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A Method for the Investigation of those Transformations under which the Visual Recognition of a Given Object is Invariant

II. An Example Experiment: The Group of Rotations $SO(2)$ Acting on a Landolt Ring

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Abstract

An experiment is described which shows in operation the program set out in Foster (1972a) for the investigation of the invariance transformations of visual recognition. The concern in the present study is with the Lie group of rotations $SO(2)$, and a certain centrally located foveal Landolt ring. By presenting to the visual system this Landolt ring and a rotated image in rapid succession, one attempted to induce a specified rotation-type phi-motion. Two subjects were employed. Both reported the existence of the required type of phi-motion for rotations ϱ_θ of the Landolt ring about the visual axis with $-2\pi/7 \leqq \theta \leqq 2\pi/7$. By appealing to the basic Proposition 2 of Foster (1972a), the conclusion is reached that the visual system appears capable of effecting upon a certain centrally located foveal annulus the local 1-parameter group of rotations about the visual axis $\varrho_\theta, \theta \in [-2\pi/7, 2\pi/7]$.

Introduction

In the paper immediately preceding this (Foster, 1972a), an investigatory procedure was put forward for obtaining information about the nature of those transformations that leave the visual recognition of a given object invariant. Briefly, the method consists of the following.

For a given visual stimulus or object, A say, one selects a local Lie group of local transformations acting upon a certain neighbourhood U of A. A transformation, τ say, is then chosen from this local Lie transformation group, and the object A and transform $\tau(A)$ presented, in succession, to the visual system. If the apparent movement effect known as phi-motion (Wertheimer, 1912) is found to take place between A and $\tau(A)$, then this is taken to imply that the visual system can perform a certain 1-parameter family of local transformations ψ_t, $t \in [0, t_1]$, which act upon U and satisfy the conditions $\psi_0(A) = A$ and $\psi_{t_1}(A) = \tau(A)$.

It is further proposed that it is possible to determine, by visual inspection of the orbit of A under ψ_t, whether ψ_t coincides locally with a local 1-parameter group of local transformations. By testing with different transformations τ, one gathers data about the composition of a certain path-connected neighbourhood of the identity element in that portion of the selected local Lie group "carried" by the visual system.

The purpose of the present study is to demonstrate this investigatory procedure in practice. The visual object under consideration is a Landolt ring (an annulus with a gap in it, like a letter C), and the selected local Lie group of local transformations is the group of rotations $SO(2)$, acting in the plane of the Landolt ring and about its centre.

Before proceeding with a description of the experimental program, we make the specification of the stimulus and transformation group a little more precise. Notation and definations are as in the previous paper (Foster, 1972a) (which, for brevity, will henceforth be referred to as Paper I). In particular, recall that the retina V is considered (roughly) as a hemisphere in 3-dimensional Euclidean space $\boldsymbol{R}^3$, and that a visual stimulus or object O is interpreted as a mapping from V into $\boldsymbol{R}$, ($O(p) \geqq 0$ is then the illumination at $p \in V$). Recall, also, that for an object $O: V \to \boldsymbol{R}$, $N(O)$ denotes the set of those p in V for which $O(p) \neq 0$.

i) The Landolt ring, which will be represented by C, has a uniform distribution of illumination, i.e.

$$C(p) = I_0 > 0 \quad \text{for all} \quad p \in N(C)$$

(The precise dimensions of C are given later.)

ii) The object C is positioned foveally such that its centre coincides with the visual axis, and such that its gap is located (A) vertically below centre (on the retina), or (B) horizontally to the temporal side of centre (on the retina).

iii) The domain (and range) of action of the transformation group $SO(2)$ is the foveal annulus U_C formed by filling in the gap in the Landolt ring C.

In the general terminology of Paper I, the restriction of $SO(2)$ to U_C is the Lie group of local transformations

$$\varrho: \boldsymbol{T}^1 \times U_C \to U_C \tag{1}$$

where $\boldsymbol{T}^1 = \boldsymbol{R}/2\pi\boldsymbol{Z}$($\boldsymbol{Z}$ the integers) is the additive group of the reals mod 2π. That is denoting each element in $\boldsymbol{T}^1$ by its representative in the interval $(-\pi, \pi]$, we have $\theta \in \boldsymbol{T}^1$, the transformation

$$\varrho_\theta : p \to \varrho(\theta, p)$$

rotates the Landolt ring C about the visual axis through the angle θ. Note that since $\det(D\varrho_\theta(p)) = 1$, for all $p \in U_C$, $\theta \in \boldsymbol{T}^1$, the two ways of extending the action of ϱ_θ to the set $F(U_C)$ of all visual objects on U_C, described in Paper I [Eqs. (1) and (2) in Paper I], now come to the same thing.

Since $\boldsymbol{T}^1$ is connected and the mapping: $\theta \in \boldsymbol{T}^1 \to \varrho_\theta \in \mathrm{Bij}(U_C)$ is injective, the restrictions imposed, in Paper I, upon the selected local Lie transformation group are satisfied. Because $\boldsymbol{T}^1$ is 1-dimensional, the investigatory program, here, simply reduces to the following steps:

(1) Choose an element θ from $(-\pi, \pi]$.

(2) Present the object C (of case A or case B) and transform $\varrho_\theta(C)$, in succession, to the visual system.

(3) If phi-motion is induced, i.e., the object appears to move from C to $\varrho_\theta(C)$, then examine the orbit to see whether the induced 1-parameter family of local transformations

$$t \in [0, t_1] \to \psi_t \in \mathrm{Inj}(U_C, V),$$

where $\psi_0(C) = C$ and $\psi_{t_1}(C) = \varrho_\theta(C)$, is such that one can write $\psi_t = \varrho_t$ for all $t \in [0, t_1]$. (Observe that $\boldsymbol{T}^1$ is not simply connected, so if the preceding were true, we could have, for $0 < \theta \leqq \pi$, t running from 0 to θ or from 0 to $\theta - 2\pi$. Similarly for $-\pi < \theta < 0$.)

(4) Repeat Steps (2) and (3) for a different $\theta \in (-\pi, \pi]$ until a sufficiently dense collection of θ in $(-\pi, \pi]$ is arrived at.

Some comment should be made in connection with the decision to use two initial objects [Step (2)], and the selection of θ from $(-\pi, \pi]$ [Step (4)]. Consider first the choice of initial object. Suppose an experiment were carried out using the Landolt ring of case A, and the following results obtained: (i) phi-motion occurs between C and $\varrho_{\theta'}(C)$ for some particular $\theta' \in (-\pi, \pi]$; (ii) for this θ', the orbit of C under the family ψ_t is visually indistinguishable from the orbit of C under the family ϱ_t. It could be argued that carrying out such an experiment with the same value θ' for the Landolt ring of case B reveals no new information – other than that obtainable by repeating with the Landolt ring of case A. However, there exists the possibility that, because of the nature of the stimulus, the observer, in arriving at result (ii), makes a decision which is based solely upon observation of the apparent motion of some part of the Landolt ring around the gap. In this case, any failure of ψ_t to coincide with ϱ_t on the rest of U_C could then go undetected. The repetition of the experiment with the gap in the initial object at a different location (case B) is designed to reveal such an event (at least in the region indicated).

Consider next the choice of θ from $(-\pi, \pi]$. Suppose, again, that phi-motion is obtained for some particular $\theta' \in (-\pi, \pi]$ (assume $t_1 = \theta', \theta' > 0$). It could then be argued that the procedure need only be repeated for values of θ not included in $[0, \theta']$, since the existence of the families ϱ_t, $t \in [0, \theta]$, for $0 < \theta \leqq \theta'$ is then already implied by Proposition 2 of the Theory (Paper I). Performing the experiment for values of θ also in $[0, \theta']$ allows us a check on the self-consistency of the latter. (It is also desirable, from the point of view of experimental technique, that all values of θ be treated independently and equally.)

In the subsequent sections, the experimental apparatus and method are described in detail. One feature to note is the inclusion in the apparatus of a *fixation monitor*. It is well known that when one attempts to hold the gaze steady, there still remain certain involuntary movements of the eye (flicks, drifts and tremor). See, for example, Ditchburn and Ginsborg (1953). In the present experiments, the steady location of the stimulus upon the retina is an important requisite, and the monitor serves not only to measure any changes in fixation that occur, but also to provide the subject with some external feedback. This takes the form of an audio-signal with varying pitch.

Experimental Apparatus

A schematic diagram of the apparatus is shown in Fig. 1. It consisted essentially of three channels each forming a Maxwellian view system. The two channels labelled LLH and RLH in the diagram gave rise to the primary stimulus: LLH produced the fixed (initial) Landolt ring C, and RLH produced the rotated (final) Landolt ring $\varrho_\theta(C)$. The alternate presentation of the two rings to the observer was effected by means of the rotating 180° sector R_1, which interrupted each channel in turn. The third channel, labelled RH in the diagram, produced a steady and spatially uniform background field upon which the primary stimulus was superimposed.

Detailed Description. The single light source P was a 12 V, 100 W quartz-iodine lamp with a compact coiled filament. This was run from a 12 V stabilised power supply. (Fluctuations of the light level were determined to be less than 0.25% of the mean.) Light was taken from both sides of P and rendered parallel by the collimating lenses L_1 and L_2. The left-hand beam was split (amplitude-division in all cases) by the semi-reflecting plate SM_1, and the two resulting beams focussed by the lenses L_3 and L_4 onto the stops S_1 and S_2,

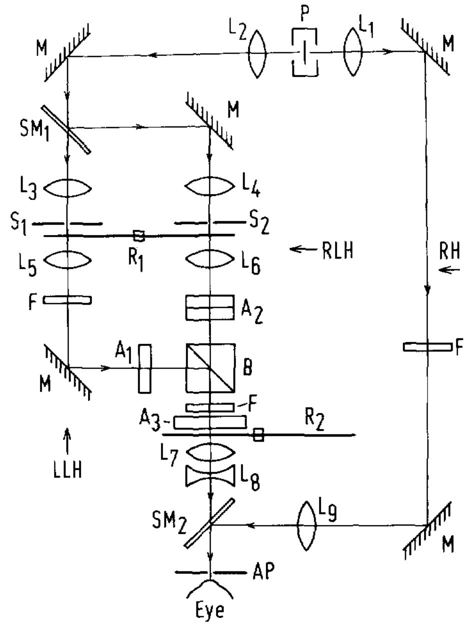

Fig. 1. The experimental apparatus: *P*, light source; *M*, mirror; *SM*, semi-reflecting plate; *L*, lens; *S*, stop; *F*, filter; *R*, rotating sector; *A*, mask; *B*, biprism; *AP*, artificial pupil.

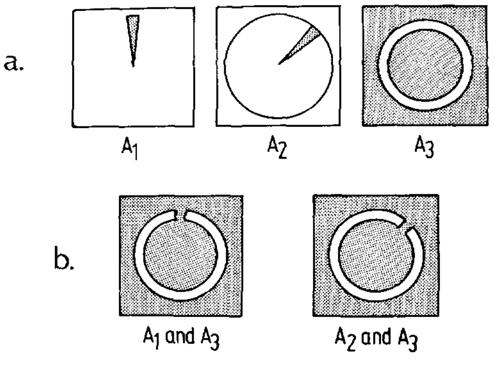

Fig. 2. a The masks A_1, A_2, and A_3 (to scale). b The results of superposing A_1 and A_3, and A_2 and A_3.

respectively. The light from S_1 and S_2 was then recollimated by the lenses L_5 and L_6. The parallel light beam in channel LLH transilluminated the mask A_1, and the parallel beam in channel RLH transilluminated the mask A_2. The two beams were brought together by the biprism *B*, after which a common path was followed. Each beam passed through the mask A_3, and, via the lenses L_7 and L_8, each was brought to a focus at the 2 mm artificial pupil *AP*. Through the lens L_9 and semi-reflecting plate SM_2, the parallel beam of channel RH was also brought to a focus at *AP*. The aperture was completely filled with light.

With the colur correcting filters *F* inserted, the channels RLH, LLH, and RH all matched (in colour and brightness) from SM_2 onwards.

The rotating sector R_1 was driven by an electric motor with an electronic feedback circuit for speed stabilization. (The speed setting could be varied continuously.) An electronic tachometer was also attached.

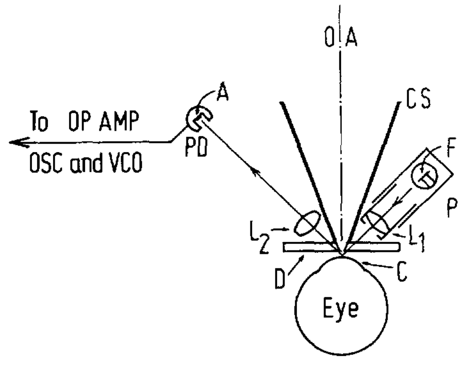

Fig. 3. The fixation monitor: *P*, projector; *F*, filament; L_1, collimating lens; L_2, collecting lens; *PD*, photodiode; *A*, photosensitive area; *CS*, conical screen; *D*, Perspex disc.

The (slow) rotating 180° sector R_2 cycled the presentation of the primary stimulus. The circular background field was visible at all times. At its centre, there was a small fixation spot.

The masks A_1, A_2, and A_3 are shown in Fig. 2a. The orientation of A_1 could be fixed at 0, 90, 180, or 270 degrees to the vertical. The orientation of A_2, on the other hand, could be continuously varied by means of a gear system with remote control. The result of superposing each of these masks with the third annular mask A_3 (so their centres coincided) is shown in Fig. 2b. A_1 and A_3 together gave the fixed Landolt ring, and A_2 and A_3 together, the rotated Landolt ring.

The drawings in Fig. 2 are to scale; the angular subtense, at the eye, of the Landolt rings (outside diameter) was 1.0 degrees, that of the circular background field (not shown in Fig. 2) was 13.0 degrees. The retinal illumination produced by the background field was determined (against a standard lamp) to be 2400 trolands, at a colour temperature of 3200° K.

Fixation Monitor. The system is sketched in Fig. 3. The projector *P* produced a parallel pencil of light from the filament *F*. This pencil was reflected from the cornea *C* (the eye oriented in the standard position as shown) into the collecting lens L_2, and brought to a focus on the small photosensitive area *A* of the silicon pin photodiode *PD*. (Any rotation of the eye away from standard position thus gave rise to a reduction in the quantity of light reaching *A*.) The signal from the photodiode passed into an operational amplifier OP AMP, and then into an oscilloscope OSC and voltage controlled oscillator VCO.

The artificial pupil was formed by a conical screen *CS*, terminating in a Perspex disc *D*. The projector *P* was constructed as indicated, with the light source a small 12 *V*, 0.1 *A* bulb with a straight filament *F* oriented along the optic axis of *P*. The diameter of the emergent pencil was 4 mm. The photodiode *PD* (Monsanto, type MD 1) had a photosensitive area *A* of 0.58 sq. mm, which was a little smaller than the image of the filament *F* formed upon it. The peak wavelength response of the photodiode was at 0.9 μ, and therefore compatible with the IR filter attached to the projector (not shown in the figure). In operation, with the IR filter in place, the light from the projector was invisible to the subject.

The amplified signal from the monitor was utilised in two ways.

(i) For the rejection of data values obtained with inadequate location of the retinal image. The time course of the subject's fixation was displayed on the oscilloscope for inspection by the experimenter. The minimum detectable angular deviation from correct fixation was less than 20 min of arc horizontally and less than 30 min of arc vertically when determined for an artificial eye.

(ii) To provide feedback for the subject. Although basically an on-off system, for small rotations of the eye away from standard position (less than 3 degrees) a graded response could be obtained from the monitor. This allowed the production of a signal which the subject could use to correct his fixation. The feedback stimulus took the form of an audio (square-wave) signal generated by the voltage controlled oscillator; as the subject's gaze shifted away from the fixation spot, the pitch of the signal increased.

By the nature of the system, deflection of the reflected beam could also be achieved by lateral displacement of the eye (i.e., translations in a plane perpendicular to the optic axis OA). To minimise such effects, the subject was provided with both the usual dental bite-bar and a head rest.

It is remarked that the system described above does not counteract the effect of all eyemovements. The effect of tremor is best neutralised using a contact lens technique [see, for example, Riggs et al. (1953)]. However, there is some reason to believe that this fine vibratory movement serves only to counter the adaptation effects which result in fade-out [see Gilbert and Fender (1969), and West (1968)]. We therefore assume, for the present purposes, that the effect of tremor can be ignored.

Experimental Procedure

First the fixation monitor was aligned. The subject fixated, monocularly, the central fixation spot, using the dental bite-bar and headrest, and the position of the pin photodiode *PD* (see Fig. 3) adjusted for maximum response on the oscilloscope. (For this procedure, the IR filter over the projector was removed.)

For the experiment proper, the subject again fixated, monocularly, the central fixation spot, using the dental bite-bar and headrest. Each of the Landolt rings, C and $\varrho_\theta(C)$, was presented alternately for 0.2 sec. The continuous viewing of this test stimulus was periodically interrupted by the rotating sector R_2 (see Fig. 1): seven seconds free observation, and seven seconds rest.

Earlier pilot experiments had indicated that for certain values of the rotation angle θ, an unequivocal result on the presence or absence of phi-motion could not always be obtained. A forced-choice technique was consequently employed. The subject was instructed to indicate (by means of a hand-held buzzer) whether or not the Landolt ring C appeared to rotate to the Landolt ring $\varrho_\theta(C)$. (Rotation was specifically asked for. The uniformity of the motion was established afterwards.) The instruction was also given that a decision was not to be made by the subject until at least one full (seven-seconds) exposure of the test stimulus had occurred free of excessive eye-movement. (This was to be determined by the subject on the basis of the response of the audio VCO. The experimenter also had available for inspection the oscilloscope display.)

Twenty-seven different values of the rotation angle θ were used ($\theta = 0°$ was not included); that is, $\theta = (360/28)\,n°$, $n = -13, -12, \ldots, -1, 1, \ldots, 13, 14$ (rotation anticlockwise, as viewed, was taken as positive). For each θ in this set, a single trial T_θ (i.e., a single test to see if phi-motion is induced or not) was performed. The order of the trials $T_{\theta_1}, T_{\theta_2}, \ldots, T_{\theta_{27}}$ was chosen at random. This test procedure was repeated four more times, with different random orderings of the T_{θ_i} in each case. For each rotation angle θ_i, five independent data values were thus obtained.

Two separate experiments were performed:

(*A*) The fixed Landolt ring was oriented with the gap (as viewed) vertically above centre.

(*B*) The fixed Landolt ring was oriented with the gap (as viewed) horizontally to the left of centre.

Two naive observers were employed: FMF who was slightly myopic and aged twenty-five years, and JC who was myopic and aged fifty-five years. In both cases, the apparent distance of the test stimuli (see Fig. 1) was within the range of accommodation of the subject's naked eye.

Results

In Fig. 4, the results of each experiment for the two observers are shown. The number of positive responses (a positive response being a report of the induction of phi-motion of the requisite type) is plotted against rotation angle θ. The maximum possible number of positive responses, at each θ, is five.

Both subjects indicated that when the rotation-type phi-motion occurred, the shorter of the two available paths was taken; for example, if C appeared to rotate to $\varrho_\theta(C)$, $0 < \theta < \pi$, then the orbit consisted of the objects $\varrho_t(C)$ with t running from 0 to θ, not from 0 to $2\pi - \theta$. (See the remark under Step (3) in the Introduction.)

Discussion

By the nature of the experimental technique employed, a probabilistic approach must be adopted in the analysis of the results. Suppose that in the long run the relative frequency of positive responses for a given θ is $p(\theta)$, i.e., if $x_N(\theta)$ is the number of positive responses in N trials, then, denoting $x_N(\theta)/N$ by $\hat{p}_N(\theta)$,

$$p(\theta) = \lim_{N \to \infty} \hat{p}_N(\theta)$$

On the basis of this value of $p(\theta)$, a decision on the existence or nonexistence of the invariance transformation mechanisms must be reached. [It is being assumed that the system suffers from noise, both

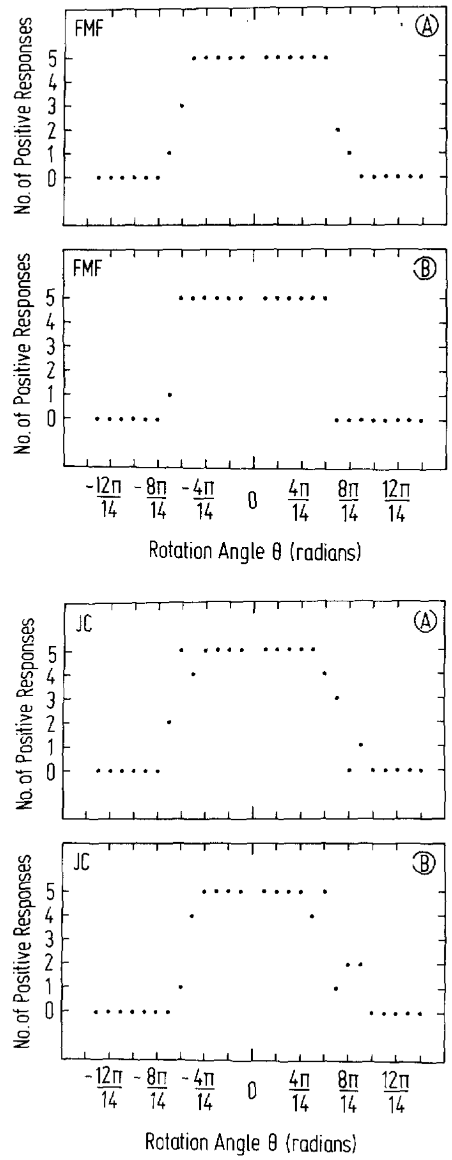

Fig. 4. The results of experiments *A* and *B*. (Two subjects: FMF and JC.) For each rotation angle θ, the maximum possible number of positive responses is five

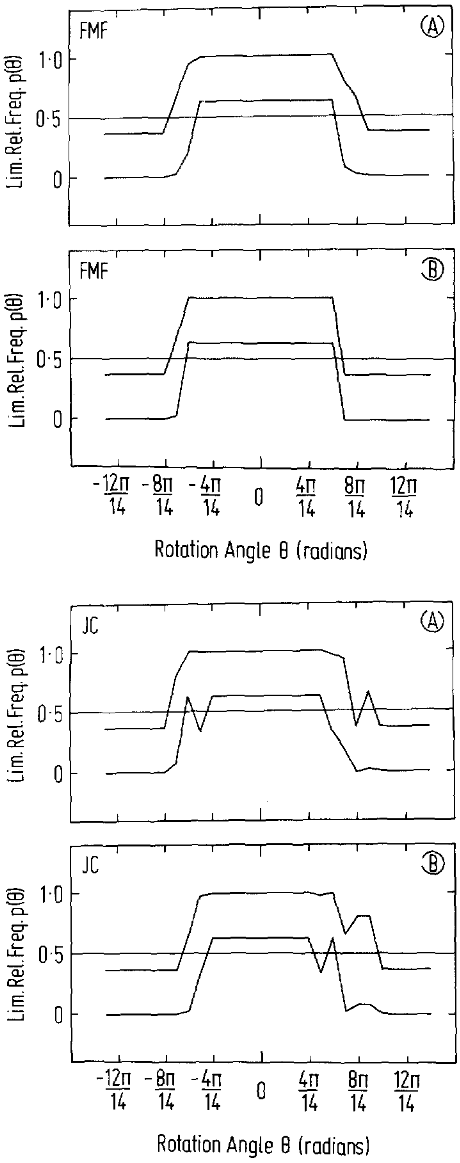

Fig. 5. 90 per cent confidence intervals for the limiting (true) relative frequency $p(\theta)$, for Experiments *A* and *B*. (Two subjects: FMF and JC.)

external (e.g., poor stimulus location) and internal (e.g., fluctuating synaptic thresholds).] The following criterion is adopted. We will say that the relevant transformation mechanisms exist (in the visual system) if, for the given θ, $p(\theta) > \frac{1}{2}$. No decision is to be made if $p(\theta) = \frac{1}{2}$. (It might be suggested that the result $p(\theta) < \frac{1}{2}$ should be taken to imply that the relevant mechanisms do *not* exist. However, this conclusion requires the converse of the basic proposition: phi-motion ⇒ invariance transformations. This point is discussed in Paper I.)

In Fig. 4, we have the recorded value of the (unnormalised) relative frequency $\hat{p}_5(\theta)$ as a function of the rotation angle θ. To obtain a measure of the accuracy of this estimate $\hat{p}_5(\theta)$ of the true relative frequency $p(\theta)$, 90% confidence intervals for $p(\theta)$ were constructed at each of the selected values of θ [see, for example, Hoel (1962)], and then extended to the whole domain $-\pi < \theta \leqq \pi$ by linear interpolation. The results obtained for each observer and experiment (*A* and *B*) are displayed in Fig. 5. We have the following conclusions. For observer FMF, the assertion holds, with confidence coefficient 90%, that $p(\theta) > \frac{1}{2}$ for $-5\pi/14 \leqq \theta \leqq 6\pi/14$ in case A, and $-6\pi/14 \leqq \theta \leqq 6\pi/14$ in case B. For observer JC, the assertion holds, with confidence coefficient 90 per cent, that $p(\theta) > \frac{1}{2}$ for $-4\pi/14 \leqq \theta \leqq 5\pi/14$ in case A, and $-4\pi/14 \leqq \theta \leqq 4\pi/14$ in case B.

It is recollected that two consistency checks were incorporated into the experimental program. The reasons for each were discussed at the end of the Introduction. Before applying to the above data the decision criterion defined earlier in the present section, we verify that the recorded results do indeed satisfy these two checks. The first concerns the domain of definition of the transformations ψ_t, $t \in [0, t_1]$. The carrying out of the experiment with the initial Landolt rings of case A and case B is designed to reveal any failure of ψ_t to coincide with ϱ_t on the annulus U_C (in the region indicated). The similarity of the observed results (see Fig. 4) for case A and case B is evident for both FMF and JC. An indication of compatibility (i.e., the extent to which the same $p(\theta)$ can govern the results for both A and B) is obtained if one takes the overlap of the confidence intervals for the two cases. If this is non-zero, for a given θ, then for any $p(\theta)$ lying in the overlap region, the observed data values $\hat{p}_5(\theta)$ for the two cases then both fall in the central 90 per cent region of the frequency distribution corresponding to this $p(\theta)$. [It does not follow, of course, that the real $p(\theta)$ need lie in this region.] For FMF (see Fig. 6) all confidence intervals overlap, and for JC (see Fig. 6) all but one overlap.

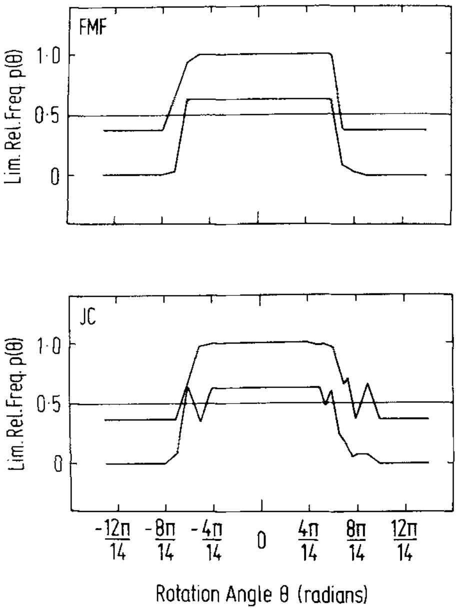

Fig. 6. Overlap of the confidence intervals for $p(\theta)$ for Experiments A and B. (Two subjects: FMF and JC.)

The second check requires that (rotation-type) phi-motion be capable of demonstration for rotations ϱ_θ, $0 < \theta < \theta' < \pi$, when it has already been established that such phi-motion exists for the rotation $\varrho_{\theta'}$. That this is true [allowing for the greater variance of $\hat{p}_5(\theta)$ when $p(\theta) \doteq \frac{1}{2}$] is also apparent from the data of Figs. 4 and 5.

We are now in a position to formulate a conclusion.

Under the decision criterion set up earlier, we have the result that the visual system (as evidenced by the two subjects) seems capable of carrying out a certain local 1-parameter group of rotations defined locally upon the retina. Specifically, the assertion holds, with confidence coefficient 90%, that, for the centrally positioned foveal Landolt ring C and annulus U_C, defined earlier, the visual system is capable of performing the local 1-parameter group of rotations about the visual axis

$$\varrho_\theta : U_C \rightarrow U_C, \quad \theta \in [-2\pi/7, 2\pi/7].$$

Summary

The aim of this study is to demonstrate, in practice, the method put forward in Paper I for the investigation of those transformations under which the visual recognition of a given object is invariant.

The selected visual object is a Landolt ring C of angular subtense 1° located centrally upon the retina. The selected local Lie group of local transformations is the group of rotations $SO(2)$ defined upon the annulus U_C formed by closing up this Landolt ring, and acting about the visual axis.

The proposed investigatory procedure reduces, here, to the successive presentation to the visual system of the object C and rotated object $\varrho_\theta(C)$, and asking for (uniform) phi-motion in the form of an apparent rotation between C and $\varrho_\theta(C)$. The existence of the latter is taken, under Proposition 2 of Paper I, to imply the capacity of the visual system to perform the corresponding local 1-parameter group of local rotations. Results obtained for the two subjects examined show that phi-motion in the form of an apparent rotation can be induced for rotations ϱ_θ of C for $-2\pi/7 \leqq \theta \leqq 2\pi/7$.

The conclusion is thus arrived at that, for the centrally located 1° annulus U_C, the visual system can apparently effect the local 1-parameter group of rotations about the visual axis

$$\varrho_\theta : U_C \rightarrow U_C, \quad \theta \in [-2\pi/7, 2\pi/7].$$

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