1. Starting from values of $f(0)$, $f(3)$ and $f(6)$, carry out five steps* of a bisection-type search to find a minimum of the function

$$f(x) = 7x^2 - 28x + 33.$$

Check your results by computing the minimum value using calculus.

2. Consider the function $f(x) = \sin x + x + x^2 - x^3$.

   (a) By considering the values of $f(x)$ for $x = -3, -2.5, -2, -1.5, -1, -0.5, 0, 0.5, 1, 1.5, 2, 2.5$ and 3, show that there is a minimum between $x = -1$ and $x = 0$ and a maximum between $x = 0.5$ and $x = 1.5$.

   (b) Using a bisection-type search* locate the minimum between $x = -1$ and $x = 0$ to two decimal places.

   (c) Using a bisection-type search* locate the maximum between $x = 0.5$ and $x = 1.5$ to two decimal places.

   (d) Compute numerical solutions to the equation $f'(x) = 0$ and verify that these agree with the maximum and the minimum found in parts (b) and (c).

3. A Newton–Raphson iteration can be constructed to find critical points of a function (by seeking a point x_* where $f'(x_*) = 0$). Hence run the iteration†

$$x_{n+1} = x_n - \frac{f'(x_n)}{f''(x_n)}$$

starting from $x_0 = 5$ and verify that it converges to the minimum of the function $f(x) = 3x^4 - 8x^3 + 24x^2 - 252x + 67$.

4. Under certain conditions, the *fugacity coefficient* of a gas is given, in terms of the compressibility factor, by

$$f(z) = z - 1 - \ln(z-1) - \ln\left(1 + \frac{1}{z}\right).$$

There is a minimum of this function between $z = 1.5$ and $z = 2.5$. Compute the value of the compressibility, correct to two significant digits, for which this minimum occurs.

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*An efficient way of doing this is to modify the function `bisectsearch.m` that is developed in Computational Exercise II.

†An efficient way of doing this is to modify the function `newtonit.m` that is defined in Computational Exercise I.

5. The height of water in a curved tank while water is flowing through it is given by $z = \left(t + t^{1/3}\right) e^{-t/10}$. The volume of fluid is zero to begin with, but increases to a maximum value before slowly decreasing back to zero.

   (a) By noting that $t^{1/3} << t$ for large t, find, using calculus, an approximate value t_0 for the time at which the maximum height occurs.

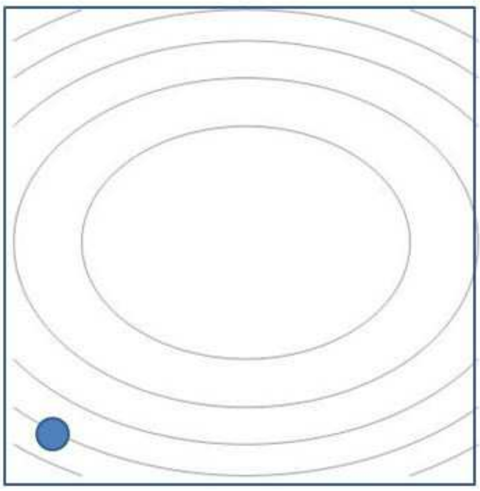
   (b) By considering t_0 and $t_0 \pm 0.1$, carry out a search for the value of t which causes the height to be maximised. (You can stop the search process when the interval points agree to four significant digits.)

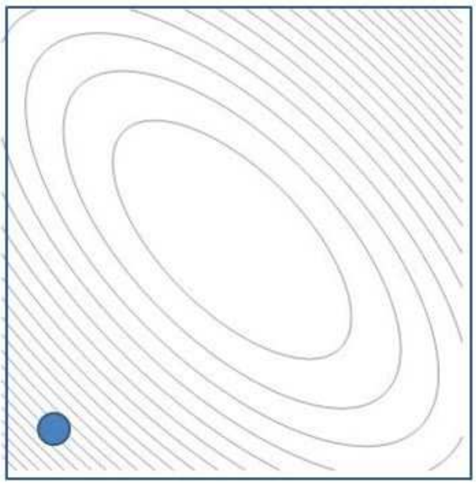
6. Consider the function

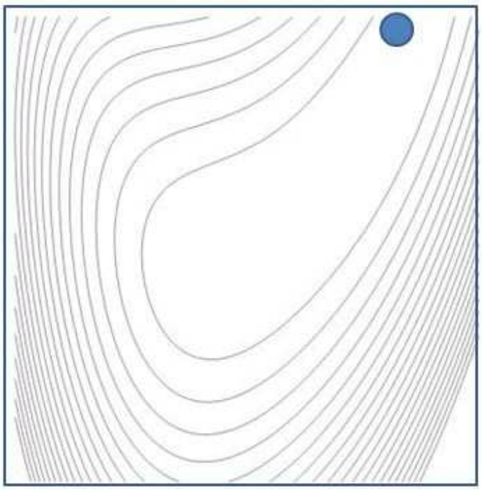
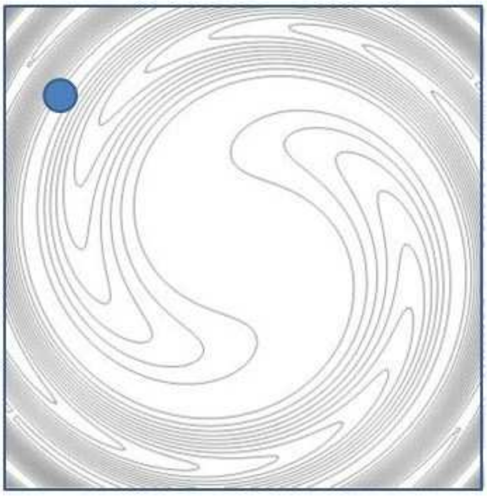
$$f(x, y) = x^2 + 3x + y^2 - 6y.$$

   Starting from the point $(0, 0)$, carry out a 'successive search' technique to find a minimum value for f. Use an initial 'step' of 0.1. [Warning: this could be a long calculation.]

7. For each of the contour plots below, there is a minimum at the centre of the diagram. Starting from the 'dot', trace the routes taken to find the minimum using the successive search technique.





8. Find the minimum value of the function $f(x, y) = 10 - x - y$ subject to the following constraints; $0 \leq x \leq 5$, $0 \leq y \leq 2$, $2x + y \leq 8$, $x + 2y \leq 20$. Are any of the conditions redundant? (Would the question be unchanged if a condition were to be removed?)

9. A tank is used for mixing of solvents. Solvent A comes in 12 litre drums while solvent B comes in 20 litre drums. It is customary not to use more than one drum of each solvent at any time.
   If $2A + B$ (amounts measured in litres) exceeds 30, some material will solidify, clearly not recommended. If $A + 4B$ exceeds 99 litres, then toxic fumes will be produced.
   What is the maximum amount of solvent that can be mixed in the tank?
   A different manufacturer offers solvent B in containers of 25 litres. What is the maximum amount of solvent that can now be mixed?