

Final Report GR/N25565 : EFFICIENT SOLVERS FOR INCOMPRESSIBLE FLOW PROBLEMS

Personnel supported:

Professor Stefan Turek (nominated Visiting Fellow; visited UMIST: 27/4/00–30/4/00).

Professor Howard Elman (University of Maryland, visited UMIST: 24/4/00–30/4/00)

Introduction. Simulation of the motion of an incompressible fluid remains an important but very challenging problem. The resources required for accurate three-dimensional simulation of practical flows test even the most advanced of supercomputer hardware. The necessity for reliable and efficient solvers is widely recognised. The aim of this project was to study the performance of a novel preconditioning methodology designed for use with Krylov subspace iteration to compute numerical solutions of the incompressible Navier-Stokes equations.

The project effort centered on five days of extensive discussion among Dr Silvester, Professor Elman and Professor Turek. Dr Andrew Wathen, Dr Daniel Loghin (both from the University of Oxford) and Dr David Kay (University of Sussex) also visited UMIST during this period. We also took the opportunity to have “on-line” discussions with Professor Turek on a variety of hardware and software issues. The main outcome of these technical exchanges was the realisation that our new preconditioning framework really did seem to offer the possibility of uniformly fast convergence independent of the problem parameters (namely; the mesh size, the time step and the Reynolds number). In contrast, conventional approaches tend to work well if the time-step is sufficiently small, but efficiency deteriorates rapidly if either the time-step or the Reynolds number is increased. It is our intention to build on these discussions in the future by seeking support for a formal collaborative project with Professor Turek’s group.

The research achievements are commensurate with those envisaged on the original proposal. Further details are given below.

Theoretical aspects. The main outcome of Professor Elman’s visit to Manchester was the subsequent completion of a report submitted to Numerische Mathematik ([1]) that discusses the influence of the mesh size and the Reynolds number on the convergence of our solver methodology in the difficult case of arbitrarily large time-steps. Using a combination of analytic and empirical results it is demonstrated that the preconditioned linearised systems that arise at each level of the outer nonlinear iteration have an eigenvalue distribution consisting of a tightly clustered set together with a small number of outliers. The structure of these distributions is independent of the mesh size, but the cardinality of the set of outliers increases slowly as the Reynolds number is increased. These characteristics are directly correlated with the convergence properties of Krylov subspace solvers.

Practical aspects. The main outcome of Professor Turek’s visit to Manchester has been the subsequent implementation of our methodology into his state-of-the-art FEATFLOW software (freely available from <http://www.featflow.de/turek>). Testing of the effectiveness of the preconditioner in practical situations is still ongoing, but preliminary results reported by Professor Turek at a meeting in Naples in May are encouraging.

Dissemination. As anticipated in the proposal, Professor Turek’s visit was timed to correspond with the occasion of the BAMC 2000 meeting at UMIST. Indeed, a half-day minisymposium devoted to the numerical analysis of Navier-Stokes equations was organized with Professor Turek the keynote speaker.

In the event, the minisymposium was well attended (attracting approximately eighty delegates) and it generated heated discussion. Professor Turek's overview was both wide ranging and informative. The slides that were presented in the talk are available via www.mathematik.uni-dortmund.de/htmldata1/featflow/ture/talks/bamc_00.ps.gz
The other talks given in the minisymposium were as follows.

Dr. Daniel Loghin (University of Oxford)

Fundamental solution preconditioners for the Navier-Stokes equations

Prof. Howard Elman (University of Maryland)

Performance and analysis of saddle-point preconditioners for the Navier-Stokes equations

Dr. Peter Jimack (University of Leeds)

An adaptive finite element algorithm for the solution of time-dependent free surface incompressible flow problems

Dr. Ron Thatcher (UMIST)

Least squares methods for incompressible flow

Dr. Jitesh Gajjar (University of Manchester)

On the solution of the Navier-Stokes equations using the GMRES method

PUBLICATIONS: GR/N25565

1. Elman, H., Silvester, D. and Wathen, A., Performance and analysis of saddle-point preconditioners for the discrete steady-state Navier-Stokes equations, University of Maryland Report UMIACS-TR-2000-54, July 2000.