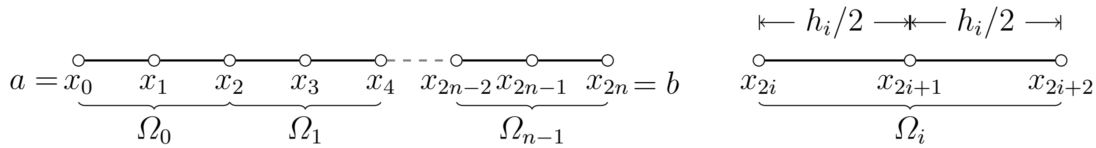
SECTION B

Answer **TWO** of the three questions

B7. This question concerns one-dimensional interpolation. The question has two distinct parts.

(a) Given the domain $a \le x \le b$, consider a partitioning into n subdomains $\{\Omega_0, \Omega_1, \dots, \Omega_{n-1}\}$.



Let $I_h f(x)$ denote the piecewise quadratic interpolant of a given function $f(x)$ such that $I_h f(x) \in C^0[a, b]$ is a quadratic polynomial in each Ω_i and $I_h f(x_i) = f_i := f(x_i)$.

(i) For the ith subdomain Ω_i, determine expressions for the basis functions $\phi_{2i}(x)$, $\phi_{2i+1}(x)$ and $\phi_{2i+2}(x)$, so that $I_h f(x) = \phi_{2i}(x) f_{2i} + \phi_{2i+1}(x) f_{2i+1} + \phi_{2i+2}(x) f_{2i+2}$.

[4 marks]

(ii) Plot the basis functions $\phi_{2i}(x)$, $\phi_{2i+1}(x)$ and $\phi_{2i+2}(x)$ for the ith subdomain Ω_i. The diagram should show the function values at the points x_{2i}, x_{2i+1}, x_{2i+2} and the values at the points of local extrema.

[2 marks]

(iii) For each x_i construct the global basis functions $\phi_i(x)$ obtained by assembly of local components and plot them over a suitable interval. (*Hint:* all $\phi_{2i}(x)$ where $i = \{1, 2, \dots, n-1\}$ and all $\phi_{2i+1}(x)$ where $i = \{0, 1, \dots, n-1\}$ will have similar plots, so draw any one from each set, together with the two end functions ϕ_0 and ϕ_{2n}.)

[2 marks]

(iv) Assume $f \in C^3[a, b]$ and $h := \max\{h_i\}$. Establish the following pointwise error estimate.

$$\|f(x) - I_h f(x)\|_{\infty,\Omega} \le \frac{h^3}{72\sqrt{3}} \left\| \frac{\mathrm{d}^3}{\mathrm{d}x^3} f(x) \right\|_{\infty,\Omega}$$

[6 marks]

(b) Given a sufficiently smooth function $f(x)$, consider a partitioning of the domain $[a, b]$ into n intervals.

(i) Write down the equation of the error in global polynomial interpolation of $f(x)$ in $[a, b]$, and use it to derive the tightest bound you can get for the pointwise interpolation error.

(ii) By referring to this pointwise error bound, explain why one might prefer to use piecewise polynomials for interpolation in place of a global polynomial.

[2+4 marks]