... continued from Exercises 1

8. The aim of this exercise is to assess the effectiveness of the adaptive refinement strategy that is built into T-IFISS by looking at a problem with a singular solution. The test problem can be set up in T-IFISS by setting `tolerance=5e-3` and then running the driver `Run8DigitChallenge`.

   You should discover that the adaptive algorithm converges in 15 steps and that the number of vertices (degrees of freedom) on the final mesh is 1901. Save the plots of the refinement path (Figure 3) and of the final mesh (Figure 2). (Hint: use the command `savefig`.)

   Next, run the driver `ell_adiff` with linear approximation with grid parameter set to `5,6,7` and `8` so as to estimate the order of convergence of the uniform grid finite element approximation in the energy norm (as a power of h). You might like to add this data to the previously saved plot to facilitate a direct comparison. (The number of vertices is given by `length(x_gal)` and the energy error estimate is given by `err_p`.)

   Finally, repeat the experiment, this time by running `ell_adiff` using quadratic approximation with the grid parameter set to `3,4,5,6` and `7`. (The number of degrees of freedom is given by `length(x_gal)` and the energy error estimate is given by `norm(elerr_p_p4)`.) The key point here is that while the uniform grid quadratic approximation is better than the linear approximation the order of convergence is exactly the same. Why is this behaviour to be expected?

9. The aim of this exercise is to compare the error estimation strategies that are built into T-IFISS. Running the driver `testconvestimators` displays results generated for two different reference problems.

   Figure 1 shows the computed error estimates generated for a problem with a (smooth) solution using linear approximation. The results may be regenerated by running `adapt_diff_testproblem` and selecting reference problem 3. It can be observed that all four estimation strategies give an adaptive refinement sequence converging at the optimal rate.

   Figure 2 shows the computed error estimates generated for a problem with a prescribed singular solution. The superior performance of quadratic approximation should be clearly evident. These results may be regenerated by running `adapt_diff_testproblem` and selecting reference problem 5. Figure 3 shows the associated effectivity indices $\eta_h/\|u-u_h\|_E$ computed using the representation formula

$$\|u-u_h\|_E^2 = \|u\|_E^2 - \|u_h\|_E^2\,.$$

   These results suggest that the hierarchical strategy EES[1] discussed in the notes leads to the most reliable estimate of the true error. The two alternative estimation strategies consistently underestimate the true error (by a factor of approximately 3/2).