David Silvester, 20 July 2020

Notes for the video tutorial on
Taylor–Hood local mass conservation

15.1 The idea of adding a piecewise constant pressure to the rectangular Taylor–Hood $\mathbb{Q}_2$–$\mathbb{Q}_1$ approximation was originally suggested during discussion around a blackboard at a conference in Banff in 1980; see Gresho et al. [gresho81]. The need for local mass conservation was motivated by competition from finite volume methods (such as the MAC scheme) in the design of effective strategies for modelling buoyancy-driven flow in the atmosphere.

15.2 The extension of the augmentation idea to $\mathbb{P}_2$–$\mathbb{P}_1$ triangular approximation was proposed in a paper presented at a conference* in Reading in 1982; see Griffiths [griffiths82].

15.3 A rigorous assessment of the augmentation strategy was undertaken by Boffi and collaborators some two decades later. Rigorous discussion and a set of convincing numerical experiments can be found in [boffi12].

References

- P.M. Gresho, R.L. Lee, S.T. Chan, and J.M. Leone. A new finite element for incompressible or Boussinesq fluids. In *Third International Conference on Finite Elements in Flow Problems*, pages 204–215. Wiley, New York, 1981.
- David Griffiths. The effect of pressure approximation on finite element calculations of incompressible flows. In K.W. Morton and M.J. Baines, editors, *Numerical Methods for Fluid Dynamics*, pages 359–374. Academic Press, San Diego, 1982.
- D. Boffi, N. Cavallini, F. Gardini, and L. Gastaldi. Local mass conservation of Stokes finite elements. *J. Sci. Comput.*, 52:383–400, 2012. http://dx.doi.org/10.1007/s10915-011-9549-4.

* Being a diligent PhD student in the audience, I made detailed notes during the talk.