

Lectures in Mathematics, ETH Zürich, Birkhäuser (500 pp)
ISBN 3-7643-2419-8

Optimal Stopping and Free-Boundary Problems

Goran Peskir & Albert Shiryaev

Contents

Introduction	ix
I. Optimal stopping: General facts	1
1. Discrete time	1
1.1. Martingale approach	1
1.2. Markovian approach	12
2. Continuous time	26
2.1. Martingale approach	26
2.2. Markovian approach	34
II. Stochastic processes: A brief review	53
3. Martingales	53
3.1. Basic definitions and properties	53
3.2. Fundamental theorems	60
3.3. Stochastic integral and Itô's formula	63
3.4. Stochastic differential equations	72
3.5. A local time-space formula	74
4. Markov processes	76
4.1. Markov sequences (chains)	76
4.2. Elements of potential theory (discrete time)	79
4.3. Markov processes (continuous time)	88
4.4. Brownian motion (Wiener process)	93
4.5. Diffusion processes	101
4.6. Lévy processes	102
5. Basic transformations	106
5.1. Change of time	106
5.2. Change of space	111
5.3. Change of measure	115
5.4. Killing (discounting)	119
III. Optimal stopping and free-boundary problems	123
6. MLS formulation of optimal stopping problems	124
6.1. Infinite and finite horizon problems	125

6.2.	Dimension of the problem	126
6.3.	Killed (discounted) problems	127
7.	MLS functionals and PIDE problems	128
7.1.	Mayer functional and Dirichlet problem	130
7.2.	Lagrange functional and Dirichlet/Poisson problem	132
7.3.	Supremum functional and Neumann problem	133
7.4.	MLS functionals and Cauchy problem	135
7.5.	Connection with the Kolmogorov backward equation	139
IV.	Methods of solution	143
8.	Reduction to free-boundary problem	143
8.1.	Infinite horizon	144
8.2.	Finite horizon	146
9.	Superharmonic characterization	147
9.1.	The principle of smooth fit	149
9.2.	The principle of continuous fit	153
9.3.	Diffusions with angles	155
10.	The method of time change	165
10.1.	Description of the method	165
10.2.	Problems and solutions	168
11.	The method of space change	193
11.1.	Description of the method	193
11.2.	Problems and solutions	196
12.	The method of measure change	197
12.1.	Description of the method	197
12.2.	Problems and solutions	198
13.	Optimal stopping of the maximum process	199
13.1.	Formulation of the problem	199
13.2.	Solution to the problem	201
14.	Nonlinear integral equations	219
14.1.	The free-boundary equation	219
14.2.	The first-passage equation	221
V.	Optimal stopping in stochastic analysis	243
15.	Review of problems	243
16.	Wald inequalities	244
16.1.	Formulation of the problem	245
16.2.	Solution to the problem	245
16.3.	Applications	249
17.	Bessel inequalities	251
17.1.	Formulation of the problem	251
17.2.	Solution to the problem	252
18.	Doob inequalities	255
18.1.	Formulation of the problem	255

18.2.	Solution to the problem	256
18.3.	The expected waiting time	263
18.4.	Further examples	268
19.	Hardy–Littlewood inequalities	272
19.1.	Formulation of the problem	272
19.2.	Solution to the problem	273
19.3.	Further examples	283
20.	Burkholder–Davis–Gundy inequalities	284
VI. Optimal stopping in mathematical statistics		287
21.	Sequential testing of a Wiener process	287
21.1.	Infinite horizon	289
21.2.	Finite horizon	292
22.	Quickest detection of a Wiener process	308
22.1.	Infinite horizon	310
22.2.	Finite horizon	313
23.	Sequential testing of a Poisson process	334
23.1.	Infinite horizon	334
24.	Quickest detection of a Poisson process	355
24.1.	Infinite horizon	355
VII. Optimal stopping in mathematical finance		375
25.	The American option	375
25.1.	Infinite horizon	375
25.2.	Finite horizon	379
26.	The Russian option	395
26.1.	Infinite horizon	395
26.2.	Finite horizon	400
27.	The Asian option	416
27.1.	Finite horizon	417
VIII. Optimal stopping in financial engineering		437
28.	Ultimate position	437
29.	Ultimate integral	438
30.	Ultimate maximum	441
30.1.	Free Brownian motion	441
30.2.	Brownian motion with drift	452
Bibliography		477
Subject Index		493
List of Symbols		499