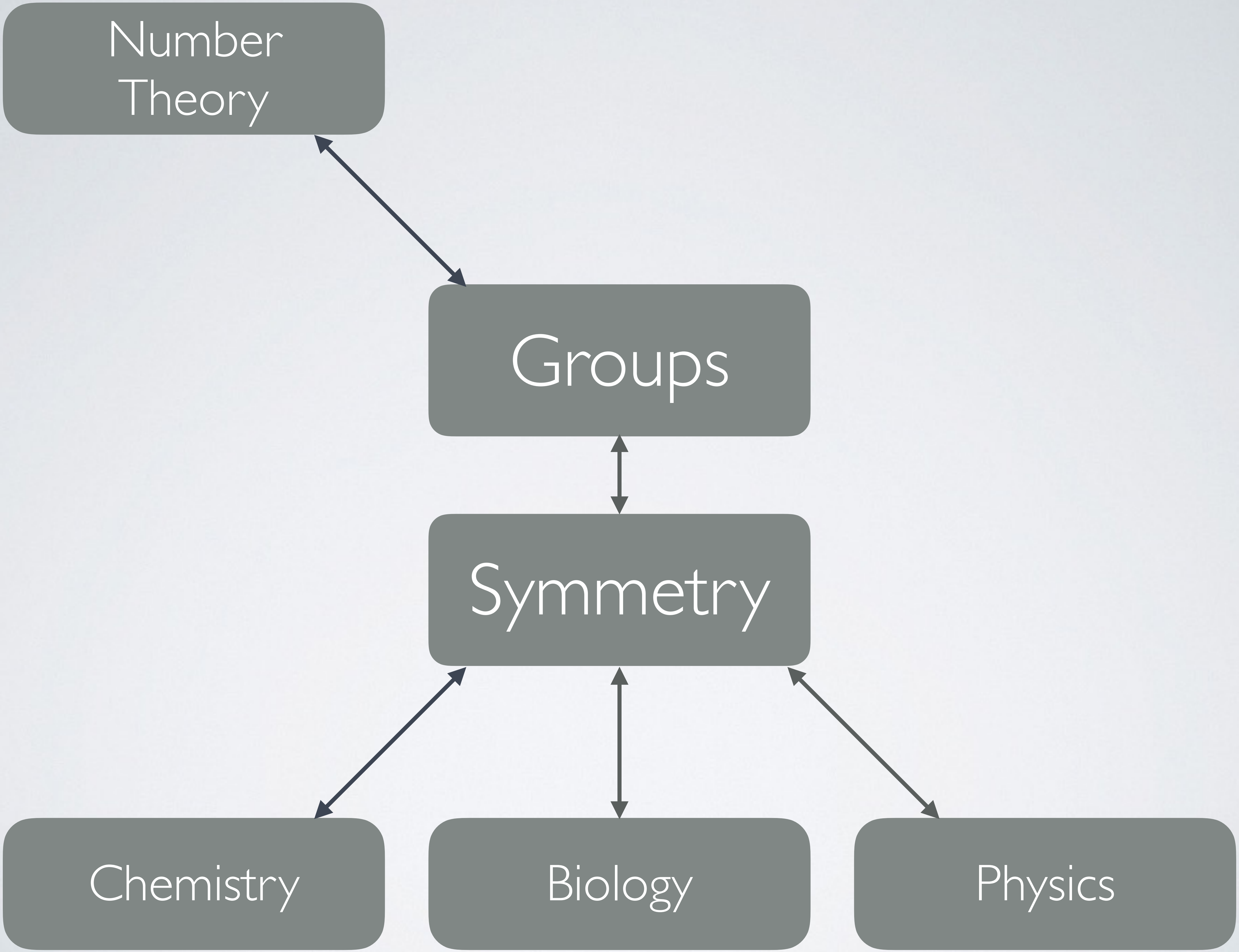
Physics


Groups
Symmetry
Chemistry
Biology
Physics


Number Theory
Groups
Symmetry
Chemistry
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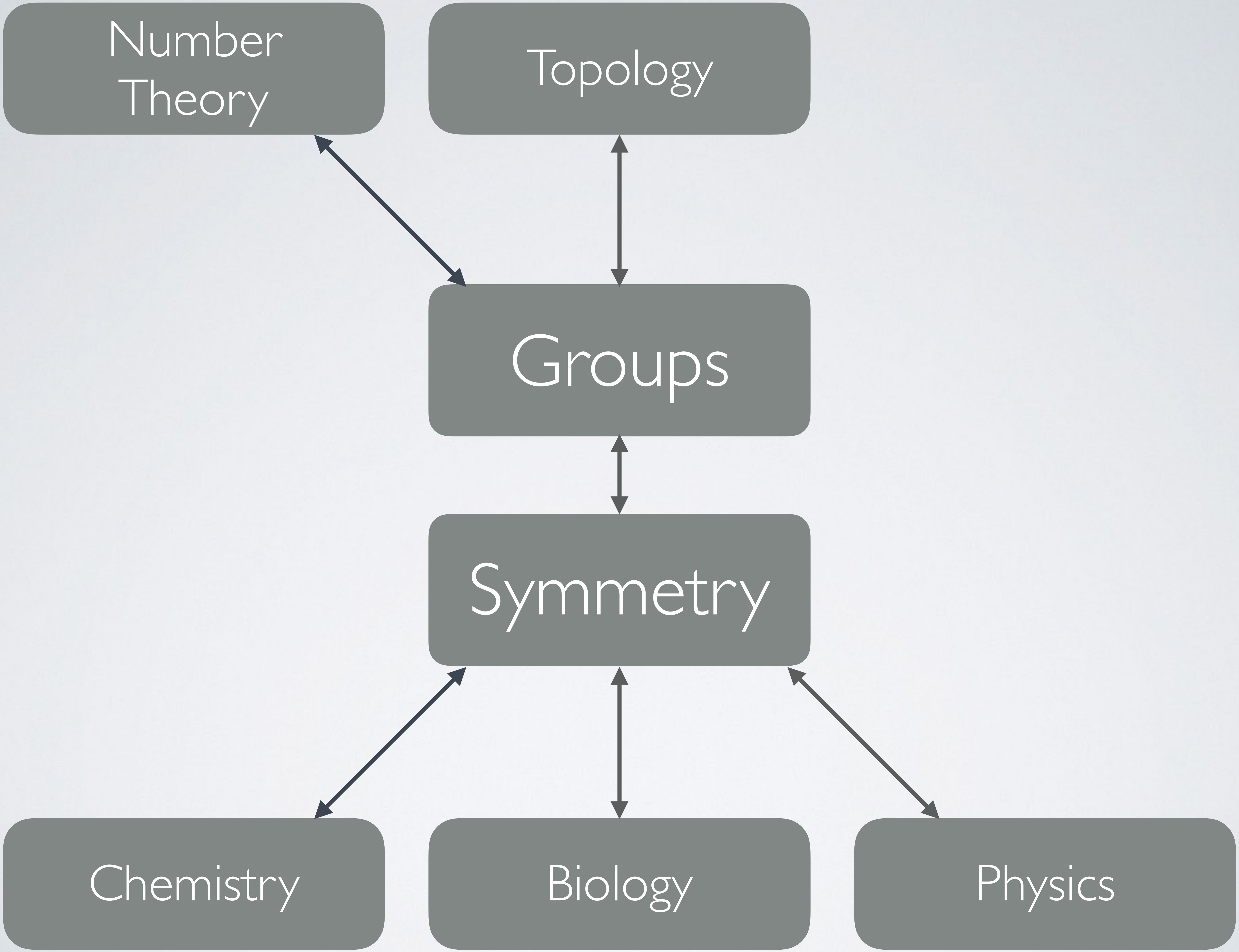
Number Theory
Topology
Groups
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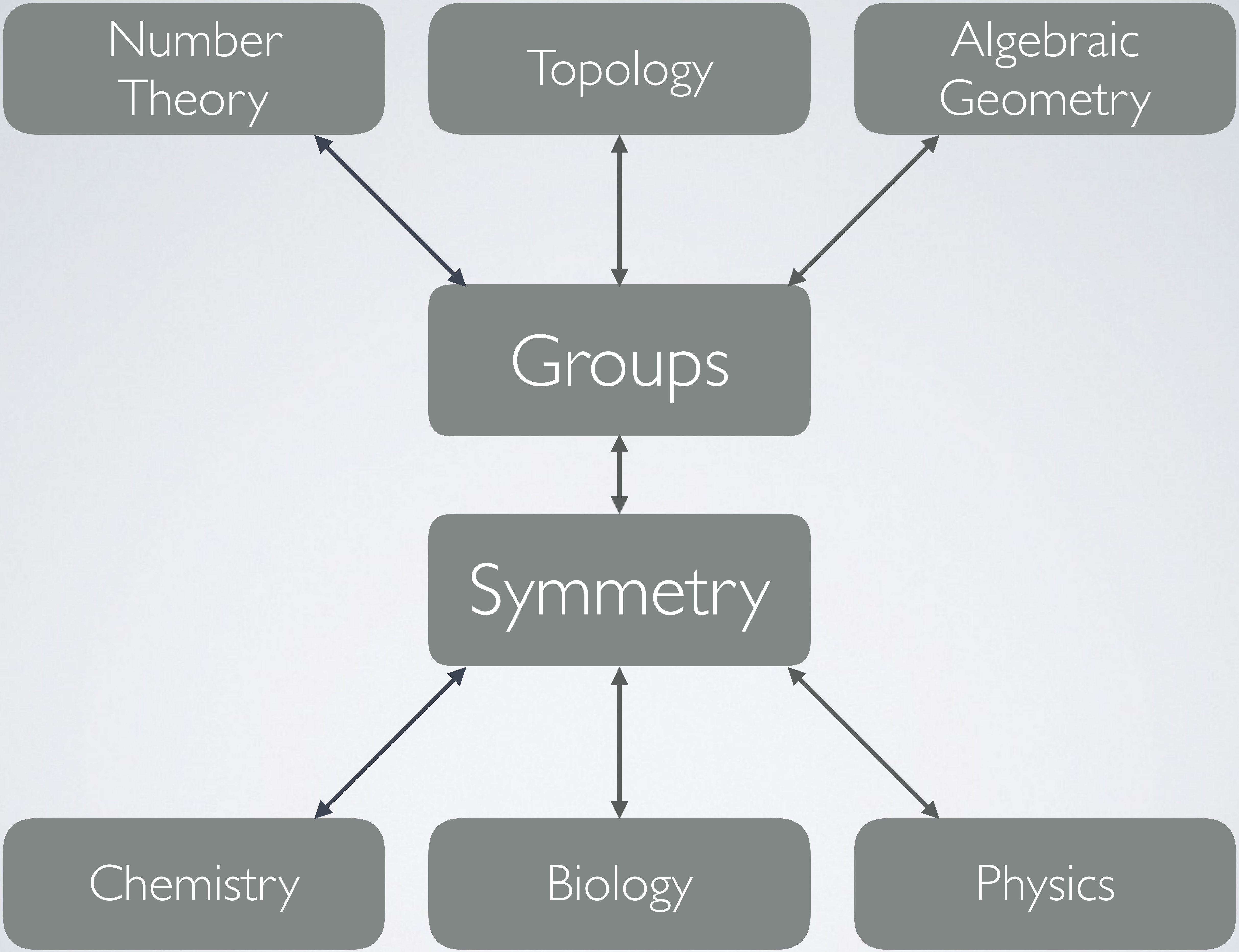
Number Theory
Topology
Algebraic Geometry
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DEFINITION

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Examples

- $(\mathbb{Z}, +)$ with $\mathbb{Z} = \{\ldots, -2, -1, 0, 1, 2, \ldots\}$,
- $(\mathbb{R}^\times, \times)$ with $\mathbb{R}^\times = \mathbb{R} \setminus \{0\}$ where $\mathbb{R}$ denote the real numbers.

DIHEDRAL GROUPS

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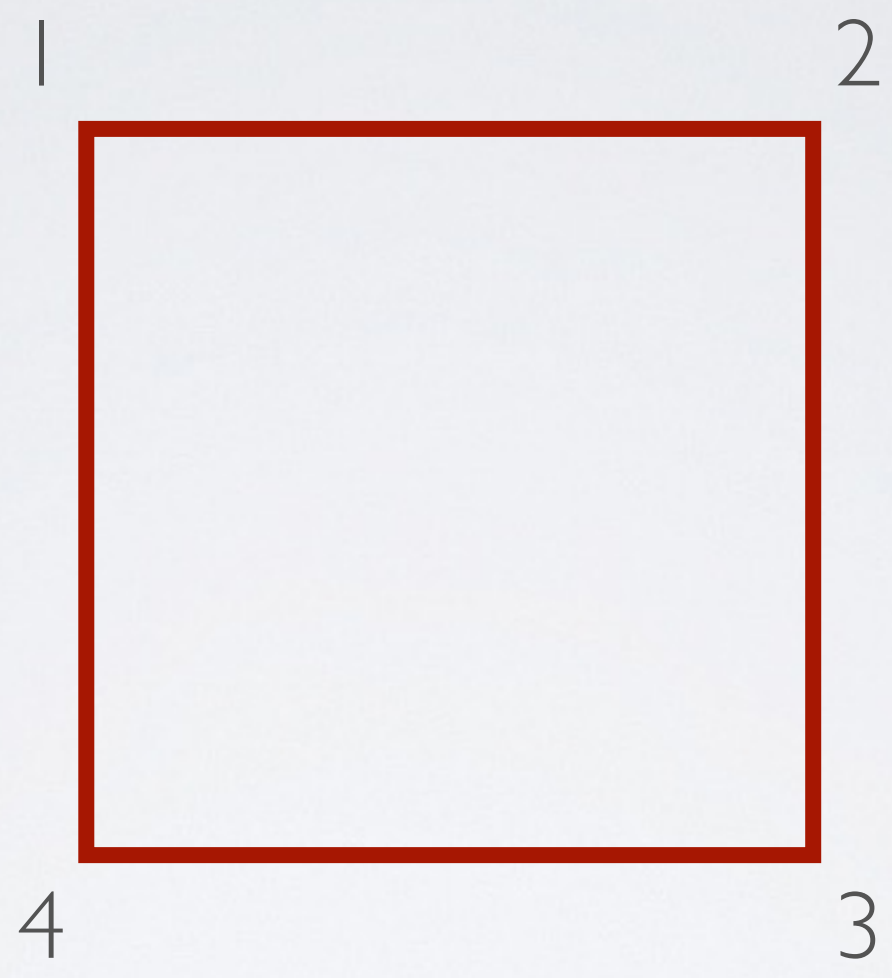

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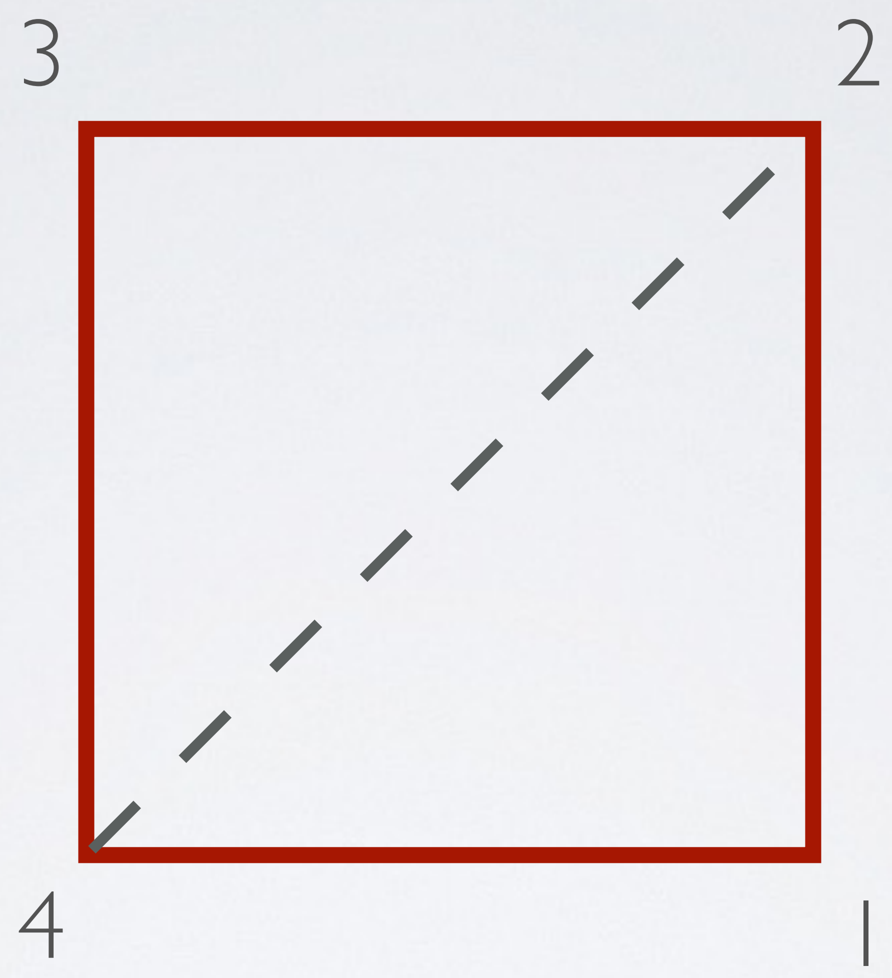

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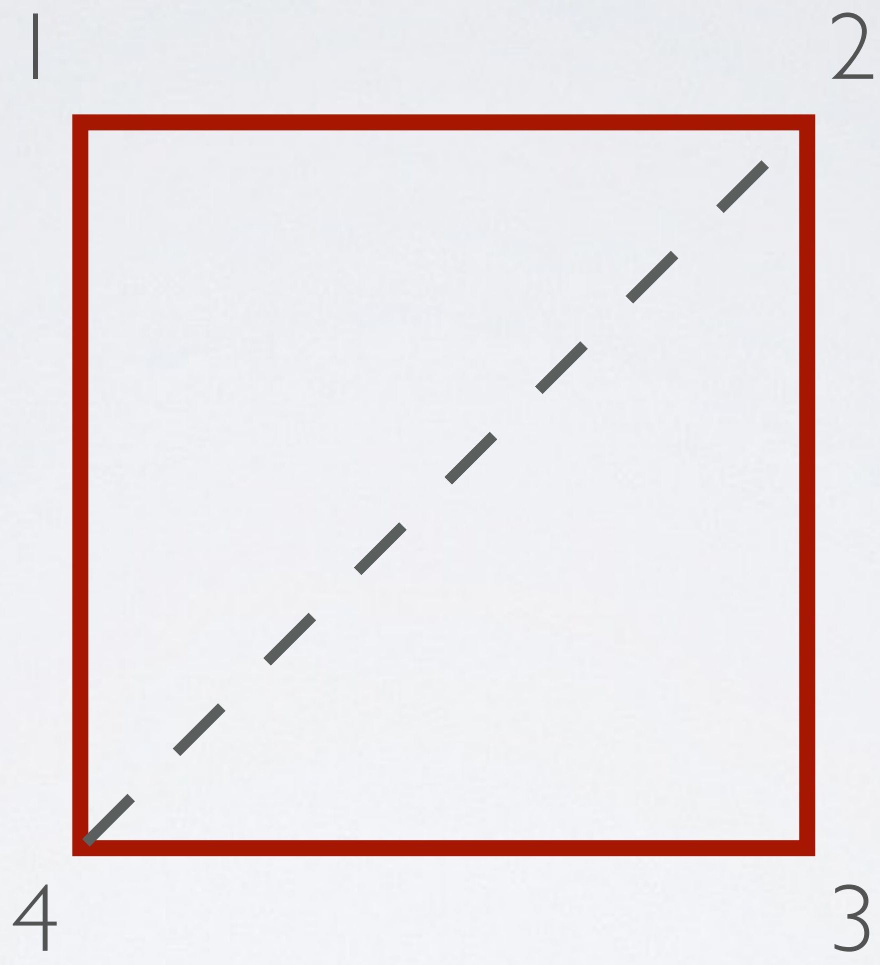

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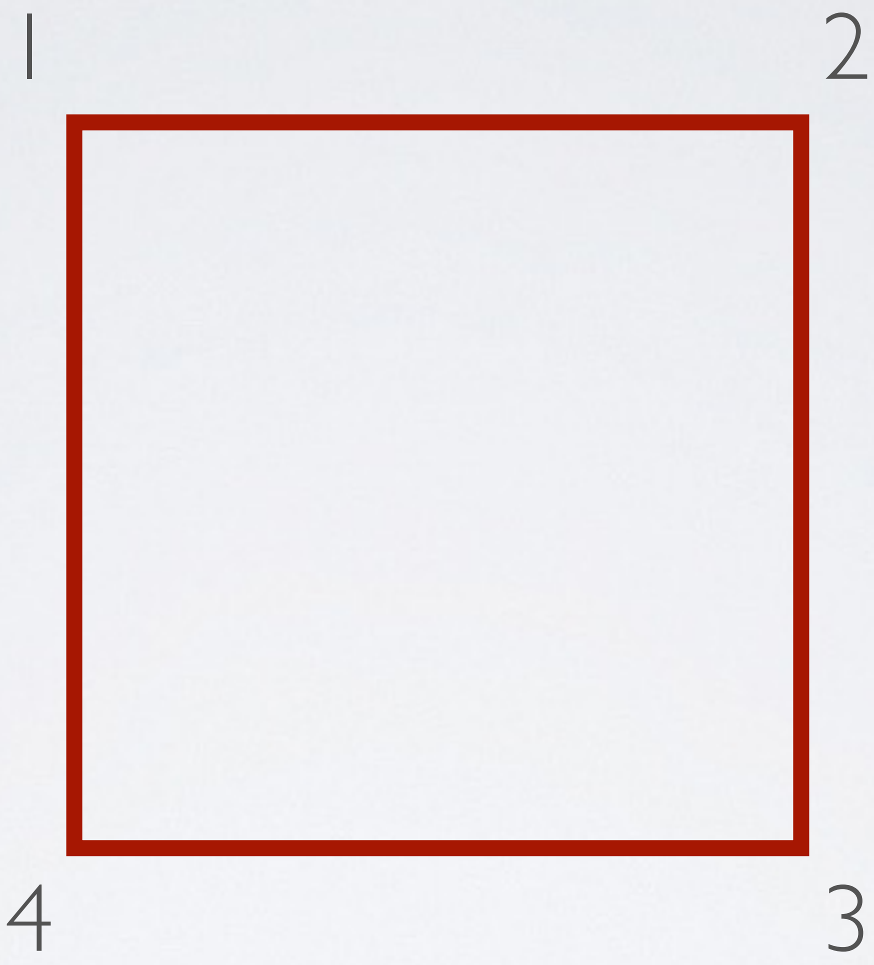

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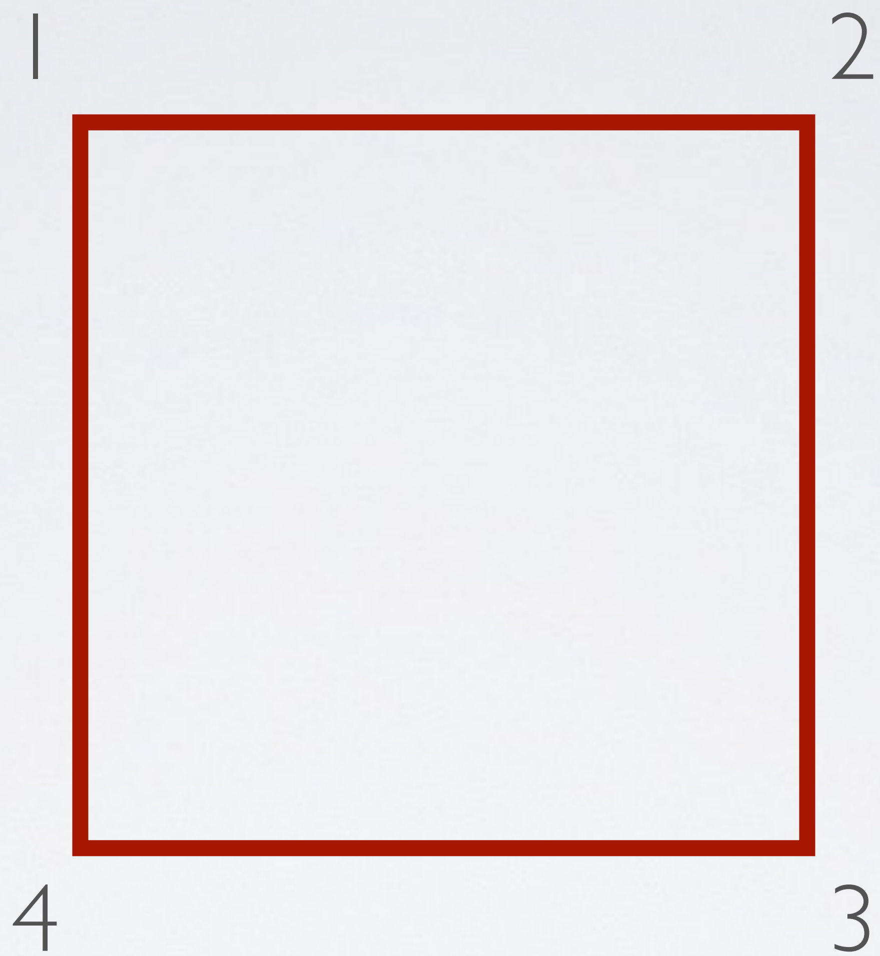

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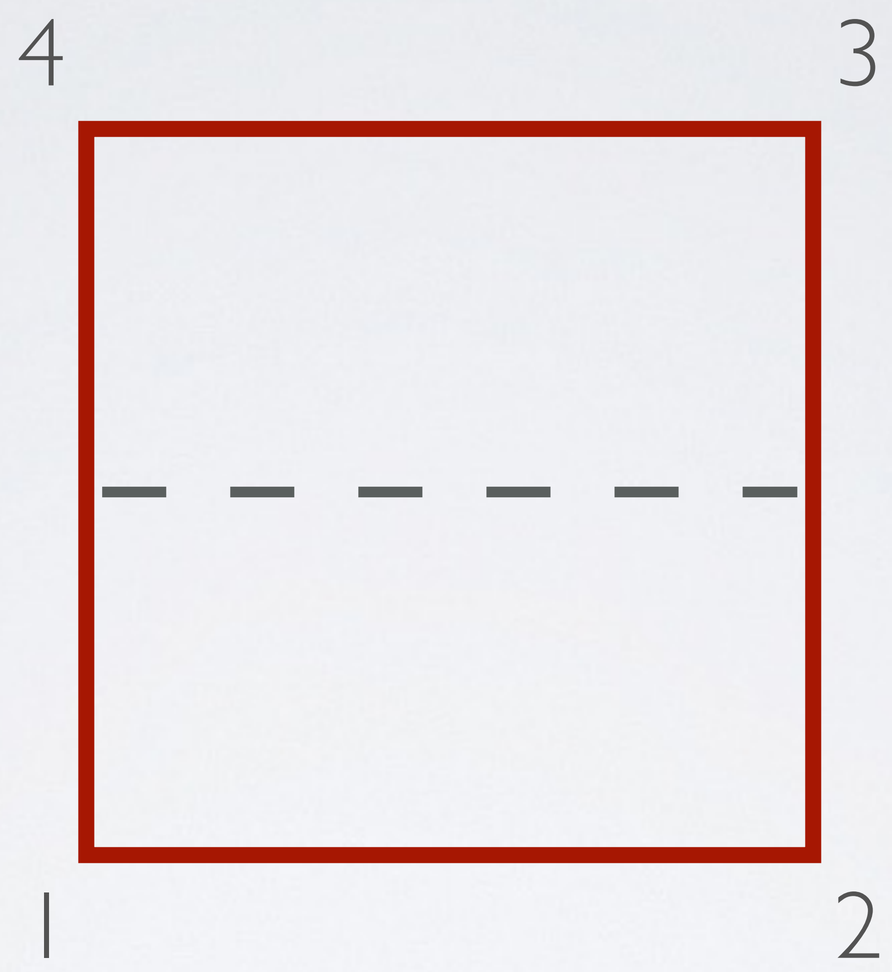

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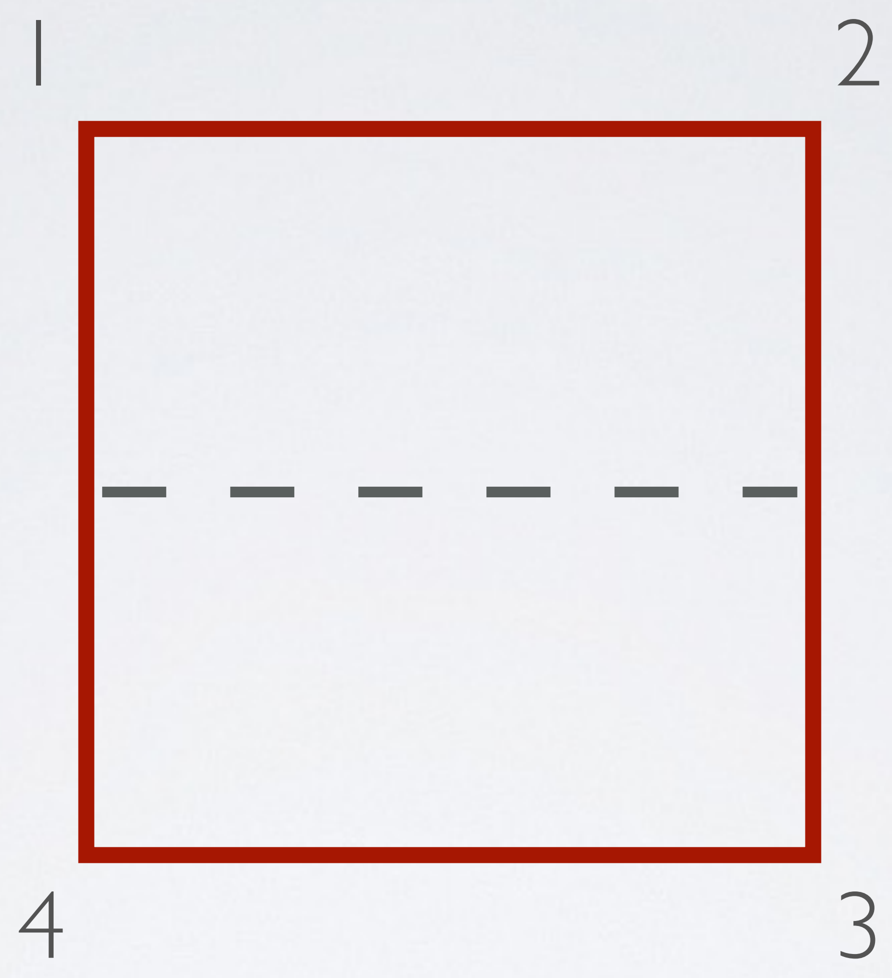

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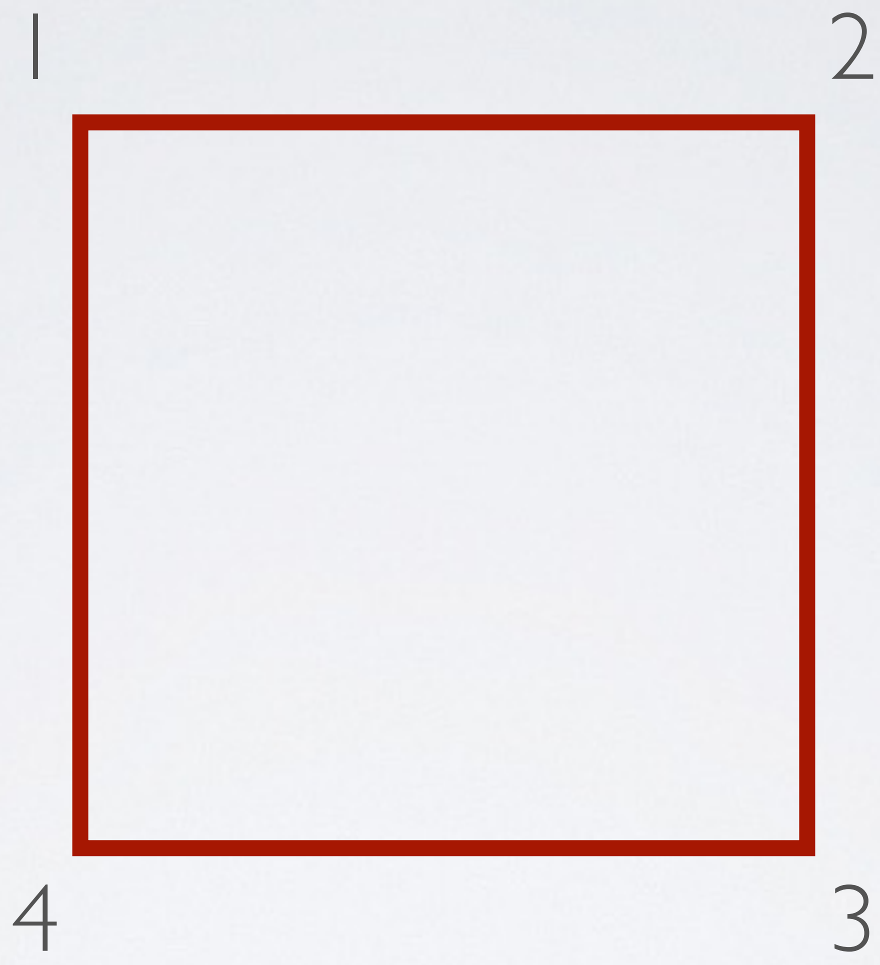

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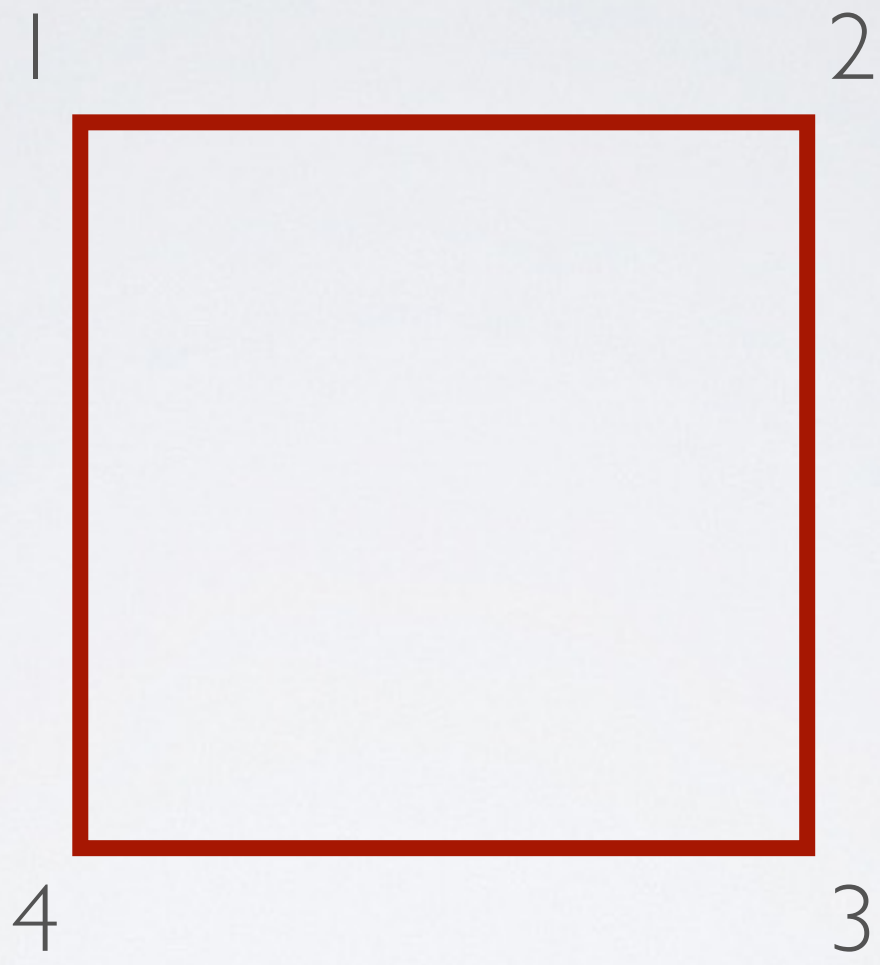

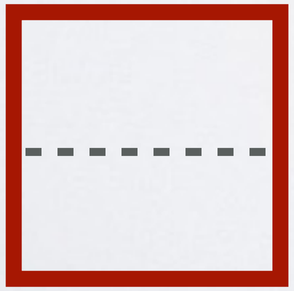

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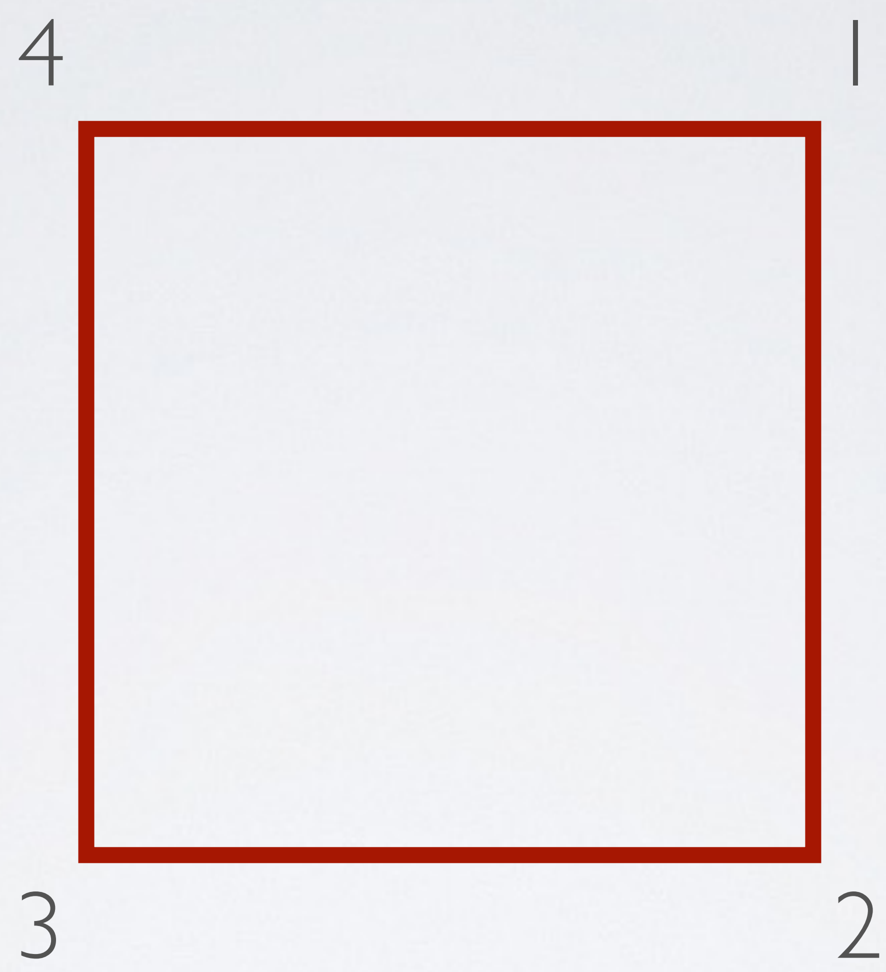

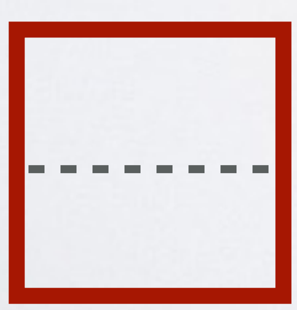

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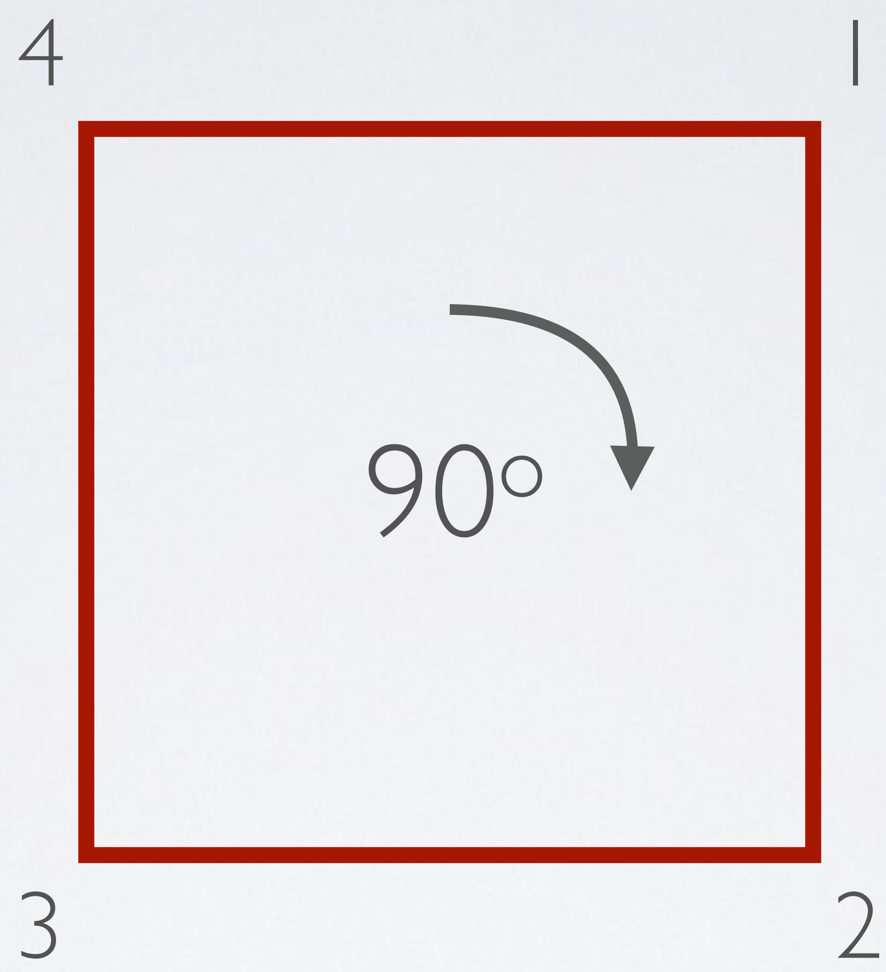

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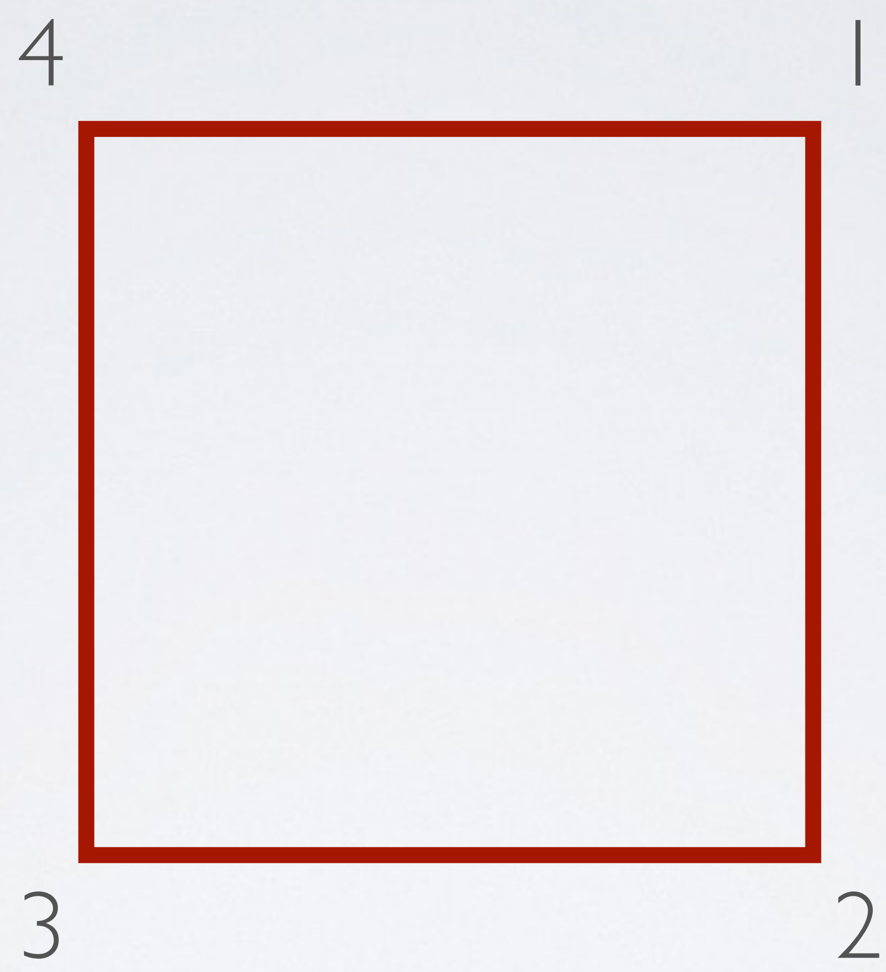

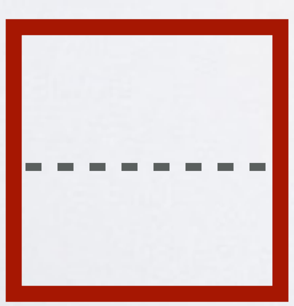

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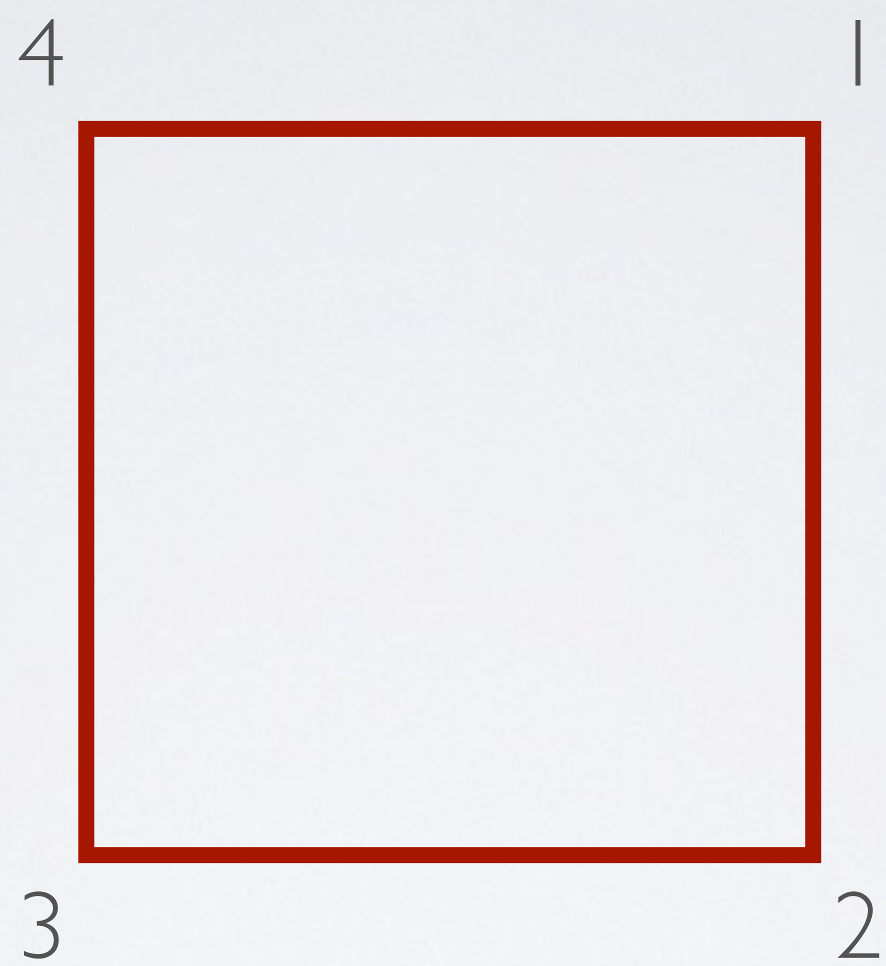

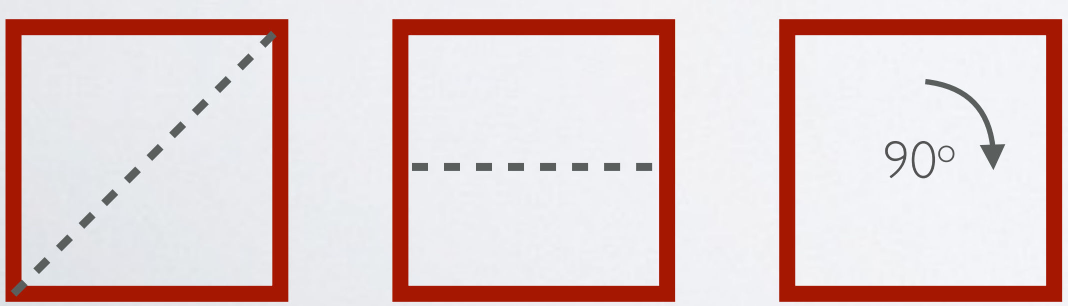

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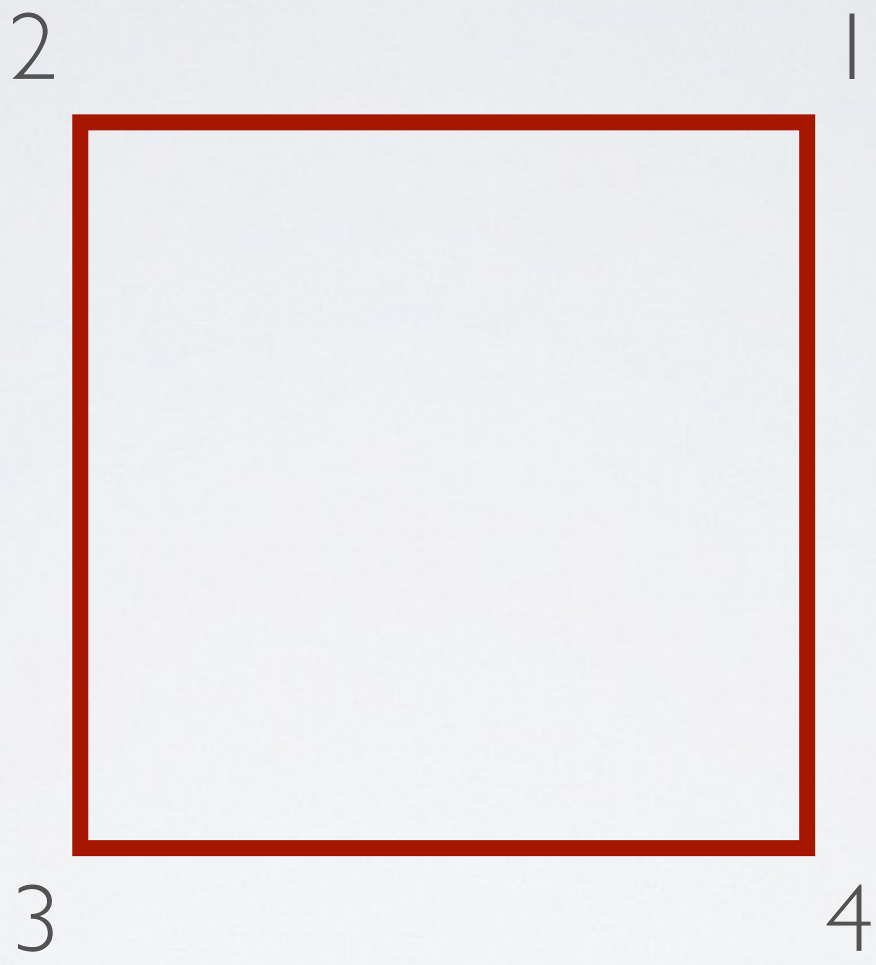

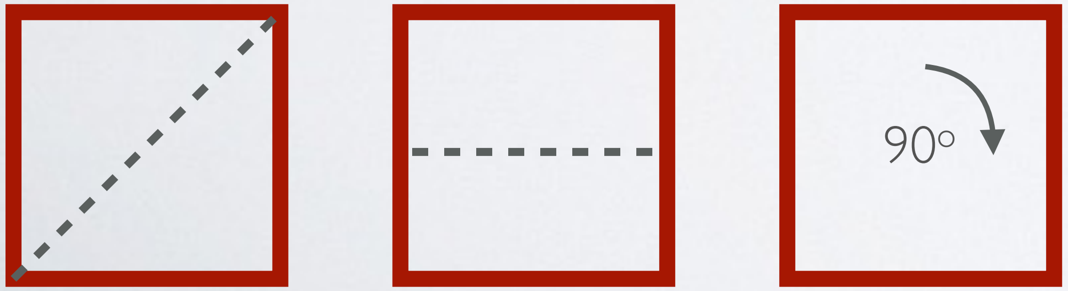

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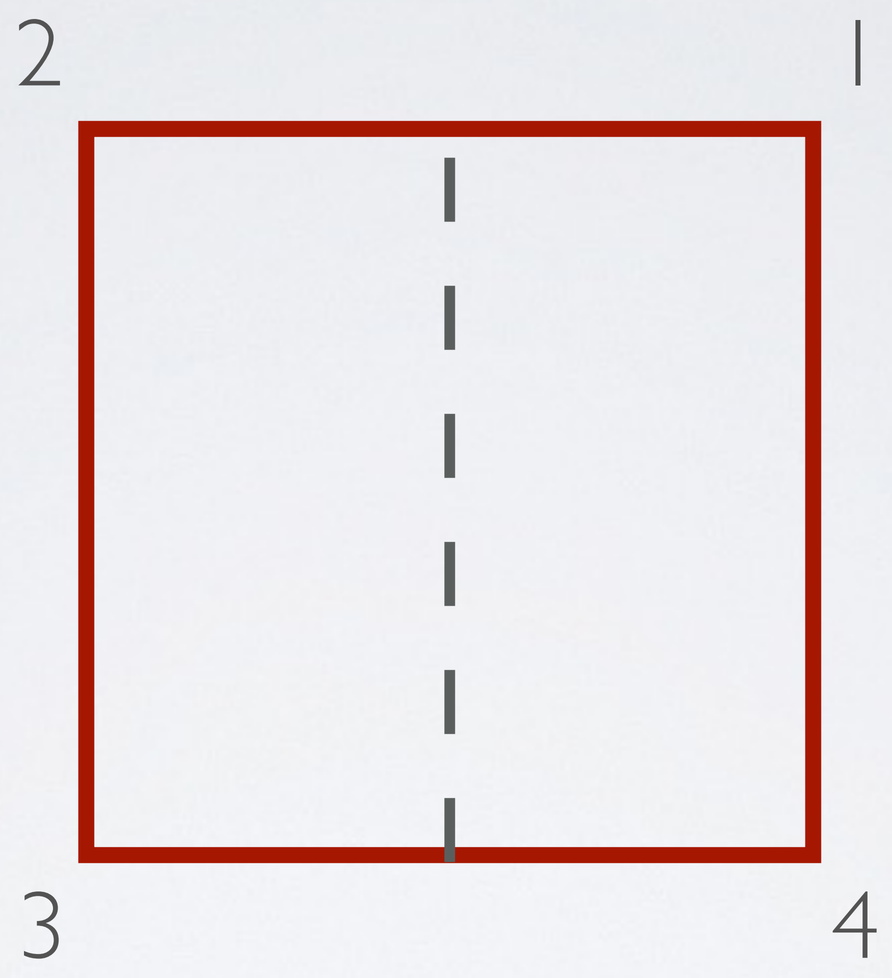

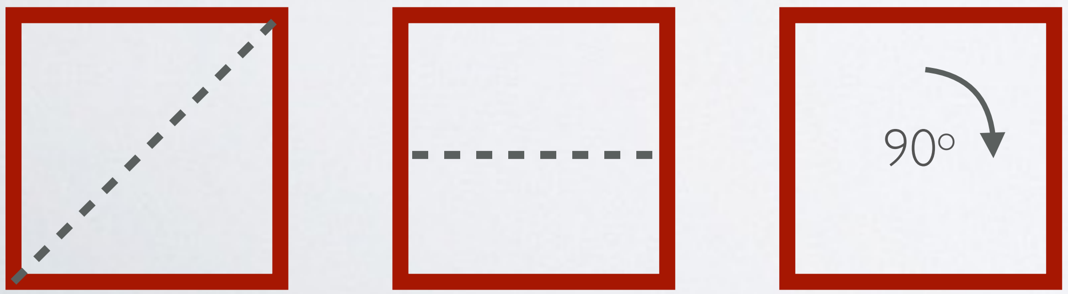

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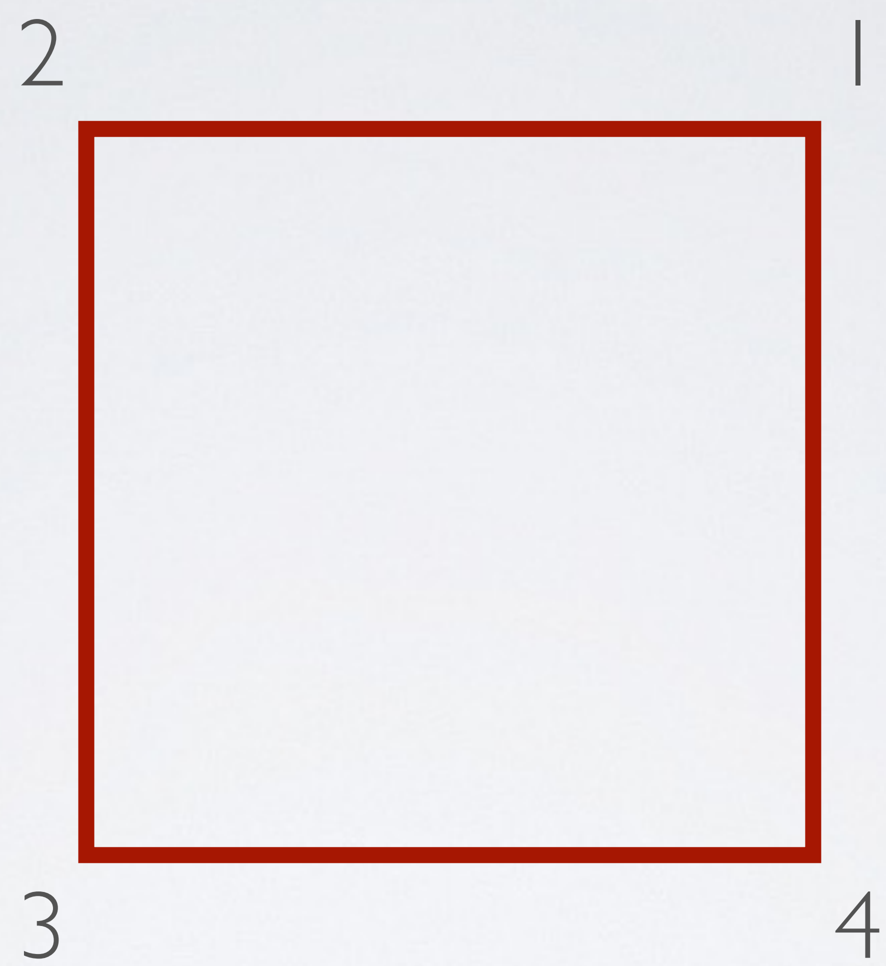

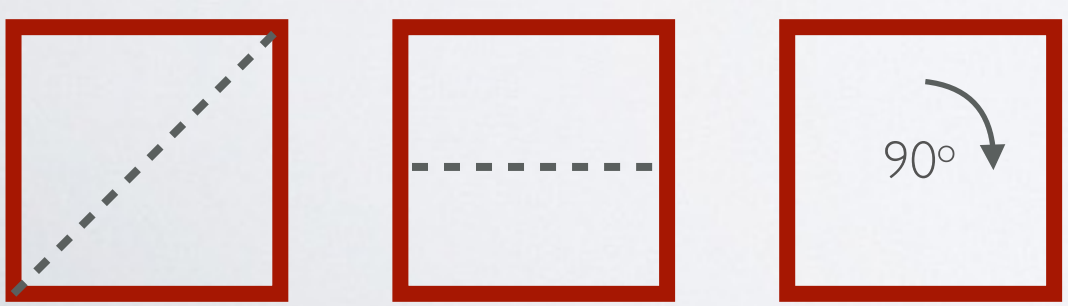

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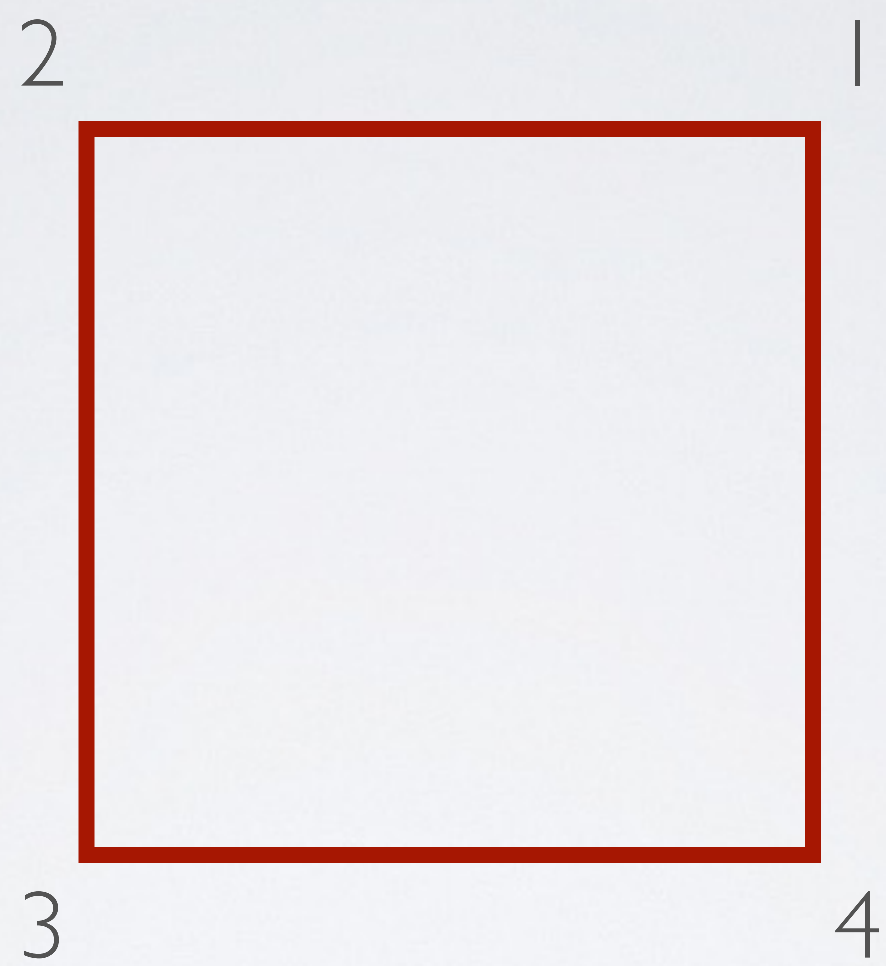

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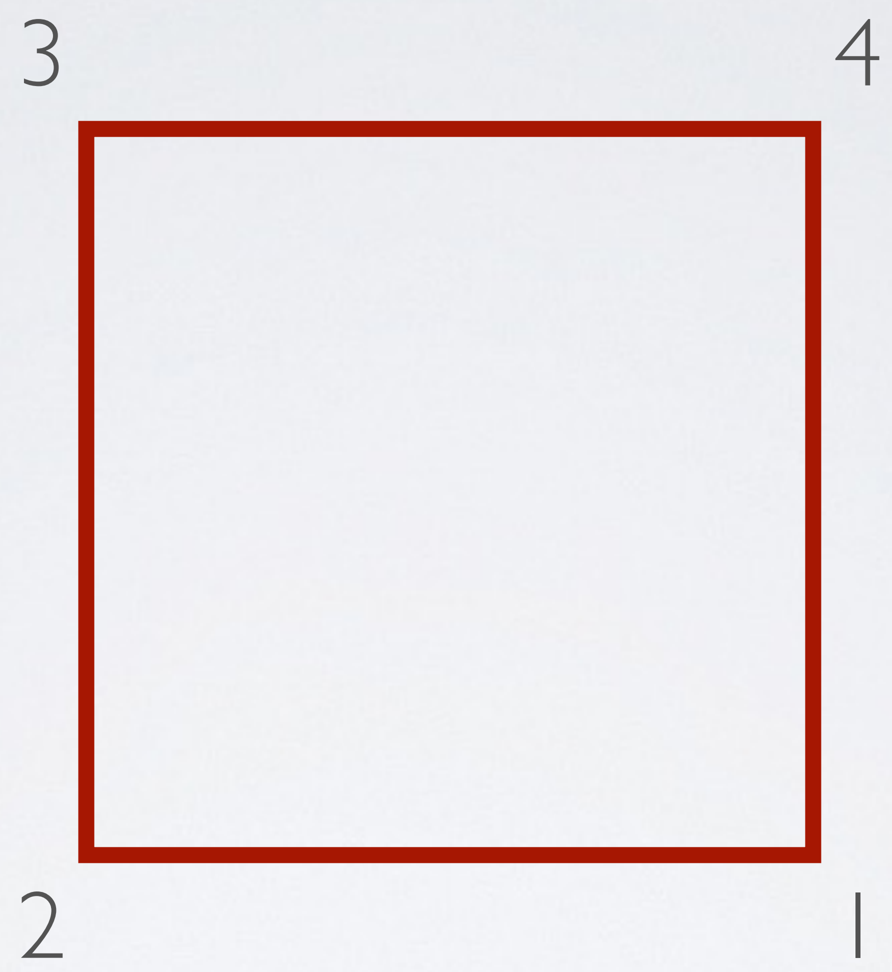

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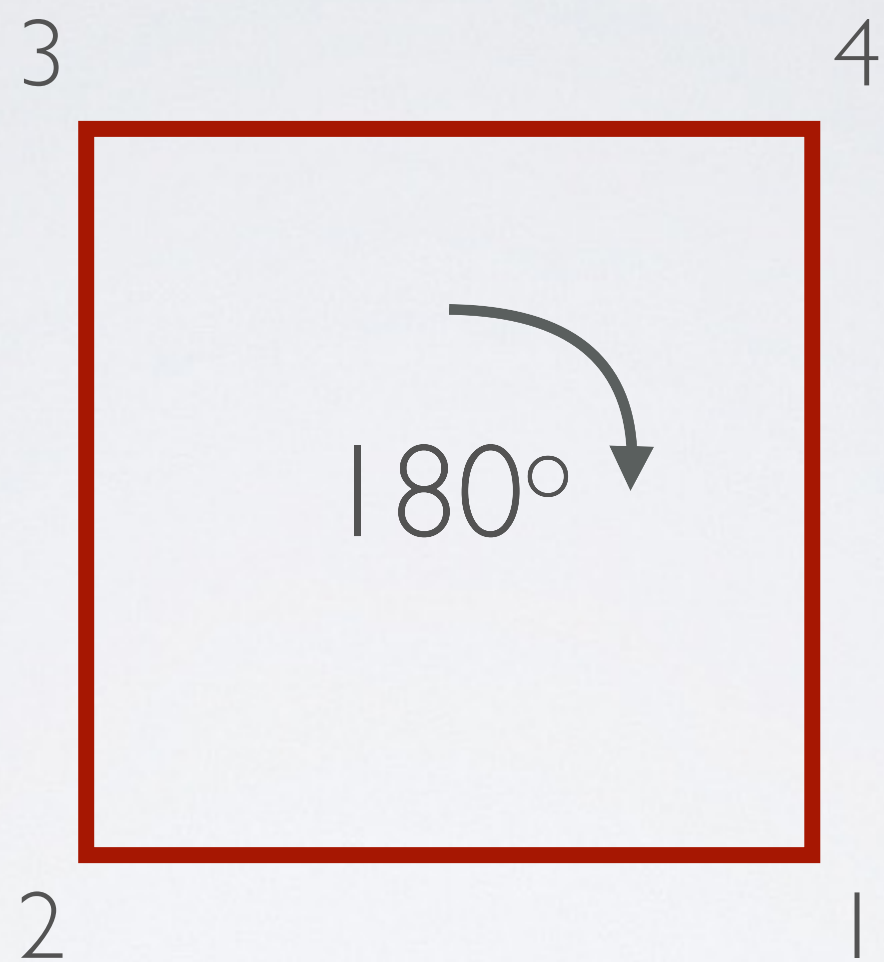

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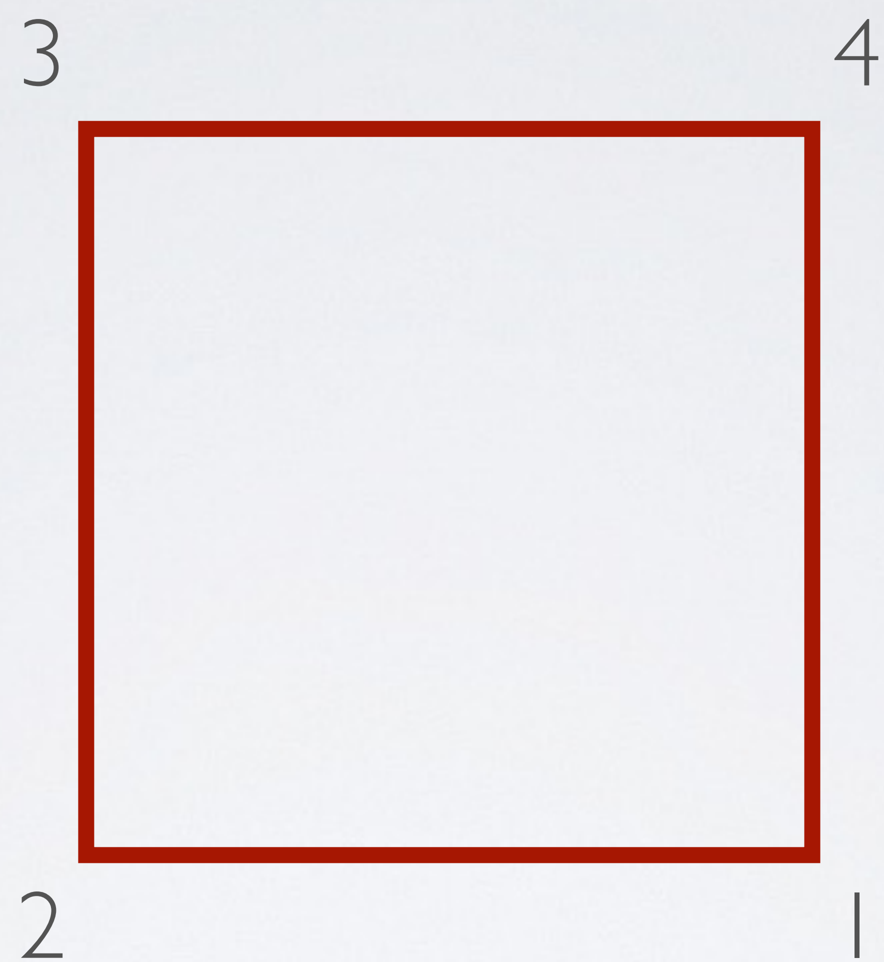

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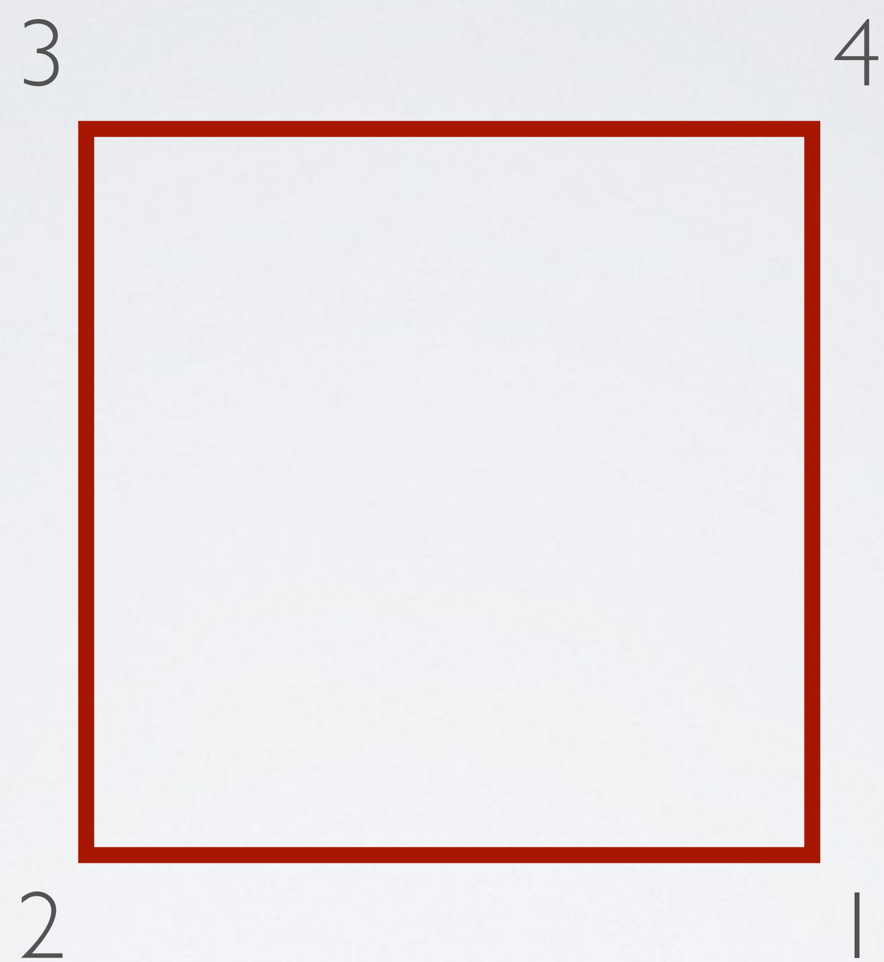

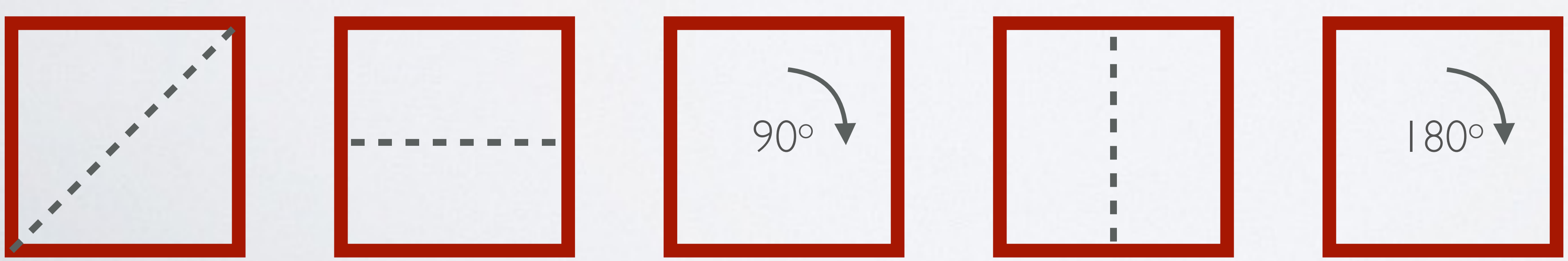

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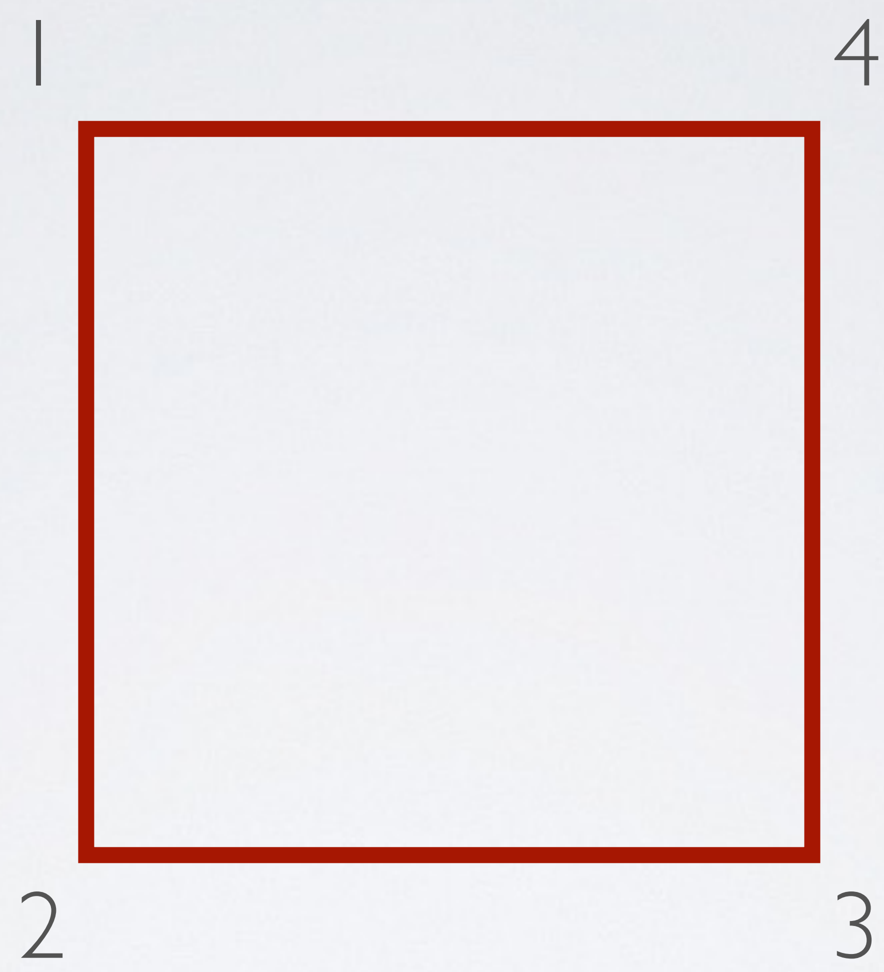

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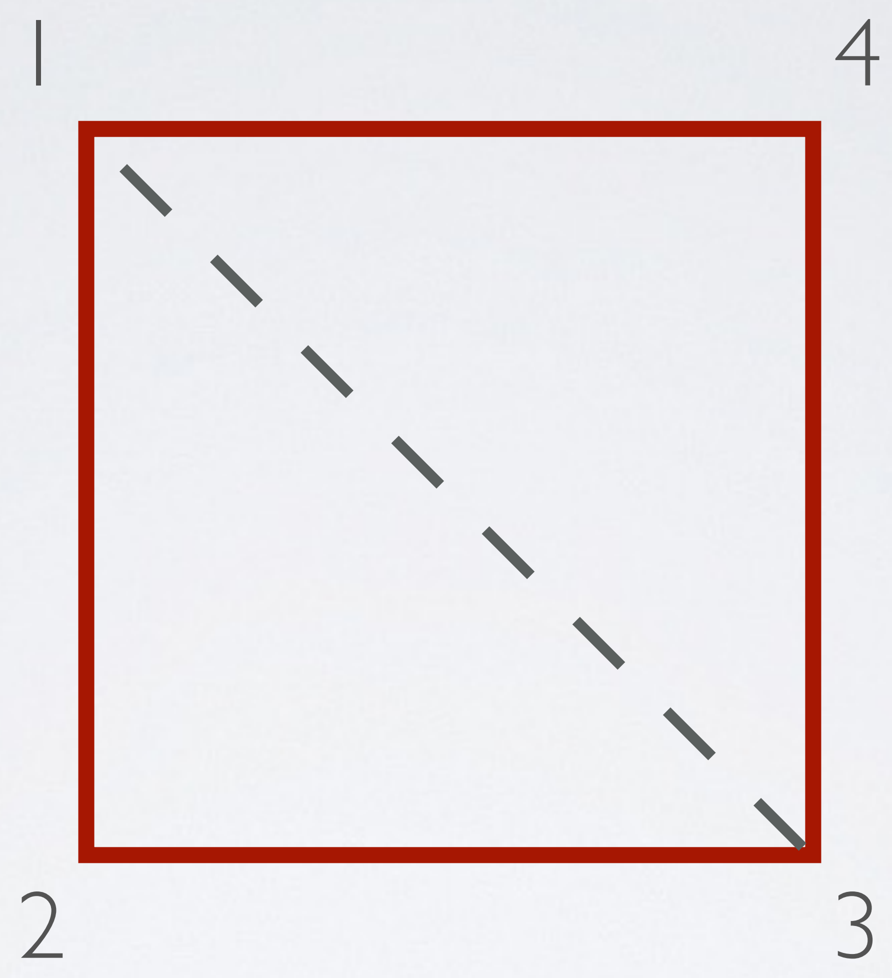

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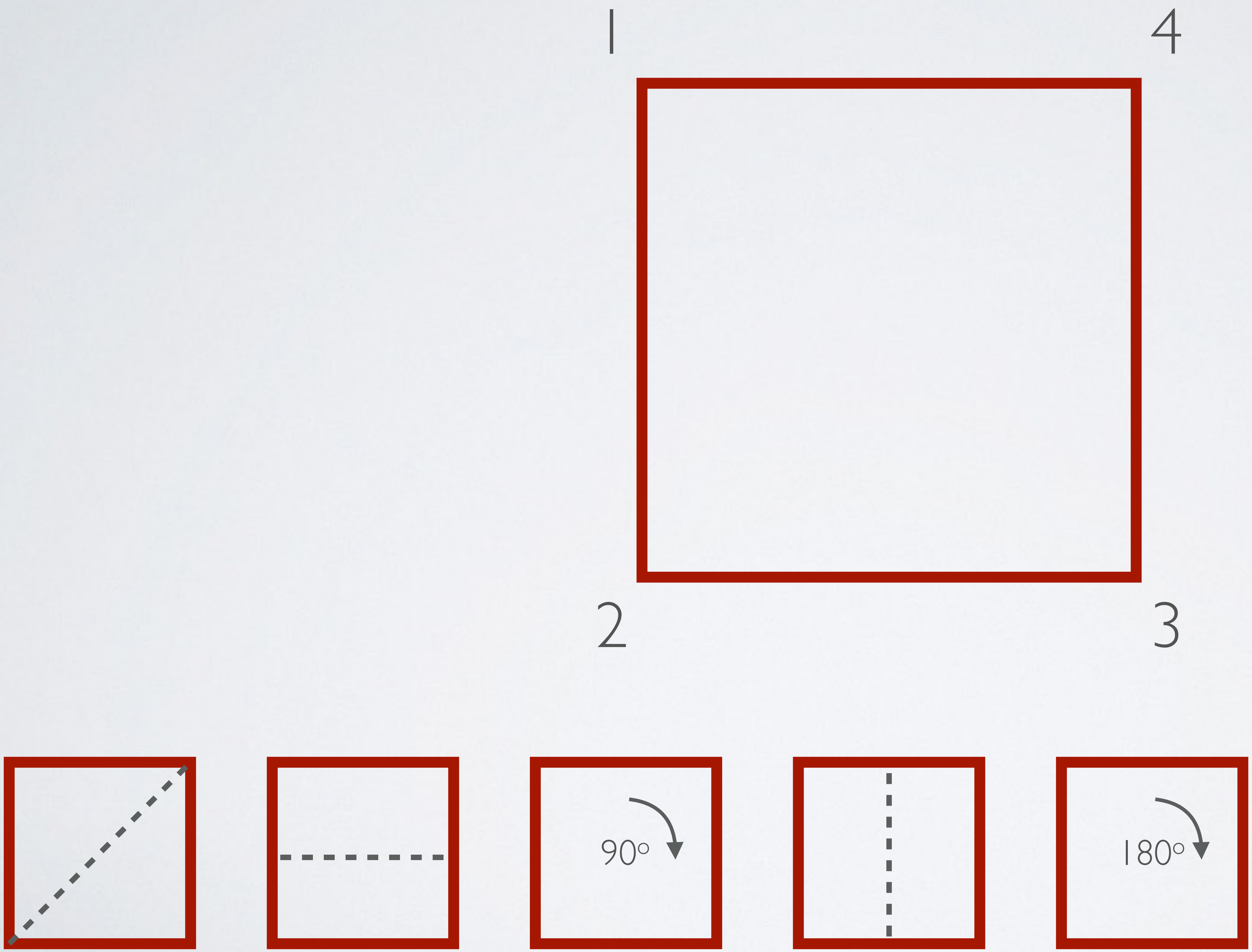

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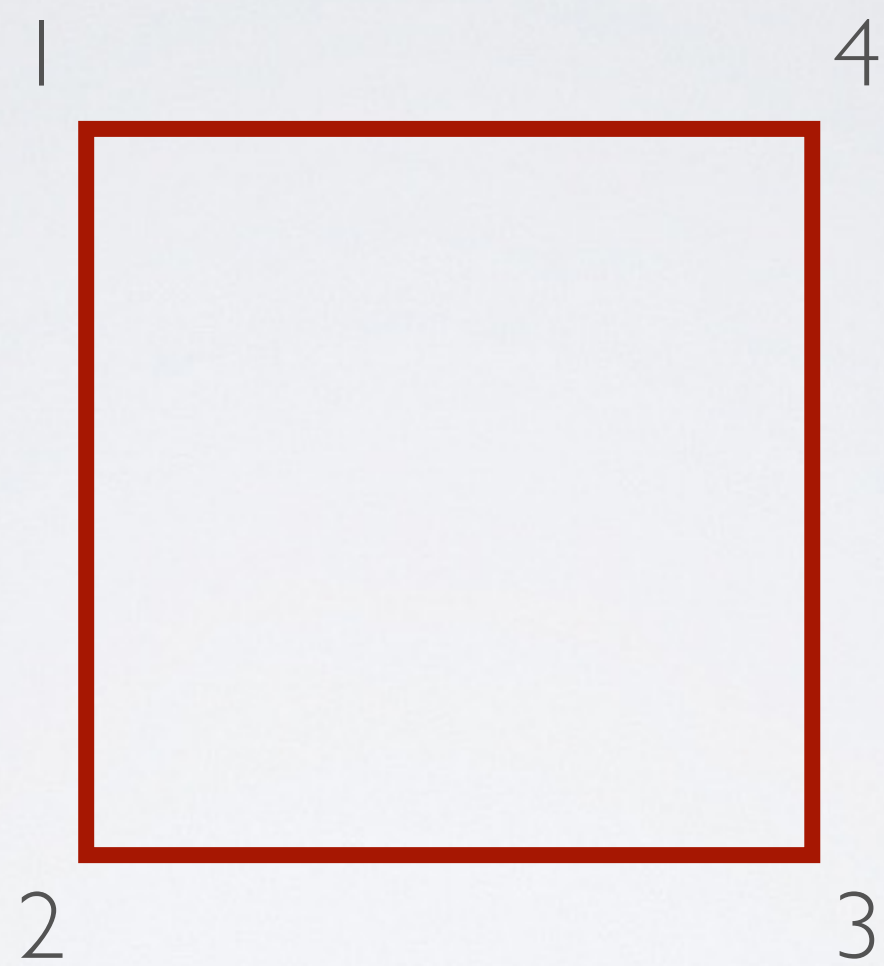

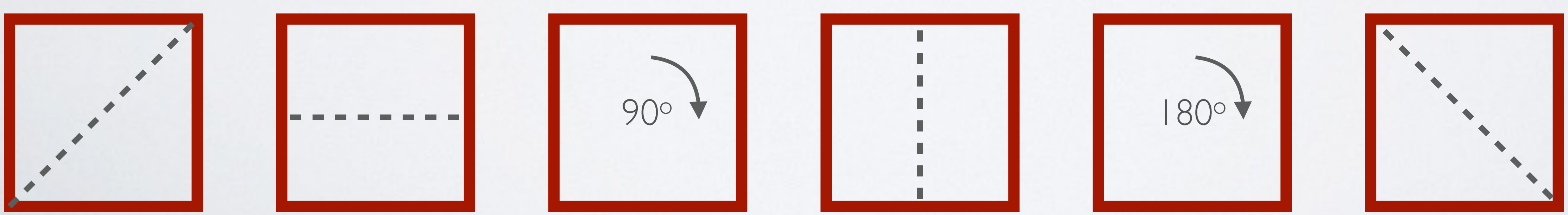

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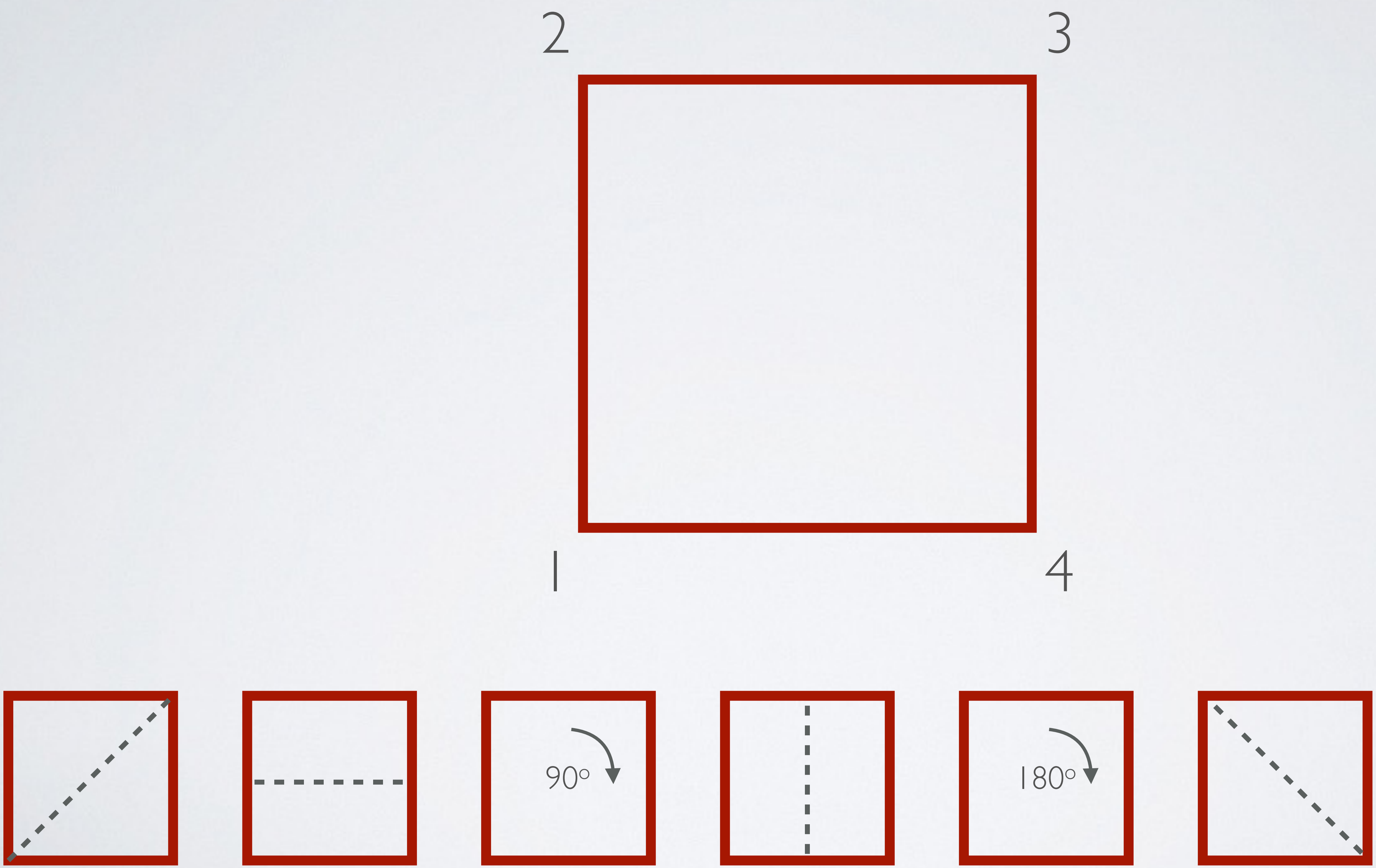

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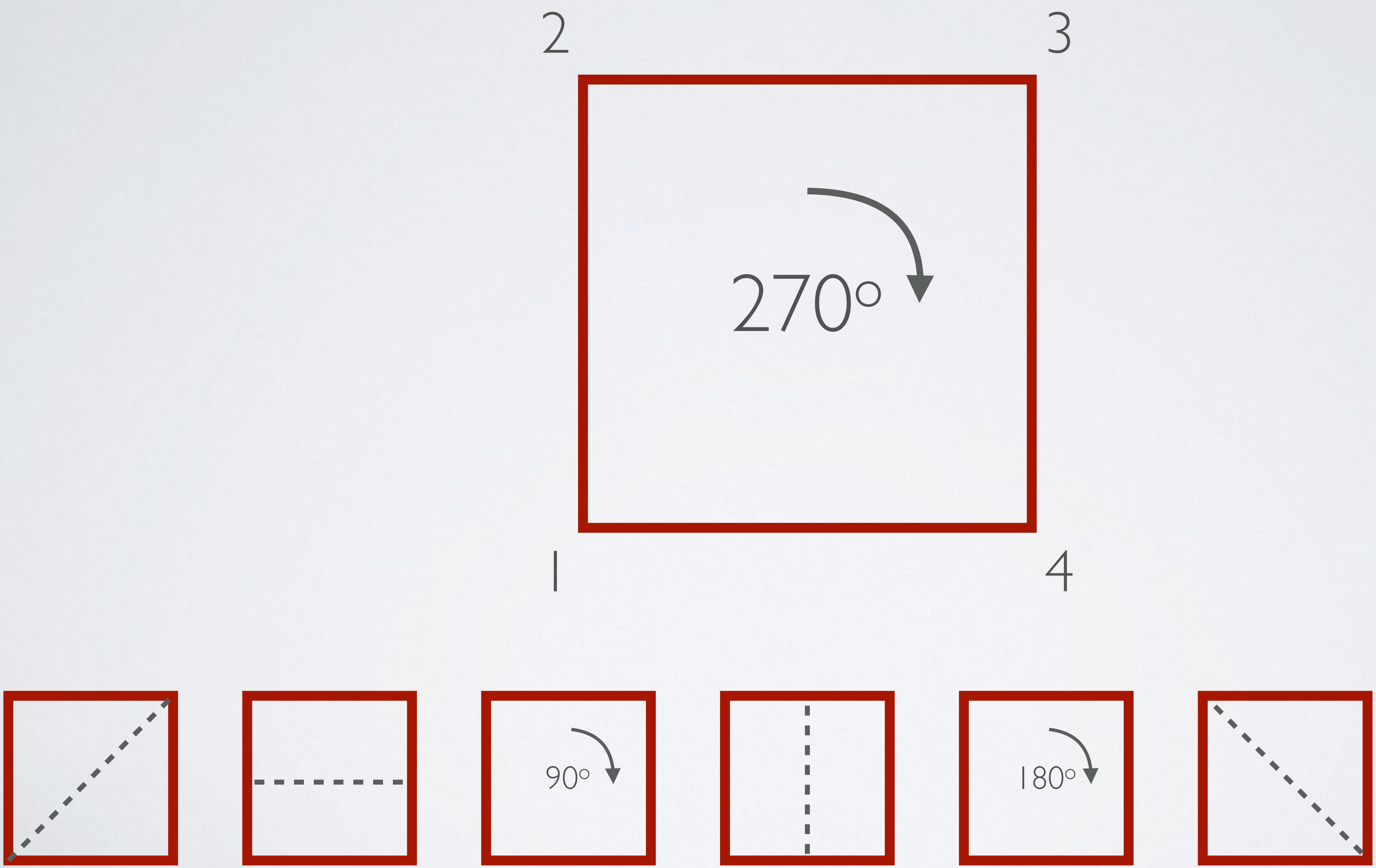

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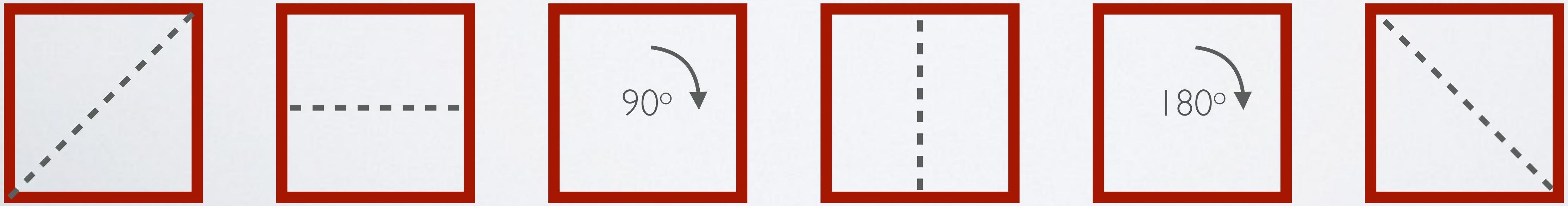

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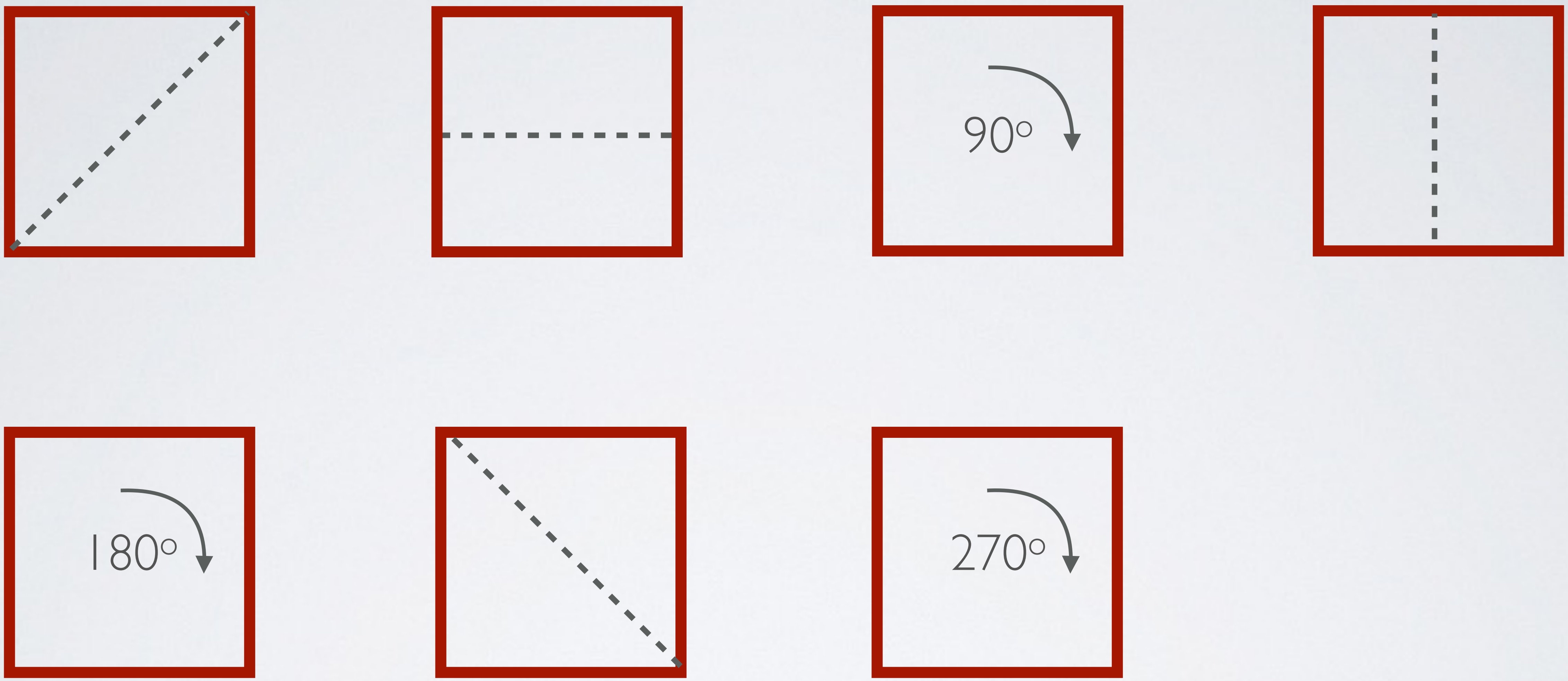

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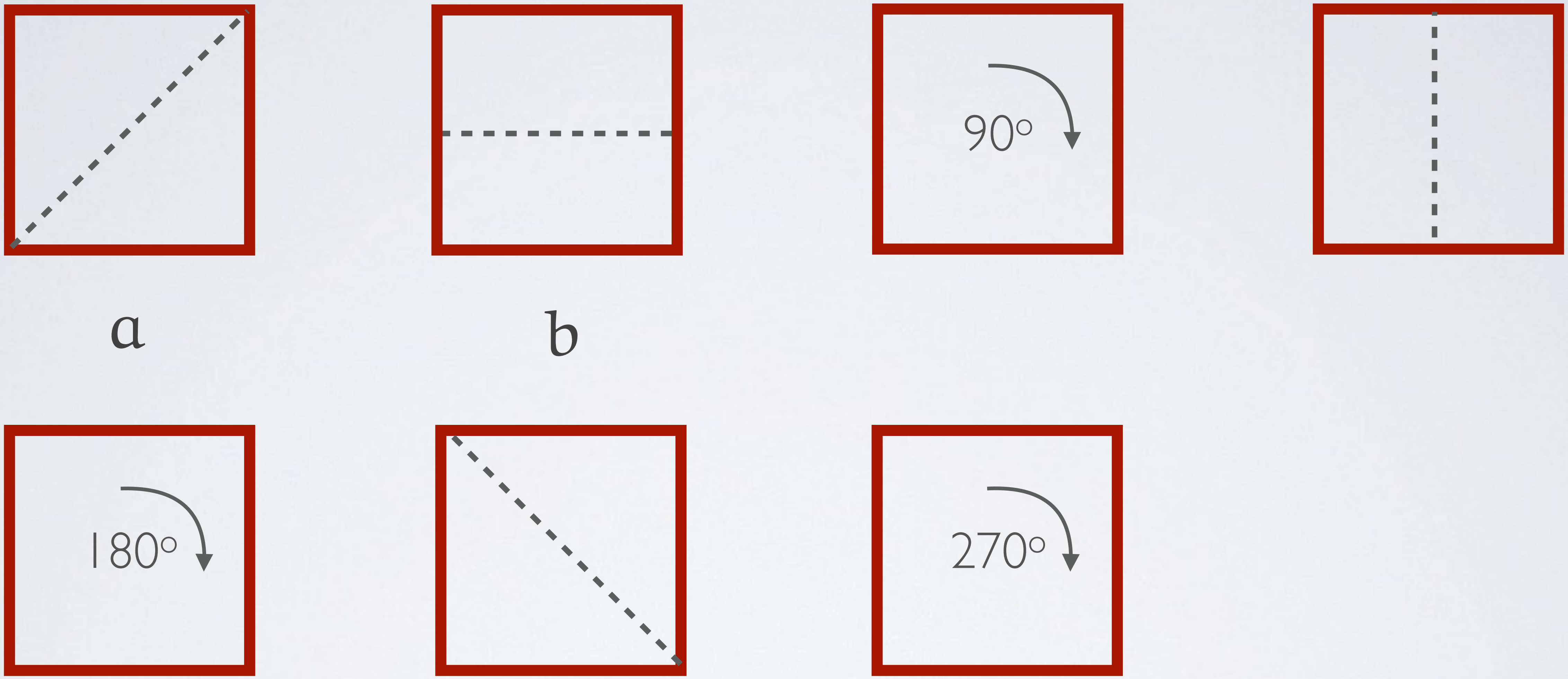

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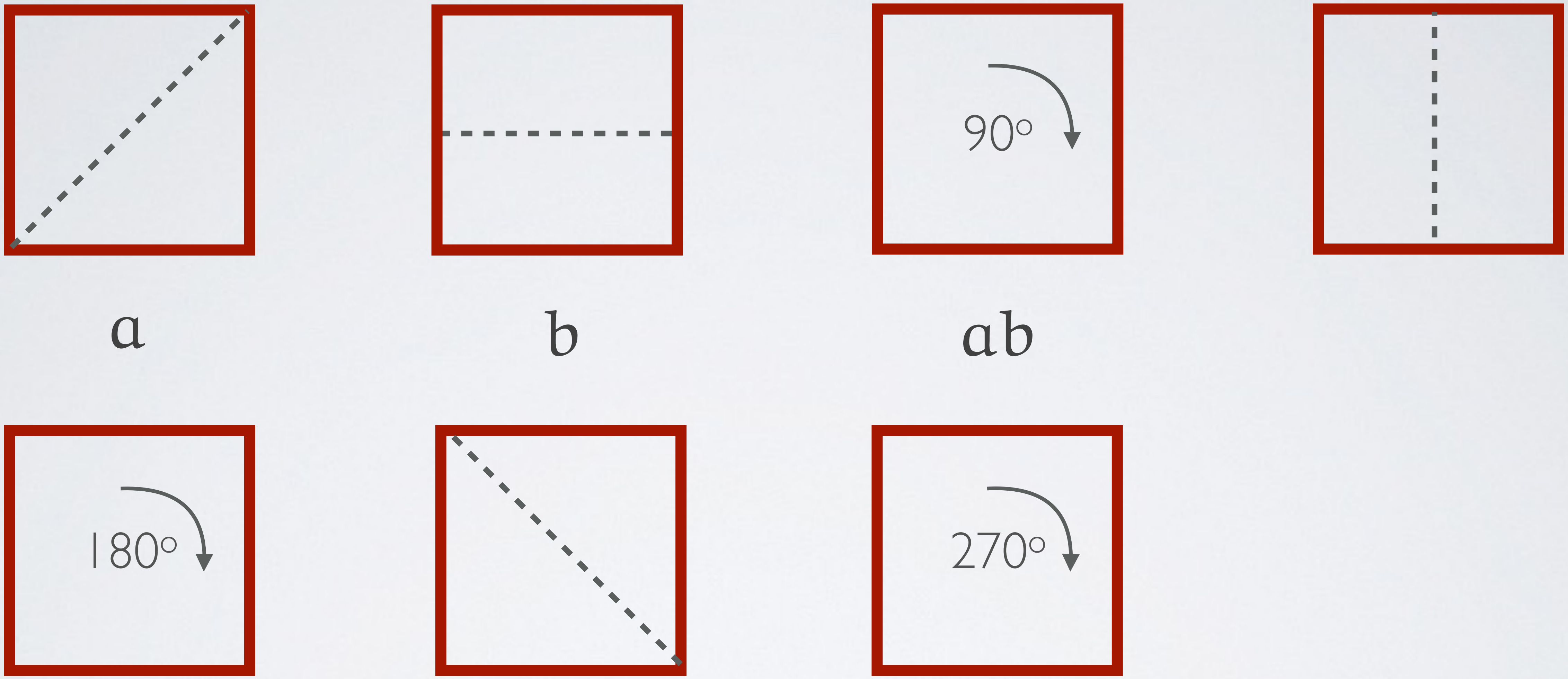

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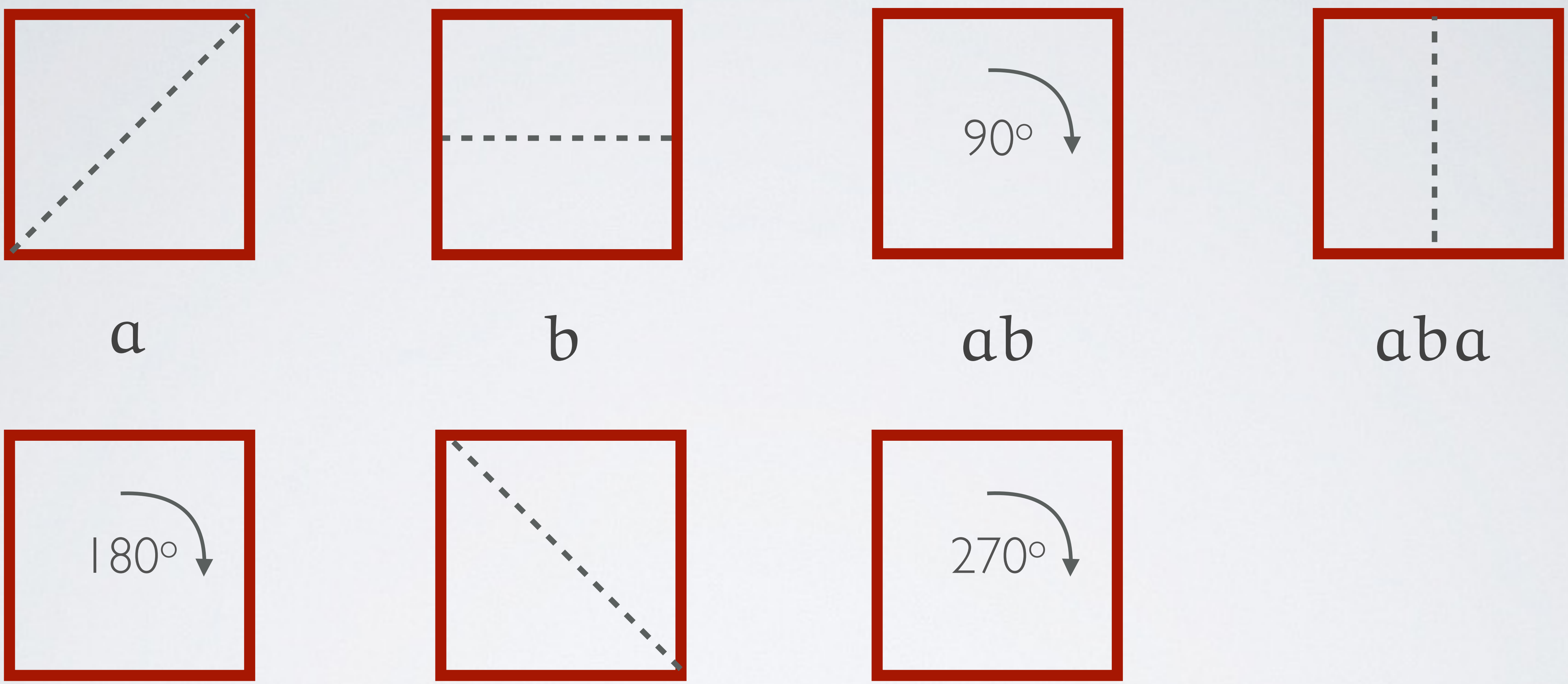

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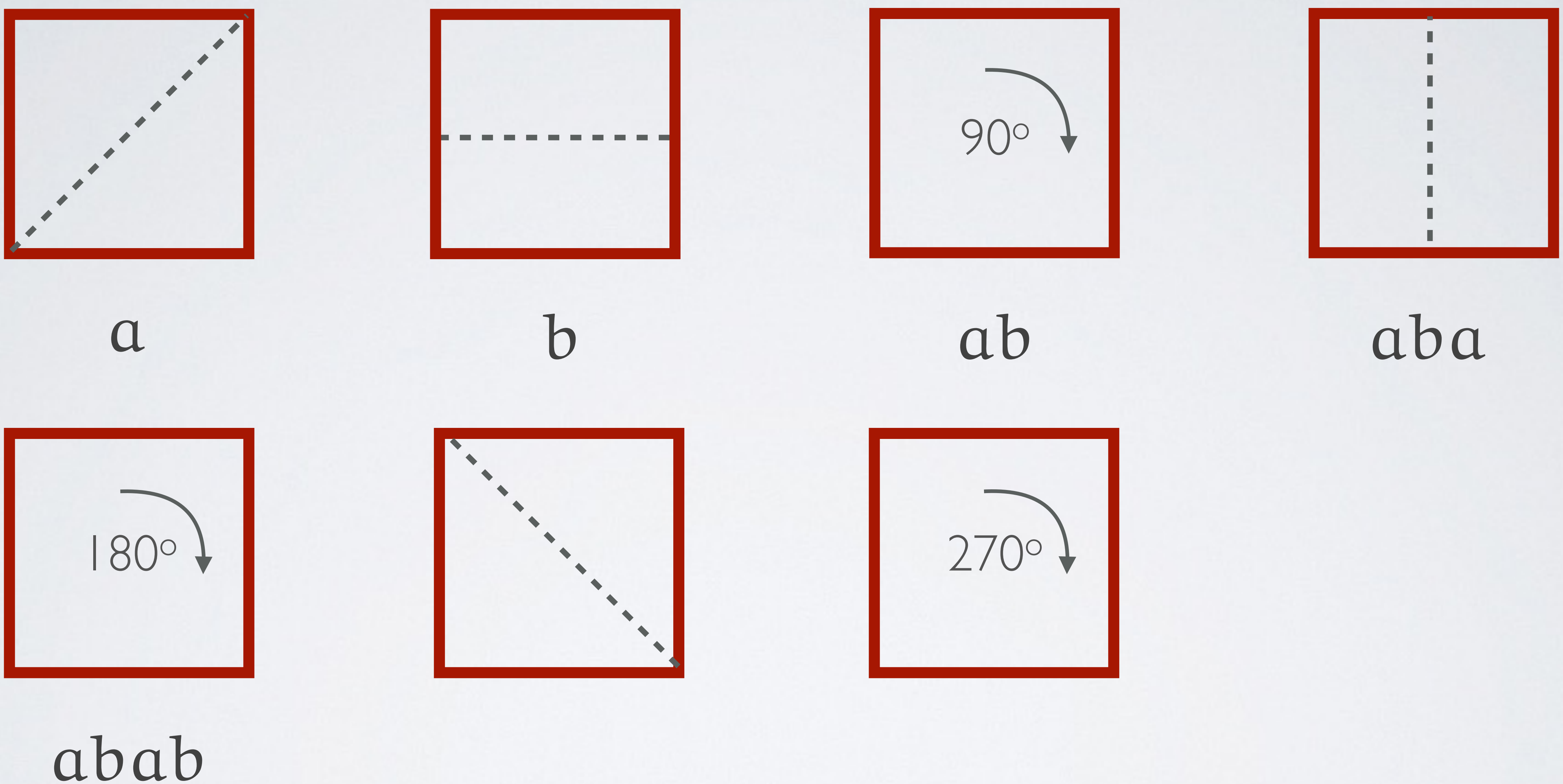

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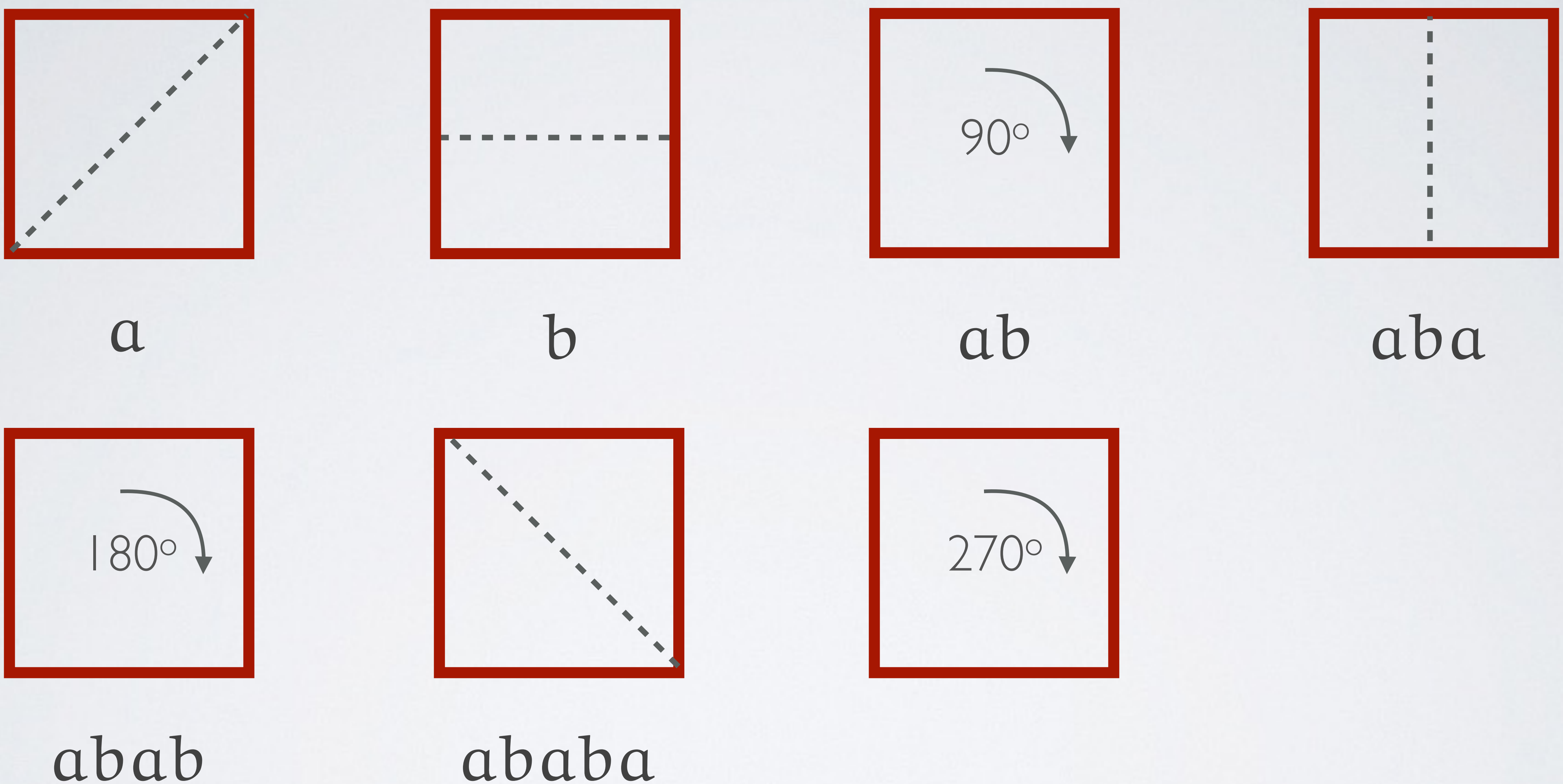

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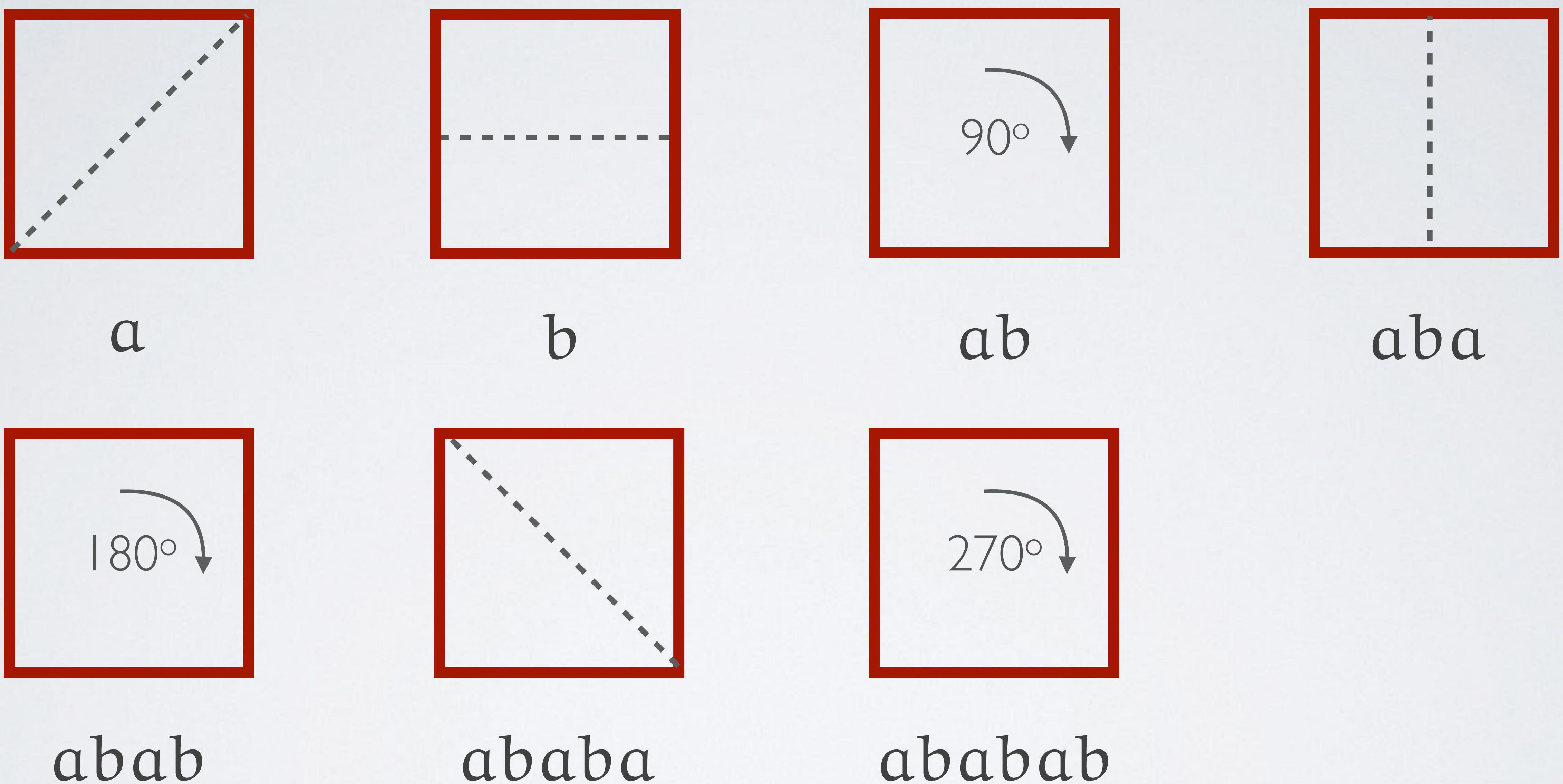

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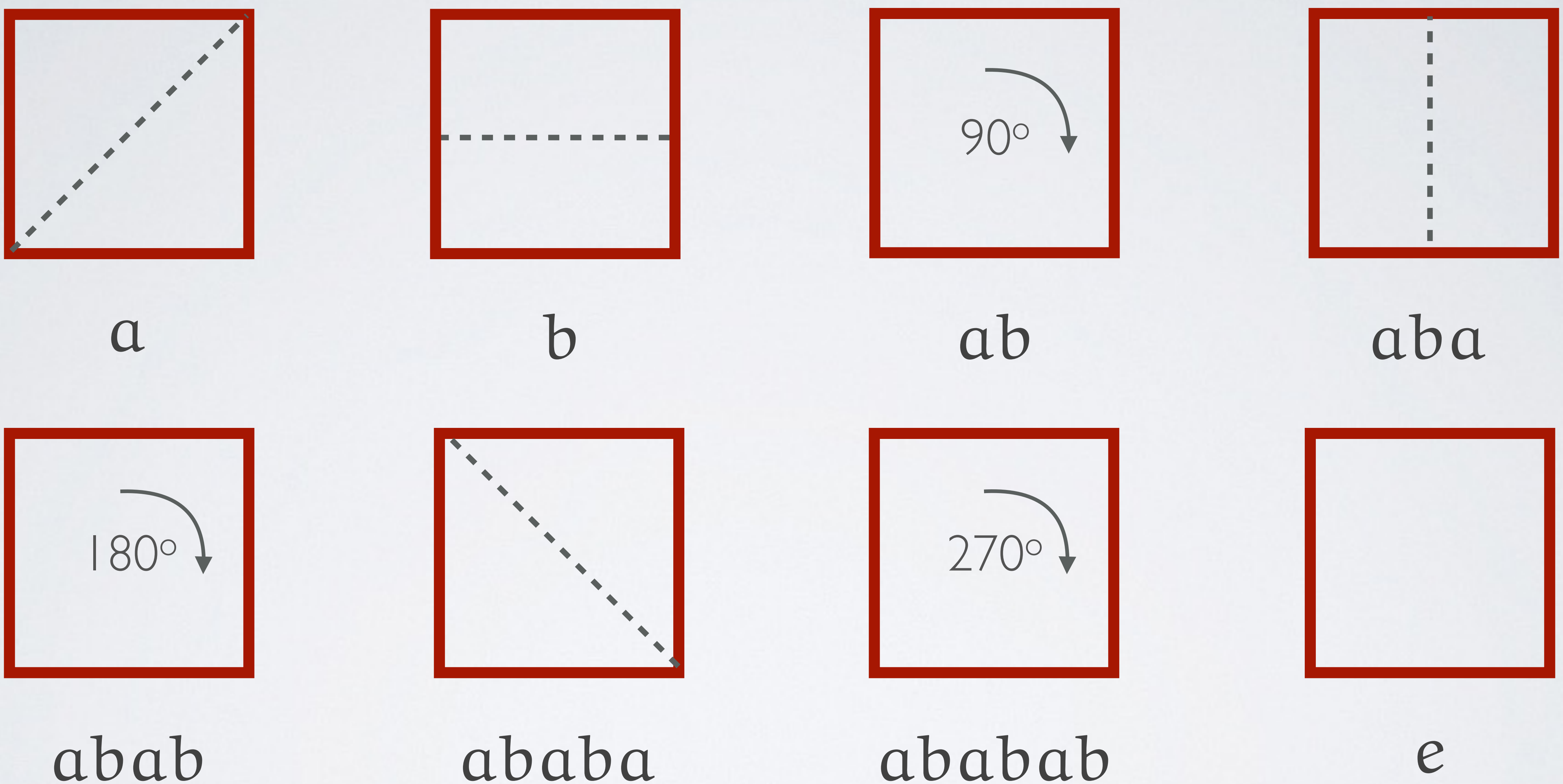

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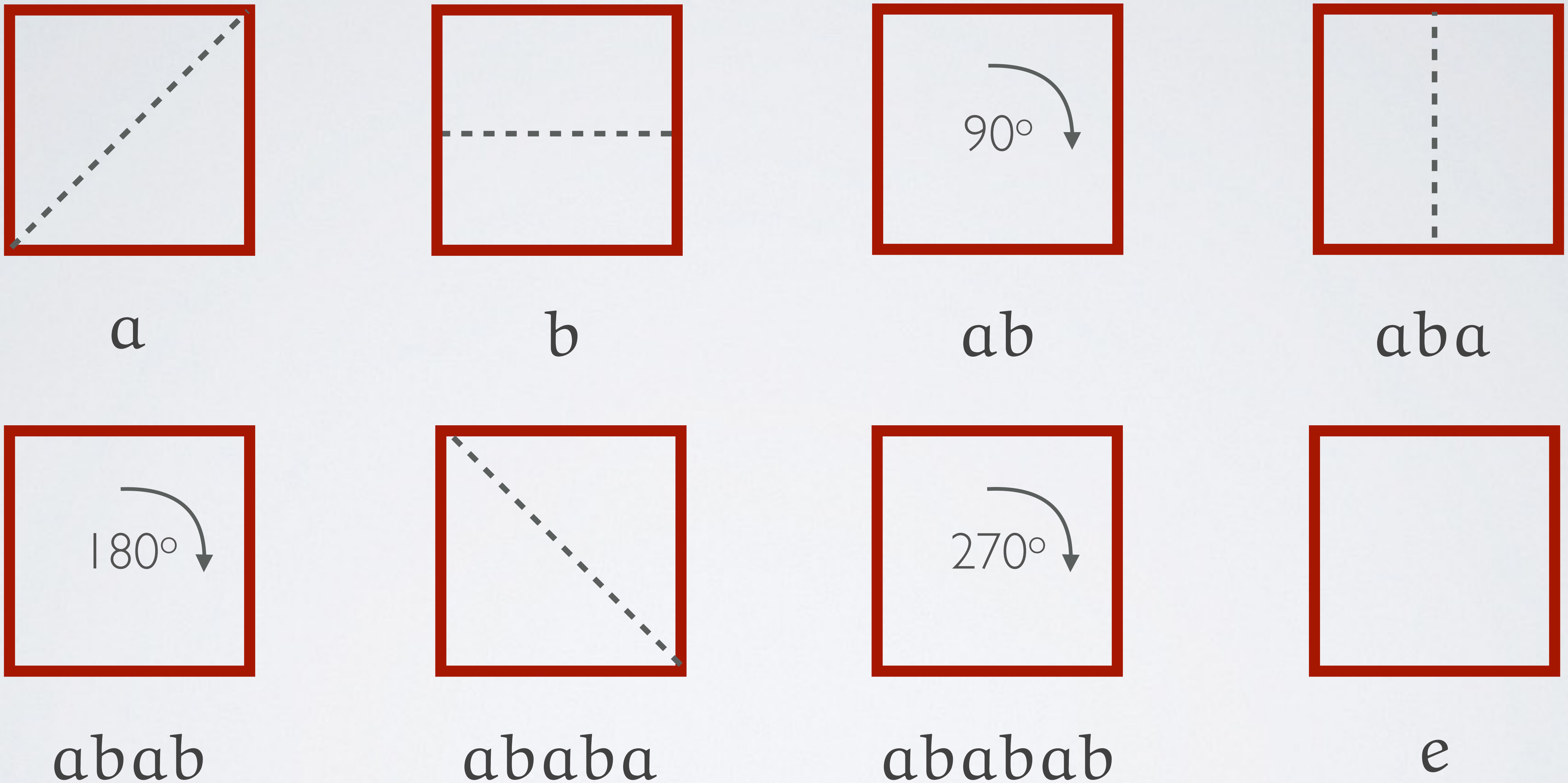
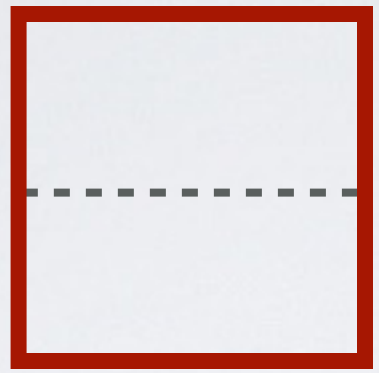


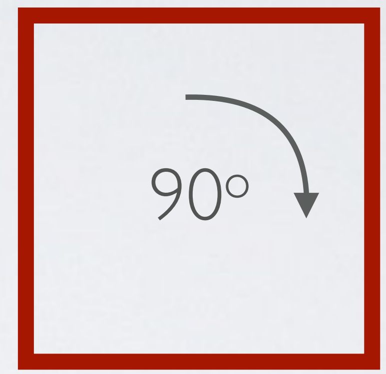
$$a^2 = e,$$

DIHEDRAL GROUPS

a

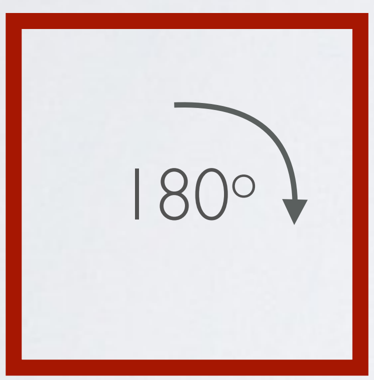


b

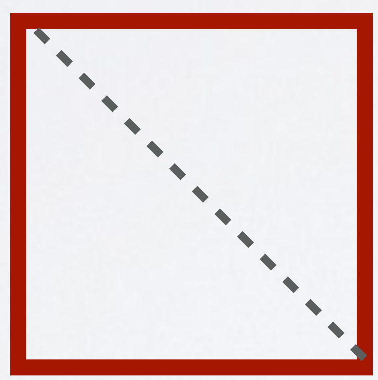


ab

aba



$abab$



$ababa$

$ababab$

e

$$a^2 = e, \quad b^2 = e,$$

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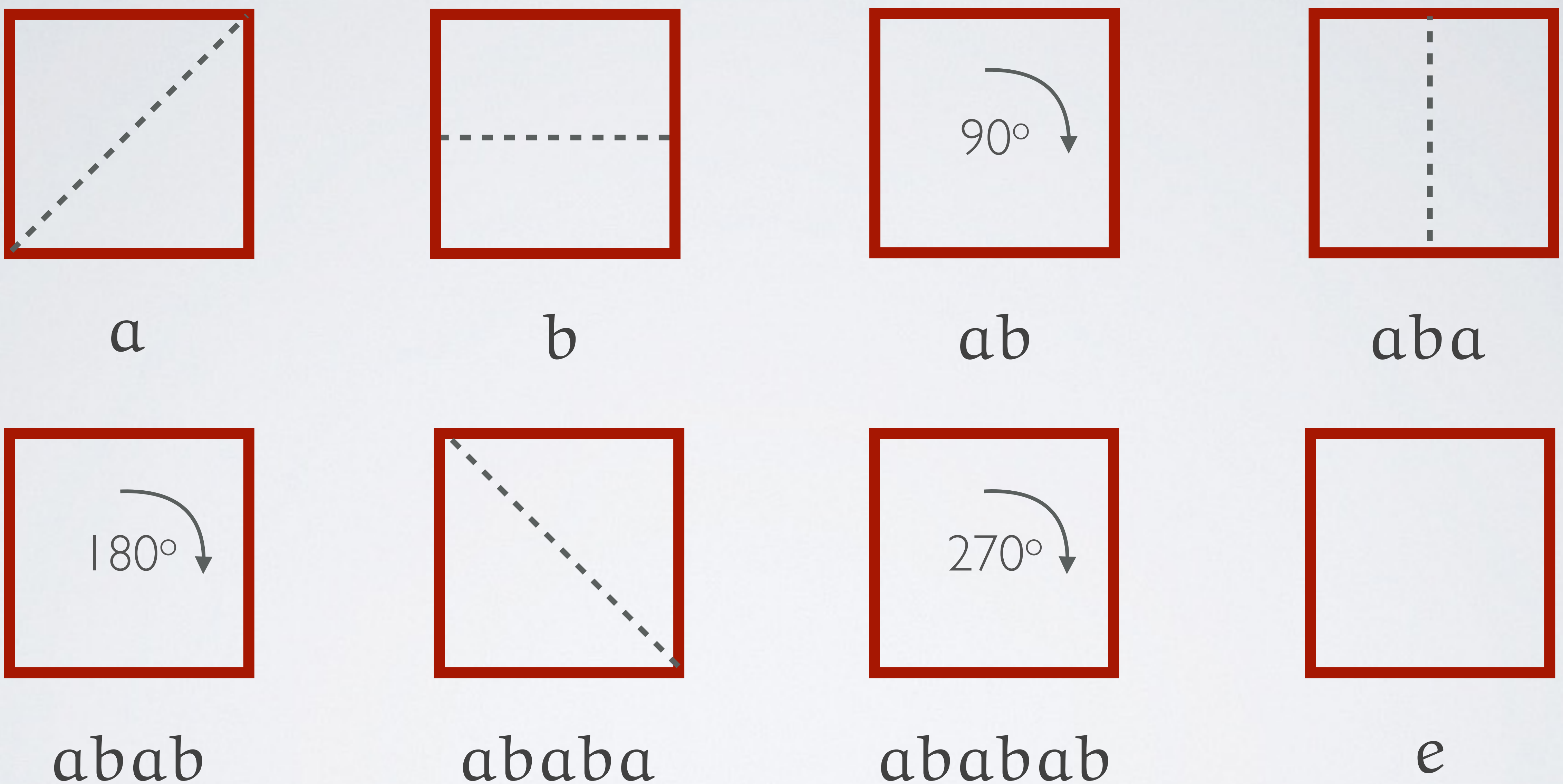

$$a^2 = e, \quad b^2 = e, \quad (ab)^4 = e$$

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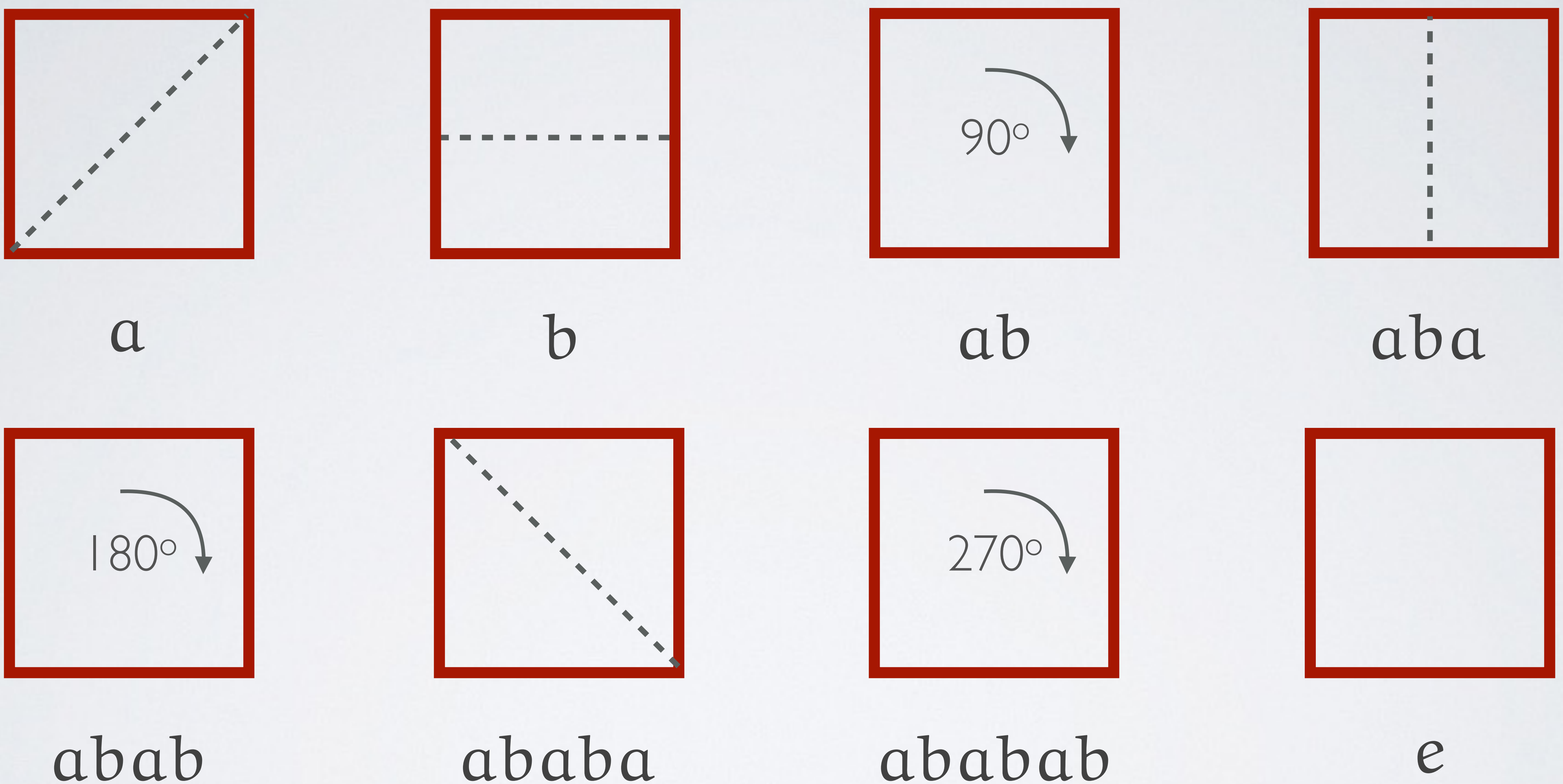

$$I_2(4) = \langle a, b \mid a^2 = e, \quad b^2 = e, \quad (ab)^4 = e \rangle$$

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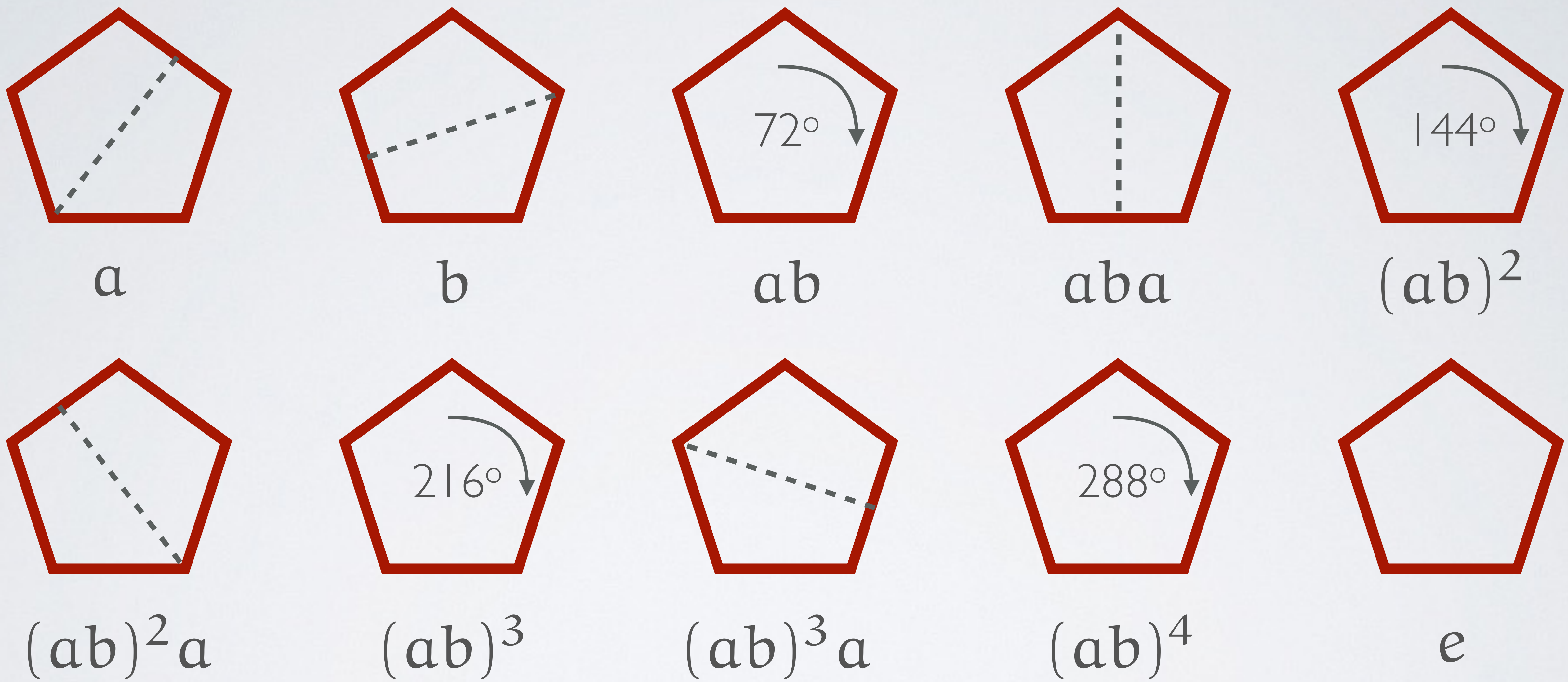

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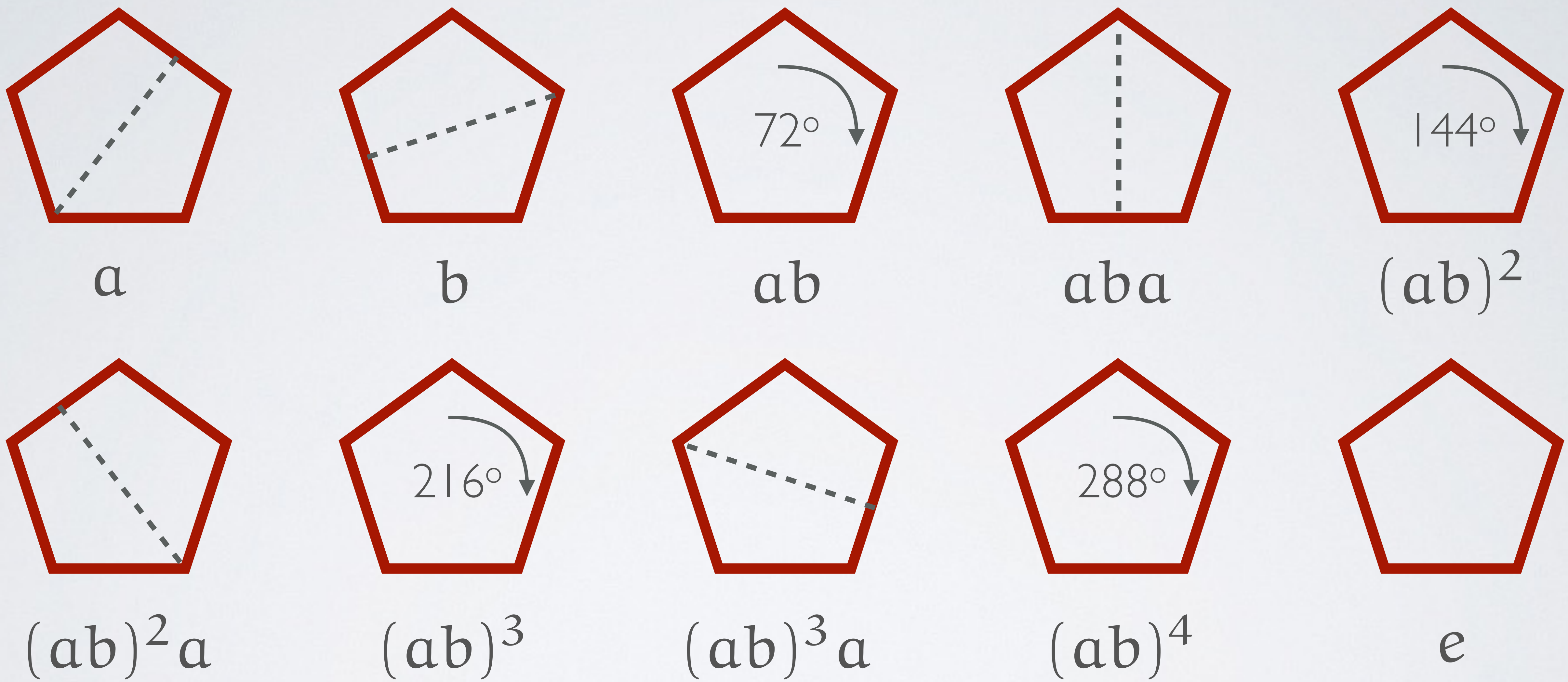

$$I_2(5) = \langle a, b \mid a^2, b^2, (ab)^5 \rangle$$

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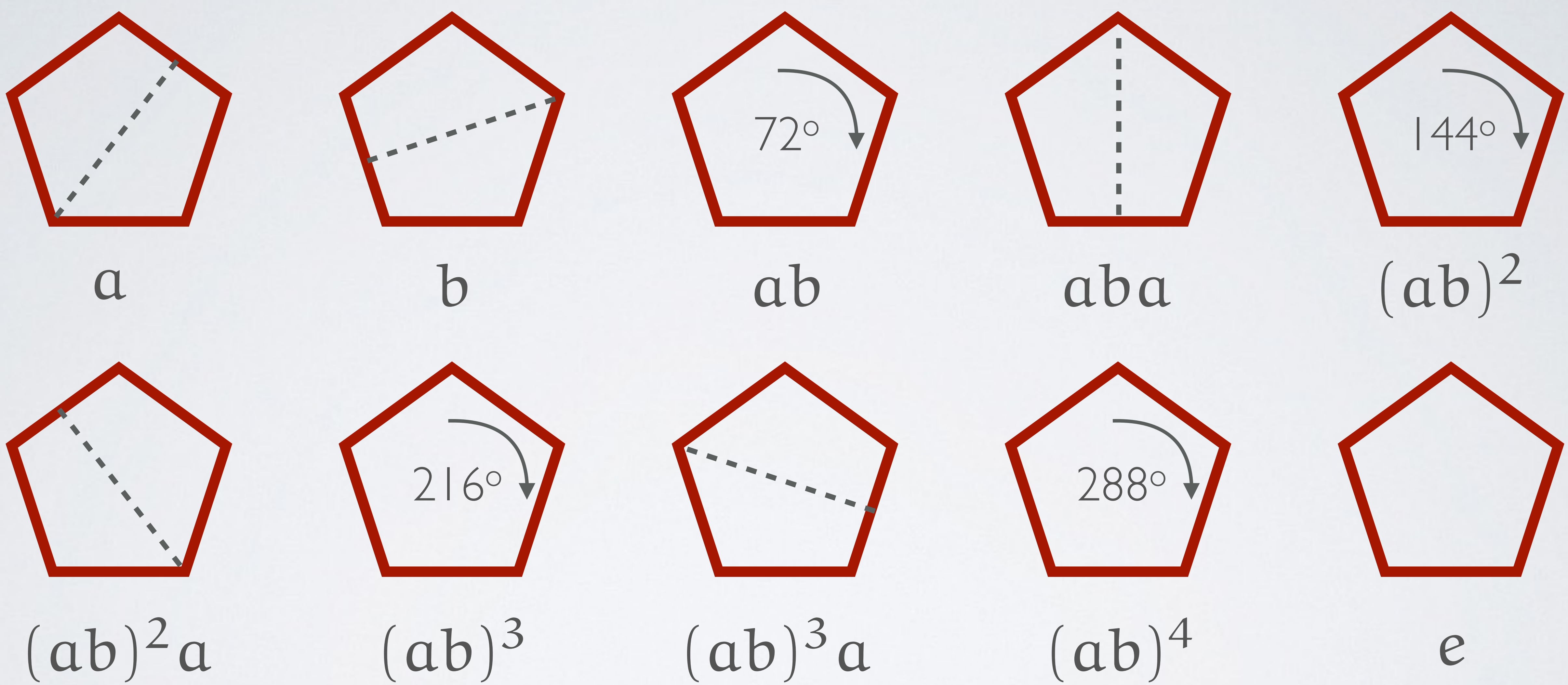

$$I_2(m) = \langle a, b \mid a^2, b^2, (ab)^m \rangle$$

MATRIX REPRESENTATION

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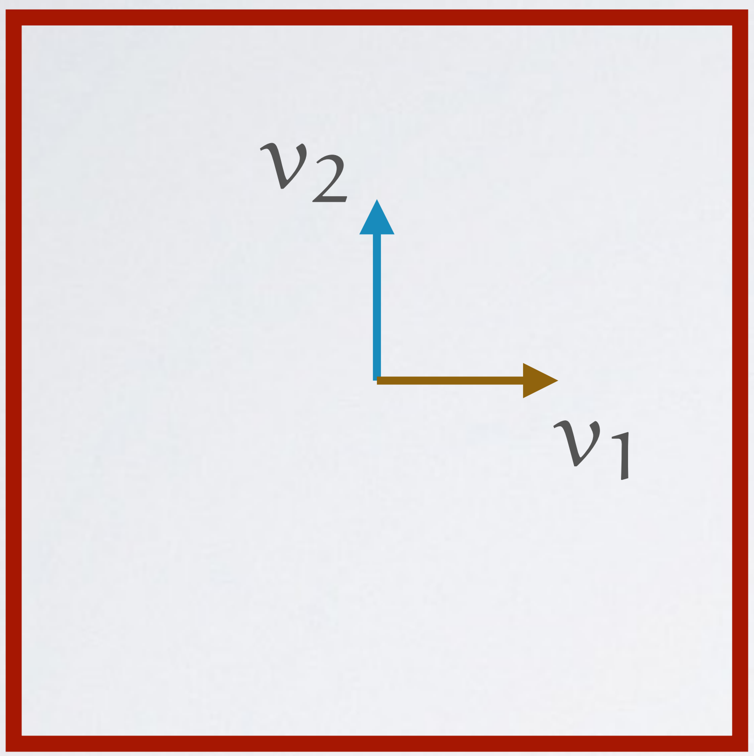

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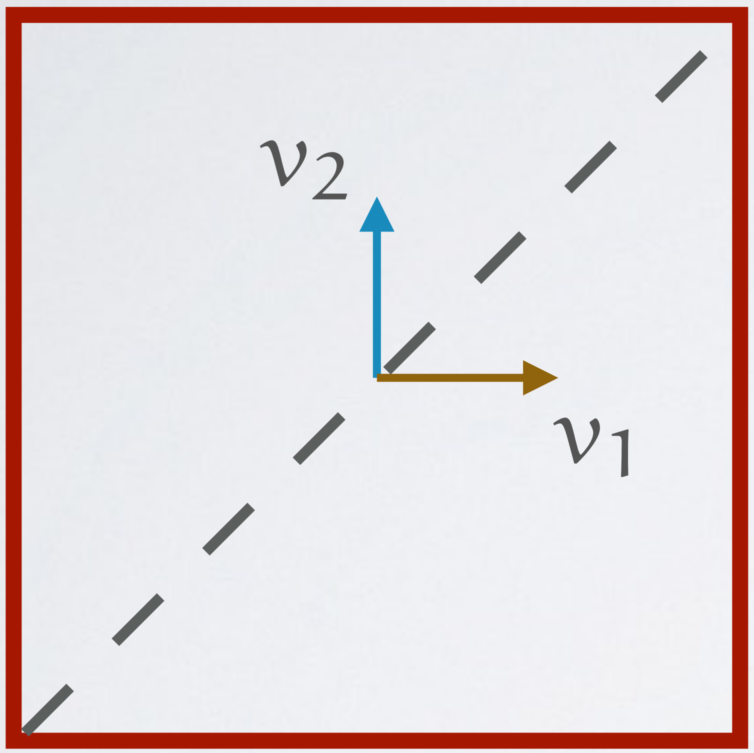

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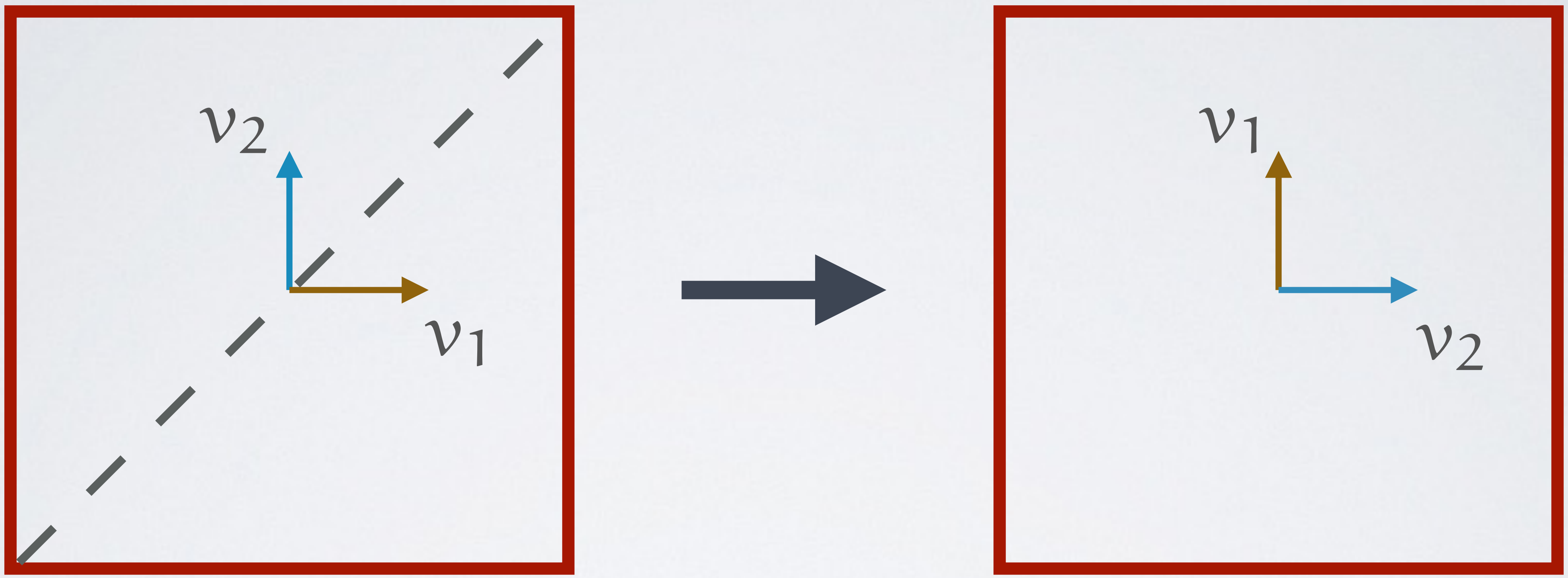

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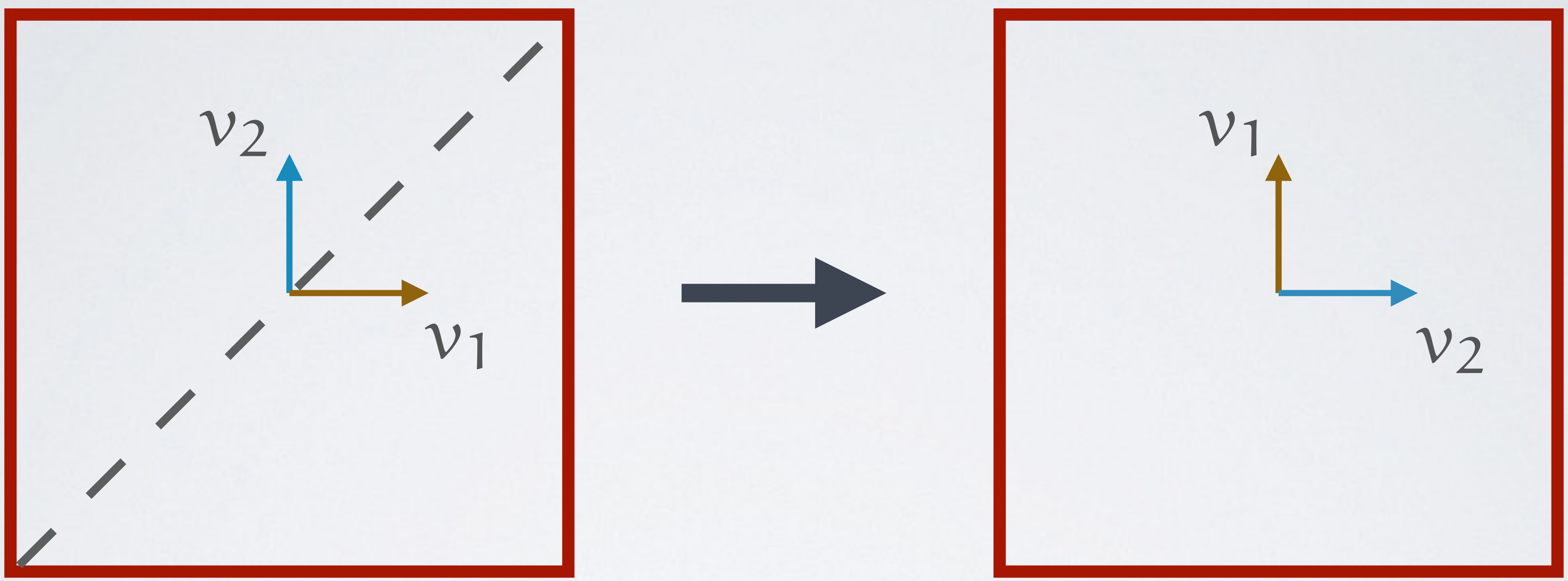

$$\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$$

a

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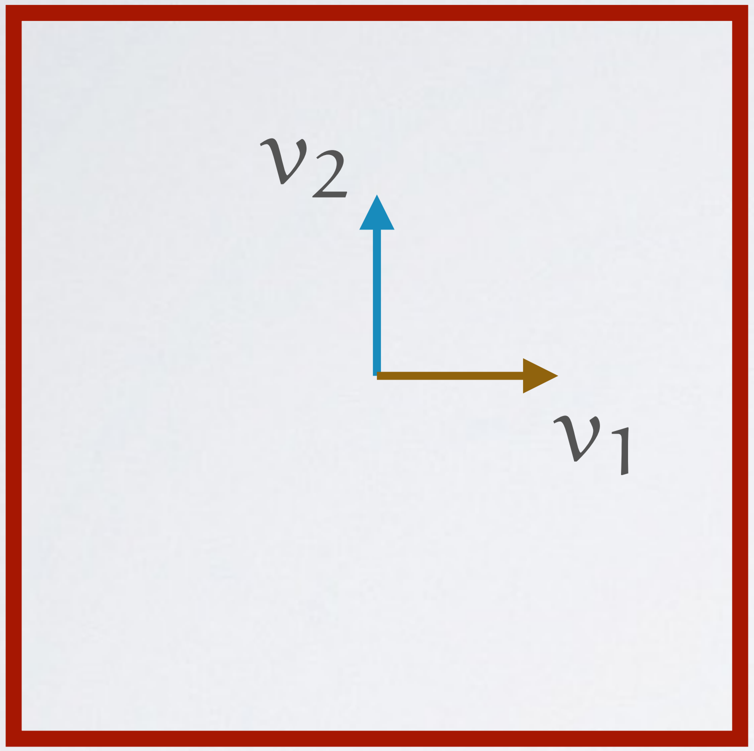

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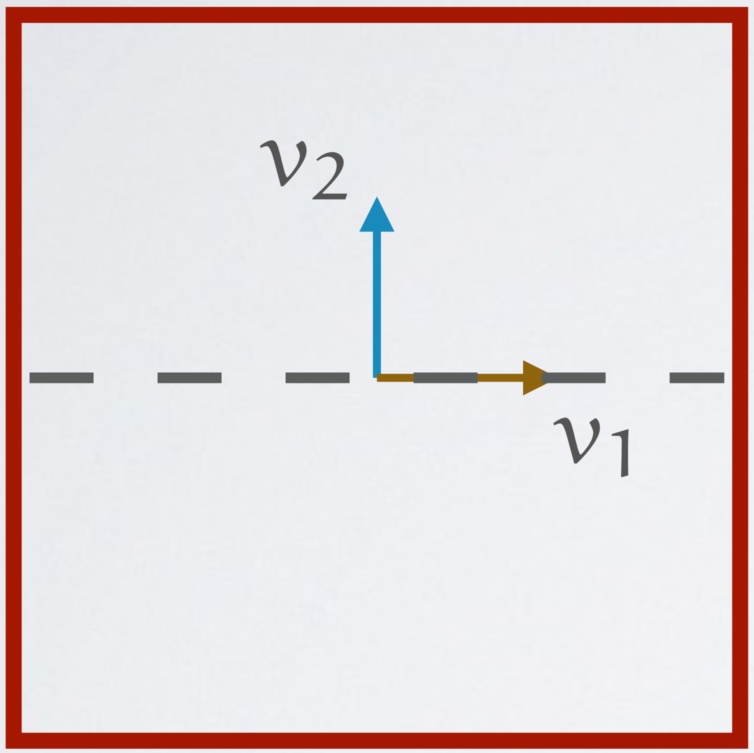

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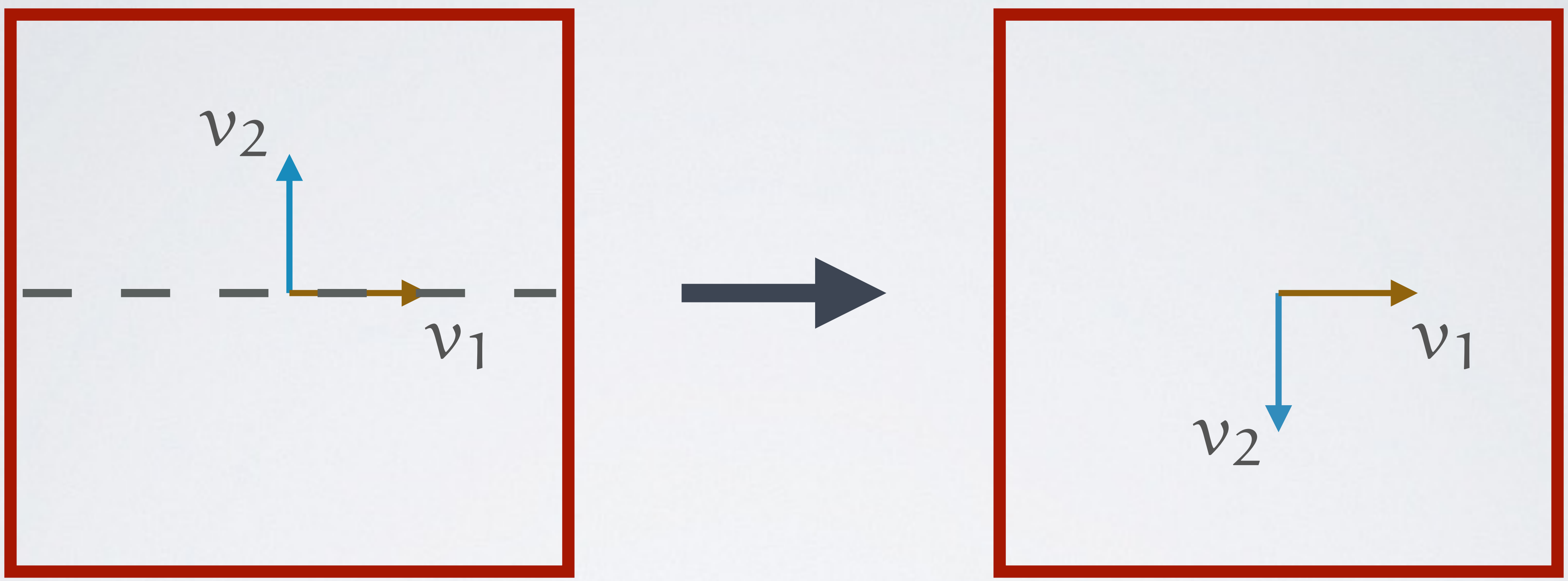

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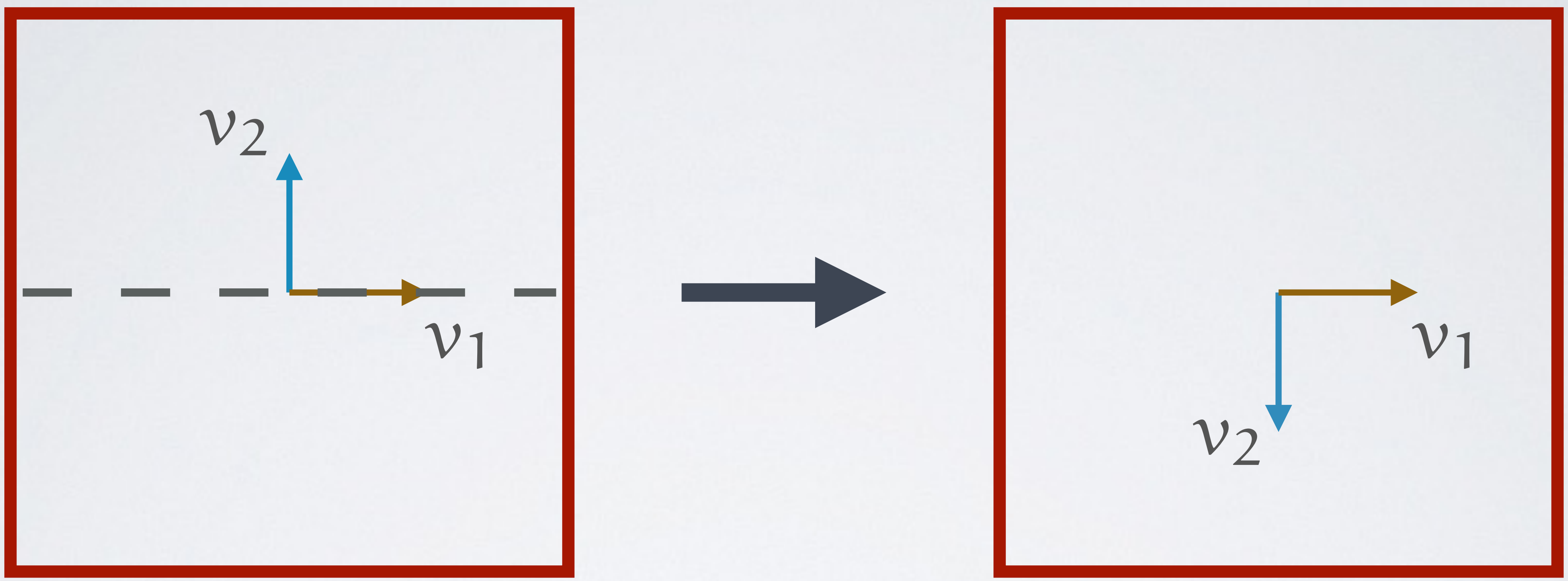

$$\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \qquad \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$$

a b

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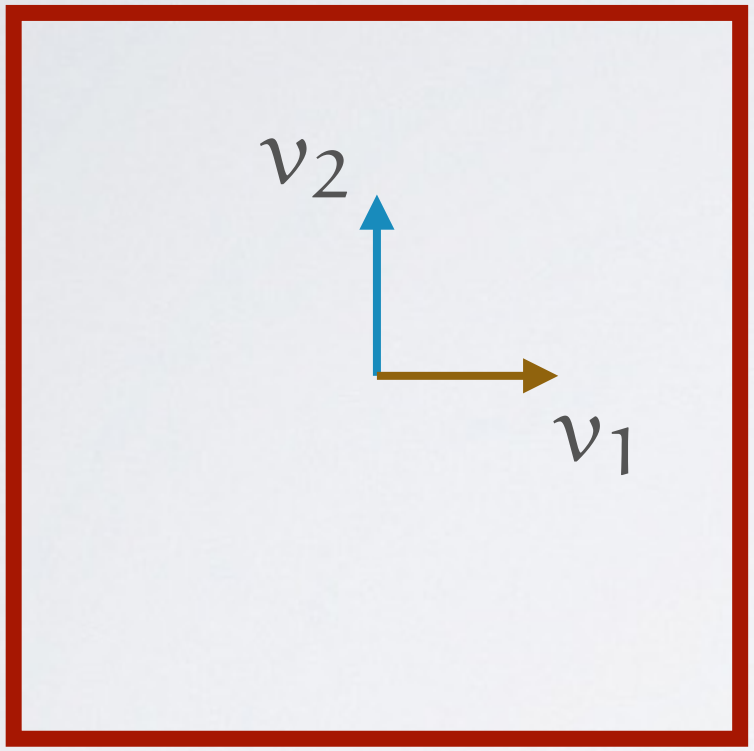

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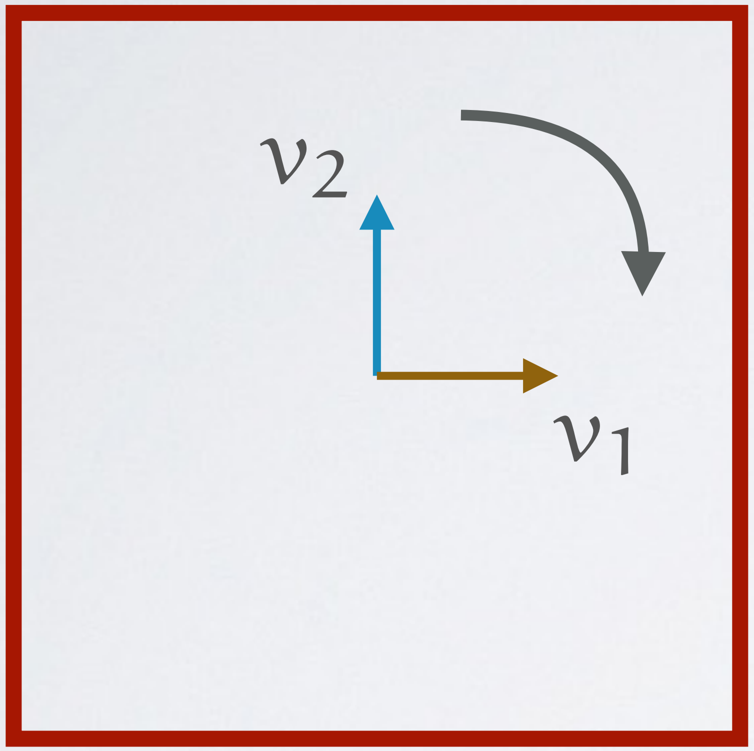

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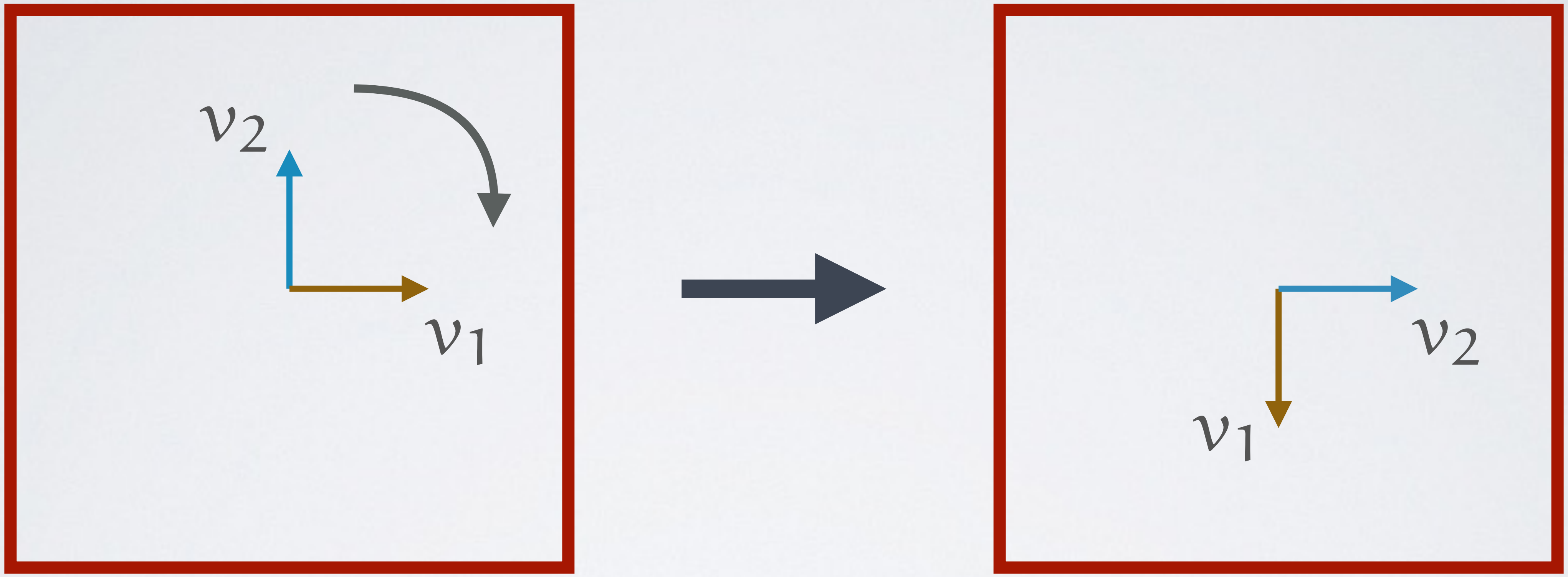

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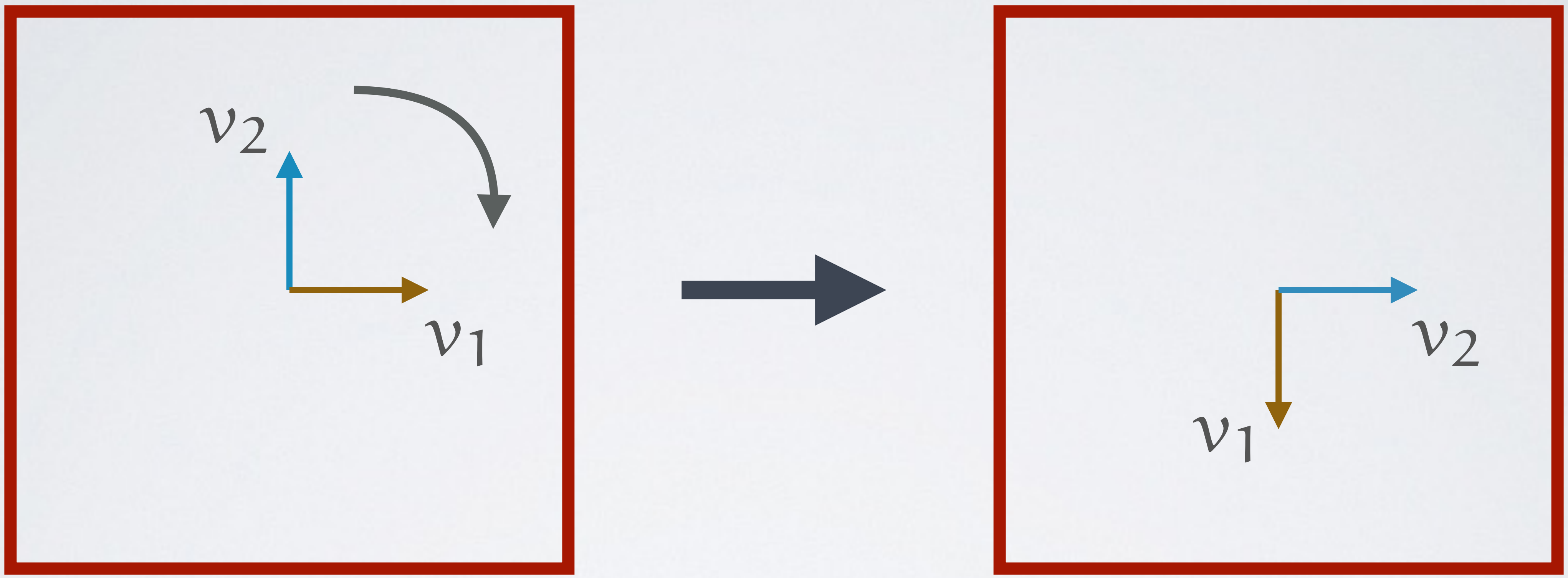

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a $\qquad$ b $\qquad$ ab

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$$a \qquad b \qquad ab \qquad aba$$

$$\begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix} \quad \begin{bmatrix} 0 & -1 \\ -1 & 0 \end{bmatrix} \quad \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix} \quad \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$abab \qquad ababa \qquad ababab \qquad e$$

REPRESENTATIONS

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- A representation of a group $(G, \star)$ is a map $\rho : G \to \mathrm{GL}(V)$ such that for all $a,\ b \in G$ we have

$$\rho(a \star b) = \rho(a) \circ \rho(b)$$

CHARACTERS

- Let $\rho : G \to GL_n(\mathbb{C})$ be a representation of a group $(G, \star)$. The function $\chi_\rho : G \to \mathbb{C}$ defined by

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e	a	b	ab	aba	$abab$	$ababa$	$ababab$
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2	0	0	0	0	−2	0	0

CONJUGACY

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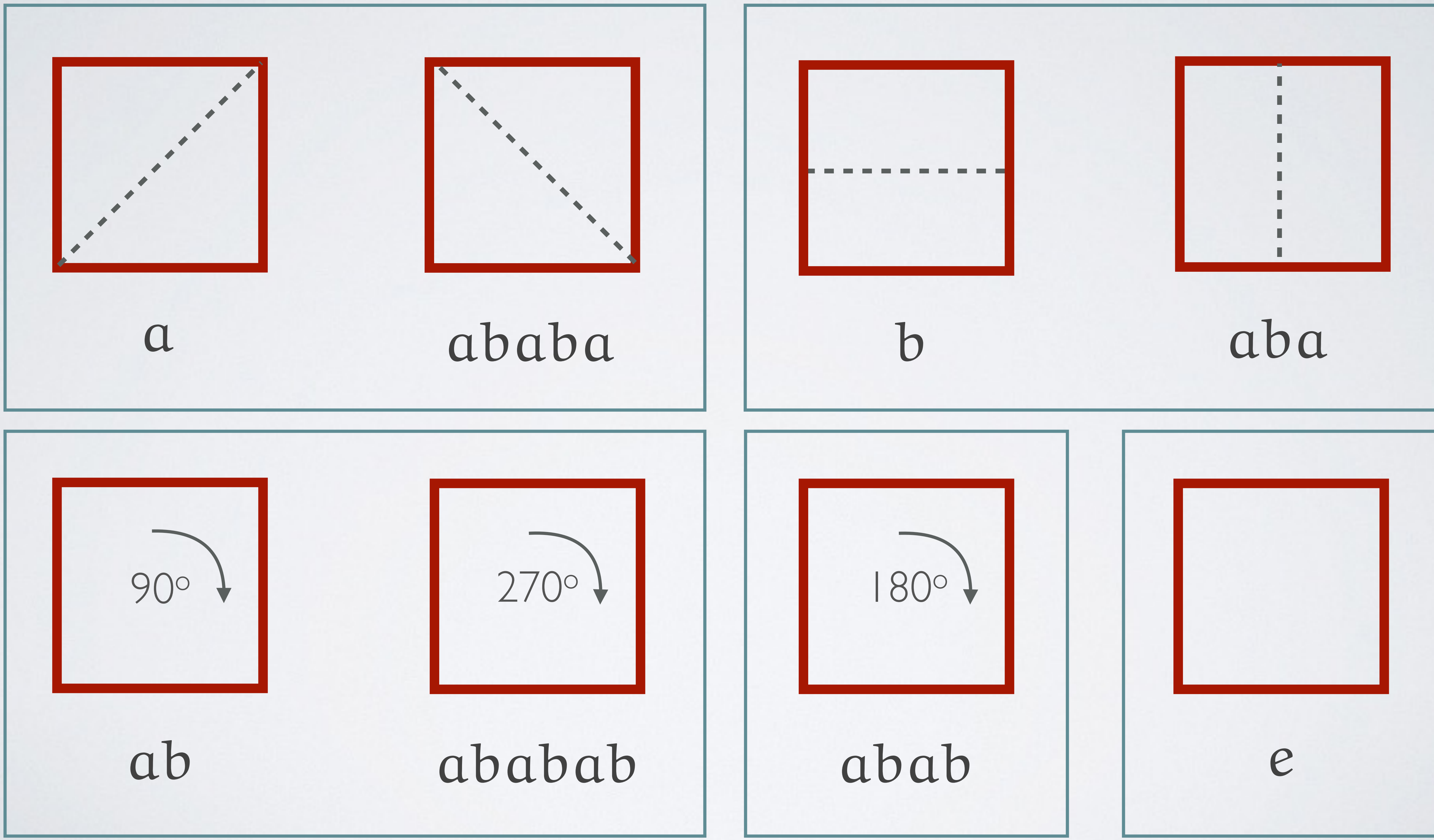
- This defines an equivalence relation on G. The resulting equivalence classes are called conjugacy classes.

CONJUGACY



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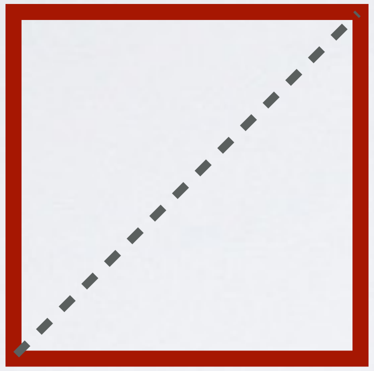
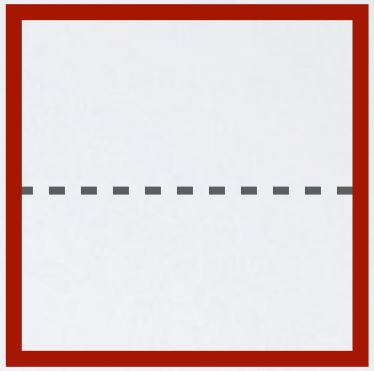
- A character with this property is also called irreducible.

IRREDUCIBLE CHARACTERS

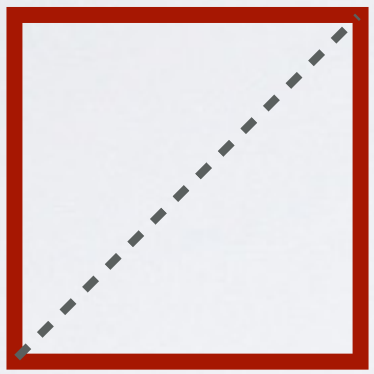


a



b

IRREDUCIBLE CHARACTERS



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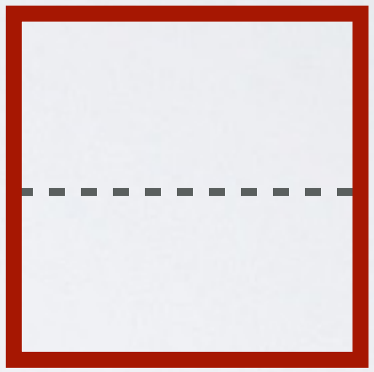

b

$$\frac{1}{8}(2^2 + 0 + 0 + 0 + 0 + (-2)^2 + 0 + 0) = 1$$

CHARACTER TABLES

Theorem

The number of distinct irreducible characters of a finite group is equal to the number of conjugacy classes.

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- Let $g_1, \ldots, g_n \in G$ be representatives for the conjugacy classes and let $\chi_1, \ldots, \chi_n$ be the irreducible characters of G. The square matrix

$$(\chi_i(g_j))_{1 \leqslant i,j \leqslant n}$$

is called the character table of G.

CHARACTER TABLES

$I_2(4)$	e	a	b	ab	$(ab)^2$
χ_1	1	1	1	1	1
χ_2	1	-1	1	-1	1
χ_3	1	1	-1	-1	1
χ_4	1	-1	-1	1	1
ψ	2	0	0	0	-2

CHARACTER TABLES

$I_2(2m)$	e	a	b	$(ab)^r$	$(ab)^m$
χ_1	1	1	1	1	1
χ_2	1	-1	1	$(-1)^r$	$(-1)^m$
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$1 \leqslant j, r \leqslant m-1$ $\qquad$ $\varepsilon = e^{\pi i/m}$

SYMMETRIC GROUPS

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- $\mathfrak{S}_n$ is the group of all bijective functions $f : \{1, \ldots, n\} \to \{1, \ldots, n\}$.

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- We call $\mathfrak{S}_n$ the symmetric group on n points.
- $|\mathfrak{S}_n| = n!$ which can be very large even for small n. For example

$$|\mathfrak{S}_{20}| = 2432902008176640000$$

SYMMETRIC GROUPS

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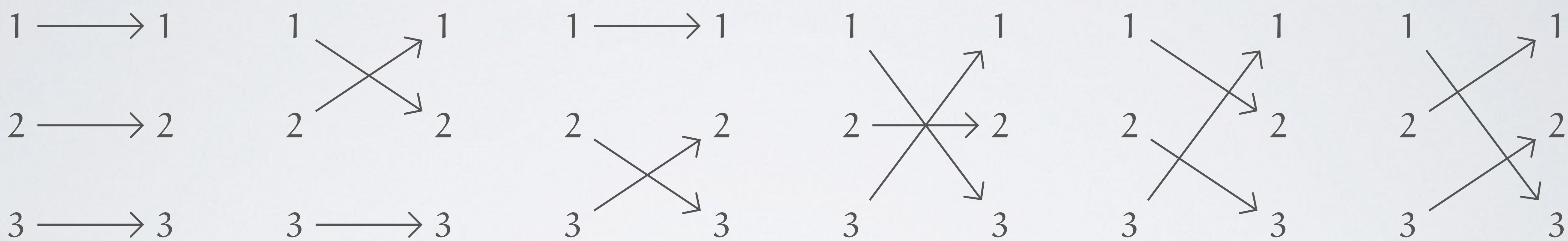
Example ($n = 3$)



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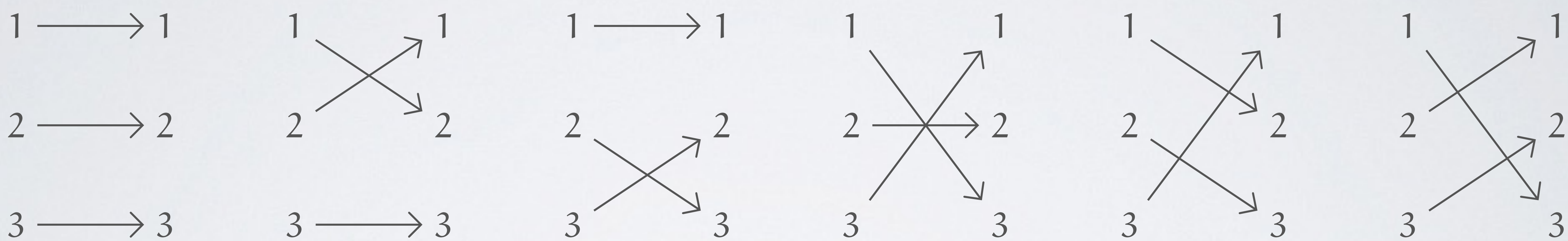

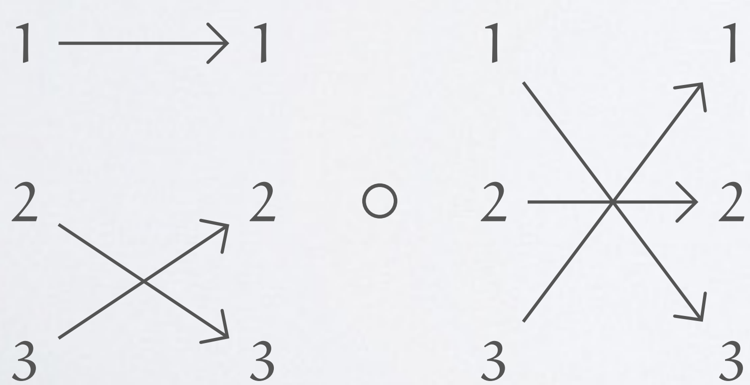

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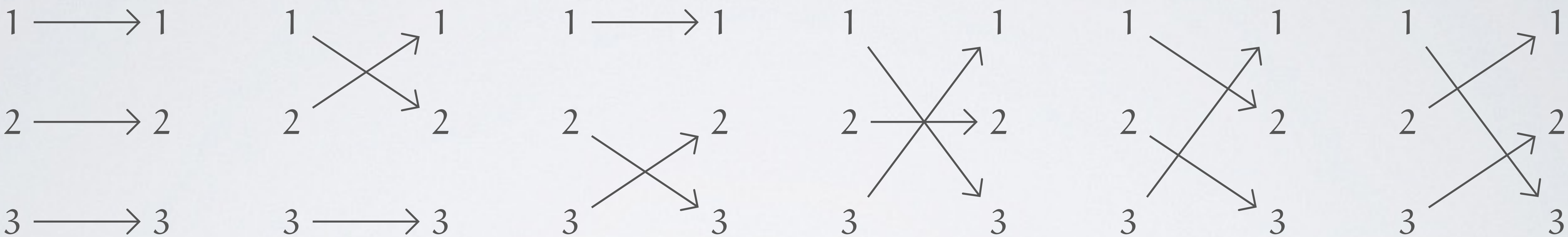

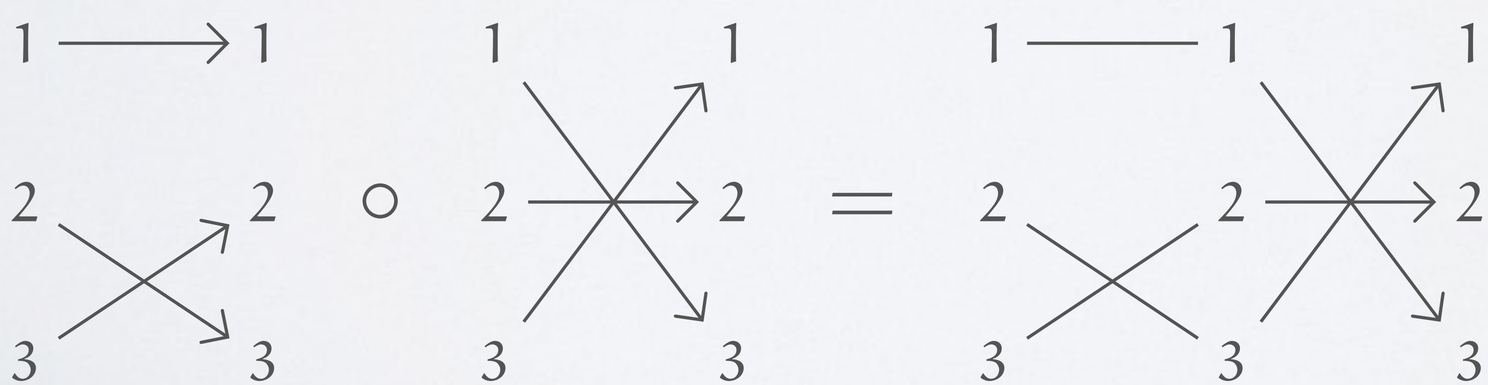

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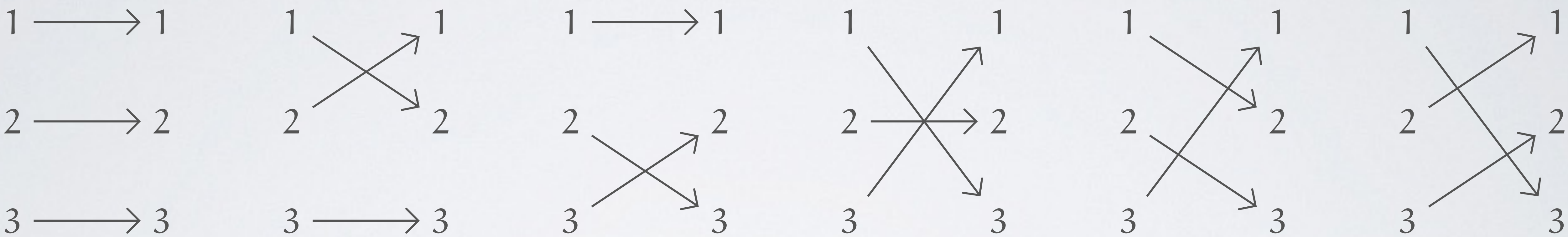

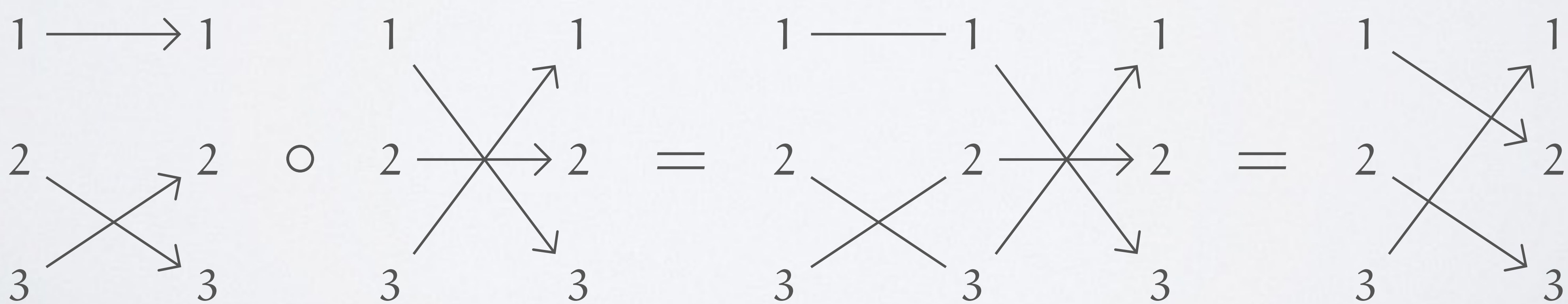

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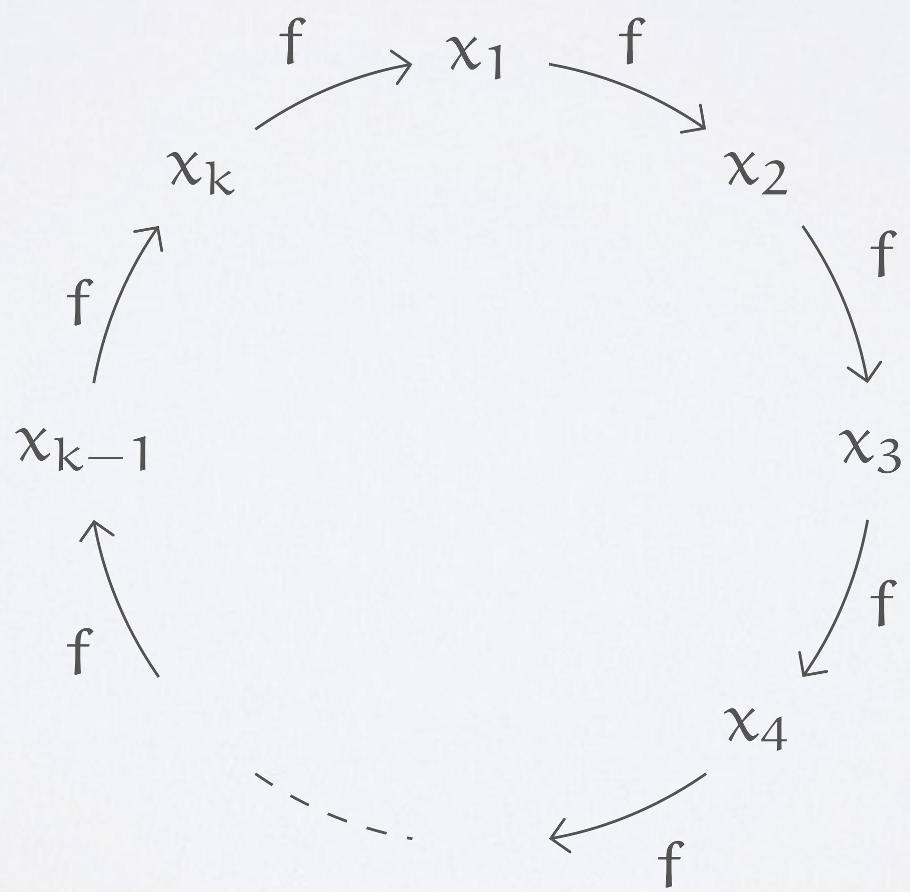
- A function $f \in \mathfrak{S}_n$ is called a cycle of length k if there exists a subset $X = \{x_1, \ldots, x_k\} \subseteq \{1, \ldots, n\}$ such that $f(i) = i$ for any integer $i \notin X$ and f acts on the elements of X in the following way



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Every element of $\mathfrak{S}_n$ is a product of disjoint cycles.

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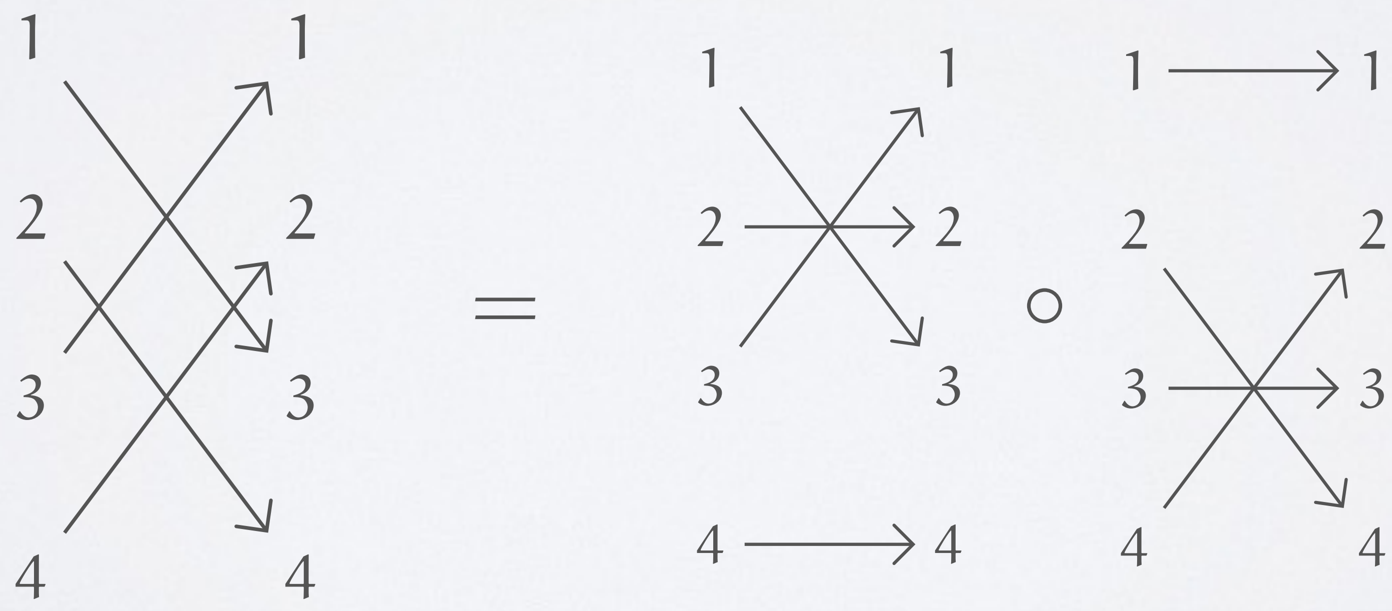

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- Given $f \in \mathfrak{S}_n$ let $f_1 \circ \cdots \circ f_k$ be a decomposition of f into a product of disjoint cycles. If μ_i denotes the length of the cycle f_i then the sequence $\mu(f) = (\mu_1, \ldots, \mu_k)$ is a partition of n, after possibly reordering the entries. We call $\mu(f)$ the cycle type of f.

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$$|P(20)| = 627$$

HOOKS OF PARTITIONS

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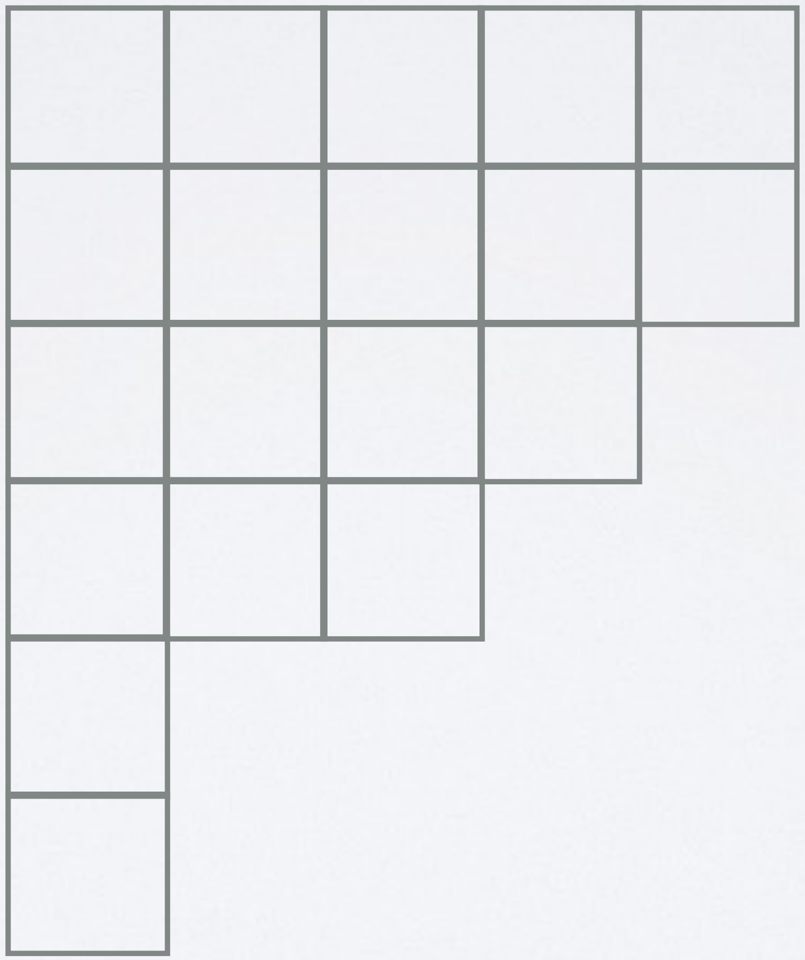

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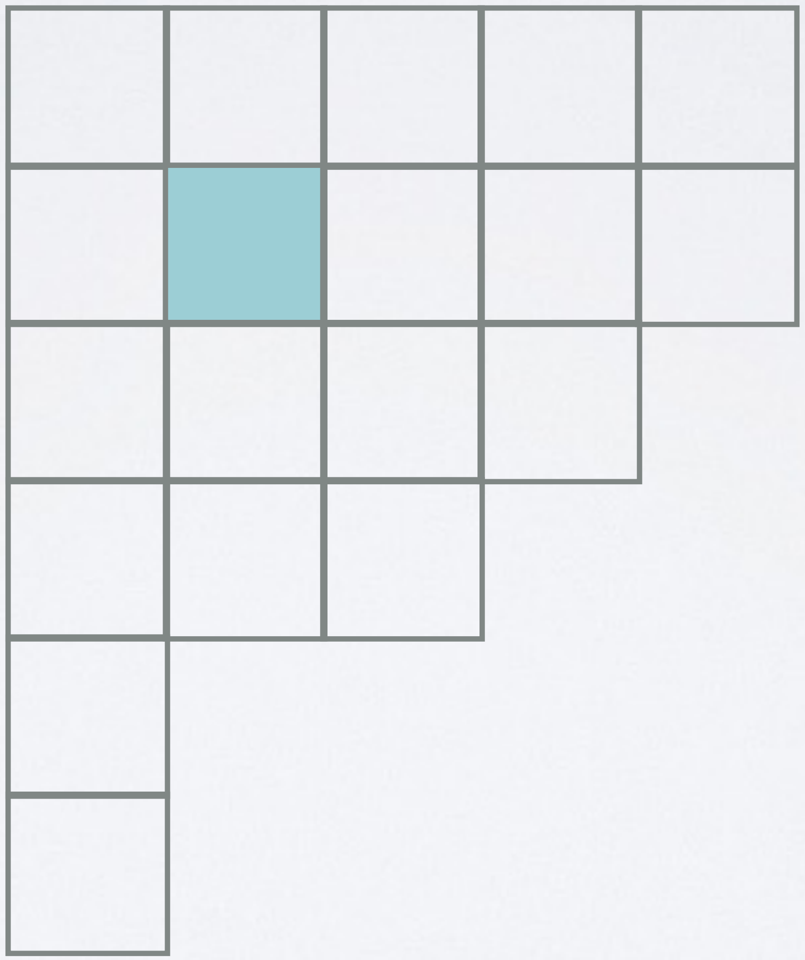

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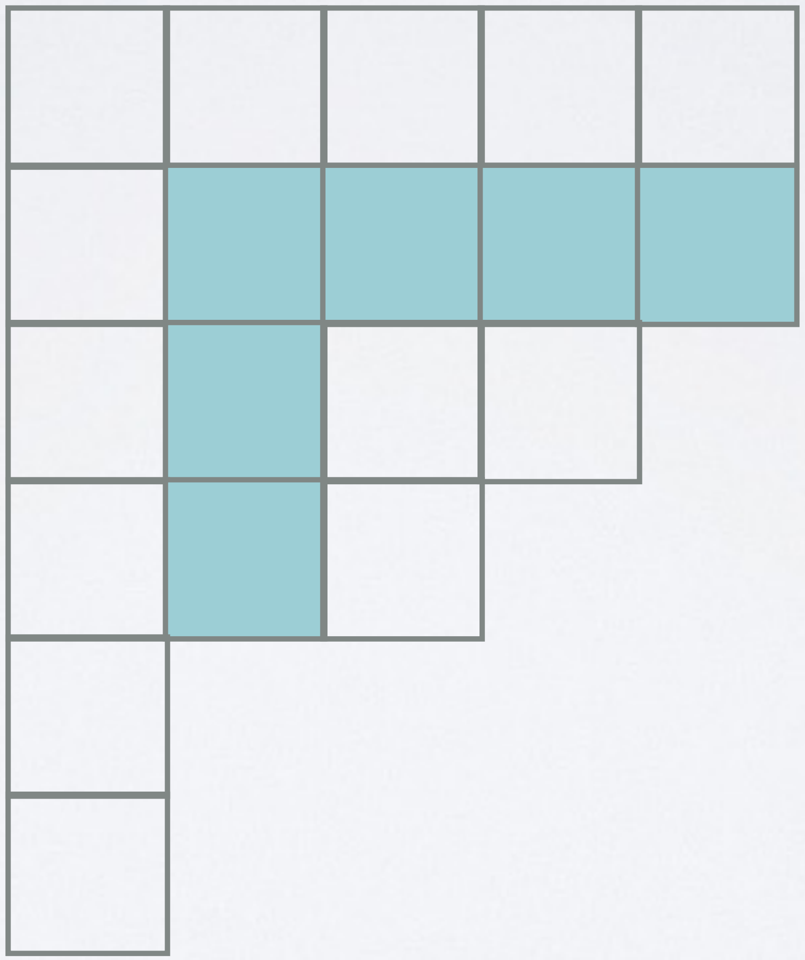

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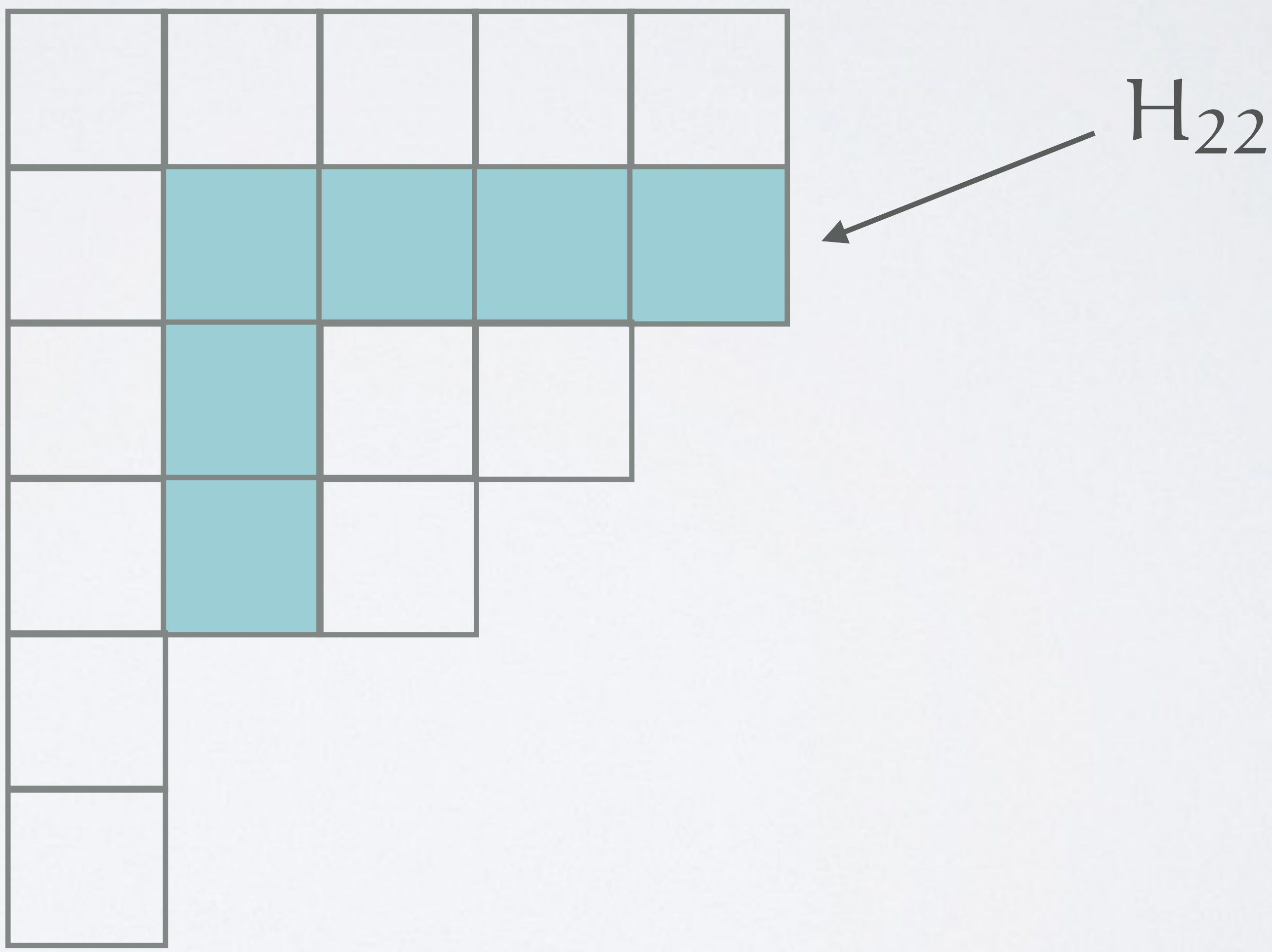

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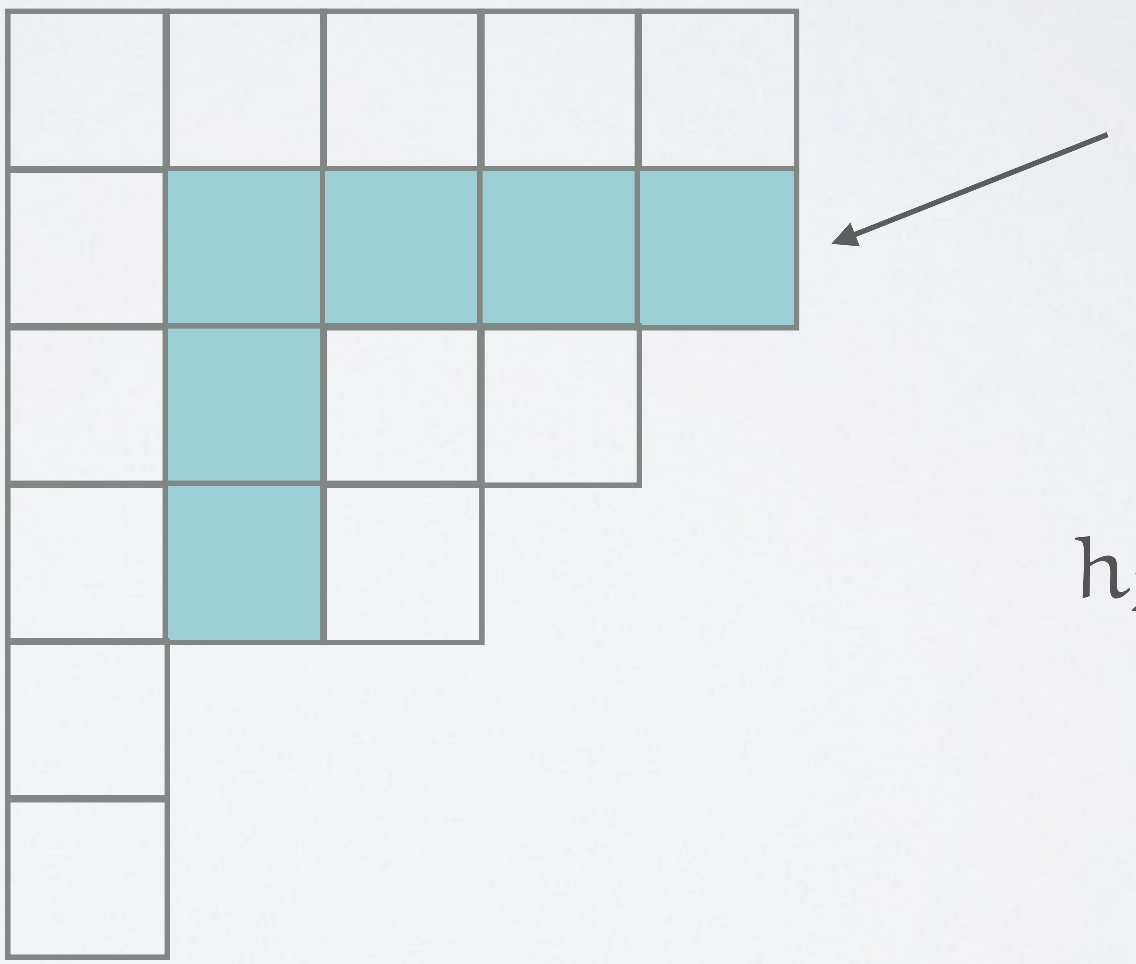

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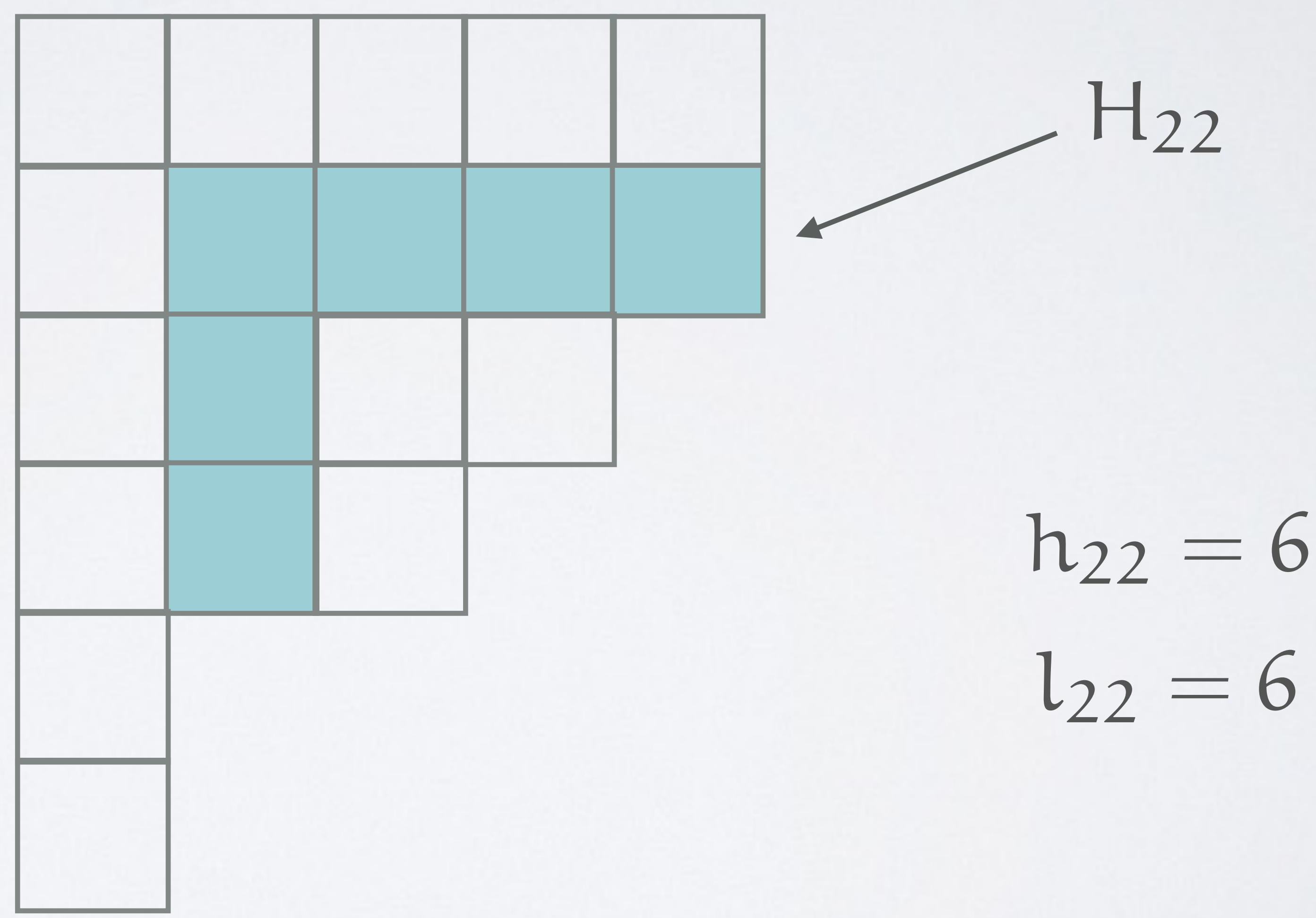

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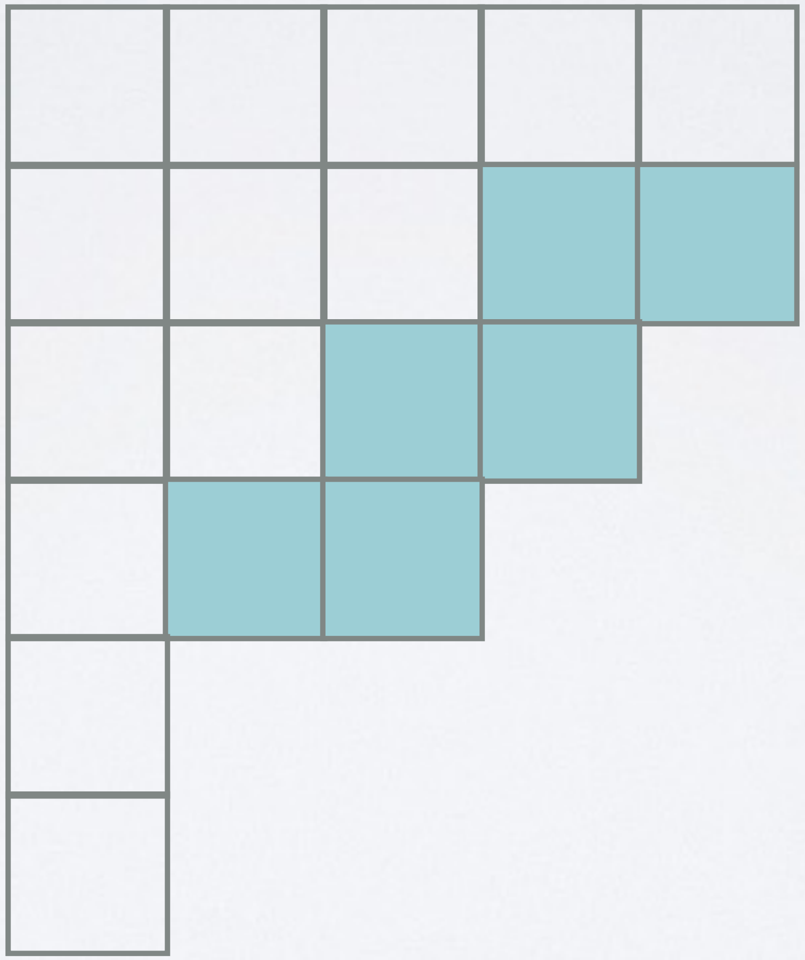

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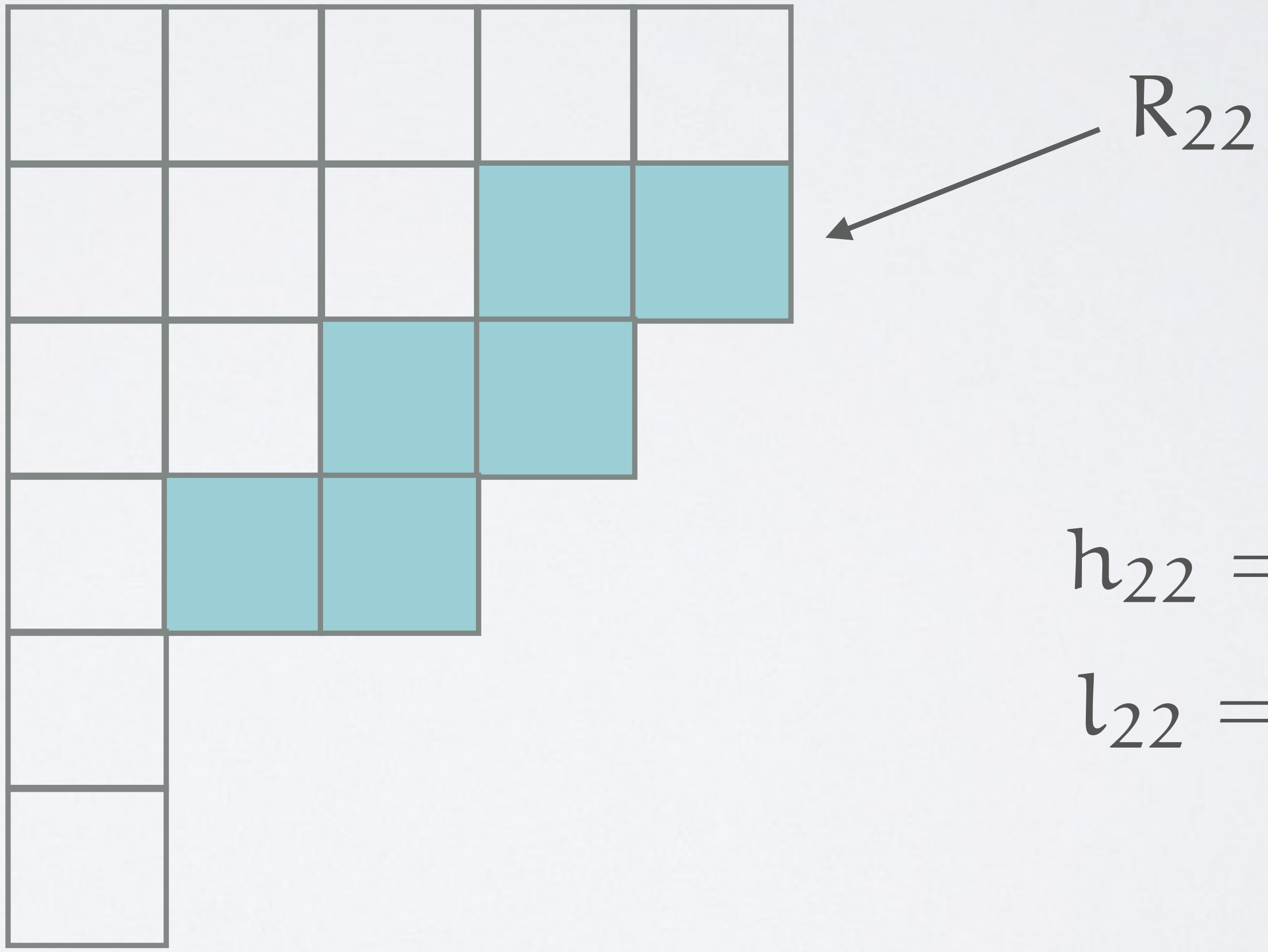

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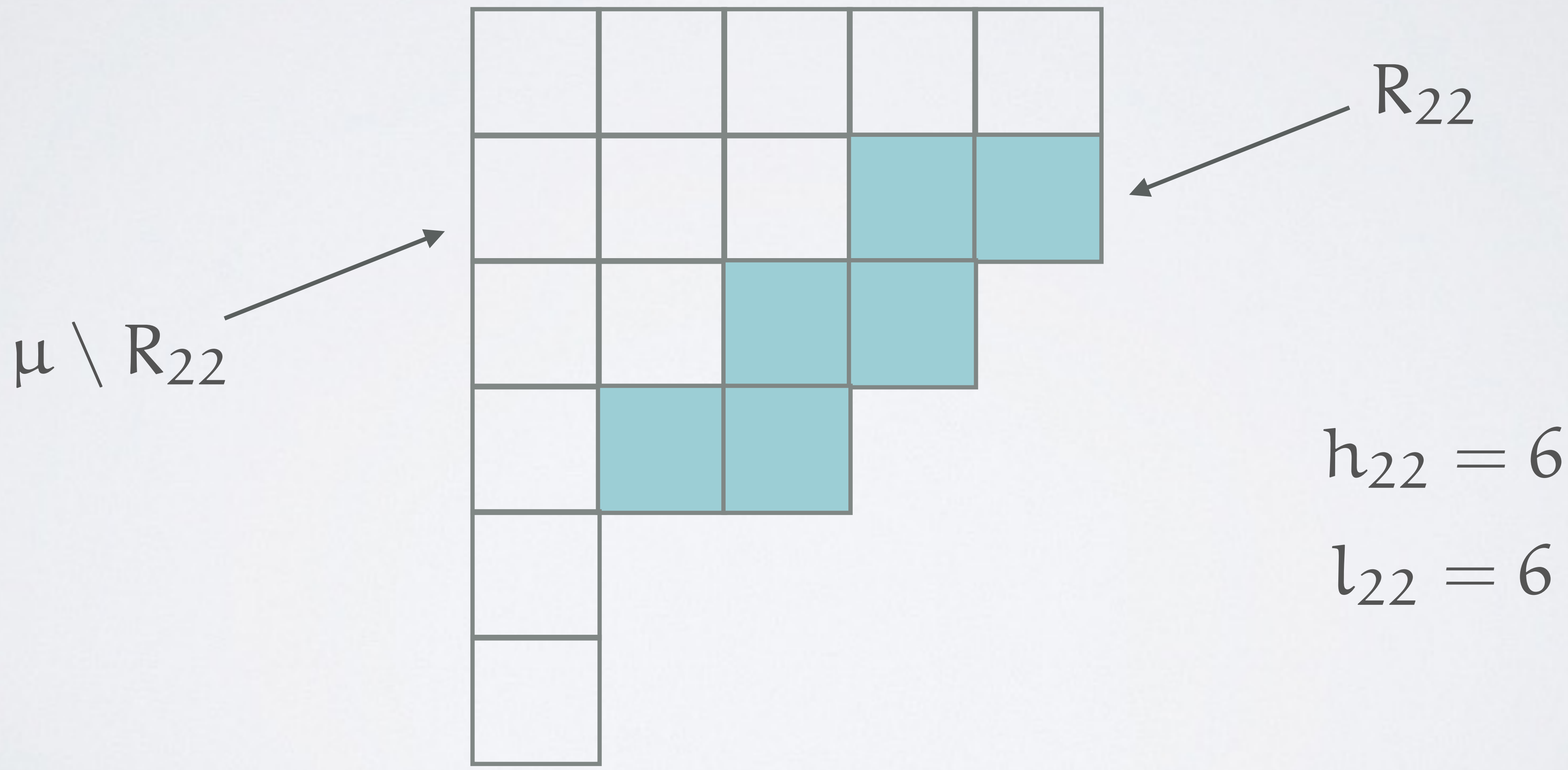

CHARACTER TABLE

Theorem (Murnaghan–Nakayama Formula)

Write $f \in \mathfrak{S}_n$ as a product $f_1 \circ \cdots \circ f_k$ of disjoint cycles. Assume f_k is a cycle of length m then the element

$$g = f_1 \circ \cdots \circ f_{k-1}$$

is contained in the symmetric group $\mathfrak{S}_{n-m}$. For any partition $\lambda \in P(n)$ we have

$$\chi^\lambda(f) = \sum_{h_{ij}=m} (-1)^{l_{ij}} \chi^{\lambda \setminus R_{ij}}(g)$$

CHARACTER TABLE

$\mathfrak{S}_5$	11111	2111	221	311	32	41	5
5	1	1	1	1	1	1	1
41	4	2	0	1	−1	0	−1
32	5	1	1	−1	1	−1	0
311	6	0	−2	0	0	0	1
221	5	−1	1	−1	−1	1	0
2111	4	−2	0	1	1	0	−1
11111	1	−1	1	1	−1	−1	1