The coupled equations

$$x^2 + y^2 - 9 = 0$$
$$x + y - 1 = 0$$

are known to have a solution near the point $x = 1$, $y = -1$. To compute an accurate estimate using Newton iteration construct an M-file `newtonit.m` consisting of the following three MATLAB functions (all in the same file).

```
function xx = newtonit(x0,nstep)
% Newton solver for n-dimensional system of equations
%   inputs are
%      x0    : starting vector (column of length n)
%      nstep : number of iterations
%   output is
%      xx    : solution vector
n = length(x0);
fprintf('Newton iteration in %g dimensions\n',n)
% iteration loop
for k = 1:nstep
    [rr,Jmatrix] = fdef(x0);  % compute vector and derivative matrix
    itntable(n,k,x0,rr);      % print current solution
    dx = - Jmatrix\rr;        % solve
    xx = x0 + dx;             % update
    x0 = xx;                  % new solution
end
fprintf('done \n')

function itntable(n,k,xx,rr)
if n==2,  % two-dimensional problem
      if k==1; % print header
      fprintf(' k     x_k      y_k        |f_k|        |g_k|\n'),
      end
   fprintf(' %g  %8.5f  %8.5f    %6.3e   %6.3e \n', ...
            k-1, xx(1), xx(2),abs(rr(1)),abs(rr(2))), pause(1)
end
return

function [rr, J] = fdef(xx)
x=xx(1);  y=xx(2);
% function values
f = x^2+y^2-9;  g = x+y-1; rr = [f;g];
% Jacobian matrix
J(1,1) = 2*x;  J(1,2 ) = 2*y;
J(2,1)= 1;     J(2,2)=1;
return
```

Having done this, run the code to compute the solution (by typing the following)

```
>> start=[1;-1];
>> xx = newtonit(start,7);
```

Next, run `newtonit` with some other initial vectors. Can you get the iteration to converge to a different root? Can you get the iteration to diverge? What happens if you start from the point $x = 1$, $y = 1$?

Simple extensions

- The M-file can be readily modified to solve any other two-dimensional problem (by simply editing the function `fdef` to reflect the functions $f(x, y)$ and $g(x, y)$ and the associated Jacobian matrix J).
- The `newtonit` functionality is very general: the M-file may can also be used to solve one-dimensional or general n-dimensional problems. There is no tabular output generated in such cases however. If you want to see tabular output when $n \neq 2$ then the function `itntable` needs to be extended appropriately ...