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We have characterized ergodicity of a distribution μ on $\mathbb{Z}_p$ via an algebraic property of its support: μ is ergodic if and only if $\mathrm{spt}(\mu)^{\oplus n} = \mathbb{Z}_p$ for some n, and this is equivalent to $\mathrm{spt}(\mu)$ not being contained in any proper subgroup's coset. Our next question is to ask whether we can determine a rate for the convergence $d(\mu^{*n}, \lambda) \to 0$ when μ is ergodic. By a rate we mean a function $\phi : \mathbb{N} \to [0,\infty)$ with $\lim_{n\to\infty} \phi(n) = 0$ such that $d(\mu^{*n}, \lambda) \leq \phi(n)$. A common and interesting rate is an exponential rate $\phi(n) = e^{-\alpha n}$ for some constant $\alpha > 0$. Exponential mixing is when $d(\mu^{*n}, \lambda) \leq e^{-\alpha n}$ for some $\alpha > 0$. The next part of the course is about harmonic analysis. Our main goal is to use it to determine a rate of mixing for an ergodic random walk.