

# **Estimating limits on colour vision performance in natural scenes**

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## **ABSTRACT**

The aim of this work was to estimate some of the basic limits on human colour vision performance over a range of natural scenes. Computational simulations of colour processing were carried out with 50 hyperspectral images of rural and urban scenes under different daylights. Three limits were estimated for each scene: the number of discriminable coloured surfaces under a single daylight, the relative frequency of metamerism across two daylights, and, as a measure of colour constancy, the mean error in colour matches across two daylights. All three limits were found to vary over scenes by 1–2 orders of magnitude. Some or all of the variation could, however, be explained by a measure of the randomness of the colours in each scene.

## **1. INTRODUCTION**

Colour vision is often characterized by the limits on its performance, for example, the smallest detectable difference in the wavelength of two spectral lights or the largest number of surface colours that can be discriminated from one another. Such estimates are based on simple experimental stimulus arrangements or theoretical abstractions which are easy to specify, but which may have uncertain relevance when used to represent the natural world. One reason is that natural scenes are generally difficult to describe or analyse theoretically, owing to their complex spectral and spatial variation. This variation affects both detection and discrimination performance.

The aim of the work reported here was to estimate some of the basic limits on human colour vision performance over a range of natural scenes. Computational simulations of colour processing were carried with 50 hyperspectral images of rural and urban scenes under different daylight illuminants. Each hyperspectral image provided a reflectance spectrum at each point or pixel in the scene, and a basis for estimating the signal arriving at the eye for any particular daylight illuminant.

For each scene, three limits on colour vision performance were considered: first, the number of visually discriminable coloured surfaces under a single daylight; second, the relative frequency of metamerism across two daylights; and third, a measure of colour constancy, namely the mean error in surface-colour matches across two daylights. Large variations in the estimated values of each of these limits were found from scene to scene. These variations were modelled by a measure of the randomness of the colours in each scene.

Part of this work is based on material in Marín-Franch and Foster (2010) and Feng and Foster (2012).

## **2. METHOD**

Details of the methods used for the simulations have been described elsewhere (Foster et al. 2006; Marín-Franch and Foster, 2010). In brief, each hyperspectral image had

dimensions  $\leq 1344 \times 1024$  pixels and spectral range 400–720 nm sampled at 10-nm intervals, providing an effective spectral reflectance  $r(\lambda; x, y)$  at each wavelength  $\lambda$  and pixel position  $(x, y)$ . For a given daylight illuminant with radiance spectrum  $e(\lambda)$ , the reflected radiance  $c(\lambda; x, y)$  at each  $(x, y)$  was simulated by the product  $c(\lambda; x, y) = e(\lambda)r(\lambda; x, y)$ . The assumptions underlying this description and their validity are discussed in Foster et al. (2006).

The reflected spectrum  $c(\lambda; x, y)$  at each  $(x, y)$  was converted to tristimulus values  $X, Y, Z$  according to the CIE 1931 standard observer and then to colour values  $J, a_C, b_C$ , corresponding to lightness and the red-green and yellow-blue components of chroma in the colour space CIECAM02 (CIE 2004). This space has the property that constant Euclidean differences  $\Delta E = (\Delta J^2 + \Delta a_C^2 + \Delta b_C^2)^{1/2}$  in colour values correspond to almost constant perceptual colour differences (Luo, Cui and Li 2006; Melgosa, Huertas and Berns 2008). The values  $J, a_C, b_C$  were calculated at each pixel in each scene according to the CIECAM02 specification (CIE 2004) with default parameters, but with full chromatic adaptation. Integrations were performed numerically over 400–720 nm with the given 10-nm sampling interval.

## 2.1 Number of discriminable surfaces

The number  $N$  of visually discriminable coloured surfaces in each scene was estimated with the aid of Shannon’s channel-coding theorem (Cover and Thomas 2006). For any particular illuminant on the scene, the values  $(J, a_C, b_C)$  may be treated as instances of a trivariate continuous random variable  $U$  and similarly the observer’s response, in the same coordinate system, as a trivariate continuous random variable  $V$  (Marín-Franch and Foster 2010). The logarithm of  $N$  is then given by the mutual information  $I$  between  $U$  and  $V$ .<sup>1</sup>

The mutual information was derived from estimates of the differential entropy of the colours of the scene under a standard daylight with correlated colour temperature (CCT) of 6500 K and the differential entropy of the observer’s response, as in Marín-Franch and Foster (2010). The differential entropy was estimated with an asymptotically bias-free,  $k$ -nearest-neighbour estimator due to Kozachenko and Leonenko (1987).

## 2.2 Frequency of metamerism

The relative frequency of metamerism in a scene is the probability of a pair of surfaces that are not visually discriminable under one illuminant being discriminable under another illuminant. Estimates of the relative frequency of metamerism were obtained as in Foster et al. (2006).

Thus, with spatially uniform random sampling with replacement,  $n = 10^9$  pairs of pixels were drawn from each scene, and from this set, the subset of  $n_0$  pairs was found with colour differences less than a nominal threshold  $\Delta E^{\text{thr}} = 0.5$  under a daylight with CCT of 4000 K. From this set of  $n_0$  pairs, the subset of  $n_1$  pairs was then found with colour differences greater than  $\Delta E^{\text{thr}}$  under a second daylight with CCT of 25,000 K (on a mired

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<sup>1</sup> The quantity  $N$  has been referred to as the number of perceptually distinct surface colours in a scene (Marín-Franch and Foster 2010). The reference here to visually discriminable coloured surfaces was intended to emphasize the physical origins of the colour signal. This quantity  $N$  is different from the number of discernible colours based on gamut estimates (Linhares et al. 2008).

scale, these two CCTs are roughly equidistant from a CCT of 6500 K). The relative frequency of metamerism in the scene was estimated by the quotient  $n_1/n$ .

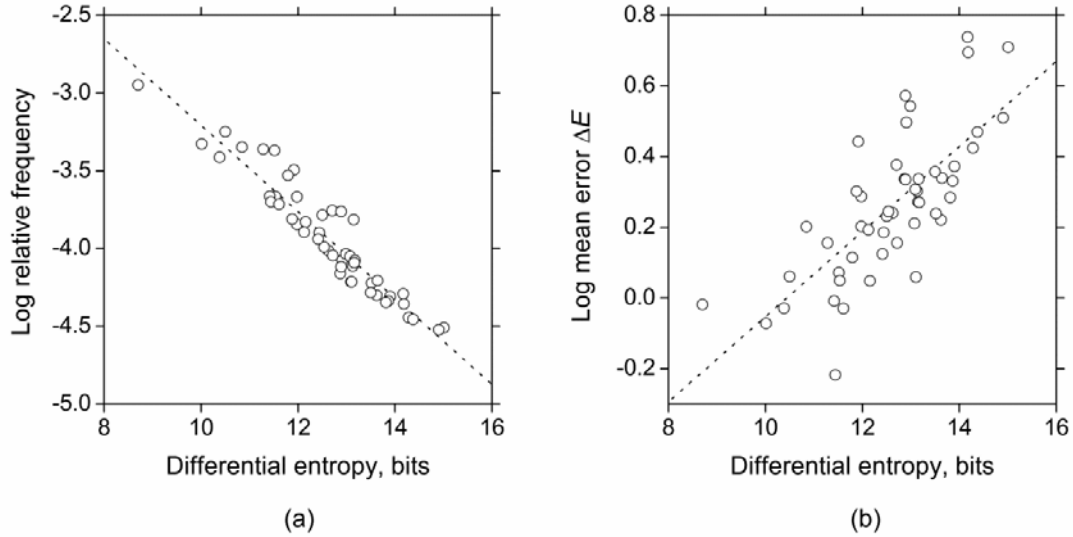


Figure 1: Logarithm of (a) estimated relative frequency of metamerism and (b) estimated mean colour-matching error as a function of differential entropy of scene colours. Each symbol represents data from a different scene and the dotted line is a linear regression.

### 2.3 Errors in colour matches

The goodness of surface-colour matching in each scene was quantified by the errors made by a standard observer matching across two daylights with CCTs of 4000 K and 25,000 K. The mean colour-matching error was estimated by the difference  $\Delta E = (\Delta J^2 + \Delta a_C^2 + \Delta b_C^2)^{1/2}$  in colour values averaged over pixels. Recall that the observer was fully adapted.

## 3. RESULTS

The estimated number  $N$  of discriminable coloured surfaces varied strongly over scenes, from  $3.5 \times 10^3$  with a scene composed of grey-white buildings to  $2.7 \times 10^5$  for a scene composed of coloured flowers. By construction of the estimator, the logarithm of  $N$  depended linearly on the differential entropy of the colours in the scene under the selected daylight. Results are not plotted here, but the proportion  $R^2$  of variance accounted for by differential entropy was 99%.

The estimated relative frequency  $n_1/n$  of metamerism also varied strongly over scenes, from  $3.0 \times 10^{-5}$  to  $1.1 \times 10^{-3}$  (cf. Foster et al. 2006). Although in principle the relative frequency, unlike the number of discriminable coloured surfaces, does not bear the same relationship to then differential entropy of the colours in a scene under a single illuminant, differential entropy remained a powerful explanatory variable. In Fig. 1(a), the logarithm of the estimated relative frequency of metamerism is plotted against the estimated differential entropy of the colours under the first selected daylight. The proportion  $R^2$  of variance accounted for was 90%.

The estimated mean error  $\Delta E$  in colour matching across the two daylights varied less strongly over scenes, from 0.61 to 5.5. In principle this measure is even less closely related to the differential entropy of the colours in a scene under a single illuminant, but differential entropy did retain some explanatory power. In Fig. 1(b), the logarithm of the estimated mean error is plotted against the estimated differential entropy of the colours under the first selected daylight. The proportion  $R^2$  of variance accounted for was 55%.

## 4. DISCUSSION

Natural scenes are complex and their properties are difficult to predict. Variations in the gamut of colours in a scene and in the relative abundances of the colours within that gamut (Burton and Moorhead 1987) both set limits on colour vision performance.

Nevertheless, some or all of the large variations over scenes in the estimated values of these limits can be explained simply by the differential entropy of the colours in each scene under a single daylight. Differential entropy accounted for 99% of the variance in the number of discriminable coloured surfaces and for 90% of the variance in the relative frequency of metamerism, despite the latter involving two daylights rather than one. But differential entropy does not offer a complete explanation, and it accounted for only 55% of the variance in the mean colour-matching error across two daylights. As indicated in Feng and Foster (2012), a more comprehensive description of colour vision performance in natural scenes requires a fuller model, with more than a single explanatory variable.

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