**Ertel’s potential vorticity theorem**

after H. Ertel, Ein Neuer Hydrodynamischer Wirbelsatz, Met. Z., 59, 277-281, 1942.

Start from the Navier-Stokes equation:

………….. 1

Substitute from the vector identity

………………….. 2

Take the curl of this equation, writing **ξ** for (2**Ω** + **∇V**), the absolute vorticity:

……………… 3

Now use the expansion:

with ∇.**ξ = 0** (div curl ≡ 0 and ∇.**Ω** is very small – it is not zero because although **Ω** is constant the local coordinate system is not).

Collecting terms and applying the continuity equation:

…………. 4

Now multiply by **∇**q where q is any scalar field:

Use

Finally, if q is a function of p and ρ, as θ is, the first term on the RHS vanishes, leaving:

The quantity (**ξ.∇**θ)/ρ = is the *Ertel potential vorticity.* The equation may also be written in flux form (Haynes and McIntyre, 1990) as: